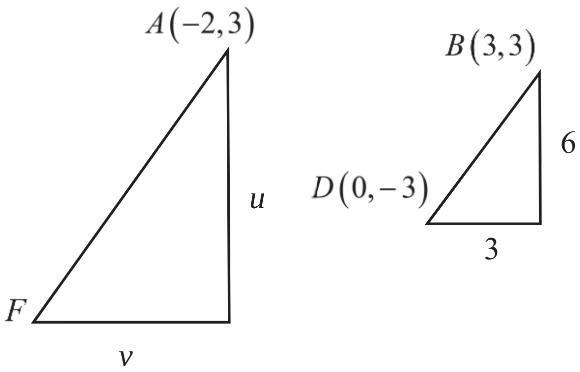
AM-2019-AHS-Prelim-P1 – Marking Scheme

1(i)	$T = A\,e^{-kt} + T_a$ $t = 0\ ,\ T_a = 31,\ T = 81$ $81 = A\,e^{-k\times 0} + 31$ $A = 50$ $t = 10\ ,\ T_a = 31,\ T = 51$ $51 = 50\,e^{-k\times 10} + 31$ $e^{-10k} = \frac{20}{50}$ $k = -\frac{1}{10}\times\ln\left(\frac{2}{5}\right)$ $= 0.091629$ $= 0.0916$
(ii)	$T = 50\,e^{-0.091629t} + 31$ $35 = 50\,e^{-0.091629t} + 31$ $e^{-0.091629t} = \frac{4}{50}$ $-0.091629t = \ln\left(\frac{2}{25}\right)$ $t = \frac{1}{-0.091629}\times\ln\left(\frac{2}{25}\right)$ $= 27.565$ $= 27.6$ Rashid must wait for at least 27.6 minutes before he can drink the soup.

2(i)	$BD = \sqrt{45}$ $\sqrt{(3-0)^2 + (k-(-3))^2} = \sqrt{45}$ $9 + k^2 + 6k + 9 = 45$ $k^2 + 6k - 27 = 0$ $(k+9)(k-3) = 0$ $k = -9(\text{NA}),$ $k = 3$
(ii)	$A(-2,3),\ B(3,3),\ D(0,-3)$



$$\frac{AF}{BD}=\frac{3}{2}$$
$$\frac{u}{6}=\frac{3}{2}$$
$$u=9$$

$$\frac{AF}{BD}=\frac{3}{2}$$
$$\frac{v}{3}=\frac{3}{2}$$
$$v=\frac{9}{2}$$

Coordinates of $F=\left(-2-\frac{9}{2},\ 3-9\right)=\left(-\frac{13}{2},-6\right)$

(iii)

Gradient of $BCDE=\dfrac{3-(-3)}{3-0}=2$

Equation of line $BCDE$ is $y=2x-3$

Gradient of $EF=-\dfrac{1}{2}$

Equation of line EF

$$y-(-6)=-\frac{1}{2}\left(x-\left(-\frac{13}{2}\right)\right)$$
$$y=-\frac{1}{2}x-\frac{37}{4}$$

Solving the pair of simultaneous equations

$$2x-3=-\frac{1}{2}x-\frac{37}{4}$$
$$\frac{5}{2}x=-\frac{37}{4}+3$$
$$x=-\frac{5}{2}$$

$$y=2\left(-\frac{5}{2}\right)-3$$
$$=-8$$

Coordinates of $E=\left(-\frac{5}{2},-8\right)$

3(i)	$\angle CAB = \angle CDA$ (Alternate Segment Theorem) And $\angle BDA = \angle CDA$ (same angle) $\angle ABC = \angle ABD$ (Common angle) Triangle *ABD* is similar to triangle *CBA*. (AA)
(ii)	$\angle CAB = \angle CDA$ (Alternate Segment Theorem) $\angle CAB = \angle ACH$ (Alternate angles, *GAB*//*FEHC*) Hence $\angle ACH = \angle CDA$ $\angle HAC = \angle DAC$ (Common angle) Triangle *ACH* is similar to triangle *ADC*. (AA)
(iii)	From (ii), $\angle ACH = \angle CDA$ $\angle ACH = \angle ADE$ (Angles in the same segment) Hence $\angle ADE = \angle CDA$ Therefore *AD* bisects angle *CDE*.
(iv)	Triangle *ABD* is similar to triangle *CBA*. $\frac{AB}{BC} = \frac{AD}{AC}$ Triangle *ACH* is similar to triangle *ADC*. $\frac{AC}{AH} = \frac{AD}{AC}$ Hence $\frac{AB}{BC} = \frac{AC}{AH}$ $AB \times AH = AC \times BC$

4(i)	$12\sin\theta\cos\theta - 8\cos^2\theta + 7 = 6(2\sin\theta\cos\theta) - 8\cos^2\theta + 7$ $= 6\sin 2\theta - 8\left(\frac{\cos 2\theta + 1}{2}\right) + 7$ $= 6\sin 2\theta - 4\cos 2\theta + 3$
(ii)	$6\sin 2\theta - 8\cos^2\theta + 7 = 0$ $6\sin 2\theta - 4\cos 2\theta + 3 = 0$ Let $6\sin 2\theta - 4\cos 2\theta = R\sin(2\theta - \alpha)$ $R = \sqrt{6^2 + 4^2} = \sqrt{52}$ $\tan\alpha = \frac{4}{6}$ $\alpha = 33.690°$ $\sqrt{52}\sin\left(2\theta - 33.690^{\circ}\right) + 3 = 0$

	$\sin\left(2\theta - 33.690^{\circ}\right) = -\frac{3}{\sqrt{52}}$ basic angle $= 24.583^{\circ}$ $2\theta - 33.690^{\circ} = -24.583^{\circ}$ or $2\theta - 33.690^{\circ} = 180^{\circ} + 24.583^{\circ}$ $\theta = 4.553^{\circ}$ $\quad$ $\theta = 119.137^{\circ}$ $\theta = 4.6^{\circ}$ $\quad$ $= 119.1^{\circ}$

5(a)	$y = \frac{x^2}{e^x}$ $\frac{dy}{dx} = \frac{e^x(2x) - x^2 e^x}{\left(e^x\right)^2}$ $\frac{dy}{dx} = \frac{2x - x^2}{e^x}$ Since y is an increasing function, $\frac{dy}{dx} > 0$ $\frac{2x - x^2}{e^x} > 0$ Since $e^x > 0$, $2x - x^2 > 0$ $x(2 - x) > 0$ $0 < x < 2$
(b)	$y = (x-1)\ln(1-x)$ $\frac{dy}{dx} = (x-1)\frac{1}{1-x}(-1) + (1)\ln(1-x)$ $\frac{dy}{dx} = 1 + \ln(1-x)$ Given that normal is parallel to the y-axis,

	$\frac{dy}{dx}=0$ $1+\ln(1-x)=0$ $\ln(1-x)=-1$ $1-x=e^{-1}$ $x=1-\frac{1}{e}$ $x=\frac{e-1}{e}$

6(i)	$v=t^2-14t+48$ $\frac{dv}{dt}=2t-14$ For minimium velocity, $\frac{dv}{dt}=0$ $2t-14=0$ $t=7$ Minimum $v=7^2-14(7)+48=-1\text{ m / s}$
(ii)	$v=t^2-14t+48$ When P is instantaneous at rest, $v=0$ $t^2-14t+48=0$ $t=6$ or $t=8$

(iii)	$v = t^2 - 14t + 48$ $s = \int t^2 - 14t + 48 \, dt$ $s = \frac{t^3}{3} - 7t^2 + 48t + c$ When $t = 0$, $s = 0$, $c = 0$ $s = \frac{t^3}{3} - 7t^2 + 48t$ When $t = 6$, $s = 108$ When $t = 8$, $s = 106\frac{2}{3}$ When $t = 10$, $s = 113\frac{1}{3}$ Total distance $= 108 + \left(108 - 106\frac{2}{3}\right) + \left(113\frac{1}{3} - 106\frac{2}{3}\right) = 116$ m
(iv)	$s = \frac{t^3}{3} - 7t^2 + 48t$ When P returns to O, $s = 0$ $\frac{t^3}{3} - 7t^2 + 48t = 0$ $\frac{t}{3}\left(t^2 - 21t + 144\right) = 0$ $t = 0$ or $t^2 - 21t + 144 = 0$ Discriminant $= (-21)^2 - 4(1)(144)$ $= -135 < 0$ The particle does not return to O.

7	By similar triangles, $\frac{10}{40} = \frac{r}{h}$ $r = \frac{h}{4}$ $V = \frac{1}{3}\pi r^2 h$ $= \frac{1}{3}\pi \left(\frac{h}{4}\right)^2 h$ $= \frac{1}{48}\pi h^3$ $\frac{dV}{dh} = \frac{1}{16}\pi h^2$ When $r = 4$, $h = 16$ cm. Rate at which the volume is decreasing, $\frac{dV}{dt} = -8$ Using chain rule, $\frac{dV}{dt} = \frac{dV}{dh} \times \frac{dh}{dt}$ $-8 = \frac{1}{16}\pi (16)^2 \times \frac{dh}{dt}$ $\frac{dh}{dt} = -\frac{1}{2\pi}$ Rate at which the water level is decreasing is $\frac{1}{2\pi}$ cm s^{-1}.

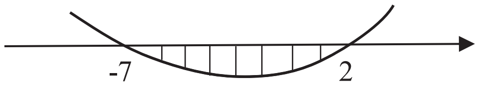
8 Determine the number of solutions for the equation

$$2+6\log_8 x=\frac{\log_5(9x-15)}{\log_5 2}$$

[5]

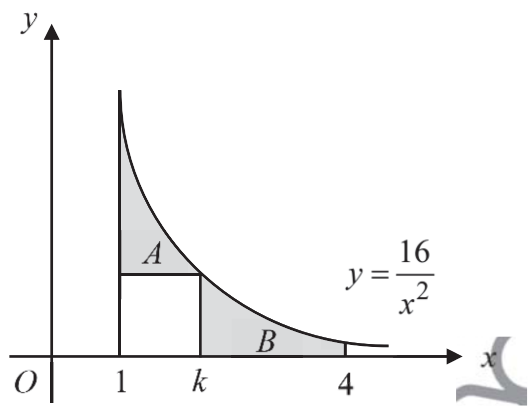
8	$2+6\log_8 x=\frac{\log_5(9x-15)}{\log_5 2}$ $2+\frac{6\log_2 x}{\log_2 8}=\log_2(9x-15)$ $2+\frac{6\log_2 x}{\log_2 2^3}=\log_2(9x-15)$ $2+\frac{6\log_2 x}{3\log_2 2}=\log_2(9x-15)$ $2+2\log_2 x=\log_2(9x-15)$ $2+2\log_2 x=\log_2(9x-15)$ $2+\log_2 x^2=\log_2(9x-15)$ $\log_2(9x-15)-\log_2 x^2=2$ $\log_2\frac{(9x-15)}{x^2}=2$ $\frac{(9x-15)}{x^2}=2^2$ $9x-15=4x^2$ $9x-15=4x^2$ $4x^2-9x+15=0$ Discriminant $=(-9)^2-4(4)(15)$ $=-159<0$ The equation has no real roots, hence there are zero solutions.	**M2 – apply rules of logarithms and change of base to obtain the quadratic equation** **A1 – quadratic equation** **M1** **A1**

9 Find the range of values of x such that the curve $y = 2x - x^2$ lies above the curve $y = 2x^2 + 17x - 42$. [4]

9	$2x - x^2 > 2x^2 + 17x - 42$	M1
	$2x - x^2 - 2x^2 - 17x + 42 > 0$	
	$-3x^2 - 15x + 42 > 0$	
	$3x^2 + 15x - 42 < 0$	
	$x^2 + 5x - 14 < 0$	
	$(x-2)(x+7) < 0$	M1
		M1
	$-7 < x < 2$	A1

10 **(a)** Find, in terms of π, the value of $\int_0^{\pi} \sin^2 x \cos^2 x \, dx$. [4]

(b) The diagram shows part of the curve $y = \dfrac{16}{x^2}$. Also shown are lines perpendicular to the x-axis at the points with x-coordinates 1, k and 4.



Given that the areas of the regions marked A and B are equal , find the value of k. [7]

10(a)	$\int_0^{\pi} \sin^2 x \cos^2 x \, dx$ $= \int_0^{\pi} (\sin x \cos x)^2 \, dx$ $= \int_0^{\pi} \left(\frac{1}{2}\sin 2x\right)^2 dx$ $= \frac{1}{4}\int_0^{\pi} \sin^2 2x \, dx$ $= \frac{1}{4}\int_0^{\pi} \frac{1-\cos 4x}{2} \, dx$ $= \frac{1}{8}\left[x - \frac{\sin 4x}{4}\right]_0^{\pi}$ $= \frac{\pi}{8}$	 M1 M1 M1 A1

(b)	When $x = k,\ y = \dfrac{16}{k^2}$	
	Area of A	
	$= \int_1^k \dfrac{16}{x^2}\,dx - (k-1)\left(\dfrac{16}{k^2}\right)$	M1
	$= \left[-\dfrac{16}{x}\right]_1^k - \left(\dfrac{16}{k} - \dfrac{16}{k^2}\right)$	
	$= -\dfrac{16}{k} - \left(-\dfrac{16}{1}\right) - \dfrac{16}{k} + \dfrac{16}{k^2}$	
	$= -\dfrac{32}{k} + 16 + \dfrac{16}{k^2}$	A1
	Area of B	
	$= \int_k^4 \dfrac{16}{x^2}\,dx$	M1
	$= \left[-\dfrac{16}{x}\right]_k^4$	
	$= -\dfrac{16}{4} - \left(-\dfrac{16}{k}\right)$	
	$= -4 + \dfrac{16}{k}$	A1
	Area of A = Area of B	
	$-\dfrac{32}{k} + 16 + \dfrac{16}{k^2} = -4 + \dfrac{16}{k}$	
	$\dfrac{16}{k^2} - \dfrac{48}{k} + 20 = 0$	
	$16 - 48k + 20k^2 = 0$	M1
	$5k^2 - 12k + 4 = 0$	
	$(k-2)(5k-2) = 0$	M1
	$k = 2$ or $k = 0.4$ (N.A.)	A1

11 The table shows experimental values of two variables, x and y, which are connected by an equation of the form $y^m x = k$, where k and m are constants.

x	2	4	6	8	10
y	6.25	1.56	0.694	0.391	0.250

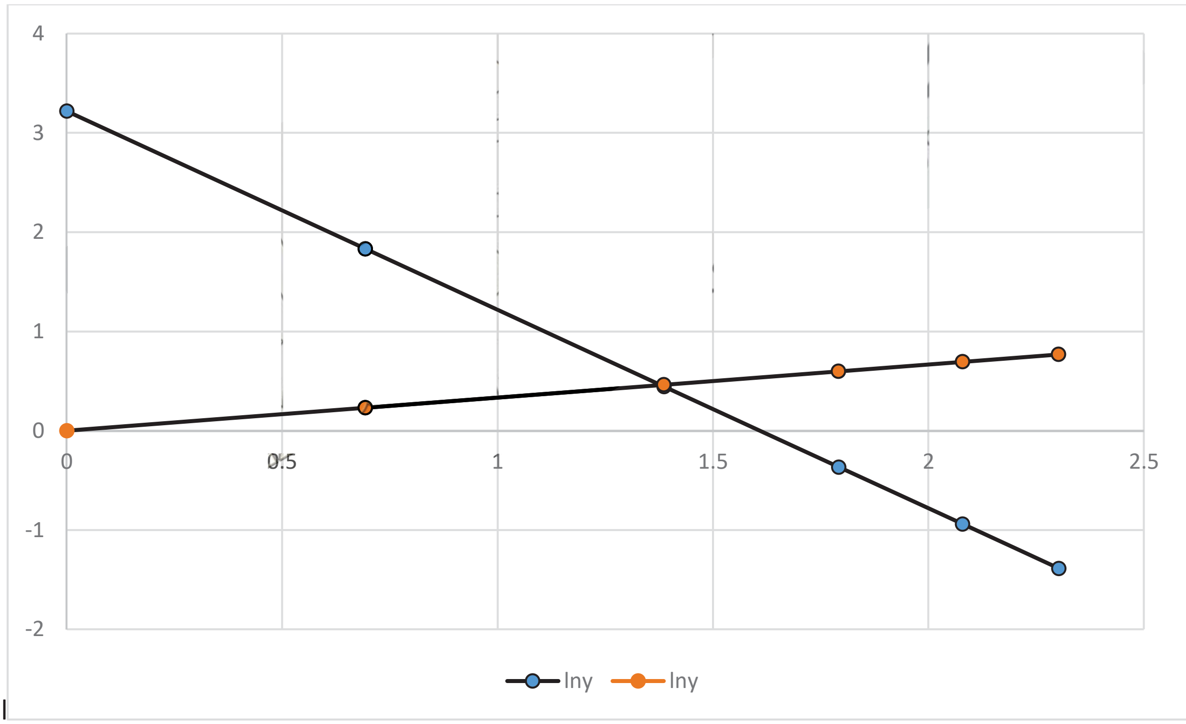
(i) Plot $\ln y$ against $\ln x$, and draw a straight line graph. [3]

(ii) Use your graph to estimate the value of k and of m. [4]

(iii) By adding a suitable straight line to the same diagram, find the solution to the pair of simultaneous equations $y^m x = k$ and $y = \sqrt[3]{x}$. [3]

11(i)

$\ln x$	0.693	1.386	1.792	2.079	2.303
$\ln y$	1.833	0.445	-0.365	-0.939	-1.386



(ii)	$y^m x = k$ $\ln\left(y^m x\right) = \ln k$ $\ln y^m + \ln x = \ln k$ $m \ln y = -\ln x + \ln k$ $\ln y = -\frac{1}{m}\ln x + \frac{1}{m}\ln k$ Gradient of the line $= -\frac{1}{m}$ Y-intercept $= \frac{1}{m}\ln k$ Gradient of the line $= -2$ $\left(\text{Accept } -1.7 \text{ to } -2.3\right)$ $-\frac{1}{m} = -2$ $m = 0.5$ $\left(\text{Accept } 0.435 \text{ to } 0.588\right)$ Y-intercept =3.217859 $\left(\text{Accept } 3.1 \text{ to } 3.3\right)$ $\frac{1}{m}\ln k = 3.217589$ $\frac{1}{0.5}\ln k = 3.217589$ $\ln k = 0.5 \times 3.217589$ $k = e^{0.5\times 3.217589}$ $= 5$ $\left(\text{Accept } 3.8 \text{ to } 6.2\right)$
(iii)	$y = \sqrt[3]{x}$ $y = x^{\frac{1}{3}}$ $\ln y = \frac{1}{3}\ln x$ <table><tr><td>$\ln x$</td><td>0</td><td>1</td><td>2</td></tr><tr><td>$\ln y$</td><td>0</td><td>0.333</td><td>0.667</td></tr></table> $\ln x = 1.2$ $\left(\text{Accept } 1.0 \text{ to } 1.4\right)$ At intersection point, $x = e^{1.2}$ $= 3.32$ $\left(\text{Accept } 2.7 \text{ to } 4.1\right)$

AM-2019-AHS-SA2-P2-Marking Scheme

1	$\alpha+\beta=\frac{6}{3}$ $\quad\quad$ $\alpha\beta=\frac{5}{3}$ $=2$ Sum of roots : $\frac{\beta^2}{\alpha}+\frac{\alpha^2}{\beta}=-p$ $-p=\frac{\alpha^3+\beta^3}{\alpha\beta}$ $-p=\frac{(\alpha+\beta)(\alpha^2-\alpha\beta+\beta^2)}{\alpha\beta}$ $-p=\frac{(\alpha+\beta)\left[(\alpha+\beta)^2-2\alpha\beta-\alpha\beta\right]}{\alpha\beta}$ $-p=\frac{2\left[(2)^2-3\times\frac{5}{3}\right]}{\frac{5}{3}}$ $p=\frac{6}{5}$ $\quad \left[\text{or} \quad 1\frac{1}{5} \quad \text{or} \quad 1.2\right]$ Product of roots: $\frac{\beta^2}{\alpha}\times\frac{\alpha^2}{\beta}=q$ $q=\frac{\alpha^2\beta^2}{\alpha\beta}$ $q=\alpha\beta$ $q=\frac{5}{3}$ $\quad \left[\text{or} \ 1\frac{2}{3}\right]$

2(i)	$D=b^2-4ac\ of\ -x^2+x-(1+h^2)$ $=1^2-4(-1)(-1-h^2)$ $=1-4-4h^2$ $=-3-4h^2$ <0	**alternative** $-x^2+x-(1+h^2)$ $=-(x^2-x)-(1+h^2)$ $=-\left[(x-\frac{1}{2})^2-\frac{1}{4}\right]-(1+h^2)$ $=-(x-\frac{1}{2})^2+\frac{1}{4}-(1+h^2)$ $=-(x-\frac{1}{2})^2-(\frac{3}{4}+h^2)$ <0

Since D (b^2- $4ac$) is always negative for all values of h and coefficient of x^2 is -1, then $-x^2+x-h^2-1$ is always negative.

2(ii)

Subst $y=2x+k$ into $y^2=1-2x^2$

$$(2x+k)^2=1-2x^2$$

$$4x^2+4kx+k^2=1-2x^2$$

$$6x^2+4kx+k^2-1=0$$

For line to be tangent to curve, discriminant =0

$$(4k)^2-4(6)(k^2-1)=0$$

$$16k^2-24k^2+24=0$$

$$8k^2=24$$

$$k=\pm\sqrt{3}$$

2(iii)

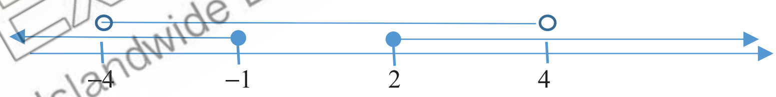
$$x+2\le x^2<16$$

$x+2\le x^2$ $x^2<16$

$x^2-x-2\ge 0$ $x^2-16<0$

$(x-2)(x+1)\ge 0$ $(x+4)(x-4)<0$

$x\le -1$ *or* $x\ge 2$ $-4<x<4$



Therefore the range of values of x are

$-4<x\le -1$ or $2\le x<4$

3(i)	$x-\sqrt{1-2x}=-7$ $x+7=\sqrt{1-2x}$ Square both sides $x^2+14x+49=1-2x$ $x^2+16x+48=0$ $(x+4)(x+12)=0$ $x=-4 \quad or\ x=-12(reject)$

3(ii)	$\frac{3^x}{9^{1-y}}=81$ and $4^x(2^{3y})=\frac{1}{\sqrt{8}}$ $\frac{3^x}{9^{1-y}}=81$ ------------------(1) $4^x(2^{3y})=\frac{1}{\sqrt{8}}$ ------------------(2) Eq(1) $3^x=9^{1-y}(81)$ $3^x=3^{2-2y}(3^4)$ $\therefore\ x=6-2y$ ----------------(3) Eq(2) $2^x(2^{3y})=2^{-\frac{3}{2}}$ $2^{2x+3y}=2^{-\frac{3}{2}}$ $\therefore\ 2x+3y=-\frac{3}{2}$ ----------------(4) Sub Eq(3) into Eq(4) $2(6-2y)+3y=-\frac{3}{2}$ $12-4y+3y=-\frac{3}{2}$ $y=\frac{27}{2}$ $\left[\text{or } 13\frac{1}{2} \text{ or } 13.5\right]$ Sub $y=\frac{27}{2}$ into Eq(3) $x=6-2(\frac{27}{2})$ $=-21$

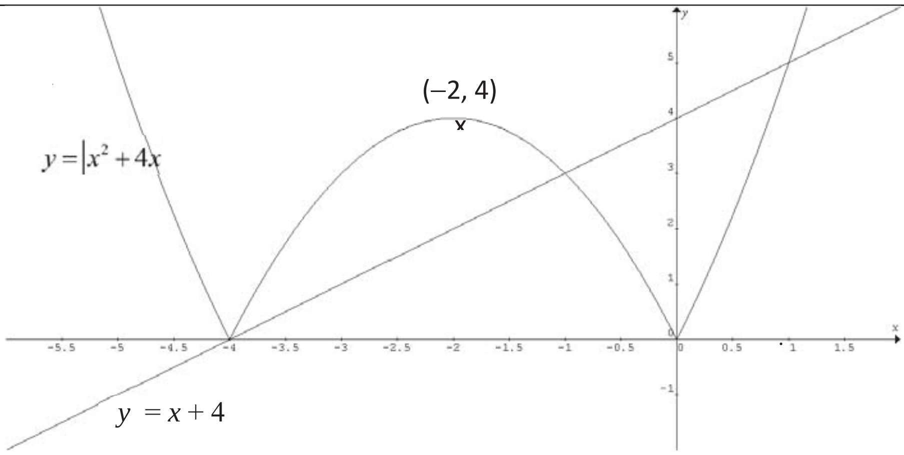
3(iii) (a)	Area of Trapezium $=12+6\sqrt{10}$ $\frac{1}{2}\left(AB+CD\right)Height=12+6\sqrt{10}$ $Height=\frac{2\left(12+6\sqrt{10}\right)}{3\left(\sqrt{2}+\sqrt{5}\right)}$ $=\frac{\left(24+12\sqrt{10}\right)}{3\left(\sqrt{2}+\sqrt{5}\right)}\times\frac{\sqrt{2}-\sqrt{5}}{\sqrt{2}-\sqrt{5}}$ $=\frac{24\sqrt{2}+12\sqrt{20}-24\sqrt{5}-12\sqrt{50}}{3(2-5)}$ $=\frac{24\sqrt{2}+12(2\sqrt{5})-24\sqrt{5}-12(5\sqrt{2})}{-9}$ $=\frac{-36\sqrt{2}}{-9}$ $=4\sqrt{2}cm$

(b)	$AD^2=\left(\sqrt{2}+\sqrt{5}\right)^2+\left(4\sqrt{2}\right)^2$ $=2+2\left(\sqrt{2}\right)\left(\sqrt{5}\right)+25+16(2)$ $=59+2\sqrt{10}$

4(i)	$f(2)=0$ $(2)^3+a(2)^2-2(2)-36=0$ $8+4a-4-36=0$ $4a=32$ $a=8$

(ii)	$x^3+5x^2-2x-24=(x+1)^2(x+b)+cx-27$ $Let\ x=-1$ $-1+5+2-24=-c-27$ $c=-27+1-5-2+24$ $c=-9$ $Let\ x=0$ $-24=(1)^2b-27$ $b=3$ ~~~~~~~~~~~~~~~~~~~~**Alternatively** ~~~~~~~~~~~~~~~~ Using comparison method, $x^3+5x^2-2x-24=(x^2+2x+1)(x+b)+cx-27$ $x^3+5x^2-2x-24=x^3+bx^2+2x^2+2bx+x+b+cx-27$ $x^3+5x^2-2x-24=x^3+(b+2)x^2+(2b+1+c)x+b-27$ By comparing constant $\therefore -24=b-27$ $b=3$ By comparing coefficient of x $-2=2(3)+1+c$ $c=-9$

4(iii)	$f(x)=w(x+3)(x+1)(x-4)$ At $(0, 24)$ $24=w(3)(1)(-4)$ $w=-2$ $\therefore\ f(x)=-2(x+3)(x+1)(x-4)$

5(i)	 Deduct 1 mark if axes and graphs are not labelled. G1 for correct shape of $y=\lvert x^2+4x\rvert$. G1 for indicating -4 and 0 as x-intercepts along the x-axis. G1 for indicating coordinates of turning point (-2,4) on the graph. G1 for correct sketch of $y=x+4$
5(ii)	B1 for stating **3 solutions. No B1 marks if graph is not drawn.**

6(i)	Middle term of $\left(x-\frac{1}{2x^2}\right)^8$ is $T_5=\binom{8}{4}x^4\left(-\frac{1}{2x^2}\right)^4$ $=\frac{35}{8x^4}$

6(ii)	$(2a+x)(1-3x)^n$ $=(2a+x)[1^n+\binom{n}{1}1^{n-1}(-3x)+\binom{n}{2}1^{n-2}(-3x)^2+...]$ $=(2a+x)[1-3nx+\frac{9n(n-1)x^2}{2}+...]$ $=2a-6anx+9an(n-1)x^2+x-3nx^2+...$ $=2a+(1-6an)x+[9an(n-1)-3n]x^2$ Comparing expression with $(4-59x+bx^2)$, Constant: $2a=4$

	$a = 2$ Coeff. of x: $1 - 6an = -59$ $1 - 6(2)n = -59$ $n = 5$ Coeff. of x^2: $9an(n-1) - 3n = b$ $9(2)(5)(5-1) - 3(5) = b$ $b = 345$

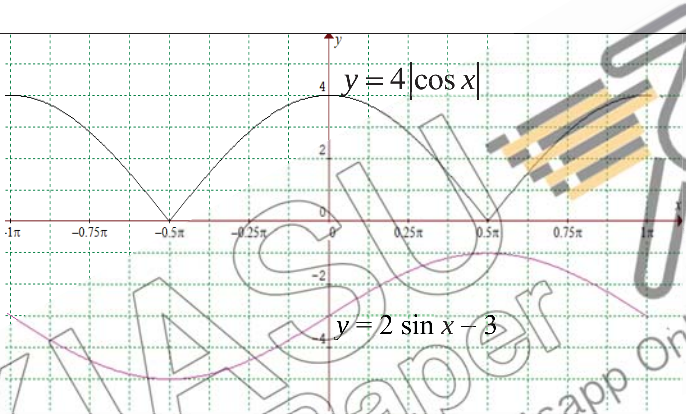
7(i)	Mid-point of $AB = \left(\frac{5+6}{2}, \frac{-7+0}{2}\right)$ $= \left(\frac{11}{2}, -\frac{7}{2}\right)$ Gradient of $AB = \frac{-7-0}{5-6}$ $= 7$ Equation of the perpendicular bisector of AB $y - \left(-\frac{7}{2}\right) = -\frac{1}{7}\left(x - \frac{11}{2}\right)$ $y = -\frac{1}{7}x - \frac{19}{7}$ $y = -\frac{1}{7}x - \frac{19}{7}$ …………..(1) $y = 5x - 13$ …………. (2) (1)=(2), $-\frac{1}{7}x - \frac{19}{7} = 5x - 13$ $36x = 72$ $x = 2$ sub $x = 2$ into (2), $y = 5(2) - 13$ $= -3$ Centre $(2, -3)$ Radius $= \sqrt{(2-6)^2 + (-3-0)^2}$ $= 5$ units Equation of the circle: $(x-2)^2 + (y+3)^2 = 25$ [OR $x^2 + y^2 - 4x + 6y - 12 = 0$]

(ii)	The exact smallest possible value of r $= \frac{1}{2} \times \sqrt{(5-6)^2 + (-7-0)^2}$ $= \frac{\sqrt{50}}{2}$ $= \frac{5\sqrt{2}}{2}$	

(iii)	Coordinates of the centre P are (5.5, –3.5) [OR $\left(\frac{11}{2}, -\frac{7}{2}\right)$]	B1

8(i)	Method 1 LHS = $\cot\theta + \tan\theta$ $= \frac{\cos\theta}{\sin\theta} + \frac{\sin\theta}{\cos\theta}$ $= \frac{\cos^2\theta + \sin^2\theta}{\sin\theta\cos\theta}$ $= \frac{1}{\sin\theta\cos\theta}$ $= \frac{2}{2\sin\theta\cos\theta}$ $= \frac{2}{\sin 2\theta}$ $= 2\operatorname{cosec} 2\theta$ $= RHS$ (shown)		Method 2 LHS = $\cot\theta + \tan\theta$ $= \frac{1}{\tan\theta} + \tan\theta$ $= \frac{1+\tan^2\theta}{\tan\theta}$ $= \frac{\sec^2\theta}{\tan\theta}$ $= \frac{1}{\cos^2\theta} \times \frac{\cos\theta}{\sin\theta}$ $= \frac{1}{\sin\theta\cos\theta}$ $= \frac{2}{2\sin\theta\cos\theta}$ $= \frac{2}{\sin 2\theta}$ $= 2\operatorname{cosec} 2\theta$ $= RHS$ (Shown)

(ii)	$\cot\frac{x}{2}+\tan\frac{x}{2}=\text{cosec}^2 x-3$ $2\,\text{cosec}\,x=\text{cosec}^2x-3$ $\text{cosec}^2x-2\,\text{cosec}\,x-3=0$ $(\text{cosec}\,x-3)(\text{cosec}\,x+1)=0$ $\text{cosec}\,x=3$ or $\text{cosec}\,x=-1$ $\frac{1}{\sin x}=3$ $\quad$ $\frac{1}{\sin x}=-1$ $\sin x=\frac{1}{3}$ $\quad$ $\sin x=-1$ $x=0.33983$ $\quad$ $x=-\frac{\pi}{2}$ (or -1.5707) Ans: $x\approx 0.340,\ -\frac{\pi}{2}$ (or -1.57)	M[1] -- QE M[1] – factorization / formula M[1] – in terms of sine A[1, 1] --- deduct 1 mk if ans in degree.

9(i)	
For $2\sin x-3=4\lvert\cos x\rvert+m$ to have 1 solution in the range $-\pi\le x\le\pi$. $m=-1$	

9(iia)	The maximum & minimum values of each curve do not occur at the same value of x. [max & min pts of each curves do not occur simultaneously.] **Accept solution where students sketch the graphs to explain.

9 (ii) Method 1		Method 2	
$\sqrt{8^2+5^2}=\sqrt{89}$, $\tan\alpha=\frac{5}{8}$ $\alpha=32.005°$ $8\sin x+5\cos x$ $=\sqrt{89}\sin(x-32.0°)$ $\therefore$ The range of $8\sin x+5\cos x$ $-\sqrt{89}\le 8\sin x+5\cos x\le\sqrt{89}$		Let $y=8\sin x+5\cos x$ $\frac{dy}{dx}=8\cos x-5\cos x$ let $\frac{dy}{dx}=0$, $8\cos x-5\sin x=0$ $\tan x=\frac{8}{5}$ $x=57.994°,\ 237.994°$	

Or $-9.4339 \le 8\sin x + 5\cos x \le 9.4339$ $-9.43 \le 8\sin x + 5\cos x \le 9.43$		when $x = 57.994^{o}$, $8\sin 57.994^{o} + 5\cos 57.994^{o}$ $= 9.4339$ when $x = 237.994^{o}$, $8\sin 237.994^{o} + 5\cos 237.994^{o}$ $= -9.4339$ $\therefore -9.43 \le 8\sin x + 5\cos x \le 9.43$	

10(i)	$\dfrac{2x-18}{x^3+6x^2+9x}=\dfrac{2x-18}{x(x+3)^2}$ Let $\dfrac{2x-18}{x(x+3)^2}=\dfrac{A}{x}+\dfrac{B}{x+3}+\dfrac{C}{(x+3)^2}$ $2x-18 \equiv A(x+3)^2+Bx(x+3)+Cx$ let $x=0$, $-18=A(9)$ $A=-2$ let $x=-3$, $2(-3)-18=C(-3)$ $C=8$ let $x=1$, $2(1)-18=(-2)(4)^2+B(1)(4)+(8)(1)$ $B=2$ $\therefore \dfrac{2x-18}{x(x+3)^2}=-\dfrac{2}{x}+\dfrac{2}{x+3}+\dfrac{8}{(x+3)^2}$

(ii)	Given that $\frac{dy}{dx}=\frac{x-9}{x^3+6x^2+9x}$ $$y=\frac{1}{2}\int\frac{2x-18}{x(x+3)^2}\,dx$$ $$=\frac{1}{2}\int(-\frac{2}{x}+\frac{2}{x+3}+\frac{8}{\left(x+3\right)^2})\,dx$$ $$=\frac{1}{2}\left[-2\ln x+2\ln(x+3)+\frac{8(x+3)^{-1}}{(-1)(1)}\right]+c$$ $$=\frac{1}{2}\left[-2\ln x+2\ln(x+3)-\frac{8}{(x+3)}\right]+c$$ $$=-\ln x+\ln(x+3)-\frac{4}{(x+3)}+c$$ $$=\ln\frac{x+3}{x}-\frac{4}{(x+3)}+c$$ Sub (3, ln 2) into the above eqn $\ln 2=\ln\frac{3+3}{3}-\frac{4}{3+3}+c$ $c=\frac{2}{3}$ Hence the equation of the curve is $y=\ln\frac{x+3}{x}-\frac{4}{(x+3)}+\frac{2}{3}$ OR $y=\ln(x+3)-\ln x-\frac{4}{(x+3)}+\frac{2}{3}$

11

$y = 3(x-1)^4 - 4(x-1)^3 + 5$

$\frac{dy}{dx} = 12(x-1)^3 - 12(x-1)^2$

Let $\frac{dy}{dx} = 0$,

$12(x-1)^3 - 12(x-1)^2 = 0$

$12(x-1)^2[(x-1)-1)] = 0$

$(x-1)^2 = 0$ or $(x-2) = 0$

$x = 1$ $\quad$ $x = 2$

when $x = 1$, $y = 3(1-1)^4 - 4(1-1)^3 + 5$

$= 5$

x	< 1	$= 1$	> 1
Sketch of tangent			

$\therefore$ (1, 5) is a point of inflexion

when $x = 2$, $y = 3(2-1)^4 - 4(2-1)^3 + 5$

$= 4$

x	< 2	$= 2$	> 2
Sketch of tangent			

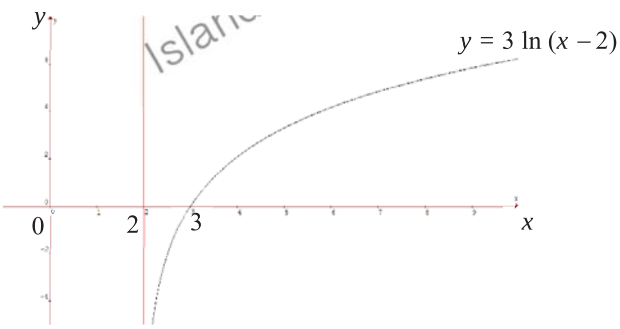
$\therefore$ (2, 4) is a minimum point

12(i)

$y = 3 \ln (x-2)$

The range of value of x is $x > 2$.

The curve crosses the x-axis at the point (3, 0)



12(ii)	$\frac{d}{dx}\left(\cos^3 \frac{x}{2}\right)$ $= (3\cos^2 \frac{x}{2})(-\sin \frac{x}{2})(\frac{1}{2})$ $= -\frac{3}{2}\cos^2 \frac{x}{2}\sin \frac{x}{2}$ $\int_0^{\pi} \cos^2 \frac{x}{2}\sin \frac{x}{2} \;\; dx$ $= -\frac{2}{3}\int_0^{\pi}\left(-\frac{3}{2}\right)\cos^2 \frac{x}{2}\sin \frac{x}{2} \;\; dx$ $= -\frac{2}{3}\left[\cos^3 \frac{x}{2}\right]_0^{\pi}$ $= -\frac{2}{3}\left[\cos^3 \frac{\pi}{2} - \cos^3 0\right]$ $= -\frac{2}{3}\left[(0)^3 - (1)^3\right]$ $= \frac{2}{3}$