<u>Answer Keys</u>

1 (ii) $x^2 - 5x + 4 = 0$

2 (i) $n = 5, a = 3$ (ii) –6

3 (i) $3e^{2x} + 6xe^{2x}$

4 (i) $p = -0.983$ (± 0.1), $q = 7.4 \pm 0.1$ (ii) $x^2 = 3.5$ (± 0.1), $x = 1.87$

5 (ii) $\left(\frac{1}{4}, -7\right)$ (ii) $x = \frac{1}{4}$

6 (ii) $-2.94, -0.201, 3.34$

7 (i) –4 (ii) $a = 4$ or $a = 26$

8 (i) $k = 5$ (ii) 13.5 m

9 (a) $3 < k < 7$

10 (i) $AE = 5\cos\theta + 2\sin\theta$ (ii) $5.39\cos(\theta - 21.8°)$ or $\sqrt{29}\cos(\theta - 21.8°)$

(iii) Maximum length of AC = 5.39 cm, corresponding value of $\theta = 21.8°$

(iv) $\theta = 21.8°$

11 (i) $\int x \sin x \, dx = \sin x - x\cos x + c$

(ii) y-coordinate of P is π and x-coordinate of Q is π. (iii) 1.82 units^2

12 (i) $(x+2)^2 + (y-3)^2 = 25$ (ii) $y = -\frac{4}{3}x + \frac{1}{3}$ (iii) 8 units

(iv) 4 units (v) $(x+2)^2 + y^2 = 16$

ST. MARGARET'S SECONDARY SCHOOL

Preliminary Examinations 2020

Founded 1842

CANDIDATE NAME

CLASS

REGISTER NUMBER

ADDITIONAL MATHEMATICS

4047/01

Paper 1

20 August 2020

Secondary 4 Express / 5 Normal (Academic)

2 hours

Candidates answer on the Question Paper.

No Additional Materials are required.

READ THESE INSTRUCTIONS FIRST

Write your name, register number and class in the spaces at the top of this page.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 80.

This document consists of **20** printed pages.

1 Solve the equation $25^{x+1} - 5(5^{x+1}) - 14 = 0$. [5]

$5^{2x+2} - 5(5^x . 5) - 14 = 0$

$5^{2x} . 5^2 - 5^x . 5^2 - 14 = 0$

Let $u = 5^x$

$25u^2 - 25u - 14 = 0$

$(5u - 7)(5u + 2) = 0$

$u = \frac{7}{5}$ or $u = -\frac{2}{5}$

$5^x = \frac{7}{5}$ or $5^x = -\frac{2}{5}$ (NA)

$\lg 5^x = \lg(\frac{7}{5})$

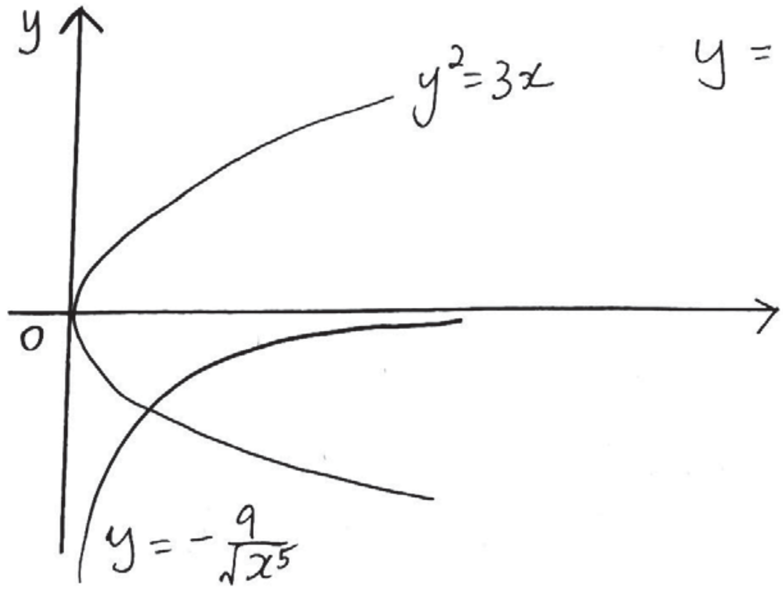
$x \lg 5 = \lg(\frac{7}{5})$

$x = \frac{\lg(\frac{7}{5})}{\lg 5}$

$\underline{\underline{x = 0.209}}$

2 **(i)** On the same axes, sketch the graphs of $y^2 = 3x$ and $y = \frac{-9}{\sqrt{x^5}}$ for $x > 0$. [2]

$y = -9x^{-\frac{5}{2}}$



(ii) Calculate the x-coordinate of the point of intersection of the two graphs, giving your answer in exact form. [2]

$$\left(-\frac{9}{\sqrt{x^5}}\right)^2 = 3x$$

$$\frac{81}{x^5} = 3x$$

$$x^6 = 27 = 3^3$$

$$x = 3^{\frac{3}{6}}$$

$$= 3^{\frac{1}{2}}$$

$$= \sqrt{3}$$

3 Express $\frac{\sqrt{2}}{3+\sqrt{2}} - \frac{1}{1+\sqrt{2}}$ in the form $\frac{a+b\sqrt{2}}{7}$, where a and b are integers. [4]

$$\frac{\sqrt{2}(1+\sqrt{2}) - (3+\sqrt{2})}{(3+\sqrt{2})(1+\sqrt{2})}$$

$$= \frac{\sqrt{2} + 2 - 3 - \sqrt{2}}{3 + 3\sqrt{2} + \sqrt{2} + 2}$$

$$= \frac{-1}{5 + 4\sqrt{2}} \times \frac{5 - 4\sqrt{2}}{5 - 4\sqrt{2}}$$

$$= \frac{-5 + 4\sqrt{2}}{25 - 32}$$

$$= \frac{-5 + 4\sqrt{2}}{-7}$$

$$= \frac{5 - 4\sqrt{2}}{7}$$

//

[Turn over

4 Given that $\log_2 a = h$ and $\log_2 b = k$, express $\log_2 \sqrt{\frac{32a}{b^3}}$ in terms of h and k. [4]

$$\log_2 \left(\frac{32a}{b^3}\right)^{\frac{1}{2}}$$

$$= \frac{1}{2}\left[\log_2 \underbrace{32}\underbrace{a} - \log_2 b^3\right]$$

$$= \frac{1}{2}\left[\log_2 32 + \log_2 a - 3\log_2 b\right]$$

$$= \frac{1}{2}\left[\log_2 2^5 + h - 3k\right]$$

$$= \underline{\underline{\frac{1}{2}\left[5 + h - 3k\right]}}$$

5 (i) Express $\frac{7x^2+16x-3}{(2x-1)(1+x)^2}$ in partial fractions. [6]

$$\frac{7x^2+16x-3}{(2x-1)(1+x)^2} = \frac{A}{2x-1} + \frac{B}{1+x} + \frac{C}{(1+x)^2}$$

$$7x^2+16x-3 = A(1+x)^2 + B(2x-1)(1+x) + C(2x-1)$$

$$\therefore \frac{7x^2+16x-3}{(2x-1)(1+x)^2} = \frac{3}{2x-1} + \frac{2}{x+1} + \frac{4}{(x+1)^2}$$

[Turn over

(ii) Hence evaluate $\int \frac{7x^2+16x-3}{(2x-1)(1+x)^2}\,dx$. [3]

$4(x+1)^{-2}$

$$\int \frac{7x^2+16x-3}{(2x-1)(1+x)^2}\,dx$$

$$= \int \frac{3}{2x-1} + \frac{2}{x+1} + \frac{4}{(x+1)^2}\,dx$$

$$= \frac{3\ln(2x-1)}{2} + 2\ln(x+1) + \frac{4(x+1)^{-1}}{-1} + C$$

$$= \frac{3\ln(2x-1)}{2} + 2\ln(x+1) - \frac{4}{(x+1)} + C$$

6 A particle moves along the curve $y = \frac{x+1}{2x-1}$ in such a way that the x-coordinate of the particle is increasing at a constant rate of 0.03 units per second. Find the x-coordinates of the particle at the instants where the y-coordinate is decreasing at 1 unit per second. [4]

given that $\frac{dx}{dt} = 0.03$ and $\frac{dy}{dt} = -1$ find x

$$\frac{dy}{dx} = -\frac{3}{(2x-1)^2}$$

$$\frac{dy}{dt} = \frac{dy}{dx} \cdot \frac{dx}{dt}$$

$$-1 = -\frac{3}{(2x-1)^2} \cdot (0.03)$$

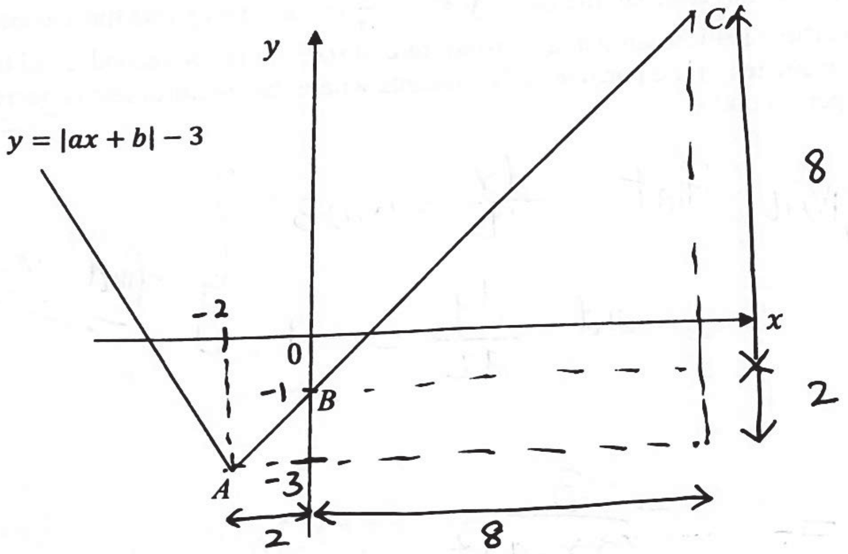
$$(2x-1)^2 = 0.09$$

$\vdots$

$x = 0.65$ $\left(\frac{13}{20}\right)$ or $x = 0.35$ $\left(\frac{7}{20}\right)$

[Turn over

7



The diagram shows the graph of $y = |ax + b| - 3$, where a and b are constants. The graph crosses the y-axis at B where the point B is $(0, -1)$.

(i) Find the y-coordinate of A. [1]

y-coord of A $= -3$

(ii) Given that the x-coordinate of A is -2, find the value of a and of b. [2]

$|b| - 3 = -1$

$|b| = 2$

$b = 2$

$-\frac{b}{a} = -2$

$\frac{b}{a} = 2$

$a = \frac{2}{2}$

$a = 1$

or $a = -1$, $b = -2$

$y = |x + 2| - 3$ or $y = |-x - 2| - 3$

(iii) Given that $AB : BC = 1 : 4$, find the coordinates of C. [2]

$x = 8$

$y = 8 - 1 = 7$

$AB : BC$
$= 1 : 4$
$= 2 : 8$

$\therefore$ C is $(8, 7)$

8 **(i)** Show that $5\cos^2 x - 3\sin^2 x$ can be written as $a\cos 2x + 1$ where a is a positive integer. [3]

$5\cos^2 x - 3(1-\cos^2 x)$

$= 8\cos^2 x - 3$

$= 8\left[\frac{\cos 2x + 1}{2}\right] - 3$

$= 4(\cos 2x + 1) - 3$

$= 4\cos 2x + 4 - 3$

$= \underline{\underline{4\cos 2x + 1}}$

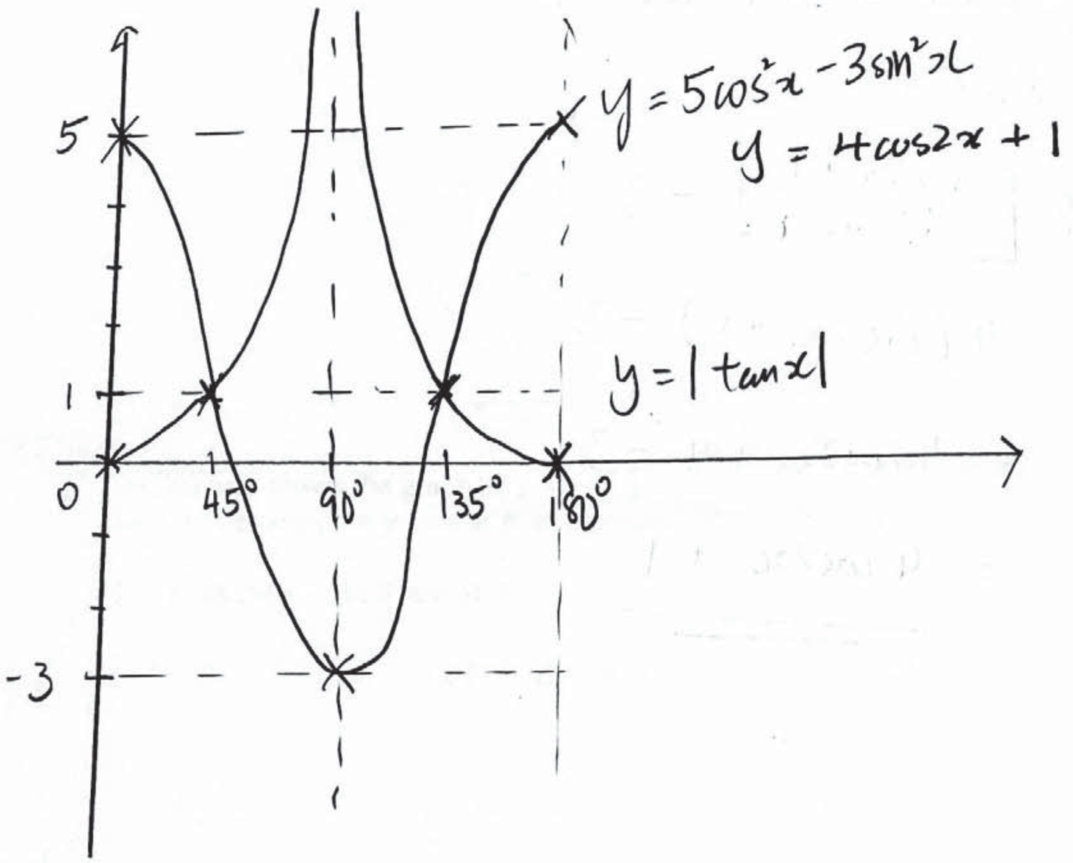
(ii) State the amplitude and period of $5\cos^2 x - 3\sin^2 x$. [2]

amp $= 4$

period $= 180^\circ$ or π radians.

[Turn over

(iii) Sketch on the same axes, the graphs of $y = 5\cos^2 x - 3\sin^2 x$ and $y = |\tan x|$ for $0° \le x \le 180°$. [3]



(iv) Hence state the coordinates of the solutions of the equation $5\cos^2 x - 3\sin^2 x = |\tan x|$, $0° \le x \le 180°$. [2]

$(45°, 1)$ and $(135°, 1)$

[Turn over

9 A curve is such that $\frac{dy}{dx} = \frac{9}{(1-2x)^4}$, $x \neq \frac{1}{2}$ and $(1, 5)$ is a point on the curve.

(i) Find the equation of the curve. [4]

$$y = \int 9(1-2x)^{-4} \, dx$$

$$= \frac{9(1-2x)^{-3}}{-3(-2)} + C$$

sub $(1,5)$, $C = \frac{13}{2}$

curve is

$$y = \frac{3}{2(1-2x)^3} + \frac{13}{2}$$

(ii) Find the range of values of x such that the gradient of the curve is decreasing. [3]

$$\frac{d^2y}{dx^2} = \frac{72}{(1-2x)^5}$$

gradient is decreasing $\Rightarrow \frac{d^2y}{dx^2} < 0$

since $72 > 0$, $\Rightarrow (1-2x)^5 < 0$

$$1 - 2x < 0$$

$$x > \frac{1}{2}$$

10 The mass m grams of a radioactive substance present at time t days after first being observed is given by the formula $m = 38e^{kt}$.
25 days after first being observed, the mass has dropped to 10.3 g.

(i) Find the initial mass of the radioactive substance. [1]

$t = 0, \quad m = 38\ g$

(ii) Find the number of days it takes after first being observed for the mass to become half its original value. [5]

$38e^{k(25)} = 10.3$

$k = -0.0522217$

$38e^{-0.0522217t} = 19$

$t = 13.274$ days

$= \underline{\underline{13.3}}$ days (3 sf)

(accept 14 days)

(iii) Find the rate at which the mass is decreasing when $t = 40$ days. [2]

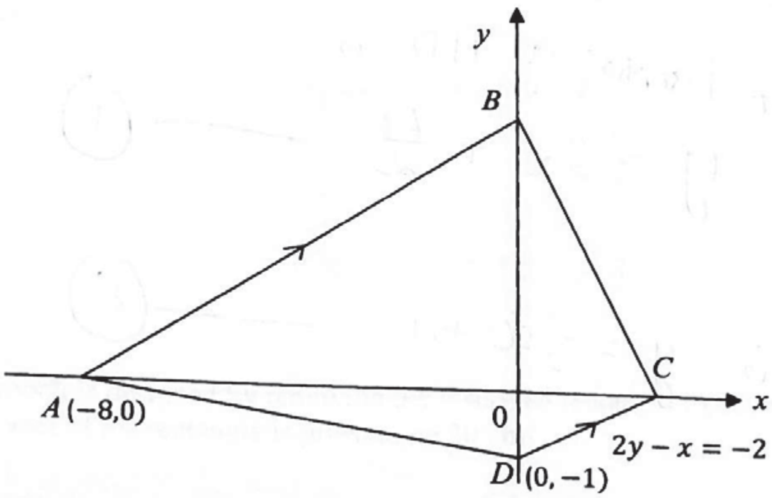
$$\frac{dm}{dt} = 38ke^{kt}$$

$$= 38(-0.052217)e^{-0.052217 \times 40}$$

$$\frac{dm}{dt} = -0.246 \quad (3sf)$$

mass is decreasing at a rate of 0.246 g/s.

11 **Solutions to this question by accurate drawing will not be accepted.**



The diagram shows a trapezium *ABCD* such that *AB* is parallel to *DC* and *AB* is perpendicular to *BC*. The point *A* is $(-8, 0)$ and *D* is $(0, -1)$. The equation of the line *CD* is $2y - x = -2$.

(i) Find the coordinates of *B*. [3]

Eqn of AB is $y = \frac{1}{2}x + 4$

B is $(0, 4)$

(ii) The perpendicular bisector of *AD* meets *AB* at point *E*.
Determine with explanation if it is possible to construct a circle with centre *E*, passing through the points *A* and *D*. [2]

Since E lies on the perpendicular bisector of AD, AE = DE

If E is the centre, then AE = DE = Radius.

hence, Yes.

(iii) Find the coordinates of E. [5]

Perpendicular bisector of AD is

$y = 8x + \frac{63}{2}$ —— (1)

line AB is $y = \frac{1}{2}x + 4$ —— (2)

$\therefore$ E is $\left(-3\frac{2}{3}, 2\frac{1}{6}\right)$.

working:

grad of AD $= \frac{0-(-1)}{-8-0} = -\frac{1}{8}$

grad of $\perp = 8$

midpt of AD is $\left(-4, -\frac{1}{2}\right)$

Eq^n of $\perp$ bisector is

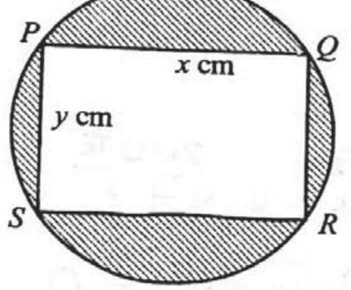
$y = 8x + c$

$-\frac{1}{2} = 8(-4) + c$

$c = \frac{63}{2}$

$\therefore \quad y = 8x + \frac{63}{2}$ //

12 The diagram shows a rectangle of length x cm and width y cm inscribed in a circle such that all the vertices of the rectangle $PQRS$ touch the circumference of the circle.



A brooch is designed by removing the inscribed rectangle $PQRS$ from the circle. The area of the rectangle is given to be 20 cm^2.

(i) Show that the radius of the circle is $\frac{\sqrt{x^4+400}}{2x}$. [3]

$$xy = 20 \Rightarrow y = \frac{20}{x}$$

$$SQ^2 = x^2 + \left(\frac{20}{x}\right)^2$$

$$= \frac{x^4+400}{x^2}$$

$$SQ = \sqrt{\frac{x^4+400}{x^2}} = \frac{\sqrt{x^4+400}}{x}$$

$$\therefore \text{radius} = \frac{\sqrt{x^4+400}}{2x} \;//$$

(ii) Ignoring the thickness of the brooch, show that the remaining area is given by

$$A = \frac{\pi x^2}{4} + \frac{100\pi}{x^2} - 20.$$

[2]

$$A = \pi r^2 - 20$$

$$= \pi\left(\frac{\sqrt{x^4+400}}{2x}\right)^2 - 20$$

$$= \pi\left(\frac{x^4+400}{4x^2}\right) - 20$$

$$= \frac{\pi x^2}{4} + \frac{100\pi}{x^2} - 20$$

(iii) Given that x can vary, find the value of x for which A has a stationary value. [3]

$$\frac{dA}{dx} = \frac{\pi x}{2} - \frac{200\pi}{x^3}$$

$$\frac{dA}{dx} = 0, \quad \frac{\pi x}{2} - \frac{200\pi}{x^3} = 0$$

$$x^4 - 400 = 0$$

$$x^4 = 400$$

$$x = 400^{\frac{1}{4}}$$

$$x = \underline{\underline{4.47}}$$

(iv) Determine the nature of the stationary value of A. [2]

$$\frac{d^2A}{dx^2} = \frac{\pi}{2} + \frac{600\pi}{x^4}$$

$$= 6.283 > 0$$

$\therefore$ Stat A is a $\underline{\underline{\text{minimum}}}$.