JUNYUAN SECONDARY SCHOOL
PRELIMINARY EXAMINATION 2023
SECONDARY FOUR EXPRESS / FIVE NORMAL (ACADEMIC)

CANDIDATE NAME

CLASS

INDEX NUMBER

ADDITIONAL MATHEMATICS

4049/01

Paper 1

29 August 2023

2 hours 15 minutes

Candidates answer on the Question Paper

No Additional Materials are required

READ THESE INSTRUCTIONS FIRST

Write your name, class and index number at the top of this page.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.

The total number of marks for this paper is 90.

For Examiner's Use
90

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{(n-r)!r!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2\sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

$$\Delta = \frac{1}{2}ab\sin C$$

1 A curve has equation $y = (x - 4)\sqrt{2x+3}$. Find the rate of change of x when $x = 3$, given that y is increasing at a constant rate of 4 units/s. [4]

2 **(i)** Show that $\log_2 p + \log_4 p = \dfrac{3\ln p}{\ln 4}$. [3]

(ii) Hence solve the equation $\log_2 p + \log_4 p = 9$. [2]

3 It is given that $2 + \lg (x - y) = \lg x$, where x and y are positive constants.

(i) Explain why $y < x$. [1]

(ii) Express y in terms of x. [4]

4 **(i)** Write down and simplify, in ascending powers of x, the first three terms in the expansion of $(2 - 3x)^5$. [2]

(ii) Given that the expansion of $(p + qx)(2 - 3x)^5$ up to the terms in x^2 is $8 + rx + 1680x^2$, find the value of p, of q and of r. [4]

5 A curve has equation $y = x^3 + 2x^2 + x + c$, where c is a constant.

(i) Find the x-coordinates of the stationary points on the curve. [4]

(ii) Hence find the value of c for which the maximum point of the curve lies on the line $y = 2$. [3]

6 **(a)** Write down the principal value of the following in radians.

(i) $\cos^{-1}(-\frac{1}{2})$, [1]

(ii) $\tan^{-1}(-\frac{1}{\sqrt{3}})$. [1]

(b) It is given that P, Q and R are the angles of a triangle.

(i) Show that $\cos P = -\cos(Q + R)$. [2]

(ii) Given that $Q = 45°$ and $R = 60°$, find $\cos P$ in the form $\frac{1}{4}(\sqrt{a} - \sqrt{b})$, where a and b are integers. [3]

7 It is given that $f(x) = \cos\left(\frac{x}{2}\right) - 1$ and $g(x) = 3\sin 2x$.

(i) State the least and greatest values of $f(x)$. [2]

(ii) State the period and amplitude of $g(x)$. [2]

(iii) Sketch the graph of $y = g(x)$ for $0° \leq x \leq 360°$. [3]

(iv) Hence state the number of solution(s) for the equation $3\sin 2x = \cos\left(\frac{x}{2}\right) - 1$ for $0° < x < 90°$. [1]

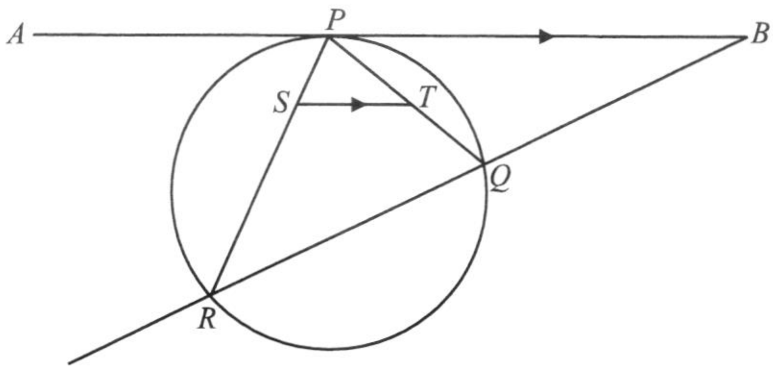
8 **(a)** What conditions must be applied to the constants a and c such that $ax^2 + 3x + c$ is always positive? [2]

(b) Find the range of values of x for which $2x(8x - 3) < 15x^2 - x + 14$. [3]

(c) Find the values of h for which the line $y = 6x + h$ is a tangent to the curve $x^2 - xy = 5$. [4]

9 The diagram, not drawn to scale, shows a circle passing through the points P, Q and R. The point Q lies on the line RB. AB is a tangent to the circle at P. The points S and T lie on PR and PQ respectively. Given that AB is parallel to ST, prove that

(i) triangle PST is similar to triangle PQR, [3]



(ii) $PQ \times PT = PR \times PS$, [2]

(iii) angle QTS + angle $QRS = 180°$, [3]

(iv) angle RST + angle $RQT = 180°$. [1]

10 The equation of a circle with centre O is given by $(x+2)^2 + y^2 = 10y + 75$.

(i) Find the radius of the circle and the coordinates of O. [3]

The point $P(4, -3)$ lies on the circle.

(ii) Find the equation of the tangent to the circle at P. [3]

(iii) The tangent of the circle at P meets the x-axis at Q. Find the coordinates of Q. [1]

(iv) Find the value of $\tan \angle OQP$. [2]

11 The equation of a polynomial is given by $f(x) = 4x^3 + 9x - 5$.

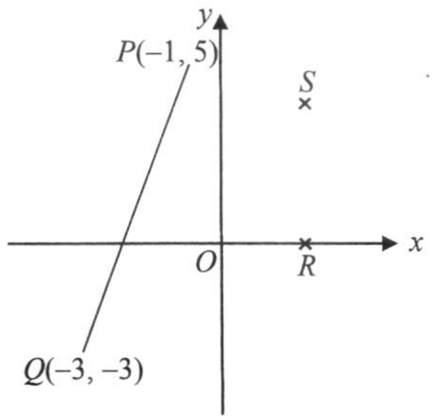
(i) Find the remainder when $f(x)$ is divided by $x + 1$. [1]

(ii) Show that $2x - 1$ is a factor of $f(x)$. [1]

(iii) Show that the equation $f(x) = 0$ has only one real root. [4]

(iv) Hence, or otherwise, solve the equation $\frac{4}{3}(3^{3y+1}) + 9(3^y) = 5$. [4]

12 **Solutions to this question by accurate drawing will not be accepted.**
The diagram, not drawn to scale, shows a line segment PQ where $P(-1, 5)$ and $Q(-3, -3)$. It is given that the point R lies on the perpendicular bisector of PQ and also lies on the x-axis. The point S is in the first quadrant.



(i) Find the coordinates of R. [6]

It is given that RS is parallel to the y-axis and the area of triangle PRS is 6 units^2.

(ii) Explain why the y-coordinate of S is 4. [3]

(iii) Hence, or otherwise, calculate the area of the quadrilateral $PQRS$. [2]

End of Paper

BLANK PAGE

BP~473

JUNYUAN SECONDARY SCHOOL
PRELIMINARY EXAMINATION 2023
SECONDARY FOUR EXPRESS / FIVE NORMAL (ACADEMIC)

CANDIDATE NAME

CLASS

INDEX NUMBER

ADDITIONAL MATHEMATICS

4049/02

Paper 2

30 August 2023

2 hours 15 minutes

Candidates answer on the Question Paper

No Additional Materials are required

READ THESE INSTRUCTIONS FIRST

Write your name, class and index number at the top of this page.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.

The total number of marks for this paper is 90.

For Examiner's Use
90

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1} a^{n-1}b + \binom{n}{2} a^{n-2}b^2 + \ldots + \binom{n}{r} a^{n-r}b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{(n-r)!\,r!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2\sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

$$\Delta = \frac{1}{2}ab\sin C$$

1 P is on the y-axis. The normal to the curve $y = 3\sin 2x - 1$ at P meets the x-axis at Q. Find

(i) the equation of the normal to the curve at P, [4]

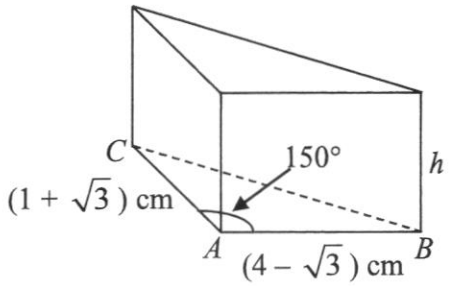
(ii) the coordinates of Q. [2]

2 **(a)** Prove that $\cot^2 x - \cos^2 x = \cos^4 x \operatorname{cosec}^2 x$. [3]

(b) Solve the equation $2\sin y \cos y = \sin y$ for $0° < y < 360°$. [3]

3 **(a)** Find the value of a and of b such that $3b + a\sqrt{75} = 6a - 10\sqrt{3}$. [3]

(b) A prism has a triangular base ABC such that the length of AB and AC are $(4 - \sqrt{3})$ cm and $(1 + \sqrt{3})$ cm respectively. Angle $BAC = 150°$. The prism has a height of h cm and a volume of $\frac{1}{2}(30 - \sqrt{3})$ cm^3. Without using a calculator, show that h can be expressed as $(p + q\sqrt{3})$ cm, where p and q are integers. [5]

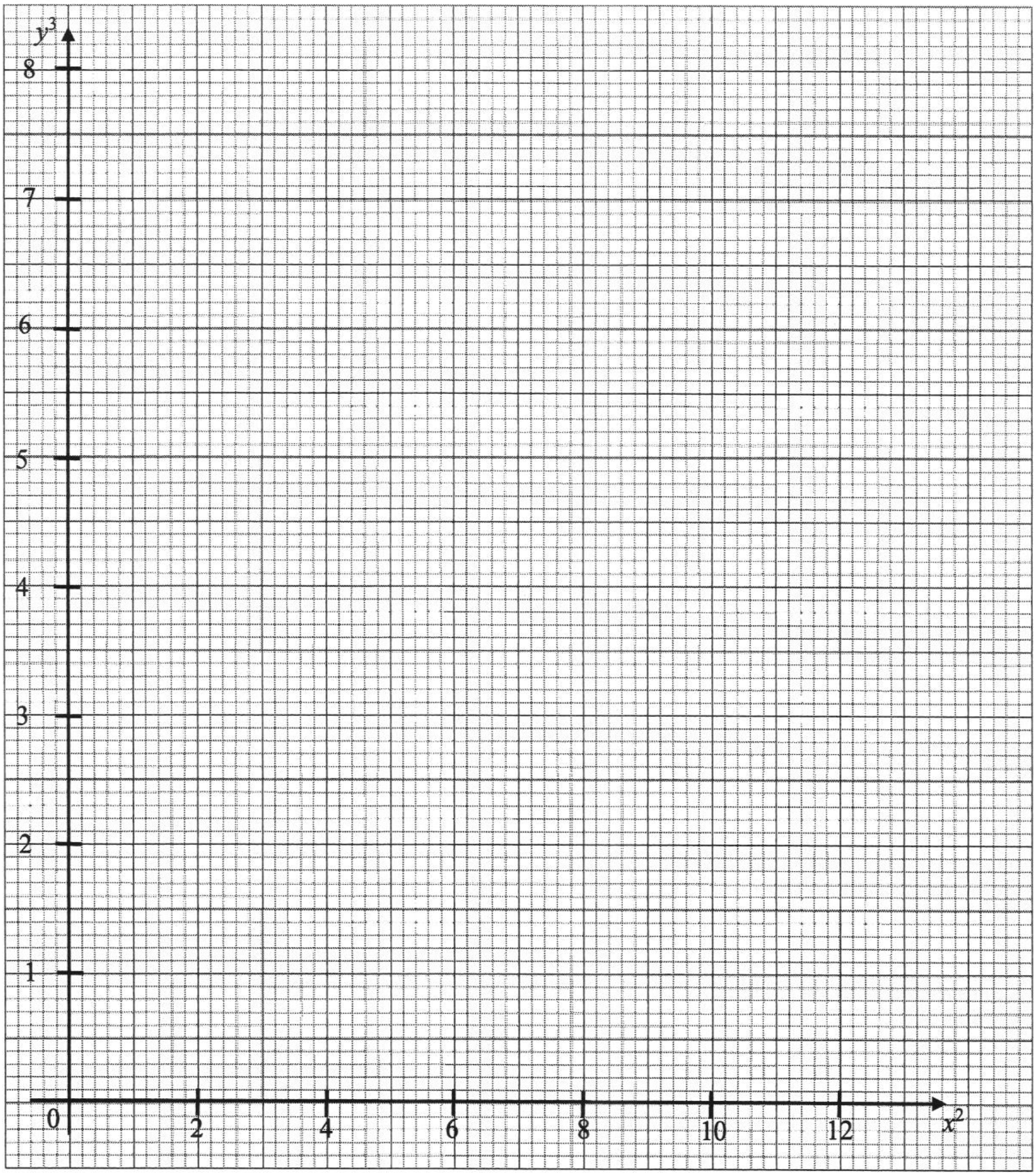


4 The table shows experimental values of two variables x and y, which are connected by the equation $py^3 - qx^2 = 1$, where p and q are constants.

x	0.5	1	2	3
y	1.94	1.89	1.69	1.14

(i) Complete the table below and draw the straight line graph of y^3 against x^2 on the grid. Give your values to 2 decimal places. [3]

x^2	0.25	1	4	9
y^3				



(ii) Use your graph to estimate

(a) the value of p and of q, [3]

(b) the value of x^2 when $y = 1.5$. [1]

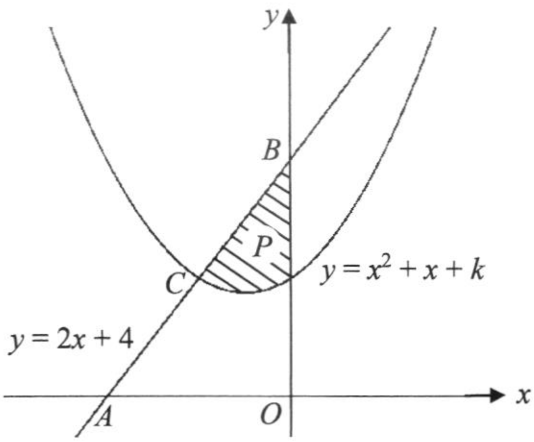
5 **(a)** Solve the equation $2^{x+2} = 2^{-x} - 3$. [4]

(b) **(i)** Given that $8^{x+1} \times 5^{3x-2} = 10^{4x}$, find the value of 10^{x}. [3]

(ii) Hence solve $8^{x+1} \times 5^{3x-2} = 10^{4x}$. [1]

6 The diagram, not drawn to scale, shows part of the curve $y = x^2 + x + k$ and the line $y = 2x + 4$. The line $y = 2x + 4$ intersects the x-axis and the y-axis at A and B respectively. C is the midpoint of AB, and also a point of intersection of the line $y = 2x + 4$ and the curve $y = x^2 + x + k$. The region P is bounded by the line, the curve and the y-axis.

(i) Find the coordinates of C. [3]

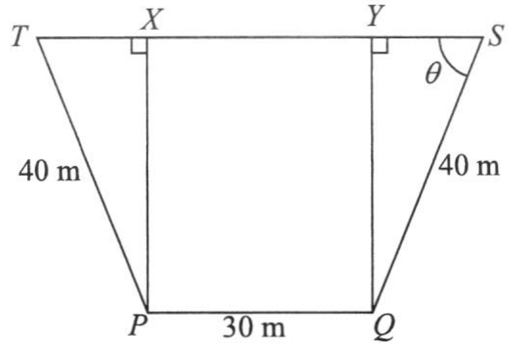


(ii) Find the value of k. [1]

(iii) Find the area of region P. [4]

7 An owner fences his private property. He puts fences around the perimeter of the trapezium *PQST*. He also puts extra fences from *P* to *X* and from *Q* to *Y*, where *PX* and *QY* are perpendicular to *TS*. Angle *QSY* = angle *PTX* = θ radians and the lengths of *PT* and *QS* are 40 m. It is also given that *PQ* = 30 m.

(i) Show that the length of fencing needed, *L* m, can be expressed in the form $a + b\cos\theta + c\sin\theta$, where *a*, *b* and *c* are constants to be found. [3]



(ii) Express L in the form $a + R\cos(\theta - \alpha)$, where $R > 0$ and α is an acute angle. [3]

(iii) Explain why L is always greater than 25 m. [2]

8 A particle travels in a straight line such that its displacement, x m, from a fixed point O is given by $x = t^3 - 12t^2 + 45t$, where t is the time in seconds after passing through O.

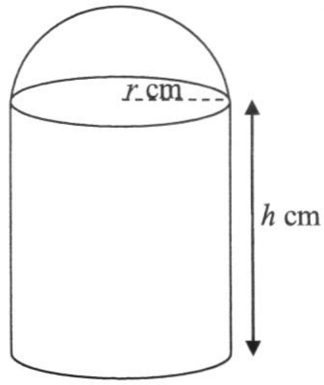
(i) Find the times when the particle is at instantaneous rest. [3]

(ii) Hence, or otherwise, find the distance travelled in the first 4 seconds of motion. [3]

(iii) Explain why the particle will never return to its starting point. [3]

9 A container has a volume of V cm^3 and is made up of a hemisphere of radius r cm on top of a cylinder of radius r cm and height h cm as shown in the diagram.

(i) Show that $h = \dfrac{V}{\pi r^2} - \dfrac{2r}{3}$. [2]



(ii) Given that $V = 2880\pi$ cm^3 and that the container is made of plastic of negligible thickness, show that the total surface area is represented by $A = \dfrac{5760\pi}{r} + \dfrac{5\pi r^2}{3}$ cm^2. [3]

(iii) Given that r can vary, find the value of r such that the total surface area is a minimum. [4]

10 **(i)** Given that $\dfrac{7x+3}{x^2(2x+1)} = \dfrac{A}{x} + \dfrac{B}{x^2} + \dfrac{C}{2x+1}$, where A, B and C are constants, find the value of A, of B and of C. [5]

(ii) Hence find $\int_1^3 \frac{7x+3}{x^2(2x+1)}\,\mathrm{d}x$ in the form $\ln a + b$, where a and b are rational numbers. [5]

11 **(a)** The function f is defined by $f(x) = \dfrac{3-x^2}{x^2+2}$, $x > 0$. Explain whether f is an increasing or decreasing function. [4]

(b) A curve is such that $\frac{d^2y}{dx^2} = 2x - 8$. The gradient of the curve at the point (3, 7) is 3.

(i) Express y in terms of x. [5]

(ii) Explain why the gradient of the curve is never less than 2. [2]

End of Paper

BLANK PAGE

BLANK PAGE

BLANK PAGE

BP~499

JUNYUAN SECONDARY SCHOOL
PRELIMINARY EXAMINATION 2023
SECONDARY FOUR EXPRESS / FIVE NORMAL (ACADEMIC)

CANDIDATE NAME

CLASS

INDEX NUMBER

ADDITIONAL MATHEMATICS

4049/01

Paper 1

29 August 2023

2 hours 15 minutes

Candidates answer on the Question Paper

No Additional Materials are required

READ THESE INSTRUCTIONS FIRST

Write your name, class and index number at the top of this page.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.

The total number of marks for this paper is 90.

For Examiner's Use
90

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{(n-r)!\,r!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2\sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

$$\Delta = \frac{1}{2}ab\sin C$$

AM P1 – Marking Scheme

1 A curve has equation $y = (x-4)\sqrt{2x+3}$. Find the rate of change of x when $x = 3$, given that y is increasing at a constant rate of 4 units/s. [4]

$$y = (x-4)\sqrt{2x+3} \quad = (x-4)(2x+3)^{\frac{1}{2}}$$

$$\frac{dy}{dx} = (x-4) \times \frac{1}{2}(2x+3)^{-\frac{1}{2}}(2) + (2x+3)^{\frac{1}{2}}(1) \quad \boxed{\text{M1 (product rule)}}$$

$$= \frac{x-4}{\sqrt{2x+3}} + \sqrt{2x+3} \quad = \frac{3x-1}{\sqrt{2x+3}} \quad \boxed{\text{M1 (simplify)}}$$

When $x = 3$, $\frac{dy}{dt} = \frac{dy}{dx} \times \frac{dx}{dt}$

$$4 = \frac{3(3)-1}{\sqrt{2(3)+3}} \times \frac{dx}{dt} \quad \boxed{\text{M1}}$$

$$\Rightarrow \quad \frac{dx}{dt} = 1.5 \text{ units/s} \quad \boxed{\text{A1}}$$

2 **(i)** Show that $\log_2 p + \log_4 p = \frac{3\ln p}{\ln 4}$. [3]

(ii) Hence solve the equation $\log_2 p + \log_4 p = 9$. [2]

(i)

$$\log_2 p + \log_4 p = \frac{\ln p}{\ln 2} + \frac{\ln p}{\ln 4} \quad \boxed{\text{M1 (change base)}}$$

$$= \frac{2\ln p}{2\ln 2} + \frac{\ln p}{\ln 4} \quad \boxed{\text{M1 (common denominator)}}$$

$$= \frac{2\ln p}{\ln 2^2} + \frac{\ln p}{\ln 4}$$

$$= \frac{2\ln p}{\ln 4} + \frac{\ln p}{\ln 4}$$

$$= \frac{3\ln p}{\ln 4} \text{ (Shown)} \quad \boxed{\text{A1}}$$

(ii) $\log_2 p + \log_4 p = 9$

From **(i)**: $\frac{3\ln p}{\ln 4} = 9$

$$\Rightarrow \quad \ln p = 3\ln 4 \quad \boxed{\text{M1 (cross multiply)}}$$

$$= \ln 4^3$$

$$\therefore p = 64 \quad \boxed{\text{A1}}$$

3 It is given that $2 + \lg(x - y) = \lg x$, where x and y are positive constants.

(i) Explain why $y < x$. [1]

(ii) Express y in terms of x. [4]

(i) For $\lg(x - y)$ to be defined, $x - y > 0$ $\boxed{\text{B1}}$
Hence $y < x$.

(ii) $2 + \lg(x - y) = \lg x$
$2\lg 10 + \lg(x - y) = \lg x$ $\boxed{\text{M1}}$

$\lg 10^2(x - y) = \lg x$ $\boxed{\text{M1}}$

$\Rightarrow$ $100x - 100y = x$ $\boxed{\text{M1}}$
$99x = 100y$
$\therefore y = \dfrac{99x}{100}$ (or $0.99x$) $\boxed{\text{A1}}$

4 (i) Write down and simplify, in ascending powers of x, the first three terms in the expansion of $(2 - 3x)^5$. [2]

(ii) Given that the expansion of $(p + qx)(2 - 3x)^5$ up to the terms in x^2 is $8 + rx + 1680x^2$, find the value of p, of q and of r. [4]

(i) $(2 - 3x)^5 = 2^5 + 5(2^4)(-3x) + 10(2^3)(-3x)^2 + \ldots$ $\boxed{\text{M1}}$
$= 32 - 240x + 720x^2 + \ldots$ $\boxed{\text{A1}}$

(ii) Given: $(p + qx)(2 - 3x)^5 = 8 + rx + 1680x^2 + \ldots$
$(p + qx)(32 - 240x + 720x^2 + \ldots) = 8 + rx + 1680x^2 + \ldots$

Equating constants: $32p = 8$ $\Rightarrow$ $p = \dfrac{1}{4}$ $\boxed{\text{B1}}$

Equating coeff. of x^2: $720p - 240q = 1680$ $\boxed{\text{M1}}$
$q = -6\dfrac{1}{4}$ $\boxed{\text{A1}}$

Equating coeff. of x: $r = 32q - 240p$
$= -260$ B1

5 A curve has equation $y = x^3 + 2x^2 + x + c$, where c is a constant.

(i) Find the x-coordinates of the stationary points on the curve. [4]

(ii) Hence find the value of c for which the maximum point of the curve lies on the line $y = 2$. [3]

(i) $y = x^3 + 2x^2 + x + c$

$$\frac{dy}{dx} = 3x^2 + 4x + 1 \quad \boxed{\text{M1}}$$

For stationary points, $\frac{dy}{dx} = 0$

$$\Rightarrow \quad 3x^2 + 4x + 1 = 0 \quad \boxed{\text{M1}}$$

$$(3x + 1)(x + 1) = 0$$

$$x = -\frac{1}{3} \text{ or } -1 \quad \boxed{\text{A1, A1}}$$

(ii) $\frac{d^2y}{dx^2} = 6x + 4 \quad \boxed{\text{M1}}$

$$\left.\begin{array}{l} \text{When } x = -\frac{1}{3},\ \frac{d^2y}{dx^2} > 0 \text{ gives a minimum.} \\ \text{When } x = -1,\ \frac{d^2y}{dx^2} < 0 \text{ gives a maximum.} \end{array}\right\} \boxed{\text{M1}}$$

Subs. $x = -1$ and $y = 2$ into curve:

$$2 = -1 + 2 - 1 + c$$

$$c = 2 \quad \boxed{\text{A1}}$$

6 **(a)** Write down the principal value of the following in radians.

(i) $\cos^{-1}(-\frac{1}{2})$, [1]

(ii) $\tan^{-1}(-\frac{1}{\sqrt{3}})$. [1]

(b) It is given that P, Q and R are the angles of a triangle.

(i) Show that $\cos P = -\cos(Q+R)$. [2]

(ii) Given that $Q = 45°$ and $R = 60°$, find $\cos P$ in the form $\frac{1}{4}(\sqrt{a} - \sqrt{b})$, where a and b are integers. [3]

(a) **(i)** $\frac{2\pi}{3}$ $\boxed{\text{B1}}$ **(ii)** $-\frac{\pi}{6}$ $\boxed{\text{B1}}$

(b) **(i)** $\cos P = \cos[180° - (Q+R)]$ $\boxed{\text{M1}}$

$= \cos 180° \cos(Q+R) + \sin 180° \sin(Q+R)$

$= -\cos(Q+R) + 0 \quad = -\cos(Q+R)$ $\boxed{\text{A1}}$

Accept supplementary angles:		
$\cos P$	$= -\cos(180° - P)$	M1
	$= -\cos(Q+R)$	A1

(ii) $\cos P = -\cos(45° + 60°)$

$= -\cos 45° \cos 60° + \sin 45° \sin 60°$ $\boxed{\text{M1}}$

$= -\frac{\sqrt{2}}{2}(\frac{1}{2}) + \frac{\sqrt{2}}{2}(\frac{\sqrt{3}}{2})$ $\boxed{\text{M1}}$

$= \frac{1}{4}(\sqrt{6} - \sqrt{2})$ $\boxed{\text{A1}}$

7 It is given that $f(x) = \cos\left(\frac{x}{2}\right) - 1$ and $g(x) = 3\sin 2x$.

(i) State the least and greatest values of $f(x)$. [2]
(ii) State the period and amplitude of $g(x)$. [2]
(iii) Sketch the graph of $y = g(x)$ for $0° \le x \le 360°$. [3]
(iv) Hence state the number of solution(s) for the equation $3\sin 2x = \cos\left(\frac{x}{2}\right) - 1$ for $0° < x < 90°$. [1]

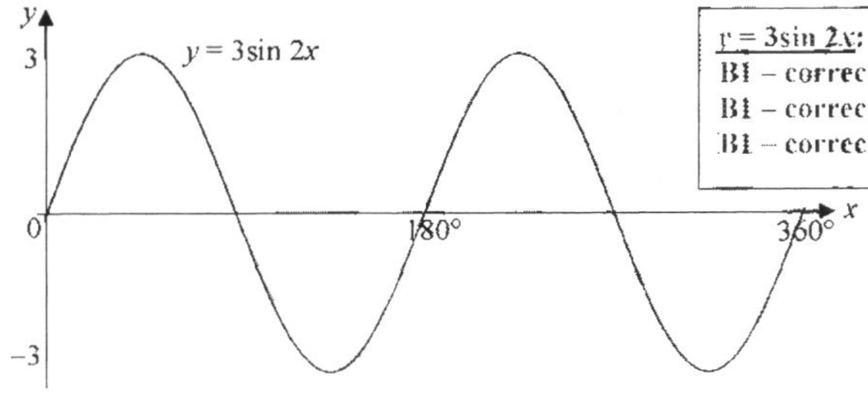
(i) Least value of $f(x) = -2$ $\boxed{\text{B1}}$

Greatest value of $f(x) = 0$ $\boxed{\text{B1}}$

(ii) Period of $g(x) = 180°$ $\boxed{\text{B1}}$

Amplitude of $g(x) = 3$ $\boxed{\text{B1}}$

(iii)



$y = 3\sin 2x$:
B1 – correct shape
B1 – correct cycle
B1 – correct max & min

(iv) For $0° < x < 90°$: from **(i)**, $f(x) \le 0$
from **(iii)**, $g(x) \ge 0$

Hence, there are no solution. $\boxed{\text{B1}}$

8 (a) What conditions must be applied to the constants a and c such that $ax^2 + 3x + c$ is always positive? [2]

(b) Find the range of values of x for which $2x(8x - 3) < 15x^2 - x + 14$. [3]

(c) Find the values of h for which the line $y = 6x + h$ is a tangent to the curve $x^2 - xy = 5$. [4]

(a) Discriminant: $9 - 4ac < 0$ (or $4ac > 9$) $\boxed{\text{B1 (o.e.)}}$

$a > 0$ $\boxed{\text{B1}}$

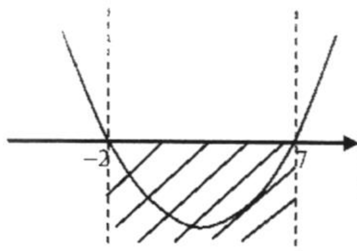
(b) $2x(8x - 3) < 15x^2 - x + 14$

$\Rightarrow \quad 16x^2 - 6x < 15x^2 - x + 14$

$x^2 - 5x - 14 < 0$

$(x + 2)(x - 7) < 0$ $\boxed{\text{M1}}$

$-2 < x < 7$ $\boxed{\text{A1}}$



$\boxed{\text{M1}}$

(c) Subs. line into curve: $x^2 - x(6x + h) = 5$ $\boxed{\text{M1}}$

$x^2 - 6x^2 - hx = 5$

$5x^2 + hx + 5 = 0$ $\boxed{\text{M1}}$

For tangent, discriminant = 0

$h^2 - 4(5)(5) = 0$ $\boxed{\text{M1}}$

$h = \pm 10$ $\boxed{\text{A1}}$

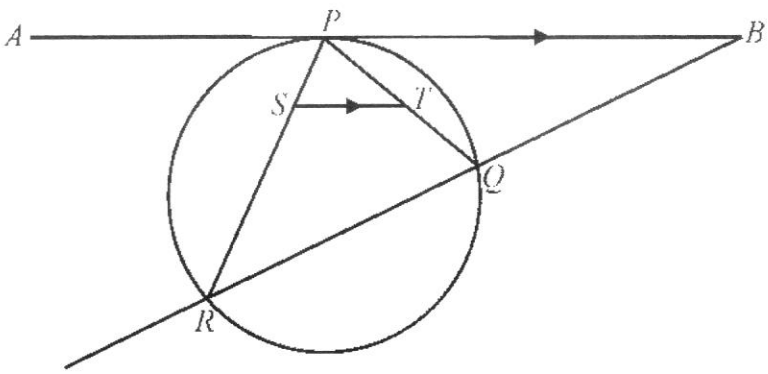
9 The diagram, not drawn to scale, shows a circle passing through the points P, Q and R. The point Q lies on the line RB. AB is a tangent to the circle at P. The points S and T lie on PR and PQ respectively. Given that AB is parallel to ST, prove that

(i) triangle PST is similar to triangle PQR, [3]

(ii) $PQ \times PT = PR \times PS$, [2]

(iii) angle QTS + angle $QRS = 180°$, [3]

(iv) angle RST + angle $RQT = 180°$. [1]



(i) $\angle SPT = \angle QPR$ (common angle) $\boxed{\text{B1}}$

$\angle PST = \angle SPA$ (alt. $\angle$s, $AB \parallel ST$)

$= \angle PQR$ (Alt. Segment Thm) $\boxed{\text{B1}}$ **OR** $\angle PTS = \angle TPB$ (alt. $\angle$s, $AB \parallel ST$) $= \angle PRQ$ (Alt. Segment Thm)

$\therefore$ triangle PST is similar to triangle PQR. $\boxed{\text{B1}}$ (by AA property)

(ii) From above result, $\dfrac{PQ}{PS} = \dfrac{PR}{PT}$ $\boxed{\text{M1}}$

$\Rightarrow \quad PQ \times PT = PR \times PS$ $\boxed{\text{A1}}$

(iii) $\angle QTS = 180° - \angle PTS$ (adj. $\angle$s on a st. line) B1

$= 180° - \angle QRS$ (from (i) result) B1

So $\angle QTS + \angle QRS = 180°$ ----- (1) B1

(iv) $\angle RST + \angle RQT = 360° - (\angle QTS + \angle QRS)$ ($\angle$ sum of a quadrilateral) $\boxed{\text{B1}}$

$= 360° - 180°$

$= 180°$

10 The equation of a circle with centre O is given by $(x+2)^2 + y^2 = 10y + 75$.

(i) Find the radius of the circle and the coordinates of O. [3]

The point $P(4, -3)$ lies on the circle.

(ii) Find the equation of the tangent to the circle at P. [3]

(iii) The tangent of the circle at P meets the x-axis at Q. Find the coordinates of Q. [1]

(iv) Find the value of $\tan \angle OQP$. [2]

(i)

$(x+2)^2 + y^2 = 10y + 75$

$(x+2)^2 + y^2 - 10y = 75$

$(x+2)^2 + (y-5)^2 - 25 = 75$

$(x+2)^2 + (y-5)^2 = 100$ $\boxed{\text{M1 for } (y-5)^2 \text{ seen}}$

Radius = 10 units $\boxed{\text{A1}}$

Centre, $O = (-2, 5)$ $\boxed{\text{A1}}$

OR

$(x+2)^2 + y^2 = 10y + 75$

$x^2 + y^2 + 4x - 10y - 71 = 0$

$2g = 4,\ 2f = -10$

$g = 2, f = -5$ $\boxed{\text{M1}}$

Radius $= \sqrt{2^2 + (-5)^2 - (-71)}$

$= 10$ units $\boxed{\text{A1}}$

Centre, $O = (-g, -f) = (-2, 5)$ $\boxed{\text{A1}}$

(ii) Grad. $OP = \dfrac{5+3}{-2-4} = -\dfrac{4}{3}$ $\boxed{\text{M1}}$

Grad. of tangent $= \dfrac{3}{4}$ $\boxed{\text{M1}}$

Equation of tangent is $y + 3 = \dfrac{3}{4}(x-4)$

$y = \dfrac{3}{4}x - 6$ $\boxed{\text{A1}}$

(iii) At Q, $y = 0 \quad \Rightarrow \quad x = 8$

$\therefore Q = (8, 0)$ $\boxed{\text{B1}}$

(iv) $PQ = \sqrt{(8-4)^2 + (0+3)^2} = 5$ $\boxed{\text{M1}}$

$\tan \angle OQP = \dfrac{OP}{PQ} = \dfrac{10}{5} = 2$ $\boxed{\text{A1}}$

11 The equation of a polynomial is given by $f(x) = 4x^3 + 9x - 5$.

(i) Find the remainder when $f(x)$ is divided by $x + 1$. [1]

(ii) Show that $2x - 1$ is a factor of $f(x)$. [1]

(iii) Show that the equation $f(x) = 0$ has only one real root. [4]

(iv) Hence, or otherwise, solve the equation $\frac{4}{3}(3^{3y-1}) + 9(3^y) = 5$. [4]

(i) Remainder $= f(-1) \quad = -4 - 9 - 5 \quad = -18$ $\boxed{\text{B1}}$

(ii) Since $f(\frac{1}{2}) = 4(\frac{1}{2})^3 + 9(\frac{1}{2}) - 5 \quad = 0$

$\therefore 2x - 1$ is a factor of $f(x)$ by factor theorem. (Shown) $\boxed{\text{B1}}$

(iii) By inspection, $f(x) = (2x - 1)(2x^2 + bx + 5)$ $\boxed{\text{M1}}$

Equating coeff. of x^2: $0 = 2b - 2 \quad \Rightarrow \quad b = 1$ $\boxed{\text{M1}}$

So $f(x) = (2x - 1)(2x^2 + x + 5)$

When $2x^2 + x + 5 = 0$, since discriminant $= 1^2 - 4(2)(5) < 0$ $\boxed{\text{B1}}$

$\Rightarrow$ there are no real roots for $2x^2 + x + 5 = 0$

$\therefore$ $f(x) = 0$ has only one real root, which is $x = \frac{1}{2}$ $\boxed{\text{B1}}$

(iv) $\frac{4}{3}(3^{3y-1}) + 9(3^y) = 5$

$\Rightarrow \quad 4(3^y)^3 + 9(3^y) - 5 = 0$

From above results, $3^y = \frac{1}{2}$ $\boxed{\text{M1}}$

$\ln 3^y = \ln \frac{1}{2}$ $\boxed{\text{M1}}$

$y = \ln \frac{1}{2} \div \ln 3$ M1 $= -0.631$ $\boxed{\text{A1}}$

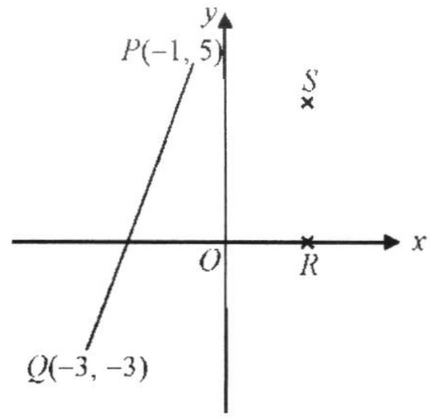
12 **Solutions to this question by accurate drawing will not be accepted.**
The diagram, not drawn to scale, shows a line segment PQ where $P(-1, 5)$ and $Q(-3, -3)$. It is given that the point R lies on the perpendicular bisector of PQ and also lies on the x-axis. The point S is in the first quadrant.

(i) Find the coordinates of R. [6]

It is given that RS is parallel to the y-axis and the area of triangle PRS is 6 units^2.

(ii) Explain why the y-coordinate of S is 4. [3]

(iii) Hence, or otherwise, calculate the area of the quadrilateral $PQRS$. [2]



(i) Grad $PQ = \dfrac{5+3}{-1+3} = 4$ $\boxed{\text{M1}}$

$\Rightarrow \quad \perp$ grad. $= -\dfrac{1}{4}$ $\boxed{\text{M1}}$

Midpt $PQ = \left(\dfrac{-1-3}{2}, \dfrac{5-3}{2}\right) = (-2, 1)$ $\boxed{\text{M1}}$

Required equation is $y - 1 = -\dfrac{1}{4}(x + 2)$ $\boxed{\text{M1 (o.e.)}}$

$y = -\dfrac{1}{4}x + \dfrac{1}{2}$

On the x-axis, $y = 0$: $0 = -\dfrac{1}{4}x + \dfrac{1}{2}$ $\boxed{\text{M1}}$

$x = 2$

$\therefore R = (2, 0)$ $\boxed{\text{A1}}$

(ii) Let $S = (2, k)$ and RS is the base of the triangle with height 3 units. $\boxed{\text{B1}}$

$\dfrac{1}{2}(RS)(3) = 6 \Rightarrow \quad RS = 4$ $\boxed{\text{B1}}$

Possible values of $k = 0 \pm 4$
Since S is in the first quadrant, y-coordinate of $S = 0 + 4 = 4$ $\boxed{\text{B1}}$

(iii) Area $= \dfrac{1}{2}\begin{vmatrix} 2 & -1 & -3 & 2 \\ 0 & 5 & -3 & 0 \end{vmatrix} + 6$

$= \dfrac{1}{2}[(10 + 3 + 0) - (0 - 15 - 6)] + 6$ $\boxed{\text{M1}}$

$= 23$ units^2 A1

Alternative:

Area $= \dfrac{1}{2}\begin{vmatrix} 2 & 2 & -1 & -3 & 2 \\ 0 & 4 & 5 & -3 & 0 \end{vmatrix}$

$= \dfrac{1}{2}[(8 + 10 + 3 + 0) - (0 - 4 - 15 - 6)]$ $\boxed{\text{M1}}$

$= 23$ units^2 $\boxed{\text{A1}}$

End of Paper

JUNYUAN SECONDARY SCHOOL
PRELIMINARY EXAMINATION 2023
SECONDARY FOUR EXPRESS / FIVE NORMAL (ACADEMIC)

CANDIDATE NAME

CLASS

INDEX NUMBER

ADDITIONAL MATHEMATICS **4049/02**

Paper 2

30 August 2023

2 hours 15 minutes

Candidates answer on the Question Paper

No Additional Materials are required

READ THESE INSTRUCTIONS FIRST

Write your name, class and index number at the top of this page.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.

The total number of marks for this paper is 90.

For Examiner's Use
90

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{(n-r)!r!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2\sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

$$\Delta = \frac{1}{2}ab\sin C$$

AM P2 – Marking Scheme

1 P is on the y-axis. The normal to the curve $y = 3\sin 2x - 1$ at P meets the x-axis at Q. Find

(i) the equation of the normal to the curve at P, [4]

(ii) the coordinates of Q. [2]

(i) $\frac{dy}{dx} = 6\cos 2x$ $\boxed{\text{M1}}$

At P, $x = 0$: $y = -1$ $\boxed{\text{M1}}$

and $\frac{dy}{dx} = 6 \Rightarrow$ grad. of normal $= -\frac{1}{6}$ $\boxed{\text{M1}}$

Equation of normal is $y + 1 = -\frac{1}{6}x$

i.e. $y = -\frac{x}{6} - 1$ $\boxed{\text{A1 (o.e.)}}$

(ii) At Q, $y = 0$ hence:

$-\frac{x}{6} - 1 = 0 \Rightarrow x = -6$ $\boxed{\text{M1}}$

$\therefore Q = (-6, 0)$ $\boxed{\text{A1}}$

2 **(a)** Prove that $\cot^2 x - \cos^2 x = \cos^4 x \operatorname{cosec}^2 x$. [3]

(b) Solve the equation $2\sin y \cos y = \sin y$ for $0° < y < 360°$. [3]

(a)

$$\begin{aligned}\cot^2 x - \cos^2 x &= \frac{\cos^2 x}{\sin^2 x} - \cos^2 x \\ &= \frac{\cos^2 x - \cos^2 x \sin^2 x}{\sin^2 x} \quad \boxed{\text{B1}} \\ &= \frac{\cos^2 x(1-\sin^2 x)}{\sin^2 x} \quad \boxed{\text{B1}} \\ &\left.\begin{aligned} &= \frac{\cos^2 x \cos^2 x}{\sin^2 x} \\ &= \cos^4 x \operatorname{cosec}^2 x \end{aligned}\right\} \boxed{\text{B1}}\end{aligned}$$

(b) $2\sin y \cos y = \sin y$

$\sin y(2\cos y - 1) = 0$ $\boxed{\text{M1}}$

$\sin y = 0$ or $\cos y = 0.5$ $\boxed{\text{M1 – both ratios}}$

$y = 180°$ $y = 60°, 300°$ $\boxed{\text{A1 – all 3 solutions}}$

B2 if students cancelled "$\sin y$" <u>and</u> solve $\cos y = 0.5$ correctly

3 **(a)** Find the value of a and of b such that $3b + a\sqrt{75} = 6a - 10\sqrt{3}$. [3]

(b) A prism has a triangular base ABC such that the length of AB and AC are $(4 - \sqrt{3})$ cm and $(1 + \sqrt{3})$ cm respectively. Angle $BAC = 150°$. The prism has a height of h cm and a volume of $\frac{1}{2}(30 - \sqrt{3})$ cm^3. Without using a calculator, show that h can be expressed as $(p + q\sqrt{3})$ cm, where p and q are integers. [5]

(a) $3b + a\sqrt{75} = 6a - 10\sqrt{3}$

$\Rightarrow \quad 3b + 5a\sqrt{3} = 6a - 10\sqrt{3}$ M1

Equating coeff. of $\sqrt{3}$:

$5a = -10$

$\Rightarrow \quad a = -2$

M1 for either equations

Equating constants:

$3b = 6(-2)$

$\Rightarrow \quad b = -4$

A1 for both $a = -2$ and $b = -4$

(b) Vol. $= \frac{1}{2}(30 - \sqrt{3})$

$\cancel{\frac{1}{2}}(4 - \sqrt{3})(1 + \sqrt{3})\sin 150° \times h = \cancel{\frac{1}{2}}(30 - \sqrt{3})$ M1 (formula)

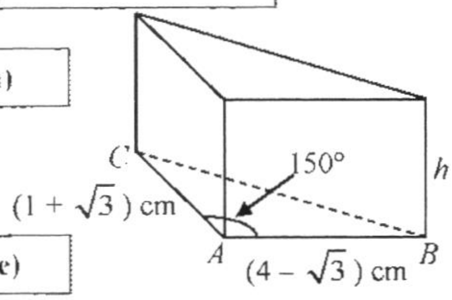
$(1 + 3\sqrt{3})\sin 30° \times h = 30 - \sqrt{3}$ M1 (expansion)

$\frac{1}{2}(1 + 3\sqrt{3}) \times h = 30 - \sqrt{3}$

$h = \frac{2(30 - \sqrt{3})}{1 + 3\sqrt{3}} \times \frac{1 - 3\sqrt{3}}{1 - 3\sqrt{3}}$ M1 (rationalise)

$= \frac{78 - 182\sqrt{3}}{1 - 27}$ M1

$= (-3 + 7\sqrt{3})$ cm A1 (o.e.)



4 The table shows experimental values of two variables x and y, which are connected by the equation $py^3 - qx^2 = 1$, where p and q are constants.

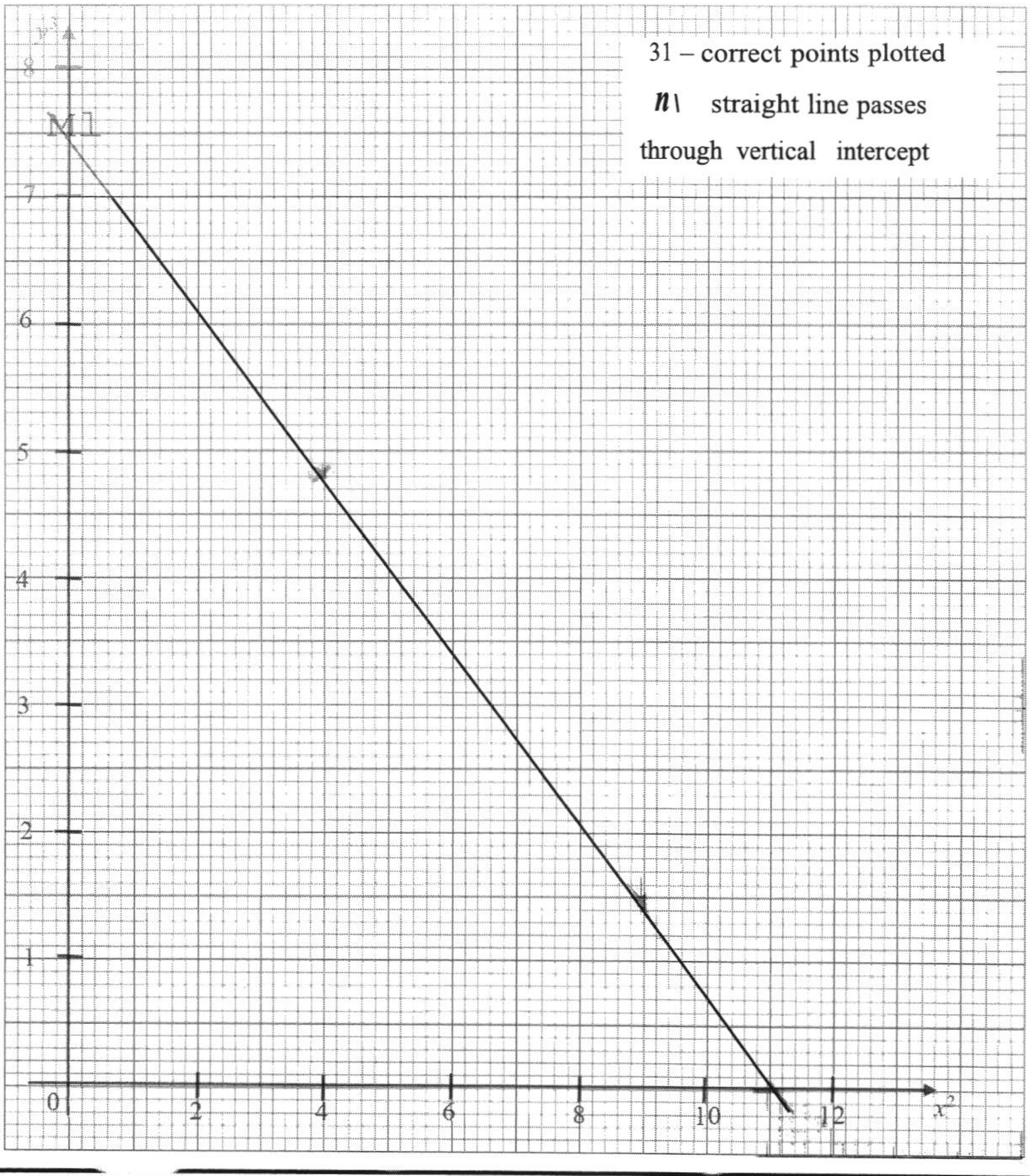
x	0.5	1	2	3
y	1.94	1.89	1.69	1.14

(i) Complete the table below and draw the straight line graph of y^3 against x^2 on the grid. Give your values to 2 decimal places. [3]

x^2	0.25	1	4	9
y^3	7.30	6.75	4.83	1.48

B1

$$py^3 - qx^2 = 1 \Rightarrow \quad y^3 = \frac{q}{p}x^2 + \frac{1}{p}$$



(ii) Use your graph to estimate
(a) the value of p and of q, [3]
(b) the value of x^2 when $y = 1.5$. [1]

(ii) From the graph,

$py^3 - qx^2 = 1 \quad \Rightarrow \quad y^3 = \frac{q}{p}x^2 + \frac{1}{p}$ $\boxed{\text{B1}}$

(a) $\frac{1}{p} \approx 7.5 \quad \Rightarrow \quad p = 0.133\ (\pm 0.05)$ $\boxed{\text{B1}}$

$\frac{q}{p} = \text{grad.} \approx -\frac{4}{6}$

$\Rightarrow \quad q = 0.0889\ (\pm 0.015)$ $\boxed{\text{B1}}$

(b) When $y = 1.5$, then $y^3 = 3.375$.
From the graph, $x^2 \approx 6.1\ (\pm 0.3)$ $\boxed{\text{B1}}$

5 **(a)** Solve the equation $2^{x+2} = 2^{-x} - 3$. [4]
(b) **(i)** Given that $8^{x+1} \times 5^{3x-2} = 10^{4x}$, find the value of 10^x. [3]
(ii) Hence solve $8^{x+1} \times 5^{3x-2} = 10^{4x}$. [1]

(a) $2^{x+2} = 2^{-x} - 3$

$4(2^x) + 3 - \frac{1}{2^x} = 0$ $\boxed{\text{M1}}$

Students may use substitution, such as $a = 2^x$.

$\times 2^x$: $4(2^x)^2 + 3(2^x) - 1 = 0$

$[4(2^x) - 1]\,[2^x + 1] = 0$ $\boxed{\text{M1 (o.e.)}}$

$2^x = \frac{1}{4}$ or $2^x = -1$ (rejected as $2^x > 0$) $\boxed{\text{M1 (both answers)}}$

$x = -2$ $\boxed{\text{A1}}$

(b) **(i)** $8^{x+1} \times 5^{3x-2} = 10^{4x}$

$8(2^{3x}) \times \frac{5^{3x}}{25} = 10^{4x}$ $\boxed{\text{M1 (split indices)}}$

$10^{3x} \times \frac{8}{25} = 10^{4x}$ $\boxed{\text{M1}}$

$\frac{8}{25} = \frac{10^{4x}}{10^{3x}}$

$\Rightarrow \quad 10^x = \frac{8}{25}$ $\boxed{\text{A1}}$

(ii) $x = \lg \frac{8}{25}$

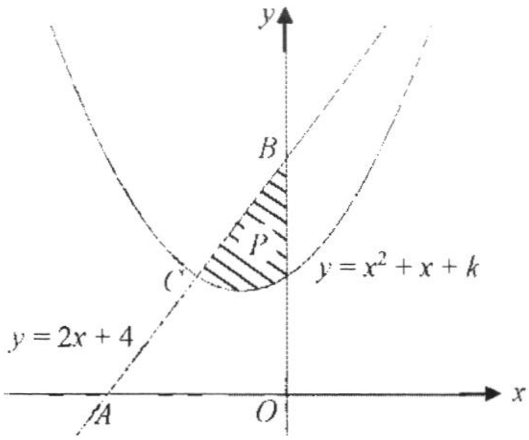
$= -0.495$ $\boxed{\text{B1}}$

6 The diagram, not drawn to scale, shows part of the curve $y = x^2 + x + k$ and the line $y = 2x + 4$. The line $y = 2x + 4$ intersects the x-axis and the y-axis at A and B respectively. C is the midpoint of AB, and also a point of intersection of the line $y = 2x + 4$ and the curve $y = x^2 + x + k$. The region P is bounded by the line, the curve and the y-axis.

(i) Find the coordinates of C. [3]

(ii) Find the value of k. [1]

(iii) Find the area of region P. [4]



(i) At A, $y = 0$, hence $2x + 4 = 0$

$\Rightarrow \quad x = -2$ $\boxed{\text{M1}}$

At B, $x = 0$, hence $y = 4$

$$\therefore C = \left(\frac{-2+0}{2}, \frac{0+4}{2}\right) \boxed{\text{M1}}$$

$$= (-1, 2) \boxed{\text{A1}}$$

(ii) At $C(-1, 2)$: $2 = 1 - 1 + k$

$\Rightarrow \quad k = 2$ B1

(iii) Area P = area of trapezium $- \int_{-1}^{0} (x^2 + x + 2)\, dx$

M1

$$= \frac{1}{2}(4+2)(1) - \left[\frac{x^3}{3} + \frac{x^2}{2} + 2x\right]_{-1}^{0} \boxed{\text{M1}}$$

$$= 3 - [0 - (-\frac{1}{3} + \frac{1}{2} - 2)] \boxed{\text{M1}}$$

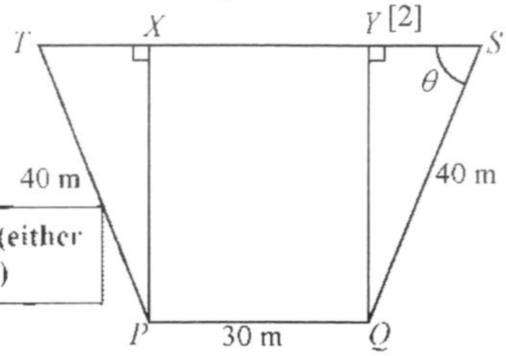
$$= 1\frac{1}{6} \text{ (or 1.17) units}^2 \boxed{\text{A1}}$$

7 An owner fences his private property. He puts fences around the perimeter of the trapezium $PQST$. He also puts extra fences from P to X and from Q to Y, where PX and QY are perpendicular to TS. Angle QSY = angle $PTX = \theta$ radians and the lengths of PT and QS are 40 m. It is also given that $PQ = 30$ m.

(i) Show that the length of fencing needed, L m, can be expressed in the form $a + b\cos\theta + c\sin\theta$, where a, b and c are constants to be found. [3]

(ii) Express L in the form $a + R\cos(\theta - \alpha)$, where $R > 0$ and α is an acute angle. [3]

(iii) Explain why L is always greater than 25 m. [2]



(i) In triangle QSY (or PTX),

$$\cos\theta = \frac{YS}{40} \Rightarrow YS = 40\cos\theta$$
$$\sin\theta = \frac{QY}{40} \Rightarrow QY = 40\sin\theta$$

M1 (either one)

$$\therefore L = 2(40) + 30 + [30 + 2(40\cos\theta)] + 2(40\sin\theta)$$ M1

$$= 140 + 80\cos\theta + 80\sin\theta$$ A1

(ii) Let $L = 140 + R\cos(\theta - \alpha)$

$R = \sqrt{80^2 + 80^2} = 80\sqrt{2}$ M1

$\alpha = \tan^{-1}\dfrac{80}{80} = \dfrac{\pi}{4}$ M1

$\therefore L = 140 + 80\sqrt{2}\cos\left(\theta - \dfrac{\pi}{4}\right)$ A1 (o.e.) [or $140 + 113\cos\left(\theta - \dfrac{\pi}{4}\right)$]

(iii) Since $-80\sqrt{2} \le 80\sqrt{2}\cos\left(\theta - \dfrac{\pi}{4}\right) \le 80\sqrt{2}$ M1

$$140 - 80\sqrt{2} \le 140 + 80\sqrt{2}\cos\left(\theta - \frac{\pi}{4}\right) \le 140 + 80\sqrt{2}$$

$\therefore 26.86 \le L \le 253.14$

As $\underline{25 < 26.86 \le L}$, hence L is always greater than 25 m.

B1 (accept min. 26.9)

8 A particle travels in a straight line such that its displacement, x m, from a fixed point O is given by $x = t^3 - 12t^2 + 45t$, where t is the time in seconds after passing through O.

(i) Find the times when the particle is at instantaneous rest. [3]

(ii) Hence, or otherwise, find the distance travelled in the first 4 seconds of motion. [3]

(iii) Explain why the particle will never return to its starting point. [3]

(i)

$x = t^3 - 12t^2 + 45t$

$v = 3t^2 - 24t + 45$ $\boxed{\text{M1}}$

At rest, $v = 0$: $3(t^2 - 8t + 15) = 0$ $\boxed{\text{M1}}$

$(t - 3)(t - 5) = 0$

$t = 3$ s or 5 s $\boxed{\text{A1}}$

(ii)

When $t = 3$, $x = 3^3 - 12(3)^2 + 45(3) = 54$ $\boxed{\text{M1}}$

When $t = 4$, $x = 4^3 - 12(4)^2 + 45(4) = 52$ $\boxed{\text{M1}}$

Required dist. $= 54 + (54 - 52)$

$= 56$ m $\boxed{\text{A1}}$

(iii)

When $x = 0$: $t^3 - 12t^2 + 45t = 0$

$t(t^2 - 12t + 45) = 0$ $\boxed{\text{M1}}$

But when $t^2 - 12t + 45 = 0$, discriminant $= (-12)^2 - 4(1)(45) < 0$ $\boxed{\text{M1}}$

$\Rightarrow$ there are no real roots

Hence the particle will never return to its starting point. $\boxed{\text{A1}}$

[illegible] B1

From **(i)** and [illegible]

B1

[illegible]

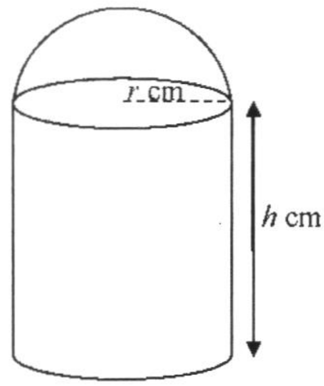
B1

9 A container has a volume of V cm^3 and is made up of a hemisphere of radius r cm on top of a cylinder of radius r cm and height h cm as shown in the diagram.

(i) Show that $h = \dfrac{V}{\pi r^2} - \dfrac{2r}{3}$. [2]

(ii) Given that $V = 2880\pi$ cm^3 and that the container is made of plastic of negligible thickness, show that the total surface area is represented by $A = \dfrac{5760\pi}{r} + \dfrac{5\pi r^2}{3}$ cm^2. [3]

(iii) Given that r can vary, find the value of r such that the total surface area is a minimum. [4]



(i) $V = \pi r^2 h + \dfrac{2\pi r^3}{3}$ $\boxed{\text{M1}}$

$\pi r^2 h = V - \dfrac{2\pi r^3}{3} \quad \Rightarrow \quad h = \dfrac{V}{\pi r^2} - \dfrac{2r}{3}$ $\boxed{\text{A1}}$ (Shown)

(ii) $A = 2\pi rh + \pi r^2 + \dfrac{4\pi r^2}{2}$ $\boxed{\text{M1}}$

$= 2\pi r\left(\dfrac{2880\pi}{\pi r^2} - \dfrac{2r}{3}\right) + 3\pi r^2$ $\boxed{\text{M1 - substitution}}$

$= \dfrac{5760\pi}{r} + \dfrac{5\pi r^2}{3}$ $\boxed{\text{A1}}$

(iii) $\dfrac{dA}{dr} = -\dfrac{5760\pi}{r^2} + \dfrac{10\pi r}{3}$ $\boxed{\text{M1}}$

For stationary values, $\dfrac{dA}{dr} = 0$

$\dfrac{5760\pi}{r^2} = \dfrac{10\pi r}{3}$ $\boxed{\text{M1 (o.e.)}}$

$r^3 = 1728$

$r = 12$ $\boxed{\text{M1}}$

$\dfrac{d^2A}{dr^2} = \dfrac{11520\pi}{r^3} + \dfrac{10\pi}{3}$, when $r = 12$, $\dfrac{d^2A}{dr^2} > 0$ $\boxed{\text{A1}}$

Hence A is a minimum when $r = 12$ cm.

10 **(i)** Given that $\dfrac{7x+3}{x^2(2x+1)} = \dfrac{A}{x} + \dfrac{B}{x^2} + \dfrac{C}{2x+1}$, where A, B and C are constants, find the value of A, of B and of C. [5]

(ii) Hence find $\int_1^3 \dfrac{7x+3}{x^2(2x+1)}\, dx$ in the form $\ln a + b$, where a and b are rational numbers. [5]

(i) Let $\dfrac{7x+3}{x^2(2x+1)} = \dfrac{A}{x} + \dfrac{B}{x^2} + \dfrac{C}{2x+1}$

$\times x^2(2x+1)$: $7x + 3 = Ax(2x+1) + B(2x+1) + Cx^2$ $\boxed{\text{M1}}$

Let $x = 0$: $3 = B$ $\boxed{\text{A1}}$

Let $x = -\dfrac{1}{2}$: $7(-\dfrac{1}{2}) + 3 = \dfrac{1}{4}C$ $\boxed{\text{M1}}$

$\Rightarrow C = -2$ $\boxed{\text{A1}}$

Equating coeff. of x^2: $0 = 2A + C \Rightarrow A = 1$ $\boxed{\text{A1}}$

$\therefore \dfrac{7x+3}{x^2(2x+1)} = \dfrac{1}{x} + \dfrac{3}{x^2} - \dfrac{2}{2x+1}$

(ii) $\int_1^3 \dfrac{7x+3}{x^2(2x+1)}\, dx = \int_1^3 \left[\dfrac{1}{x} + 3x^{-2} - \dfrac{2}{2x+1}\right] dx$ $\boxed{\text{M1}}$

$= \left[\ln x + \dfrac{3x^{-1}}{(-1)} - \dfrac{2\ln(2x+1)}{2}\right]_1^3$

M1 integration of $3x^{-2}$
M1 – integration of $\dfrac{1}{ax+b}$ terms

$= \ln 3 - 1 - \ln 7 - (0 - 3 - \ln 3)$ $\boxed{\text{M1}}$

$= \ln\dfrac{9}{7} + 2$ $\boxed{\text{A1}}$

11 **(a)** The function f is defined by $f(x) = \dfrac{3-x^2}{x^2+2}$, $x > 0$. Explain whether f is an increasing or decreasing function. [4]

(b) A curve is such that $\dfrac{d^2y}{dx^2} = 2x - 8$. The gradient of the curve at the point (3, 7) is 3.

(i) Express y in terms of x. [5]

(ii) Explain why the gradient of the curve is never less than 2. [2]

(a) $f(x) = \dfrac{3-x^2}{x^2+2}$

$f'(x) = \dfrac{(x^2+2)(-2x)-(3-x^2)(2x)}{(x^2+2)^2}$ $\boxed{\text{M1 – Quotient Rule (o.e.)}}$

$= \dfrac{-10x}{(x^2+2)^2}$ $\boxed{\text{M1 – simplify}}$

For $x > 0$, $-10x < 0$, also $(x^2+5)^2 > 0$ $\boxed{\text{B1 – either seen}}$

So $f'(x) < 0$ $\Rightarrow$ f is a decreasing function. B1

(b) $\dfrac{d^2y}{dx^2} = 2x - 8$

(i) $\dfrac{dy}{dx} = \int (2x-8)\,dx$

$= x^2 - 8x + c$ $\boxed{\text{M1}}$

At (3, 7), $\dfrac{dy}{dx} = 3$

$3^2 - 8(3) + c = 3$

$c = 18$ M1

$y = \int (x^2 - 8x + 18)\,dx$

$= \dfrac{x^3}{3} - 4x^2 + 18x + d$ $\boxed{\text{M1}}$

At (3, 7), $7 = \dfrac{3^3}{3} - 4(3)^2 + 18(3) + d$ $\boxed{\text{M1}}$

$d = -20$

$\therefore y = \dfrac{x^3}{3} - 4x^2 + 18x - 20$ $\boxed{\text{A1}}$

(ii) $\dfrac{dy}{dx} = x^2 - 8x + 18 = (x-4)^2 + 2$ $\boxed{\text{B1}}$

Since $(x-4)^2 \geq 0$, then $(x-4)^2 + 2 \geq 2$ $\boxed{\text{B1}}$

Hence the gradient is never less than 2.

End of Paper