2019 Additional Mathematics Prelim Paper 1 Solutions

1 ΔSQR: $\tan 30° = \dfrac{\frac{4}{\sqrt{3}+1}}{QR} \Rightarrow \dfrac{1}{\sqrt{3}} = \dfrac{\frac{4}{\sqrt{3}+1}}{QR}$

$$\Rightarrow QR = \frac{4\sqrt{3}}{\sqrt{3}+1}$$
$$= \frac{4\sqrt{3}}{\sqrt{3}+1} \times \frac{\sqrt{3}-1}{\sqrt{3}-1}$$
$$= \frac{4\sqrt{3}\left(\sqrt{3}-1\right)}{3-1}$$
$$= 2\left(3-\sqrt{3}\right)$$
$$\therefore\ PR = 2\sqrt{3} + 4\left(3-\sqrt{3}\right)$$
$$= 12 - 2\sqrt{3}$$
$$\therefore \text{Area of trapezium} = \frac{1}{2}\left[2\sqrt{3}+12-2\sqrt{3}\right] \times \frac{4}{\sqrt{3}+1}$$
$$= \frac{24}{\sqrt{3}+1}$$
$$= \frac{24\left(\sqrt{3}-1\right)}{\left(\sqrt{3}+1\right)\left(\sqrt{3}-1\right)}$$
$$= \frac{24\left(\sqrt{3}-1\right)}{3-1}$$
$$= \underline{\underline{12\left(\sqrt{3}-1\right)\text{ units}^2}}$$ [5]

2 $\dfrac{4x^3+x^2+6}{(x-2)\left(x^2+2\right)} = 4 + \dfrac{A}{x-2} + \dfrac{Bx+C}{x^2+2}$

Multiplying by $(x-2)\left(x^2+2\right)$, we obtain

$4x^3 + x^2 + 6 = 4(x-2)(x^2+2) + A(x^2+2) + (Bx+C)(x-2)$

Sub $x = 2$: $4\times 8 + 4 + 6 = A(4+2)$

$42 = 6A \Rightarrow A = 7$

Sub $x = 0$: $6 = -16 + 2(7) + C(-2)$

$-2C = 8 \Rightarrow C = -4$

Compare x^2: $1 = -8 + 7 + B \Rightarrow B = 2$

$\therefore\ \dfrac{4x^3+x^2+6}{(x-2)\left(x^2+2\right)} = \underline{\underline{4 + \dfrac{7}{x-2} + \dfrac{2-4x}{x^2+2}}}$. [5]

3 **(i)** $8x^3 - (x-1)^3 = (2x)^3 - (x-1)^3$

$= \left[2x-(x-1)\right]\left[(2x)^2 + 2x(x-1) + (x-1)^2\right]$

$= (x+1)(4x^2 + 2x^2 - 2x + x^2 - 2x + 1)$

$= (x+1)(7x^2 - 4x + 1)$ [3]

(ii) $8x^3 - (x-1)^3 = 0$

$\Rightarrow (x+1)(7x^2 - 4x + 1) = 0$

$\Rightarrow x = -1$ since for $7x^2 - 4x + 1 = 0$,

$D = (-4)^2 - 4(7) = -12 < 0$

$\therefore\ 7x^2 - 4x + 1 = 0$ has no real roots.

Thus $8x^3 - (x-1)^3 = 0$ has only one real root, -1. [2]

4 **(i)** The minimum point (h, k) lies on the line of symmetry: $x = \dfrac{1+3}{2} = 2$.

$\therefore\ h = 2$. [1]

(ii) $y = -\left|a(x-h)^2 + k\right|$ where $a > 0$

For $(1, 0)$: $-|a+k| = 0 \Rightarrow a + k = 0$

$\Rightarrow k = -a$ . . . (1)

For $(0, -9)$: $-|4a+k| = -9 \Rightarrow |3a| = 9$

$\Rightarrow 3a = 9$

$a = 3$

$k = -3$

(iii) We find the value of m where the line $y = mx$ is tangent to the curve.

$$\begin{cases} y = mx & \ldots (1) \\ y = 3(x-2)^2 - 3 & \ldots (2) \end{cases}$$

(1) = (2): $mx = 3x^2 - 12x + 9$

$3x^2 - (12+m)x + 9 = 0$

Since the line is tangent to the curve, then $D = 0$,

i.e. $\left[-(12+m)\right]^2 - 4\times 3\times 9 = 0$

$(12+m)^2 = 108$

$12 + m = \pm\sqrt{108}$

$= \pm\, 6\sqrt{3}$

$\therefore\ m = -12 \pm 6\sqrt{3}$

$\therefore\ m = -12 + 6\sqrt{3}$ since $m > -3$

Thus, the line intersects the curve at four distinct points when $\underline{\underline{-12+6\sqrt{3} < m < 0}}$. [2]

5 **(i)** gradient of $OQ = \dfrac{2-0}{4-0} = \dfrac{1}{2}$

$\therefore$ gradient of $\perp$ bisector of $OQ = -2$

Midpoint of $OQ = \left(\dfrac{4+0}{2}, \dfrac{2+0}{2}\right)$

$= (2, 1)$

Thus, equation of the perpendicular bisector of OQ is $y - 1 = -2(x-2)$

$\underline{\underline{y = -2x + 5}}$ [3]

(ii) When $x = 0$, $y = 5$.

When $x = 3$, $y = -1$.

These results show that R and P lie on the perpendicular bisector of OQ.

i.e., RP is the perpendicular bisector of OQ.

Thus, the quadrilateral $OPQR$ is a <u>kite</u>. [2]

(iii) Let $T = (a, b)$

Since $OPQT$ is a rhombus, then midpoint of OQ is the midpoint of RP.

$\therefore\ \left(\dfrac{a+3}{2}, \dfrac{b-1}{2}\right) = (2, 1)$

$\Rightarrow a = 1,\ b = 3$

$\therefore\ \underline{\underline{T = (1, 3)}}$. [2]

6 $4x^2 + px + q = 0$

(i) Since the roots are $\dfrac{1}{\alpha}$ and $\dfrac{1}{\beta}$, then

$\dfrac{1}{\alpha} + \dfrac{1}{\beta} = -\dfrac{p}{4} \Rightarrow \dfrac{\beta+\alpha}{\alpha\beta} = -\dfrac{p}{4}$

$\Rightarrow \dfrac{5}{2} = -\dfrac{p}{4}$

$\therefore\ p = -10$

$\dfrac{1}{\alpha} \times \dfrac{1}{\beta} = \dfrac{q}{4} \Rightarrow \dfrac{1}{2} = \dfrac{q}{4}$

$\therefore\ q = 2$

$\underline{\underline{p = -10,\ q = 2}}$ [4]

(ii) For the new equation, the roots are $\dfrac{2\alpha^2}{\beta}$ and $\dfrac{2\beta^2}{\alpha}$.

$$\text{Sum of roots} = \frac{2\alpha^2}{\beta} + \frac{2\beta^2}{\alpha}$$
$$= \frac{2\left(\alpha^3 + \beta^3\right)}{\alpha\beta}$$
$$= \frac{2(\alpha+\beta)\left[\alpha^2 - \alpha\beta + \beta^2\right]}{\alpha\beta}$$
$$= \frac{2(\alpha+\beta)\left[(\alpha+\beta)^2 - 3\alpha\beta\right]}{\alpha\beta}$$
$$= \frac{2(5)\left[5^2 - 3\times 2\right]}{2}$$
$$= 95$$

$$\text{Product of roots} = \left(\frac{2\alpha^2}{\beta}\right)\left(\frac{2\beta^2}{\alpha}\right)$$
$$= 4\alpha\beta$$
$$= 4 \times 2$$
$$= 8$$

Thus the equation is $\underline{x^2 - 95x + 8 = 0}$. [5]

7 (a) Given: $x^2 + ax + 2(a-1) > 1$

$x^2 + ax + (2a-3) > 0$

For a positive quadratic function, $D < 0$

$\therefore\ a^2 - 4(2a-3) < 0$

$\therefore\ a^2 - 8a + 12 < 0$

$\therefore\ (a-2)(a-6) < 0$

$\therefore\ \underline{2 < a < 6}$ [3]

(b) $y = 3x^2 + 4x + 6$

(i) $y > 6 \Rightarrow 3x^2 + 4x + 6 > 6$

$3x^2 + 4x > 0$

$x(3x+4) > 0$

$\therefore\ \underline{\underline{x < -\frac{4}{3} \text{ or } x > 0}}$ [2]

(ii) $y = 3x^2 + 4x + 6$

$y = -8x - 6$

$\therefore\ 3x^2 + 4x + 6 = -8x - 6$

$3x^2 + 12x + 12 = 0$

$3(x+2)^2 = 0$

The equation has equal roots $x = -2$

So the line is tangent to the curve. [2]

8 (a) Given: $y = 2x^3 - 9x^2 - 1$

gradient: $\dfrac{dy}{dx} = 6x^2 - 18x$

$$\frac{d^2y}{dx^2} = 12x - 18$$

For minimum gradient, $\dfrac{d^2y}{dx^2} = 0$,

i.e., $12x - 18 = 0 \Rightarrow x = \dfrac{3}{2}$

Since $\dfrac{d^3y}{dx^3} = 12\ (>0)$, gradient is minimum at $x = \dfrac{3}{2}$.

$\therefore$ Minimum gradient $= 6\left(\dfrac{3}{2}\right)^2 - 18\left(\dfrac{3}{2}\right)$

$= \underline{-13.5}$ [3]

(b) When $y = x^3 - 6x^2 + k$ touches the positive x-axis at A

[This means that the x-axis is tangent to the curve at A and so A is a stationary point.]

(i) $\dfrac{dy}{dx} = 3x^2 - 12x$

At A, $\dfrac{dy}{dx} = 0 \Rightarrow 3x(x-4) = 0$

$\therefore\ x = 0$ or $x = 4$

For A, $x > 0$, $\therefore\ \underline{A = (4, 0)}$ [2]

(ii) A also lies on the curve,

$\therefore\ 0 = 4^3 - 6 \times 4^2 + k$

$\therefore\ \underline{k = 32}$ [1]

(iii) $\dfrac{d^2y}{dx^2} = 6x - 12$

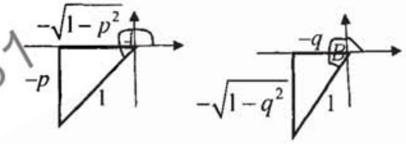
At A, $\dfrac{d^2y}{dx^2} = 6(4) - 12$

$= 12\ (>0)$

Thus A is a minimum point. [2]

9 (a)
$$\frac{5 - 10\cos^2 A}{\sin A - \cos A} = \frac{5\left(\sin^2 A + \cos^2 A\right) - 10\cos^2 A}{\sin A - \cos A}$$
$$= \frac{5\left(\sin^2 A - \cos^2 A\right)}{\sin A - \cos A}$$
$$= \frac{5(\sin A + \cos A)(\sin A - \cos A)}{\sin A - \cos A}$$
$$= 5(\sin A + \cos A)$$ [3]

(b) Given: $\sin A = -p$ $\cos B = -q$ and A and B lie in the same quadrant, then they both lie in 3[rd] quadrant.



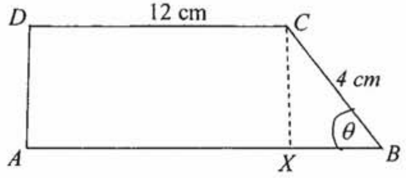
(i) $\sin(-A) = -\sin A$

$= \underline{p}$ [1]

(ii)
$$\tan(45^\circ - A) = \frac{\tan 45^\circ - \tan A}{1 + \tan 45^\circ \tan A}$$
$$= \frac{1 - \tan A}{1 + \tan A} \quad \text{since } \tan 45^\circ = 1$$
$$= \frac{1 - \frac{-p}{-\sqrt{1-p^2}}}{1 + \frac{-p}{-\sqrt{1-p^2}}}$$
$$= \underline{\underline{\frac{\sqrt{1-p^2} - p}{\sqrt{1-p^2} + p}}}$$ [3]

(iii)
$$\sec 2B = \frac{1}{\cos 2B}$$
$$= \frac{1}{2\cos^2 B - 1}$$
$$= \frac{1}{2(-q)^2 - 1}$$
$$= \underline{\underline{\frac{1}{2q^2 - 1}}}.$$ [2]

10 (i)



$\sin\theta = \dfrac{CX}{4} \quad \Rightarrow \quad CX = 4\sin\theta$

$\cos\theta = \dfrac{BX}{4} \quad \Rightarrow BX = 4\cos\theta$

Area of trapezium $= \dfrac{1}{2}[12+12+4\cos\theta]\times 4\sin\theta$

$= (12 + 2\cos\theta)\, 4\sin\theta$

$= 48\sin\theta + 4(2\sin\theta\cos\theta)$

$= 48\sin\theta + 4\sin 2\theta$ (shown)

[3]

(ii) $\dfrac{\mathrm{d}A}{\mathrm{d}\theta} = 48\cos\theta + 8\cos 2\theta$

For maximum A, $\dfrac{\mathrm{d}A}{\mathrm{d}\theta} = 0$.

$\therefore 48\cos\theta + 8\cos 2\theta = 0$

$48\cos\theta + 8(2\cos^2\theta - 1) = 0$

$8(2\cos^2\theta + 6\cos\theta - 1) = 0$

$\cos\theta = \dfrac{-6\pm\sqrt{6^2-4\times2\times(-1)}}{4}$

$\cos\theta = \dfrac{-6\pm\sqrt{44}}{4}$

$\cos\theta = \dfrac{-6\pm2\sqrt{11}}{4}$

$\cos\theta = \dfrac{-3+\sqrt{11}}{4}$ since θ is acute.

$\therefore \theta = 1.491\ 55$

$\dfrac{\mathrm{d}^2A}{\mathrm{d}\theta^2} = -48\sin\theta - 16\sin 2\theta$

When $\theta = 1.491\ 55$, $\dfrac{\mathrm{d}^2A}{\mathrm{d}\theta^2} < 0$,

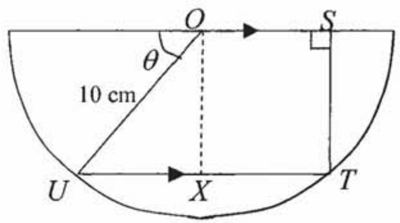
Thus A is maximum when $\underline{\underline{\theta = 1.49}}$. [5]

(iii) Maximum A

$= 48\sin(1.491\ 55) + 4\sin 2(1.491\ 55)$

$= \underline{53.3 \text{ sq units}}$ [1]

11 (i)



Consider $\Delta\, OUX$,

$\angle OUX = \theta$ (alternate angles)

$\sin\theta = \dfrac{OX}{10} \Rightarrow OX = 10\sin\theta$

$\cos\theta = \dfrac{UX}{10} \quad \Rightarrow \quad UX = 10\cos\theta$

$L = 10 + UT + ST + OS$

$= 10 + 2\times10\cos\theta + 10\sin\theta + 10\cos\theta$

$= 10 + 30\cos\theta + 10\sin\theta$ (shown) [3]

(ii) Let $30\cos\theta + 10\sin\theta = R\cos(\theta-\alpha)$

Then $R = \sqrt{30^2+10^2} = 10\sqrt{10}$

$\tan\alpha = \dfrac{10}{30} \Rightarrow \alpha = 18.434°$

$\therefore \underline{\underline{L = 10 + 10\sqrt{10}\cos(\theta - 18.4°)}}$ [3]

(iii) Since max $L = 10 + 10\sqrt{10} = 41.6$, it is impossible for the perimeter of $OSTU$ to be 50 cm. [1]

(iv) Given: $L = 35$

$10 + 10\sqrt{10}\cos(\theta - 18.434°) = 35$

$\cos(\theta - 18.434°) = \dfrac{35-10}{10\sqrt{10}} = \dfrac{5}{2\sqrt{10}}$

Basic angle $= 37.761°$

$\therefore \theta - 18.434° = 37.761°$

$\therefore \underline{\underline{\theta = 56.2°}}$ [3]

1 Given: $\dfrac{d^2y}{dx^2} = 6x - 2$

$\therefore$ Integrating, $\dfrac{dy}{dx} = 3x^2 - 2x + c$

Given that gradient of normal at $P(2, -8) = -\dfrac{1}{2}$, this means that gradient of tangent at $P = 2$, i.e. when $x = 2$, $\dfrac{dy}{dx} = 2$

$\therefore\ 2 = 3(2)^2 - 2(2) + c$

$\Rightarrow c = -6$

$\therefore\ \dfrac{dy}{dx} = 3x^2 - 2x - 6$

$\therefore$ Integrating, $y = x^3 - x^2 - 6x + c_2$

Since $P(2, -8)$ lies on the curve, i.e. when $x = 2$, $y = -8$,

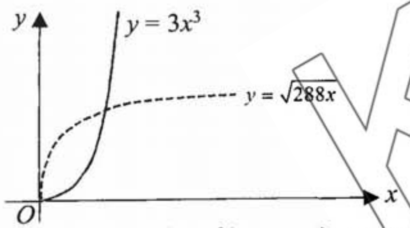
$\therefore\ -8 = 2^3 - 2^2 - 6(2) + c_2$

$\Rightarrow c_2 = 0$

Thus the equation of the curve is

$\underline{\underline{y = x^3 - x^2 - 6x}}$ [6]

2 **(i)**



[2]

(ii) To find the point of intersection, we solve the equations simultaneously:

$y = 3x^3$ … (1)

$y = \sqrt{288x}$ … (2)

(1) = (2): $3x^3 = \sqrt{288x}$

$9x^6 = 288x$

$9x^6 - 288x = 0$ **[Do not divide by variable x]**

$9x(x^5 - 32) = 0$

$\therefore x = 0$ or $x = 2$

$y = 0$ or $x = 24$

So the 2 points of intersection are (0, 0) and (2, 24).

Gradient of line joining these 2 points $= \dfrac{24-0}{2-0}$

$= 12$

$y = 3x^3$

$\dfrac{dy}{dx} = 9x^2$

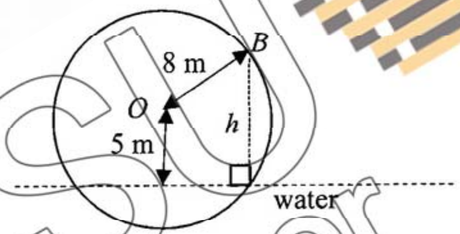
Since gradient of tangent to the curve at A is parallel to the line passing through the 2 points of intersection,

$\therefore 9x^2 = 12$

$x^2 = \dfrac{4}{3}$

$\therefore\ \underline{\underline{x = \dfrac{2}{\sqrt{3}} = \dfrac{2\sqrt{3}}{3}}}$ since $x \geq 0$. [4]

3



Given: $h = a\cos bt + c$

(i) Starting point is when B is at its highest point, i.e., when $t = 0$, $h = 13$

$\therefore 13 = a(1) + c \Rightarrow a + c = 13$ … (1)

and lowest point is when B is 3 m below water level.

$\therefore -3 = a(-1) + c \Rightarrow -a + c = -3$ … (2)

Solving (1) and (2),

$\underline{a = 8,\ c = 5}$

Given: 5 revolutions take 1 minute

$\therefore$ 1 revolution takes $\dfrac{1}{5}$ minute (= 12 seconds)

So, period: $\dfrac{2\pi}{b} = 12 \Rightarrow \underline{\underline{b = \dfrac{\pi}{6}}}$ [3]

$\therefore\ h = 8\cos\left(\dfrac{\pi}{6}t\right) + 5$

(ii) $h < 0 \Rightarrow 8\cos\left(\dfrac{\pi}{6}t\right) + 5 < 0$

When $h = 0$, $8\cos\left(\dfrac{\pi}{6}t\right) + 5 = 0$

$\cos\left(\dfrac{\pi}{6}t\right) = -\dfrac{5}{8}$

Basic angle $= \cos^{-1}\left(\dfrac{5}{8}\right) = 0.895\ 66$

The variable angle $\dfrac{\pi}{6}t$ lies in the 2[nd] and 3[rd] quadrants,

$\therefore \dfrac{\pi}{6}t = \pi - 0.895\ 66$ or $\pi + 0.895\ 66$ in the 1[st] revolution

$t = 4.289$ or 7.710

so duration $= 7.710 - 4.289$

$= \underline{3.42 \text{ seconds}}$ [3]

4 **(i)** For $x\left(2x + \dfrac{k}{x}\right)^8$, $T_{r+1} = x\dbinom{8}{r}(2x)^{8-r}\left(\dfrac{k}{x}\right)^r$

Power of $x = 1 + (8 - r) + (-r)$

$= 9 - 2r$

For term in x^3, power of $x = 3$

$\therefore 9 - 2r = 3$

$\Rightarrow r = 3$

$\therefore$ Term in $x^3 = x\dbinom{8}{3}(2x)^5\left(\dfrac{k}{x}\right)^3$

$= \ldots$

$= 1\ 792\ k^3$

Since coefficient of $x^3 = 28$,

$1\ 792\ k^3 = 28$

$\Rightarrow k = \dfrac{1}{4}$ (shown) [4]

(ii) For constant term, power of $x = 0$,

$\therefore 9 - 2r = 0$

$\Rightarrow r = \dfrac{9}{2}$

Since r is not a <u>whole number</u> (or <u>positive integer</u>), then we can conclude that there is no constant term. [1]

(iii) In the expansion of $x\left(2x + \dfrac{k}{x}\right)^8 + k(1-x)^{10}$,

Term in $x^3 = 28x^3 + \dfrac{1}{4}\times\dbinom{10}{3}(-x)^3$

$= 28x^3 + \dfrac{1}{4}\times(-120x^3)$

$= -2x^3$

$\therefore$ $\underline{\text{coefficient of } x^3 = -2}$ [2]

5 **(i)** To show: AB is parallel to DC

This means that we need only to show that there are 2 equal angles that satisfy either alternate angles, corresponding angles or interior angles
Here we can show $\angle CDT = \angle BAT$]

$\angle CDT = \angle DTP$ (alternate segment theorem)
$= \angle ATP$ (common angle)
$= \angle BAT$ (alternate segment theorem)

Since this result satisfies the properties of corresponding angles, then $AB \parallel DC$. (shown) [2]

(ii) To show: the line TE bisects $\angle ATB$

[This means that we need to show: $\angle ATE = \angle ETB$]

$\angle ATE = \angle DTE$ (common angle)
$= \angle DCE$ (angle in the same segment)
$= \angle CEB$ (alternate angles since $AB//DC$)
$= \angle ETB$ (alternate segment theorem)

Thus, TE bisects $\angle ATB$. (show) [3]

(iii) To show: $\Delta DFT \equiv \Delta EFC$ if $DF = EF$

From (ii), $\angle DTE = \angle DCE$ (angle in same segment)
i.e. $\angle DTF = \angle ECF$ (common angles) **A**
$DF = EF$ (given) **S**
and $\angle DFT = \angle EFC$ (vertically opp angles) **A**

Thus, $\Delta DFT \equiv \Delta EFC$ (AAS) [2]

6 Given: $\underbrace{y - x}_{Y} = \underbrace{\frac{b}{a}}_{m} \underbrace{x^2}_{X} + \underbrace{b}_{c}$ **[note that this is already in linear form]**

(i) Plotting $y - x$ against x^2 will give a straight line.

x^2	1	4	9	16
$y - x$	1.73	5.5	11.75	20.5

(ii) $b = Y$ – intercept
$= 0.5$

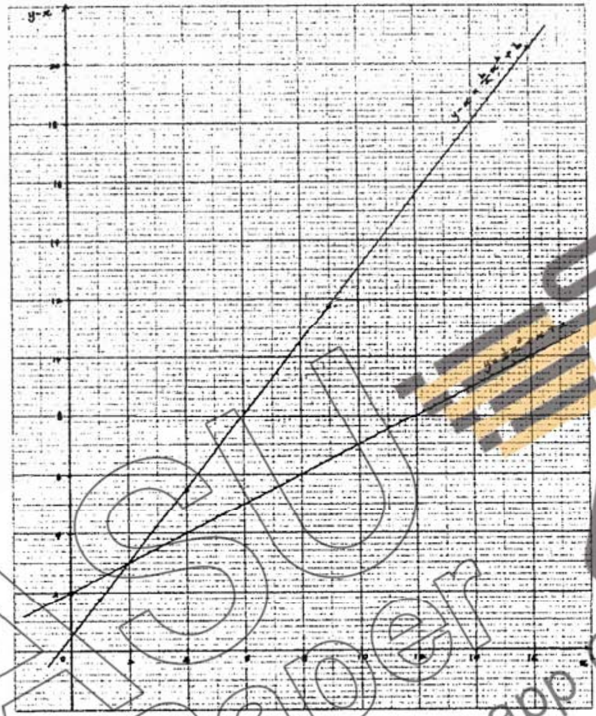
$\frac{b}{a}$ = gradient of line

$= \frac{20.5 - 0.5}{16 - 0}$

$= 1.25$

$\therefore \frac{0.5}{a} = 1.25 \Rightarrow a = 0.4$

$\therefore$ $\underline{a = 0.4, b = 0.5}$ [2]



[3]

(iii) $y = \frac{1}{2}x^2 + x + 2 \Rightarrow y - x = \frac{1}{2}x^2 + 2$

Using the same axes, $Y = y - x$ and $X = x^2$,

Then we need to draw the line $Y = \frac{1}{2}X + 2$.

When $X = 0$, $Y = 2$ and when $X = 16$, $Y = 10$

So the line passes through (0, 2) and (16, 10)

The point of intersection between the 2 lines is approximately (2, 3).

i.e. $X = 2 \Rightarrow x^2 = 2$, $\therefore x = 1.41$ (3 sf)
$Y = 3 \Rightarrow y - x = 3$, $\therefore y = 3 + 1.41 = 4.41$

$\underline{x = 1.41 \text{ and } y = 4.41}$ [2]

7 **(i)** Since $(2x^2 + x - 1)$ is a factor of P(x), then we can write P(x) = $(2x^2 + x - 1)\,Q(x)$. However, we know that $Q(x)$ is a polynomial of degree 2.

So, P(x) = $(2x^2 + x - 1)(ax^2 + bx + c)$.

However, we know that for P(x), coeff of x^2 = and constant term = –3.

This tells us that $a = 1$ and $c = 3$.

So, P(x) = $(2x^2 + x - 1)(x^2 + bx + 3)$.

[Writing P(x) in this form, we only need to find one unknown, i.e. b.]

Given: when divided by $(x - 1)$, P(x) has remainder 4, i.e. P(1) = 4.

$\therefore 4 = (2 + 1 - 1)(1 + b + 3)$
$= 2(b + 4)$
$b + 4 = 2 \Rightarrow b = -2$.

$\therefore$ P(x) = $(2x^2 + x - 1)(x^2 - 2x + 3)$
$= \underline{(2x - 1)(x + 1)(x^2 - 2x + 3)}$ [4]

(ii) P(x) = 0

$\Rightarrow$ $\underline{x = \frac{1}{2} \text{ or } -1}$ since $x^2 - 2x + 3 = 0$ has no real roots as D = $(-2)^2 - 4\times3 = -8 < 0$. [1]

(iii) P($1 - x$) = 0

$\Rightarrow 1 - x = \frac{1}{2}$ or -1

$\therefore$ $\underline{x = \frac{1}{2} \text{ or } 2}$ [2]

8 **(a)** $y = x^2 e^{3x}$

$\therefore \frac{dy}{dx} = 2xe^{3x} + 3x^2e^{3x}$
$= xe^{3x}(2 + 3x)$

For increasing function, $\frac{dy}{dx} > 0$

$\therefore xe^{3x}(2 + 3x) > 0$

Since $e^{3x} > 0$ for all real values of x.

$\therefore x(2 + 3x) > 0$

$\therefore$ $\underline{x < -\frac{2}{3} \text{ or } x > 0}$ [4]

(b) $y = \frac{x}{\sqrt{2x^2-1}}$ where $x > 0$

$$\frac{dy}{dx} = \frac{\sqrt{2x^2-1} - x \times \frac{1}{2} \times \frac{4x}{\sqrt{2x^2-1}}}{2x^2-1}$$

$$= \frac{\sqrt{2x^2-1} - x \times \frac{1}{2} \times \frac{4x}{\sqrt{2x^2-1}}}{2x^2-1} \times \frac{\sqrt{2x^2-1}}{\sqrt{2x^2-1}}$$

$$= \frac{(2x^2-1) - 2x^2}{\sqrt{(2x^2-1)^3}}$$

$$= -\frac{1}{\sqrt{(2x^2-1)^3}}$$

Given: $\frac{dy}{dt} = -\frac{9}{8}\frac{dx}{dt} \Rightarrow \frac{dy}{dx} = -\frac{9}{8}$

$$\therefore -\frac{1}{\sqrt{(2x^2-1)^3}} = -\frac{9}{8}$$

$$\therefore \sqrt{(2x^2-1)^3} = \frac{8}{9}$$

$$(2x^2-1)^3 = \frac{64}{81}$$

$$2x^2 - 1 = \sqrt[3]{\frac{64}{81}}$$

$$x^2 = \frac{1}{2}\left(1 + \frac{4}{\sqrt[3]{81}}\right)$$

$$\therefore x = \sqrt{\frac{1}{2} + \frac{2}{\sqrt[3]{81}}} \text{ since } x > 0$$

$= 0.981$ (3 sf) [4]

9 Given: $y = x \ln x - x$

(i) $\frac{dy}{dx} = \left(\ln x + x \times \frac{1}{x}\right) - 1$

$= (\ln x + 1) - 1$

$= \ln x$ (shown) [2]

(ii) From (i), we know that $\int \ln x \, dx = x \ln x - x + c$

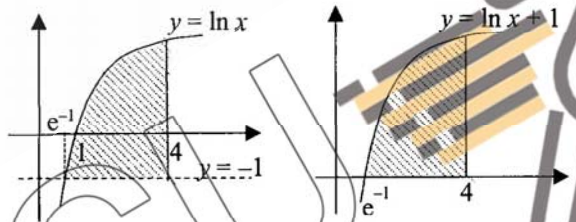
$$\int_a^b (\ln x + 1)\, dx = \int_a^b \ln x \, dx + \int_a^b 1 \, dx$$

$$= [x \ln x - x]_a^b + [x]_a^b$$

$$= (b \ln b - b) - (a \ln a - a) + (b - a)$$

$= b \ln b - a \ln a$ (shown) [3]

(iii) The shaded area is the same in both diagrams:



When $\ln x = -1$, $x = e^{-1} = \frac{1}{e}$.

Shaded area $= \int_{\frac{1}{e}}^{4} (\ln x + 1)\, dx$

$= 4 \ln 4 - \frac{1}{e} \ln \frac{1}{e}$ [using (ii)]

$= 5.91$ units^2 [3]

10 (a) $\cos\left(\frac{3x}{2}\right) = -\cos\frac{\pi}{10}$ for $0 < x < \pi$

basic angle $= \frac{\pi}{10}$

The variable angle $\frac{3x}{2}$ lies in 2nd and 3rd quadrants.

$$\therefore \frac{3x}{2} = \pi - \frac{\pi}{10} \text{ or } \pi + \frac{\pi}{10}$$

$$= \frac{9\pi}{10} \text{ or } \frac{11\pi}{10}$$

$\therefore x = \frac{3\pi}{5}$ or $\frac{11\pi}{15}$ [3]

(b) $\frac{1}{1-\sin x} - \frac{1}{1+\sin x} = \frac{(1+\sin x) - (1-\sin x)}{(1-\sin x)(1+\sin x)}$

$$= \frac{2\sin x}{1-\sin^2 x}$$

$$= \frac{2\sin x}{\cos^2 x}$$

$$= 2\frac{\sin x}{\cos x} \times \frac{1}{\cos x}$$

$= 2 \tan x \sec x$ (shown) [3]

(c) $\sin 4x + 3 \sin 2x = 0$ for $-180° \le x \le 180°$

$2 \sin 2x \cos 2x + 3 \sin 2x = 0$

$\sin 2x (2 \cos 2x + 3) = 0$

$\therefore \sin 2x = 0$ or $\cos 2x = -\frac{3}{2}$ (no solution)

basic angle $= 0°$

$\therefore 2x = -360°, -180°, 0°, 180°$ or $360°$

$x = -180°, -90°, 0°, 90°$ or $180°$ [3]

11 Given: $a = \frac{24}{(2t+1)^2}$

(i) $v = \frac{24(2t+1)^{-1}}{2(-1)} + c$

$= -\frac{12}{2t+1} + c$

when $t = 0$, $v = -10$ m/s

$\therefore c = 2$

$\therefore v = 2 - \frac{12}{2t+1}$

At P, $v = 0 \Rightarrow 2 - \frac{12}{2t+1} = 0$

$\Rightarrow t = 2.5$ s [4]

(ii) $s = 2t - 12\frac{\ln(2t+1)}{2} + c_1$

$= 2t - 6\ln(2t+1) + c_1$

when $t = 0$, $s = 0$, $\therefore c_1 = 0$

$\therefore s = 2t - 6\ln(2t+1)$

$t = 0$, $s = 0$

$t = 2.5$, $s = 2(2.5) - 6 \ln 6 = -5.750\,5$

$t = 3$, $s = 2(3) - 6 \ln 7 = -5.675\,4$

Distance travelled $= 5.750\,5 + (5.750\,5 - 5.675\,4)$

$= 5.83$ m (3 sf) [3]

(iii) **[9th second means from $t = 8$ s to $t = 9$ s]**

When $t = 8$, $s = 2(8) - 6\ln 17 = -0.999\ 28$ m

When $t = 9$, $s = 2(9) - 6\ln 19 = +0.333\ 36$ m

$\therefore s = 0$ for $8 < t < 9$

i.e. The particle is again at O during the 9th sec. [2]

12 (i) centre, X = midpoint of AB

$$= \left(\frac{-2+12}{2}, \frac{5+11}{2}\right)$$

$$= (5, 8)$$

diameter = AB

$$= \sqrt{(12+2)^2 + (11-5)^2}$$

$$= \sqrt{232}$$

$\therefore$ Equation of circle is

$$(x-5)^2 + (y-8)^2 = \left(\frac{\sqrt{232}}{2}\right)^2$$

$$\underline{\underline{(x-5)^2 + (y-8)^2 = 58}}$$ [3]

(ii) Since $D(8, k)$ lies on the circle, then

$(8-5)^2 + (k-8)^2 = 58$

$(k-8)^2 = 49$

$\therefore k - 8 = 7$ or -7

$k = 15$ or 1

Since $k > 1$, $\therefore \underline{k = 15}$. [1]

(iii) gradient of $DX = \frac{15-8}{8-5} = \frac{7}{3}$

Since line l is perpendicular to radius DX,

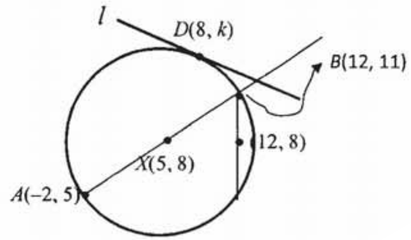
$\therefore$ gradient of $l = -\frac{3}{7}$

So, equation of line l is

$$y - 15 = -\frac{3}{7}(x-8)$$

$$y = -\frac{3}{7}x + \frac{24}{7} + 15$$

$$\underline{\underline{y = -\frac{3}{7}x + \frac{129}{7}}}$$ [3]



(iv) Given that (12, 8) is the midpoint of the chord, and since $B(12, 11)$ is vertically above (12, 8), then the chord is a vertical line with $x = 12$.

The other end-point of the chord is (12, 5)

Thus the points of intersection are

$\underline{(12, 5) \text{ and } (12, 11)}$. [2]

13 (a) Given: $9^x + 8 = 3^{x+2}$

$3^{2x} + 8 = 3^x \times 3^2$

Let $u = 3^x$, then the equation becomes

$u^2 + 8 = 9u$

$u^2 - 9u + 8 = 0$

$(u-1)(u-8) = 0$

$u = 1$ or $u = 8$

$3^x = 1$ or $3^x = 8$

$\therefore x = 0$ or $x = \log_3 8$

$$= \frac{\lg 8}{\lg 3}$$

$= 1.89$ (3 sf) [4]

(b) $40^{2p-1} = 5^{2-p} \Rightarrow \frac{40^{2p}}{40} = \frac{5^2}{5^p}$

$\therefore 40^{2p} \times 5^p = 5^2 \times 40$

$(40^2 \times 5)^p = 1000$

$(8000)^p = 1000$

$(20)^{3p} = 10^3$

$\therefore \underline{20^p = 10}$ [3]

(c) $\log_4(2y) = \log_{16}(y-3) + 3\log_9 3$

$$\log_4(2y) = \frac{\log_4(y-3)}{\log_4 16} + 3\frac{\log_3 3}{\log_3 9}$$

$$\log_4(2y) = \frac{\log_4(y-3)}{2} + \frac{3}{2}$$

$2\log_4(2y) = \log_4(y-3) + 3$

$\log_4(2y)^2 - \log_4(y-3) = 3$

$$\log_4 \frac{(2y)^2}{y-3} = 3$$

$$\therefore \frac{4y^2}{y-3} = 4^3$$

$4y^2 = 64(y-3)$

$y^2 = 16(y-3)$

$y^2 - 16y + 48 = 0$

$(y-4)(y-12) = 0$

$\therefore \underline{y = 4 \text{ or } y = 12}$. [4]