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1. ALGEBRA*Quadratic Equation*

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial Theorem

$$(a + b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \dots + \binom{n}{r}a^{n-r}b^r + \dots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{(n-r)!r!} = \frac{n(n-1)\dots(n-r+1)}{r!}$.

2. TRIGONOMETRY*Identities*

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2 \cos^2 A - 1 = 1 - 2 \sin^2 A$$

$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

Formulae for $\triangle ABC$

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2}bc \sin A$$

- 1 (i) Sketch the graph of $y = -|3x - 5|$, indicating the intercepts clearly. [2]

- (ii) Explain why the equation $-2 = |3x - 5|$ has no real roots using a graphical approach. [1]

2 It is known that $\log_2 b = y$ and $\log_b y = 3$.

(i) Find an expression for $\log_{\sqrt{2}} b - \log_2 2b$ in terms of y .

(ii) By considering a pair of simultaneous equations, show that $\lg b = b^3 \lg 2$.

- 3 (i) On the same diagram, sketch the graphs of $\frac{y^2}{25} = x$ and $y = -3x^{\frac{3}{2}}$. [3]

- (ii) Find the coordinates of the points of intersection of the two curves. [3]

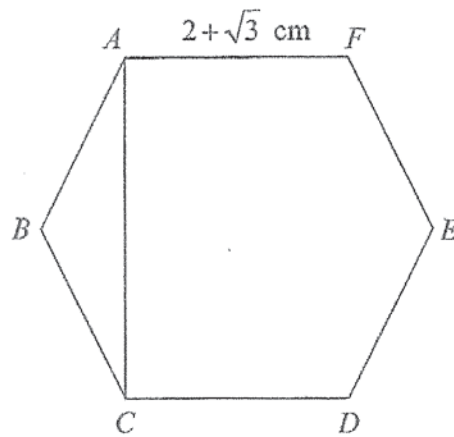
- 4 (i) Factorise $8x^3 + 27$.

- (ii) Express $\frac{8x^3 + 27}{(2x^2 + 3x)(x-1)^2}$ in partial fractions.

5 (i) Given that $u = 2^{2^x}$, solve the equation $8^x = 7(2^x) + \frac{8}{2^x}$. [4]

(ii) Hence, solve the equation $64^{-x} = 7(4^{-x}) + \frac{8}{4^{-x}}$. [2]

- 6 A regular hexagon $ABCDEF$ with sides $(2 + \sqrt{3})$ cm is shown below.



- (i) Show that $AC = (2\sqrt{3} + 3)$ cm.
- (ii) If the line segment AC has length $(\sqrt{27} - \sqrt{3})r$ cm, find the value of r , expressing answer in the form $a + b\sqrt{3}$, where a and b are rational numbers.

- 7 The roots of $x^2 - 7x + 4 = 0$ are α and β . Given that α and β are opposite in sign,
- (i) Find two possible values of $\alpha + \beta$. [4]

- (ii) Find two non-equivalent quadratic equations whose roots are α and β . [2]

8 (i) Show that $\frac{d}{dx}(2x \sin^2 x) = 2 \sin^2 x + 4x \sin x \cos x$.

(ii) Hence find $\int x \sin 2x \, dx$.

- 9 (i) Using $\tan 3x = \tan(2x + x)$, show that $\tan 3x$ may be expressed as

$$\frac{3 \tan x - \tan^3 x}{1 - 3 \tan^2 x}.$$

[3]

- (ii) Find all the values of x between 0 and π for which $\tan 3x = -5 \tan x$.

[5]

- 10 (i) Show that the curve $y = \frac{1}{4}x^2 - hx + 4$ meets the line $y = 2x - (2h - 1)$ for any real value of h .

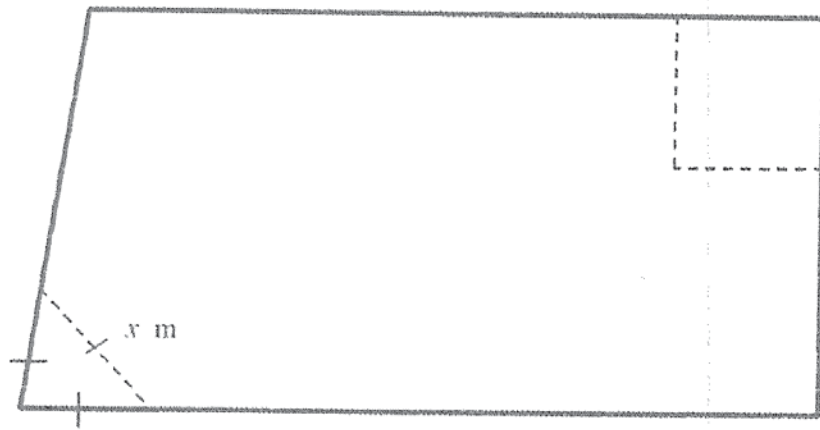
- (ii) State the value of h if the line $y = 2x - (2h - 1)$ is a tangent to the curve $y = \frac{1}{4}x^2 - hx + 4$.

- (iii) Using your answer from part (ii), find the values of p and q if the line $y = 2x - (2h - 1)$ intersects the line $y = px + q$ at an infinite number of points.

- 11 A semi-circle has radius r m, area A m² and perimeter P m. At the instant when its radius is a m, its area is increasing at a rate of 2π m²/s.

(i) Find an expression in terms of a , for the rate of increase of the radius at this instant. [3]

(ii) Find an expression in terms of a and π , for the rate of increase of the perimeter at the same instant. [2]



The diagram shows a room (crime scene) surrounded by solid walls. Two bodies are found at extreme corners of the room. To facilitate forensic work, a tape of total length p m is used to off these two extreme corners. The dotted lines represent the length of tape used. One corner room is an equilateral triangle of side x m and the other corner is a square.

- (i) Show that the area of the triangle is $\frac{\sqrt{3}}{4}x^2$.

- (ii) Show that the total area, A m², of the two cordoned-off corners is given by

$$A = \frac{p^2}{4} - \frac{p}{2}x + \frac{\sqrt{3}+1}{4}x^2.$$

- (iii) Given that x can vary depending on how taut the tape is, find an expression for x in terms of p for which the area A is stationary. [2]

- (iv) Write down, but do not simplify, an expression for the stationary value of A in terms of p and determine the nature of this stationary value. [3]

- (v) In the case where $p = \sqrt{3} + 1$, sketch a graph of A against x , indicating the turning point and the y -intercept clearly. [3]

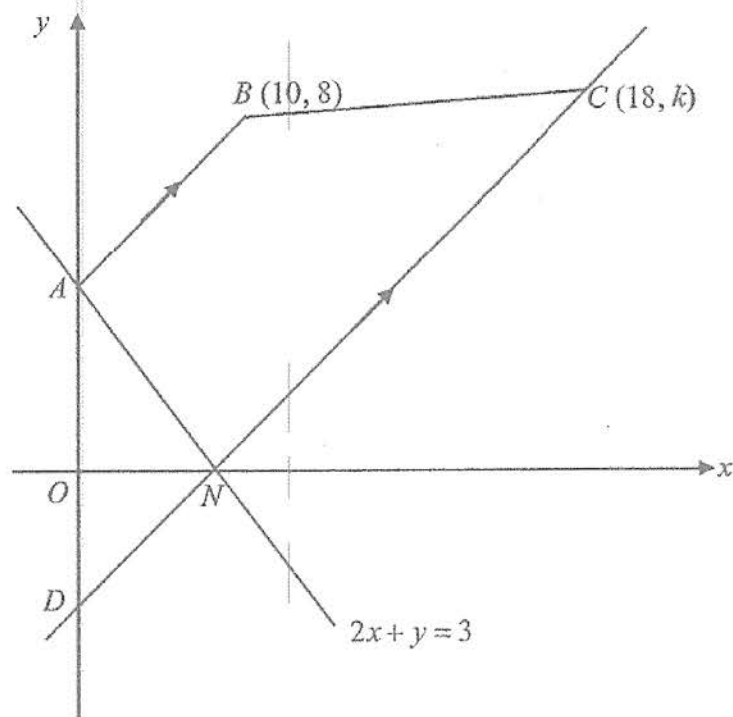
- 1 (i) Find the remainder when the polynomial $f(x) = 4x^3 - x + 3$ is divided by $2x - 1$. |

- (ii) Given that $f(x) - 5 = (2x - 1)A(x) + b$, where $A(x)$ is a polynomial, state the value of b the degree of $A(x)$. |

- (iii) A constant c is added to $f(x)$ to make it divisible by $2x - 1$. State the value of c . |

2

Solutions to this question by accurate drawing will not be accepted.



The diagram shows a trapezium $ABCD$ in which AB is parallel to DC . The coordinates of the points B and C are $(10, 8)$ and $(18, k)$ respectively. The line with equation $2x + y = 3$ intersects the x -axis and y -axis at N and A respectively. The line CN intersects the y -axis at D .

- (i) Determine whether $\angle ANC = 90^\circ$.

[4]

- (ii) Write down the equation of the line CD .
- (iii) Find the value of k and hence, find the area of the trapezium $ABCD$.

3 The curve $y = f(x)$ is such that $f''(x) = x^2 - \frac{1}{2}$.

(i) Find the range of values of x for which $f'(x)$ is a decreasing function. [3]

The point $P(3, 10)$ lies on the curve. The gradient of the curve at P is $\frac{5}{2}$.

(ii) Find the equation of the curve. [5]

- 4 (i) Given that the coefficient of the $\frac{1}{x^2}$ term in the binomial expansion of $\left(\frac{1}{x^3} - kx^2\right)^9$ is 4032, show that $k = -2$.

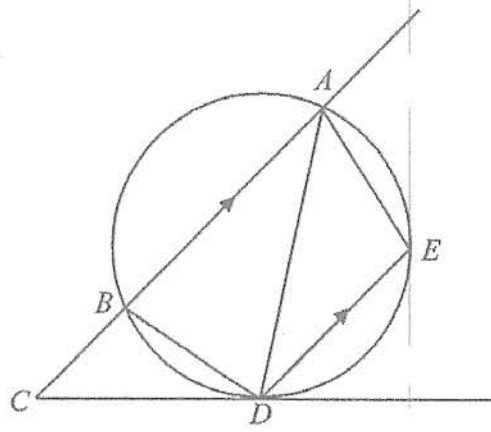
- (ii) Find the constant term in the expansion of $(x^{12} + x^7)\left(\frac{1}{x^3} - kx^2\right)^9$.

- 5 A curve C has equation $y = \ln(1-2x)$, $x < \frac{1}{2}$. The point P on C has coordinates $\left(\frac{1}{3}, \ln \frac{1}{3}\right)$. The tangent and normal to C at P meet the y -axis at Q and R respectively.

(i) Find the equations of the tangent and normal to the curve C at P . [4]

(ii) Find the coordinates of Q and of R . [2]

(ii) Find the area of triangle OPR . [2]



In the diagram, $ABDE$ is a cyclic quadrilateral in which BA is parallel to DE . The tangent to the circle at D meets AB produced at C . The chord AD bisects angle BAE .

- (i) Prove that $\angle BCD = \frac{1}{2} \angle BAE$.

- (ii) If AD is the diameter of the circle,
 (a) prove that $\triangle ABD$ is similar to $\triangle DEA$,

- (b) state the name of the geometric shape given to the quadrilateral $ABDE$.

- 7 A boy runs along the coastline of a beach and passes a fixed point A . The velocity, v m/s, that he runs in t seconds after he passes A is given by

$$v = 20e^{-0.9t} - 3.$$

- (i) Find the distance that the boy ran 60 seconds after he passes A .

[5]

- (ii) Find the boy's acceleration when he is instantaneously at rest.

[4]

- (iii) Explain what the sign of the acceleration indicates.

[1]

- (iv) Explain whether the boy will be running at his maximum velocity at any point of time of his run.

[2]

The function $f(x) = 2x^3 + bx^2 - 14x - 20$ has a factor of $a(x+5)(x+1)$, where a and b are positive constants.

(i) Find the value of b . [2]

(ii) If $(x-2)$ is also a factor of $f(x)$, state the value of a . [1]

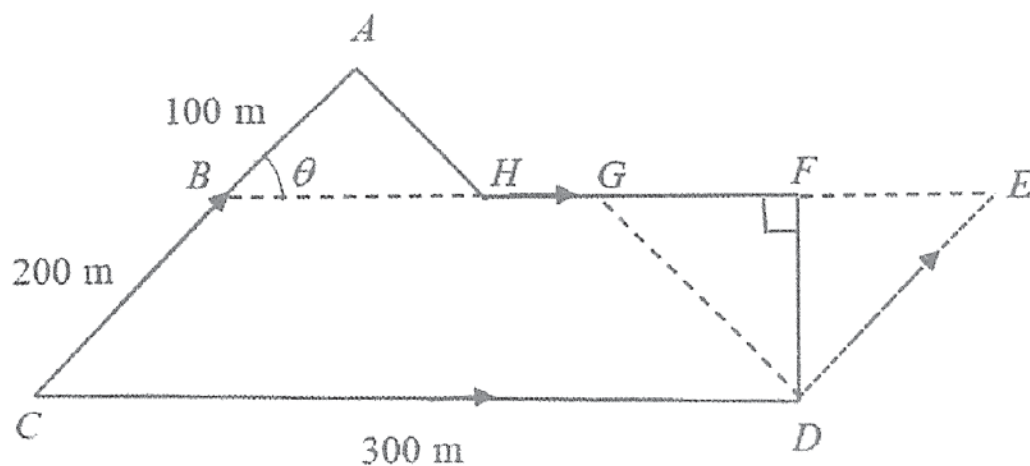
(iii) Hence, deduce another function $g(x)$ whose coefficient of x^3 is 3 and values of the roots of $g(x) = 0$ are twice the values of the roots of $f(x) = 0$. [1]

- 9 The function $f(x) = a \cos bx + 2$ has a period of 6π and a range of $-2 \leq f(x) \leq 6$.
- (i) State the value of a and of b .

- (ii) Sketch the graph of $y = f(x)$ for $0 \leq x \leq 9\pi$.

- (iii) On the same diagram, sketch $g(x)$ where $g(x) = -f(x)$ for $0 \leq x \leq 9\pi$.

- (iv) State the equation of the line of symmetry between $f(x)$ and $g(x)$.



The diagram shows the running path ($A \rightarrow B \rightarrow C \rightarrow D \rightarrow F \rightarrow G \rightarrow H \rightarrow A$) of Ali. $\triangle ABH$ and $\triangle DEG$ are isosceles triangles. CA is parallel to DE and CD is parallel to HF . $AB = 100$ m, $BC = 200$ m and $CD = 300$ m. It is also given that angle $GFD = 90^\circ$ and angle $ABH = \theta^\circ$, where $0^\circ < \theta < 90^\circ$.

- (i) Given that Ali runs at a uniform speed of 10 m/s throughout, show that the time taken t s for the run can be expressed as $100 + 20\sin\theta - 40\cos\theta$. [5]

- (ii) Express t in the form of $100 + R \sin(\theta - \alpha)$, where $R > 0$ and α is an acute angle. [3]

- (iii) Using your answer from part (ii), justify with working whether Ali can complete his run in the shortest possible time, assuming that Ali completes his run. [2]

A circle, C_1 , has equation $x^2 + y^2 - 4x + 6y = 12$.

- (i) Find the radius and coordinates of the centre of the circle C_1 . [3]

- (ii) Determine whether $(3, 1)$ lies inside, outside or on the circle. [2]

Another circle, C_2 , has centre $(2, -8)$ and the same radius as C_1 .

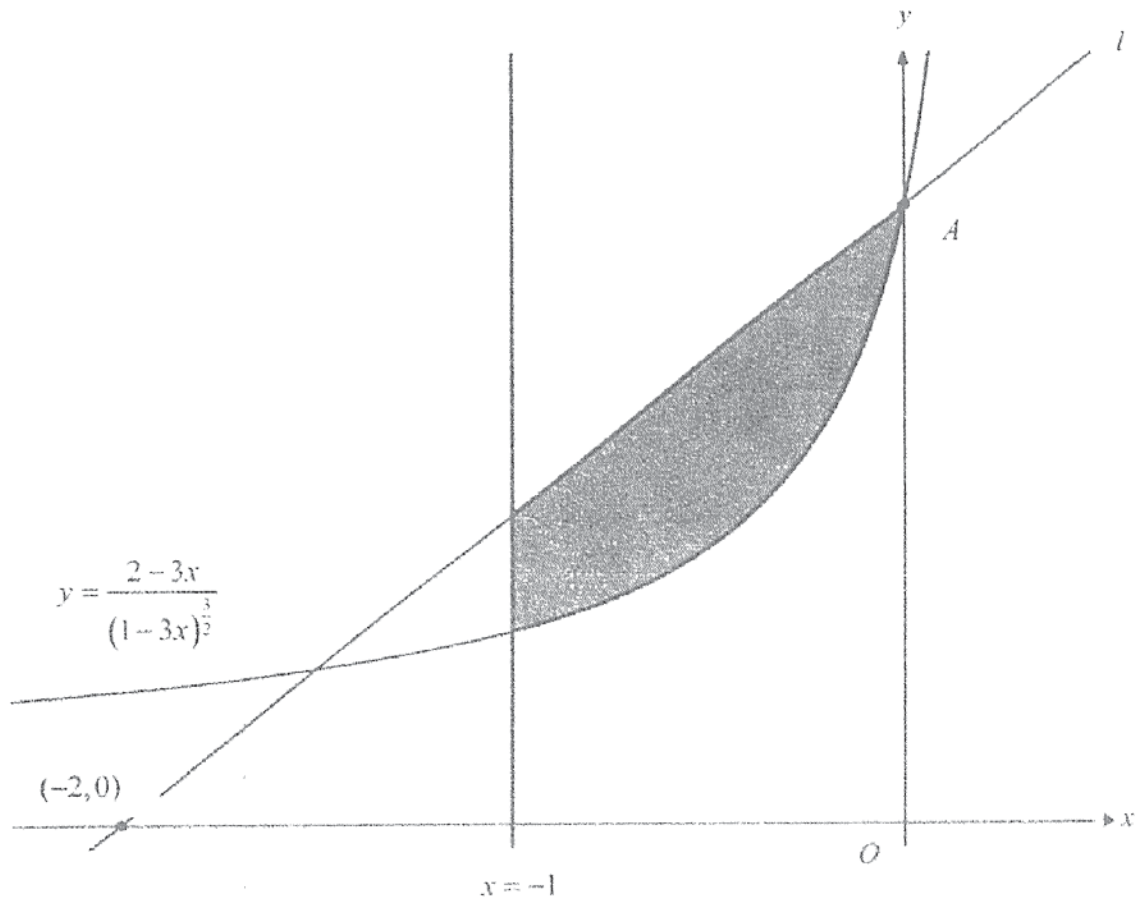
- (iii) State the equation of C_2 . [1]

The line which is parallel to the x -axis and passes through $(2, -8)$ intersects C_2 at A and B .

- (iv) State the equations of the tangents to C_2 at A and B . [2]

12 (a) Show that $\frac{d}{dx} \left(\frac{2x}{\sqrt{1-3x}} \right) = \frac{2-3x}{(1-3x)^{\frac{3}{2}}}.$

(b)



The diagram shows the line $x = -1$ and part of the curve $y = \frac{2-3x}{(1-3x)^{\frac{3}{2}}}$. The curve intersects the y-axis at point A . The line l through A intersects the x-axis at $(-2, 0)$.

- (i) Determine the area of the shaded region bounded by the curve, the line $x = -1$ and the line l . [4]



- (ii) The area bounded by the x -axis, the curve, the line $x = -1$ and the line $x = a$ is four times the area of the shaded region in part (i), where $a > 0$. Find the value of a . [3]

- 3 The population, P , in millions of a country on 1st January has been increasing every year from 1960 to 2000. The increase is exponential and so can be modelled by an equation of the form

$$P = P_0 e^{kt},$$

where P_0 and k are constants and t is the time in years since 1st January 1960. The table below gives values of P and t for some of the years 1960 to 2000.

Year	1960	1970	1980	1990	2000
t years	0	10	20	30	40
P	5.75	9.97	17.3	30.0	51.9

- (i) Plot a suitable straight line graph to show that the model is valid for years 1960 to 2000. [3]
- (ii) Estimate the value of k . [2]
- (iii) Assuming that the model is still appropriate, use your graph to estimate the population in 1st January 2005. [2]

