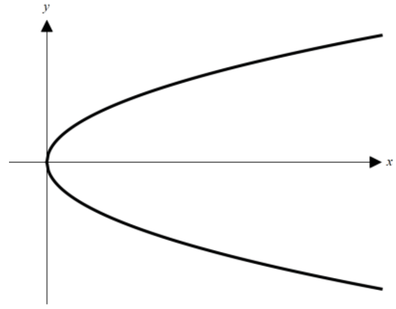
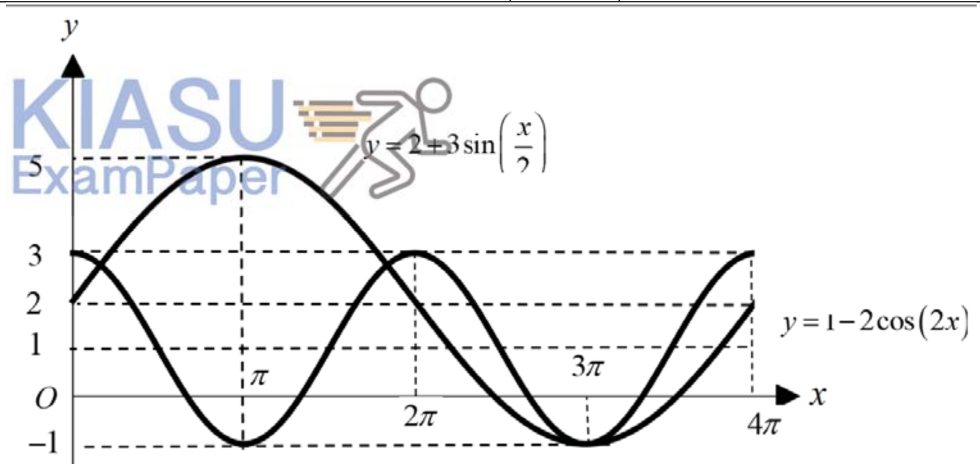
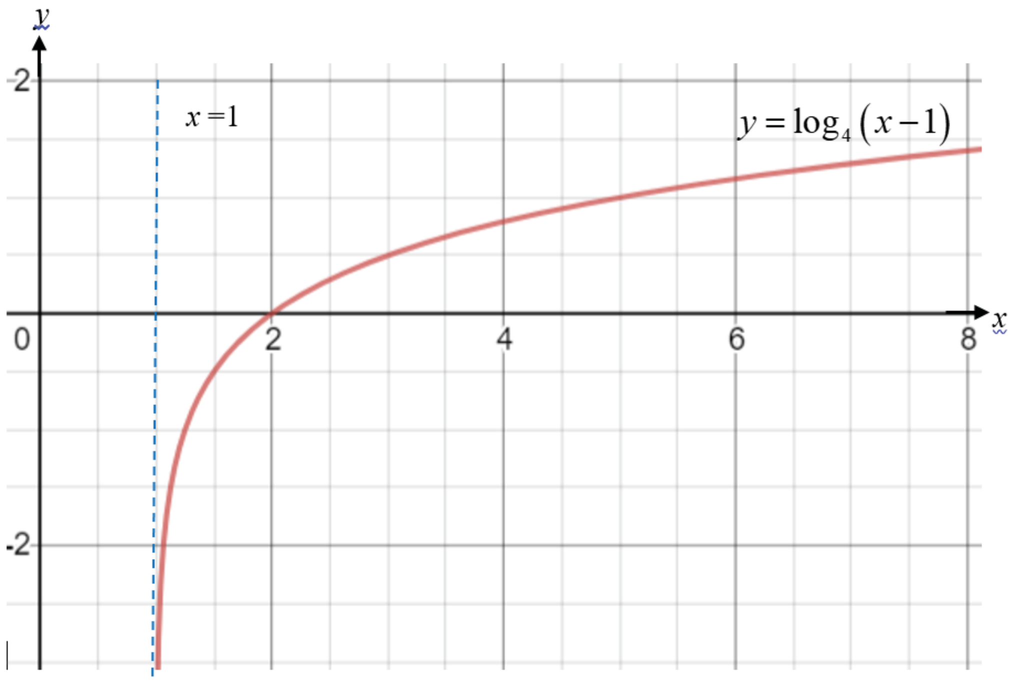
Answer Key

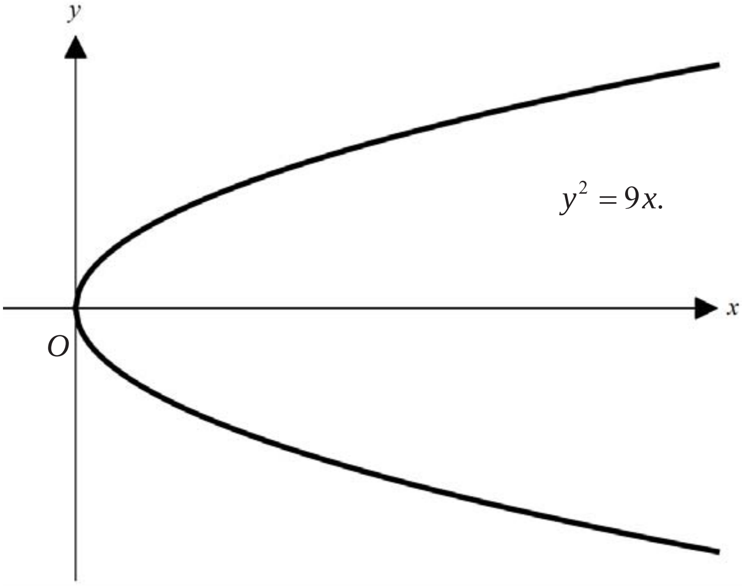
1	$8-4\sqrt{3}$ **2**	**2(i)**	
2(ii)	$k=\pm 48$	**3**	$-\frac{4}{5}$
4	$k<-3$ or $k>1$	**5(i)**	$\therefore \mathrm{f}(x)=\ln(x-2)-\frac{1}{2e^{2x}}-\frac{1}{2e^{6}}$
5(ii)	The function $\mathrm{f}'(x)=\frac{1}{x-2}+e^{-2x}$ is decreasing.		
7(a)	$\therefore x=1, x=1.69$ or -1.19	**7(b)**	256
8	$x^2+x+\frac{1}{4}=0$ or $4x^2+4x+1=0$	**9(ii)**	$\int \frac{\ln x}{x^4}\,dx=-\frac{1}{9x^3}-\frac{\ln x}{3x^3}+c$
10(i)	$y=-x+2$	**10(ii)**	10 unit^2
11(ii)	$\frac{1}{a}=-1\pm 0.1$ $a=-1$ (accept -1.11 to -0.909) $-\frac{b}{a}=\text{Gradient}$ $=2\pm 0.1$ $b=a\times gradient$	**11(iii)**	Accept from 1.24 to 1.28
12(i)	$(10,-5)$	**12(ii)**	$y=0$ or $y=-\frac{4}{3}x$
12(iii)	$(10,0)$ and $(6,-8)$	**13(i)**	$\therefore A=\frac{3\pi}{2}$, 0.253 or 2.89
13(ii)			
13(iii)	Replacing x with 2A.		

Answers

1(a)	0.528	1(b)	0.630, 1.41	1(ci)	$p=4,\ q=0$
1 (cii)					
2(i)	$P=750$	2(ii)	$k=0.0522$		
2(iii)	As t becomes very large, $e^{10-0.3t}$ approaches zero. $20+e^{10-0.3t}$ approaches 20 $\frac{164000}{20+e^{10-0.3t}}$ approaches $\frac{164000}{20}$ … therefore 8200 **Thus, the largest possible number of bacteria B that can be present is 8200.**				
3(i)	$3-\frac{4x}{x^2+3}+\frac{2}{x+1}$	3(ii)	$\frac{2x}{x^2+3}$	3(iii)	$3x+2\ln(x+1)-2\ln(x^2+3)+c$
3(b)	$g(x)=f(x)-7$	4(i)	$n=2$	4(ii)	-972
5a(i)	$a=4$	5a(ii)	$b=3$	5a(iii)	$k=4$
5a(iv)	$-2<m<\frac{5}{3}$	5(b)	-6, 10		
6(i)	$y=2x-20$	6(ii)	150	7(ii)	$\frac{4\sqrt{3}}{3}-\frac{\pi}{2}$
8a(i)	$3e^{-2x}(-2x+1)$	8a(ii)	$p=-9$	8b(ii)	$\frac{\pi}{6}$ rad
9(i)	$H=17\cos(\theta-61.9^\circ)+20$	10(ii)	$x=15.6$	10(iii)	$\frac{d^2A}{dx^2}=-76.8$, Maximum
11(i)	3	11(ii)	10	11(iii)	-2
11(iv)	11				

	Solutions
1	$\cot 75° = \dfrac{1-\tan 45° \tan 30°}{\tan 45° + \tan 30°}$ $= \dfrac{1-1\times\frac{\sqrt{3}}{3}}{1+\frac{\sqrt{3}}{3}}$ $= \dfrac{3-\sqrt{3}}{3+\sqrt{3}} \times \dfrac{3-\sqrt{3}}{3-\sqrt{3}}$ $= 2-\sqrt{3}$ $\operatorname{cos}ec^2 75° = 1+\left(2-\sqrt{3}\right)^2$ $= 1+4+3-4\sqrt{3}$ $= 8-4\sqrt{3}$
	Alternative Solution 1: $\sin 75° = \sin 45° \cos 30° + \cos 45° \sin 30°$ $= \dfrac{\sqrt{2}}{2}\times\dfrac{\sqrt{3}}{2}+\dfrac{\sqrt{2}}{2}\times\dfrac{1}{2}$ $= \dfrac{\sqrt{6}+\sqrt{2}}{4}$ $\operatorname{cos}ec^2 75° = \dfrac{16}{\left(\sqrt{6}+\sqrt{2}\right)^2}$ $= \dfrac{16}{8+4\sqrt{3}}$ $= \dfrac{4}{2+\sqrt{3}}\times\dfrac{2-\sqrt{3}}{2-\sqrt{3}}$ $= 8-4\sqrt{3}$
	Alternative Solution 2: $\operatorname{cos}ec^2 75° = \dfrac{1}{\sin^2 75°}$ $= \dfrac{2}{1-\cos 150°}$ $= \dfrac{2}{1+\frac{\sqrt{3}}{2}}$ $= \dfrac{4}{2+\sqrt{3}}\times\dfrac{2-\sqrt{3}}{2-\sqrt{3}}$ $= 8-4\sqrt{3}$
	Total Marks: 4m

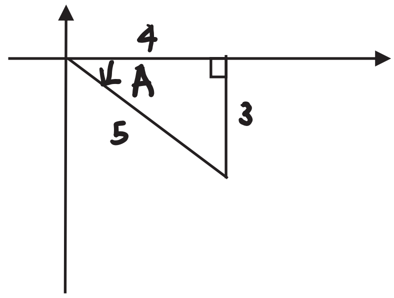
This paper consists of 12 printed pages.

2	**(i)**	
	(ii)	$y^2 = 9x ----(1)$ $y = kx^{-\frac{3}{2}} ----(2)$ Equating equations (1) and (2) gives, $9x = \left(kx^{-\frac{3}{2}}\right)^2$ Since the curves meet at a point $x = 4$. $36 = k^2 \times 4^{-3}$ $k = \pm 48$ **Alternatively,** From (1), $y = \pm 3x^{\frac{1}{2}}$ $3\sqrt{x} = kx^{-\frac{3}{2}}$ or $-3\sqrt{x} = kx^{-\frac{3}{2}}$ Since the curves meet at a point $x = 4$. $12 = k \times 4^{-\frac{3}{2}}$ or $-12 = k \times 4^{-\frac{3}{2}}$ $k = 48$ $k = -48$
		Total Marks: 4m

3

$\tan A = -\frac{3}{4}$

A is in the 4th quadrant.



$$\sin\left(A-\frac{\pi}{2}\right)=\sin\left[-\left(\frac{\pi}{2}-A\right)\right]$$
$$=-\sin\left(\frac{\pi}{2}-A\right)$$
$$=-\cos A$$
$$=-\frac{4}{5}$$

Alternatively,

$$\sin\left(A-\frac{\pi}{2}\right)=\sin\left[-\left(\frac{\pi}{2}-A\right)\right]$$
$$=-\sin\left(\frac{\pi}{2}-A\right)$$
$$=-\left(\sin\frac{\pi}{2}\cos A-\cos\frac{\pi}{2}\sin A\right)$$
$$=-\left(1\times\frac{4}{5}-0\times-\frac{3}{5}\right)$$
$$=-\frac{4}{5}$$

Alternatively,

$$\sin\left(A-\frac{\pi}{2}\right)=\sin A\cos\frac{\pi}{2}-\cos A\sin\frac{\pi}{2}$$
$$=-\frac{3}{5}\times 0-\left(-\frac{4}{5}\right)\times 1$$
$$=-\frac{4}{5}$$

Total Marks: 3m

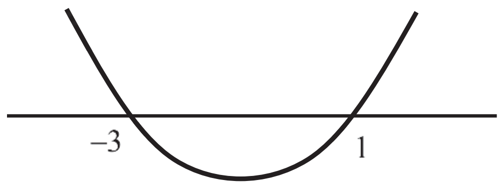
4

For the equation $x^2-(k+1)x+1=0$ to have 2 distinct and real roots p and q,

$(k+1)^2-4\times 1>0$

$k^2+2k-3>0$

$(k+3)(k-1)>0$



$k<-3$ or $k>1$

Total Marks: 4m

5	**(i)**	$f'(x)=\frac{1}{x-2}+e^{-2x}$ for $x>a$ $\int \frac{1}{x-2}+e^{-2x}\,dx=\ln(x-2)-\frac{e^{-2x}}{2}+c$, where c is an arbitrary constant At $\left(3,-\frac{1}{e^6}\right)$, $-e^{-6}=\ln(3-2)-\frac{e^{-2\times3}}{2}+c$ $-e^{-6}=-\frac{e^{-6}}{2}+c$ $c=-\frac{e^{-6}}{2}$ or -0.00124 $\therefore f(x)=\ln(x-2)-\frac{1}{2e^{2x}}-\frac{1}{2e^6}$ or $\therefore f(x)=\ln(x-2)-\frac{1}{2e^{2x}}-0.00124$
	(ii)	$f'(x)=\frac{1}{x-2}+e^{-2x}$ for $x>a$ $f''(x)=-\frac{1}{(x-2)^2}-2e^{-2x}$ Since $(x-2)^2>0$ and $e^{-2x}>0$ for $x>2$ $f''(x)=-\frac{1}{(x-2)^2}-2e^{-2x}<0$ Thus, the function $f'(x)=\frac{1}{x-2}+e^{-2x}$ is decreasing.
		Total Marks: 7m
6	For the curve $y=mx^2+(m+n)x+n$ to lie entirely above the x-axis, it would not have any x-intersections. $b^2-4ac=(m+n)^2-4mn$ $=m^2+2mn+n^2-4mn$ $=m^2-2mn+n^2$ $=(m-n)^2$ Since $(m-n)^2\geq 0$ for all values of m and n, the curve would intersect with the x-axis. The curve will not lie completely above the x-axis.	
		Total Marks: 3m

7	**(a)**	Let $\mathrm{f}(x) = 2x^3 - 3x^2 - 3x + 4$ $\mathrm{f}(1) = 2(1)^3 - 3(1)^2 - 3(1) + 4 = 0$ By Factor Theorem, $(x-1)$ is a factor of $\mathrm{f}(x)$. Let $2x^3 - 3x^2 - 3x + 4 = (x-1)(2x^2 + bx - 4)$ Compare the coefficient of x $-3 = -4 - b$ $b = -1$ $\mathrm{f}(x) = (x-1)(2x^2 - x - 4)$ $(x-1)(2x^2 - x - 4) = 0$ $x = 1$, or $x = \dfrac{1 \pm \sqrt{1+32}}{4}$ $\therefore x = 1, x = 1.69$ or -1.19
7	**(b)**	By Factor Theorem, $\mathrm{p}(5) + 1 = 0$ $\mathrm{p}(5) = -1$ By Remainder Theorem, $\mathrm{g}(5) = 2(5)^3 - \mathrm{p}(5) + 5$ $=256$ **Alternatively,** $\mathrm{g}(x) = 2(x)^3 + 6 - \mathrm{p}(x) - 1$ $\mathrm{g}(5) = 2(5)^3 + 6 - \mathrm{p}(5) - 1$ $=256$
		Total Marks: 6m

8	$\frac{1}{\alpha}\times\frac{1}{\beta}=\frac{1}{4}---(1)$ $\frac{1}{\alpha}+\frac{1}{\beta}=\frac{1}{2}$ $\frac{\alpha+\beta}{\alpha\beta}=\frac{1}{2}$ From(1), $\alpha\beta=4$ $\frac{1}{\alpha}+\frac{1}{\beta}=\frac{1}{2}$ $\frac{\alpha+\beta}{\alpha\beta}=\frac{1}{2}$ $\alpha+\beta=2$ Sum of roots is $\frac{\beta}{\alpha^2}+\frac{\alpha}{\beta^2}$ $=\frac{\alpha^3+\beta^3}{(\alpha\beta)^2}$ $=\frac{(\alpha+\beta)\left[(\alpha+\beta)^2-3\alpha\beta\right]}{(\alpha\beta)^2}$ $=\frac{2\left[(2)^2-3(4)\right]}{4^2}$ $=-1$ Product of roots is $\frac{\beta}{\alpha^2}\times\frac{\alpha}{\beta^2}=\frac{1}{\alpha\beta}$ $=\frac{1}{4}$ The quadratic equation $x^2+x+\frac{1}{4}=0$ or $4x^2+4x+1=0$
	Total Marks: 5m

9	**(i)**	$\frac{d}{dx}\left(\frac{2\ln x}{x^3}\right)=\frac{x^3\left(\frac{2}{x}\right)-2\ln x\left(3x^2\right)}{x^6}$ $=\frac{2x^2-6x^2\ln x}{x^6}$ $=\frac{2x^2}{x^6}-\frac{6x^2\ln x}{x^6}$ $=\frac{2}{x^4}-\frac{6\ln x}{x^4}$
	(ii)	$\int \frac{2}{x^4}-\frac{6\ln x}{x^4}\,dx=\frac{2\ln x}{x^3}+c$ where c is an arbitrary constant $\int \frac{2}{x^4}\,dx-6\int\frac{\ln x}{x^4}\,dx=\frac{2\ln x}{x^3}+c$ $6\int\frac{\ln x}{x^4}\,dx=\int\frac{2}{x^4}\,dx-\frac{2\ln x}{x^3}+c_1$ where c_1 is an arbitrary constant $=-\frac{2}{3x^3}-\frac{2\ln x}{x^3}+c_2$, where c_2 is an arbitrary constant $\int\frac{\ln x}{x^4}\,dx=-\frac{1}{9x^3}-\frac{\ln x}{3x^3}+c_3$, where c_3 is an arbitrary constant or $=-\frac{1+3\ln x}{9x^3}+c_3$
		Total Marks: 6m

10	**(i)**	$y = \frac{32}{(4-x)^2} - 8$ $\frac{dy}{dx} = \frac{64}{(4-x)^3}$ when $x = 8$, $\frac{dy}{dx} = -1 \Rightarrow$ gradient $= -1$ When $x = 8$, $y = \frac{32}{(4-8)^2} - 8 = -6$ $\therefore (8, -6)$ Equation of tangent $y + 6 = -(x - 8)$ $y = -x + 2$
	(ii)	When $y = 0$, $0 = \frac{32}{(4-x)^2} - 8$ $x = 2$ or 6 $\therefore A(2,0)$ and $B(6,0)$ Area of shaded region $= \frac{1}{2} \times (8-2) \times 6 - \left\| \int_6^8 \frac{32}{(4-x)^2} - 8 \, dx \right\|$ $= 18 - \left\| \left[\frac{32}{(4-x)} - 8x \right]_6^8 \right\|$ $= 18 - \left\| \left(\frac{32}{(4-8)} - 64 \right) - \left(\frac{32}{(4-6)} - 48 \right) \right\|$ $= 18 - \left\| -72 + 64 \right\| - (2) + 2\frac{5}{8}$ $= 10 \text{ unit}^2$
		Total Marks: 9m

11 **(i)**

x	1	1.22	1.29	1.32	1.34
y	1	2	3	4	5
$\mathrm{Y} = x^2 y$	1	3.00	4.99	6.97	8.98
X=y	1	2	3	4	5

(ii)

$$y = \frac{1}{ax^2 + b}$$

$$\left(ax^2 + b\right) y = 1$$

$$ax^2 y + by = 1$$

$$x^2 y = \frac{1}{a} - \frac{b}{a} y$$

$$\frac{1}{a} = -1 \pm 0.1$$

$a = -1$ (accept -1.11 to -0.909)

$$-\frac{b}{a} = \text{Gradient}$$

$$= \frac{8-2}{4.5-1.5}$$

$$= 2 \pm 0.1$$

$$b = a \times gradient$$

(iii)

$$2.5 = \frac{1}{ax^2 + b}$$

Draw the vertical line $\mathrm{X} = 2.5$

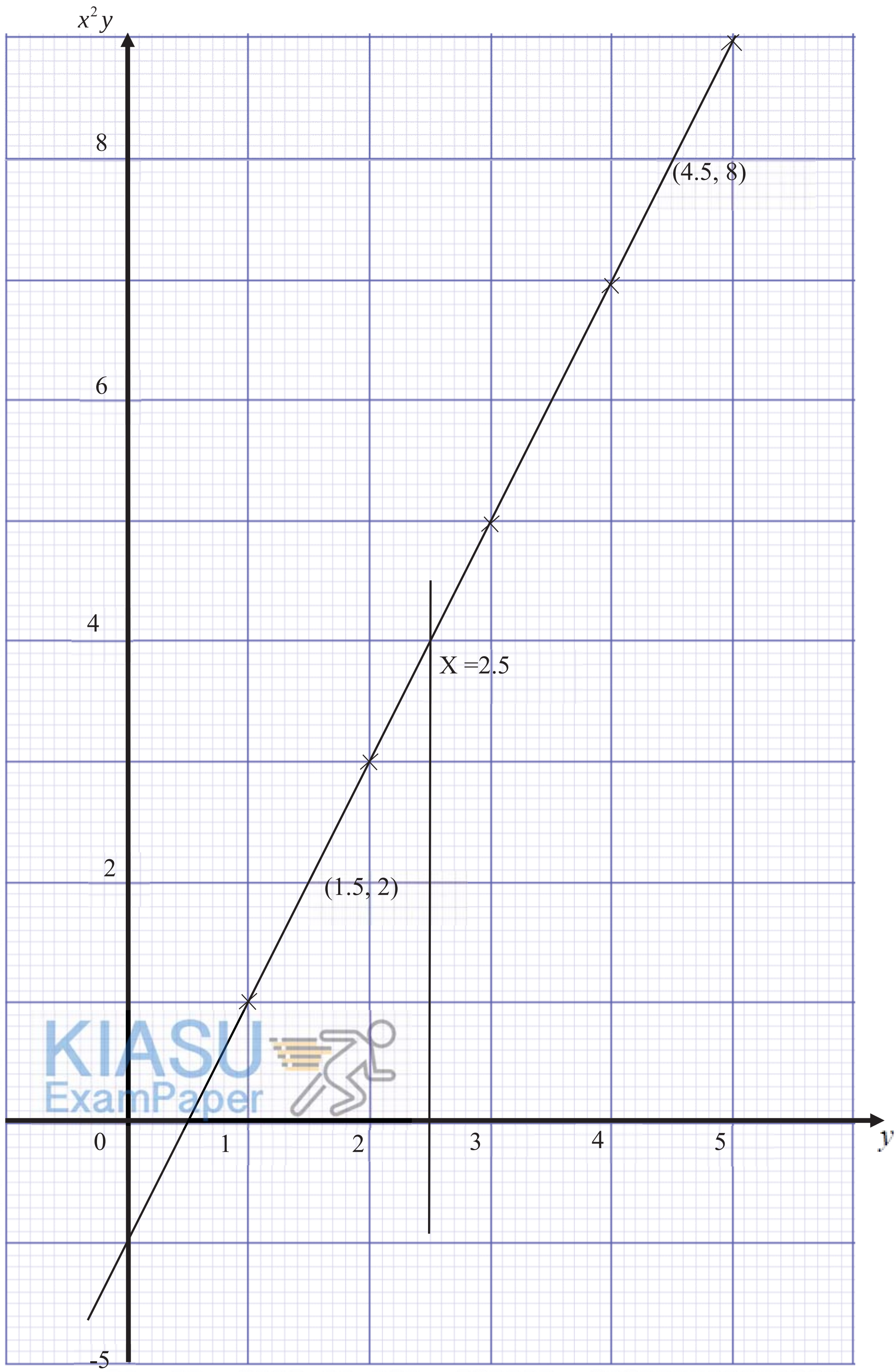
$$x^2 y = 4$$

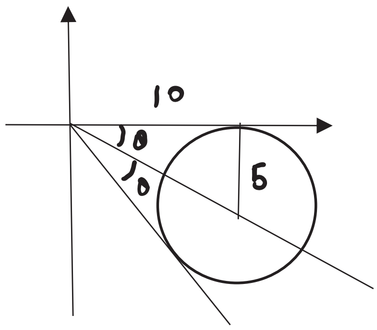
$$x = \sqrt{\frac{4}{2.5}} \quad (x > 0)$$

$$= 1.26$$

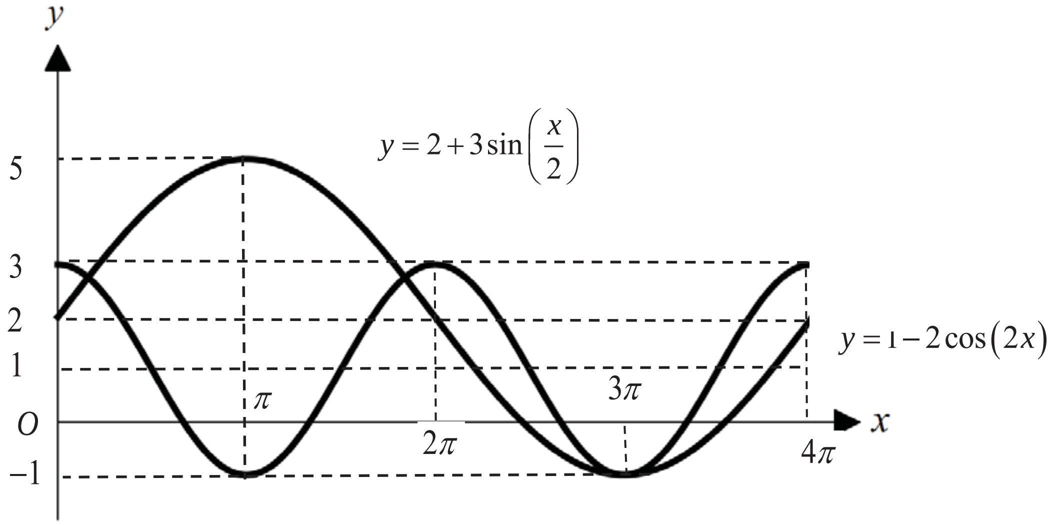
(accept from 1.24 to 1.28)

Total Marks: 8m


x^2y
8
6
4
2
-5
0
1
2
3
4
5
y
(4.5, 8)
X =2.5
(1.5, 2)

<table>
<tr><td>12</td><td>(i)</td><td>
$x^2+y^2-20x+10y+100=0$
$x^2-20x+10^2+y^2+10y+5^2=-100+10^2+5^2$
$(x-10)^2+(y+5)^2=25$
Radius of the circle =5 units
Centre of the cirlce: is $(10,-5)$
</td></tr>
<tr><td></td><td>(ii)</td><td>
Let the equation(s) of the tangent be $y=mx$
$x^2+(mx)^2-20x+10(mx)+100=0$
$(m^2+1)x^2+(10m-20)x+100=0$
$b^2-4ac=0$
$(10m-20)^2-4(m^2+1)\times 100=0$
$-3m^2-4m=0$
$m(-3m-4)=0$
$m=0$ or $m=-\frac{4}{3}$
Hence, equations of tangent are $y=0$ or $y=-\frac{4}{3}x$.
Alternatively,
Equation of the tangent $y=0$.
$\tan\theta=\frac{1}{2}$
$\tan 2\theta=\frac{1}{1-\frac{1}{4}}$ (using double angle formula)
$=\frac{4}{3}$


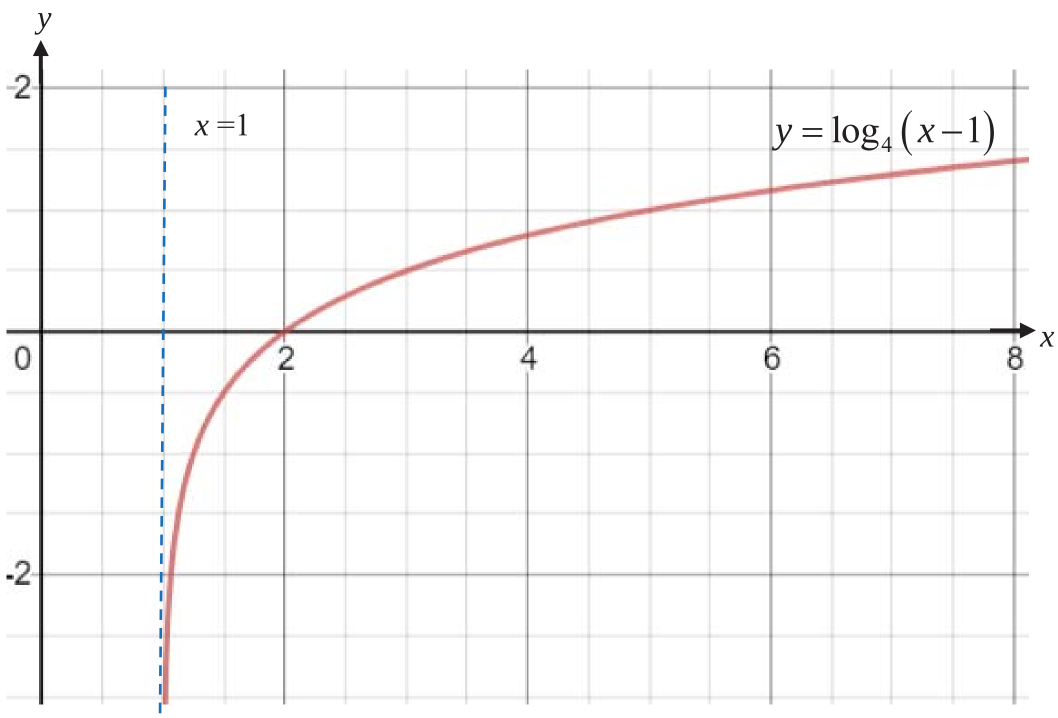
Hence the equation of the other tangent is $y=-\frac{4}{3}x$.
</td></tr>
<tr><td></td><td>(iii)</td><td>
Since the tangent $y=0$ is x-axis, and tangent $\perp$ radius, the point required is $(10,0)$
For the other tangent line $y=-\frac{4}{3}x$, let the point required be $\left(a,-\frac{4}{3}a\right)$
$\frac{-5-\left(-\frac{4}{3}a\right)}{10-a}=\frac{3}{4}$
$a=6$
Hence, the point required is $(6,-8)$.
</td></tr>
<tr><td colspan="3" align="right">Total Marks: 11m</td></tr>
</table>

13	**(i)**	$2\cos 2A - 3\sin A - 1 = 0$ $2(1-2\sin^2 A) - 3\sin A - 1 = 0$ $-4\sin^2 A - 3\sin A + 1 = 0$ $(\sin A + 1)(-4\sin A + 1) = 0$ $\sin A = -1$ or $4\sin A + 1 = 0$ $A = \frac{3\pi}{2}$ or $\sin A = \frac{1}{4}$ basic $\angle$=0.25268 $A = 0.253,\ 2.89$ $\therefore A = \frac{3\pi}{2},\ 0.253$ or 2.89
	(ii)	
	(iii)	$2 + 3\sin\left(\frac{x}{2}\right) = 2\cos(x) + 1$ $2\cos(x) - 3\sin\left(\frac{x}{2}\right) - 1 = 0$ sub x = 2A $2\cos 2A - 3\sin A - 1 = 0$ From Part(i), $A = \frac{3\pi}{2},\ 0.253$ or 2.89 $x = 3\pi, 0.506, 5.78$
		Total Marks: 10m

2020 S4 Prelim AM P2 Solution

Solutions

1	**(a)**	$2^{6x}+\dfrac{7\left(2^{6x}\right)}{2^{3x}} = 30$ $2^{6x}+7\left(2^{3x}\right)-30=0$ Let $u=2^{3x}$ $u^2+7u-30=0$ $(u-3)(u+10)=0$ $u-3=0$ OR $u+10=0$ $u=3$ $\quad$ $u=-10$ $2^{3x}=3$ $\quad$ $2^{3x}=-10$ (Reject, no solution) $3x=\dfrac{\lg 3}{\lg 2}$ $x=\dfrac{1}{3}\left(\dfrac{\lg 3}{\lg 2}\right)$ $x=0.528$ (3sf)
	(b)	$\log_x 2=\log_7\sqrt{7}+3\log_2 x$ $\dfrac{1}{\log_2 x}=\dfrac{1}{2}+3\log_2 x$ Let $u=\log_2 x$ $\dfrac{1}{u}=\dfrac{1}{2}+3u$ $2=u+6u^2$ $6u^2+u-2=0$ $(2u-1)(3u+2)=0$ $u=\dfrac{1}{2}$ OR $u=-\dfrac{2}{3}$ $\log_2 x=\dfrac{1}{2}$ $\quad$ $\log_2 x=-\dfrac{2}{3}$ $x=2^{\frac{1}{2}}$ $\quad$ $x=2^{-\frac{2}{3}}$ $x=1.41$ $\quad$ $x=0.630$ (3sf)

	c(i)	$y=\log_p(x-1)$ Sub (5, 1) $1=\log_p(5-1)$ $p=4$ Sub $p = 4$, $x = 2$, $y= q$ $q=\log_4(2-1)$ $q=0$
	c(ii)	
2	**(i)**	When $t = 0$, $P=300e^{k(0)}+450$ $P=750$
	(ii)	$1500=300e^{24k}+450$ $300e^{24k}=1050$ $e^{24k}=3.5$ $24k=\ln(3.5)$ $k=\dfrac{\ln(3.5)}{24}$ $k=0.0522$ (3sf)

	(iii)	As t becomes very large, $e^{10-0.3t}$ approaches zero. $20+e^{10-0.3t}$ approaches 20 $\dfrac{164000}{20+e^{10-0.3t}}$ approaches $\dfrac{164000}{20}$ … therefore 8200 Thus, the largest possible number of bacteria B that can be present is 8200.
3	**(i)**	$$3x^3-3x^2+3x+3\overline{\big)3x^3+x^2+5x+15}$$ quotient 3 $$\underline{-(3x^3+3x^2+9x+9)}$$ $$\underline{-2x^2-4x+6}$$ $\dfrac{3x^3+x^2+5x+15}{\left(x^2+3\right)\left(x+1\right)}=3+\dfrac{\left(-2x^2-4x+6\right)}{\left(x^2+3\right)\left(x+1\right)}$ Let $\dfrac{-2x^2-4x+6}{\left(x^2+3\right)\left(x+1\right)}=\dfrac{Ax+B}{x^2+3}+\dfrac{C}{x+1}$ $-2x^2-4x+6=\left(Ax+B\right)\left(x+1\right)+C\left(x^2+3\right)$ Sub $x=-1$, $-2\left(-1\right)^2-4\left(-1\right)+6=C\left[\left(-1\right)^2+3\right]$ $C=2$ By comparison of coefficient, Constant: $6=B+3C$ $6=B+3(2)$ $B=0$ x^2 term: $-2=A+C$ $A=-4$ $\dfrac{3x^3+x^2+5x+15}{\left(x^2+3\right)\left(x+1\right)}=3-\dfrac{4x}{x^2+3}+\dfrac{2}{x+1}$

		Alternative Solution: $\dfrac{3x^3+x^2+5x+15}{(x^2+3)(x+1)}=3-\dfrac{2x^2+4x-6}{(x^2+3)(x+1)}$ Let $\dfrac{2x^2+4x-6}{(x^2+3)(x+1)}=\dfrac{Ax+B}{x^2+3}+\dfrac{C}{x+1}$ $2x^2+4x-6=(Ax+B)(x+1)+C(x^2+3)$ Sub $x=-1$, $2(-1)^2+4(-1)-6=C\left[(-1)^2+3\right]$ $C=-2$ By comparison of coefficient, Constant: $-6=B+3C$ $-6=B+3(-2)$ $B=0$ x^2 term: $2=A+C$ $2=A-2$ $A=4$ $\dfrac{3x^3+x^2+5x+15}{(x^2+3)(x+1)}=3-\dfrac{4x}{x^2+3}+\dfrac{2}{x+1}$
	(ii)	$\dfrac{d}{dx}\ln(x^2+3)=\dfrac{2x}{x^2+3}$
	(iii)	$\int\dfrac{3x^3+x^2+5x+15}{(x^2+3)(x+1)}\,dx$ $=\int 3+\dfrac{2}{x+1}-\dfrac{4x}{x^2+3}\,dx$ $=\int 3\,dx+\int\dfrac{2}{x+1}\,dx-\int\dfrac{4x}{x^2+3}\,dx$ $=\int 3\,dx+2\int\dfrac{1}{x+1}\,dx-2\int\dfrac{2x}{x^2+3}\,dx$ $=3x+2\ln(x+1)-2\ln(x^2+3)+c$

	(b)	Since $f'(x)=g'(x)$, $f(x)+c_1=g(x)+c_2$ $g(x)=f(x)+c_1-c_2$ Since (3, 6) is 7 units above $(3,-1)$ $g(x)=f(x)-7$
4	**(i)**	$\mathrm{T}_{r+1}=\binom{6}{r}(3x)^{6-r}\left(-\frac{n}{x^2}\right)^r$ $\mathrm{T}_{r+1}=\binom{6}{r}3^{6-r}(-n)^r x^{6-3r}$ $6-3r=0$ $r=2$ When $r = 2$, $\binom{6}{2}3^{6-2}(-n)^2=4860$ $n^2=4$ $n=-2$ (reject, since n is positive) $n=2$
	(ii)	For x^3 term in $\left(3x-\frac{n}{x^2}\right)^6$, $6-3r=3$ $r=1$ When $r = 1$, $\mathrm{T}_2=\binom{6}{1}3^{6-1}(-2)^1x^{6-3}$ $\mathrm{T}_2=-2916x^3$ $\left(1+\frac{2}{x^3}\right)\left(3x-\frac{n}{x^2}\right)^6 =\left(1+\frac{2}{x^3}\right)\left(4860-2916x^3+...\right)$ $=(1)(4860)+\left(\frac{2}{x^3}\right)\left(-2916x^3\right)+...$ $=-972+...$ Constant term = -972

5	**a(i)**	$a=4$
	a(ii)	$y=a-\|2x-2b\|$ $-2b=-6$ $b=3$
	a(iii)	$k=4$
	a(iv)	Gradient of left arm of graph = $\frac{5}{3}$ Gradient of right arm of graph = -2 $-2<m<\frac{5}{3}$
	(b)	$2\|x-2\|=\|3x-6\|-8$ $2\|x-2\|=\|3\|\|x-2\|-8$ $\|x-2\|=8$ $x-2=8$ OR $x-2=-8$ $x=10$ $x=-6$ <u>Check:</u> When $x=-6$, LHS = $2\|-6-2\|+3=19$ RHS = $\|6-3(-6)\|-5=19$ LHS = RHS When $x=10$, LHS = $2\|10-2\|+3=19$ RHS = $\|6-3(10)\|-5=19$ LHS = RHS $x=10$ OR $x=-6$
6	**(i)**	Gradient of OQ = $\frac{-8-0}{16-0}$ $=-\frac{1}{2}$ $OQ\perp PS$ $\therefore$ Gradient of $PS=2$ Since $OPQR$ is a rhombus, $\left(\frac{0+16}{2},\frac{0-8}{2}\right)=(8,-4)$ $y+4=2(x-8)$ $y=2x-20$ Hence, the equation is $y=2x-20$

	(ii)	Gradient of PR $=2$ (from part (i)) $y=2(10)+c$ $c=-20$ Equation of PR: $y=2x-20$ $\therefore S(0,-20)$ When y = 0, $0=2x-20$ $x=10$ $P(10,\ 0)$ $T(0,-10)$ Area of OPQS $=\frac{1}{2}\begin{vmatrix} 0 & 0 & 16 & 10 & 0 \\ -10 & -20 & -8 & 0 & -10 \end{vmatrix}$ = 150 units^2
7	**(i)**	LHS = $\dfrac{2\sin 2x-\sin 4x}{2\sin 2x+\sin 4x}$ $=\dfrac{2\sin 2x-2\sin 2x\cos 2x}{2\sin 2x+2\sin 2x\cos 2x}$ $=\dfrac{2\sin 2x(1-\cos 2x)}{2\sin 2x(1+\cos 2x)}$ $=\dfrac{1-\cos 2x}{1+\cos 2x}$ $=\dfrac{1-(1-2\sin^2 x)}{1+(2\cos^2 x-1)}$ $=\dfrac{2\sin^2 x}{2\cos^2 x}$ $=\tan^2 x$ = RHS

	(ii)	$\int_{-\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{2\sin 2x(1-\cos 2x)}{2\sin 2x(1+\cos 2x)}\,dx$ $=\int_{-\frac{\pi}{6}}^{\frac{\pi}{3}} \tan^2 x\,dx$ $=\int_{-\frac{\pi}{6}}^{\frac{\pi}{3}} (\sec^2 x-1)\,dx$ $=[\tan x - x]_{-\frac{\pi}{6}}^{\frac{\pi}{3}}$ $=\left[\tan\left(\frac{\pi}{3}\right)-\frac{\pi}{3}\right]-\left[\tan\left(-\frac{\pi}{6}\right)+\frac{\pi}{6}\right]$ $=\sqrt{3}-\frac{\pi}{3}-\left(-\frac{1}{\sqrt{3}}+\frac{\pi}{6}\right)$ $=\sqrt{3}+\frac{1}{\sqrt{3}}-\frac{\pi}{2}$ $=\frac{4\sqrt{3}}{3}-\frac{\pi}{2}$
8	**a(i)**	$\frac{dy}{dx}=(3x)(-2e^{-2x})+(e^{-2x})(3)$ $\frac{dy}{dx}=3e^{-2x}(-2x+1)$
	a(ii)	$\frac{dy}{dx}=3e^{-2x}(-2x+1)$ $\frac{d^2y}{dx^2}=3e^{-2x}(-2)+(-2x+1)(3e^{-2x})(-2)$ $\frac{d^2y}{dx^2}=-6e^{-2x}(2-2x)$ $\frac{d^2y}{dx^2}=12e^{-2x}(x-1)$ $p=e^{2x}\left(\frac{d^2y}{dx^2}+\frac{dy}{dx}-2y\right)$ $p=e^{2x}\left[(12xe^{-2x}-12e^{-2x})+(3e^{-2x}-6xe^{-2x})-2(3xe^{-2x})\right]$ $p=12x-12+3-6x-6x$ $p=-9$

	b(i)	$y=\ln\left(\frac{1-\cos x}{\sin x}\right)$ $y=\ln(1-\cos x)-\ln(\sin x)$ $\frac{dy}{dx}=\frac{\sin x}{1-\cos x}-\frac{\cos x}{\sin x}$ $\frac{dy}{dx}=\frac{\sin^2 x-\cos x(1-\cos x)}{\sin x(1-\cos x)}$ $\frac{dy}{dx}=\frac{\sin^2 x-\cos x+\cos^2 x}{\sin x(1-\cos x)}$ $\frac{dy}{dx}=\frac{1-\cos x}{\sin x(1-\cos x)}$ $\frac{dy}{dx}=\frac{1}{\sin x}$ $\frac{dy}{dx}=\operatorname{cosec} x$
		Alternative Solution $y=\ln\left(\frac{1-\cos x}{\sin x}\right)$ $\frac{dy}{dx}=\frac{\frac{d}{dx}\left(\frac{1-\cos x}{\sin x}\right)}{\left(\frac{1-\cos x}{\sin x}\right)}$ $\frac{dy}{dx}=\frac{\left[\frac{\sin x(\sin x)-(1-\cos x)(\cos x)}{\sin^2 x}\right]}{\left(\frac{1-\cos x}{\sin x}\right)}$ $\frac{dy}{dx}=\frac{\sin x(\sin x)-(1-\cos x)(\cos x)}{\sin x(1-\cos x)}$ $\frac{dy}{dx}=\frac{\sin^2 x-\cos x+\cos^2 x}{\sin x(1-\cos x)}$ $\frac{dy}{dx}=\frac{1-\cos x}{\sin x(1-\cos x)}$ $\frac{dy}{dx}=\frac{1}{\sin x}$ $\frac{dy}{dx}=\operatorname{cosec} x$

	(ii)	$\frac{dy}{dt}=2\frac{dx}{dt}$ $\frac{dy}{dt}=\left(\frac{dy}{dx}\right)\left(\frac{dx}{dt}\right)$ $2\frac{dx}{dt}=\left(\frac{dy}{dx}\right)\left(\frac{dx}{dt}\right)$ $\frac{dy}{dx}=2$ $\operatorname{cosec} x=2$ $\sin x=\frac{1}{2}$ $x=\frac{\pi}{6}$ rad
9	**(i)**	Let $8\cos\theta+15\sin\theta=R\cos(\theta-\alpha)$ $8\cos\theta+15\sin\theta=R\cos\theta\cos\alpha+R\sin\theta\sin\alpha$ By comparison, $R\cos\alpha=8$ ------ (1) $R\sin\alpha=15$ ------- (2) $(1)^2+(2)^2$: $R^2\cos^2\alpha+R^2\sin^2\alpha=8^2+15^2$ $R=\sqrt{8^2+15^2}$ $R=17$ (2) / (1): $\tan\alpha=\frac{15}{8}$ $\alpha=61.928^\circ$ $H=17\cos(\theta-61.9^\circ)+20$
	(ii)	$H=15\sin\theta+8\cos\theta+20$ $H=17\cos(\theta-61.9^\circ)+20$ $\cos(\theta-61.9^\circ)\le 1$ for all values of θ $17\cos(\theta-61.9^\circ)\le 17$ $17\cos(\theta-61.9^\circ)\le 17$ $17\cos(\theta-61.9^\circ)+20\le 37$ Since maximum height < 50 m, the design will NOT exceed the stipulated height limit.

10	**(i)**	Area of figure = $6xy+\frac{1}{2}(6x)(6x)\sin 60^{\circ}$ $A=6xy+9\sqrt{3}x^2$ ------ (1) erimeter of figure = $2y+3(6x)$ $400=2y+18x$ $y=200-9x$ ------ (2) Sub (2) into (1) $A=6x(200-9x)+9\sqrt{3}x^2$ $A=\left(9\sqrt{3}-54\right)x^2+1200x$ (Shown)
	(ii)	$\frac{dA}{dx}=2\left(9\sqrt{3}-54\right)x+1200$ $\frac{dA}{dx}=0$ $2\left(9\sqrt{3}-54\right)x+1200=0$ $x=-\frac{600}{9\sqrt{3}-54}$ $x=15.6$ (3sf)
	(iii)	$\frac{d^2A}{dx^2}=2\left(9\sqrt{3}-54\right)$ $\frac{d^2A}{dx^2}=-76.8$ $\frac{d^2A}{dx^2}<0$ Value obtained in (ii) is a maximum.
11	**(i)**	When t = 0, $XY=-(0)^2+6(0)+3$ $XY=3$ m

	(ii)	$v=-2t+6$ At instantaneous rest, $v = 0$ $-2t+6=0$ $t=3$ When $t=3$, $s=-(3)^2+6(3)+3$ $s=12$ m When $t=4$, $s=-(4)^2+6(4)+3$ $s=11$ m Total distance travelled = 9 + 1 =10 m
	(iii)	When $t=4$, $v=-2(4)+6$ $v=-2$ m/s
	(iv)	$a=-2t+k$ $v=\int(-2t+k)\,dt$ $v=-t^2+kt+c$ When $t = 4$, $-2=-(4)^2+4k+c$ $c=14-4k$ $v=-t^2+kt+14-4k$ When $t = 5$, $v = 0$, $0=-(5)^2+5k+14-4k$ $k=11$