NCHS Sec 4 Mid-Year Exam 2019
Additional Mathematics Paper 1

1

$\sqrt{64^x} = \frac{8^{x+1}}{32^{1-2x}}$

$2^{6x} = \left(\frac{2^{3x+3}}{2^{5-10x}}\right)^2$

$2^{6x} = \frac{2^{6x+6}}{2^{10-20x}}$

$6x = 6x + 6 - (10 - 20x)$

$20x = 4$

$x = \frac{1}{5}$ or 0.2

2

$(\tan x + \sec x)^2$

$= \left(\frac{\sin x}{\cos x} + \frac{1}{\cos x}\right)^2$

$= \left(\frac{\sin x + 1}{\cos x}\right)^2$

$= \frac{(\sin x + 1)^2}{\cos^2 x}$

$= \frac{(\sin x + 1)^2}{1 - \sin^2 x}$

$= \frac{(\sin x + 1)^2}{(1 - \sin x)(1 + \sin x)}$

$= \frac{1 + \sin x}{1 - \sin x}$

3i

$y = \frac{2x^2}{1-3x}$

$\frac{dy}{dx} = \frac{4x(1-3x) - 2x^2(-3)}{(1-3x)^2}$

$= \frac{4x - 12x^2 + 6x^2}{(1-3x)^2}$

$= \frac{4x - 6x^2}{(1-3x)^2}$

$= \frac{2x(2-3x)}{(1-3x)^2}$

3ii

$\frac{dx}{dt} = 0.05$ unit/s , $x = 3$

$\frac{dy}{dt} = \frac{dy}{dx} \times \frac{dx}{dt}$

$= \frac{2(3)(2-9)}{(1-9)^2} \times 0.05$

$= -\frac{21}{640}$ or -0.0328 units/s

4

$\frac{-4x^3 + 11x^2 - 16x + 9}{x(2x-1)(x^2+3)} = \frac{A}{x} + \frac{B}{2x-1} + \frac{Cx+D}{x^2+3}$

$-4x^3 + 11x^2 - 16x + 9$

$= A(2x-1)(x^2+3) + Bx(x^2+3) + x(Cx+D)(2x-1)$

Let $x = 0$, $9 = -3A$

$A = -3$

Let $x = \frac{1}{2}$, $\frac{13}{8}B = \frac{13}{4}$

$B = 2$

Compare x^3; $-4 = 2A + B + 2C$

$C = 0$

Let $x = 1$, $D = 4$

$\therefore$ $\frac{-3}{x} + \frac{2}{2x-1} + \frac{4}{x^2+3}$

5i

$\left(\frac{\sqrt{5}+\sqrt{3}}{\sqrt{5}-\sqrt{3}}\right)^2$

$= \frac{5 + 2\sqrt{15} + 3}{5 - 2\sqrt{15} + 3}$

$= \frac{8 + 2\sqrt{15}}{8 - 2\sqrt{15}} \times \frac{8 + 2\sqrt{15}}{8 + 2\sqrt{15}}$

$= \frac{64 + 32\sqrt{15} + 60}{64 - 60}$

$= \frac{124 + 32\sqrt{15}}{4} = 31 + 8\sqrt{15}$

5ii

$2x^2 - px + 8 < 0$

$2[x - (\sqrt{3} - 1)](x - k) < 0$

$2x^2 - 2kx - 2\sqrt{3}x + 2x + 2\sqrt{3}k - 2k < 0$

$2x^2 - (2k + 2\sqrt{3} - 2)x + 2\sqrt{3}k - 2k < 0$

Compare with $2x^2 - px + 8 < 0$,

$2\sqrt{3}k - 2k = 8$

$k(2)(\sqrt{3} - 1) = 8$

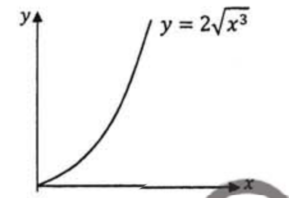
$k = \frac{4}{\sqrt{3}-1} \times \frac{\sqrt{3}+1}{\sqrt{3}+1} = 2\sqrt{3} + 2$

$p = (2k + 2\sqrt{3} - 2)$

$= 2(2\sqrt{3} + 2) + 2\sqrt{3} - 2$

$= 6\sqrt{3} + 2$

6i



6ii

$2\sqrt{x^3} = -2x + 4$

Square both sides,

$4x^3 = (-2x + 4)^2$

$4x^3 = 4x^2 - 16x + 16$

$4x^3 - 4x^2 + 16x - 16 = 0$

Let $f(x) = 4x^3 - 4x^2 + 16x - 16$

$f(1) = 4 - 4 + 16 - 16 = 0$

$x - 1$ is a factor of $f(x)$

By inspection, $f(x) = (x - 1)(4x^2 + bx + 16)$ where b is a constant.

Compare x^2, $-4 + b = -4$

$b = 0$

$4x^3 - 4x^2 + 16x - 16 = (x - 1)(4x^2 + 16) = 0$

$x - 1 = 0$ or $4x^2 + 16 = 0$

$x = 1$ (rejected)

When $x = 1$, $y = -2 + 4 = 2$

Coordinates $= (1, 2)$

7i

$10^{\log_5 x} = 5$

$\log_5 x = \log_{10} 5$

$x = 5^{\lg 5}$

$= 3.08$

7ii

$\log_2(6 - x) - \log_2(x - 2) = 3 - \log_2(2x + 1)$

$\log_2(6 - x) - \log_2(x - 2) + \log_2(2x + 1) = 3$

$\log_2 \frac{(6-x)}{(x-2)} \times (2x + 1) = 3$

$\frac{(6-x)(2x+1)}{(x-2)} = 2^3$

$12x + 6 - 2x^2 - x = 8(x - 2)$

$2x^2 - 3x - 22 = 0$

$x = \frac{3 \pm \sqrt{(-3)^2 - 4(2)(-22)}}{2(2)}$

$= \frac{3+\sqrt{185}}{4}$ (4.15) or $\frac{3-\sqrt{185}}{4}$ (−2.65)(reject)

8

$2x^2 - x - 10 = 0$

$(x + 2)(2x - 5) = 0$

$x = -2$ or 2.5

$\frac{d^2y}{dx^2} = 4x - 1$

$\frac{d^2y}{dx^2} = 4(2.5) - 1 = 9 > 0$, min y when x = 2.5

$\frac{d^2y}{dx^2} = 4(-2) - 1 = -9 < 0$, max y when x = −2

$y = \int 2x^2 - x - 10 \, dx$

$y = \frac{2x^3}{3} - \frac{x^2}{2} - 10x + c$

$13 = \frac{2(-2)^3}{3} - \frac{(-2)^2}{2} - 10(-2) + c$

$c = \frac{1}{3}$

$y = \frac{2}{3}x^3 - \frac{1}{2}x^2 - 10x + \frac{1}{3}$

9i

$\frac{dy}{dx} = \frac{1}{9}\left(\frac{1}{3x-2}\right)(3) - 2$

$= -\frac{1}{3(3x-2)} - 2$

Since $3x - 2 > 0$, $-\frac{1}{3(3x-2)} < 0$,

$\frac{dy}{dx} < 0$ or $-\frac{1}{3(3x-2)} - 2 < 0$

Therefore y is decreasing for $x > \frac{2}{3}$.

9ii

$\frac{d^2y}{dx^2} = -\frac{1}{3}(-1)(3x - 2)^{-2}(3)$

$= \frac{1}{(3x-2)^2}$

$\frac{1}{(3x-2)^2} > 0$

Therefore $x > \frac{2}{3}$.

10i

$y_p = -1 + \frac{10}{5}(3) = 5$

$5 = 3x + 2$

$x = 1$

P (1, 5)

10ii

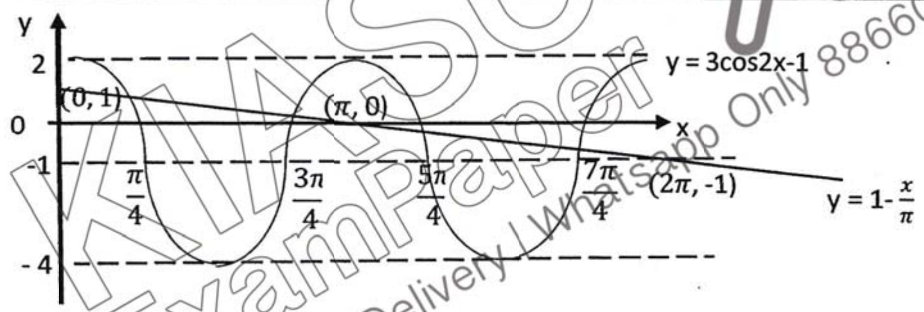
$k = \frac{1-(-5)}{2}(5) - 5 = 10$

$\left(\frac{9-h}{-5-2}\right)\left(\frac{-1-h}{10-2}\right) = -1$

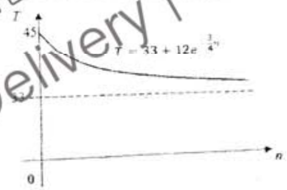
$(9 - h)(1 + h) = -56$

$h^2 - 8h - 65 = 0$

$h = 13\ (rej)\ or -5$

10iii	$A = \frac{1}{2}\begin{vmatrix} -5 & 2 & 10 & -5 \\ 9 & -5 & -1 & 9 \end{vmatrix}$ $= 70$	11iib	$0 = -3\lvert 4x-3\rvert + 5$ $\lvert 4x-3\rvert = \frac{5}{3}$ $4x - 3 = \frac{5}{3}$ or $4x - 3 = -\frac{5}{3}$ $x = \frac{7}{6}$ or $x = \frac{1}{3}$ $\left(\frac{7}{6}, 0\right)$ and $\left(\frac{1}{3}, 0\right)$
11i	$(2x-3)^2 > 49$ $x^2 - 3x - 10 > 0$ $(x-5)(x+2) > 0$ $x < -2$ or $x > 5$ OR $2x - 3 > 7$ or $2x - 3 < -7$ $x > 5$ or $x < -2$	12i	$y = 2cos^2x - 4(1 - cos^2x)$ $y = 6cos^2x - 4$ $y = 3(cos2x + 1) - 4$ $y = 3cos2x - 1$
11iia	$y = a\lvert 4x-3\rvert + 5$ $-4 = a\lvert -3\rvert + 5$ $a = -3$, $b = 4$, $c = 5$	12ii	Period $= \pi$ Amplitude $= 3$
12iii			
12iv	$y = 3cos2x - 1$ $\frac{x}{\pi} = 2 - 3cos2x$ $3\,cos2x = 2 - \frac{x}{\pi}$ $3\,cos2x - 1 = 1 - \frac{x}{\pi}$ $y = 1 - \frac{x}{\pi}$		

ICHS Sec 4 Mid-Year Exam 2019
Additional Mathematics Paper 2

1ai	$-90^\circ < tan^{-1}x < 90^\circ$ or $-\frac{\pi}{2} < tan^{-1}x < \frac{\pi}{2}$
1aii	$0^\circ \le cos^{-1}x \le 180^\circ$ or $0 \le cos^{-1}x \le \pi$
1b	$\tan 105^\circ$ $= \tan(60^\circ + 45^\circ)$ $= \frac{tan60^\circ + tan45^\circ}{1 - tan60^\circ tan45^\circ}$ $= \frac{\sqrt{3}+1}{1-\sqrt{3}} \times \frac{1+\sqrt{3}}{1+\sqrt{3}}$ $= \frac{\sqrt{3}+3+1+\sqrt{3}}{1-3}$ $= \frac{2(\sqrt{3}+2)}{-2}$ $= -\sqrt{3} - 2$
2	$\frac{d^2y}{dx^2} = \frac{36}{(1-2x)^3}$ $\frac{dy}{dx} = \int 36(1-2x)^{-3}\,dx$ $= \frac{36(1-2x)^{-2}}{(-2)(-2)} + c$ $= \frac{9}{(1-2x)^2} + c$ Sub $x = -1, \frac{dy}{dx} = \frac{1}{2}$, $c = -\frac{1}{2}$ $\frac{dy}{dx} = \frac{9}{(1-2x)^2} - \frac{1}{2}$ $y = \int 9(1-2x)^{-2} - \frac{1}{2}dx$ $y = \frac{9}{2(1-2x)} - \frac{1}{2}x + d$ Sub $x = -1, y = 3$, $3 = \frac{9}{2(3)} + \frac{1}{2} + d$ $d = 1$ $y = \frac{9}{2(1-2x)} - \frac{1}{2}x + 1$
3	$\alpha + \beta = -m$ $\alpha\beta = n$ $(\alpha+\beta)(\alpha-\beta) = \alpha^2 - \beta^2$ $(\alpha+\beta)(-1) = 13$ $\therefore m = 13$ $2\beta^2 = 72$ $\beta = 6$ or -6 Since $\alpha - \beta = -1$, $\alpha = 5$ (rejected) or $\alpha = -7$ $n = \alpha\beta = (-7)(-6) = 42$
4	$y = (k-2)x^2 - (k+1)x + (k-2)$ $b^2 - 4ac < 0$ $[-(k+1)]^2 - 4(k-2)(k-2) < 0$ $k^2 + 2k + 1 - 4(k^2 - 4k + 4) < 0$ $-3k^2 + 18k - 15 < 0$ $-k^2 + 6k - 5 < 0$ $(k-5)(k-1) > 0$ $k < 1$ or $5 < k$ $k - 2 > 0$ $k > 2$ Answer: $\therefore k > 5$
5i	Sub $n = 0$, $T_0 = 45^\circ$
5ii	$37 = 33 + 12e^{-\frac{3}{4}n}$ $n = 1.46$
5iii	$\frac{dT}{dn} = 12(-\frac{3}{4})e^{-\frac{3}{4}}$ $-0.67 = -9e^{-\frac{3}{4}n}$ $n = 3.46$
5iv	$12e^{-\frac{3}{4}n} > 0$ $33 + 12e^{-\frac{3}{4}n} > 33$ $T > 33^\circ C$ (shown)
5v	
6i	$f(-2) = 0$ $-8a - 2b + 10 = 0$ $8a + 2b = 10$ $4a + b = 5$ ------ {1} $f(1) = 18$ $a + b = 11$ ----- {2} {1} - {2} : $3a = -6$ $a = -2$ $-2 + b = 11$ $b = 13$
6ii	$f(x) = -2x^3 + x^2 + 13x + 6$ $= (x+2)(-2x^2 + 5x + 3)$ $= (x+2)(3-x)(2x+1)$
6iii	Let $x = y + 1$ $-(y - 1 + 2)(y - 1 + 3)(2(y - 1) + 1) = 0$ $-(y+1)(y-4)(2y-1) = 0$ $y = -1, 4, 0.5$
7i	$\frac{d}{dx}(sin^3 2x) = 3sin^2 2x(2cos2x)$ $\frac{d}{dx}(sin^3 2x) = 6sin^2 2x cos2x$
7iia	$\int_0^{\frac{\pi}{8}} sin^2 2x cos2x\,dx$ $= \frac{1}{6}\int_0^{\frac{\pi}{8}} \frac{d}{dx}(sin^3 2x)dx$ $= \frac{1}{6}[sin^3 2x]_0^{\frac{\pi}{8}} = 0.0589$ (3sf)
7iib	$\int_0^{\frac{\pi}{8}} cos^3 2x\,dx$ $= \int_0^{\frac{\pi}{8}} cos^2 2x cos2x\,dx$ $= \int_0^{\frac{\pi}{8}} (1 - sin^2 2x)cos2x\,dx$ $= \int_0^{\frac{\pi}{8}} cos2x\,dx - \frac{1}{6}\int_0^{\frac{\pi}{8}} 6\,sin^2 2x cos2x\,dx$ $= [\frac{1}{2}sin2x - \frac{1}{6}sin^3 2x]_0^{\frac{\pi}{8}}$ $= 0.295$ (3sf)
8i	$\binom{n}{2}(3)^{n-2}(5x)^2 = \frac{n(n-1)}{2}3^{n-2}(25x^2)$ $\binom{n}{3}(3)^{n-3}(5x)^3 = \frac{n(n-1)(n-2)}{6}3^{n-3}(125x^3)$ $\frac{\frac{n(n-1)}{2}3^{n-2}(25x^2)}{\frac{n(n-1)(n-2)}{6}3^{n-3}(125x^3)} = \frac{3}{10}$ $\frac{3}{\frac{5(n-2)}{3}} = \frac{3}{10}$ $30 = 5(n-2)$ $6 = n - 2$ $n = 8$
8ii	$(3+5x)^8(1-\frac{2}{x})^2$ $= (3+5x)^8(1 - \frac{4}{x} + \frac{4}{x^2})$ x term : $\binom{8}{1}(3)^7(5x)$ $= 87480\,x$ x^2 term: $\binom{8}{2}(3)^6(5x)^2$ $= 510300\,x^2$ Term independent of x $= 3^8(1) - 4\,(87480) + 4\,(510300)$ $= 1697841$

9i

$angle\ DEC = \theta$ (angles in same segment)

$15\cos\theta + 12\sin\theta$

$= \sqrt{15^2 + 12^2}\cos\left(\theta - \tan^{-1}\left(\frac{12}{15}\right)\right)$

$= \sqrt{369}\cos(\theta - 38.65981°)$

$= 3\sqrt{41}\cos(\theta - 38.7°)$

$= 19.2\cos(\theta - 38.7°)$

9ii

$16.5 = 3\sqrt{41}\cos(\theta - 38.65981)$

$\cos(\theta - 38.65981) = 0.85896$

Basic angle $= 30.80°$

$\theta = -30.80°, 69.46028°$

$= 7.9°, 69.5°$

Full mark was given even though students missed out 7.9°.

9iii

$-1 \le \cos(\theta - 38.65981) \le 1$

Max value $= 3\sqrt{41}$

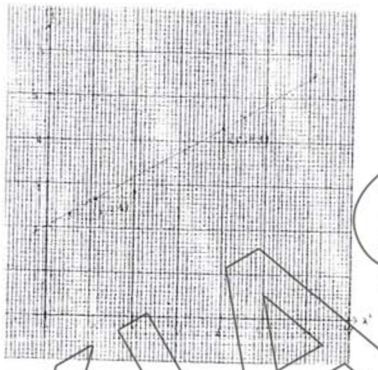
$\cos(\theta - 38.65981) = 1$

$\theta - 38.65981 = 0$

$\theta = 38.7°$

10i

x^2	1	2.25	4	6.25	9	5.50
$\sqrt{y}$	2.29	2.64	2.81	3.78	4.57	12.25



10ii

$b\sqrt{y} = ab + ax^2$,

$\sqrt{y} = \frac{a}{b}x^2 + a$

$m = \frac{4.3-2.6}{8-2} = \frac{17}{60} = 0.28333 = \frac{a}{b}$

Y-intercept $= a = 2$

$\frac{17}{60} = \frac{2}{b}$

$b = 7.0588$

$= 7.06$

Accept 6.9 - 7.1

10iii

Abnormal reading when $x^2 = 4$, $\sqrt{y} = 2.81$

Correct $\sqrt{y}$ should be 3.15, $y = 9.92$ (3.s.f)

Accept $\sqrt{y}$ - 3.05 – 3.15

Accept y = 9.3025 – 10.5625

11a

$y = 5x^2e^{-3x}$ ………(1)

$\frac{dy}{dx} = 10xe^{-3x} + 5x^2e^{-3x}(-3)$

$= 5xe^{-3x}(2 - 3x)$

At stationary point, $\frac{dy}{dx} = 0$

$5xe^{-3x}(2 - 3x) = 0$

$x = 0$, $2 - 3x = 0$ or $e^{-3x} = 0$ (rejected)

$x = 0$, $x = \frac{2}{3}$

Sub $x = 0$ into (1),

$y = 0$

Sub $x = \frac{2}{3}$ into (1),

$y = \frac{20}{9e^2}$

x	-0.1	0	0.1
Sign of $\frac{dy}{dx}$	–	0	+
Sketch of tangent			

(0, 0) is a minimum point

x	0.57	0.67	0.77
Sign of $\frac{dy}{dx}$	+	0	–
Sketch of tangent			

$\left(\frac{2}{3}, \frac{20}{9e^2}\right)$ is a maximum point

11b

$y = \frac{x^3+2}{x^2}$ ………(1)

$= x + 2x^{-2}$

$\frac{dy}{dx} = 1 - \frac{4}{x^3}$

Equation of normal: $y + \frac{27}{23}x = k$

$y = -\frac{27}{23}x + k$ ………(2)

Gradient of normal $= -\frac{27}{23}$

Gradient of tangent $= \frac{23}{27}$

$\frac{23}{27} = 1 - \frac{4}{x^3}$

$x = 3$

Sub $x = 3$ into (1),

$y = 3\frac{2}{9}$

Sub $(3, 3\frac{2}{9})$ into (2)

$3\frac{2}{9} = -\frac{27}{23}(3) + k$

$k = \frac{1396}{207}$ *or* $6\frac{154}{207}$

12i

$2x^2 - 3x + 2y^2 - \frac{1}{2}(4y - 3) = 0$

$x^2 + y^2 - 1.5x - y + 0.75 = 0$

Centre $= \left(\frac{-1.5}{-2}, \frac{-1}{-2}\right) = (0.75, 0.5)$

Radius $= \sqrt{0.75^2 + 0.5^2 - 0.75}$

$= 0.25$

12ii

Distance of P from centre

$= \sqrt{(0.75 + 1)^2 + (0.5 - 2)^2}$

$= 2.3049 >$ radius

Hence, P lies outside the circle.

12iii

$y = x + 3$

Gradient = 1

Perpendicular gradient = -1

Equation of the line joining the two centres:

$y - 0.5 = -(x - 0.75)$

$y = -x + 1.25$ …… (1)

$y = x + 3$ ………… (2)

Sub (2) into (1),

$x + 3 = -x + 1.25$

$x = -\frac{7}{8}$

Sub $x = -\frac{7}{8}$ into (2).

$y = 2\frac{1}{8}$

Let centre of C_2 be (x, y),

$\left(-\frac{7}{8}, 2\frac{1}{8}\right) = \left(\frac{x+0.75}{2}, \frac{y+0.5}{2}\right)$

$x = -2.5$

$y = 3\frac{3}{4}$

Equation of C_2:

$(x + 2.5)^2 + \left(y - 3\frac{3}{4}\right)^2 = \frac{1}{16}$