2019 Additional Mathematics Paper 1 Sec 4 MYE (Solutions)

1 A triangle has an area of $(58+8\sqrt{5})$ cm^2 and a height of $(7+3\sqrt{5})$ cm. Without using a calculator, find the exact length of its base, expressing in the form $+b\sqrt{5}$, where a and b are integers. [4]

1 Length of the base

$$= \frac{2(58+8\sqrt{5})}{7+3\sqrt{5}}$$

$$= \frac{116+16\sqrt{5}}{7+3\sqrt{5}} \times \frac{7-3\sqrt{5}}{7-3\sqrt{5}}$$

$$= \frac{812-348\sqrt{5}+112\sqrt{5}-240}{49-45}$$

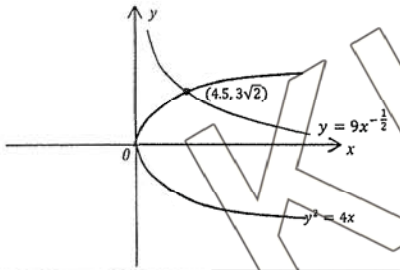
$$= \frac{572-236\sqrt{5}}{4}$$

$$= 143-59\sqrt{5} \text{ cm}$$

2 (i) On the same diagram, sketch the curves $y = 9x^{-\frac{1}{2}}$ and $y^2 = 4x$. [2]

(ii) Find the coordinates of the point(s) of intersection of the two curves. [2]

2(i)



(ii)

$y = 9x^{-\frac{1}{2}}$ -----(1)

$y^2 = 4x$ -----(2)

Sub (1) into (2): $(9x^{-\frac{1}{2}})^2 = 4x$

$81x^{-1} = 4x$

$x^2 = \frac{81}{4}$

$x = \frac{9}{2}$ or $-\frac{9}{2}$ (reject)

When $x = \frac{9}{2}, y = 9(\frac{9}{2})^{-\frac{1}{2}}$

$= 9 \div \frac{\sqrt{9}}{\sqrt{2}}$

$= 3\sqrt{2}$

The coordinates of the point of intersection is

$(4\frac{1}{2}, 3\sqrt{2})$.

3 The equation of a curve is $y = 2xe^{x-k}$, where k is a constant. The curve passes through the point $(5, 10)$.

(i) Find the value of k. [2]

(i) For what values of x is y an increasing function of x. [3]

3(i)

$y = 2xe^{x-k}$

When $x = 5, y = 10$,

$10 = 2(5)e^{5-k}$

$1 = e^{5-k}$

$e^0 = e^{5-k}$

$5-k = 0$

$k = 5$

(ii)

$y = 2xe^{x-5}$

$\frac{dy}{dx} = 2xe^{x-5} + 2e^{x-5}$

$= 2e^{x-5}(x+1)$

For y to be an increasing function of x,

$\frac{dy}{dx} > 0$

Since $2e^{x-5} > 0$, $x+1 > 0$

$x > -1$

4 Express $\frac{16x^2-9x+18}{x^3+3x^2}$ in partial fractions. [5]

4

$$\frac{16x^2-9x+18}{x^3+3x^2} = \frac{16x^2-9x+18}{x^2(x+3)}$$

Let $\frac{16x^2-9x+18}{x^3+3x^2} = \frac{A}{x} + \frac{B}{x^2} + \frac{C}{x+3}$

$16x^2-9x+18 = Ax(x+3)+B(x+3)+Cx^2$

Let $x=-3$, $16(-3)^2-9(-3)+18 = 9C$

$9C = 189$

$C = 21$

Let $x = 0$, $18 = 3B$

$B = 6$

Comparing x^2 term, $16x^2 = Ax^2 + Cx^2$

$A + C = 16$

$A + 21 = 16$

$A = -5$

$$\frac{16x^2-9x+18}{x^3+3x^2} = \frac{-5}{x} + \frac{6}{x^2} + \frac{21}{x+3}$$

5 The function f is given by $f(x) = -3\sin\frac{x}{2} + 2$.

(i) State the amplitude and period of f. [2]

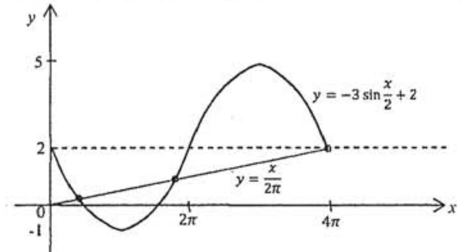
(ii) Sketch the graph of $y = f(x)$ for $0 \le x \le 4\pi$. By drawing a suitable straight line on the same axes, state the number of solutions to the equation $4\pi - x - 6\pi\sin\frac{x}{2} = 0$ for $0 \le x \le 4\pi$. [5]

5(i) Amplitude = 3

Period $= 2\pi \div \frac{1}{2}$

$= 4\pi$

(ii)



$4\pi - x - 6\pi\sin\frac{x}{2} = 0$

$\frac{4\pi - x - 6\pi\sin\frac{x}{2}}{2\pi} = \frac{0}{2\pi}$

$2 - \frac{x}{2\pi} - 3\sin\frac{x}{2} = 0$

$-3\sin\frac{x}{2} + 2 = \frac{x}{2\pi}$

Since there are 3 points of intersection between the graphs $y = -3\sin\frac{x}{2} + 2$ and $y = \frac{x}{2\pi}$, there are 3 solutions.

6 (i) Given that $\cos(A + B) = 3\cos(A - B)$ and $\tan A = -\frac{5}{2}$, find the value of $\cot B$. [3]

(ii) Prove that $\frac{1+\tan^2 x}{1-\tan^2 x} = \sec 2x$. [3]

6(i) $\cos(A + B) = 3\cos(A - B)$

$\cos A\cos B - \sin A\sin B = 3(\cos A\cos B + \sin A\sin B)$

$\cos A\cos B - \sin A\sin B = 3\cos A\cos B + 3\sin A\sin B$

$-4\sin A\sin B = 2\cos A\cos B$

$\frac{-4\sin A\sin B}{\cos A\cos B} = \frac{2\cos A\cos B}{\cos A\cos B}$

$-4\tan A\tan B = 2$

Sub $\tan A = -\frac{5}{2}$,

$-4\left(-\frac{5}{2}\right)\tan B = 2$

$10\tan B = 2$

$\tan B = \frac{1}{5}$

$\cot B = 5$

(ii) LHS $= \frac{1+\tan^2 x}{1-\tan^2 x} = \frac{1+\frac{\sin^2 x}{\cos^2 x}}{1-\frac{\sin^2 x}{\cos^2 x}}$

$= \frac{1+\frac{\sin^2 x}{\cos^2 x}}{1-\frac{\sin^2 x}{\cos^2 x}} \times \frac{\cos^2 x}{\cos^2 x}$

$= \frac{\cos^2 x + \sin^2 x}{\cos^2 x - \sin^2 x}$

$= \frac{1}{\cos 2x}$

$= \sec 2x$

$= RHS$ (Proven)

7 The roots of the quadratic equation $2x^2 + x + 6 = 0$ are α and β.

(i) Express $\alpha^2 - \alpha\beta + \beta^2$ in terms of $(\alpha + \beta)$ and $\alpha\beta$. [1]

(ii) Form a quadratic equation whose roots are α^3 and β^3. [5]

7(i) $\alpha^2 - \alpha\beta + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta - \alpha\beta$

$= (\alpha + \beta)^2 - 3\alpha\beta$

(ii) Sum of roots: $\alpha + \beta = -\frac{1}{2}$

Product of roots: $\alpha\beta = \frac{6}{2} = 3$

For an equation whose roots are α^3 and β^3,

Sum of roots: $\alpha^3 + \beta^3 = (\alpha + \beta)(\alpha^2 - \alpha\beta + \beta^2)$

$= (\alpha + \beta)[(\alpha + \beta)^2 - 3\alpha\beta]$

$= \left(-\frac{1}{2}\right)\left[\left(-\frac{1}{2}\right)^2 - 3(3)\right]$

$= \frac{35}{8}$

Product of roots: $\alpha^3\beta^3 = (3)^3$

$= 27$

The equation is $x^2 - \left(\frac{35}{8}\right)x + 27 = 0$

or $8x^2 - 35x + 216 = 0$

8 An antique grandfather clock manufactured using the finest wood in 1850 was valued at \$2000. The clock appreciated in its value such that its value \$$V$ can be modelled by the equation $V = 20000 - Ae^{kt}$, where t was the number of years after its manufacture date.

(i) Find the value of A. [2]

(ii) In the year 1880, the clock reached five times its initial value. Show that $k = -0.01959$ correct to 4 significant figures. [3]

(iii) Explain why the value of the clock will not exceed \$20000. [2]

8(i) When $t = 0$, $V = 2000$

$2000 = 20000 - Ae^{k(0)}$

$A = 20000 - 2000$

$= 18000$

(ii) $V = 20000 - 18000e^{kt}$

In the year 1880, $t = 30$, $V = 5(2000)$

$20000 - 18000e^{30k} = 10000$

$-18000e^{30k} = -10000$

$e^{30k} = \frac{5}{9}$

$$\ln e^{30k} = \ln\frac{5}{9}$$

$$30k = \ln\frac{5}{9}$$

$$k = \frac{\ln\frac{5}{9}}{30}$$

$$= -0.019592 \ldots \text{ or } 3 \ldots.$$

$$= -0.01959\ (4\ sf)(shown)$$

(iii) For all values of $t \geq 0$, $\quad e^{-0.01959t} > 0$

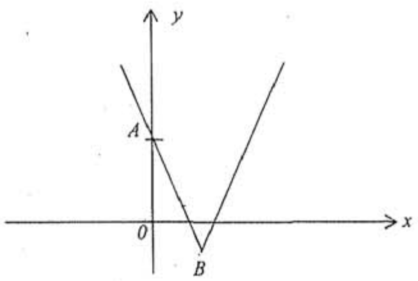
$$-18000e^{-0.01959t} < 0$$

$$20000 - 18000e^{-0.01959t} < 20000$$

$$V < 20000$$

Hence the value of the clock will not exceed \$20000.

9 The diagram shows the graph of $y = |6 - 2x| - 1$.



(i) Find the coordinates of A and of B. [2]

(ii) By solving the equation $|6 - 2x| = 3x + 1$, find the x-coordinate of the point(s) of intersection between the graphs $y = |6 - 2x| - 1$ and $y = 3x$. [3]

(iii) State the range of values of m for the equation $|6 - 2x| = mx + 1$ to have no solution. [2]

9(i) When $x = 0$, $y = |6 - 2(0)| - 1$

$$= 5$$

$A(0, 5)$

At B, y is minimum when $6 - 2x = 0$

$$x = 3$$

$$y = -1$$

$B(3, -1)$

(ii) $|6 - 2x| = 3x + 1$

$6 - 2x = 3x + 1 \quad or \quad 6 - 2x = -(3x + 1)$

$-5x = -5 \qquad\qquad x = -7$ (reject)

$x = 1$

(iii) For $|6 - 2x| = mx + 1$ to have no solutions

$$-2 \leq m < -\frac{1}{3}$$

10 A circle passes through the points $P(0, 8)$ and $Q(8, 12)$. The y-axisis a tangent to the circle at P.

(i) Find the equation of the circle. [5]

The tangent to the circle at Q intersects the x-axis and y-axis at A and B respectively.

(ii) Find the ratio of $AQ : QB$. [3]

10(i) $y -$ coordinate of centre of circle $= 8$

Midpoint of $PQ = \left(\frac{0+8}{2}, \frac{8+12}{2}\right)$

$$= (4, 10)$$

Gradient of $PQ = \frac{12-8}{8-0}$

$$= \frac{1}{2}$$

Gradient of perpendicular bisector of $PQ = -2$

Equation of perpendicular bisector of PQ:

$$y - 10 = -2(x - 4)$$

$$y = -2x + 18$$

Sub $y = 8$, $\quad 8 = -2x + 18$

$$x = 5$$

Centre of the circle is $(5, 8)$.

$Radius^2 = (5 - 0)^2$

$$= 25$$

The equation of the circle is

$$(x - 5)^2 + (y - 8)^2 = 25$$

(ii) Gradient of line from Q to centre of circle$= \frac{12-8}{8-5}$

$$= \frac{4}{3}$$

Equation of tangent at $Q(8, 12)$:

$$y - 12 = -\frac{3}{4}(x - 8)$$

$$y = -\frac{3}{4}x + 18$$

When $y = 0$, $x = 24$

$A(24, 0)$

When $x = 0$, $y = 18$.

$B(0, 18)$

For the points $A(24,0)$, $Q(8,12)$ and $B(0,18)$,

$AQ : QB = 24 - 8 : 8 - 0$ (Comparing difference in x or y-coordinates)

$= 2 : 1$

11 (i) Expand $(1 - 2x)^9$ in ascending powers of x up to the term in x^3. [2]

(ii) Find the value of k, given that the coefficient of x in the expansion of $\left(3x + \frac{1}{kx^2}\right)(1 - 2x)^9$ is -53. [3]

11(i) $(1 - 2x)^9 = \binom{9}{0}(-2x)^0 + \binom{9}{1}(-2x)^1 + \binom{9}{2}(-2x)^2 + \binom{9}{3}(-2x)^3 + \cdots$

$$= 1 - 18x + 144x^2 - 672x^3 + \cdots$$

(ii)

$$\left(3x+\frac{1}{kx^2}\right)(1-2x)^9$$
$$=\left(3x+\frac{1}{kx^2}\right)(1-18x+144x^2-672x^3+\cdots)$$

Term in $x = 3x(1) + \frac{1}{kx^2}(-672x^3)$

coefficient of $x = -53$

$$3-\frac{672}{k}=-53$$
$$k=12$$

12 The equation of a curve is given by $y=\ln\sqrt{\frac{5x}{9x+4}}$.

(i) Find $\frac{dy}{dx}$, expressing it as a single fraction. [3]

(ii) Find the rate at which x is changing when the graph crosses the x-axis, given that y is increasing at a rate of 0.3 units per second. [4]

(i)

$$y=\ln\sqrt{\frac{5x}{9x+4}}$$
$$=\frac{1}{2}[\ln 5x-\ln(9x+4)]$$
$$\frac{dy}{dx}=\frac{1}{2}\left[\frac{5}{5x}-\frac{9}{9x+4}\right]$$
$$=\frac{1}{2}\left[\frac{9x+4}{x(9x+4)}-\frac{9x}{x(9x+4)}\right]$$
$$=\frac{1}{2}\left[\frac{4}{x(9x+4)}\right]$$
$$=\frac{2}{x(9x+4)}$$

(ii)

Let $y=0$, $\quad \ln\sqrt{\frac{5x}{9x+4}}=0$

$$\frac{1}{2}[\ln 5x-\ln(9x+4)]=0$$
$$\ln 5x-\ln(9x+4)=0$$
$$\ln 5x=\ln(9x+4)$$
$$5x=9x+4$$
$$x=-1$$
$$\frac{dy}{dt}=\frac{dy}{dx}\times\frac{dx}{dt}$$
$$0.3=\frac{2}{x(9x+4)}\times\frac{dx}{dt}$$

When $x=-1$, $\frac{dx}{dt}=0.3\div\frac{2}{(-1)(-9+4)}$

$$=\frac{3}{4}$$

x is increasing at a rate of $\frac{3}{4}$ units per second.

13 **Solutions to this question by accurate drawing will not be accepted.**

The diagram, which is not drawn to scale, shows a triangle PQR in which $PQ=QR$.

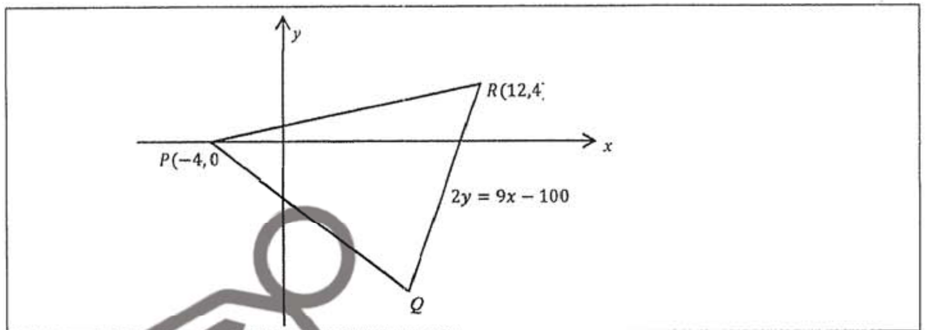
The coordinates of the points P and R are $(-4,0)$ and $(12,4)$ respectively.

(i) Find the equation of the perpendicular bisector of PR. [3]

The equation of the line QR is $2y=9x-100$.

(ii) Find the coordinates of Q. [2]

(iii) Find the coordinates of S if $PQRS$ forms a rhombus. Hence, or otherwise, find the area of rhombus $PQRS$. [4]



(i)

Midpoint of $PR=\left(\frac{-4+12}{2},\frac{0+4}{2}\right)$

$$=(4,2)$$

Gradient of $PR=\frac{4-0}{12-(-4)}$

$$=\frac{1}{4}$$

Gradient of perpendicular bisector of $PR=-4$

Equation of perpendicular bisector of PR:

$$y-2=-4(x-4)$$
$$y=-4x+18$$

(ii)

$y=-4x+18$ ------(1)

$2y=9x-100$-----(2)

Sub (1) into (2): $2(-4x+18)=9x-100$

$$-8x+36=9x-100$$
$$x=8$$
$$y=-14$$

The coordinates of Q are $(8,-14)$.

(iii)

Let the coordinates of S be (x_S,y_S). If $PQRS$ forms a rhombus, then

Midpoint of QS = Midpoint of PR

$$\left(\frac{8+x_S}{2},\frac{-14+y_S}{2}\right)=(4,2)$$
$$\frac{8+x_S}{2}=4$$
$$x_S=0$$
$$\frac{-14+y_S}{2}=2$$
$$y_S=18$$

The coordinates of S are $(0,18)$.

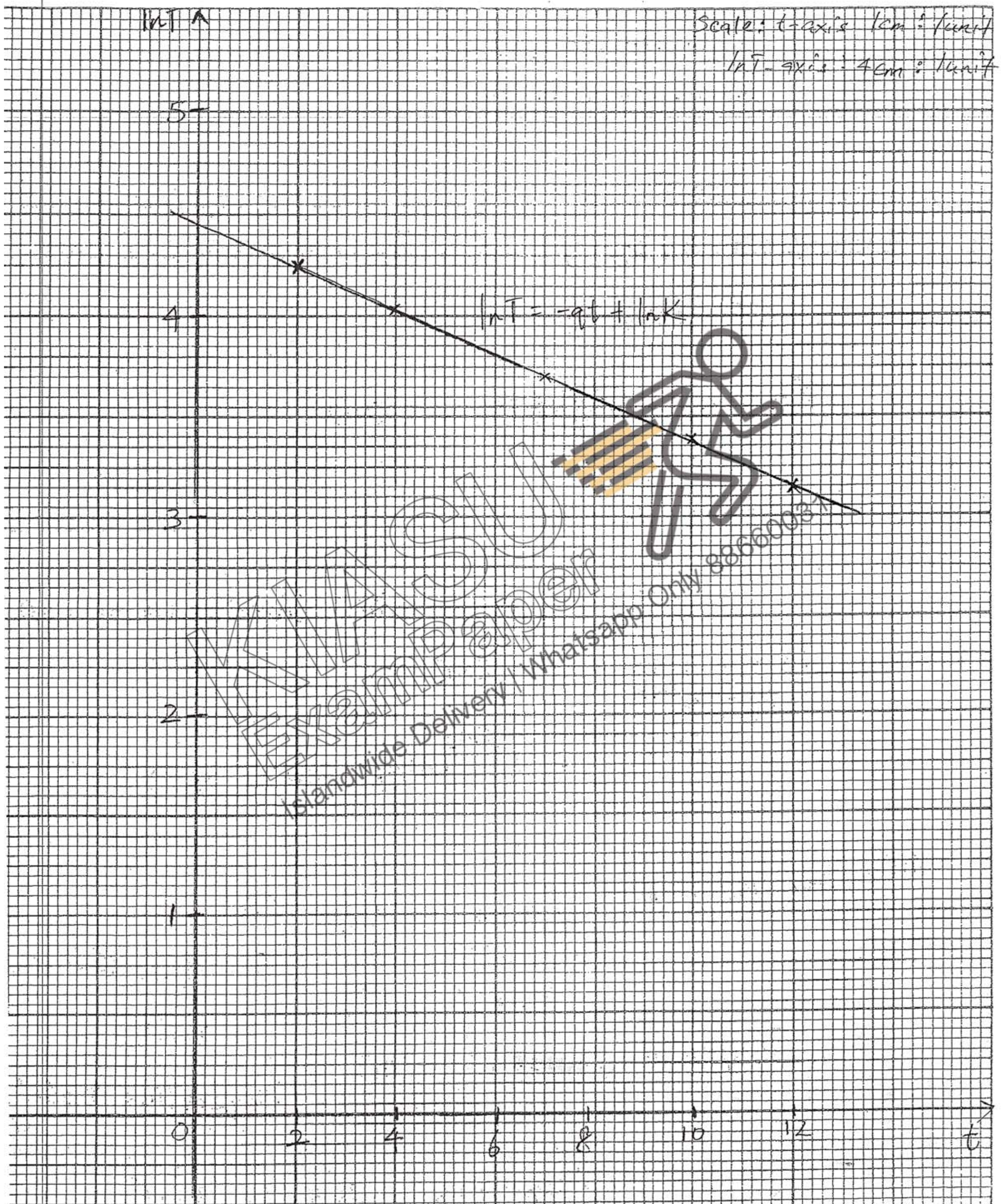
Area of the rhombus $PQRS$

$$=\frac{1}{2}\begin{vmatrix}-4 & 8 & 12 & 0 & -4\\ 0 & -14 & 4 & 18 & 0\end{vmatrix}$$
$$=\frac{1}{2}|(56+32+216+0)-(0-168+0-72)|$$
$$=\frac{1}{2}|544|$$
$$=272 \text{ unit sq}$$

海 星 中 学 Maris Stella High School

姓 名 ______________________ (   )　　科目／Subject: ______________________

(Q4) Name: ______________________　　班级／Class: ______________________



1 The polynomial $f(x)=2x^3+ax^2+bx+8$, where a and b are constants, has a factor $(x+2)$ and leaves a remainder of 10 when divided by $(2x-1)$.

(i) Find the value of a and of b. [4]

(ii) Using the values of a and of b found in part (i), explain why the equation $f(x)=0$ has only one real root. Find this root. [4]

(iii) Hence, solve $x^3+3x^2+4x+32=0$. [2]

1	$f(x)=2x^3+ax^2+bx+8$
(i)	$f(-2)=2(-2)^3+a(-2)^2+b(-2)+8=0$ $4a-2b=8$ --------- Eqn (1)
	$f\left(\frac{1}{2}\right)=2\left(\frac{1}{2}\right)^3+a\left(\frac{1}{2}\right)^2+b\left(\frac{1}{2}\right)+8=10$ $a+2b=7$ ------------ Eqn (2)
	Solving the equations, $b=2$, $a=7-2(2)=3$
(ii	$f(x)=2x^3+3x^2+2x+8$ $=(x+2)(2x^2+bx+4)$ Term in x^2: $3x^2=bx^2+4x^2$, $b=-1$ $f(x)=2x^3+3x^2+2x+8$ $=(x+2)(2x^2-x+4)$ [Getting Quadratic factor by long division also allowed]
	For the factor $2x^2-x+4$, Discriminant $=1-4(2)(4)$ $=-31<0$ Hence, the equation $2x^2-x+4=0$ has no real roots. Therefore $f(x)=0$ has only 1 real root. The root is $x=-2$
(iii	$2x^3+3x^2+2x+32=0$ $2\left(\frac{x}{2}\right)^3+3\left(\frac{x}{2}\right)^2+2\left(\frac{x}{2}\right)+8=0$ $\left(\frac{x}{2}+2\right)\left(2\left(\frac{x}{2}\right)^2-\left(\frac{x}{2}\right)+4\right)=0$ $x=-4$

2019 Mid-Year Sec 4 AM Paper 2 Solution

2 (a) Find the range of values of k for which $(k-3)x^2+4x+k$ is always positive for all real values of x. [4]

(b) Show that the roots of the equation $6x^2+4(m-1)=2(x+m)$ are real if $m\le 2\frac{1}{12}$. [3]

2(a)	$(k-3)x^2+4x+k>0$ for all values of x Discriminant <0 $16-4k(k-3)<0$ $4k^2-12k-16>0$ $k^2-3k-4>0$ $(k-4)(k+1)>0$ $k>4$ or $k<-1$ Since $k-3>0$, $k>3$ $\therefore k>4$
(b)	$6x^2+4(m-1)=2(x+m)$ $6x^2-2x+2m-4=0$ Discriminant $=100-48m$ Since $m\le 2\frac{1}{12}$ $25-12m\ge 0$ $100-48m\ge 0$ Since discriminant ≥ 0, $6x^2+4(m-1)=2(x+m)$ has real roots if $m\le 2\frac{1}{12}$

3 (a) Simplify $\frac{9^{x+1}+18(3^{2x})}{3^{2-x}\times 27^{x+1}}$ without the use of a calculator. [4]

(b) Solve the equation $4^{x+1}=18(2^x)-8$. [4]

(c) Solve the equation $\log_3(2x-1)-\frac{1}{2}\log_3(x^2+2)=\log_{25}5$. [5]

3 (a)

$$\frac{9^{x+1}+18(3^{2x})}{3^{2-x}\times 27^{x+1}}$$

$$=\frac{3^{2(x+1)}+18(3^{2x})}{3^{2-x}\times 3^{3(x+1)}}$$

$$=\frac{3^{2x}(3^2+18)}{3^{2x+5}}$$

$$=\frac{3^{2x}(3^3)}{3^{2x}(3^5)}$$

$$=\frac{1}{3^2}$$

$$=\frac{1}{9}$$

(b) $4^{x+1}=18(2^x)-8$

$4(2^{2x})=18(2^x)-8$

$4(2^x)^2-18(2^x)+8=0$

Let $2^x=A$,

$4A^2-18A+8=0$

$2A^2-9A+4=0$

$(2A-1)(A-4)=0$

$A=\frac{1}{2}$ or $A=4$

$2^x=\frac{1}{2}$ $\quad 2^x=4$

$x=-1$ or $x=2$

(c) $\log_3(2x-1)-\frac{1}{2}\log_3(x^2+2)=\log_{25}5$

$2\log_3(2x-1)-\log_3(x^2+2)=1$

$\log_3\left(\frac{(2x-1)^2}{x^2+2}\right)=1$

$(2x-1)^2=3(x^2+2)$

$x^2-4x-5=0$

$(x-5)(x+1)=0$

$x=5$ or $x=-1$ (rej)

4 In a Science experiment, a container of liquid was heated to a temperature of K °C.

It was then left to cool in a chiller such that its temperature, T °C, t minutes after removing the heat, is given by $T=Ke^{-qt}$, where q is a constant.

Measured values of t and T are given in the following table.

t (minutes)	2	4	7	10	12
T °C	71.1	57.0	40.8	29.3	23.4

(i) Using a scale of 1 cm to 1 unit on the t-axis and 4 cm to 1 unit on the ln$T$- axis, plot ln T against t and draw a straight line graph. [2]

ln T	4.26	4.04	3.71	3.38	3.15
t	2	4	7	10	12

(ii) Use the graph to estimate the value of K and of q. [4]

(iii) Estimate the temperature of the liquid 8 minutes after it was left to cool. [2]

4(i) Plot a straight line passing all the points with correct scale etc

(ii) $T=Ke^{-qt}$

$\ln T=-qt+\ln K$

$$\text{Gradient}=\frac{3.15-4.45}{12-0}$$

$$=-\frac{13}{120}$$

$$-q=-\frac{13}{120}$$

$$q\approx\frac{13}{120}$$

$\ln K=4.475$

$K=e^{4.475}$

≈ 87.8

(iii) $T=87.8e^{-\frac{13}{120}(8)}$

≈ 36.9

Temperature is 36.9°.

Alternatively from graph,

$t=8, \ln T=3.6$

$T=e^{3.6}\approx 36.6$

Temperature is 36.6°.

5 (a) (i) Prove that $\frac{\sin x}{\sec x+1}+\frac{\sin x}{\sec x-1}=2\cot x$. [4]

(ii) Hence find, for $0 \le x \le 4$, the exact solution of the equation
$$\frac{\sin x}{\sec x+1}+\frac{\sin x}{\sec x-1}=\frac{2\tan x}{3}.$$ [3]

(b) Given that θ is obtuse and that $\sin\theta=\frac{1}{\sqrt{3}}$, express, **without the use of a calculator**, $\frac{1}{\sin\theta-\cos\theta}$ in the form $\sqrt{a}-\sqrt{b}$ where a and b are integers. [4]

5(a)(i)	$LHS=\frac{\sin x}{\sec x+1}+\frac{\sin x}{\sec x-1}$ $=\frac{\sin x(\sec x-1)+\sin x(\sec x+1)}{\sec^2 x-1}$ $=\frac{\tan x-\sin x+\tan x+\sin x}{\sec^2 x-1}$ $=\frac{2\tan x}{\sec^2 x-1}$ $=\frac{2\tan x}{\tan^2 x}$ $=2\cot x$ $=RHS$ (proved)
(ii)	$\frac{\sin x}{\sec x+1}+\frac{\sin x}{\sec x-1}=\frac{2\tan x}{3}$ $2\cot x=\frac{2\tan x}{3}$ $\tan^2 x=3$ $\tan x=\pm\sqrt{3}$ Basic angle $=\frac{\pi}{3}$ For $0\le x\le 4$, $x=\frac{\pi}{3},\frac{2\pi}{3}$

5b.	$1^2+x^2=\left(\sqrt{3}\right)^2$
	$x=\sqrt{2}$
	$\cos\theta=-\frac{\sqrt{2}}{\sqrt{3}}$
	$\frac{1}{\sin\theta-\cos\theta}=\frac{1}{\frac{1}{\sqrt{3}}+\frac{\sqrt{2}}{\sqrt{3}}}$ $=\frac{\sqrt{3}}{1+\sqrt{2}}\times\frac{1-\sqrt{2}}{1-\sqrt{2}}$ $=\frac{\sqrt{3}-\sqrt{6}}{-1}$ $=\sqrt{6}-\sqrt{3}$

6 The equation of a curve is $y=\frac{a}{x}+bx-1$, where a and b are constants. The normal to the curve at the point $Q(1,-1)$ is parallel to the line $4y-x=20$. This normal meets the curve again at point P.

(i) Find the value of a and of b. [5]

(ii) Find the coordinates of point P. [3]

6 (i)	Equation of line : $y=\frac{1}{4}x+5$ At $x=1$, gradient of normal $=\frac{1}{4}$ Gradient of tangent $=-4$ $\frac{dy}{dx}=-\frac{a}{x^2}+b$ $-a+b=-4$ ------ Eqn (1) sub $(1,-1)$ into $y=\frac{a}{x}+bx-1$ $a+b=0$ --------- Eqn (2) Solving : $a=2, b=-2$

(ii)	$y=\frac{2}{x}-2x-1$ Equation of normal is : $y+1=\frac{1}{4}(x-1)$ $y=\frac{1}{4}x-\frac{5}{4}$ $\frac{2}{x}-2x-1=\frac{1}{4}x-\frac{5}{4}$ $8-8x^2-4x=x^2-5x$ $9x^2-x-8=0$ $(9x+8)(x-1)=0$ $x=-\frac{8}{9}$ or $x=1$ $y=2\left(-\frac{9}{8}\right)-2\left(-\frac{8}{9}\right)-1$ $=-\frac{53}{36}$ Coordinates of P is $\left(-\frac{8}{9},-\frac{53}{36}\right)$

7 The equation of a curve is $y=\frac{x^2}{x-1}$, where $x\neq 1$.

(i) Obtain an expression for $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$. [4]

(ii) Find the coordinates of the stationary points of the curve and determine their nature. [4]

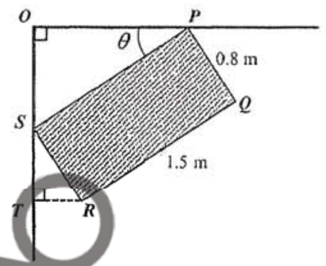
7(i)	$y=\frac{x^2}{x-1}$ $\frac{dy}{dx}=\frac{(x-1)(2x)-x^2(1)}{(x-1)^2}$ $=\frac{x^2-2x}{(x-1)^2}$ $\frac{d^2y}{dx^2}=\frac{(x-1)^2(2x-2)-2(x-1)(x^2-2x)}{(x-1)^4}$ $=\frac{(x-1)(2x^2-4x+2-2x^2+4x)}{(x-1)^4}$ $=\frac{2}{(x-1)^3}$
(ii)	For stationary point, $\frac{dy}{dx}=0$ $\frac{x^2-2x}{(x-1)^2}=0$ $x^2-2x=0$ $x(x-2)=0$ $x=0$ or 2 when $x=0, y=0$ $\frac{d^2y}{dx^2}=\frac{2}{(0-1)^3}$ $=-2<0$ $(0,0)$ is a maximum point. when $x=2, y=4$ $\frac{d^2y}{dx^2}=\frac{2}{(2-1)^3}$ $=2>0$ $(2,4)$ is a minimum point.

8 **(a)** Differentiate $\cot^4\left(\frac{\pi}{2}-2x\right)$ with respect to x. [3]

(b) Given that the curve has the equation $y=3\sin 2x-\cos x$, find the gradient of the curve when $x=\frac{\pi}{3}$, leaving your answer in exact form. [3]

8 (a)	Let $y=\cot^4\left(\frac{\pi}{2}-2x\right)$
	$y=\dfrac{1}{\tan^4\left(\frac{\pi}{2}-2x\right)}$ $=\dfrac{1}{\cot^4(2x)}$ $=\tan^4 2x$ $\frac{dy}{dx}=4\tan^3(2x)\left[2\sec^2(2x)\right]$ $=8\tan^3(2x)\sec^2(2x)$
	OR $y=\dfrac{1}{\tan^4\left(\frac{\pi}{2}-2x\right)}$ $=\tan^{-4}\left(\frac{\pi}{2}-2x\right)$ $\frac{dy}{dx}=-4\tan^{-5}\left(\frac{\pi}{2}-2x\right)\left[-2\sec^2\left(\frac{\pi}{2}-2x\right)\right]$ $=8\tan^{-5}\left(\frac{\pi}{2}-2x\right)\sec^2\left(\frac{\pi}{2}-2x\right)$
8(b)	$y=3\sin 2x-\cos x$,
	$\frac{dy}{dx}=6\cos 2x+\sin x$ At $x=\frac{\pi}{3}$, Gradient $=6\cos\frac{2\pi}{3}+\sin\frac{\pi}{3}$ $=-3+\frac{\sqrt{3}}{2}$

9



The diagram shows the top view of a rectangular desk, $PQRS$, in a corner of a room. The desk has a length of 1.5 m and width 0.8 m, $\angle POS=\angle STR=90^\circ$ and $\angle OPS=\theta$.

(i) Show that $OT=(1.5\sin\theta+0.8\cos\theta)$ m. [3]

(ii) Express OT in the form $R\sin(\theta+\alpha)$, where $R>0$ and α is acute. [3]

(iii) Given that θ can vary, find the maximum value of OT and the corresponding value of θ. [3]

9(i)	$\angle TSR=\theta$, $\cos\theta=\frac{ST}{0.8}$ $ST=0.8\cos\theta$ $\sin\theta=\frac{OS}{1.5}$ $OS=1.5\sin\theta$ $OT=OS+ST$ $=(1.5\sin\theta+0.8\cos\theta)$ m (Shown)
(ii)	$OT=1.5\sin\theta+0.8\cos\theta=R\sin(\theta+\alpha)$ where $R=\sqrt{1.5^2+0.8^2}=1.7$ $\tan\alpha=\frac{0.8}{1.5},\Rightarrow\alpha=28.072^\circ$ $\therefore\ OT=1.7\sin(\theta+28.1^\circ)$ (correct to 1 d.p.)
(iii)	Maximum value of $OT=1.7m$ when $\sin(\theta+28.072^\circ)=1$ $\theta+28.072^\circ=90^\circ$ $\theta=61.9^\circ$ (1 dp)