SCHOOL OF SCIENCE AND TECHNOLOGY, SINGAPORE

ADDITIONAL MATHEMATICS 4047/1

PAPER 1
SECONDARY 4
2020 PRELIMINARY EXAMINATION

14 September 2020 (Monday) **2 hours**

CANDIDATE NAME	

CLASS		INDEX NUMBER		

READ THESE INSTRUCTIONS FIRST
Do not turn over the page until you are told to do so.
Write your name, class and index number in the spaces above.
Write in dark blue or black pen in the space provided for each question.

You may use a pencil for any diagrams or graphs.
Do not use paper clips, highlighters, glue or correction fluid.

INFORMATION FOR CANDIDATES
Answer **all** the questions in the space provided.

If working is needed for any question it must be shown with the answer.

Omission of essential working will result in loss of marks.

The use of an approved scientific calculator is expected, where appropriate.

Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.

For π, use either your calculator value or 3.142, unless the question requires the answer in terms of π.

The number of marks is given in brackets [ ] at the end of each question or part question.

The total number of marks for this paper is 80.

For Examiner's Use		
Q1	4	
Q2	4	
Q3	5	
Q4	4	
Q5	7	
Q6	7	
Q7	7	
Q8	5	
Q9	10	
Q10	8	
Q11	8	
Q12	11	
Total		/80

This document consists of **20** printed pages including the Cover Sheet.

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial Expansion

$$(a+b)^n = a^n + \binom{n}{1} a^{n-1} b + \binom{n}{2} a^{n-2} b^2 + \ldots + \binom{n}{r} a^{n-r} b^r + \ldots + b^n$$

where n *is a positive integer and* $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

Formulae for $\Delta\ ABC$

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2} ab \sin C$$

Answer **all** questions.

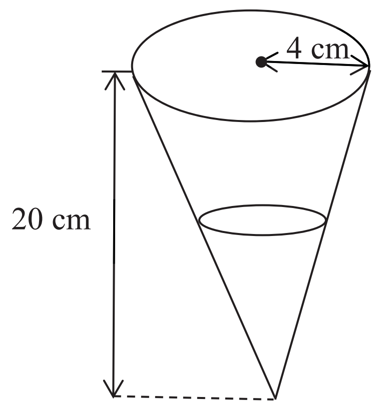
1 Given that $\tan\alpha = p$, where $\pi < \alpha < \frac{3\pi}{2}$, find, in terms of p,

(i) $\tan(-\alpha)$ [1]

(ii) $\tan\left(\frac{\pi}{2} - \alpha\right)$ [1]

(iii) $\sec\alpha$ [2]

2



The diagram above shows an inverted circular cone of radius 4 cm and height of 20 cm. Liquid leaks out through a small hole in the vertex at a constant rate of 5 cm^3/s. Find the rate at which the height of the liquid in the cone is decreasing when the height of liquid is 12 cm. [4]

3 The number of tulips, y, that have bloomed in a huge flower bed can be modelled by the equation $y = (2 + 5t)(900 - t^2)$, where t is the number of days that have passed from the day that the seeds of the tulips have been planted.

(i) Using this model, find the number of seeds that were initially planted. [1]

(ii) Find the rate of increase in the number of tulips that have bloomed when $t = 10$. [2]

(iii) Adam commented that the model will not be valid after 30 days. Is he correct? Explain your answer. [2]

4 Given that $y = (1 + 3x)\mathrm{e}^{3x}$, find the value of the constant k such that

$$\frac{\mathrm{d}^2 y}{\mathrm{d}x^2} - 3\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right) - 9y = kx\mathrm{e}^{3x}\ .$$ [4]

5 The equation of a curve is $y = x^3 + kx^2 - 3x + 1$, where k is a constant.

(a) Find, in terms of k, the gradient of tangent to the curve at the point $x = 1$. [2]

(b) Explain why the curve has two stationary points, for any real values of k. [3]

(c) If $k = -4$, find the range of values of x for which y is a decreasing function. [2]

6 The number of cells, $N(t)$, in an experiment can be modelled by a function $N(t) = 50000(1.6^{0.5t})$ where t is the number of hours that have passed from the start of the experiment.

(i) Find the number of cells present at the start of the experiment. [1]

(ii) Find the time taken for the number of cells to be ten times the initial number of cells, giving your answer to the nearest hour. [3]

(iii) If the number of cells at time $t = t_2$ is double the number of cells at the time $t = t_1$, prove that the difference between the two timings ($t_2 - t_1$) is approximately 2.95 when converted to 3 significant figures. [3]

7 **(i)** Write down and simplify the first 3 terms in the expansion of $(1 - 3x)^5$ in ascending powers of x. [2]

(ii) Given that the coefficient of x^2 in the expansion of $(1 - 3x)^5(1 + 5x)^n$ is 90, find the value of n, where n is a positive integer. [5]

8 The line L_1 has equation $y = ax + 5$. Line L_1 intersects the curve $y = x^2 + 2x + 4$ at M, the turning point of the curve.

(i) Show that $a = 2$. [2]

(ii) Given that L_1 meets the curve $y = x^2 + 2x + 4$ at another point N, find the equation of a line parallel to L_1 and passing through midpoint of M and N. [3]

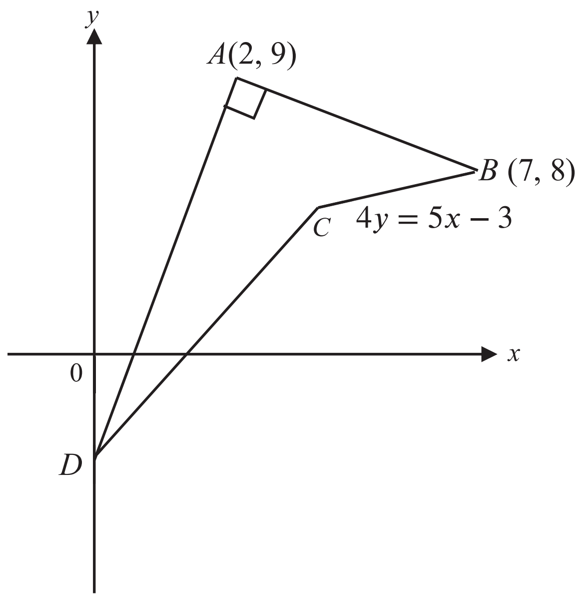
9 Two particles, A and B, leave the origin O at the same time and travel along the positive x-axis. Particle A starts with a velocity of 10 m/s and moves with a constant acceleration of $\frac{2}{3}$ m/s^2. Particle B starts from rest and its acceleration at time, t s, is given by $\left(1+\frac{t}{3}\right)$ m/s^2.

(i) Express the velocity, v_A m/s, and the displacement, s_A m, of particle A in terms of t. [3]

(ii) Express the velocity, v_B m/s, and the displacement, s_B m, of particle B in terms of t. [3]

(iii) Hence, find the distance from O when A and B meet after leaving O. [4]

10 Solutions to this question by accurate drawing will not be accepted.



The diagram above shows a quadrilateral $ABCD$ where the point C lies on the perpendicular bisector of AB and the point D lies on y-axis. The equation of the line BC is $4y = 5x - 3$. Given the coordinates of A and B are (2, 9) and (7, 8) respectively, find

(i) the equation of AD, [2]

(ii) the equation of perpendicular bisector of AB, [2]

(iii) the coordinates of C, [2]

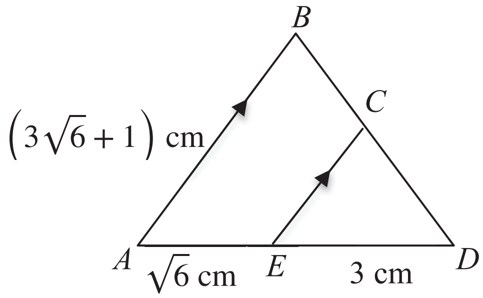
(iv) the area of quadrilateral *ABCD*. [2]

11 Solve the following equations.

(a) $3^{x+2} = \frac{10}{3} - 3^{2x+1}$, [2]

(b) $e^{3x} + 2e^{x} = 3e^{2x}$. [3]

(c)



The diagram above shows a pair of similar triangles *ABD* and *ECD* where *AB* is parallel to *EC*. Given that $AB = \left(3\sqrt{6}+1\right)$ cm, $AE = \sqrt{6}$ cm and $ED = 3$ cm, express the length of *EC* in the form $\left(a\sqrt{6}+b\right)$ cm where a and b are rational numbers. [3]

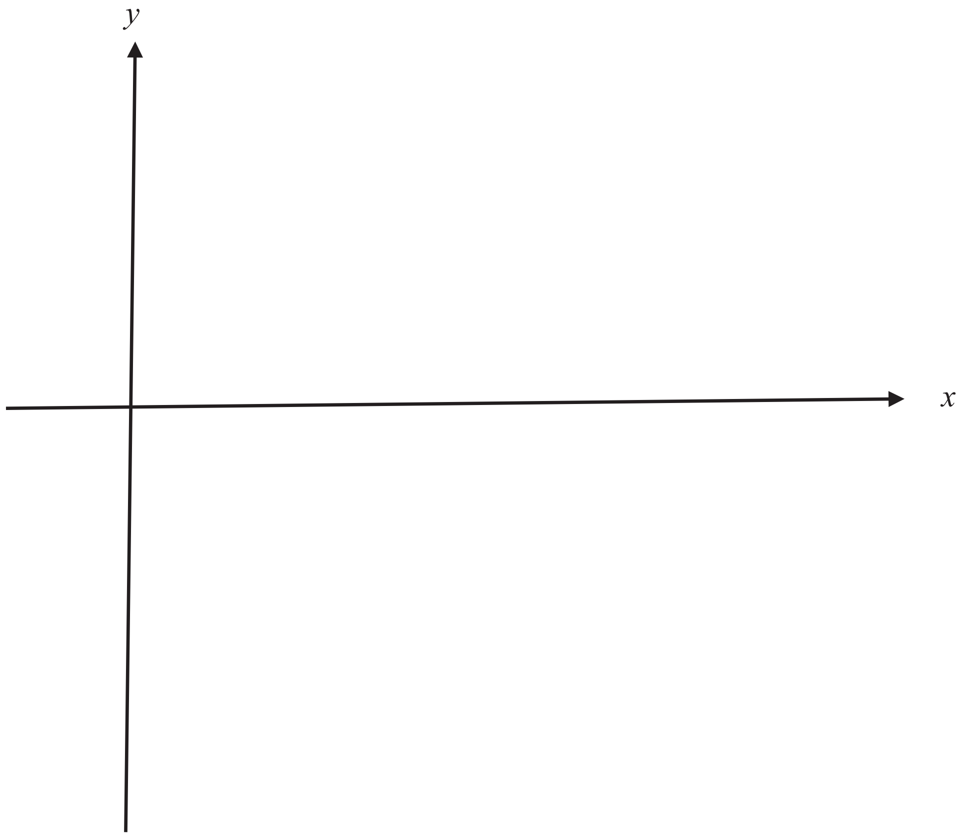
12 It is given that $y_2 = \frac{\cos 2x}{2}$ and $y_1 = \sin x - 1$.

(i) State the amplitude and period, in degrees of y_1, [2]

For the interval of $0° \leq x \leq 360°$,

(ii) solve the equation $y_1 = y_2$, [3]

(iii) sketch, on the same diagram, the graphs of y_1 and y_2 , [4]



(i) find the set of values of x for which $y_1 - y_2 < 0$. [2]

End of Paper

SCHOOL OF SCIENCE AND TECHNOLOGY, SINGAPORE

SECONDARY 4
PRELIMINARY EXAMINATION
ADDITIONAL MATHEMATICS
Paper 2 4047/2

16 September 2020 (Wednesday) **2 hours 30 minutes**

CANDIDATE NAME	

CLASS		INDEX NUMBER		

READ THESE INSTRUCTIONS FIRST
Do not turn over the page until you are told to do so.
Write your name, class and index number in the spaces above.
Write in dark blue or black pen in the writing papers provided.

You may use a pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

INFORMATION FOR CANDIDATES
Answer **all** the questions in the space provided.

Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.

The use of an approved scientific calculator is expected, where appropriate.

You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your answer scripts securely together.

The number of marks is given in brackets [ ] at the end of each question or part question.

For Examiner's Use		
Q1	5	
Q2	5	
Q3	6	
Q4	5	
Q5	8	
Q6	8	
Q7	9	
Q8	10	
Q9	10	
Q10	10	
Q11	11	
Q12	13	
Total		/100

The total number of marks for this paper is **100.**

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial Expansion

$$(a+b)^n = a^n + \binom{n}{1} a^{n-1} b + \binom{n}{2} a^{n-2} b^2 + \ldots + \binom{n}{r} a^{n-r} b^r + \ldots + b^n$$

where n *is a positive integer and* $\binom{n}{r} = \dfrac{n!}{r!(n-r)!} = \dfrac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

Formulae for $\Delta\ ABC$

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2} ab \sin C$$

1 **(i)** Differentiate $x^2 \ln 3x$ with respect to x. [2]

(ii) Hence find $\int x \ln 3x \, dx$. [3]

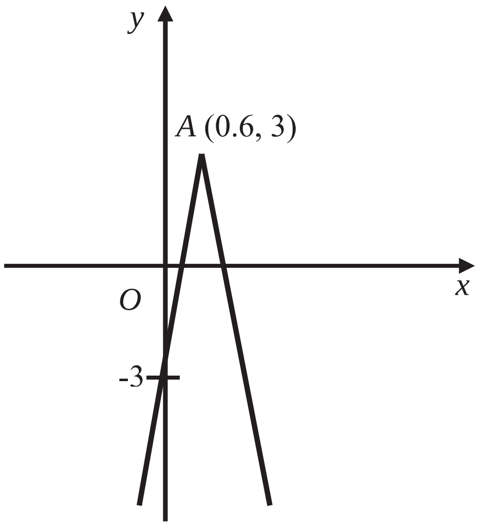
2 Express $\dfrac{x^3+1}{\left(x^2+1\right)\left(x-2\right)}$ in partial fractions. [5]

3 The quadratic equation $3x^2 + 2x + 1 = 0$ has roots α and β.

(i) Show that the value of $\alpha^3 + \beta^3$ is $\dfrac{10}{27}$. [3]

(ii) Find a quadratic equation whose roots are $\dfrac{1}{\alpha^3}$ and $\dfrac{1}{\beta^3}$. [3]

4



The diagram shows part of the graph $y = r - q\left|3 - px\right|$, where p, q and r are positive constants. The graph has a vertex at A (0.6, 3) and y-intercept of -3.

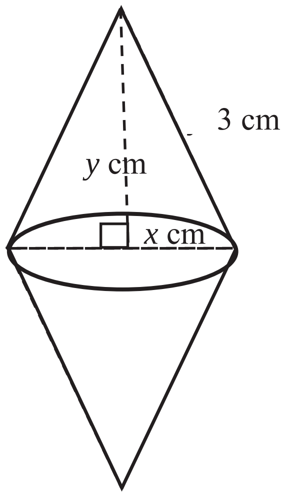
(i) Determine the values of p, q and r. [3]

(ii) State the value or range of values of k such that $k = r - q\left|3 - px\right|$ has

(a) 1 solution, [1]

(b) 2 solutions. [1]

5 A buoy is formed by two identical right circular cones of sheet iron joined by its bases with a radius of x cm. The buoy has a vertical height of y cm and a slant height of 3 cm.



(i) Express y in terms of x. [1]

(ii) Given that x can vary, find the exact value of x for which the volume, V, of the buoy is stationary. [4]

(iii) Determine with reasons whether this value of V is a maximum or minimum. [2]

(iv) Find the exact surface area of the buoy when V is stationary, leaving your answer in terms of π. [1]

6 The equation of a polynomial is given by $\mathrm{p}(x) = 2x^3 + 2ax^2 - x^2 + ax - a$ where a is a constant.

(i) Find the remainder when $\mathrm{p}(x)$ is divided by $(x + 1)$. [1]

(ii) Show that $(2x - 1)$ is a factor of $\mathrm{p}(x)$. [2]

(iii) Showing your working clearly, factorise $\mathrm{p}(x)$ completely, leaving your answer in terms of a. [2]

(iv) Find the range of values of a for which the equation $\mathrm{p}(x) = 0$ has only one real root. [3]

7 The table below shows the data obtained from an experiment on the vertical motion based on the oscillation of a spring with different masses attached to it.

Mass, x kg	0.02	0.03	0.04	0.05	0.15
Frequency of oscillations, y	16	13	11.4	10	6

It is known that the mass, x kg, and the frequency of oscillations per second, y, are related by the equation $xy^2 = k$, where k is a constant.

(a) Plot y^2 against $\frac{1}{x}$ and draw a straight line graph. [3]

(b) Use your graph to estimate

(i) the frequency of oscillations when a mass of 0.08 kg is attached to the spring, [1]

(ii) the mass which produces 15 oscillations per second, [1]

(iii) the value of k. [1]

(c) When the spring is replaced by a second spring, the relation between y and x is represented by $y^2 = \frac{2}{x} + 80$.

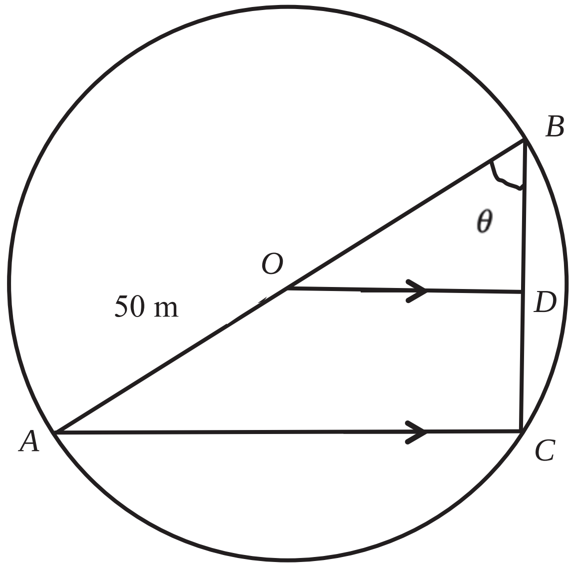
(i) On the same diagram, draw the line representing the second spring. [1]

(ii) Hence, explain how to find the mass which produces the same frequency of oscillations by both springs. [2]

8 **(i)** Prove that $\dfrac{\sec A+\tan A-1}{1-\sec A+\tan A}\equiv\dfrac{1+\sin A}{\cos A}$. [5]

(ii) Hence solve the equation $\dfrac{\sec A+\tan A-1}{1-\sec A+\tan A}=3\cos A$ for $0<A<2\pi$. [5]

9



The diagram shows a circular field with centre O and radius 50 m. A, B and C are points on the circumference of the field and angle $ABC = \theta$. D is a point on BC such that OD is parallel to AC. The trapezium $AODC$ is the jogging path of a man.

(i) Show that $BD = DC = 50\cos\theta$. [2]

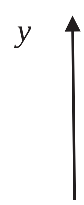
(ii) Show that the perimeter, L m, of the jogging path $AODC$ can be expressed in the form $p + q\cos\theta + r\sin\theta$, where p, q and r are constants to be found. [3]

(iii) Express L in the form of $p + R\cos(\theta - \alpha)$ where $R > 0$

and $0^\circ < \alpha < 90^\circ$. [3]

(iv) Hence state the maximum perimeter of the jogging path $AODC$.
Find the value of θ at which this occurs. [2]

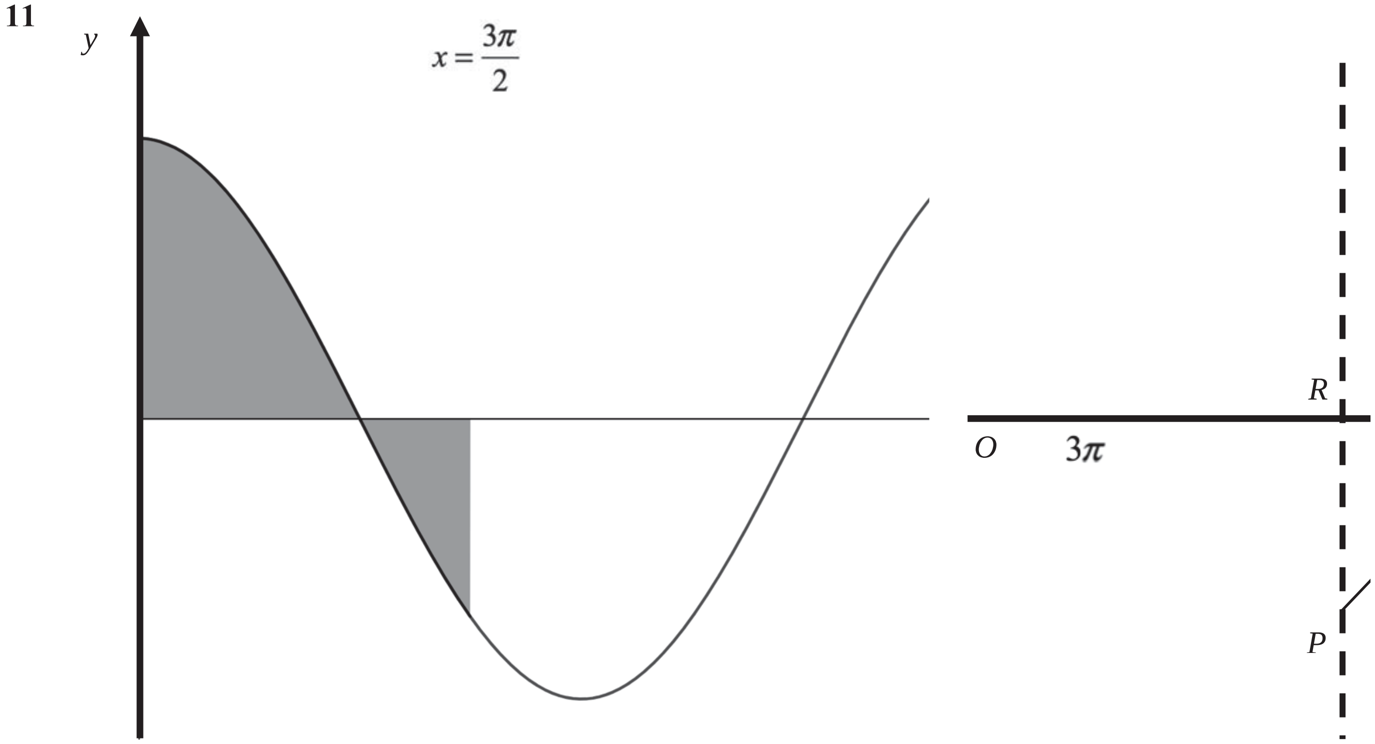
10 **(a)** Sketch the graph of $y = \log_3 x$. [2]



(b) Express $\log_9 4 + \log_3(x-4) = 2\log_3 x$ as a quadratic equation in x and explain why there are no real solutions. [4]

(c) Given that $\log_b(x^2y) = m$ and $\log_b(x^3y) = n$, express, $\log_b x$ and $\log_b y$ in terms of m and n. [4]

11



The diagram shows part of the curve $y = 4\cos\left(\frac{x}{2}\right)$ that meets the x-axis at $x = \pi$ and $x = 3\pi$. The line $x = \frac{3\pi}{2}$ meets the x-axis at R and the curve at P. The normal to the curve at P meets the x-axis at Q.

(i) Find the equation of the normal at P, expressing your answer in exact form. [4]

(ii) Find the exact coordinates of Q. [2]

(iii) Find the exact area of the shaded region. [5]

12 A circle, C_1 has equation $x^2 + y^2 + 8x - 12y + 16 = 0$.

(i) Find the radius and the coordinates of the centre of C_1. [3]

(ii) The lowest point on the circle is A. Explain why A lies on the x-axis. [1]

A second circle, C_2 , has a diameter PQ. The point P has coordinates (-1, 3) and the equation of the tangent to C_2 at Q is $2y = x - 18$.

(iii) Find the equation of the diameter PQ and hence the coordinates of Q. [4]

(iv) Find the equation of the circle, C_2. [3]

(v) Determine whether the circles C_1 and C_2 intersect each other. [2]

END OF PAPER