Name:	Class:	Class Register Number:

中正中学

CHUNG CHENG HIGH SCHOOL (MAIN)

PRELIMINARY EXAMINATION 2020
SECONDARY 4

ADDITIONAL MATHEMATICS 4047/01

Paper 1 **Tuesday 15 September 2020**

2 hours

Candidates answer on the Question Paper.

READ THESE INSTRUCTIONS FIRST

Write your name, class and index number in the spaces at the top of this page.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staple, paper clips, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.

The total number of marks for this paper is 80.

For Candidate's Use	For Examiner's Use
Question Number	Marks Obtained
1	
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Total Marks	

This document consists of **20** printed pages.

[Turn over

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$.

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

$$\Delta = \frac{1}{2}ab\sin C$$

1 Find the range of values of the constant k for which the curve $y = x^2 + 3x + 4$ lies entirely above the line $y = kx + 3$. [4]

2 Sketch the graph of $y = |x^2 - 1|$, showing the **coordinates** of the points where the graph meets the x-axis and the turning point of the graph. [3]

3 A polynomial $\mathrm{f}(x) = 2x^3 - 5x^2 + ax + b$, where a and b are constants, leaves a remainder of -4 when divided by $x - 1$. The graph of $y = \mathrm{f}(x)$ has a stationary point at $x = 2$.

(i) Show that $a = -2$ and $b = 3$. [5]

(ii) Show that $x+1$ is a factor of $\mathrm{f}(x)$ and solve the equation $\mathrm{f}(x)=0$. [4]

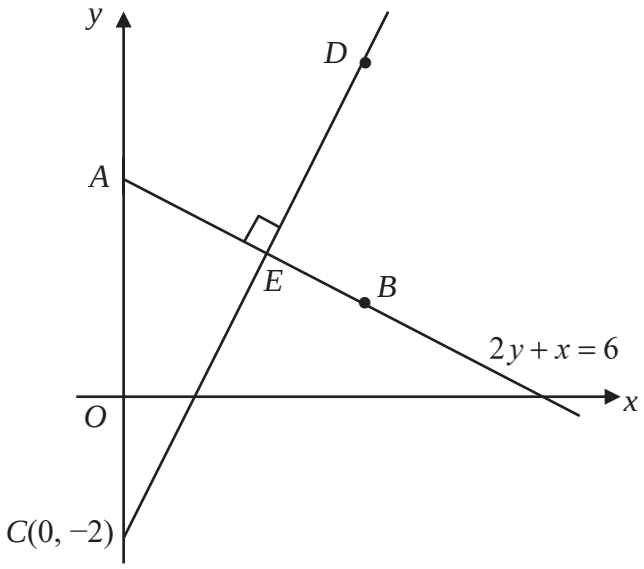
4 **(i)** By completing the square, express $x^2 - x + 1$ in the form $(x - p)^2 + q$ where p and q are constants. [2]

(ii) Show that the curve $y = x^2 - 2px + p - 1$ will always cut the x-axis at two distinct points for all real values of p. [3]

[Turn over

5 The equation of a curve is $y = \ln\left(\frac{x-3}{x+6}\right)$ for $x > 3$. Determine whether y is an increasing or decreasing function. [5]

6 In the diagram, the line $2y + x = 6$ cuts the y-axis at point A and passes through point B. The line CD cuts the y-axis at $(0, -2)$ and intersects line AB at point E. The two lines AB and CD are perpendicular to each other.



(i) Find the coordinates of E. [4]

[Turn over

(ii) The ratio of AE : AB is 3 : 5. Find the coordinates of B. [4]

It is given that BD is parallel to the y-axis.

(iii) Find the coordinates of D. [2]

(iv) Find the area of quadrilateral *ACBD*. [2]

7 Variables x and y are related by the equation $y = ab^x$ where a and b are constants.

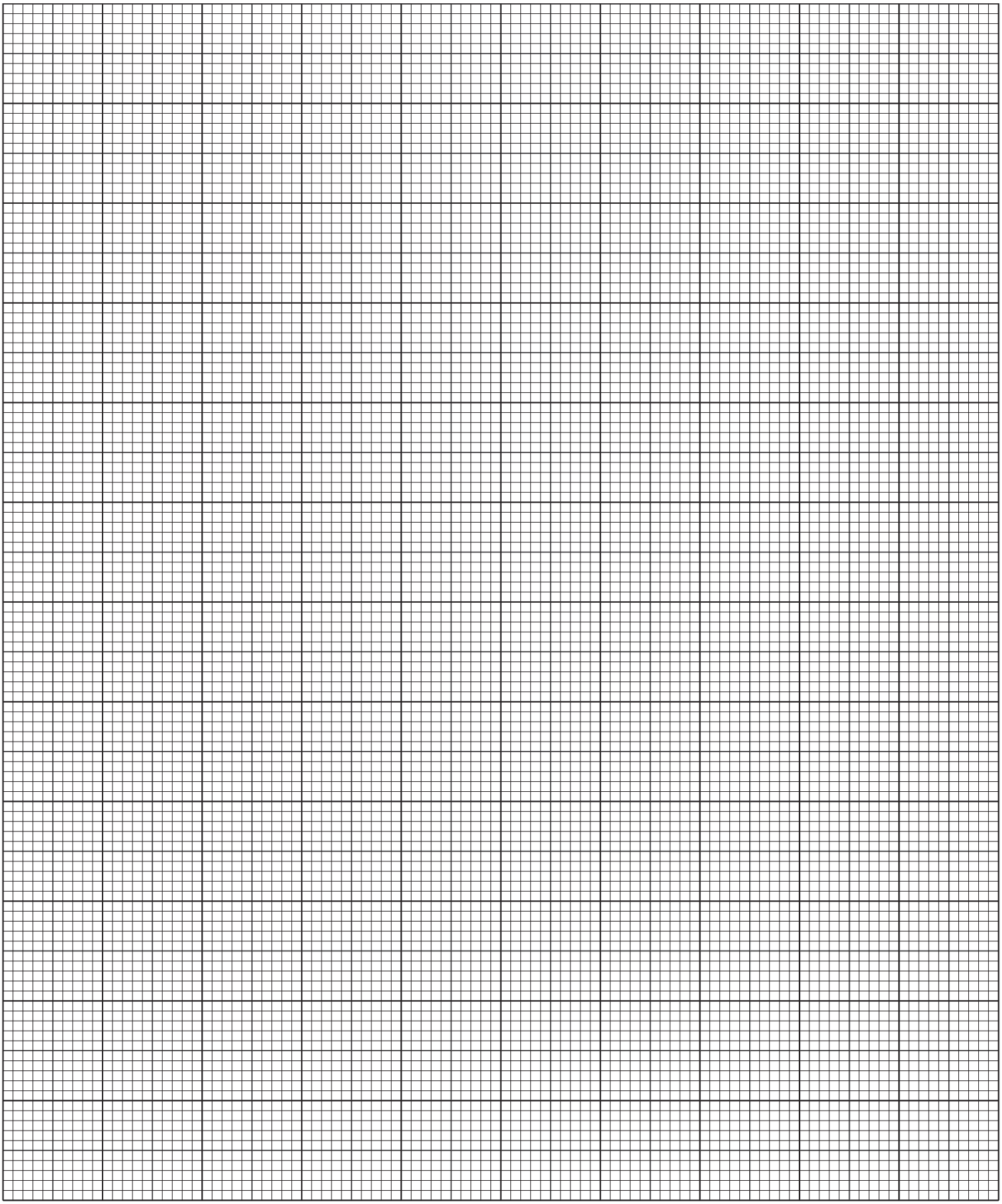
The table below shows corresponding values of x and y.

x	1	2	3	4	5	6
y	4.8	9.6	19.2	38.4	76.8	153.6

(i) Draw the graph of $\lg y$ plotted against x, using a scale of 2 cm for 1 unit on the x-axis and 1 cm for 0.1 unit on the $\lg y$-axis. [3]

(ii) Use the graph to estimate the value of a and of b. [3]

[Turn over



(iii) By adding a suitable straight line to your graph in **part (i)**, estimate the solution to the equation $ab^x = 10^{2-\frac{x}{3}}$. [2]

(iv) The point $(14, k)$ lies on the graph of $\lg y$ against x, using the values of a and b that were found in part **(ii)**, find the value of k. [1]

8 The surface area of an object, A cm^2, is given by $A = 300x - \left(\frac{\pi + 4}{8}\right)x^2$. Given that x cm can vary, find the stationary value of A and determine whether it is a maximum or minimum. [5]

[Turn over

9 **(a)** Solve the equation $\sqrt{8-4x}+\sqrt{2-x}=3x$. [4]

(b) A cuboid of volume $\left(54-11\sqrt{3}\right)$ cm^3 has a base area $\left(7+2\sqrt{3}\right)$ cm^2 and height h cm.

Without using a calculator, obtain an expression for h in the form $a+b\sqrt{3}$, where a and b are integers. [4]

10 It is given that $6\sin^2 A - 4\cos^2 A = 5\sin 2A$ where $0 \le A \le 90°$.

(i) Show that $\tan A = 2$. [3]

(ii) **Hence,** find the value of $\cos\left(60^\circ + A\right)$, leaving your answer in the form $\frac{a+b\sqrt{3}}{2\sqrt{5}}$. [3]

(iii) Without finding the value of A, explain whether $60° + A$ is acute or obtuse. [1]

[Turn over

11 Differentiate $x\cos(2x+1)$ with respect to x. Hence, find $\int 4x\sin(2x+1)\,dx$. [5]

12 A curve has the equation $y = -2\left(1 - \frac{1}{4}x\right)^2 + p$.

(i) Find $\frac{dy}{dx}$. Hence, find the equation of the line that is parallel to the tangent of the curve at $x = 1$ and passes through the origin. [3]

The normal to the curve at $x = 1$ passes through the point $\left(-1,\ 1\frac{1}{2}\right)$.

(ii) Find the value of the constant p, [4]

[Turn over

Continuation of working space for Question 12

Name:	Class:	Class Register Number:

CHUNG CHENG HIGH SCHOOL (MAIN)

PRELIMINARY EXAMINATION 2020

SECONDARY 4

ADDITIONAL MATHEMATICS 4047/02

Paper 2 **Thursday 17 September 2020**

2 hours 30 minutes

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1 It is given that $y = \dfrac{2}{x-1}$ and y is increasing at a constant rate of 5 units per second.

(i) Find the rate of change of x when $x = 4$. [3]

(ii) Find the value of the constant k for which $(x-1)^3\left(\dfrac{\mathrm{d}^2y}{\mathrm{d}x^2} - 2\dfrac{\mathrm{d}y}{\mathrm{d}x}\right) = kx$. [4]

2 **(a)** Solve the equation $\log_3(x-2) = \log_3(12-x) - 2$. [4]

(b) Given that $(2\log_5 y)(\log_x 5) = 8$, express y in terms of x. [3]

(c) Express $e^{4x} + 7 = 3e^{2x}$ as a quadratic equation in e^{2x} and explain why there are no real solutions. [3]

3 The roots of the quadratic equation $2x^2 + 3x + 5 = 0$ are α and β.

(i) Find the value of $\alpha^2 + \beta^2$. [3]

(ii) Show that the value of $\dfrac{\alpha^3 + \beta^3}{\alpha\beta}$ is $\dfrac{63}{20}$. [2]

(iii) Find a quadratic equation whose roots are $\dfrac{\alpha^2}{\beta}$ and $\dfrac{\beta^2}{\alpha}$. [3]

4 **(i)** Show that $\dfrac{\text{cosec}\,\theta + \sin\theta}{\text{cosec}\,\theta - \sin\theta} = 2\sec^2\theta - 1$. [4]

(ii) Hence, solve the equation $\dfrac{\text{cosec}\,2x + \sin 2x}{\text{cosec}\,2x - \sin 2x} = 3$ for $0 \le x \le \dfrac{\pi}{2}$, leaving your answers in terms of π. [4]

5 **(i)** Write down, and simplify, the first 4 terms in the expansion $(1+x)^7$ in ascending powers of x. [2]

(ii) Write down the general term in the binomial expansion of $\left(x^2-\frac{2}{x^3}\right)^9$. [1]

(iii) Write down the power of x in this general term. [1]

(iv) Hence, or otherwise, determine the coefficient of x^3 in the expansion of $(1+x)^7+\left(x^2-\frac{2}{x^3}\right)^9$. [2]

6 It is given that $f(x) = 3\cos\left(\frac{x}{2}\right) + k$, where k is a constant. The graph of $y = f(x)$ passes through the point $(\pi, -1)$.

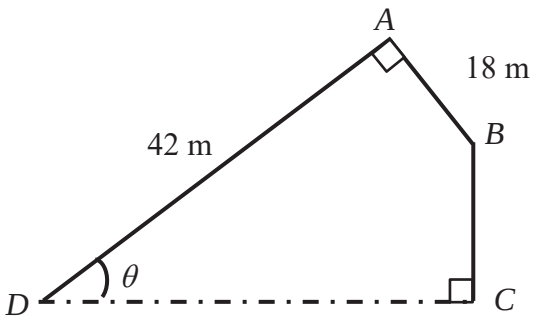
(i) State the amplitude and period of $f(x)$. [2]

(ii) Find the value of k. [2]

(iii) Sketch the graph of $y = f(x)$ for $0 \le x \le 3\pi$. [3]

(iv) By drawing a suitable straight line to the sketch in **(iii)**, find the number of solutions to the equation $f(x) + 4 = \frac{x}{2\pi}$ for $0 \le x \le 3\pi$. [1]

7



The diagram shows a plot of land being fenced up along AB, BC and AD, where $AB = 18$ m, $AD = 42$ m, angle DAB = angle $BCD = 90^{\circ}$ and the acute angle $ADC = \theta$ can vary.

(i) Show that L m, the length of the fence, can be expressed as $L = 60 + 42\sin\theta - 18\cos\theta$. [2]

(ii) Express L in the form $p + R\sin(\theta - \alpha)$, where p and $R > 0$ are constants and $0° < \alpha < 90°$. [4]

(iii) Given that the exact length of the fence used is 92 m, find the value of θ. [3]

8 Given that $\frac{4x^2 - x - 3}{2x^2 - x} = a + \frac{bx + c}{2x^2 - x}$,

(i) find the value of each of the integers a, b and c. [2]

(ii) Using the values of b and c obtained in part **(i)**, express $\frac{bx + c}{2x^2 - x}$ in partial fractions. [4]

Hence, using parts **(i)** and **(ii)**, find

(iii) $\int \frac{8x^2 - 2x - 6}{2x^2 - x} \, dx.$ [4]

9 A bee leaves its hive and flies towards a flower in a straight path. The velocity, v m/s, that it flies in t s after it leaves the hive is given by

$$v = 4e^{-\frac{t}{5}} - 2.$$

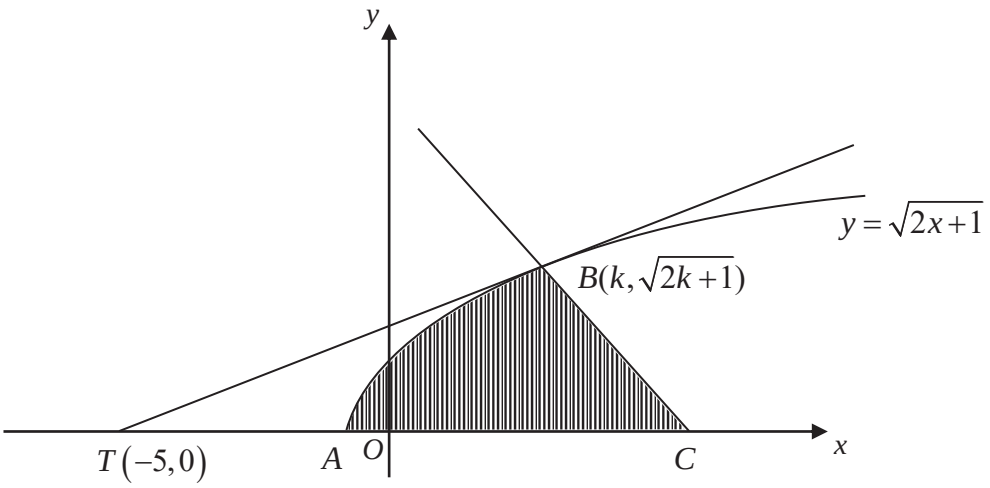
(i) Find the bee's acceleration 3 seconds after it leaves the hive. [3]

The bee comes to an instantaneous rest when it reaches the flower.

(ii) Show that the time when the bee reaches the flower is $t = \ln 32$. [3]

(iii) Calculate the distance of the flower from the hive. [4]

10



The diagram shows part of the curve $y = \sqrt{2x+1}$ passing through the point $B(k, \sqrt{2k+1})$, where k is a constant, $k > -\frac{1}{2}$. The curve meets the x-axis at the point A. The tangent to the curve at B meets the x-axis at the point $T(-5, 0)$. Another line passing through B meets the $x-$axis at C.

(i) Prove that $k = 4$. [5]

(ii) Given that the area of triangle BTC is $\frac{33}{2}$ units^2, show that the x-coordinate of C is 6. [2]

The curve meets the x-axis at the point $A\left(-\frac{1}{2}, 0\right)$.

(iii) Find the area of the shaded region BCA. [5]

11 A tangent to a circle at the point $A(-6,12)$ intersects the y-axis at $y = \frac{15}{2}$. The normal to the circle at point B has equation $3y + 4x = -12$.

(i) Showing all your working, find the equation of the circle. [7]

(ii) The tangent at A meets the x-axis at C. Find the coordinates of C. [2]

(iii) The point B is to the right and below the centre of the circle, and the normal to the circle at B intersects the y-axis at the point D.
Find the distance BD. [3]

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