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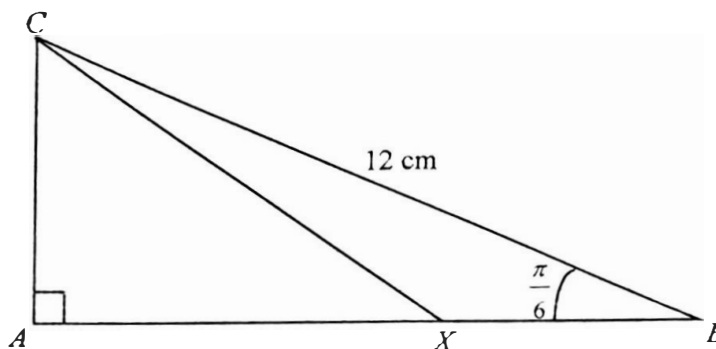


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1



In the diagram, the right-angle triangle  $ABC$  is such that  $BC = 12$  cm,  
 $\angle ABC = \frac{\pi}{6}$  and  $AX = \frac{2}{3} AB$ .

Show that  $\cos \angle BXC = -\frac{2\sqrt{7}}{7}$ . [4]

2 Solve the equation  $6\cos x = 4\sec x - \tan x$  for  $0 < x < 5$ . [5]

3 Air leaks from a spherical balloon at a constant rate of  $25\pi$  cm<sup>3</sup> per second. Given that the initial volume is  $5000\pi$  cm<sup>3</sup>,

(i) calculate the radius of the balloon after 20 seconds, [3]

(ii) find the rate of change of radius at this instant. [2]

4 A curve is such that  $\frac{d^2y}{dx^2} = 6x - 6$ . The gradient of the curve at the point  $(2, -1)$  is 4.

(i) Show that  $y$  is an increasing function for all real values of  $x$ . [4]

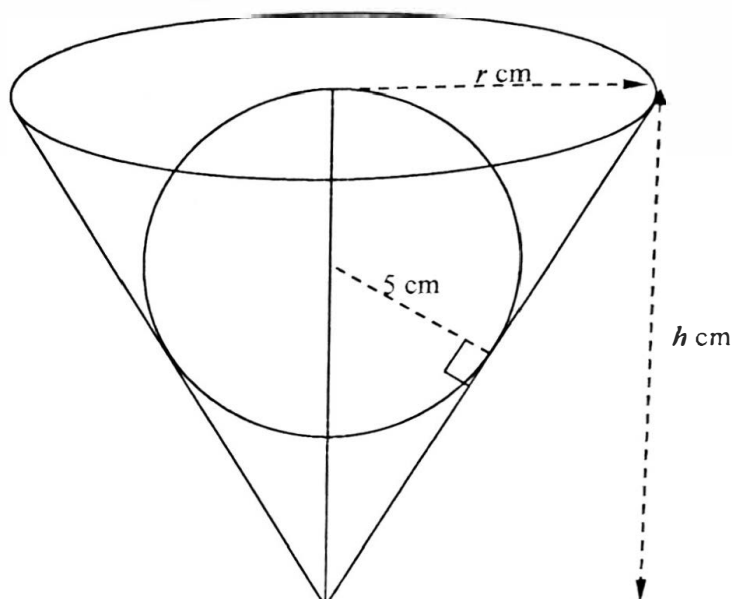
(ii) Find the equation of the curve. [2]

[Turn over...]

- 5 Given the cubic expression  $f(x) = x^3 + px^2 + qx + 4$  has a factor  $(x + 2)$  and leaves a remainder of 6 when divided by  $(x + 1)$ ,
- (i) find the value of  $p$  and of  $q$ , [4]
- (ii) factorize  $f(x)$  completely. [2]
- 6 (a) Simplify the expression  $\frac{3^{n-2} - 3^{n+1}}{3^{n+2} - 3^{n-1}}$ . [3]
- (b) Solve the equation  $\log_2 8x = 4 \log_x 2$ . [4]
- 7 Given that the roots of the equation  $2x^2 - 2x + 5 = 0$  are  $\alpha$  and  $\beta$ .
- (i) Show that  $\alpha^2 + \beta^2 = -4$ . [2]
- (ii) Find the value of  $\alpha^3 + \beta^3$ . [2]
- (iii) Find a quadratic equation whose roots are  $\frac{\alpha}{2\beta^2}$  and  $\frac{\beta}{2\alpha^2}$ . [4]
- 8 The equation of the curve is given by  $y = 3 \cos 3x - 2$  for  $0 \leq x \leq \pi$ .
- (i) Write down the amplitude and period of  $y$ . [2]
- (ii) Find the coordinates of the maximum and minimum points for  $0 < x < \pi$ . [2]
- (iii) Calculate the values of  $x$  for which the curve cuts the  $x$ -axis. [2]
- (iv) Sketch the curve  $y = 3 \cos 3x - 2$  for  $0 \leq x \leq \pi$ . [–]
- (v) State the range of values of  $x$  for which  $y$  is decreasing between 0 and  $\pi$ . [2]

[ Turn over ...

- 9 A solid spherical ball is dropped into a cone of height  $h$  cm and radius  $r$  cm.



Given that the radius of the spherical ball is 5 cm,

- (i) show that the volume of the cone,  $V$  is given by  $V = \frac{25\pi h^2}{3(h-10)}$ . [3]
- (ii) Given that  $h$  can vary, find the value of  $h$  for which  $V$  has a stationary value. [3]
- (iii) Calculate this stationary value of  $V$  and determine if the volume is a maximum or minimum value. [3]
- 10 (i) Express  $\frac{4x^3 + 7x^2 + 4x - 2}{(2x-1)(x^2+2)}$  in partial fractions. [5]
- (ii) Differentiate  $\ln(x^2 + 2)$  with respect to  $x$ . [1]
- (iii) Hence evaluate  $\int_1^2 \frac{4x^3 + 7x^2 + 4x - 2}{(2x-1)(x^2+2)} dx$ . [4]

[Turn over...]

- 11 The table show experimental values of two variables  $x$  and  $y$ .

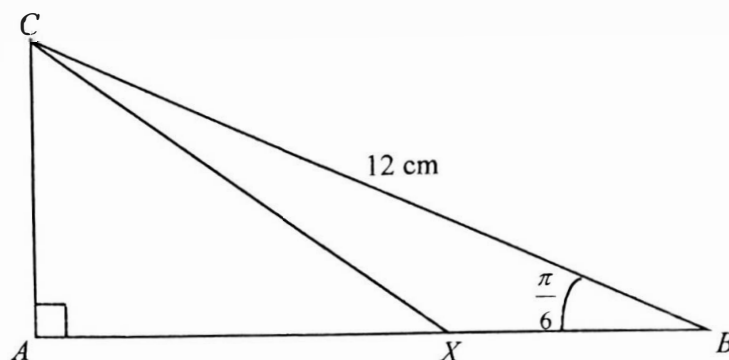
|     |      |      |   |       |       |
|-----|------|------|---|-------|-------|
| $x$ | 2    | 3    | 4 | 6     | 10    |
| $y$ | 3.24 | 5.79 | 9 | 17.05 | 38.43 |

It is known that  $x$  and  $y$  are related by the equation  $\frac{y-b}{x} = a\sqrt{x} - 1$  for  $x > 0$  where  $a$  and  $b$  are constants.

- (i) Using a scale of 1 cm to 2 units on the horizontal axis and 2 cm to 5 units on the vertical axis, draw a straight line graph of  $x + y$  against  $x\sqrt{x}$ . [3]
- (ii) Use your graph to estimate, to 2 decimal places, the value of  $a$  and of  $b$  [4]
- (iii) On the same diagram, draw a straight line representing the equation  $y + x + 2x\sqrt{x} = 36$ .  
Hence find the value of  $x$  that satisfies the equation  $(a+2)x\sqrt{x} = 36 - b$ . [3]

~ End of Paper ~

1



In the diagram, the right-angle triangle  $ABC$  is such that  $BC = 12$  cm,

$$\angle ABC = \frac{\pi}{6} \text{ and } AX = \frac{2}{3} AB.$$

$$\text{Show that } \cos \angle BXC = -\frac{2\sqrt{7}}{7}.$$

[4]

[soln]  $\cos \angle BXC = -\cos \angle AXC$

$$\sin \frac{\pi}{6} = \frac{AC}{12} \Rightarrow AC = 6$$

$$AB = \sqrt{144 - 36} = \sqrt{108} = 6\sqrt{3}$$

$$AX = 4\sqrt{3}$$

$$CX = \sqrt{36 + 48} = \sqrt{84} = 2\sqrt{21}$$

$$\cos \angle BXC = -\cos \angle AXC = -\frac{4\sqrt{3}}{2\sqrt{21}} = -\frac{2}{\sqrt{7}} = -\frac{2\sqrt{7}}{7}$$

2 Solve the equation  $6 \cos x = 4 \sec x - \tan x$  for  $0 < x < 5$ .

[5]

[soln]  $6 \cos x = \frac{4}{\cos x} - \tan x$

$$6 \cos^2 x = 4 - \sin x$$

$$6(1 - \sin^2 x) = 4 - \sin x$$

$$6 \sin^2 x - \sin x - 2 = 0$$

$$(3 \sin x - 2)(2 \sin x + 1) = 0$$

$$\sin x = \frac{2}{3} \quad \text{or} \quad \sin x = -\frac{1}{2}$$

$$\text{Basic angle} = 0.7297$$

$$\text{Basic angle} = 0.5236$$

$$x = 0.730, 2.41$$

$$x = 2.62, 5.76 \text{ (NA)}$$

- 3 Air leaks from a spherical balloon at a constant rate of  $25\pi \text{ cm}^3$  per second. Given that the initial volume is  $5000\pi \text{ cm}^3$ ,

- (i) calculate the radius of the balloon after 20 seconds, [3]  
 (ii) find the rate of change of radius at this instant. [2]

[soln]  $\frac{dV}{dt} = -25\pi$   
 After 20s, volume =  $5000\pi - 25\pi \times 20 = 4500\pi$   
 $\frac{4}{3}\pi r^3 = 4500\pi$   
 $r^3 = 3375$   
 $r = 15$

$$\frac{dV}{dt} = \frac{dV}{dr} \times \frac{dr}{dt}$$

$$-25\pi = 4\pi r^2 \times \frac{dr}{dt}$$

$$\frac{dr}{dt} = -\frac{25}{4 \times 225} = -\frac{25}{900} = -\frac{1}{36} \text{ cm/s}$$

- 4 A curve is such that  $\frac{d^2y}{dx^2} = 6x - 6$ . The gradient of the curve at the point  $(2, -1)$  is 4.

- (i) Show that  $y$  is an increasing function for all real values of  $x$ . [4]  
 (ii) Find the equation of the curve. [2]

[soln]  $\frac{d^2y}{dx^2} = 6x - 6$   
 $\frac{dy}{dx} = 3x^2 - 6x + c$   
 At  $(2, -1)$ ,  $\frac{dy}{dx} = 4$   
 $12 - 12 + c = 4$   
 $c = 4$   
 $\frac{dy}{dx} = 3x^2 - 6x + 4$   
 $\frac{dy}{dx} = 3(x^2 - 2x) + 4$   
 $\frac{dy}{dx} = 3(x - 1)^2 + 1$

For all values of  $x$ ,  $\frac{dy}{dx} > 0$ ,  $y$  is increasing.

$$y = x^3 - 3x^2 + 4x + d$$

$$8 - 12 + 8 + d = -1$$

$$d = -5$$

$$y = x^3 - 3x^2 + 4x - 5$$

- 5** Given the cubic expression  $f(x) = x^3 + px^2 + qx + 4$  has a factor  $(x + 2)$  and leaves a remainder of 6 when divided by  $(x + 1)$ ,

(i) find the value of  $p$  and of  $q$ , [4]

(ii) factorize  $f(x)$  completely. [2]

[soln]

$$-8 + 4p - 2q + 4 = 0$$

$$2p - q = 2$$

$$-1 + p - q + 4 = 6$$

$$p - q = 3$$

$$p = -1, q = -4$$

$$f(x) = x^3 - x^2 - 4x + 4$$

$$f(x) = (x + 2)(x^2 - 3x + 2)$$

$$f(x) = (x + 2)(x - 2)(x - 1)$$

- 6** (a) Simplify the expression  $\frac{3^{n-2} - 3^{n+1}}{3^{n+2} - 3^{n-1}}$ . [3]

(b) Solve the equation  $\log_2 8x = 4 \log_x 2$ . [4]

[soln]

$$(a) \quad \frac{3^{n-2} - 3^{n+1}}{3^{n+2} - 3^{n-1}} = \frac{3^n \left( \frac{1}{9} - 3 \right)}{3^n \left( 9 - \frac{1}{3} \right)} = -\frac{1}{3}$$

$$(b) \quad \begin{aligned} \log_2 8x &= 4 \log_x 2 \\ \log_2 8 + \log_2 x &= \frac{4 \log_2 2}{\log_2 x} \end{aligned}$$

$$3 + \log_2 x = \frac{4}{\log_2 x}$$

Let  $y = \log_2 x$

$$y^2 + 3y - 4 = 0$$

$$(y + 4)(y - 1) = 0$$

$$\log_2 x = -4 \text{ or } \log_2 x = 1$$

$$x = \frac{1}{16} \text{ or } x = 2$$

7 Given that the roots of the equation  $2x^2 - 2x + 5 = 0$  are  $\alpha$  and  $\beta$ .

(i) Show that  $\alpha^2 + \beta^2 = -4$ . [2]

(ii) Find the value of  $\alpha^3 + \beta^3$ . [2]

(iii) Find a quadratic equation whose roots are  $\frac{\alpha}{2\beta^2}$  and  $\frac{\beta}{2\alpha^2}$ . [4]

[soln]

$$\alpha + \beta = 1 \quad \text{and} \quad \alpha\beta = \frac{5}{2}$$

$$\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta = 1 - 2 \times \frac{5}{2} = -4$$

$$(\alpha + \beta)^3 = \alpha^3 + 3\alpha^2\beta + 3\alpha\beta^2 + \beta^3$$

$$\alpha^3 + \beta^3 = (\alpha + \beta)^3 - 3\alpha\beta(\alpha + \beta)$$

$$\alpha^3 + \beta^3 = 1 - 3 \times \frac{5}{2} = -\frac{13}{2}$$

$$\frac{\alpha}{2\beta^2} + \frac{\beta}{2\alpha^2} = \frac{\alpha^3 + \beta^3}{2(\alpha\beta)^2} = \left(-\frac{13}{2}\right) \div \frac{25}{2} = -\frac{13}{25}$$

$$\frac{\alpha}{2\beta^2} \times \frac{\beta}{2\alpha^2} = \frac{1}{4\alpha\beta} = \frac{1}{10}$$

$$\text{Quadratic equation is } x^2 + \frac{13}{25}x + \frac{1}{10} = 0 \text{ or } 50x^2 + 26x + 5 = 0$$

- 8 The equation of the curve is given by  $y = 3 \cos 3x - 2$  for  $0 \leq x \leq \pi$ .
- (i) Write down the amplitude and period of  $y$ . [2]
  - (ii) Find the coordinates of the maximum and minimum points for  $0 < x < \pi$ . [2]
  - (iii) Calculate the values of  $x$  for which the curve cuts the  $x$ -axis. [2]
  - (iv) Sketch the curve  $y = 3 \cos 3x - 2$  for  $0 \leq x \leq \pi$ . [2]
  - (v) State the range of values of  $x$  for which  $y$  is decreasing between 0 and  $\pi$ . [2]

[soln]

$$\text{amplitude} = 3, \text{ period} = \frac{2\pi}{3}$$

$$\text{Minimum point is } \left(\frac{\pi}{3}, -5\right) \text{ and Maximum point is } \left(\frac{2\pi}{3}, 1\right)$$

$$\cos 3x = \frac{2}{3}$$

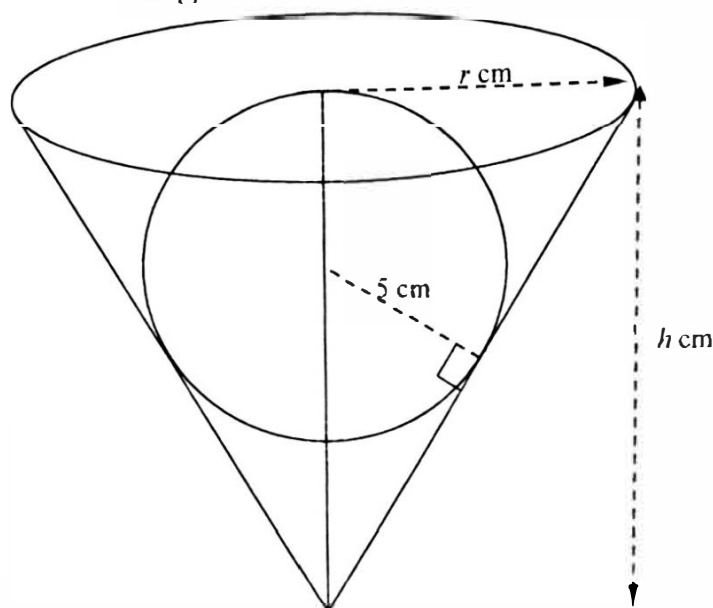
$$\text{Basic angle} = 0.841$$

$$3x = 0.841, 5.4421, 7.124$$

$$x = 0.280, 1.81, 2.37$$

$$y \text{ is decreasing for } 0 < x < \frac{\pi}{3} \text{ and } \frac{2\pi}{3} < x < \pi$$

- 9 A solid spherical ball is dropped into a cone of height  $h$  cm and radius  $r$  cm.



Given that the radius of the spherical ball is 5 cm,

- (i) show that the volume of the cone,  $V$  is given by  $V = \frac{25\pi h^2}{3(h-10)}$ . [3]
- (ii) Given that  $h$  can vary, find the value of  $h$  for which  $V$  has a stationary value. [3]
- (iii) Calculate this stationary value of  $V$  and determine if the volume is a maximum or minimum value. [3]

[soln]

$$\frac{r}{\sqrt{h^2 + r^2}} = \frac{5}{h-5}$$

$$\frac{r^2}{h^2 + r^2} = \frac{25}{h^2 - 10h + 25}$$

$$r^2 h^2 - 10r^2 h + 25r^2 = 25h^2 + 25r^2$$

$$r^2 = \frac{25h^2}{h^2 - 10h} = \frac{25h}{h-10}$$

$$V = \frac{1}{3} \pi h \times \frac{25h}{h-10} = \frac{25\pi h^2}{3(h-10)}$$

$$\frac{dV}{dh} = \frac{25\pi}{3} \left[ \frac{(h-10) \times 2h - h^2}{(h-10)^2} \right]$$

$$\frac{dV}{dh} = \frac{25\pi}{3} \left[ \frac{h^2 - 20h}{(h-10)^2} \right]$$

For stationary value,

$$\frac{dV}{dh} = 0 \Rightarrow h = 20$$

$$V = \frac{25\pi \times 400}{3 \times 10} = \frac{1000\pi}{3} = 1047.20 \text{ (minimum volume)}$$

| $x$             | $< 20$   | $20$ | $> 20$   |
|-----------------|----------|------|----------|
| $\frac{dV}{dh}$ | negative | 0    | positive |

10 (i) Express  $\frac{4x^3 + 7x^2 + 4x - 2}{(2x-1)(x^2+2)}$  in partial fractions. [5]

(ii) Differentiate  $\ln(x^2 + 2)$  with respect to  $x$ . [1]

(iii) Hence evaluate  $\int_1^2 \frac{4x^3 + 7x^2 + 4x - 2}{(2x-1)(x^2+2)} dx$ . [4]

[soln] 
$$\frac{4x^3 + 7x^2 + 4x - 2}{(2x-1)(x^2+2)} = 2 + \frac{9x^2 - 4x + 2}{(2x-1)(x^2+2)}$$

$$\frac{9x^2 - 4x + 2}{(2x-1)(x^2+2)} = \frac{A}{2x-1} + \frac{Bx+C}{x^2+2}$$

$$9x^2 - 4x + 2 = A(x^2 + 2) + (Bx + C)(2x - 1)$$

Subst  $x = \frac{1}{2}$ ,  $\frac{9}{4}A = \frac{9}{4}$   $A = 1$

Coefficient of  $x^2$ :  $B = 4$

Constant term:  $C = 0$

$$\frac{9x^2 - 4x + 2}{(2x-1)(x^2+2)} = \frac{1}{2x-1} + \frac{4x}{x^2+2}$$

$$\frac{d}{dx} \ln(x^2 + 2) = \frac{2x}{x^2 + 2}$$

$$\begin{aligned}
 \int_1^2 \frac{4x^3 + 7x^2 + 4x - 2}{(2x-1)(x^2+2)} dx &= \int_1^2 \left( 2 + \frac{1}{2x-1} + \frac{4x}{x^2+2} \right) dx \\
 &= \left[ 2x + \frac{1}{2} \ln(2x-1) + 2 \ln(x^2+2) \right]_1^2 = \left[ 4 + \frac{1}{2} \ln 3 + 2 \ln 6 \right] - \left[ 2 + \frac{1}{2} \ln 1 + 2 \ln 3 \right] \\
 &= 2 - \frac{3}{2} \ln 3 + 2 \ln 6 \\
 &= 3.94
 \end{aligned}$$

- 11 The table show experimental values of two variables  $x$  and  $y$ .

|     |      |      |   |       |       |
|-----|------|------|---|-------|-------|
| $x$ | 2    | 3    | 4 | 6     | 10    |
| $y$ | 3.24 | 5.79 | 9 | 17.05 | 38.43 |

It is known that  $x$  and  $y$  are related by the equation  $\frac{y-b}{x} = a\sqrt{x} - 1$  for  $x > 0$  where  $a$  and  $b$  are constants.

- (i) Using a scale of 1 cm to 2 units on the horizontal axis and 2 cm to 5 units on the vertical axis, draw a straight line graph of  $x + y$  against  $x\sqrt{x}$ . [3]
- (ii) Use your graph to estimate, to 2 decimal places, the value of  $a$  and of  $b$ . [4]
- (iii) On the same diagram, draw a straight line representing the equation  $y + x + 2x\sqrt{x} = 36$ .  
Hence find the value of  $x$  that satisfies the equation  $(a+2)x\sqrt{x} = 36 - b$ . [3]

[soln]

$$\frac{y-b}{x} = a\sqrt{x} - 1$$

$$y - b = ax\sqrt{x} - x$$

$$x + y = ax\sqrt{x} + b$$

|             |      |      |    |       |       |
|-------------|------|------|----|-------|-------|
| $x\sqrt{x}$ | 2.83 | 5.20 | 8  | 14.70 | 31.62 |
| $x + y$     | 5.24 | 8.79 | 13 | 23.05 | 48.43 |

$$a = 1.5 \text{ and } b = 0.994$$

$$ax\sqrt{x} + 2x\sqrt{x} = 36 - b$$

$$ax\sqrt{x} + b = -2x\sqrt{x} + 36 \quad (\text{gradient} = -2, \text{intercept} = 36)$$

~ End of Paper ~

1. (a) (i) Sketch the graph of the curve  $y^2 = kx$ , where  $k$  is a positive constant. [1]
- (ii) Given that the line  $y = 2x + 1$  meets the curve  $y^2 = kx$ , find the range of values of  $k$ . [4]
- (b) Determine the conditions for  $p$  and  $q$  such that the curve  $y = px^2 - 2x + 3q$  lies entirely above the  $x$ -axis, where  $p$  and  $q$  are constants. [3]
2. (i) Sketch the curve  $y = 2\ln(x - 3)$  for  $x > 3$ . [2]
- (ii) The tangent to the curve  $y = 2\ln(x - 3)$  at the point  $P$  where  $x = 5$  intersects the  $x$ -axis at  $A$  and the normal to the curve at  $P$  intersects the  $x$ -axis at  $B$ . Calculate the area of  $\triangle APB$ . [5]
3. (a) Write down and simplify the first three terms in the expansion of  $(2 - 3x)^6$ , in ascending powers of  $x$ . [2]
- (b) Hence
- (i) using a suitable value of  $x$ , find the estimated value of  $(1.997)^6$ , correct to 3 decimal places. [2]
- (ii) determine the coefficient of  $x^2$  in the expansion of  $(2 - 3x)^7 - (2 - 3x)^6$ . [3]
4. A curve has the equation  $y = f(x)$ , where  $f(x) = \frac{2 + \cos x}{\sin x}$  for  $-\pi \leq x \leq \pi$ .
- (i) Obtain an expression for  $f'(x)$ . [2]
- (ii) Find the **exact** value of the  $x$ -coordinates of the stationary points of the curve, and determine the nature of each stationary point. [6]

5. (a) (i) Show that  $\frac{\cot x - \tan x}{\cot x + \tan x} = \cos 2x$ . [3]

(ii) Hence solve the equation  $\frac{\cot x - \tan x}{\cot x + \tan x} = \cos x$  for  $0^\circ < x < 360^\circ$  [3]

(b) Without using a calculator, express  $\sin 15^\circ$  in the form  $\frac{1}{k}(\sqrt{a} - \sqrt{b})$ , where  $a, b$  and  $k$  are integers. [3]

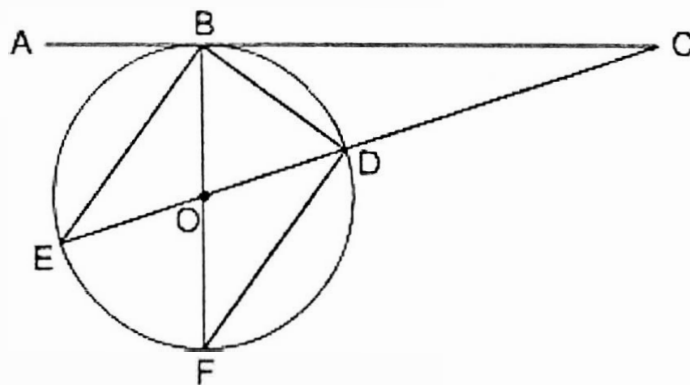
6. (i) Sketch the graph of  $y = 1 - |x - 3|$ . [3]

A line  $y = mx + 1$  is drawn on the same axes with the graph  $y = 1 - |x - 3|$ .

(ii) In the case where  $m = 2$ , find the coordinates of the point of intersection of the line and the graph of  $y = 1 - |x - 3|$ . [2]

(iii) Determine the set of values of  $m$  for which the line does not intersect the graph of  $y = 1 - |x - 3|$ . [2]

7.



In the diagram,  $BF$  and  $DE$  are the diameters of the circle with centre  $O$ .

The tangent at  $B$  meets  $ED$  produced at  $C$ . Prove that

(i)  $BE = DF$  [3]

(ii)  $DF \times BC = BD \times CE$  [3]

(iii)  $\angle BCE + 2\angle CBD = 90^\circ$ . [2]

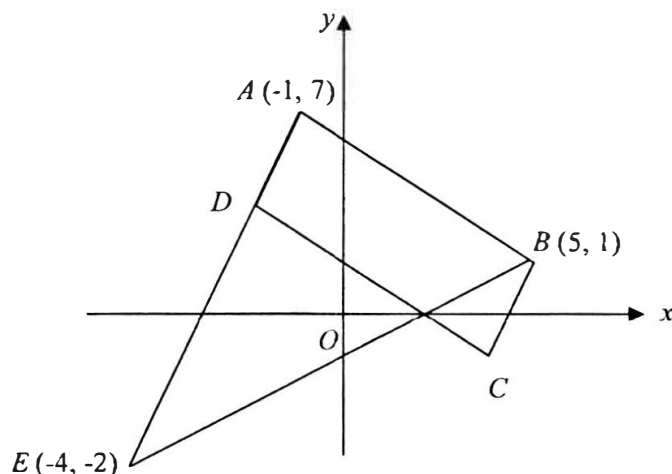
8. The equation of a circle  $C_1$  is  $x^2 + y^2 - 4x - 8y + 4 = 0$ .

(a) Find the coordinates of the centre and the radius of the circle. [3]

(b) The highest point on the circle is  $A$ .  
State the coordinates of  $A$ . [1]

(c) Another circle,  $C_2$ , touches  $C_1$  at the point  $A$ . Given that both circles do not overlap and the area of  $C_2$  is four times that of the area of  $C_1$ , find the equation of  $C_2$  in the form of  $x^2 + y^2 + 2gx + 2fy + c = 0$ , stating the value of  $f$ ,  $g$  and  $c$ . [4]

9. Solutions to this question by accurate drawing will not be accepted.



The diagram, not to scale, shows a parallelogram,  $ABCD$ .  $ADE$  and  $BE$  are straight lines.  $D$  divides  $AE$  such that  $AD : DE$  is in the ratio  $1 : 2$ .

$A$ ,  $B$  and  $E$  have coordinates  $(-1, 7)$ ,  $(5, 1)$  and  $(-4, -2)$  respectively.

(a) (i) Find the equation of the perpendicular bisector of  $AB$  and show that it passes through  $E$ . [3]

(ii) Hence deduce the geometrical property of triangle  $ABE$ . [1]

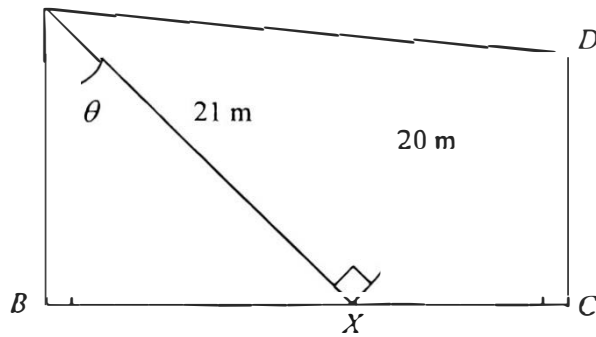
(b) Find the coordinates of  $D$ . [2]

(c) Find the area of the parallelogram  $ABCD$ . [2]

10. A particle starts from rest at 5 m from a fixed point  $O$  and moves in a straight line with a velocity,  $v = 12t - 3t^2$  m/s where  $t$  is the time in seconds after leaving from the initial rest position.

- (i) Calculate the acceleration when the particle is instantaneously at rest. [3]
- (ii) Calculate the maximum velocity. [2]
- (iii) Express the displacement,  $s$ , from point  $O$  in terms of  $t$ . [1]
- (iv) Find the average speed of the particle during the first five seconds. [3]

11.



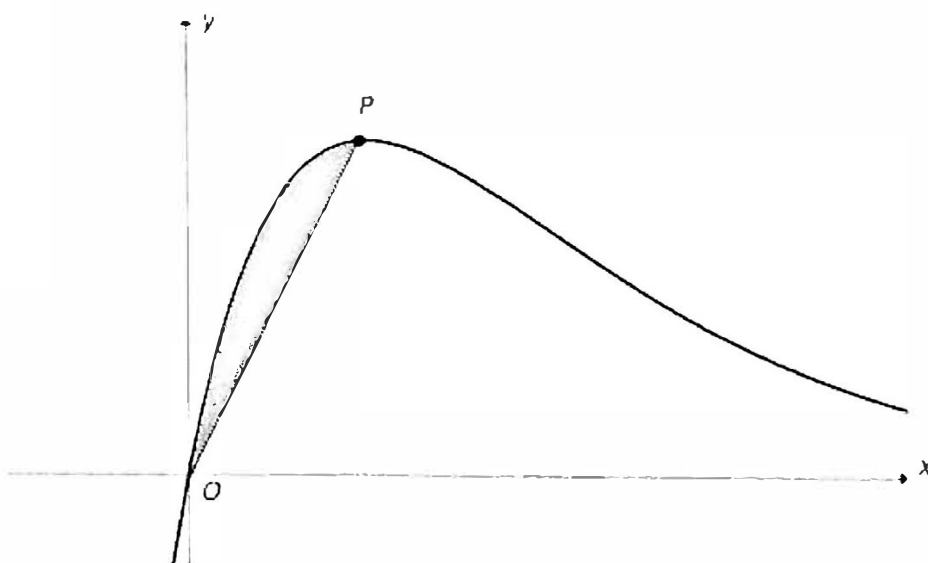
The diagram shows a trapezium field  $ABCD$ . The point  $X$  lies on the side  $BC$  such that  $AX = 21$  m,  $DX = 20$  m,  $\angle AXD = \angle ABX = \angle DCX = 90^\circ$  and  $\angle BAX = \theta$ .

- (i) Show that the length of fencing required for the perimeter of the field,  $L$  m, can be expressed in the form of  $p + q \sin \theta + r \cos \theta$ , where  $p$ ,  $q$  and  $r$  are constants to be determined. [3]
- (ii) Express  $L$  in the form  $p + R \cos(\theta - \alpha)$ , where  $R > 0$  and  $\alpha$  is an acute angle. [2]
- (iii) State the maximum value of  $L$  and the corresponding value of  $\theta$ . [2]
- (iv) Given that the fencing used is 80 m, find the value(s) of  $\theta$ . [3]

12. (a) (i) Given that  $y = xe^{-2x}$ ,  $x > 0$ , show that  $\frac{dy}{dx} = (1 - 2x)e^{-2x}$ . [1]
- (ii) Hence, find  $\int xe^{-2x} dx$ . [3]

- (b) The diagram, which is not drawn to scale, shows part of the curve  $y = xe^{-2x}$

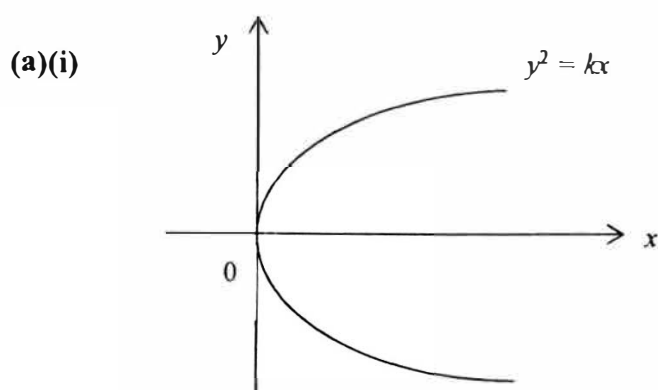
A line drawn from the origin meets the curve at the maximum point  $P$ .



- (i) Find the coordinates of  $P$ . [3]
- (ii) Calculate the area of the region bounded by the curve and the line  $OP$ . [4]

--- END OF PAPER ---

1. (a) (i) Sketch the graph of the curve  $y^2 = kx$ , where  $k$  is a positive constant. [1]
- (ii) Given that the line  $y = 2x + 1$  meets the curve  $y^2 = kx$ , find the range of values of  $k$ . [4]
- (b) Determine the conditions for  $p$  and  $q$  such that the curve  $y = px^2 - 2x + 3q$  lies entirely above the  $x$ -axis, where  $p$  and  $q$  are constants. [3]



[D1]

(a)(ii)  $y = 2x + 1$  ..... (1)

$y^2 = kx$  ..... (2)

(1) in (2):  $(2x + 1)^2 = kx$

$4x^2 + (4 - k)x + 1 = 0$  [A1]

For line meets the curve,  $D \geq 0$ .

$(4 - k)^2 - 4(4)(1) \geq 0$  [M1]

$16 - 8k + k^2 - 16 \geq 0$

$k(k - 8) \geq 0$  [M1A1]

$\therefore k \leq 0 (NA) \text{ or } k \geq 8$

- (b) Curve lies entirely above line,  $D < 0$  and  $p > 0$ .

$(-2)^2 - 4p(3q) < 0$

$4 - 12pq < 0$  [M1]

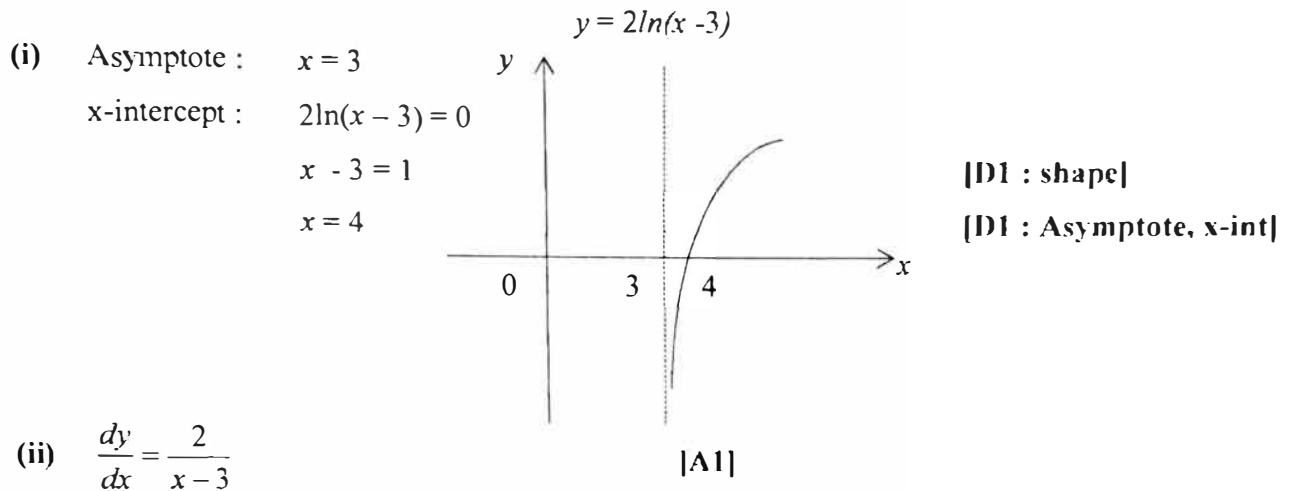
$pq > \frac{1}{3}$

$\therefore p > 0 \text{ and } pq > \frac{1}{3}$  [A2]

2. (i) Sketch the curve  $y = 2 \ln(x - 3)$  for  $x > 3$ . [2]

(ii) The tangent to the curve  $y = 2 \ln(x - 3)$  at the point  $P$  where  $x = 5$  intersects the  $x$ -axis at  $A$  and the normal to the curve at  $P$  intersects the  $x$ -axis at  $B$ .

Calculate the area of  $\triangle APB$ . [5]



When  $x = 5$ , gradient of tangent at  $P = 1$

When  $x = 5$ ,  $y = 2 \ln 2$

$P(5, 2 \ln 2)$

Equation of tangent at  $P$  :  $y - 2 \ln 2 = x - 5$

$$\therefore y = x - 5 + 2 \ln 2$$

At  $x$ -axis,  $y = 0$  :  $x = 5 - 2 \ln 2$

$$\therefore A(5 - 2 \ln 2, 0) \quad [A1]$$

Gradient of normal at  $P = -1$  [M1]

Equation of normal at  $P$  :  $y - 2 \ln 2 = -1(x - 5)$

$$\therefore y = -x + 5 + 2 \ln 2$$

At  $x$ -axis,  $y = 0$  :  $x = 5 + 2 \ln 2$

$$\therefore B(5 + 2 \ln 2, 0) \quad [A1]$$

$$\therefore \text{Area of } \triangle APB = \frac{1}{2}(5 + 2 \ln 2 - 5 + 2 \ln 2)(2 \ln 2)$$

$$= 1.92 \text{ units}^2 \quad [A1]$$

3. (a) Write down and simplify the first three terms in the expansion of  $(2 - 3x)^6$ , in ascending powers of  $x$ . [2]

(b) Hence

- (i) using a suitable value of  $x$ , find the estimated value of  $(1.997)^6$ , correct to 3 decimal places. [2]
- (ii) determine the coefficient of  $x^2$  in the expansion of  $(2 - 3x)^7 - (2 - 3x)^6$ . [3]

$$\begin{aligned} \text{(a)} \quad (2 - 3x)^6 &= 2^6 + \binom{6}{1} 2^5 (-3x) + \binom{6}{2} 2^4 (-3x)^2 + \dots \\ &= 64 - 576x + 2160x^2 - \dots \text{ (up to 1st 3 terms) [M1A1]} \end{aligned}$$

$$\begin{aligned} \text{(b)(i)} \quad \text{Put } 2 - 3x &= 1.997 \\ x &= 0.001 \quad \quad \quad \text{[M1]} \\ (1.997)^6 &= 64 - 576(0.001) + 2160(0.001)^2 + \dots \\ &= 63.42616 = 63.426 \text{ (correct to 3dp)} \quad \quad \quad \text{[A1]} \end{aligned}$$

$$\begin{aligned} \text{(b)(ii)} \quad (2 - 3x)^7 - (2 - 3x)^6 &= (2 - 3x)^6 [2 - 3x - 1] \\ &= (1 - 3x)(2 - 3x)^6 \quad \quad \quad \text{[M1]} \\ &= (1 - 3x)(64 - 576x + 2160x^2 - \dots) \\ \text{Coefficient of } x^2 &= 1(2160) - 3(-576) = 3888 \quad \quad \quad \text{[M1A1]} \end{aligned}$$

4. A curve has the equation  $y = f(x)$ , where  $f(x) = \frac{2 + \cos x}{\sin x}$  for  $-\pi \leq x \leq \pi$ .

(i) Obtain an expression for  $f'(x)$ . [2]

(ii) Find the **exact** value of the  $x$ -coordinates of the stationary points of the curve, and determine the nature of each stationary point. [6]

$$\begin{aligned} \text{(i)} \quad f'(x) &= \frac{\sin x(-\sin x) - (2 + \cos x)(\cos x)}{\sin^2 x} \quad \quad \quad \text{[M1]} \\ &= \frac{-\sin^2 x - 2\cos x - \cos^2 x}{\sin^2 x} \\ &= \frac{-1 - 2\cos x}{\sin^2 x} \quad \quad \quad \text{[A1]} \end{aligned}$$

(ii) For stationary points,  $f'(x) = 0$ .

$$\frac{-1 - 2 \cos x}{\sin^2 x} = 0$$

$$-1 - 2 \cos x = 0$$

[M1]

$$\cos x = -\frac{1}{2}$$

$$x = \frac{2\pi}{3} \text{ or } \pi + \frac{2\pi}{3} - 2\pi$$

$$\therefore x = \frac{2\pi}{3} \text{ or } -\frac{2\pi}{3}$$

[A2]

| x       | -2.1                                                                              | $-\frac{2\pi}{3}$                                                                 | -2                                                                                | 2                                                                                 | $\frac{2\pi}{3}$                                                                  | 2.1                                                                               |
|---------|-----------------------------------------------------------------------------------|-----------------------------------------------------------------------------------|-----------------------------------------------------------------------------------|-----------------------------------------------------------------------------------|-----------------------------------------------------------------------------------|-----------------------------------------------------------------------------------|
| $f'(x)$ | +ve                                                                               | 0                                                                                 | -ve                                                                               | -ve                                                                               | 0                                                                                 | +ve                                                                               |
| Tangent |  |  |  |  |  |  |

[M1]

$\therefore x = -\frac{2\pi}{3}$  is a maximum point and  $x = \frac{2\pi}{3}$  is a minimum point. [A2]

**Alternate Mtd :**

$$f''(x) = \frac{\sin^2 x (2 \sin x) - (-1 - 2 \cos x)(2 \sin x \cos x)}{\sin^4 x}$$

$$= \frac{2(\sin^2 x + \cos x + 2 \cos^2 x)}{\sin^3 x}$$

$$f''\left(-\frac{2\pi}{3}\right) = -2.31 < 0 \Rightarrow \text{max point}$$

$$f''\left(\frac{2\pi}{3}\right) = 2.31 > 0 \Rightarrow \text{min point}$$

5. (a) (i) Show that  $\frac{\cot x - \tan x}{\cot x + \tan x} = \cos 2x$ . [3]

(ii) Hence solve the equation  $\frac{\cot x - \tan x}{\cot x + \tan x} = \cos x$  for  $0^\circ < x < 360^\circ$ . [3]

(b) Without using a calculator, express  $\sin 15^\circ$  in the form  $\frac{1}{k}(\sqrt{a} - \sqrt{b})$ , where  $a, b$  and  $k$  are integers. [3]

(a)(i) LHS : 
$$\frac{\cot x - \tan x}{\cot x + \tan x} = \frac{\frac{\cos x}{\sin x} - \frac{\sin x}{\cos x}}{\frac{\cos x}{\sin x} + \frac{\sin x}{\cos x}} \quad [\text{M1}]$$
$$= \frac{\cos^2 x - \sin^2 x}{\cos^2 x + \sin^2 x} \quad [\text{A1}]$$
$$= \cos 2x = \text{RHS} \quad [\text{A1}]$$

(ii) 
$$\frac{\cot x - \tan x}{\cot x + \tan x} = \cos x$$
$$\cos 2x = \cos x$$
$$2 \cos^2 x - \cos x - 1 = 0 \quad [\text{M1}]$$
$$(2 \cos x + 1)(\cos x - 1) = 0$$
$$\cos x = -\frac{1}{2} \text{ or } \cos x = 1$$
$$\therefore x = 120^\circ, 240^\circ \quad [\text{A2}]$$

(b)  $\sin 15^\circ = \sin(45^\circ - 30^\circ)$  Alt Mtd :  $\sin 15^\circ = \sin(60^\circ - 45^\circ)$ 
$$= \sin 45^\circ \cos 30^\circ - \sin 30^\circ \cos 45^\circ \quad [\text{M1}]$$
$$= \frac{\sqrt{2}}{2} \left( \frac{\sqrt{3}}{2} \right) - \frac{\sqrt{2}}{2} \left( \frac{1}{2} \right) \quad [\text{A1}]$$
$$= \frac{\sqrt{6} - \sqrt{2}}{4} \quad [\text{A1}]$$

6. (i) Sketch the graph of  $y = 1 - |x - 3|$ . [3]

A line  $y = mx + 1$  is drawn on the same axes with the graph  $y = 1 - |x - 3|$ .

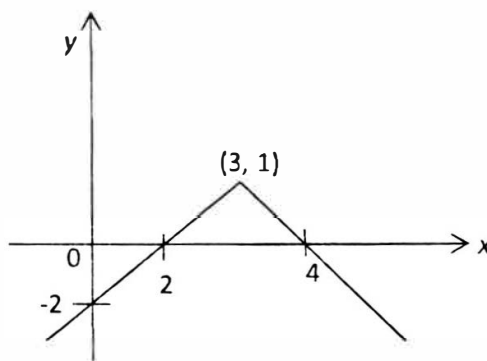
- (ii) In the case where  $m = 2$ , find the coordinates of the point of intersection of the line and the graph of  $y = 1 - |x - 3|$ . [2]

- (iii) Determine the set of values of  $m$  for which the line does not intersect the graph of  $y = 1 - |x - 3|$ . [2]

- (i) y-int : Put  $x = 0 : y = -2$

$$\begin{aligned} \text{x-int : } 1 - |x - 3| &= 0 \\ x &= 4 \text{ or } x = 2 \end{aligned}$$

$$\text{Max pt} = (3, 1)$$



D1 : Correct shape

D1 : intercepts

D1 : max pt

- (ii)  $2x + 1 = 1 - |x - 3|$   
 $|x - 3| = -2x$

$$x - 3 = -2x \text{ or } x - 3 = 2x$$

[M1]

$$x = 1 \text{ (NA) or } x = -3$$

$$\text{When } x = -3, y = -5$$

$$\text{Pt of intersection is } (-3, -5)$$

[A1]

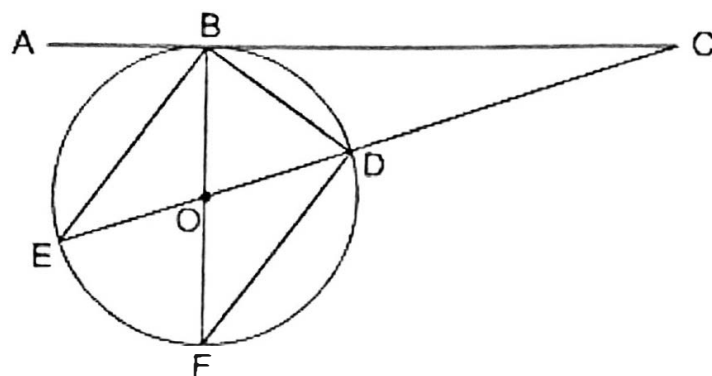
- (iii) For line not to intersect graph of  $y = 1 - |x - 3|$ , line must be parallel to the left arm.

$$\text{Gradient of left arm} = \frac{1 - (-2)}{3 - 0} = 1$$

$$\text{Set of values of } m : 0 < m \leq 1$$

[B2]

7.



In the diagram,  $BF$  and  $DE$  are the diameters of the circle with centre  $O$ .

The tangent at  $B$  meets  $ED$  produced at  $C$ . Prove that

- (i)  $BE = DF$  [3]
- (ii)  $DF \times BC = BD \times CE$  [3]
- (iii)  $\angle BCE + 2\angle CBD = 90^\circ$ . [2]

- (i)  $\angle BED = \angle DFB$  (Angles in the same segment)  
 $\angle DBE = \angle BDF = 90^\circ$  (right angle in a semi-circle)  
 $DE = BF$  (diameter) [M1]  
 $\therefore \triangle BDE \equiv \triangle DBF$  (AAS) [M1]  
 $\therefore BE = DF$  [A1]

**Alt Mtd :** Show  $\triangle BOE \equiv \triangle DOF$

- (ii)  $\angle DBC = \angle BEC$  (Alternate segment theorem)  
 $\angle DCB = \angle BCE$  (Common angle)  
 $\therefore \triangle BEC$  is similar to  $\triangle DBC$  (AA Similarity Test) [M1A1]  
 $\frac{BE}{DB} = \frac{EC}{BC}$   
 $BE \times BC = EC \times DB$  [M1]  
 $\therefore DF \times BC = BD \times CE$
- (iii)  $\angle BCE + \angle BEC + 90^\circ + \angle CBD = 180^\circ$  [M1]  
 $\angle BCE + 2\angle CBD = 180^\circ - 90^\circ$   
 $\therefore \angle BCE + 2\angle CBD = 90^\circ$ . [A1]

8. The equation of a circle  $C_1$  is  $x^2 + y^2 - 4x - 8y + 4 = 0$ .

(a) Find the coordinates of the centre and the radius of the circle. [3]

(b) The highest point on the circle is  $A$ .  
State the coordinates of  $A$ . [1]

(c) Another circle,  $C_2$ , touches  $C_1$  at the point  $A$ . Given that both circles do not overlap and the area of  $C_2$  is four times that of the area of  $C_1$ , find the equation of  $C_2$  in the form of  $x^2 + y^2 + 2gx + 2fy + c = 0$ , stating the value of  $f$ ,  $g$  and  $c$ . [4]

(a)  $C_1 : x^2 + y^2 - 4x - 8y + 4 = 0$ .

$$x^2 - 4x + \left(-\frac{4}{2}\right)^2 + y^2 - 8y + \left(-\frac{8}{2}\right)^2 = -4 + \left(-\frac{4}{2}\right)^2 + \left(-\frac{8}{2}\right)^2 \quad [\text{M1}]$$

$$(x - 2)^2 + (y - 4)^2 = 16$$

Centre = (2, 4) and radius = 4 units [A2]

(b)  $x$ -coordinate of  $A = 2$  (radius  $\perp$  tangent)

$$\therefore A = (2, 4 + 4) = (2, 8) \quad [\text{A1}]$$

(c) Radius of  $C_2 = 8$

Centre of  $C_2 = (2, 8 + 8) = (2, 16)$  [B1]

$$\text{Equation of } C_2 : (x - 2)^2 + (y - 16)^2 = 8^2 \quad [\text{M1}]$$

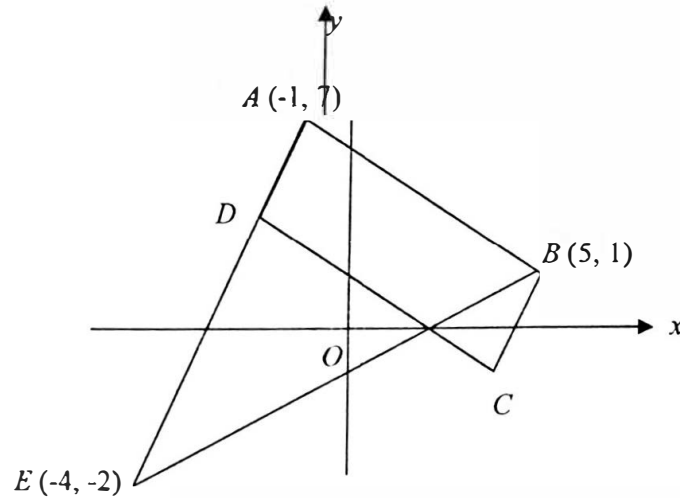
$$x^2 - 4x + 4 + y^2 - 32y + 256 = 0$$

$$x^2 + y^2 - 4x - 32y + 196 = 0 \quad [\text{A1}]$$

$$2g = -4, \quad 2f = -32 \text{ and } c = -196$$

$$\therefore g = -2, \quad f = 16, \quad c = 196 \quad [\text{A1}]$$

9. Solutions to this question by accurate drawing will not be accepted.



The diagram, not to scale, shows a parallelogram,  $ABCD$ .  $ADE$  and  $BE$  are straight lines.  $D$  divides  $AE$  such that  $AD : DE$  is in the ratio  $1 : 2$ .

$A$ ,  $B$  and  $E$  have coordinates  $(-1, 7)$ ,  $(5, 1)$  and  $(-4, -2)$  respectively.

- (a) (i) Find the equation of the perpendicular bisector of  $AB$  and show that it passes through  $E$ . [3]

- (ii) Hence deduce the geometrical property of triangle  $ABE$ . [1]

- (b) Find the coordinates of  $D$ . [2]

- (c) Find the area of the parallelogram  $ABCD$ . [2]

(a)(i) Gradient of  $AB = \frac{7-1}{-1-5} = -1$

Gradient of perpendicular bisector of  $AB = 1$

Mid-point of  $AB = \left( \frac{-1+5}{2}, \frac{7+1}{2} \right) = (2, 4)$  [A1]

Equation of perpendicular bisector of  $AB$ :  $y - 4 = x - 2$

$\therefore y = x + 2$  [A1]

When  $x = -4$ ,  $y = -4 + 2 = -2$ .

$\therefore$  perpendicular bisector of  $AB$  passes through  $E$ . (Shown) [M1]

- (ii)  $\triangle ABE$  is an isosceles triangle. [A1]

(b)  $\overrightarrow{AD} = \frac{1}{3} \overrightarrow{AE} = \begin{pmatrix} -1 \\ -3 \end{pmatrix}$

$D = (-1-1, 7-3) = (-2, 4)$  [M1A1]

(c) Area of  $\triangle ABD = \frac{1}{2} \begin{vmatrix} -1 & -2 & 5 \\ 7 & 4 & 1 \end{vmatrix} = 12 \text{ units}^2$  [M1]

Area of parallelogram  $ABCD = 12 \times 2 = 24 \text{ units}^2$  [A1]

10. A particle starts from rest at 5 m from a fixed point  $O$  and moves in a straight line with a velocity,  $v = 12t - 3t^2$  m/s where  $t$  is the time in seconds after leaving from the initial rest position.

- (i) Calculate the acceleration when the particle is instantaneously at rest. [3]  
 (ii) Calculate the maximum velocity. [2]  
 (iii) Express the displacement,  $s$ , from point  $O$  in terms of  $t$ . [1]  
 (iv) Find the average speed of the particle during the first five seconds. [3]

(i)  $a = \frac{dv}{dt} = 12 - 6t$  [A1]

When particle is instantaneously at rest,  $v = 0$

$$12t - 3t^2 = 0$$

$$3t(4 - t) = 0$$

$$t = 0 \text{ (NA)} \quad \text{or } t = 4 \quad \text{[M1]}$$

$$\text{Acceleration} = 12 - 6(4) = -12 \text{ m/s}^2. \quad \text{[A1]}$$

- (ii) For max or min velocity,  $a = 0$

$$12 - 6t = 0$$

$$t = 2 \quad \text{[M1]}$$

$$\frac{d^2v}{dt^2} = -6 < 0 \Rightarrow \text{max velocity}$$

$$\text{Max velocity} = 12(2) - 3(4) = 12 \text{ m/s} \quad \text{[A1]}$$

- (iii)  $S = \int (12t - 3t^2) dt$   
 $= 6t^2 - t^3 + C$  where  $C$  is an arbitrary constant.

$$\text{Subst } t = 0, s = 5 : C = 5.$$

$$\therefore s = 6t^2 - t^3 + 5 \quad \text{[A1]}$$

- (iv) When  $t = 0$ ,  $s = 5$  m

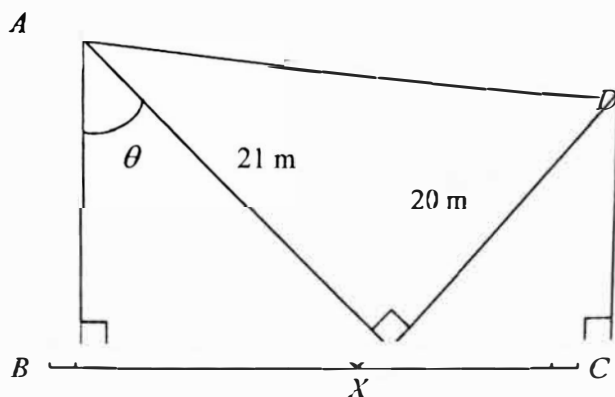
$$\text{When } t = 4, s = 37 \text{ m}$$

$$\text{When } t = 5, s = 30 \text{ m} \quad \text{[A1]}$$

$$\text{Total distance} = (37 - 5) + (37 - 30) = 39 \text{ m} \quad \text{[M1]}$$

$$\text{Average speed} = \frac{39}{5} = 7.8 \text{ m/s} \quad \text{[A1]}$$

11.



The diagram shows a trapezium field  $ABCD$ . The point  $X$  lies on the side  $BC$  such that  $AX = 21$  m,  $DX = 20$  m,  $\angle AXD = \angle ABX = \angle DCX = 90^\circ$  and  $\angle BAX = \theta$ .

- (i) Show that the length of fencing required for the perimeter of the field,  $L$  m, can be expressed in the form of  $p + q \sin \theta + r \cos \theta$ , where  $p$ ,  $q$  and  $r$  are constants to be determined. [3]
- (ii) Express  $L$  in the form  $p + R \cos(\theta - \alpha)$ , where  $R > 0$  and  $\alpha$  is an acute angle. [2]
- (iii) State the maximum value of  $L$  and the corresponding value of  $\theta$ . [2]
- (iv) Given that the fencing used is 80 m, find the value(s) of  $\theta$ . [3]

$$(i) \quad AD = \sqrt{21^2 + 20^2} = 29m$$

$$\sin \theta = \frac{BX}{21}$$

$$BX = 21 \sin \theta$$

$$\cos \theta = \frac{AB}{21}$$

$$AB = 21 \cos \theta$$

$$\angle DXC = \theta$$

$$\sin \theta = \frac{DC}{20}$$

$$DC = 20 \sin \theta$$

[M1A1]

$$\cos \theta = \frac{XC}{20}$$

$$XC = 20 \cos \theta$$

$$L = AB + BC + CD + AD$$

$$= 21 \cos \theta + 21 \sin \theta + 20 \cos \theta + 20 \sin \theta + 29$$

$$\therefore L = 41 \cos \theta + 41 \sin \theta + 29 \quad [A1]$$

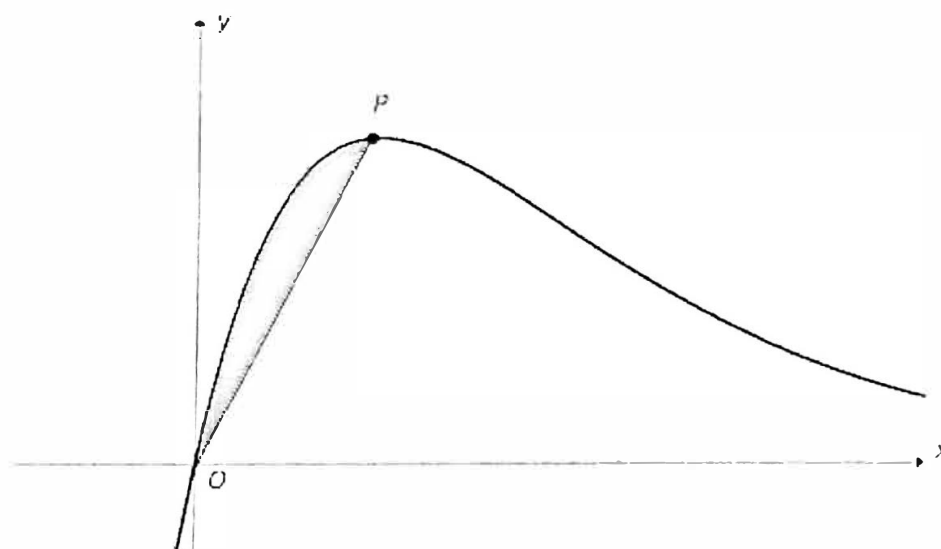
- (ii) Let  $41 \cos \theta + 41 \sin \theta = R \cos(\theta - \alpha)$   
 $R = \sqrt{41^2 + 41^2} = \sqrt{3362}$   
 $\tan \alpha = 1$   
 $\alpha = 45^\circ$  [M1A1]  
 $\therefore L = 29 + \sqrt{3362} \cos(\theta - 45^\circ)$
- (iii) Max value of  $L = 29 + \sqrt{3362} = 87.0m$  [A1]  
 $\cos(\theta - 45^\circ) = 1$   
 $\theta - 45^\circ = 0$  [A1]  
 $\therefore \theta = 45^\circ$
- (iv)  $29 + \sqrt{3362} \cos(\theta - 45^\circ) = 80$   
 $\cos(\theta - 45^\circ) = \frac{51}{\sqrt{3362}}$   
 $\theta - 45^\circ = 28.4^\circ, 331.6^\circ (NA), -28.4^\circ$  [M1A2]  
 $\therefore \theta = 73.4^\circ, 16.6^\circ$

12. (a) (i) Given that  $y = xe^{-2x}$ ,  $x > 0$ , show that  $\frac{dy}{dx} = (1 - 2x)e^{-2x}$ . [1]

(ii) Hence, find  $\int xe^{-2x} dx$ . [3]

(b) The diagram, which is not drawn to scale, shows part of the curve  $y = xe^{-2x}$

A line drawn from the origin meets the curve at the maximum point  $P$ .



[3]

(ii) Calculate the area of the region bounded by the curve and the line  $OP$ . [4]

(a)(i)  $y = xe^{-2x}$

$$\begin{aligned}\frac{dy}{dx} &= e^{-2x} - 2xe^{-2x} \\ &= (1 - 2x)e^{-2x}\end{aligned}\quad \text{[M1]}$$

(ii)  $\int e^{-2x} dx - 2 \int xe^{-2x} dx = [xe^{-2x}]$  [M1]

$$\begin{aligned}\int xe^{-2x} dx &= \frac{1}{2} \int e^{-2x} dx - \frac{1}{2} xe^{-2x} \\ \therefore \int xe^{-2x} dx &= -\frac{1}{4} e^{-2x} - \frac{1}{2} xe^{-2x} + C\end{aligned}\quad \text{[M1A1]}$$

(b)(i) For stationary points,  $\frac{dy}{dx} = 0$

$$(1 - 2x)e^{-2x} = 0$$

$$1 - 2x = 0$$

$$x = \frac{1}{2}$$

[M1A1]

When  $x = \frac{1}{2}$ ,  $y = \frac{1}{2}e^{-1} = \frac{1}{2e}$

$\therefore P\left(\frac{1}{2}, \frac{1}{2e}\right)$  [A1]

(iii) Required area  $= \int_0^{\frac{1}{2}} xe^{-2x} dx - \frac{1}{2} \left( \frac{1}{2} \right) \left( \frac{1}{2e} \right)$  [M1]

$= \left[ -\frac{1}{4}e^{-2x} - \frac{1}{2}xe^{-2x} \right]_0^{\frac{1}{2}} - \frac{1}{8e}$  [M1]

$= \left[ -\frac{1}{4}e^{-1} - \frac{1}{4}e \right] - \left( -\frac{1}{4} \right) - \frac{1}{8e}$  [M1]

$= \frac{5}{8}e^{-1} + \frac{1}{4}$  or 0.480 units<sup>2</sup> (3sf) [A1]