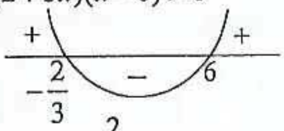
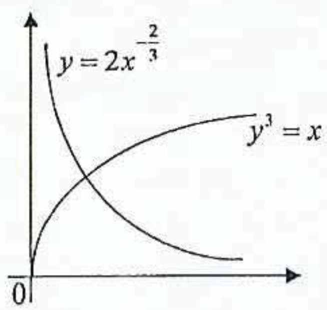
2019 4E5N MYE Additional Mathematics Paper 1 Solutions

Qn	Solution
1	$(2+3x)(x-5)>2+3x$ $(2+3x)(x-5)-(2+3x)>0$ $(2+3x)(x-6)>0$  $\therefore x<-\frac{2}{3}$ or $x>6$
2	$\frac{d^2y}{dx^2}=9x+1$ $\frac{dy}{dx}=\int(9x+1)\ dx$ $=\frac{9}{2}x^2+x+c$ When $x=2$, $\frac{dy}{dx}=18$ $\frac{9}{2}(2)^2+2+c=18$ $c=-2$ $\therefore \frac{dy}{dx}=\frac{9}{2}x^2+x-2$ $y=\int\left(\frac{9}{2}x^2+x-2\right)dx$ $=\frac{3}{2}x^3+\frac{1}{2}x^2-2x+d$ When $x=2$, $y=16$, $16=\frac{3}{2}(2)^3+\frac{1}{2}(2)^2-2(2)+d$ $d=6$ $\therefore y=\frac{3}{2}x^3+\frac{1}{2}x^2-2x+6$
3(i)	

3(ii)	$x^{\frac{1}{3}}=2x^{-\frac{2}{3}}$ $x=2$ $y=\sqrt[3]{2}$ $=1.26$ (3s.f.) Coordinates of the point of intersection are $(2,1.26)$
4	$\frac{x}{y}=mx+c------(1)$ $m=\tan 60°=\sqrt{3}$ Sub $m=\sqrt{3}$, $(3\sqrt{3},4)$ into (1): $4=\sqrt{3}\left(3\sqrt{3}\right)+c$ $c=-5$ $\frac{x}{y}=\sqrt{3}x-5$ $y=\frac{x}{\sqrt{3}x-5}$
5(i)	$\begin{array}{r} 2x^2 \qquad +1 \\ x-3\overline{)2x^3-6x^2+x-3} \\ \underline{-\left(2x^3-6x^2\right)} \\ x-3 \\ \underline{-(x-3)} \\ 0 \end{array}$ $\frac{2x^3-6x^2+x-3}{x-3}=2x^2+1$

5(ii)	$\dfrac{3x^2+21x+5}{2x^3-6x^2+x-3}=\dfrac{3x^2+21x+5}{(x-3)(2x^2+1)}$ let $\dfrac{3x^2+21x+5}{(x-3)(2x^2+1)}=\dfrac{A}{x-3}+\dfrac{Bx+C}{2x^2+1}$ $3x^2+21x+5=A(2x^2+1)+(Bx+C)(x-3)$ Sub $x=3$: $95=19A$ $A=5$ Sub $x=0$: $5=5-3C$ $C=0$ Sub $x=1$: $29=15-2B$ $B=-7$ $\dfrac{3x^2+21x+5}{2x^3-6x^2+x-3}=\dfrac{5}{x-3}-\dfrac{7x}{2x^2+1}$
6(i)	$y=\dfrac{1}{2}e^{2x}-7e^{x}+6x$ $\dfrac{dy}{dx}=e^{2x}-7e^{x}+6$ $\dfrac{d^2y}{dx^2}=2e^{2x}-7e^{x}$
6(ii)	For stationary points, $\dfrac{dy}{dx}=0$ $e^{2x}-7e^{x}+6=0$ let $e^{x}=u$ $u^2-7u+6=0$ $(u-6)(u-1)=0$ $\Rightarrow u=6$ or $u=1$ $e^{x}=6$ or $e^{x}=1$ $x=\ln 6$ or $x=0$
6(iii)	$\left.\dfrac{d^2y}{dx^2}\right\vert_{x=\ln 6}=2e^{2\ln 6}-7e^{\ln 6}=30>0$ The curve has a minimum point at $x=\ln 6$. $\left.\dfrac{d^2y}{dx^2}\right\vert_{x=0}=2e^{0}-7e^{0}=-5<0$ The curve has a maximum point at $x=0$.
7(i)	$\int\left(3x^4-5\cos 6x\right)\,dx=\dfrac{3}{5}x^5-\dfrac{5}{6}\sin 6x+c$
7(ii)	$\int\left(\sin x+\sec^2(5+2\pi x)\right)\,dx=-\cos x+\dfrac{1}{2\pi}\tan(5+2\pi x)+c$

8(i)	Area of lawn: $lr-\frac{\pi r^2}{4}=360$ $l=\frac{360-\frac{\pi r^2}{4}}{r}$ $l=\frac{360}{r}+\frac{\pi r}{4}$ --------(1) Perimeter of lawn: $P=l+r+(l-r)+\frac{1}{4}(2\pi r)$ $P=2l+\frac{\pi r}{2}$ ------------(2) Sub (1) into (2): $P=2\left(\frac{360}{r}+\frac{\pi r}{4}\right)+\frac{\pi r}{2}$ $P=\frac{720}{r}+\pi r$ (shown)
8(ii)	$\frac{dP}{dr}=-\frac{720}{r^2}+\pi$ $\frac{dP}{dr}=0$ for stationary values $-\frac{720}{r^2}+\pi=0$ $r^2=\frac{720}{\pi}$ $r=15.1$ (3s.f.) (rej. -15.1) $\frac{d^2P}{dr^2}=1440r^{-3}$ $\left.\frac{d^2P}{dr^2}\right\|_{r=15.138}=0.415$ (3s.f.) >0 $\Rightarrow$ Shortest perimeter $l=\frac{360}{15.138}+\frac{\pi(15.138)}{4}$ $=35.7$ (3s.f.) The shortest hedge can be planted when the rectangular space is 15.1cm by 35.7cm.

9(i)	$\dfrac{\tan^2 x-1}{\tan^2 x+1}$ $=\dfrac{\dfrac{\sin^2 x}{\cos^2 x}-1}{\dfrac{\sin^2 x}{\cos^2 x}+1}$ $=\dfrac{\sin^2 x-\cos^2 x}{\sin^2 x+\cos^2 x}$ $=\sin^2 x-\cos^2 x$ $=1-\cos^2 x-\cos^2 x$ $=1-2\cos^2 x$ (shown)
9(ii)	$1-2\cos^2 x=\dfrac{1}{2}$ $\cos^2 x=\dfrac{1}{4}$ $\cos x=\pm\dfrac{1}{2}$ $\alpha=\cos^{-1}\left(\dfrac{1}{2}\right)=\dfrac{\pi}{3}$ $x=\dfrac{\pi}{3},\dfrac{2\pi}{3},\dfrac{4\pi}{3}$

10	$4x^2-6x+3=0$ $\frac{1}{\alpha}+\frac{1}{\beta}=\frac{3}{2}$ ---------(1) $\left(\frac{1}{\alpha}\right)\left(\frac{1}{\beta}\right)=\frac{3}{4}$ ------(2) From (1): $\frac{\alpha+\beta}{\alpha\beta}=\frac{3}{2}$ ------(3) From (2): $\alpha\beta=\frac{4}{3}$ ----------(4) Sub (3) into (4): $\alpha+\beta=2$ -------(5) $\alpha^3+\beta^3=(\alpha+\beta)(\alpha^2-\alpha\beta+\beta^2)$ $=(\alpha+\beta)\left((\alpha+\beta)^2-3\alpha\beta\right)$ $=(2)\left((2)^2-3\left(\frac{4}{3}\right)\right)$ $=0$ $(\alpha)^3(\beta)^3=\left(\frac{4}{3}\right)^3$ $=\frac{64}{27}$ The equation is $x^2+\frac{64}{27}=0$.
11(i)	By Pythagora's Theorem, Square of Diagonal $=18^2+(5-\sqrt{3})^2$ $=324+25-10\sqrt{3}+3$ $=352-10\sqrt{3}$
11(ii)	$\frac{(1+\sqrt{3})^2}{18(5-\sqrt{3})}=\frac{4+2\sqrt{3}}{18(5-\sqrt{3})}$ $=\frac{4+2\sqrt{3}}{18(5-\sqrt{3})}\left(\frac{5+\sqrt{3}}{5+\sqrt{3}}\right)$ $=\frac{20+10\sqrt{3}+4\sqrt{3}+6}{18(25-3)}$ $=\frac{26+14\sqrt{3}}{396}$ $=\frac{13+7\sqrt{3}}{198}$

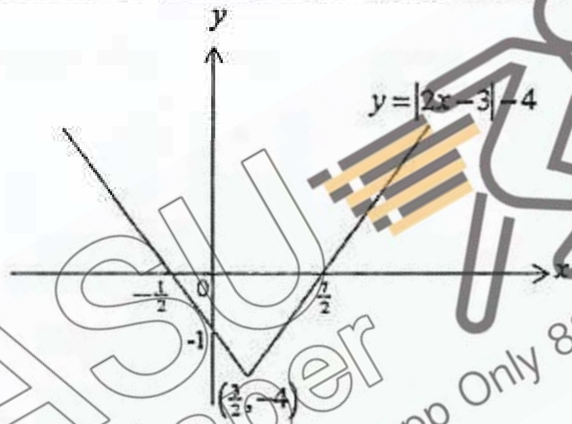
12(i)	$y = x\sqrt{5x^2-6}$ $\frac{dy}{dx} = \left(5x^2-6\right)^{\frac{1}{2}} + x\left(\frac{1}{2}\right)\left(5x^2-6\right)^{-\frac{1}{2}}(10x)$ $= \frac{5x^2-6+5x^2}{\sqrt{5x^2-6}}$ $= \frac{2\left(5x^2-3\right)}{\sqrt{5x^2-6}}$
12(ii)	$\int_2^4 \frac{5x^2-3}{\sqrt{5x^2-6}}\,dx$ $= \frac{1}{2}\int_2^4 \frac{2\left(5x^2-3\right)}{\sqrt{5x^2-6}}\,dx$ $= \frac{1}{2}\left[x\sqrt{5x^2-6}\right]_2^4$ $= \frac{1}{2}\left[4\sqrt{74}-2\sqrt{14}\right]$ $= 13.5$ (3s.f.)
13(i)	$4y+3x=60$ $y = -\frac{3}{4}x+15$ $m_{tangent} = -\frac{3}{4}$ $m_{normal} = \frac{4}{3}$ $y-9 = \frac{4}{3}(x-8)$ The equation of the normal is $y = \frac{4}{3}x - \frac{5}{3}$. -----(1) $y = 4x-7$ -------(2) (1) = (2): $\frac{4}{3}x - \frac{5}{3} = 4x-7$ $x = 2$ $y = 1$ $P(2,1)$
13(ii)	$(x-2)^2+(y-1)^2 = r^2$ Sub $(8,9)$: $r^2 = 100$ Equation of circle is $(x-2)^2+(y-1)^2 = 100$.

13(iii)	When $x=0,\ 4y+3(0)=60$ $y=15$ $B(0,15)$ Centre of circle $=\left(\dfrac{2+0}{2},\dfrac{1+15}{2}\right)$ $=(1,8)$ $BP=\sqrt{(2-0)^2+(1-15)^2}$ $=10\sqrt{2}$ Radius $=5\sqrt{2}$ $(x-1)^2+(y-8)^2=\left(5\sqrt{2}\right)^2$ Equation of circle is $(x-1)^2+(y-8)^2=50$.

CWSS 4E AM 2019 MYE P2 SOLUTION

1(i)	$-1 \le \sin x \le 1$ $-1 \le -\sin x \le 1$ $-1+3 \le -\sin x+3 \le 1+3$ $2 \le f'(x) \le 4$ Since $f'(x)$ is always more than 0, there are no stationary points on the curve.
1(ii)	$f(x)=\int 3-\sin x\,dx$ $f(x)=3x+\cos x+c$ Since curve passes through origin, $0=3(0)+\cos(0)+c$ $c=-1$ $f(x)=3x+\cos x-1$
2(i)	$y=\dfrac{2x+1}{x-4}$ $\dfrac{dy}{dx}=\dfrac{2(x-4)-1(2x+1)}{(x-4)^2}$ $\dfrac{dy}{dx}=-\dfrac{9}{(x-4)^2}$ Since $(x-4)^2 \ge 0$ for all real values of x, $\dfrac{dy}{dx}<0$ for all real values of x. Hence, y is a decreasing function.
2(ii)	$\dfrac{dy}{dt}=\dfrac{dy}{dx}\times\dfrac{dx}{dt}$
	$0.5=-\dfrac{9}{(2-4)^2}\times\dfrac{dx}{dt}$
	$\dfrac{dx}{dt}=-\dfrac{2}{9}$ units per second
3(i)	$\sin 3x=\sin 2x\cos x+\cos 2x\sin x$ $\sin 3x=2\sin x\cos x\cos x+(1-2\sin^2 x)\sin x$ $\sin 3x=\sin x(2\cos^2 x+1-2\sin^2 x)$ $\sin 3x=\sin x\left[2(1-\sin^2 x)+1-2\sin^2 x\right]$ $\sin 3x=3\sin x-4\sin^3 x$

3(ii)	$\sin 3x = \sin^2 x$ $3\sin x - 4\sin^3 x = \sin^2 x$ $\sin x(4\sin x - 3)(\sin x + 1) = 0$ $\sin x = 0,\ \sin x = \frac{3}{4},\ \sin x = -1$ $x = \pi,\ x = 0.848(3sf),\ x = \frac{3\pi}{2}$
4(i)	$R = \sqrt{\left(\sqrt{3}\right)^2 + 1^2}$ $R = 2$ $\alpha = \tan^{-1}\frac{1}{\sqrt{3}}$ $\alpha = \frac{\pi}{6}$ $\sqrt{3}\sin\theta + \cos\theta = 2\sin\left(\theta + \frac{\pi}{6}\right)$
4(ii)	$\int_0^{\pi} \sqrt{3}\sin x + \cos x \,dx$ $= \int_0^{\pi} 2\sin\left(x + \frac{\pi}{6}\right)dx$ $= 2\left[-\cos\left(x + \frac{\pi}{6}\right)\right]_0^{\pi}$ $= 2\left\{\left[-\cos\left(\pi + \frac{\pi}{6}\right)\right] - \left[-\cos\left(0 + \frac{\pi}{6}\right)\right]\right\}$ $= 2\left[-\left(-\frac{\sqrt{3}}{2}\right) - \left(-\frac{\sqrt{3}}{2}\right)\right]$ $= 2\sqrt{3}$
5(i)	$\left(kx - \frac{1}{x^3}\right)^7 = \binom{7}{r}(kx)^{7-r}\left(-\frac{1}{x^3}\right)^r + \ldots$ $\left(kx - \frac{1}{x^3}\right)^7 = \binom{7}{r}(k)^{7-r}(-1)^r x^{7-r}\left(x^{-3}\right)^r + \ldots$ $\left(kx - \frac{1}{x^3}\right)^7 = \binom{7}{r}(k)^{7-r}(-1)^r x^{7-r-3r} + \ldots$ $\left(kx - \frac{1}{x^3}\right)^7 = \binom{7}{r}(k)^{7-r}(-1)^r x^{7-4r} + \ldots$ Since the power of x is $7 - 4r = 2(3-2r)+1$ will always be odd, there are no even powers of x for this expansion.

5(ii)	$\binom{7}{2}(k)^{7-2}(-1)^2 = 3\binom{7}{1}(k)^{7-1}(-1)^1$ $21(k)^5 = -3(7)(k)^6$ $k=-1$
6(i)	When $x=0$, $y=\lvert 0-3\rvert-4$ $y=-1$ When $y=0$, $0=\lvert 2x-3\rvert-4$ $\lvert 2x-3\rvert=4$ $2x-3=4$ or $2x-3=-4$ $x=\frac{7}{2}$, $x=-\frac{1}{2}$ Points are $(0,-1)$, $\left(-\frac{1}{2},0\right)$ and $\left(\frac{7}{2},0\right)$.
6(ii)	
6(iii)	$x+4=\lvert 2x-3\rvert$ $x+4=3-2x$ or $x+4=2x-3$ $3x=-1$ or $x=2x-3-4$ $x=-\frac{1}{3}$, $x=7$
7(i)	$f(-1)=2(-1)^3+6(-1)^2+6(-1)+5$ $f(-1)=3$
7(ii)	$f(x)=(x+1)(2x^2+4x+2)+3$ $f(x)=2(x+1)^3+3$

7(iii)

$f'(x)=6x^2+12x+6$ or $f'(x)=6(x+1)^2$

$6x^2+12x+6=0$ or $(x+1)^2=0$

$x^2+2x+1=0$

$(x+1)^2=0$

$x=-1$

$y=2(-1+1)^3+3$

$y=3$

By first derivative test,

x	-1^-	-1	-1^+
$\frac{dy}{dx}$	positive	0	positive
shape	/	—	/

$(-1,3)$ is a point of inflexion.

7(iv)

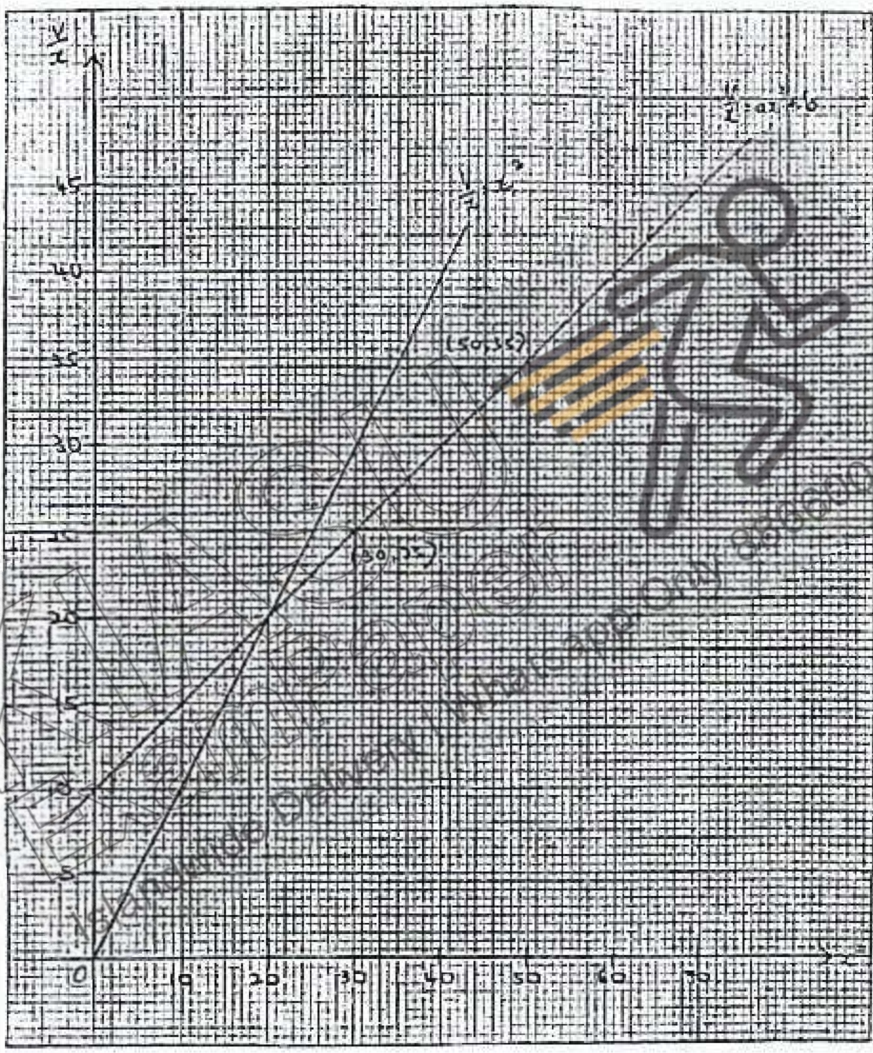
As the graph is an increasing function with no turning points, it will always only cut the x-axis only once.

8(i)

$$V = x\left(ax^2 + b\right)$$

$$\frac{V}{x} = ax^2 + b$$

x^2	4	16	36	64
$\frac{V}{x}$	12	18	28	42

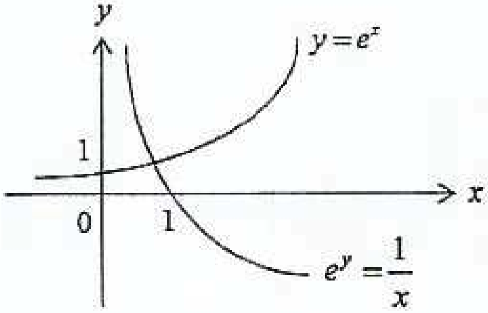


From the graph

$$a = \frac{35-25}{50-30}$$

$$a = \frac{1}{2} \text{ and } b = 10$$

8(ii)	$V = x^3$ Add the line $\frac{V}{x} = x^2$ onto graph From the graph, $x^2 = 20$ Diameter of the ball $= \sqrt{20} = 2\sqrt{5}$ cm
8(iii)	$x^3 = x\left(\frac{1}{2}x^2 + 10\right)$ $\frac{1}{2}x^3 - 10x = 0$ $x = 0$ or $x^2 = 20$ Since $x \neq 0$, the diameter of the ball $= \sqrt{20} = 2\sqrt{5}$ cm
9(a)	$\log_2(2x+1) - \log_4(x+1) = 1$ $\log_2(2x+1) - \frac{\log_2(x+1)}{\log_2 4} = \log_2 2$ $\log_2(2x+1) - \frac{1}{2}\log_2(x+1) = \log_2 2$ $\log_2 \frac{(2x+1)}{\sqrt{x+1}} = \log_2 2$ $2x+1 = 2\sqrt{x+1}$ $4x^2 + 4x + 1 = 4x + 4$ $4x^2 = 3$ $x = \frac{\sqrt{3}}{2}$ or $x = -\frac{\sqrt{3}}{2}$ (reject)
9(b)	$(\log_b e)(\log_b a)(\ln a) = 16$ $\left(\frac{\ln e}{\ln b}\right)\left(\frac{\ln a}{\ln b}\right)(\ln a) = 16$ $(\ln a)^2 = 16(\ln b)^2$ $\ln a = 4\ln b$ or $\ln a = -4\ln b$ $a = b^4$ or $a = \frac{1}{b^4}$

9(c)	$y=e^x$ $e^y=\dfrac{1}{x}$ $y\ln e=\ln x^{-1}$ $y=-\ln x$  $e^x+\ln x=0$ $e^x=-\ln x$ From the graph, there is only one point of intersection, hence there is only one solution.
10(a)	$\dfrac{\sqrt{5}+\sqrt{3}}{\sqrt{10}-\sqrt{6}}=\dfrac{\left(\sqrt{5}+\sqrt{3}\right)\left(\sqrt{10}+\sqrt{6}\right)}{\left(\sqrt{10}\right)^2-\left(\sqrt{6}\right)^2}$ $\dfrac{\sqrt{5}+\sqrt{3}}{\sqrt{10}-\sqrt{6}}=\dfrac{\sqrt{50}+\sqrt{30}+\sqrt{30}+\sqrt{18}}{10-6}$ $\dfrac{\sqrt{5}+\sqrt{3}}{\sqrt{10}-\sqrt{6}}=\dfrac{5\sqrt{2}+2\sqrt{30}+3\sqrt{2}}{4}$ $\dfrac{\sqrt{5}+\sqrt{3}}{\sqrt{10}-\sqrt{6}}=2\sqrt{2}+\dfrac{1}{2}\sqrt{30}$
10(b)(i)	$5^{2x+3}=5^{x-1}+1$ $5^3\left(5^x\right)^2=\dfrac{5^x}{5}+1$ Let $y=5^x$ $625y^2-y-5=0$ $y=0.090246$ and $y=-0.088646$ $5^x=0.090246$ and $5^x=-0.088646$ (reject) $x=\dfrac{\ln 0.090246}{\ln 5}$ $x=-1.49$ (2 *dp*)

10(b)(ii)	Let $y=5^x$ $125y^2-\frac{y}{5}+k=0$ $625y^2-y+5k=0$ $(-1)^2-4(625)(5k)<0$ $1-12500k<0$ $k>\frac{1}{12500}$
11(i)	Length of $AO=\sqrt{6^2+8^2}=10$ Length of $OB=15-10=5$ $\sqrt{x^2+y^2}=5$ and $\frac{y}{x}=\frac{4}{3}$ By substitution or by Pythagoras triplets, $x=-3$ and $y=-4$ $B(-3,-4)$
11(ii)	Gradient of $BC=\frac{-1}{\frac{4}{3}}$ $-4=-\frac{3}{4}(-3)+c$ Equation of BC is $y=-\frac{3}{4}x-\frac{25}{4}$ $4y+3x+25=0$
11(iii)	From equation of BC, C is y-int. Hence, $C\left(0,-\frac{25}{4}\right)$ Equation of CD is $y=\frac{4}{3}x-\frac{25}{4}$ since AB is parallel to DC. $0=\frac{4}{3}x-\frac{25}{4}$ $x=\frac{75}{16}$ $D\left(\frac{75}{16},0\right)$