NAME: ______________________________ (   )   CLASS: _______

FAIRFIELD METHODIST SCHOOL (SECONDARY)

PRELIMINARY EXAMINATION 2023
SECONDARY 4 EXPRESS

ADDITIONAL MATHEMATICS   4049/01

Paper 1

Date: 24 August 2023   **Duration: 2 hours 15 minutes**

Candidates answer on the Question Paper.

No Additional Materials are required.

READ THESE INSTRUCTIONS FIRST

Write your name, index number and class on all the work you hand in.
Write in dark blue or black pen.
You may use a HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

The number of marks is given in brackets [ ] at the end of each question or part question.

The total number of marks for this paper is 90.

For Examiner's Use

Table of Penalties		**Question Number**		
Presentation	□ 1 □ 2			
Rounding off	□ 1		Parent's/Guardian's Signature	90

Setter: Mdm Haliza

This question paper consists of <u>21</u> printed pages including the cover page.

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Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

$$\Delta = \frac{1}{2}ab\sin C$$

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Answer **all** the questions.

1 Without using a calculator, find the exact value of

(i) $\operatorname{cosec}\theta$, given that θ is acute and $\cos\theta = \dfrac{3}{4}$, [1]

(ii) $\cos 30°(\tan 45° + \sin 60°)$. [2]

2 Find the set of values of *a* for which the curve $y = 2x^2 + 7$ lies entirely above the line $y = ax - 3$. [4]

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3 The equation of a curve is $y = Ae^{\frac{1}{2}x} + Be^{-\frac{1}{2}x}$, and that $\frac{dy}{dx} + \frac{3}{2}y = 2e^{\frac{1}{2}x} - 5e^{-\frac{1}{2}x}$.

Find the value of each of the constants A and B. [5]

4 Water is leaking from a container at a rate of 8 cm^3/s. If the volume of water, V cm^3, in the container is given by $V = \frac{3}{2}\left(h^2 + 8h\right)$ where h is the depth of the water, in cm, remaining in the container, find the rate of change of h when the volume of water is 13.5 cm^3. [5]

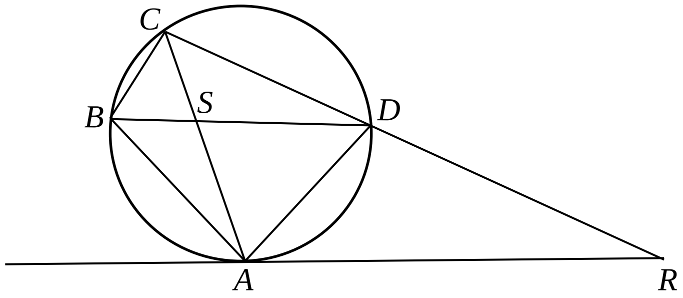
5 A certain radioactive material, radium-226, decomposes according to the formula $A = A_0 e^{kt}$ where A is the remaining mass in grams, after decomposition, A_0 is the original mass in grams, t is the time in years and k is a constant. A radioactive substance is often described in terms of its half-life, which is the time required for half the material to decompose.

(i) Given that after 400 years, a sample of radium-226 has decayed to 84.1% of its original mass, show that $k = -0.000433$, rounded off to 3 significant figures. [2]

(ii) Hence, find the half-life of radium-226, to the nearest whole number. [2]

(iii) If a sample of radium-226 has an initial mass of 100 grams, what is the remaining mass after 3200 years? [2]

6 In the diagram, A, B, C and D are points on the circle. The tangent at A meets CD produced at R. The chords AC and BD intersect at S. The line BSD bisects angle ABC.



Prove that

(i) $\angle DAR = \angle CAD$, [3]

(ii) ΔRAD is similar to ΔRCA, [2]

(iii) $RA^2 = RC \times RD$. [1]

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7 **(i)** Explain why there is no constant term in the expansion of $\left(x^3 - \frac{1}{x^2}\right)^8$. [2]

(ii) Show that the coefficient of x^{-6} in the expansion of $\left(x^3 - \frac{1}{x^2}\right)^8\left(1 + x^5\right)$ is 20. [4]

8 The equation of a curve is $y = 2x^2 - 4x + 9$.

(i) By expressing $2x^2 - 4x + 9$ in the form $a(x+b)^2 + c$, where a, b and c are constants, find the coordinates of the stationary point on the curve. [2]

8 **(ii)** The line $y = 3x + 3$ intersects the curve at points A and B. Find the value of h for which the distance AB can be expressed as $\sqrt{h}$. [4]

9 The equation of a curve is $y = x - \dfrac{2x+1}{1-2x}$.

(i) State the value of x for which y is not defined. [1]

(ii) Find $\dfrac{dy}{dx}$ and $\dfrac{d^2y}{dx^2}$. [4]

9 **(iii)** Find the coordinates of the stationary points of the curve. [3]

(iv) Using the second derivative test, find the nature of each stationary point. [3]

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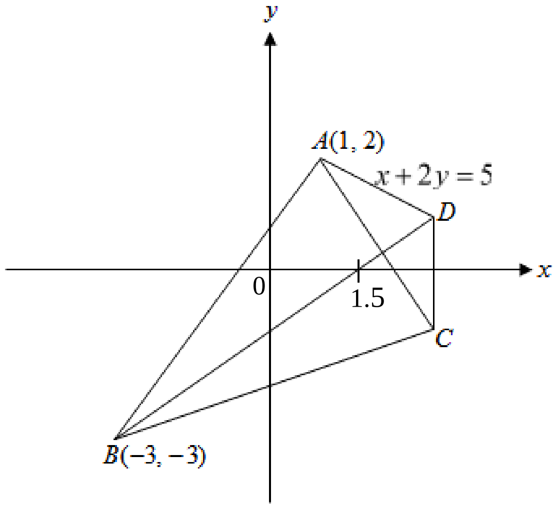
10 Solutions to this question by accurate drawing will not be accepted.



The diagram, not drawn to scale, shows a kite *ABCD*, where *A* is (1, 2) and *B* is $(-3,-3)$. The line *BD* cuts the *x*-axis at *x* = 1.5. The equation of the line *AD* is $x+2y=5$. Find the

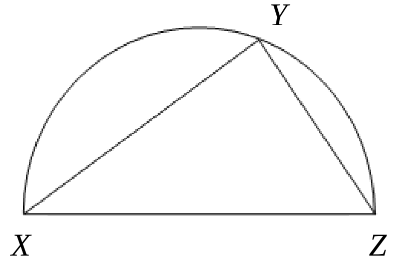
(i) coordinates of *D*, [4]

10 **(ii)** equation of AC, [2]

(iii) coordinates of C. Hence, find the area of $ABCD$. [4]

11 **(a)** Show that there is no solution for the equation $9^{x-1} = 3^x - 8$. [3]

11 **(b)** The diagram shows a semicircle XYZ with XZ as the diameter, $XY = \left(\sqrt{50}+\sqrt{2}\right)$ cm and $YZ = \left(\sqrt{28}-\sqrt{2}\right)$ cm.



(i) Show that $XZ^2 = 102 - 4\sqrt{14}$. [2]

(ii) Express $\tan\angle YXZ$ in the form $a\sqrt{14}+b$, where a and b are constants. [3]

12 A computer animation shows a cartoon giraffe moving in a straight line so that t seconds after it passes a fixed point O, its velocity, v cm s^{-1}, is given by $v = pt - qt^2$, where p and q are constants.

(i) Given that the giraffe attains a maximum speed of 48 cm s^{-1} after 2 seconds, show that the values of p and q are 48 and 12 respectively. [5]

12 **(ii)** Explain clearly why the total distance travelled by the giraffe in the interval $t = 0$ to $t = 7$ is not obtained by finding the value of the displacement, s, when $t = 7$. [1]

(iii) Find the total distance travelled by the giraffe in the first 7 seconds. [5]

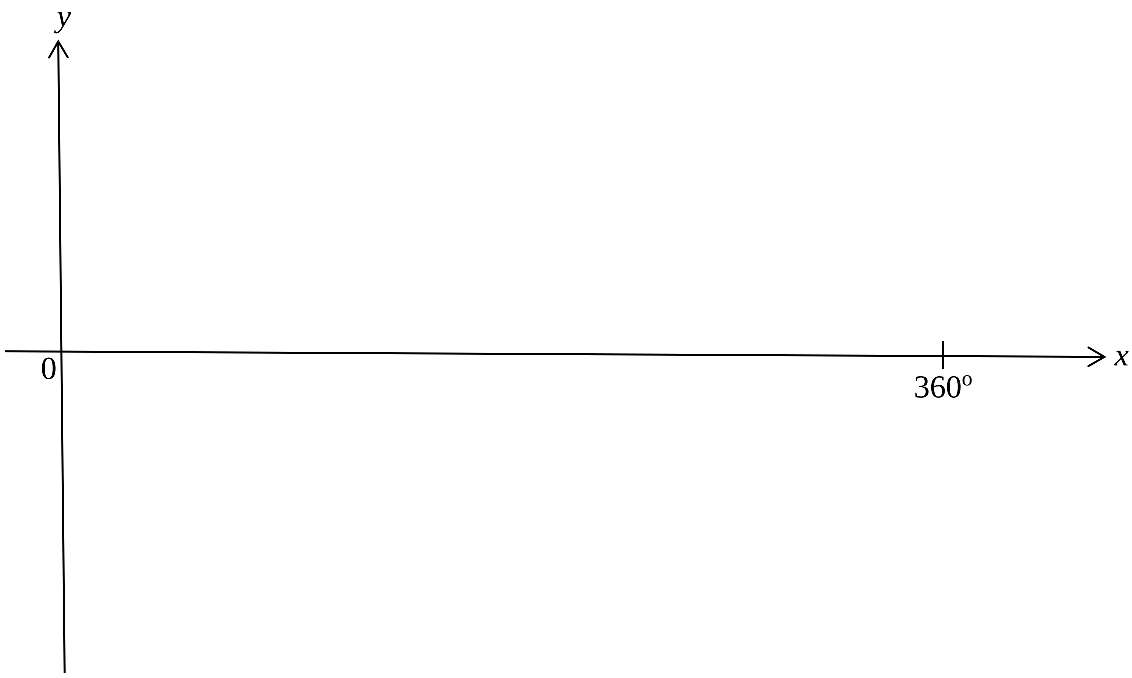
13 **(i)** It is given that $f(x) = 3\sin\left(\frac{x}{2}\right) + 1$.

(a) State the least and greatest values of $f(x)$. [2]

(b) State the period of $f(x)$. [1]

(ii) The graph of $g(x) = \tan ax$, where a is a constant, has a period of 480°. Find the value of a. [1]

13 **(iii)** On the same axes, sketch the graphs of f(x) and g(x) for $0° \le x \le 360°$. [4]



(iv) State the number of solutions of the equation $3\sin\left(\frac{x}{2}\right) = \tan ax - 1$ for $0° \le x \le 360°$. [1]

~ End of Paper ~

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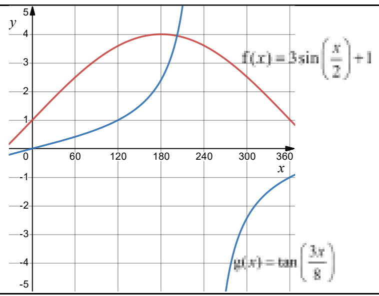
FMS(S) Sec 4 Exp Prelim Examination 2023
Additional Mathematics Paper 1

Answer Key

1(i)	$\frac{4}{\sqrt{7}}$	**1(ii)**	$\frac{\sqrt{3}}{2}+\frac{3}{4}$ or $\frac{2\sqrt{3}+3}{4}$
2	$-4\sqrt{5}<a<4\sqrt{5}$ or $-\sqrt{80}<a<\sqrt{80}$	**3**	$A=1,\ B=-5$
4	$-\frac{8}{15}$ cm/s or -0.533 cm/s (to 3 s.f.)	**5(ii)**	1601 years (to nearest whole number)
5(iii)	25.0 grams (to 3 s.f.)	**7(i)**	$r=4.8$ Hence, there is no constant term because r must be a positive integer/whole number.
8(i)	(1, 7)	**8(ii)**	$h=\frac{5}{2}$ or 2.5
9(i)	$x=\frac{1}{2}$	**9(ii)**	$\frac{dy}{dx}=1-\frac{4}{(1-2x)^2}$ or $\frac{(2x+1)(2x-3)}{(1-2x)^2}$ or $1-\frac{4x+2}{(1-2x)^2}+\frac{2}{1-2x}$
9(iii)	$\left(\frac{3}{2},\frac{7}{2}\right)$ and $\left(-\frac{1}{2},-\frac{1}{2}\right)$	**9(iv)**	y has a minimum point at $x=\frac{3}{2}$ and a maximum point at $x=-\frac{1}{2}$.
10(i)	D (3, 1)	**10(ii)**	$y=-\frac{3}{2}x+\frac{7}{2}$ or $2y+3x=7$
10(iii)	$C\left(\frac{41}{13},-\frac{16}{13}\right)$, 14 units^2	**11(a)**	$b^2-4ac=-207$. Since the discriminant is negative, there is no solution for the equation.
11(b)(ii)	$\frac{\sqrt{14}}{6}-\frac{1}{6}$	**12(ii)**	The giraffe changes direction/ moves in the opposite direction at $t=4$. So the total distance travelled by the giraffe in the interval $t=0$ to $t=7$ is not obtained by finding the value of s when $t=7$.
12(iii)	452 cm	**13(i)(a)**	Greatest value = 4 Least value $=-2$
13(i)(b)	720° or 4π	**13(ii)**	$\frac{3}{8}$
13(iii)		**13(iv)**	1 solution

2023 Sec 4 Express Additional Mathematics Paper 1 Preliminary Examinations Marking Scheme

No.	Solution	Marks	AO
1(i)	$\operatorname{cosec}\theta$ $=\dfrac{1}{\sin\theta}$ $=\dfrac{1}{\left(\dfrac{\sqrt{7}}{4}\right)}$ $=\dfrac{4}{\sqrt{7}}$	B1	AO 1
1(ii)	$\cos 30°(\tan 45° + \sin 60°)$ $=\dfrac{\sqrt{3}}{2}\left(1+\dfrac{\sqrt{3}}{2}\right)$ $=\dfrac{\sqrt{3}}{2}+\dfrac{3}{4}$ or $\dfrac{2\sqrt{3}+3}{4}$	B1 (special angle for cos 30 and sin 60) B1	AO 1
2	Sub. $y = ax - 3$ into $y = 2x^2 + 7$ $2x^2 + 7 = ax - 3$ $2x^2 - ax + 10 = 0$ Let $b^2 - 4ac < 0,$ $(-a)^2 - 4(2)(10) < 0$ $a^2 - 80 < 0$ $\left(a-4\sqrt{5}\right)\left(a+4\sqrt{5}\right)<0$ or $\left(a-\sqrt{80}\right)\left(a+\sqrt{80}\right)<0$ $-4\sqrt{5} < a < 4\sqrt{5}$ or $-\sqrt{80} < a < \sqrt{80}$	M1 (Form quadratic equation) M1 (Discriminant is negative) M1 (Factorise using surds) A1	AO 1

3	$y = Ae^{\frac{1}{2}x} + Be^{-\frac{1}{2}x}$ $\frac{dy}{dx} = \frac{1}{2}Ae^{\frac{1}{2}x} - \frac{1}{2}Be^{-\frac{1}{2}x}$ $\frac{dy}{dx} + \frac{3}{2}y = 2e^{\frac{1}{2}x} - 5e^{-\frac{1}{2}x}$ $\frac{3}{2}\left(Ae^{\frac{1}{2}x} + Be^{-\frac{1}{2}x}\right) = 2e^{\frac{1}{2}x} - 5e^{-\frac{1}{2}x} - \frac{1}{2}Ae^{\frac{1}{2}x} + \frac{1}{2}Be^{-\frac{1}{2}x}$ $\frac{3}{2}Ae^{\frac{1}{2}x} + \frac{3}{2}Be^{-\frac{1}{2}x} = \left(2 - \frac{1}{2}A\right)e^{\frac{1}{2}x} + \left(-5 + \frac{1}{2}B\right)e^{-\frac{1}{2}x}$ By comparing coefficients, $\frac{3}{2}A = 2 - \frac{1}{2}A$ $2A = 2$ $A = 1$ $\frac{3}{2}B = -5 + \frac{1}{2}B$ $B = -5$	B1 (Differentiate y correctly) M1 M1 (Compare coeff. for A or B correctly. FT from previous M1) A1 A1	AO 2
4	$V = \frac{3}{2}\left(h^2 + 8h\right)$ Sub. $V = 13.5$, $13.5 = \frac{3}{2}\left(h^2 + 8h\right)$ $3h^2 + 24h - 27 = 0$ $(3h + 27)(h - 1) = 0$ $h = -9$ (reject) or 1 $\frac{dV}{dh} = \frac{3}{2}\left(2h + 8\right)$ $= 3h + 12$ Sub. $h = 1$, $\frac{dV}{dh} = 3(1) + 12$ $= 15$ $\frac{dh}{dt} = \frac{dh}{dV} \times \frac{dV}{dt}$ $= \frac{1}{15} \times -8$ $= -\frac{8}{15}$ cm/s or -0.533 cm/s (to 3 s.f.)	M1 (Simplify to get quadratic equation) A1 B1 (Differentiate correctly) M1 (Substitute into Chain Rule. FT for dV/dh.) A1	AO 2

5(i)	$A = A_0 e^{kt}$ $0.841A_0 = A_0 e^{k(400)}$ $400k = \ln 0.841$ $k \simeq -0.00043290$ $= -0.000433$ (to 3 s.f.) (Shown)	M1 (Sub. A correctly) AG1	AO 3
5(ii)	$0.5A_0 = A_0 e^{-0.00043290t}$ $-0.00043290t = \ln 0.5$ $t = \dfrac{\ln 0.5}{-0.00043290}$ $\simeq 1601.2$ $= 1601$ years (to nearest whole number) OR $0.5A_0 = A_0 e^{-0.000433t}$ $-0.000433t = \ln 0.5$ $t = \dfrac{\ln 0.5}{-0.000433}$ $\simeq 1600.8$ $= 1601$ years (to nearest whole number)	M1 (Sub. A correctly) A1	AO 2
5(iii)	$A = 100e^{-0.00043290(3200)}$ $\simeq 25.025$ $= 25.0$ grams (to 3 s.f.) OR $A = 100e^{-0.000433(3200)}$ $\simeq 25.017$ $= 25.0$ grams (to 3 s.f.)	M1 (Substitute correctly) A1	AO 2
6(i)	$\angle DAR = \angle ABD$ (alternate segment theorem) $\angle CBD = \angle ABD$ (BSD bisects angle ABC) $\angle CBD = \angle CAD$ (angles in the same segment) $\therefore\ \angle DAR = \angle CAD$ (proven)	B1 (2 statements correct) B1 (3 statements correct) AG1	AO 3
6(ii)	$\angle ARD = \angle ARC$ (common angle) $\angle RAD = \angle RCA$ (Alternate segment theorem) $\therefore \Delta RAD$ is similar to ΔRCA (AA similarity or 2 pairs of corresponding angles are equal)	M1 (Both statements correct) AG1 (Similarity test must be stated)	AO 3

6(iii)	$\frac{RA}{RC}=\frac{RD}{RA}$ $\therefore RA^2 = RC \times RD$ (proven)	Form proportional ratios and conclude AG1	AO 3
7(i)	For $\left(x^3-\frac{1}{x^2}\right)^8$, $T_{r+1}=\binom{8}{r}\left(x^3\right)^{8-r}\left(-\frac{1}{x^2}\right)^r$ $=\binom{8}{r}(-1)^r x^{24-5r}$ For constant term, $24-5r=0$ $r=4.8$ (N.A.) <u>Hence, there is no constant term because *r* must be a positive integer/whole number.</u>	M1 (Form r +1 term) AG1 (Show that power of x is not 0 and <u>conclude accordingly</u>)	AO 3
7(ii)	For $\left(x^3-\frac{1}{x^2}\right)^8\left(1+x^5\right)$, $24-5r=-6$ $r=\frac{30}{5}$ $=6$ and $24-5r+5=-6$ $r=\frac{35}{5}$ $=7$ $\binom{8}{6}(-1)^6 x^{-6}(1)+\binom{8}{7}(-1)^7 x^{-11}\left(x^5\right)$ $=(28-8)x^{-6}$ $=20x^{-6}$ <u>Hence the coefficient of x^{-6} is 20 (Shown)</u>	M1 M1 M1 AG1	AO 3

8(i)	$2x^2-4x+9$ $=2(x^2-2x)+9$ $=2\left(x^2-2x+1-1\right)+9$ $=2\left[(x-1)^2-1\right]+9$ $=2(x-1)^2-2+9$ $=2(x-1)^2+7$ Stationary point is (1, 7).	B1(completed sq) B1	AO 1
8(ii)	Sub. $y=3x+3$ into $y=2x^2-4x+9$: $2x^2-4x+9=3x+3$ $2x^2-7x+6=0$ $(2x-3)(x-2)=0$ $x=1.5$ or $x=2$ $y=7.5$ $\quad y=9$ $AB=\sqrt{(2-1.5)^2+(9-7.5)^2}$ $=\sqrt{\dfrac{5}{2}}$ or $\sqrt{2.5}$ $\therefore h=\dfrac{5}{2}$ or 2.5	M1 (Equate and factorise) A1 (For both coordinates) M1 (Apply distance formula) A1	AO 2
9(i)	$y=x-\dfrac{2x+1}{1-2x}$ Since $1-2x\neq 0$ $x\neq\dfrac{1}{2}$ y is not defined at $x=\dfrac{1}{2}$.	B1	AO 1

9(ii)	$y = x - \frac{2x+1}{1-2x}$ $\frac{dy}{dx} = 1 - \frac{(1-2x)(2)-(2x+1)(-2)}{(1-2x)^2}$ $= 1 - \frac{2-4x+4x+2}{(1-2x)^2}$ $= 1 - \frac{4}{(1-2x)^2}$ or $\frac{(2x+1)(2x-3)}{(1-2x)^2}$ or $1 - \frac{4x+2}{(1-2x)^2} + \frac{2}{1-2x}$ $\frac{d^2y}{dx^2} = 8(1-2x)^{-3}(-2)$ $= -\frac{16}{(1-2x)^3}$ or $\frac{16}{(2x-1)^3}$	[Quotient Rule M1 –correct] M2 – *dy/dx* fully correct] A1 B1	AO 1
9(iii)	For stationary points, $\frac{dy}{dx} = 0.$ $1 - \frac{4}{(1-2x)^2} = 0$ $(1-2x)^2 = 4$ or $4x^2 - 4x - 3 = 0$ $1-2x = \pm 2$ $(2x-3)(2x+1) = 0$ $x = \frac{3}{2}, \quad -\frac{1}{2}$ $y = \frac{7}{2}, \quad -\frac{1}{2}$ Coordinates of stationary points are $\left(\frac{3}{2}, \frac{7}{2}\right)$ and $\left(-\frac{1}{2}, -\frac{1}{2}\right)$ (or equivalent)	M1 FT (Equate derivative to 0) A1, A1	AO 1
9(iv)	$\frac{d^2y}{dx^2} = -\frac{16}{(1-2x)^3}$ Sub. $x = \frac{3}{2}$, $\frac{d^2y}{dx^2} = 2 > 0$ Sub. $x = -\frac{1}{2}$, $\frac{d^2y}{dx^2} = -2 < 0$ Hence, y has a <u>minimum point</u> at $x = \frac{3}{2}$ and a <u>maximum point</u> at $x = -\frac{1}{2}$.	<u>Note: By 2nd derivative test only</u> M1 FT (Find second derivative <u>value</u> for either point) A1 A1	AO 1

10(i)	$m_{BD}=\dfrac{-3-0}{-3-1.5}$ $=\dfrac{2}{3}$ Equation of *BD* is $y-(-3)=\dfrac{2}{3}(x-(-3))$ $y+3=\dfrac{2}{3}x+2$ $y=\dfrac{2}{3}x-1$ or $3y=2x-3$ Equation of *AD*: $y=-\dfrac{1}{2}x+\dfrac{5}{2}$ $\dfrac{2}{3}x-1=-\dfrac{1}{2}x+\dfrac{5}{2}$ $\dfrac{7}{6}x=\dfrac{7}{2}$ $x=3$ $y=1$ $\therefore$ coordinates of *D* are (3, 1).	B1 M1(FT for value of gradient) M1 (FT from equation of BD) A1 (no FT)	AO 2
10 (ii)	$m_{AC}=-\dfrac{3}{2}$ Equation of *AC* is $y-2=-\dfrac{3}{2}(x-1)$ $y=-\dfrac{3}{2}x+\dfrac{7}{2}$ or $2y+3x=7$	B1 (FT from gradient of BD) B1 (no FT)	AO 2
10 (iii)	Note *CD* is not parallel to the *y*-axis. Let *E* be the mid-point of *AC*. *AC*: $y=-\dfrac{3}{2}x+\dfrac{7}{2}$ ----- (1) *BD*: $y=\dfrac{2}{3}x-1$ ----- (2) $(1)-(2):-\dfrac{3}{2}x+\dfrac{7}{2}-\left(\dfrac{2}{3}x-1\right)=0$ $-\dfrac{13}{6}x=-\dfrac{9}{2}$ $x=\dfrac{27}{13}$	M1 (FT from eqn. of AC and BD) OE	AO 2

<table>
<tr><td></td><td>$\therefore y = \frac{2}{3}\left(\frac{27}{13}\right) - 1$
$= \frac{5}{13}$
$\therefore E$ is $\left(\frac{27}{13}, \frac{5}{13}\right)$.
Let C be (x, y),
$\left(\frac{1+x}{2}, \frac{2+y}{2}\right) = \left(\frac{27}{13}, \frac{5}{13}\right)$
$\frac{1+x}{2} = \frac{27}{13}$ and $\frac{2+y}{2} = \frac{5}{13}$
$x = \frac{41}{13}$ $\quad y = -\frac{16}{13}$
$\therefore C$ is $\left(\frac{41}{13}, -\frac{16}{13}\right)$

Hence, area of $ABCD = \frac{1}{2}\begin{vmatrix} 1 & -3 & \frac{41}{13} & 3 & 1 \\ 2 & -3 & -\frac{16}{13} & 1 & 2 \end{vmatrix}$
$= \frac{1}{2}\left|-3 + \frac{48}{13} + \frac{41}{13} + 6 - (-6 - \frac{123}{13} - \frac{48}{13} + 1)\right|$
$= 14 \text{ units}^2$</td><td>A1 (no FT)

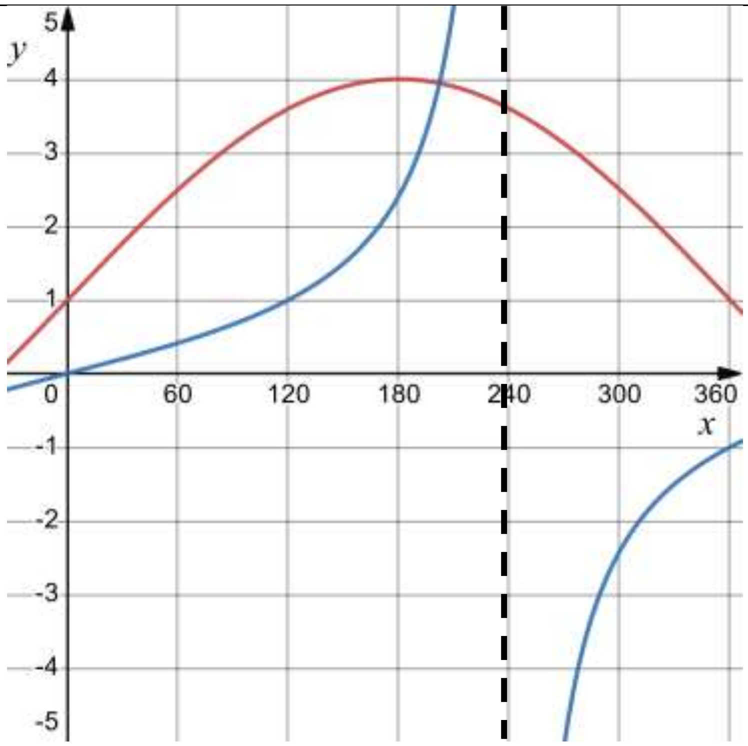
M1 (evaluate the ‘shoelace’. FT for coordinates of C and D)
A1 (no FT)</td><td></td></tr>
<tr><td>11(a)</td><td>$9^{x-1} = 3^x - 8$
$\frac{3^{2x}}{9} - 3^x = -8$
Let 3^x be y,
$\frac{y^2}{9} - y = -8$
$y^2 - 9y + 72 = 0$
$y = \frac{-(-9) \pm \sqrt{(-9)^2 - 4(1)(72)}}{2(1)}$ OR $b^2 - 4ac$
$= \frac{9 \pm \sqrt{-207}}{2}$ $\quad = (-9)^2 - 4(1)(72)$
$\quad = -207$
OR</td><td>M1 (or equivalent method)

M1 (solve for y or discriminant)</td><td>AO 3</td></tr>
</table>

	$\frac{1}{9}y^2 - y + 8 = 0$ $y = \dfrac{1 \pm \sqrt{-\frac{23}{9}}}{\frac{2}{9}}$ or $b^2 - 4ac = -\frac{23}{9}$ Since the discriminant is negative, there is <u>no solution</u> for the equation.	AG1 (mention "<u>no solution</u>")	
11 (b) (i)	Since $\angle XYZ = 90°$ (angle in a semi-circle), by Pythagoras' Theorem, $XZ^2 = \left(\sqrt{50}+\sqrt{2}\right)^2 + \left(\sqrt{28}-\sqrt{2}\right)^2$ $= 50 + 2\left(5\sqrt{2}\right)\left(\sqrt{2}\right) + 2 + 28 - 2\left(2\sqrt{7}\right)\left(\sqrt{2}\right) + 2$ $= 102 - 4\sqrt{14}$ (Shown)	B1 (state circle property for angle XYZ) M1 (apply Pythagoras' Theorem) AG0	AO 3
11 (b) (ii)	Gradient $= \tan \angle YXZ$ $= \dfrac{\sqrt{28}-\sqrt{2}}{\sqrt{50}+\sqrt{2}}$ $= \dfrac{2\sqrt{7}-\sqrt{2}}{5\sqrt{2}+\sqrt{2}} \times \dfrac{5\sqrt{2}-\sqrt{2}}{5\sqrt{2}-\sqrt{2}}$ $= \dfrac{10\sqrt{14}-2\sqrt{14}-10+2}{(25)(2)-2}$ $= \dfrac{8\sqrt{14}-8}{48}$ $= \dfrac{\sqrt{14}}{6} - \dfrac{1}{6}$	B1 (tangent ratio) M1 (rationalise denominator) A1	AO 2

12(i)	$v = pt - qt^2$ Sub. $v = 48$ when $t = 2$, $p(2) - q(2)^2 = 48$ $2p - 4q = 48$ $p - 2q = 24$ ---------(1) $\frac{dv}{dt} = p - 2qt$ At max. speed, $a = 0$ when $t = 2$, $p - 2q(2) = 0$ $p - 4q = 0$ -----------(2) $(1) - (2)$: $2q = 24$ $q = 12$ Sub. $q = 12$ into (1): $p - 2(12) = 24$ $p = 48$ $\therefore p = 48$ and $q = 12$ (Shown)	M1 (Form equation 1) M1 (Differentiate v) M1 (Form equation 2) M1 (solve simultaneously) AG1	AO 3
12 (ii)	At instantaneous rest, $v = 0$, $48t - 12t^2 = 0$ $12t(4 - t) = 0$ $t = 0$ or $t = 4$ The giraffe changes direction/ moves in the opposite direction at $t = 4$. So the total distance travelled by the giraffe in the interval $t = 0$ to $t = 7$ is not obtained by finding the value of s when $t = 7$.	B1 (Mention underlined phrase)	AO 3

12 (iii)	$s=\int(48t-12t^2)dt$ $=\dfrac{48t^2}{2}-\dfrac{12t^3}{3}+c$ $=24t^2-4t^3+c$ When $t = 0$, $s = 0$, $\therefore c=0$. $\therefore s=24t^2-4t^3$ When $t=4$, $s=24(4)^2-4(4)^3$ $= 128$ cm When $t=7$, $s=24(7)^2-4(7)^3$ $=-196$ cm Total distance travelled = $128+128+196$ = 452 cm	M1 (Integrate with + c) A1 (Find value of c) M1 (FT from s obtained) M1 (FT from s obtained) A1	AO 1
13(i) (a)	$\mathrm{f}(x)=3\sin\left(\dfrac{x}{2}\right)+1$ Greatest value = 3 + 1 = 4 Least value $=-3+1=-2$	B1 B1	AO 1
13(i) (b)	Period = $\dfrac{360°}{1/2}=720°$ or 4π	B1	AO 1
13 (ii)	$\mathrm{g}(x)=\tan ax$ $a=\dfrac{180}{480}$ $=\dfrac{3}{8}$	B1	AO 1

13 (iii)		For $\mathrm{f}(x) = 3\sin\left(\frac{x}{2}\right)+1$, S1 (shape correct and above x-axis) P1 (passes through (0,1), (180,4), (360,1) (FT from (i)(a) greatest value) For $\mathrm{g}(x) = \tan\left(\frac{3x}{8}\right)$, S1 (shape correct and on both sides of the asymptote $x = 240°$) P1 (passes through $(0,0)$ and symptote labelled at 240°)	AO 1
13 (iv)	1 solution	B1	AO 1

NAME: ___________________________ (   )   CLASS: _______

FAIRFIELD METHODIST SCHOOL (SECONDARY)

PRELIMINARY EXAMINATION 2023
SECONDARY 4 EXPRESS

ADDITIONAL MATHEMATICS **4049/02**
Paper 2

Date: 25 August 2023 **Duration: 2 hours 15 minutes**

READ THESE INSTRUCTIONS FIRST

Write your name, index number and class on all the work you hand in.
Write in dark blue or black pen.
You may use a pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** questions.
Write your answers on the space provided.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of a scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 90.

For Examiner's Use

Table of Penalties		Question Number		
Presentation	□ 1 □ 2			
Rounding off	□ 1		Parent's/Guardian's Signature	90

Setter : Mr Wilson Ho

This paper consists of 19 printed pages including this cover page.

Name: ________________________________ (   )   Class: __________

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2\sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

$$\Delta = \frac{1}{2}ab\sin C$$

Name: ________________________________ (   )   Class: __________

1 **(a)** By using long division, divide $2x^3+6x^2+x+3$ by $x+3$. [1]

(b) Express $\dfrac{9x^2-10x-16}{2x^3+6x^2+x+3}$ in partial fractions. [5]

Name: ______________________________________ (   )   Class: ___________

2 **(a)** Given that $y = e^{2x}(5x-4)$, show that $\dfrac{\mathrm{d}y}{\mathrm{d}x} = e^{2x}(10x-3)$. [3]

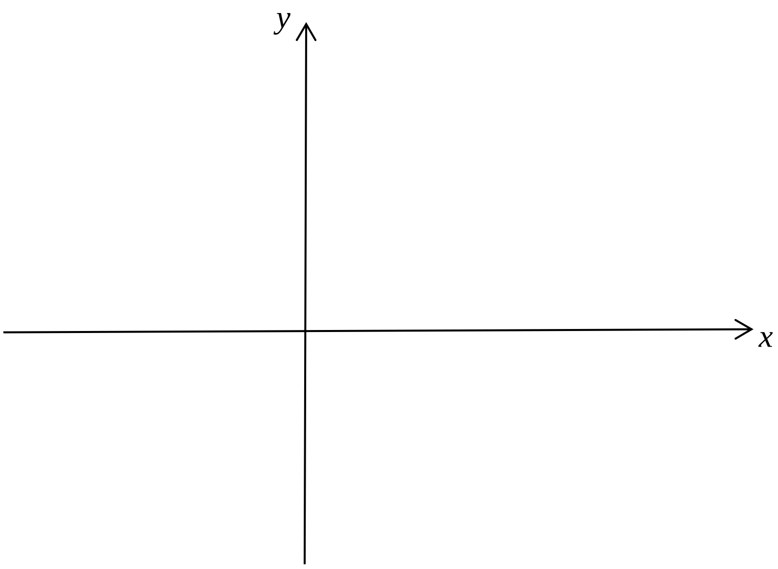
(b) Hence find $\int 4xe^{2x}dx$ and evaluate $\int_0^3 4xe^{2x}\ dx$. [5]

Name: ________________________________ (   )   Class: __________

3 The graph of $y = \log_p x$ passes through the points $(27, 3)$ and $(q, -2)$.

(a) Find the value of p and of q. [2]

(b) Sketch, on the same diagram, the graphs of $y = \log_p x$ and $y = 3^{-x}$. [3]



(c) State the number of solutions for $\log_p x = 3^{-x}$. [1]

Name: ____________________________________ (   )   Class: ___________

4 **(a)** Solve the equation $\log_6\left(2^y+1\right)-\log_6\left(2^y-4\right)=1$. [4]

(b) Given that $\left(\log_x xy\right)\left(\log_y x^6\right)-8=0$, express y in terms of x. [4]

5 It is given that $\mathrm{f}(x)$ is such that $\mathrm{f}''(x) = 4\cos 4x + 2\sin 2x$. Given also that $\mathrm{f}(0) = 0$ and $\mathrm{f}(\frac{\pi}{4}) = \frac{3}{4}$.

(a) Find $\mathrm{f}(x)$. [5]

5 **(b)** Show that $\mathrm{f}(\frac{\pi}{6}) = \frac{7-2\sqrt{3}}{8}$. [3]

Name: ______________________________ (   )   Class: __________

6 A circle, C_1, has equation $x^2 + y^2 - 4x + 6y = 12.$ The equation of the normal to this circle at a point is $3y - 4x = k.$

(a) Find the value of the constant k and the radius of C_1. [4]

A second circle, C_2, centre (14, 2), just touches C_1.

(b) Find the equation of C_2. [3]

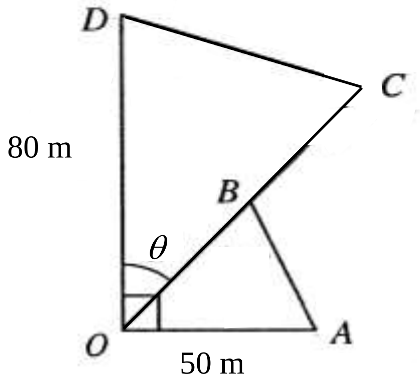
Name: ___________________________________ (   )   Class: __________

7 **(a)** Prove that $\dfrac{2\tan x+\sec^2 x}{1-\tan^2 x}=\dfrac{\cos x+\sin x}{\cos x-\sin x}$. [4]

Name: ________________________________ (   )   Class: __________

7 **(b)** Find all the values of x between $0°$ and $360°$ for which $\operatorname{cosec}^2 x - 5\cot x = -5$. [4]

8



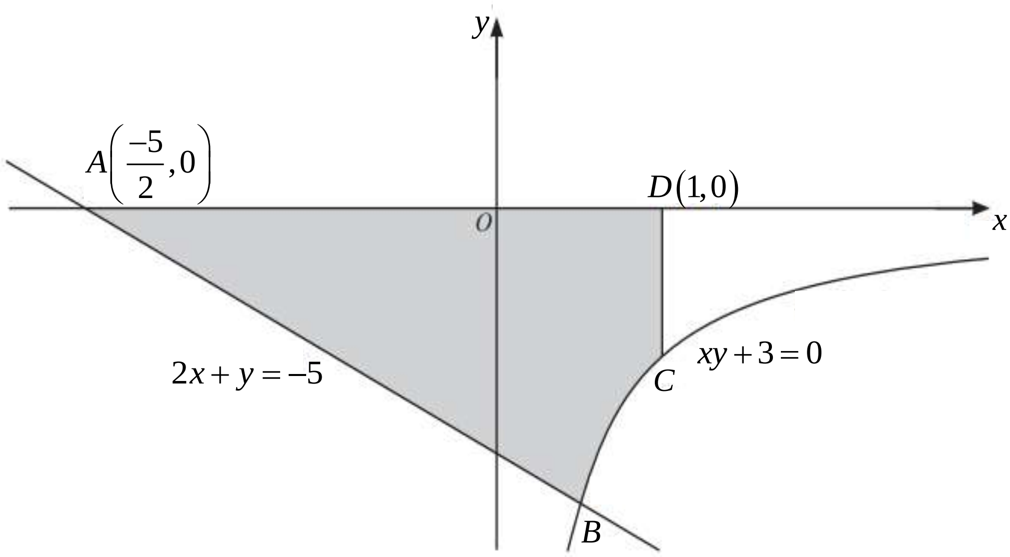
The diagram shows two triangular plots of land *OAB* and *OCD*. It is given that triangle *OAB* and triangle *OCD* are isosceles triangles with $OA = OB = 50$ m and $OC = OD = 80$ m.
Angle $AOD = 90^{\circ}$ and angle $COD = \theta$.
The sum of the areas of the two plots of land is S m^2 .

(a) Show that $S = 3200\sin\theta + 1250\cos\theta$. [2]

8 **(b)** Express S in the form $R\sin(\theta+\alpha)$ where $R>0$ and α is an acute angle. [4]

(c) Given that θ can vary, find the value of θ for which S will be a maximum. [2]

9



The diagram shows the straight line $2x+y=-5$ and part of the curve $xy+3=0$. The straight line intersects the *x*-axis at the point $A\left(\frac{-5}{2},0\right)$ and intersects the curve at the point *B*. The point *C* lies on the curve. The point *D* has coordinates (1, 0). The line *CD* is parallel to the *y*-axis.

(a) Find the coordinates of point *B*. [3]

9 **(b)** Find the area of the shaded region, giving your answer in the form $p + \ln q$ where p and q are positive integers. [5]

(c) Find the equation of the normal at point C. [4]

10 Given that $x^2 + 2x - 3$ is a factor of the function f(x) = $x^4 + 6x^3 + 2ax^2 + bx - 3a$,

(a) find the value of a and of b, [6]

(b) find the other quadratic factor of f(x), [2]

10 **(c)** show that the equation $f(x) = 0$ has two real distinct roots. [3]

Name: ________________________________ (   )   Class: __________

11 The variables x and y are related by the equation $y = 10^{-M} n^x$, where M and n are constants. The table shows values of x and y.

x	15	20	25	30	35	40
y	0.15	0.38	0.95	2.32	5.90	14.80

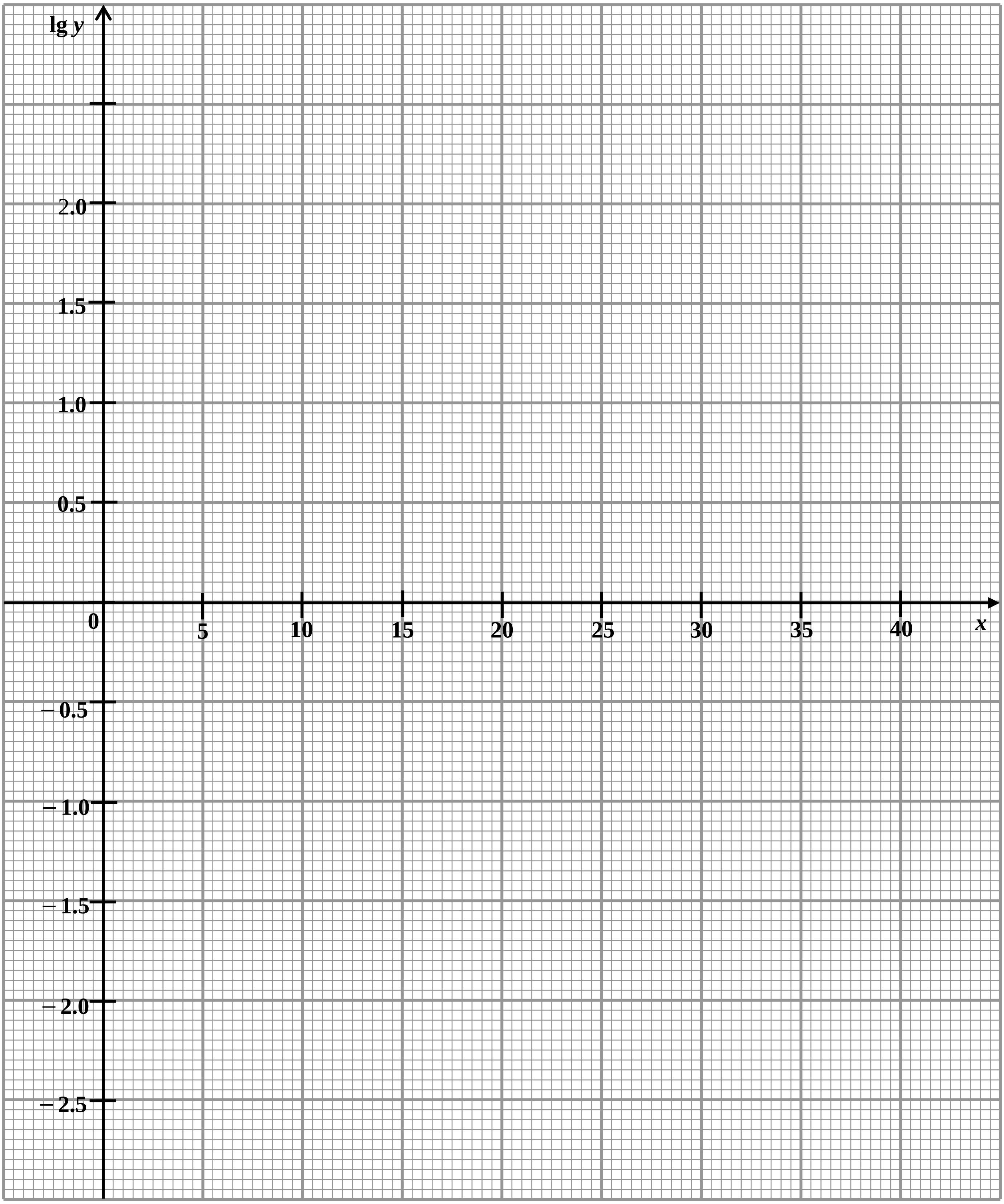
(a) Show your working clearly and draw a straight line graph of lg y against x on the grid provided. [2]

(b) Use your graph to estimate

(i) the value of M and of n, [4]

(ii) the value of x when $y = 10$. [2]

Name: ________________________________ (   )   Class: __________

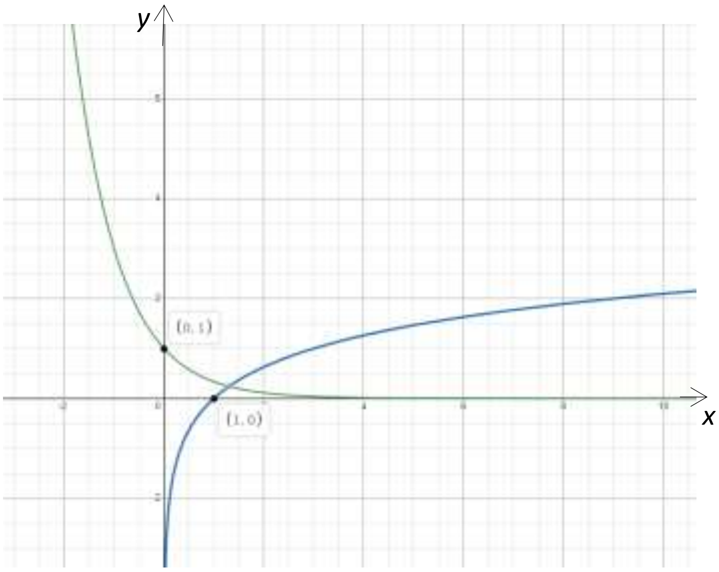


~ End-of-paper ~

Sec 4 Add Math Preliminary Exam 2023 P2 Marking Scheme

Qn. No.	Solution	Marks	AO
1(a)	$\begin{array}{r} 2x^2+1 \\ x+3\overline{)2x^3+6x^2+x+3} \\ -(2x^3+6x^2) \\ \hline x+3 \\ -(x+3) \\ \hline 0 \end{array}$ Or $2x^3+6x^2+x+3 \div x+3 = 2x^2+1$	B1 B1(either presentation)	AO1
1(b)	$\frac{9x^2-10x-16}{2x^3+6x^2+x+3} = \frac{9x^2-10x-16}{(x+3)(2x^2+1)} = \frac{A}{(x+3)}+\frac{Bx+C}{(2x^2+1)}$ $9x^2-10x-16 = A(2x^2+1)+(Bx+C)(x+3)$ Let $x=-3$, $81+30-16=19A$ $95=19A$ $A=5$ Let $x=0$, $-16=5+3C$ $-21=3C$ $C=-7$ Let $x=1$, $-17=15+4(B-7)$ $-17=-13+4B$ $B=-1$ $\frac{9x^2-10x-16}{2x^3+6x^2+x+3} = \frac{5}{(x+3)}+\frac{(-x-7)}{(2x^2+1)}$ or $\frac{5}{(x+3)}-\frac{x+7}{(2x^2+1)}$	M1 M1(substitution or comparison method) A1(1st unknown) Allow FT2 A1(for the remaining unknown) B1	AO1
2(a)	$\frac{d}{dx}\left(e^{2x}(5x-4)\right) = (5x-4)2e^{2x}+5e^{2x}$ $= e^{2x}(10x-8+5)$ $= e^{2x}(10x-3)$ (shown)	M2(M1 for each part of using product rule) AG1	AO3

Qn. No.	Solution	Marks	AO
2(b)	$\int\left[e^{2x}(10x-3)\right]dx = e^{2x}(5x-4)+C$ $\int 10xe^{2x} - 3e^{2x}\ dx = e^{2x}(5x-4)+C$ $10\int xe^{2x}dx - 3\int e^{2x}\ dx = e^{2x}(5x-4)+C$ $10\int xe^{2x}dx\ = e^{2x}(5x-4)+3\int e^{2x}dx + C$ $10\int xe^{2x}dx\ = e^{2x}(5x-4)+3\left(\frac{e^{2x}}{2}\right)+C$ $\frac{10}{4}\int 4xe^{2x}dx\ = e^{2x}(5x-4)+3\left(\frac{e^{2x}}{2}\right)+C$ $= e^{2x}\left(5x-4+\frac{3}{2}\right)+C$ $= e^{2x}\left(5x-\frac{5}{2}\right)+C$ $\int 4xe^{2x}dx\ = \frac{2}{5}e^{2x}\left(5x-\frac{5}{2}\right)+C$ or $= e^{2x}(2x-1)+C$ $\int_0^3 4xe^{2x}\ dx = \left[e^{2x}(2x-1)\right]_0^3$ $= \left[5e^{6}-(-1)\right]$ $= 5e^{6}+1$ or 2020 (3sf)	M1 M1(correct integration) M1(manipulation) M1(simplify and obtain $\int 4xe^{2x}dx$) A1	AO2
3(a)	$3 = \log_p 27$ $p^3 = 27$ $p = 3$ $-2 = \log_3 q$ $3^{-2} = q$ $q = \frac{1}{9}$	 B1 B1	AO1

Qn. No.	Solution	Marks	AO
3(b)		C2(for sketch of 2 curves correctly) P1(the x-intercept and y-intercept clearly indicated) Minus 1 mark if axes not labelled	AO1
3(c)	1 solution	B1	AO2
4(a)	$\log_6(2^y+1)-\log_6(2^y-4)=1$ $\log_6\frac{(2^y+1)}{(2^y-4)}=1$ or $\log_6 6$ $\frac{(2^y+1)}{(2^y-4)}=6^1$ $2^y+1=6(2^y-4)$ $2^y+1=6(2^y)-24$ $5(2^y)=25$ $2^y=5$ $y\ln 2=\ln 5$ $y=\frac{\ln 5}{\ln 2}$ $y=2.32$ (3 sf)	M1(quotient law) M1(simplify) M1(ln on both sides and power law) A1	AO1

Qn. No.	Solution	Marks	AO
4(b)	$\left(\log_x x+\log_x y\right)\left(\dfrac{\log_x x^6}{\log_x y}\right)=8$ $\left(1+\log_x y\right)\left(\dfrac{6}{\log_x y}\right)=8$ $\dfrac{6}{\log_x y}+6=8$ $\dfrac{6}{\log_x y}=2$ $\log_x y=\dfrac{6}{2}$ $y=x^3$	M1(product law) M1(change of base) M1(change log. to exponential form) A1	AO1
5(a)	$f'(x)=\int\left(4\cos 4x+2\sin 2x\right)\,dx$ $=\sin 4x-\cos 2x+C_1$ $f(x)=\int\left(\sin 4x-\cos 2x+C_1\right)\,dx$ $=\dfrac{-\cos 4x}{4}-\dfrac{\sin 2x}{2}+C_1x+C_2$ $f(0)=\dfrac{-1}{4}+C_2=0$ $C_2=\dfrac{1}{4}$ $f(\dfrac{\pi}{4})=\dfrac{1}{4}-\dfrac{1}{2}+C_1\left(\dfrac{\pi}{4}\right)+\dfrac{1}{4}=\dfrac{3}{4}$ $C_1\left(\dfrac{\pi}{4}\right)=\dfrac{3}{4}$ $C_1=\dfrac{3}{\pi}$ $f(x)=\dfrac{-\cos 4x}{4}-\dfrac{\sin 2x}{2}+\dfrac{3}{\pi}x+\dfrac{1}{4}$	M1(award marks even without C_1) M1(award marks even without C_2) M1(for substitution) A1(for either C_1 or C_2 correct) A1	AO2

Qn. No.	Solution	Marks	AO
5(b)	$f\left(\frac{\pi}{6}\right) = \frac{-\cos\frac{2\pi}{3}}{4} - \frac{\sin\left(\frac{\pi}{3}\right)}{2} + \frac{1}{2} + \frac{1}{4}$ $= \frac{\frac{1}{2}}{4} - \frac{\frac{\sqrt{3}}{2}}{2} + \frac{3}{4}$ $= \frac{1}{8} - \frac{\sqrt{3}}{4} + \frac{3}{4}$ $= \frac{7}{8} - \frac{\sqrt{3}}{4}$ $= \frac{7-2\sqrt{3}}{8}$ (shown)	M1(for basic angles) M1 AG1(depends on $\frac{7}{8} - \frac{\sqrt{3}}{4}$)	AO3
6(a)	(a) 2g = –4 and 2f = 6 g = –2 and f = 3 Centre (2, – 3) Sub $x = 2$ and $y = -3$, $3(-3) - 4(2) = k$. $k = -17$ Radius = $\sqrt{4+9+12}$ = 5 units	B1(for centre) M1(substitution) A1 B1	AO2
6(b)	Length between centre of C_1 to centre (14, 2) $\sqrt{(14-2)^2 + (2+3)^2}$ = 13 units Radius of C_2 = 8 units Eqn. of C_2 ----- $(x-14)^2 + (y-2)^2 = 64$ Or $x^2 + y^2 - 28x - 4y + 136 = 0$	M1 A1 A1	AO2

<table>
<tr><th>Qn. No.</th><th>Solution</th><th>Marks</th><th>AO</th></tr>
<tr><td>7(a)</td><td>
<u>LHS</u> $= \dfrac{2\tan x + 1 + \tan^2 x}{1 - \tan^2 x}$
$= \dfrac{(1+\tan x)^2}{(1-\tan x)(1+\tan x)}$
$= \dfrac{(1+\tan x)}{(1-\tan x)}$
$= \dfrac{1+\dfrac{\sin x}{\cos x}}{1-\dfrac{\sin x}{\cos x}}$
$= \dfrac{\dfrac{\cos x + \sin x}{\cos x}}{\dfrac{\cos x - \sin x}{\cos x}}$
$= \dfrac{\cos x + \sin x}{\cos x - \sin x}$
= RHS (proved)
</td><td>M1(change sec^2x)

M1(either factorization of numerator or denominator)

M1(change tan x)

AG1</td><td>AO3</td></tr>
<tr><td>Or
7(a)</td><td>
<u>LHS</u> $= \dfrac{2\dfrac{\sin x}{\cos x} + \dfrac{1}{\cos^2 x}}{1 - \dfrac{\sin^2 x}{\cos^2 x}}$
$= \dfrac{\dfrac{2\sin x\cos x + 1}{\cos^2 x}}{\dfrac{\cos^2 x - \sin^2 x}{\cos^2 x}}$
$= \dfrac{2\sin x\cos x + 1}{\cos^2 x - \sin^2 x}$
$= \dfrac{2\sin x\cos x + \sin^2 x + \cos^2 x}{(\cos x + \sin x)(\cos x - \sin x)}$
$= \dfrac{(\cos x + \sin x)(\cos x + \sin x)}{(\cos x + \sin x)(\cos x - \sin x)} = \dfrac{\cos x + \sin x}{\cos x - \sin x}$ = RHS(proved)
</td><td>M1(change tan x & sec^2 x)

M1(simplify fractions)

M1(factorization of either numerator or denominator</td><td></td></tr>
</table>

Qn. No.	Solution	Marks	AO
7(b)	$\operatorname{cosec}^2 x - 5\cot x = -5$ $\left(1+\cot^2 x\right) - 5\cot x + 5 = 0$ $\cot^2 x - 5\cot x + 6 = 0$ $\left(\cot x - 2\right)\left(\cot x - 3\right) = 0$ $\cot x = 2$ or $\cot x = 3$ $\tan x = \frac{1}{2}$ or $\tan x = \frac{1}{3}$ $\alpha = 26.56°$ or $18.43°$ $x = 18.4°,\ 26.6°,\ 198.4°,\ 206.6°$ (1 dp)	M1(sub. $1+\cot^2 x$) M1(factorization or use quadratic formula) A2(all values of x) Or A1(for 2 angles)	AO1
8(a)	Area of triangle OAB = $\frac{1}{2} \times 50 \times 50 \times \sin(90^o - \theta)$ $= 1250\cos\theta$ Area of triangle ODC = $\frac{1}{2} \times 80 \times 80 \times \sin\theta$ $= 3200\sin\theta$ Total area S = 3200 sinθ + 1250 cosθ (shown)	M1 AG1	AO3

8(b)	(a) Let $3200\sin\theta+1250\cos\theta=R\sin(\theta+\alpha)$ $=R\sin\theta\cos\alpha+R\cos\theta\sin\alpha$ By comparing, $R\cos\alpha=3200$ (1), $R\sin\alpha=1250$ (2) (2)/(1) $\frac{R\sin\alpha}{R\cos\alpha}=\frac{1250}{3200}$ $\tan\alpha=\frac{25}{64}$ $\alpha=21.3^o$ (3 sf) $(1)^2+(2)^2$, $R^2=10240000+1562500$ $R=\sqrt{11802500}=50\sqrt{4721}$ or 3440 (3sf) $S=\sqrt{11802500}\sin(\theta+21.3^o)$ or $50\sqrt{4721}\sin(\theta+21.3^o)$ or $3435.5\sin(\theta+21.3^o)$ or $3440\sin(\theta+21.3^o)$	M1(for two eqns) B1(for α) B1(for R) A1	AO1
8(c)	<u>Max. value of *S*</u> When $\sin(\theta+21.34^o)=1$ $\theta+21.34^o=90^o$ $\theta=68.66^o$ *or* 68.7^o	M1 A1	AO1
9(a)	$2x+y=-5$ ----- (1) From (1), $y=-5-2x$ Sub, into $x(-5-2x)+3=0$ $-2x^2-5x+3=0$ $2x^2+5x-3=0$ $(x+3)(2x-1)=0$ $x=-3$ (NA) or $x=\frac{1}{2}$ When $x=\frac{1}{2}$, $y=-6$ B($\frac{1}{2}$, -6)	M1(substitution) M1(factorization or using quad. formula) A1	AO1

9(b)	Area of triangle = $\frac{1}{2}\times\left(\frac{5}{2}+\frac{1}{2}\right)\times 6=9$ $\int_{\frac{1}{2}}^{1}-\frac{3}{x}\,dx = -3\left[\ln x\right]_{\frac{1}{2}}^{1}$ $= 3\ln\frac{1}{2}$ $= -3\ln 2$ or $\ln\frac{1}{8}$ $= -\ln 8$ Area above curve = ln 8 Total area = 9 + $\ln 8$	M1(allow FT using their values) M1 M1(correct application of limits) M1(apply law of log. get $-\ln 8$) A1(total area)	AO2
9(c)	$y=\frac{-3}{x}$ $\frac{dy}{dx}=-3\left(-x^{-2}\right)$ $= 3x^{-2}$ At x = 1, $\frac{dy}{dx}=3$ Gradient of normal = $-\frac{1}{3}$ When x =1 , y = -3 <u>Equation of normal</u> $y+3=-\frac{1}{3}\left(x-1\right)$ or $y=-\frac{1}{3}x-\frac{8}{3}$	B1 B1 B1(find y coordinate of pt. C) A1	AO2

10(a)	$x^2+2x-3 = (x-1)(x+3)$ f(1) = $1+6+2a+b-3a=0$ $b-a=-7$ -------- (1) f(−3) = $(-3)^4+6(-3)^3+2a(-3)^2+b(-3)-3a=0$ $15a-3b-81=0$ $5a-b=27$ ----------- (2) (1) + (2), $4a=20$ $a=5$ Sub. $a=5$ into (1), $b=-2$	M1(factorization to obtain 2 factors) M1(sub x =1 and obtain eqn) M1(sub x =-3 and obtain eqn) M1(solve simultaneous eqns) A2	AO1
10(b)	f(x) = $x^4+6x^3+10x^2-2x-15$ $=(x^2+2x-3)(x^2+kx+5)$ By comparing coeff. of x^3, $6=k+2$ $k=4$ The other quadratic factor is x^2+4x+5.	M1(comparison or long division method) A1	AO1
10(c)	$(x^2+2x-3)(x^2+4x+5)=0$ $(x-1)(x+3)(x^2+4x+5)=0$ $x=1$ or $x=-3$ $b^2-4ac = 4^2-4(1)(5)$ $= -4<0$ No real roots There is only 2 real distinct roots.	M1(two real roots) M1(discriminant) AG1	AO3
11(a)	See attached graph. <table><tr><td>x</td><td>15</td><td>20</td><td>25</td><td>30</td><td>35</td><td>40</td></tr><tr><td>$lg\ y$</td><td>−0.82</td><td>−0.42</td><td>−0.022</td><td>0.37</td><td>0.77</td><td>1.17</td></tr></table>	P1 L1	AO1

11(b) (i)	$\lg y = \lg\left(10^{-M} n^x\right)$ $= \lg 10^{-M} + \lg n^x$ $= -M \lg 10 + x \lg n$ $= -M + x \lg n$ y-intercept = $-2.025 \Rightarrow -M = -2.025$ $M = 2.025 \pm 0.05$ Gradient = $\dfrac{1.175 - 0.775}{40 - 35} = 0.08 \pm 0.01$ $\lg n = 0.08 \Rightarrow n = 10^{0.08} = 1.20 \pm 0.03$	 M1(product law) B1(for M) M1(gradient) A1	AO2
11(b) (ii)	$y = 10$ $\lg y = \lg 10 = 1$ When $\lg y = 1$, from the graph, $x = 37.75 \pm 1$	 M1(find lg y) A1	AO1