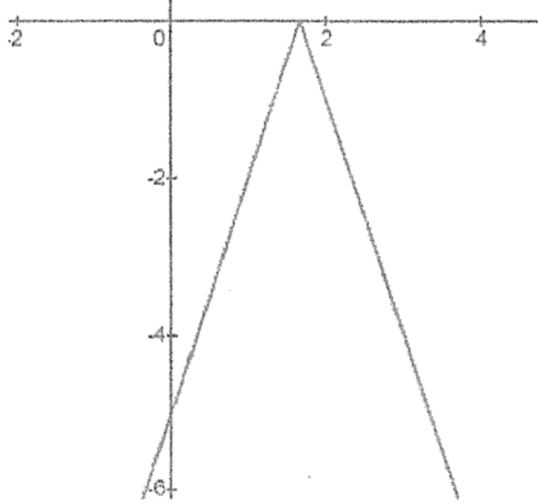
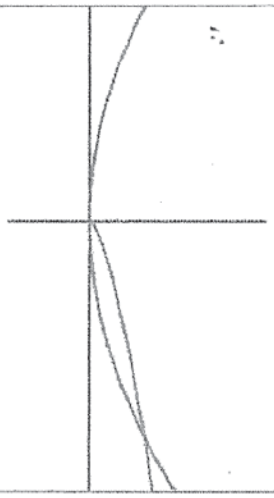
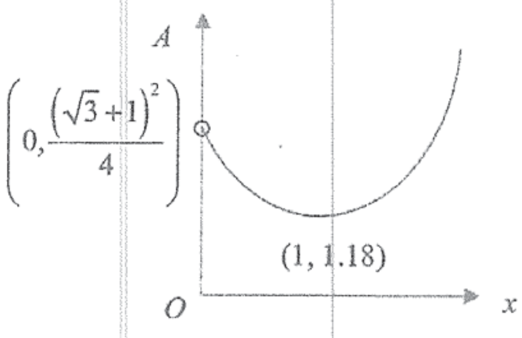
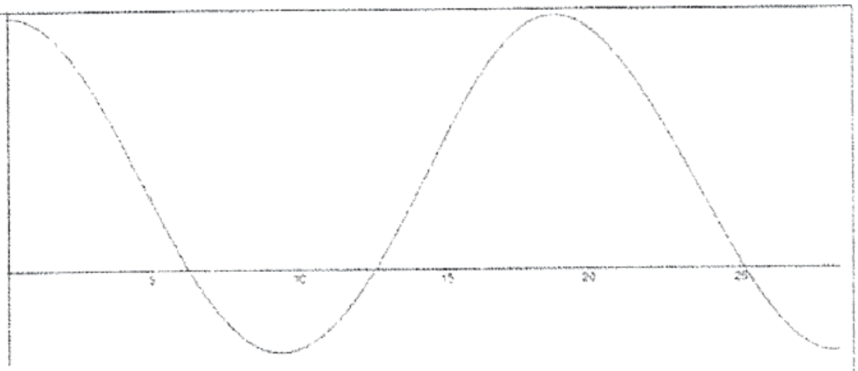
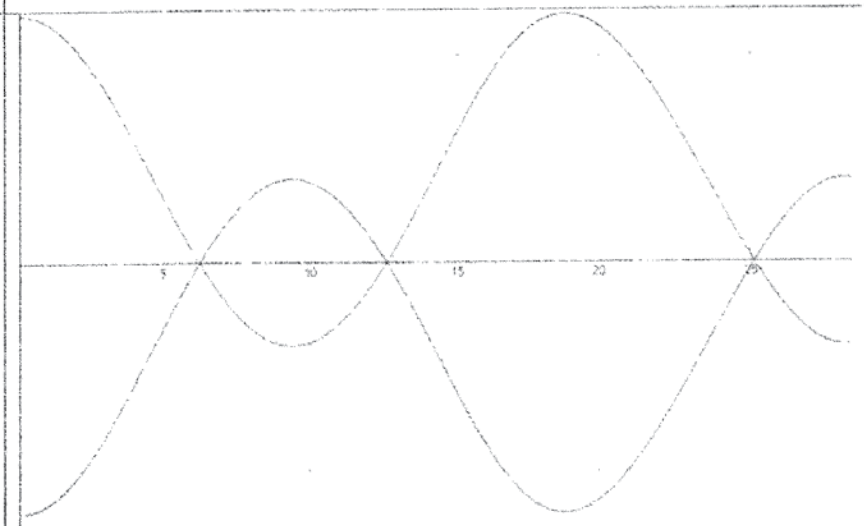
2019 4E5N PRELIM AM P1 ANSWER KEY

Q	Solution
1i	
1ii	Since the line y = 2 does not intersect the graph, there are no real roots for the equation $-2=\lvert 3x-5\rvert$.
2i	$y-1$
3i	
3ii	$(0,0)$ and $(\frac{5}{3},-6.45)$
4i	$8x^3+27=(2x+3)(4x^2-6x+9)$
4ii	$\frac{8x^3+27}{(2x^2+3x)(x-1)^2}=\frac{9}{x}-\frac{5}{x-1}+\frac{7}{(x-1)^2}$
5i	$y=\frac{3}{2}$
5ii	$x=-\frac{3}{4}$
6i	$AC=2\sqrt{3}+3$
6ii	$r=1+\frac{1}{2}\sqrt{3}$
7i	$\alpha+\beta=\sqrt{3}$ or $\alpha+\beta=-\sqrt{3}$

7ii	$x^2-\sqrt{3}x-2=0$ or $x^2+\sqrt{3}x-2=0$
8i	$2\sin^2 x+4x(\sin x)(\cos x)$
8ii	$\int x(\sin 2x)\mathrm{d}x = x\sin^2 x-\frac{x}{2}+\frac{1}{4}\sin 2x+C_3$
9i	$\frac{3\tan x-\tan^3 x}{1-3\tan^2 x}$
9ii	$x=0.615,\ 2.53$
10i	Since $(h+1)^2\geq 0$ for all values of h, there will always intersections between the line and curve, hence $y=\frac{1}{4}x^2-hx+4$ and $y=2x-(2h-1)$ meet.
10ii	$h=-1$
10iii	$p=2$ $q=3$
11i	$\frac{\mathrm{d}r}{\mathrm{d}t}=\frac{2}{a}$
11ii	$\frac{4+2\pi}{a}$
12i	$\left(\frac{\sqrt{3}}{4}\right)x^2$
12ii	$\frac{p^2}{4}-\frac{p}{2}x+\frac{\sqrt{3}+1}{4}x^2$
12iii	$x=\frac{p}{\sqrt{3}+1}$
12iv	$\frac{\mathrm{d}^2A}{\mathrm{d}x^2}=\frac{\sqrt{3}+1}{2}>0$ $\therefore A$ is minimum
12v	When $p=\sqrt{3}+1$, $x=1$ $A=1.18$ 

Qns	Answer Key
1i	3
1ii	$b=-2$ Degree = 2
1iii	$c=-3$
2i	$m_{AN}\times m_{CN}=-2\times\frac{1}{2}=-1$, therefore $\angle ANC=90^\circ$
2ii	Equation of line *CD* $y=\frac{1}{2}x-\frac{3}{4}$
2iii	52.5 units^2
3i	$-\frac{1}{\sqrt{2}}<x<\frac{1}{\sqrt{2}}$ (Accept: $-0.707<x<0.707$)
3ii	$\mathrm{f}(x)=\frac{x^4}{12}-\frac{1}{4}x^2-5x+20.5$
4i	$r=5$ $k=-2$
4ii	2688
5i	$y=\frac{1}{6}x-\frac{1}{18}+\ln\frac{1}{3}$
5ii	$R(0,\ \ln\frac{1}{3}-\frac{1}{18})$ [Accept: R (0, -1.15)
5iii	0.192 units^2 (3 s.f)
6iib	Rectangle
7i	Distance = 158m
7ii	$-2.70\,\mathrm{m/s^2}$
7iii	It means that his velocity is decreasing/ boy is slowing down
7iv	$\frac{\mathrm{d}v}{\mathrm{d}t}=-18\mathrm{e}^{-0.9t}$ For $t\geq 0$, $-18<0$ and $\mathrm{e}^{-0.9t}>0$, $\therefore\frac{\mathrm{d}v}{\mathrm{d}t}=-18\mathrm{e}^{-0.9t}<0,\therefore\frac{\mathrm{d}v}{\mathrm{d}t}\neq 0$. Hence, the boy will be **NOT** running at his maximum velocity at any point of time of his run.
8i	$b=8$
8ii	$\mathrm{f}(x)=2x^3+8x^2-14x-20=a(x+5)(x+1)(x-2)$ $\therefore a=2$
8iii	$\mathrm{g}(x)=3(x+10)(x+2)(x-4)$
9i	$a=4,\ b=\frac{1}{3}$

9ii	
9iii	
9iv	$y=0$
10i	$100+20\sin\theta-40\cos\theta$
10ii	$t=100+\sqrt{2000}\sin(\theta-63.4^\circ)$
10iii	Since no θ within the range of $0^\circ<\theta<90^\circ$ can be found, Ali cannot complete his run in the shortest possible time.
11i	Radius $r=\sqrt{(-2)^2+(3)^2-(-12)}=5$
11ii	Distance of $(3,1)$ to centre $=\sqrt{(3-2)^2+(1+3)^2}=\sqrt{17}<5$ Since distance of $(3,1)$ to centre is smaller than the radius of the circle, $(3,1)$ lies inside the circle.
11iii	$(x-2)^2+(y+8)^2=25$
11iv	$x=-3, x=7$
12a	$\dfrac{2-3x}{(1-3x)^{\frac{3}{2}}}$
12bi	$\dfrac{1}{2}$ square units
12bii	$a=0.25$ or $a=-1$ (rejected since $a>0$)
13ii	$k=0.055$ (Allow $0.05\le k\le 0.06$)
13iii	Population is 67.4 **millions**