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|--|---|
| <p>1 (i) $b = -4$</p> <p>(ii) $a = -1$</p> <p>(iii) $c = -2$</p> <p>2 (i) Show $p\left(\frac{1}{2}\right) = 0$</p> <p>(ii) $x = \frac{1}{2}$</p> <p>$(x^2 + 8) = 0$ no real solution</p> <p>(iii) $\frac{5}{2x-1} - \frac{2x}{x^2+8}$</p> <p>(iv) $\frac{5}{2}\ln(2x-1) - \ln(x^2+8) + c$</p> <p>3 (a) $x = 1.68, 0.447$ (rejected)</p> <p>(b) $a = 5, b = 3, k = 632$</p> <p>4 (i) $d = 8 \sin \theta - 5 \cos \theta$</p> <p>(ii) $d = \sqrt{89} \sin(\theta - 32.0^\circ)$</p> <p>(iii) Max value of $d = \sqrt{89} = 9.43$
when $\sin(\theta - 32.0^\circ) = 1$. So
not possible for d to be 10 m.</p> <p>(iv) $71.5^\circ, 172.5^\circ$</p> <p>5 (a) (i) $f'(x) = \frac{10}{3(25-x^2)}$</p> <p>$f''(x) = \frac{20x}{3(25-x^2)^2}$</p> <p>(ii) $0 < x < 5$</p> <p>(b) $\frac{d}{dx}\left[4\sin^2\left(\frac{x}{2} + \pi\right)\right] = 2\sin x$</p> | <p>6 (ii) $1.41, 2.98, 4.55$</p> <p>7 $16x^2 - 24x - 7 = 0$</p> <p>8 (i) x-coordinate of $A = \frac{\pi}{4}$</p> <p>x-coordinate of $B = \frac{\pi}{3}$</p> <p>(ii) $\left(1 - \sqrt{3} + \frac{\pi}{2}\right) \text{ units}^2$</p> <p>9 (i) $A(-1, 0), B\left(\frac{1}{2}, 3\right)$</p> <p>$C(2, 0)$</p> <p>(ii) $\tan \theta = -2$</p> <p>(iii) $x = 2$</p> <p>(iv) Max value of $3 - 1 - 2x$ is 3
when $x = \frac{1}{2}$.</p> <p>Min value of $4 + x^2$ is 4
when $x = 0$.</p> <p>The curve and line do not
intersect so there is no real
solution.</p> <p>10 (i) $C_1 : (x+1)^2 + (y+1)^2 = 5$</p> <p>$C_2 : (x+2)^2 + (y+2)^2 = 5$</p> <p>(ii) $x = -3$</p> <p>(iii) $\sqrt{2} + 2\sqrt{5}$ or 5.89 units</p> <p>11 (ii) $a = -2, b = 2.5$</p> <p>(iii) Draw the line $\frac{y}{\sqrt{x}} = \frac{3}{x}$.</p> <p>Point of intersection is
(0.5, 1.5)</p> <p>$x = 2, y = 2.12$</p> |
|--|---|

Name: Solution Class _____ Index No _____



BUKIT PANJANG GOVERNMENT HIGH SCHOOL
PRELIMINARY EXAMINATION 2020
SECONDARY FOUR EXPRESS
SECONDARY FIVE NORMAL ACADEMIC

ADDITIONAL MATHEMATICS

4047/1

Paper 1

Date: 21 August 2020

Candidates answer on the question paper.

Time: 0800 – 1000

No additional materials are required.

Duration: 2h

READ THESE INSTRUCTIONS FIRST

Write your name, class and index number on all the work you hand in.

Write in dark blue or black pen on both sides of the paper.

You may use an HB pencil for any diagrams or graphs.

Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer all questions.

Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.

The use of an approved scientific calculator is expected, where appropriate.

You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.

The number of marks is given in brackets [] at the end of each question or part question.

The total number of marks for this paper is 80.

This paper has a total of 22 printed pages.

Setter: Ms Penny Goh

[Turn over]

1. ALGEBRA

Quadratic Equation

For the quadratic equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

Binomial Theorem

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \dots + \binom{n}{r}a^{n-r}b^r + \dots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\dots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2 \cos^2 A - 1 = 1 - 2 \sin^2 A$$

$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

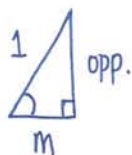
$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2} ab \sin C$$

- 1 It is given that $\cos A = -m$, where $m > 0$, and that A is obtuse. Find the value of each of the following in terms of m .

(a) $\tan A$

[2]



$$\text{opp.} = \sqrt{1-m^2} \quad [\text{M1}]$$

$$\tan A = -\frac{\sqrt{1-m^2}}{m} \quad [\text{A1}]$$

(b) $\cot(180-A) = \frac{1}{\tan(180-A)}$

[1]

$$= \frac{m}{\sqrt{1-m^2}} \quad [\text{B1}]$$

(c) $\cos\left(\frac{A}{2}\right)$

[3]

$$\cos A = 2\cos^2 \frac{A}{2} - 1$$

$$-m = 2\cos^2 \frac{A}{2} - 1 \quad [\text{M1}]$$

$$\cos^2 \frac{A}{2} = \frac{1-m}{2} \quad [\text{M1}]$$

$$\cos \frac{A}{2} = \sqrt{\frac{1-m}{2}} \quad \text{or} \quad -\sqrt{\frac{1-m}{2}} \quad (\text{rejected})$$

[A1]

- 2 (a) Find the range of values of p for which the line $y = 2x + 5$ will meet the curve $y^2 = px$. [4]

$$y = 2x + 5 \quad \text{--- (1)}$$

$$y^2 = px \quad \text{--- (2)}$$

sub (1) into (2),

$$(2x+5)^2 = px$$

$$4x^2 + 20x + 25 = px$$

$$4x^2 + 20x - px + 25 = 0 \quad \text{[M1]}$$

$$a = 4, \quad b = 20 - p, \quad c = 25$$

$$b^2 - 4ac \geq 0$$

$$(20-p)^2 - 4(4)(25) \geq 0 \quad \text{[M1]}$$

$$400 - 40p + p^2 - 400 \geq 0$$

$$p^2 - 40p \geq 0$$

$$p(p-40) \geq 0 \quad \text{[M1]}$$

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$$p \leq 0 \quad \text{or} \quad p \geq 40 \quad \text{[A1]}$$

- (ii) Hence, on the same axes, sketch the graphs of $y = 2x + 5$ and $y^2 = 5x$. [2]

$$y = 2x + 5$$

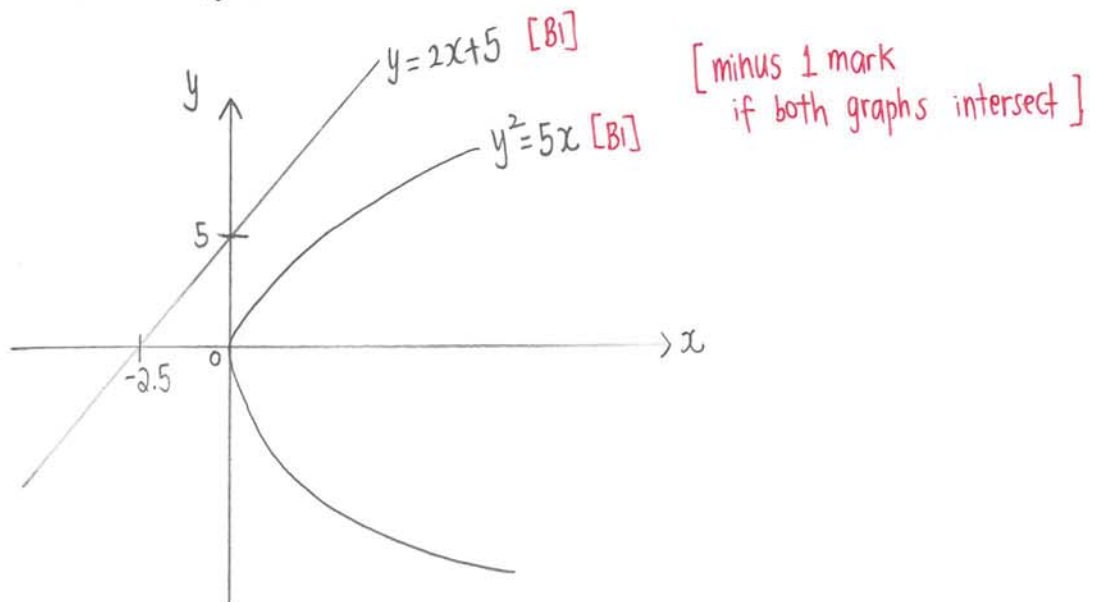
$$\text{When } x = 0,$$

$$y = 5$$

$$\text{when } y = 0,$$

$$2x + 5 = 0$$

$$x = -2.5$$



- 3 The gradient of a curve is $\frac{dy}{dx} = p + \frac{q}{x^3}$, where p and q are constants. The gradient of the normal at $A(1, 4)$ on the curve is -1 and the tangent to the curve at $B(2, 10)$ is $y = 8x - 6$.

(i) Calculate the value of p and of q .

[4]

Gradient of tangent at A = 1

When $x=1$, $\frac{dy}{dx} = 1$,

$$p + q = 1 \quad \text{[M1]}$$

$$q = 1 - p \quad \text{--- (1)}$$

When $x=2$, $\frac{dy}{dx} = 8$,

$$p + \frac{q}{8} = 8 \quad \text{[M1]}$$

$$8p + q = 64$$

$$q = 64 - 8p \quad \text{--- (2)}$$

$$\textcircled{1} = \textcircled{2} : \quad 1 - p = 64 - 8p$$

$$7p = 63$$

$$p = 9 \quad \text{[A1]}$$

Sub $p=9$ into $\textcircled{1}$,

$$q = 1 - 9$$

$$= -8 \quad \text{[A1]}$$



Using the value of p and of q found⁸ in (i),

- (b) Find the equation of the curve.

[2]

$$\frac{dy}{dx} = 9 - 8x^{-3}$$

$$\begin{aligned} y &= \int 9 - 8x^{-3} dx \\ &= 9x - \frac{8x^{-2}}{-2} + C \quad [M1] \end{aligned}$$

$$\text{Sub } x=1, y=4,$$

$$4 = 9 - \frac{8}{-2} + C$$

$$C = -9$$

$$y = 9x + \frac{4}{x^2} - 9 \quad [A1]$$

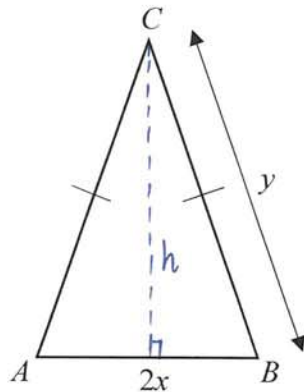
- (c) Show that gradient increases as x increases.

[2]

$$\begin{aligned} \frac{d^2y}{dx^2} &= 24x^{-4} \\ &= \frac{24}{x^4} > 0 \quad [M1] \end{aligned}$$

Since $\frac{d^2y}{dx^2} > 0$, gradient increases as x increases. [A1]

- 4 In the diagram below, $\triangle ABC$ is an isosceles triangle with $BC = y$ cm and $AB = 2x$ cm. It is also given that the perimeter of $\triangle ABC$ is 50 cm.



- (a) Show that the area of $\triangle ABC$ is given by $A = x\sqrt{625 - 50x}$. [3]

$$2x + 2y = 50$$

$$x + y = 25$$

$$y = 25 - x \quad \text{[M1]}$$

$$h = \sqrt{y^2 - x^2}$$

$$= \sqrt{(25 - x)^2 - x^2}$$

$$= \sqrt{625 - 50x} \quad \text{[M1]}$$

$$\text{Area of } \triangle ABC = \frac{1}{2} \times \sqrt{625 - 50x} \times 2x \quad \text{[A1]}$$


 $\sqrt{625 - 50x}$ (shown)

- (b) Given that x is increasing at a rate of 0.2 cm/s , find the rate at which the area is increasing at the instant when $x = 3$. [3]

$$\frac{dx}{dt} = 0.2$$

$$A = x \sqrt{625 - 50x}$$

$$\begin{aligned} \frac{dA}{dx} &= (625 - 50x)^{\frac{1}{2}} + x \left(\frac{1}{2}\right) (625 - 50x)^{-\frac{1}{2}} (-50) \quad [M1] \\ &= \sqrt{625 - 50x} - \frac{25x}{\sqrt{625 - 50x}} \end{aligned}$$

$$\begin{aligned} \frac{dA}{dt} &= \frac{dA}{dx} \times \frac{dx}{dt} \\ &= \left[\sqrt{625 - 50(3)} - \frac{25(3)}{\sqrt{625 - 50(3)}} \right] \times 0.2 \quad [M1] \end{aligned}$$

$$= 3.67 \text{ cm}^2/\text{s} \quad (\text{to 3 s.f.}) \quad [A1]$$

- 5 (a) The sum of the coefficients of the first two terms in the expansion, in descending powers of x , of $(1+2x)\left(2x-\frac{1}{x^2}\right)^n$ is 768, where n is a positive integer greater than 2. Show that n is 8.

[4]

$$\left(2x - \frac{1}{x^2}\right)^n = \binom{n}{0}(2x)^n + \binom{n}{1}(2x)^{n-1}\left(-\frac{1}{x^2}\right) + \dots \quad [\text{M1}]$$

$$= 2^n x^n - n(2^{n-1})(x^{n-1})(x^{-2}) + \dots$$

$$= 2^n x^n - n(2^{n-1})(x^{n-3}) + \dots$$

$$(1+2x)\left(2x - \frac{1}{x^2}\right)^n = (1+2x)(2^n x^n - n2^{n-1}x^{n-3} + \dots)$$

$$= \underbrace{2^n x^n}_{T_2} - n2^{n-1}x^{n-3} + \underbrace{2^{n+1}x^{n+1}}_{T_1} - n2^n x^{n-2} + \dots \quad [\text{M1}]$$

$$2^{n+1} + 2^n = 768 \quad [\text{M1}]$$

$$2^n(2+1) = 768$$

$$2^n = 256 \quad [\text{A1}]$$

$$n = 8 \quad (\text{shown})$$



term containing

(b) Hence, find the x^{-7} term in the expansion of $\left(2x - \frac{1}{x^2}\right)^8$.

[4]

$$\left(2x - \frac{1}{x^2}\right)^8$$

$$T_{r+1} = \binom{8}{r} (2x)^{8-r} \left(-\frac{1}{x^2}\right)^r \quad [M1]$$

$$= \binom{8}{r} (2)^{8-r} (x)^{8-r} (-1)^r (x)^{-2r}$$

$$= \binom{8}{r} (2)^{8-r} (-1)^r (x)^{8-3r} \quad [M1]$$

$$8-3r = -7$$

$$3r = 15$$

$$r = 5$$

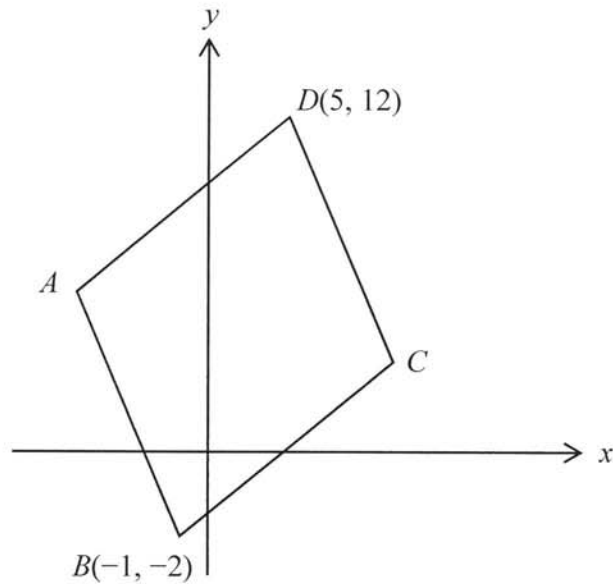
[M1]

$$T_6 = \binom{8}{5} (2)^{8-5} (-1)^5 (x)^{-7}$$

$$= -448x^{-7}$$

[A1]

6 Solutions to this question by accurate drawing will not be accepted.



In the diagram, $ABCD$ is a rhombus. The points B and D have coordinates $(-1, -2)$ and $(5, 12)$ respectively.

(a) Show that the equation of AC is $7y = -3x + 41$.

[4]

$$\text{midpoint of } BD = \left(\frac{5-1}{2}, \frac{12-2}{2} \right) \quad [M1]$$

$$= (2, 5)$$

$$\text{gradient of } BD = \frac{12-2}{5-1} \quad [M1]$$

$$= \frac{7}{3}$$

$$\text{gradient of } AC = -\frac{3}{7}$$

$$\text{Equation of } AC: y = -\frac{3}{7}x + c \quad [M1]$$

$$5 = -\frac{3}{7}(2) + c$$

$$c = \frac{41}{7}$$

$$y = -\frac{3}{7}x + \frac{41}{7}$$

$$7y = -3x + 41 \quad (\text{shown})$$

[A1]

- (b) Given that a line $5y = 3x + 55$ passes through point A , find the coordinates of A . [2]

$$7y = -3x + 41 \quad \text{--- (1)}$$

$$5y = 3x + 55 \quad \text{--- (2)}$$

$$\textcircled{1} + \textcircled{2} : \quad 12y = 96 \quad \text{[M1]}$$

$$y = 8$$

$$\text{Sub } y = 8 \text{ into } \textcircled{2},$$

$$5(8) = 3x + 55$$

$$3x = -15$$

$$x = -5$$

$$A(-5, 8) \quad \text{[A1]}$$

- (c) Find the coordinates of C . [2]

$$\text{let } C \text{ be } (x, y).$$

$$\left(\frac{-5+x}{2}, \frac{8+y}{2} \right) = (2, 5) \quad \text{[M1]}$$

$$\frac{-5+x}{2} = 2$$

$$-5+x = 4$$

$$x = 9$$

$$\frac{8+y}{2} = 5$$

$$8+y = 10$$

$$y = 2$$

$$\therefore C(9, 2) \quad \text{[A1]}$$

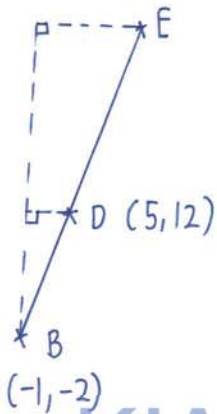
- (d) Find the area of the rhombus $ABCD$.

[2]

$$\begin{aligned}
 \text{Area} &= \frac{1}{2} \begin{vmatrix} -5 & -1 & 9 & 5 & -5 \\ 8 & -2 & 2 & 12 & 8 \end{vmatrix} \quad [\text{M1}] \\
 &= \frac{1}{2} \begin{vmatrix} 10 - 2 + 108 + 40 + 8 + 18 - 10 + 60 \end{vmatrix} \\
 &= \frac{1}{2} (232) \\
 &= 116 \text{ units}^2 \quad [\text{A1}]
 \end{aligned}$$

- (e) If point E lies on BD produced such that $BD : DE = 2 : 3$, find the coordinates of E .

[2]



diff. in x between B and $D = 6$

diff. in y between B and $D = 14$

diff. in x between D and $E = 9$

diff. in y between D and $E = 21$

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$$\begin{aligned}
 E &= (5+9, 12+21) \\
 &= (14, 33)
 \end{aligned}$$

[B2]

- 7 (a) Given that $3^{n+2} - 3^n = \frac{5^{n+1}}{25^n}$, find the value of 15^n .

[3]

$$3^n 3^2 - 3^n = \frac{5^{n+1}}{5^{2n}}$$

$$\underbrace{3^n (3^2 - 1)}_{\text{[M1]}} = 5^{n+1-2n}$$

$$3^n (8) = 5^{1-n}$$

$$= 5^1 5^{-n} \quad \text{[M1]}$$

$$3^n (5^n) = \frac{5}{8}$$

$$15^n = \frac{5}{8} \quad \text{[A1]}$$

- (b) Without using a calculator, find the root of the equation $x\sqrt{80} = \sqrt{20} - x\sqrt{48}$ in the form $\frac{a+b\sqrt{c}}{4}$. [3]

$$x\sqrt{80} = \sqrt{20} - x\sqrt{48}$$

$$4x\sqrt{5} = 2\sqrt{5} - 4x\sqrt{3}$$

[M1] - simplify surds

$$4x\sqrt{5} + 4x\sqrt{3} = 2\sqrt{5}$$

$$x(4\sqrt{5} + 4\sqrt{3}) = 2\sqrt{5}$$

$$x = \frac{2\sqrt{5}}{4\sqrt{5} + 4\sqrt{3}}$$

[M1]

$$= \frac{\sqrt{5}}{2\sqrt{5} + 2\sqrt{3}} \times \frac{2\sqrt{5} - 2\sqrt{3}}{2\sqrt{5} - 2\sqrt{3}}$$

$$= \frac{10 - 2\sqrt{15}}{(2\sqrt{5})^2 - (2\sqrt{3})^2}$$

$$= \frac{10 - 2\sqrt{15}}{20 - 12}$$

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$$= \frac{5 - \sqrt{15}}{4}$$

[A1]

=

- 8 A particle moves in a straight line from a point O such that t seconds after leaving O , its velocity, v m/s, is given by $v = 5(4t-1)^2 - 125$. Find

- (a) the initial acceleration of the particle, [2]

$$\begin{aligned}
 a &= \frac{dv}{dt} \\
 &= 10(4t-1)(4) \quad [M1] \\
 \text{sub } t=0, \\
 a &= 10(-1)(4) \\
 &= -40 \text{ m/s}^2 \quad [A1]
 \end{aligned}$$

- (b) the value of t at which the particle is instantaneously at rest, [2]

$$\begin{aligned}
 \text{when } v &= 0, \\
 5(4t-1)^2 - 125 &= 0 \quad [M1] \\
 (4t-1)^2 &= 25 \\
 4t-1 &= 5 \quad \text{or} \quad -5 \quad (\text{rej.}) \\
 t &= 1.5 \text{ s} \quad [A1]
 \end{aligned}$$

- (c) the minimum velocity of the particle. [1]

$$\text{minimum velocity} = -125 \text{ m/s} \quad [B1]$$

(d) the distance travelled by the particle in the second second,

[5]

$$s = \int 5(4t-1)^2 - 125 \, dt$$

$$= \frac{5(4t-1)^3}{3(4)} - 125t + C \quad [M1]$$

$$\text{Sub } t=0, \quad s=0,$$

$$0 = \frac{5(-1)^3}{12} + C$$

$$C = \frac{5}{12}$$

$$s = \frac{5(4t-1)^3}{12} - 125t + \frac{5}{12} \quad [M1]$$

$$\text{Sub } t=1,$$

$$s = \frac{5(3)^3}{12} - 125 + \frac{5}{12}$$

$$= -113\frac{1}{3}$$

$$\text{Sub } t=1.5,$$

$$s = \frac{5(4(1.5)-1)^3}{12} - 125(1.5) + \frac{5}{12}$$

$$= -135 \quad [M1]$$

$$\text{Sub } t=2,$$

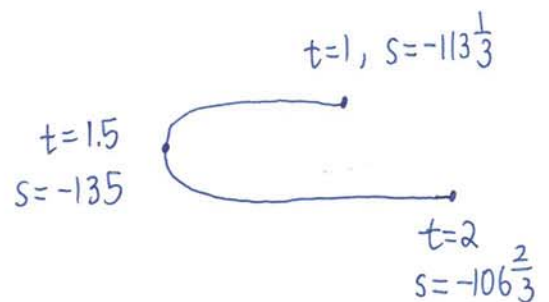
$$s = \frac{5(7)^3}{12} - 125(2) + \frac{5}{12}$$

$$= -106\frac{2}{3}$$

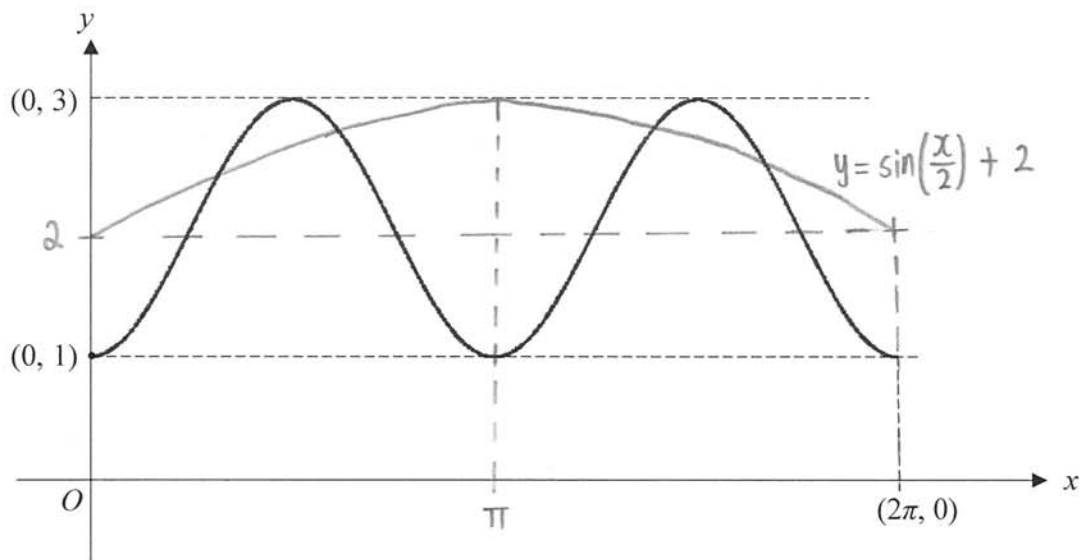
$$\text{Distance travelled} = (135 - 113\frac{1}{3}) + (135 - 106\frac{2}{3}) \quad [M1]$$

$$= 50\text{m} \quad [A1]$$

=



9



The diagram shows the curve $y = a + b \cos(cx)$ for $0 \leq x \leq 2\pi$.

- (a) Write down the value of a , of b and of c .

[3]

$$\text{period} = \pi$$

$$\frac{2\pi}{c} = \pi$$

$$c = 2 \quad [B1]$$

$$a = 2 \quad [B1]$$

$$b = -1 \quad [B1]$$

- (b) Sketch, on the same axes above, the graph of $y = \sin\left(\frac{x}{2}\right) + 2$ for $0 \leq x \leq 2\pi$. [3]

[1] shape

[1] period

[1] maximum.

$$\begin{aligned} \text{period} &= 2\pi \div \frac{1}{2} \\ &= 4\pi \end{aligned}$$

$$-1 \leq \sin\left(\frac{x}{2}\right) \leq 1$$

$$1 \leq \sin\left(\frac{x}{2}\right) + 2 \leq 3$$

- (c) Deduce the largest integer value of k such that $a + b \cos(cx) > \sin\left(\frac{x}{2}\right) + k$ for $0 \leq x \leq 2\pi$. [1]

$$k = -1 \quad [B1]$$

- 10 A curve has the equation $y = x^3 e^{2x}$.

Find the x -coordinates of the stationary points and determine the nature of each point [7]

$$\frac{dy}{dx} = 3x^2 e^{2x} + 2x^3 e^{2x} \quad [M1]$$

$$= x^2 e^{2x} (3 + 2x)$$

$$\frac{dy}{dx} = 0$$

$$x^2 e^{2x} (3 + 2x) = 0 \quad [M1]$$

$$x = 0 \quad [A1] \text{ or } x = -\frac{3}{2} \quad [A1]$$

By First Derivative Test,

x	-0.1	0	0.1		-1.6	-1.5	-1.4
$\frac{dy}{dx}$	+ve	0	+ve		-ve	0	+ve
	/	-	/		\	-	/

[M1]

point of inflexion [A1]

at $x=0$


minimum point [A1]
 at $x = -\frac{3}{2}$.

- 11 In an experimental environment, the population of a type of insect was observed. Over a period of 10 days from the start of the experiment, the number of insects decreased from 1100 to 600. The insect population is given by the formula $P = A + 900e^{kt}$, where A and k are constants and t is the number of days from the start of the experiment.

(a) Find the value of A and of k .

[4] [3]

$$P = A + 900e^{kt}$$

when $t=0$, $P=1100$,

$$1100 = A + 900$$

$$A = 200. \quad [B1]$$

when $t=10$, $P=600$,

$$600 = 200 + 900e^{10k}$$

$$e^{10k} = \frac{4}{9} \quad [M1]$$

$$10k = \ln \frac{4}{9}$$

$$k = -0.0811 \quad (\text{to 3 s.f.}) \quad [A1]$$

- (b) Explain why the population of the insects approaches the value of A after a long period of time.

[1]

As t increases, e^{kt} will approach 0,
so P will approach 200. } [B1]

End of Paper

1(i)	$\begin{aligned} f(-2) &= -8 + 4a - 2b + c \\ f(2) &= 8 + 4a + 2b + c \\ -8 + 4a - 2b + c &= 8 + 4a + 2b + c \\ -8 - 2b &= 8 + 2b \\ 4b &= -16 \\ b &= -4 \end{aligned}$	M1 A1
(ii)	$\begin{aligned} f(1) &= 1 + a - 4 + c = a + c - 3 \\ f(1) &= f(2) \\ a + c - 3 &= 8 + 4a - 8 + c \\ a - 3 &= 4a \\ a &= -1 \end{aligned}$	M1 A1
(iii)	$\begin{aligned} f(x) &= x^3 - x^2 - 4x + c \\ f(3) &= 27 - 9 - 12 + c \\ 6 + c &= 4 \\ c &= -2 \end{aligned}$	A1
2(i)	$\begin{aligned} p\left(\frac{1}{2}\right) &= 2\left(\frac{1}{2}\right)^3 - \left(\frac{1}{2}\right)^2 + 16\left(\frac{1}{2}\right) - 8 \\ &= 2\left(\frac{1}{8}\right) - \frac{1}{4} + 8 - 8 \\ &= 0 \end{aligned}$ Since remainder is 0, $2x - 1$ is a factor of $p(x)$.	A1
2(ii)	$\begin{aligned} 2x^3 - x^2 + 16x - 8 &= 0 \\ x^2(2x - 1) + 8(2x - 1) &= 0 \\ (2x - 1)(x^2 + 8) &= 0 \\ 2x - 1 = 0 &\quad x^2 + 8 = 0 \\ x = \frac{1}{2} &\quad \text{no real solution since } x^2 + 8 > 0 \end{aligned}$ There is only one real root. $\begin{array}{r} \underline{x^2} + 8 \\ 2x-1 2x^3 - x^2 + 16x - 8 \\ \underline{-(2x^3 - x^2)} \\ 0 + 16x - 8 \\ \underline{-(16x-8)} \\ 0 \end{array}$	M1 A1 , A1

2(iii)	$\frac{x^2 + 2x + 40}{2x^3 - x^2 + 16x - 8} = \frac{A}{2x-1} + \frac{Bx+C}{x^2+8}$ $= \frac{A(x^2+8) + (2x-1)(Bx+C)}{(2x-1)(x^2+8)}$ $x^2 + 2x + 40 = A(x^2+8) + (2x-1)(Bx+C)$ $\text{Sub } x = \frac{1}{2}$ $\left(\frac{1}{2}\right)^2 + 2\left(\frac{1}{2}\right) + 40 = A\left(\frac{1}{4} + 8\right)$ $A = 5$ $\text{Sub } x = 0, A = 5$ $40 = 40 - C$ $C = 0$ $\text{Sub } x = 1$ $1 + 2 + 40 = 5(9) + B$ $B = -2$ $\text{Ans : } \frac{5}{2x-1} - \frac{2x}{x^2+8}$	M1 M1 A1 A1 A1 Deduct 1 m if final ans not shown.
2(iv)	$\int \frac{x^2 + 2x + 40}{2x^3 - x^2 + 16x - 8} dx$ $= \int \frac{5}{2x-1} - \frac{2x}{x^2+8} dx$ $= \frac{5}{2} \ln(2x-1) - \ln(x^2+8) + c$	A1 (1st term) A2 (2nd term) Deduct 1 m if c is missing
3(a)	$\log_2(x+2) - \frac{\log_2(x-1)}{\log_2 \sqrt{2}} = 3$ $\log_2(x+2) - 2\log_2(x-1) = 3$ $\log_2 \frac{(x+2)}{(x-1)^2} = 3$ $\frac{x+2}{(x-1)^2} = 8$ $x+2 = 8(x-1)^2$ $8x^2 - 17x + 6 = 0$ $x = \frac{17 \pm \sqrt{(-17)^2 - 4(8)(6)}}{2(8)}$ $x = \frac{17 \pm \sqrt{97}}{16}$ $x = 1.68, 0.447 \text{ (rejected)}$	M1 M1 M1 M1 A1 [deduct 1 m if did not reject 0.447]

3(b)	Sub $x = 2, y = 47$ $a \times 2^b + 7 = 47$ $a \times 2^b = 40$ ----- (1) Sub $x = -3, y = -128$ $a \times (-3)^b + 7 = -128$ $a \times (-3)^b = -135$ ----- (2) $\frac{(1)}{(2)} \quad \frac{2^b}{(-3)^b} = \frac{40}{-135}$ $\left(\frac{2}{-3}\right)^b = -\frac{8}{27} = \left(\frac{2}{-3}\right)^3$ $b = 3$ Sub $b = 3$ into (1) $a \times 2^3 = 40$ $a = 5$ $y = 5x^3 + 7$ Sub $x = 5, y = k$ $k = 5(5)^3 + 7 = 632$	M1 for eqn (1) & (2) M1 A1 A1 A1
4(i)	$d = 8 \sin \theta - 5 \cos \theta$	A2 [1 m for each term]
4(ii)	$d = R \sin(\theta - \alpha)$ $R = \sqrt{8^2 + 5^2} = \sqrt{89}$ $\tan \alpha = \frac{5}{8}$ $\alpha = 32.0^\circ$ $d = \sqrt{89} \sin(\theta - 32.0^\circ)$	A1 A1 A1
4(iii)	Max value of $d = \sqrt{89} = 9.43 \text{ m}$ when $\sin(\theta - 32.0^\circ) = 1$ or $\theta = 122^\circ$ Not possible for d to be 10 m	A1 A1
4(iv)	$\sqrt{89} \sin(\theta - 32.0^\circ) = 6$ $\sin(\theta - 32.0^\circ) = \frac{6}{\sqrt{89}}$ $\theta - 32.0^\circ = 39.49^\circ, 140.51^\circ$ $\theta = 71.5^\circ, 172.5^\circ$	A1, A1

6(i)	$\text{LHS} = \frac{\cos \sec^2 \theta - 2}{\cos \sec^2 \theta}$ $= 1 - \frac{2}{\cos \sec^2 \theta}$ $= 1 - 2 \sin^2 \theta$ $= \cos 2\theta$	B1 B1 A1
6(ii)	$\cos 2\theta + 3 \sin 2\theta = 0$ $3 \sin 2\theta = -\cos 2\theta$ $\tan 2\theta = -\frac{1}{3}$ $\text{basic } \angle = 0.3218$ $2\theta = \pi - 0.3218, 2\pi - 0.3218, 3\pi - 0.3218$ $\theta = 1.41, 2.98, 4.55$	M1 M1 A2 Deduct 1 m for each wrong ans
7	$\alpha - \beta = 2$ $\alpha^2 - \beta^2 = 3$ $(\alpha + \beta)(\alpha - \beta) = 3$ $\alpha + \beta = \frac{3}{2}$ $(\alpha + \beta)^2 = \frac{9}{4}$ $\alpha^2 + \beta^2 + 2\alpha\beta = \frac{9}{4}$ $(\alpha - \beta)^2 + 4\alpha\beta = \frac{9}{4}$ $4 + 4\alpha\beta = \frac{9}{4}$ $\alpha\beta = -\frac{7}{16}$ Equation is $x^2 - \frac{3}{2}x - \frac{7}{16} = 0$ $16x^2 - 24x - 7 = 0$	A1 M1 M1 A1 A1
8(i)	At A, $\sec^2 x - 2 = 0$ $\cos^2 x = \frac{1}{2}$ $\cos x = \sqrt{\frac{1}{2}}$ $x = \frac{\pi}{4}$	A1

	At B, $\sec^2 x - 2 = 2$ $\sec^2 x = 4$ $\cos^2 x = \frac{1}{4}$ $\cos x = \sqrt{\frac{1}{4}}$ $x = \frac{\pi}{3}$	M1 A1
8(ii)	Area of $\triangle OBC = \frac{1}{2} \times \frac{\pi}{3} \times 2 = \frac{\pi}{3} \text{ units}^2$, C is $\left(\frac{\pi}{3}, 0\right)$ Area bounded by curve, x-axis and line $x = \frac{\pi}{3}$ $= \int_{\frac{\pi}{4}}^{\frac{\pi}{3}} (\sec^2 x - 2) dx$ $= \left[\tan x - 2x \right]_{\frac{\pi}{4}}^{\frac{\pi}{3}}$ $= \tan \frac{\pi}{3} - \frac{2\pi}{3} - \left(\tan \frac{\pi}{4} - \frac{\pi}{2} \right)$ $= \sqrt{3} - 1 - \frac{\pi}{6}$ Required area = $\frac{\pi}{3} - \left(\sqrt{3} - 1 - \frac{\pi}{6} \right)$ $= \left(1 - \sqrt{3} + \frac{\pi}{2} \right) \text{ units}^2$	A1 M1 A1 M1 A1
9(i)	When $y = 0$, $1 - 2x = 3$ $1 - 2x = 3$ or $1 - 2x = -3$ $x = -1$ or $x = 2$ A(-1 , 0) C(2 , 0) B$\left(\frac{1}{2}, 3\right)$	A1 , A1 A1
9(ii)	$\tan \theta = -2$ ($\tan \theta = \text{gradient of BC}$)	A1
9(iii)	$3 - 1 - 2x = 2x - x^2$ $x^2 - 2x + 3 = 1 - 2x $ $x^2 - 2x + 3 = 1 - 2x$ $x^2 + 2 = 0$ No real solution as $x^2 + 2 > 0$ for all real values of x .	B1 A1

	$x^2 - 2x + 3 = 2x - 1$ $x^2 - 4x + 4 = 0$ $(x - 2)^2 = 0$ $x = 2$	B1 A1												
9(iv)	Max value of $3 - 1 - 2x $ is 3 when $x = \frac{1}{2}$. Min value of $4 + x^2$ is 4 when $x = 0$. The curve and line <u>do not intersect</u> so there is no real solution.	B1 B1 A1												
10(i)	Let the centres of C_1 and C_2 be (a, a) $a^2 + (a + 3)^2 = 5$ $a^2 + a^2 + 6a + 9 - 5 = 0$ $2a^2 + 6a + 4 = 0$ $(2a + 2)(a + 2) = 0$ $a = -1$ or $a = -2$ $C_1 : (x + 1)^2 + (y + 1)^2 = 5$ $C_2 : (x + 2)^2 + (y + 2)^2 = 5$	M1 M1 A1 A1 A1												
10(ii)	$C_1 : \text{Sub } y = 0$ $(x + 1)^2 + 1 = 5$ ----- (1) $C_2 : \text{Sub } y = 0$ $(x + 2)^2 + 4 = 5$ ----- (2) $(x + 1)^2 + 1 = (x + 2)^2 + 4$ $x^2 + 2x + 1 + 1 = x^2 + 4x + 4 + 4$ $2x + 6 = 0$ $x = -3$	A1 (both (1) & (2) correct) M1 A1												
10(iii)	Dist between 2 centres = $\sqrt{(-1 + 2)^2 + (-1 + 2)^2} = \sqrt{2}$ Greatest dist = $\sqrt{5} + \sqrt{2} + \sqrt{5}$ $= \sqrt{2} + 2\sqrt{5}$ or 5.89 units	A1 M1 A1												
11(i)	$y = \frac{a}{\sqrt{x}} + b\sqrt{x}$ $\frac{y}{\sqrt{x}} = \frac{a}{x} + b$ <table border="1"><tr><td>$\frac{1}{x}$</td><td>1</td><td>0.5</td><td>0.33</td><td>0.25</td><td>0.20</td></tr><tr><td>$\frac{y}{\sqrt{x}}$</td><td>0.5</td><td>1.50</td><td>1.84</td><td>2.0</td><td>2.10</td></tr></table>	$\frac{1}{x}$	1	0.5	0.33	0.25	0.20	$\frac{y}{\sqrt{x}}$	0.5	1.50	1.84	2.0	2.10	A1 (table) B2 (4 to 5 correct points for line of best fit)
$\frac{1}{x}$	1	0.5	0.33	0.25	0.20									
$\frac{y}{\sqrt{x}}$	0.5	1.50	1.84	2.0	2.10									

11(ii)	$a = -2$ $b = 2.5$	M1 , A1 A1								
11(iii)	$y\sqrt{x} = 3$ $\frac{y}{\sqrt{x}} = \frac{3}{x}$ Draw $\frac{y}{\sqrt{x}}$ against $\frac{1}{x}$. <table border="1"><tr><td>$\frac{1}{x}$</td><td>1</td><td>0.5</td><td>0.25</td></tr><tr><td>$\frac{y}{\sqrt{x}}$</td><td>3</td><td>1.5</td><td>0.75</td></tr></table> Point of intersection is (0.5 , 1.5) $\frac{1}{x} = 0.5$ $x = 2$ $\frac{y}{\sqrt{x}} = 1.5$ $y = 1.5 \times \sqrt{2} = 2.12$	$\frac{1}{x}$	1	0.5	0.25	$\frac{y}{\sqrt{x}}$	3	1.5	0.75	A1 B1 (str line graph) A1 A1 (correct x value) A1 (correct y value)
$\frac{1}{x}$	1	0.5	0.25							
$\frac{y}{\sqrt{x}}$	3	1.5	0.75							

