Candidate Name: ____________________ **Index No:** ____ **Class:** ______

ZHENGHUA SECONDARY SCHOOL
PRELIMINARY EXAMINATION 2023
SECONDARY FOUR EXPRESS/ FIVE NORMAL (ACADEMIC)
ADDITIONAL MATHEMATICS
Paper 1 **4049/01**

Candidates answer on the Question Paper.
No Additional Materials are required.

24 AUG 2023

2 hours 15 minutes

READ THESE INSTRUCTIONS FIRST

Write your index number and name in the spaces at the top of this page.
Write in dark blue or black pen.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

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The number of marks is given in brackets [ ] at the end of each question or part question.

The total number of marks for this paper is 90.

Name of Setter :

This document consists of 19 printed pages and 1 blank page.

[Turn over

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1} a^{n-1} b + \binom{n}{2} a^{n-2} b^2 + \ldots + \binom{n}{r} a^{n-r} b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \dfrac{n!}{r!(n-r)!} = \dfrac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2} bc \sin A$$

1 The polynomial f is given by $\mathrm{f}(x) = (2x+1)(x^2+ax-b)$.

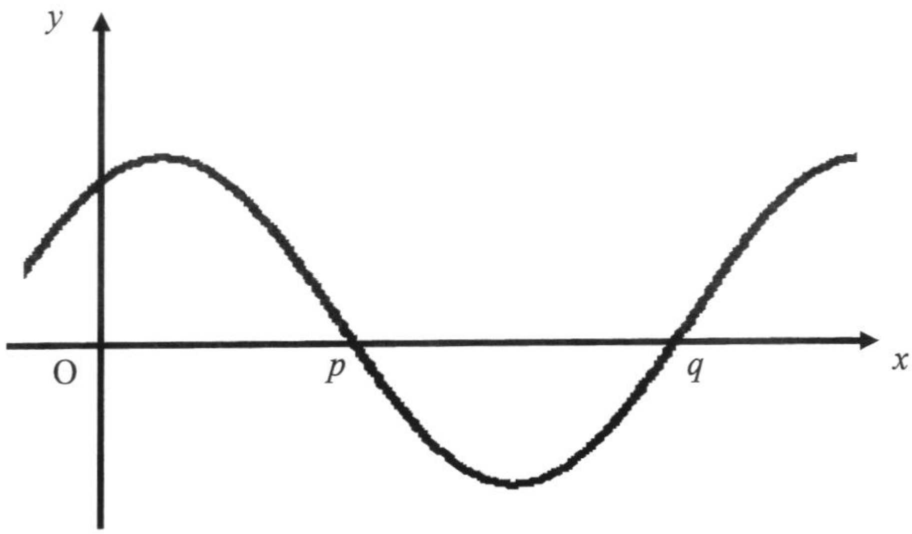
(i) What conditions must apply to the constants a and b if $\mathrm{f}(x) = 0$ has exactly one real root? [2]

(ii) Give an example of integer values of a and b which satisfy the conditions found in part **(i)**. [1]

2 **(a)** Divide $2x^3 - 4$ by $2x^3 - x^2$. [1]

(b) Express $\dfrac{2x^3 - 4}{2x^3 - x^2}$ in partial fractions. [4]

3



The diagram shows the curve with equation $y = k\cos(x - 30^\circ)$, $0^\circ \le x \le 360^\circ$, where k is a constant. The curve meets the y-axis at $(0, \sqrt{3})$ and passes through the points $(p^\circ, 0)$ and $(q^\circ, 0)$.

(i) Show that $k = 2$. [2]

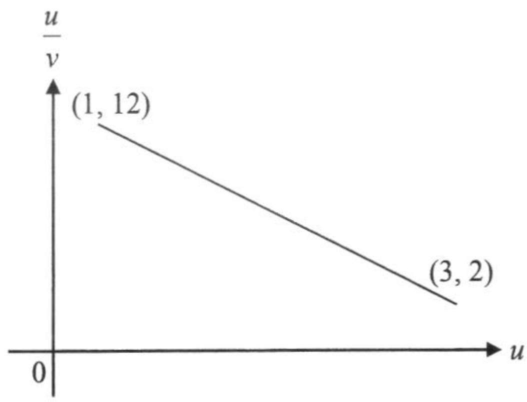
(ii) Calculate the value of p and q. [3]

4 The equation of a curve is $y = 3x^2 + ax - 12$, where a is a negative constant.

(i) Given that the coordinates of the minimum point is $(1, -15)$, show that value of a is -6. [2]

(ii) This curve meets another curve $y = -x^2 - 2$ at the points P and Q. Calculate the gradient of line segment PQ. [5]

5 Two variables u and v are related such that when a straight line graph of $\frac{u}{v}$ against u is plotted, the line passes through (1, 12) and (3, 2) as shown in the diagram.



(a) Express v in terms of u. [4]

(b) If $\frac{1}{v}$ is plotted against $\frac{1}{u}$, state the value of the gradient and the vertical intercept. [2]

6 The parallelogram $ABCD$ is such that A is $(2,4)$, B is $(2,10)$, C is (p,r) and D is (p,q) where $p > 0$. The area of parallelogram $ABCD$ is 30 units^2 and $AC = \sqrt{250}$ units.

(a) Explain why $p = 7$. [3]

(b) Find the possible values of q. [3]

7 A bungee jumper falls vertically from rest from a point O. Her velocity, v m/s, t seconds, after leaving O, is such that $\frac{dv}{dt} = 10$. After 4 s, she reaches a point X.

Find

(i) her velocity at X, [2]

(ii) her distance OX. [2]

On reaching X, she then slows so that her velocity, V m/s, T seconds after reaching X, is such that $\frac{dV}{dt} = 10 - kT$, where k is a constant.

(iii) Given that she first comes to rest at a point Y when $T = 3$, show that $k = \frac{140}{9}$. [3]

8 Given the polynomial $P(x) = -x^3 + ax^2 + bx + c$, where a, b and c are constants.

(a) State the condition in terms of a and b for $P(x)$ to be decreasing. [3]

(b) In the case where $a = 5$, $b = -2$ and $c = -2$, justify with reasoning, the number of real root(s) for $P(x) = 0$. [5]

9 A curve is such that $\frac{\mathrm{d}^2y}{\mathrm{d}x^2} = \sin\left(\frac{x}{2} - 1\right)$. The gradient of the curve at the point (2, 7) is 5.

(i) Express y in terms of x. [5]

(ii) Show that the gradient of the curve is never less than 5. [2]

10 Given a curve $y = \frac{9}{x} - 2$ and the line $x + y = k$ where k is a constant.

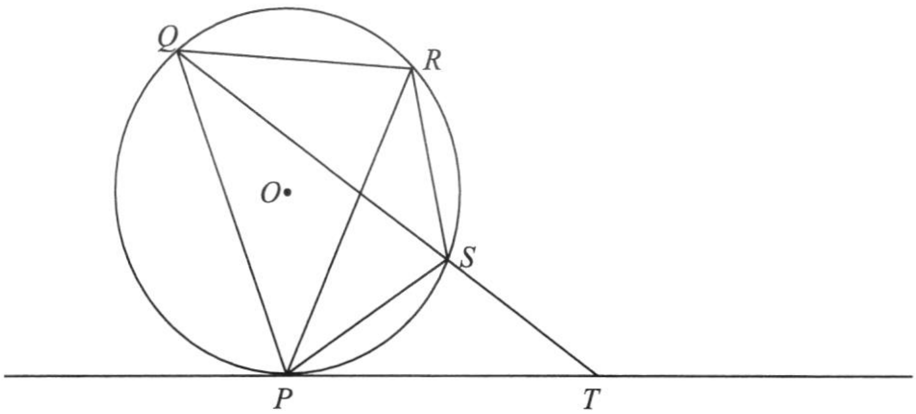
The line intersects the curve at the points A and B.

(a) In the case where $k = 8$, find the coordinates of A and B. [5]

(b) Find the value of k, in the case where the line is a tangent to the curve. [4]

11 In the figure below, the line PT is a tangent to the circle at P.

Line QT bisects angle PQR and cuts the circle at point S.



Show that

(i) $\angle PSR + 2\angle SPT = 180°$, [3]

(ii) triangle PRS is isosceles, [2]

(iii) $PT \times PQ = QT \times RS$. [3]

12 **(i)** Write down, in simplest form, the general term in the binomial expansion of $\left(\frac{x^2}{3}+\frac{2}{x}\right)^8$. [3]

(ii) Explain why the x^{-1} term does not exist in the expansion. [2]

(iii) Find the term independent of x in the expansion of $\left(x^2 + 3x - \frac{5}{x}\right)\left(\frac{x^2}{3} + \frac{2}{x}\right)^8$. [4]

13 Given a curve with equation $2xy - x + y + 2 = 0$ cuts the x-axis at point P.

(i) Find the gradient of the curve at P. [5]

(ii) Hence, determine the angle that the tangent to the curve at P makes with the positive x-axis. [2]

(iii) Find the coordinates of Q at which the normal at P cuts the y-axis. [3]

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Candidate Name: ________________________ **Index No:** ____ **Class:** _______

ZHENGHUA SECONDARY SCHOOL
PRELIMINARY EXAMINATIONS 2023
SECONDARY FOUR EXPRESS/FIVE NORMAL (ACADEMIC)
ADDITIONAL MATHEMATICS **4049/02**
Paper 2

25 AUGUST 2023

Candidates answer on the Question Paper. 2 hours 15 min

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$$a^2=b^2+c^2-2bc\cos A$$
$$\Delta=\frac{1}{2}bc\sin A$$

1 **(a)** **Without using a calculator,** show that $\sin 75° = \frac{\sqrt{2}}{4}\left(1+\sqrt{3}\right)$. [3]

(b) Hence find the exact value of $\sin 285°$. [2]

2 Solve the equation $3^{y+2} = 2\left(3^{-y}\right) + 17$. [4]

3 In a town whose population is 95 000, a disease created an epidemic. The total number of people, N, infected t days after the disease has begun, is given by

$$N = \frac{95000}{2 + 498e^{-0.6t}}.$$

(a) Find the number of people infected 5 days after the disease has begun. [1]

(b) The authority will issue home quarantine orders when 8% of the population is infected. After how many days will the authority issue the order? [3]

(c) Explain whether the entire town of 95 000 people will be infected with the disease after a long period of time. [1]

4 The expression $2x^3 + x^2 + ax + b$ is exactly divisible by $(x-2)$ and has a remainder of -4 when divided by $(x-1)$.

(a) Find the value of a and of b. [4]

(b) Hence, factorise the polynomial completely. [3]

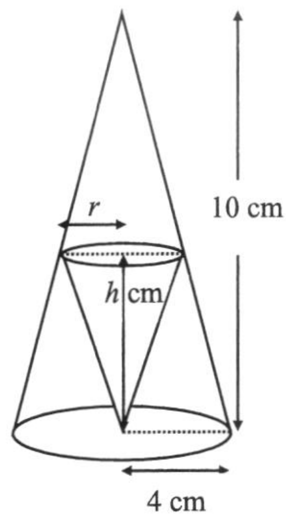
5 **(a)** Find $\dfrac{\mathrm{d}}{\mathrm{d}x}\left(\ln\sqrt{\dfrac{3x+1}{3x-1}}\right)$, leaving your answer as a single fraction in its simplest form. [4]

(b) Hence, find $\displaystyle\int \frac{2}{9x^2-1}\,\mathrm{d}x$. [3]

6 **(a)** Prove the identity $\dfrac{\tan A - \cot A}{\tan A + \cot A} + 1 = 2\sin^2 A$. [4]

(b) Hence, solve the equation $\dfrac{\tan A - \cot A}{\tan A + \cot A} + 1 = 5\sin A \cos A$ for $0° < A < 360°$. [5]

7 The diagram shows an inverted smaller cone inside a bigger upright cone of base radius 4 cm and height 10 cm. It is given that the vertex of the smaller cone touches the centre of the base of the bigger cone. The smaller cone has radius r cm and height h cm.



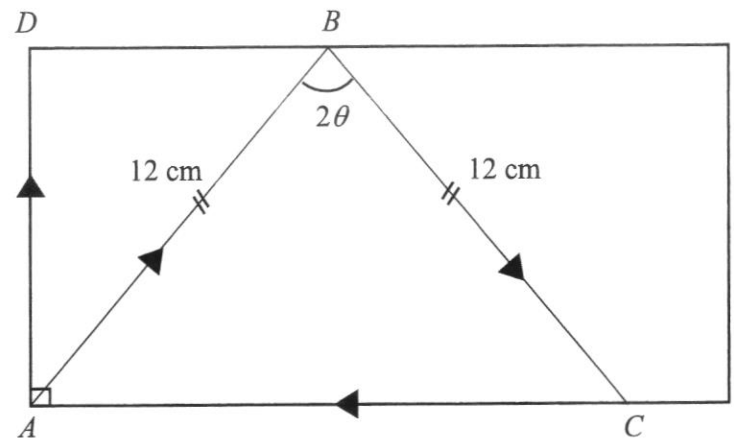
(a) Express r in terms of h. [2]

(b) Show that the volume, V cm^3, of the smaller cone is

$$V = \frac{4\pi h\left(h^2 - 20h + 100\right)}{75}.$$ [2]

(c) If h varies, find the value of h for which V has a maximum value. [5]

8 An ant crawls on a flat rectangular piece of cardboard. It starts at point A and moves to points B, C, A and D in sequence. It stops at point D and does not move anymore at that point.



It is given $\angle ABC = 2\theta$, $\angle CAD = 90°$ and $AB = BC = 12$ cm for $0° < \theta < 90°$.

(a) Show that S cm, the total distance travelled by the ant, can be expressed as $p + q \sin\theta + r\cos\theta$, where p, q and r are constants to be found. [3]

(b) Express S in the form $p + R\sin(\theta + \alpha)$, where $R > 0$ and α is an acute angle. [3]

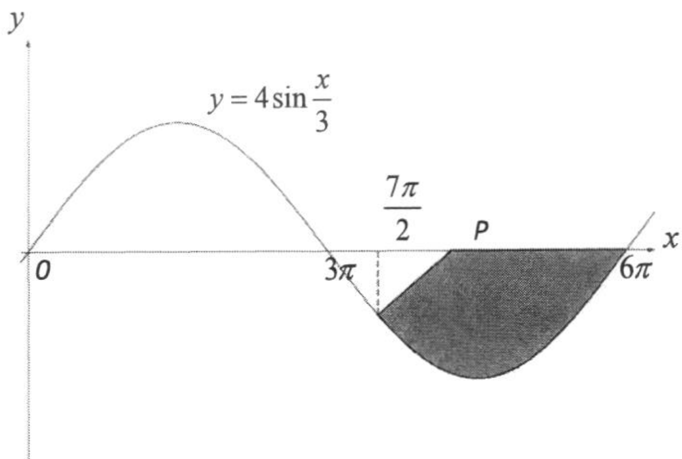
(c) Given that the ant travelled a total distance of 50 cm, find the value(s) of θ. [3]

9 **(a)** Given $y = \dfrac{5x}{\sqrt{4x+3}}$, find $\dfrac{\mathrm{d}y}{\mathrm{d}x}$. [4]

(b) A particle P moves along this such that the y-coordinate of P decreases at a constant rate of 0.4 units per second when $x = 3$. Find the rate at which the x-coordinate of P is changing at that instant. [3]

(c) Use the result from part (a) to evaluate $\int_0^3 \frac{20x+32}{(4x+3)^{\frac{3}{2}}}\,dx$. [4]

10 The graph shows part of the curve $y = 4\sin\frac{x}{3}$. The normal to the curve at $x = \frac{7\pi}{2}$ cuts the x-axis at point P.



(a) Find the **exact** coordinates of P. [6]

(b) Hence, find the area of the shaded region. [4]

11 The equation of a circle C is $x^2 + y^2 - 10x + 6y + 25 = 0$.

(a) Find the radius and the coordinates of the centre of the circle C. [3]

(b) Find the values of k if the circle touches the line $y = k$. [2]

(c) The line L with equation $x + y = 5$ intersects the circle at the points P and Q. Find the coordinates of P and Q and hence find the shortest distance of the centre of the circle from L. [7]

(d) Find the equation of the circle which is a reflection of circle C in the x-axis. [2]

- End of Paper –

Candidate Name: ______________________ Index No: ____ Class: ______

ZHENGHUA SECONDARY SCHOOL
PRELIMINARY EXAMINATION 2023
SECONDARY FOUR EXPRESS/ FIVE NORMAL (ACADEMIC)
ADDITIONAL MATHEMATICS

Paper 1 **(ANSWER KEY)** 4049/01

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24 AUG 2023

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$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

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$$a^2 = b^2 + c^2 - 2bc\cos A$$

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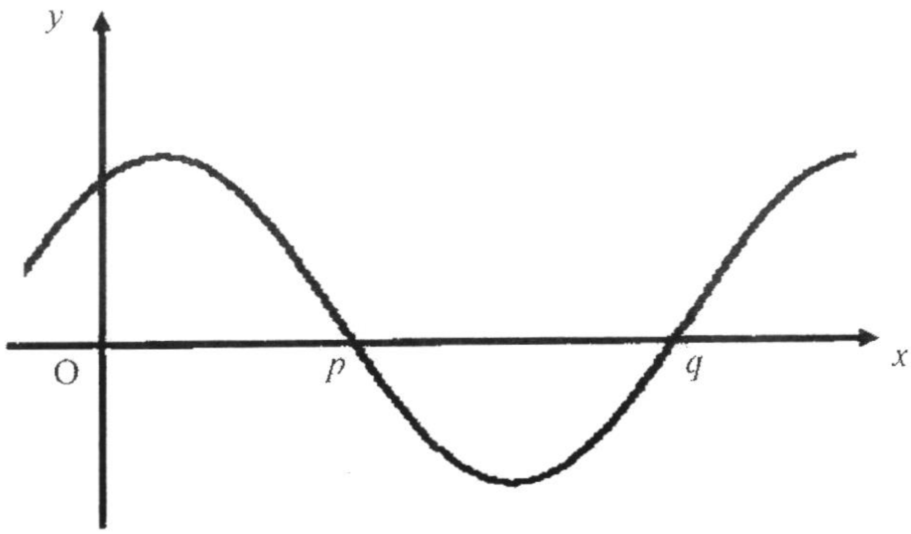
1	The polynomial f is given by $f(x) = (2x+1)(x^2 + ax - b)$.		
	(i)	What conditions must apply to the constants a and b if f(x) = 0 has exactly one real root?	[2]
		$a^2 + 4b < 0$ $b < 0$ (to highlight to students! No mark awarded for this)	
	(ii)	Give an example of integer values of a and b which satisfy the conditions found in part **(i)**.	[1]
		$b = -1$, $a = 1$	
2	**(a)**	Divide $2x^3 - 4$ by $2x^3 - x^2$.	[1]
		$\dfrac{2x^3-4}{2x^3-x^2} = 1 + \dfrac{x^2-4}{2x^3-x^2}$	
	(b)	Express $\dfrac{2x^3-4}{2x^3-x^2}$ in partial fractions.	[4]
		$\dfrac{2x^3-4}{2x^3-x^2} = 1 + \dfrac{x^2-4}{2x^3-x^2}$ $2x^3 - x^2 = x^2(2x-1)$ Consider $\dfrac{x^2-4}{2x^3-x^2} = \dfrac{A}{x} + \dfrac{B}{x^2} + \dfrac{C}{2x-1}$ $x^2 - 4 = Ax(2x-1) + B(2x-1) + Cx^2$ @ $x = 0$, $B = 4$ @ $x = \frac{1}{2}$, $C = -15$ @ $x = 1$, $A = 8$ $\dfrac{2x^3-4}{2x^3-x^2} = 1 + \dfrac{8}{x} + \dfrac{4}{x^2} - \dfrac{15}{2x-1}$	

3



The diagram shows the curve with equation $y = k\cos(x - 30^\circ)$, $0^\circ \le x \le 360^\circ$, where k is a constant. The curve meets the y-axis at $(0, \sqrt{3})$ and passes through the points $(p^\circ, 0)$ and $(q^\circ, 0)$.

(i) Show that $k = 2$. [2]

@$(0, \sqrt{3})$,

$\sqrt{3} = k\cos(0 - 30^\circ)$

$\sqrt{3} = \frac{\sqrt{3}}{2}k$

$k = 2$

(ii) Calculate the value of p and q. [3]

$y = 2\cos(x - 30^\circ)$

$2\cos(x - 30^\circ) = 0$

Basic angle $= \cos^{-1}(0)$

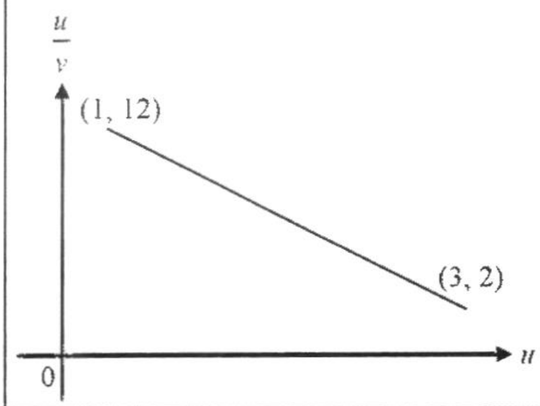
$= 90^\circ$

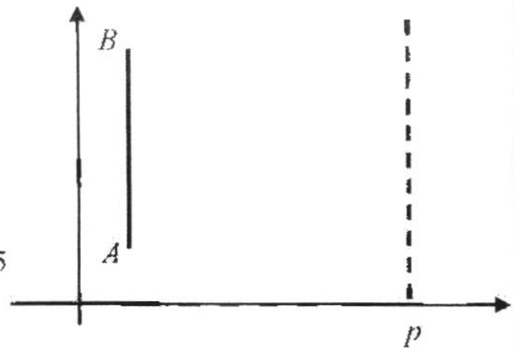
$x - 30^\circ = 90^\circ, 270^\circ$

$x = 120^\circ, 300^\circ$

$p = 120$ & $q = 300$

4	The equation of a curve is $y = 3x^2 + ax - 12$, where a is a negative constant.			
	(i)	Given that the coordinates of the minimum point is $(1, -15)$, show that value of a is -6.		[2]
		Method 1 : By differentiation $y = 3x^2 + ax - 12$ $\frac{dy}{dx} = 6x + a$ For $\frac{dy}{dx} = 0$, $6(1) + a = 0$ $a = -6$ Method 2 : By completing the square $y = 3x^2 + ax - 12$ $y = 3(x^2 + \frac{a}{3}x - 4)$ $y = 3\left[\left(x + \frac{a}{6}\right)^2 - \frac{a^2}{36} - 4\right]$ $1 + \frac{a}{6} = 0 \Rightarrow a = -6$		
	(ii)	This curve meets another curve $y = -x^2 - 2$ at the points P and Q. Calculate the gradient of line segment PQ.		[5]
		$3x^2 - 6x - 12 = -x^2 - 2$ $4x^2 - 6x - 10 = 0$ $2x^2 - 3x - 5 = 0$ $(2x - 5)(x + 1) = 0$ $x = \frac{5}{2}, \ -1$ $y = -\frac{33}{4}, \ -3$ $P(-1, -3)$ & $Q(\frac{5}{2}, -\frac{33}{4})$ gradient $= \dfrac{-3 + \frac{33}{4}}{-1 - \frac{5}{2}}$ gradient $= -\frac{3}{2}$		

5		Two variables u and v are related such that when a straight line graph of $\frac{u}{v}$ against u is plotted, the line passes through (1, 12) and (3, 2) as shown in the diagram. 		
	(a)	Express v in terms of u.		[4]
		$m=\frac{12-2}{1-3}$ $=-5$ $Y=-5X+C$ @ (3, 2), $C=17$ $Y=-5X+17$ $\frac{u}{v}=-5u+17$ $\frac{v}{u}=\frac{1}{-5u+17}$ $v=\frac{u}{17-5u}$		
	(b)	If $\frac{1}{v}$ is plotted against $\frac{1}{u}$, state the value of the gradient and the vertical intercept.		[2]
		$\frac{u}{v}=-5u+17$ Divide by u throughout, $\frac{1}{v}=-5+\frac{17}{u}$ $\frac{1}{v}=17\left(\frac{1}{u}\right)-5$ Gradient = 17 Vertical intercept = -5		

6	The parallelogram $ABCD$ is such that A is (2, 4), B is (2, 10), C is (p, r) and D is (p, q), where $p > 0$. The area of parallelogram $ABCD$ is 30 units^2 and $AC = \sqrt{250}$ units.		
	(a)	Explain why $p = 7$.	[3]
		x-coordinate of A and B are the same. Hence, AB is a vertical line. Length of AB = 6 units. Given area of ABCD is 30 $\Rightarrow \frac{1}{2} \times 6 \times h \times 2 = 30$ $\Rightarrow h = 5$ $\therefore p = 2+5$ or $p = 2-5$ $p = 7$ or $p = -3\ (rej)$  	
	(b)	Find the possible values of r.	[3]
		$\sqrt{(2-7)^2 + (4-r)^2} = \sqrt{250}$ $25 + (4-r)^2 = 250$ $(4-r)^2 = 225$ $4 - r = \pm 15$ $r = 19$ or -11	
7	A bungee jumper falls vertically from rest from a point O. Her velocity, v m/s, t seconds, after leaving O, is such that $\frac{dv}{dt} = 10$. After 4 s, she reaches a point X. Find		
	(i)	her velocity at X,	[2]
		$\frac{dv}{dt} = 10$ $v = \int 10\, dt$ $= 10t + c$ @ $t = 0, v = 0,$ $c = 0$ $v = 10t$ @ $t = 4, v = 40$	

	(ii)	her distance OX.	[2]
		$v = 10t$ $s = \int 10t\,dt$ $= 10\left(\frac{t^2}{2}\right) + c$ $s = 5t^2 + c$ @ $t = 0, s = 0$ $\therefore c = 0$ @ $t = 4, \quad s = 80\text{m}$	
	On reaching X, she then slows so that her velocity, V m/s, T seconds after reaching X, is such that $\frac{dV}{dt} = 10 - kT$, where k is a constant.		
	(iii)	Given that she first comes to rest at a point Y when $T = 3$, show that $k = \frac{140}{9}$.	[3]
		$\frac{dV}{dt} = 10 - kT$ $V = \int (10 - kT)\,dT$ $= 10T - \frac{kT^2}{2} + c$ @ $T = 0, V = 40$ $\therefore \quad c = 40$ $V = 10T - \frac{kT^2}{2} + 40$ @ $T = 3, V = 0$ $\therefore \quad k = \frac{140}{9}$	

8	Given the polynomial $P(x) = -x^3 + ax^2 + bx + c$, where a, b and c are constants.			
	(a)	Find the equation of the tangent to the curve at $(0, 3)$, leaving your answer in terms of b.		[3]
		$P'(x) = -3x^2 + 2ax + b$ Gradient $= b$ at $(0,3)$ Equation : $y = bx + 3$		
	(b)	In the case where $a = 5$, $b = -2$ and $c = -2$, justify with reasoning, the number of real root(s) for $P(x) = 0$.		[5]
		$P(x) = -x^3 + 5x^2 - 2x - 2$ $P(1) = -1 + 5 - 2 - 2 = 0$ $(x-1)$ is a factor of $P(x)$. $P(x) = (x-1)(-x^2 + 4x + 2)$ For $P(x) = 0$ $x - 1 = 0$ & $-x^2 + 4x + 2 = 0$ $x = 1$ & $b^2 - 4ac = 4^2 - 4(-1)(2)$ $= 24 > 0$ Hence, $-x^2 + 4x + 2 = 0$ has 2 real and distinct roots. The number of real roots are 3.		

9	A curve is such that $\frac{d^2y}{dx^2} = \sin\left(\frac{x}{2} - 1\right)$. The gradient of the curve at the point (2, 7) is 5.		
	(i)	Express y in terms of x.	[5]
		$\frac{d^2y}{dx^2} = \sin\left(\frac{x}{2}-1\right)$ $\frac{dy}{dx} = -2\cos\left(\frac{x}{2}-1\right)+c$ $@x=2,\quad \frac{dy}{dx}=5$ $\therefore\quad c=7$ $\frac{dy}{dx} = -2\cos\left(\frac{x}{2}-1\right)+7$ $y=\int\left(-2\cos\left(\frac{x}{2}-1\right)+7\right)dx$ $y=-4\sin\left(\frac{x}{2}-1\right)+7x+d$ @ (2, 7), $7=-4\sin\left(\frac{2}{2}-1\right)+7(2)+d$ $\therefore\quad d=-7$ $y=-4\sin\left(\frac{x}{2}-1\right)+7x-7$	
	(ii)	Show that the gradient of the curve is never less than 5.	[2]
		Since $-1 \le \cos\left(\frac{x}{2}-1\right) \le 1$, $-2 \le -2\cos\left(\frac{x}{2}-1\right) \le 2$. Least value $= -2+7$ $= 5$ Hence, gradient of curve is at least 5. $\therefore$ Gradient is never less than 5.	

10 Given the curve $y=\frac{9}{x}-2$ and the line $x+y=k$ where k is a constant.

The line intersects the curve at the points A and B.

(a) In the case where $k=8$, find the coordinates of A and B. [5]

$y=\frac{9}{x}-2 \quad \cdots \quad (1) \quad \& \quad y=8-x \quad \cdots \quad (2)$

$\frac{9}{x}-2=8-x$

$x^2-10x+9=0$

$(x-1)(x-9)=0$

$x=1, \quad 9$

$y=7, \quad -1$

$A(1,7) \quad \& \quad B(9,-1)$

(b) Find the value of k, in the case where the line is a tangent to the curve. [4]

Method 1: By differentiation

$y=\frac{9}{x}-2 \quad \Rightarrow \quad \frac{dy}{dx}=-\frac{9}{x^2}$

$x+y=k \quad \Rightarrow \quad y=-x+k$

$-\frac{9}{x^2}=-1 \quad \Rightarrow \quad x=\pm 3$

$\Rightarrow \quad y=1, \quad y=-5$

$\therefore \quad k=x+y \quad \&$

$\therefore \quad k=4 \quad \& \quad k=-8$

Method 2: By discriminant

$\frac{9}{x}-2=-x+k$

$9-2x=-x^2+kx$

$x^2-kx-2x+9=0$

$x^2+(-k-2)x+9=0$

$b^2-4ac=0 \quad \Rightarrow \quad (-k-2)^2-4(1)(9)=0$

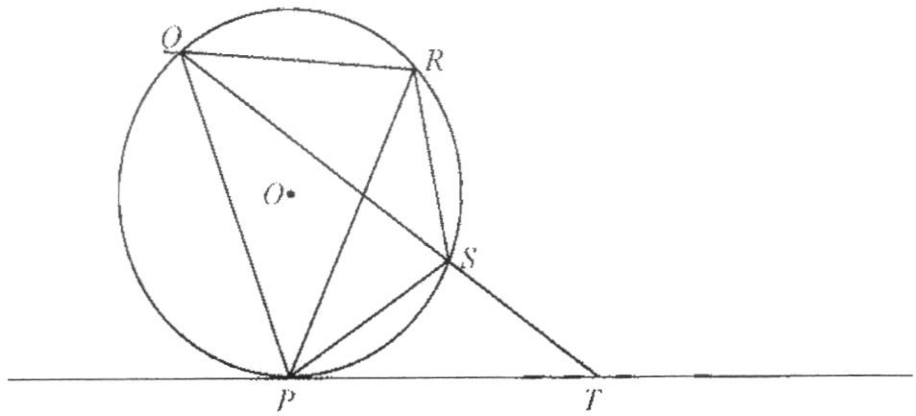
$(-k-2)^2=36$

$-k-2=6 \quad or \quad -k-2=-6$

$k=-8 \quad or \quad k=4$

11 In the figure below, the line PT is a tangent to the circle at P.

Line QT bisects angle PQR and cuts the circle at point S.



Show that

(i)	$\angle PSR + 2\angle SPT = 180°$,		[3]
	$\angle SPT = \angle PQS$ (alternate segment theorem) $\angle SQR = \angle PQS$ (QT bisect $\angle PQR$) $\therefore \angle SQR = \angle SPT$ $\angle PSR + \angle PQR = 180°$ (angle in opposite segment) $\angle PSR + 2\angle SPT = 180°$ (shown)		
(ii)	triangle PRS is isosceles,		[2]
	$\angle PRS = \angle PQS$ (angle in same segment) $\angle RPS = \angle RQS$ (angle in same segment) $\angle RPS = \angle PRS$ triangle PRS is isosceles		
(iii)	$PT \times PQ = QT \times RS$.		[3]
	$\angle SPT = \angle PQS$ (alternate segment theorem) $\angle PTS = \angle QTP$ (common angle) ΔSPT is similar to ΔPQT (AA) $\frac{SP}{PT} = \frac{PQ}{QT}$ $PT \times PQ = QT \times SP$ $PT \times PQ = QT \times RS$		

12	**(i)**	Write down, in simplest form, the general term in the binomial expansion of $\left(\frac{x^2}{3}+\frac{2}{x}\right)^8$.	[3]
	$T_{r+1}=\binom{8}{r}\left(\frac{x^2}{3}\right)^{8-r}\left(\frac{2}{x}\right)^r$ $=\binom{8}{r}\left(\frac{x^{2(8-r)}}{3^{8-r}}\right)\left(\frac{2^r}{x^r}\right)$ $=\binom{8}{r}\left(\frac{2^r}{3^{8-r}}\right)x^{16-2r-r}$ $=\binom{8}{r}\left(\frac{2^r}{3^{8-r}}\right)x^{16-3r}$		
	(ii)	Explain why the x^{-1} term does not exist in the expansion.	[2]
		$16-3r=-1$ $r=\frac{17}{3}$ But $r=5\frac{2}{3}$ which is not an integer, hence the x^{-1} term does not exist in the expansion.	

	(iii)	Find the term independent of x in the expansion of $\left(x^2+3x-\frac{5}{x}\right)\left(\frac{x^2}{3}+\frac{2}{x}\right)^8$.	[4]
		Let $16-3r=-2$ $r=6$ $T_7=\binom{8}{6}\left(\frac{2^6}{3^2}\right)x^{-2}$ $=\frac{1792}{9x^2}$ Let $16-3r=1$ $r=5$ $T_7=\binom{8}{5}\left(\frac{2^5}{3^3}\right)x$ $=\frac{1792}{27}x$ $For\left(x^2+3x-\frac{5}{x}\right)\left(\frac{1792}{9x^2}+\frac{1792}{27}x+\ldots\right)$, Term independent of x: $=1\times\frac{1792}{9}-5\times\frac{1792}{27}$ $=-132\frac{20}{27}$	

13	Given a curve with equation $2xy - x + y + 2 = 0$ cuts the x-axis at point P.		
	(i)	Find the gradient of the curve at P.	[5]
		$2xy - x + y + 2 = 0$ $y = \dfrac{x-2}{2x+1}$ $\dfrac{dy}{dx} = \dfrac{(2x+1)(1)-(x-2)(2)}{(2x+1)^2}$ $= \dfrac{5}{(2x+1)^2}$ @ $y = 0$, $-x + 2 = 0$ $x = 2$ @ $x = 2$, $\dfrac{dy}{dx} = \dfrac{1}{5}$	
	(ii)	Hence, determine the angle that the tangent to the curve at P makes with the positive x-axis.	[2]
		Let the angle be α. $\tan\alpha = \dfrac{1}{5}$ $\alpha = \tan^{-1}\left(\dfrac{1}{5}\right)$ $= 11.3^\circ$	
	(iii)	Find the coordinates of Q at which the normal at P cuts the y-axis.	[3]
		$m_{normal} = -5$ Eqn of normal : $y = -5x + c$ @ $(2, 0)$, $c = 10$ Eqn of normal : $y = -5x + 10$ $Q(0, 10)$	

****** The End ********

Candidate Name: ____________________________ Index No: ____ Class: _______

ZHENGHUA SECONDARY SCHOOL
PRELIMINARY EXAMINATIONS 2023
SECONDARY FOUR EXPRESS/FIVE NORMAL (ACADEMIC)
ADDITIONAL MATHEMATICS 4049/02
Paper 2

MARK SCHEME

25 AUGUST 2023

Candidates answer on the Question Paper. 2 hours 15 min

READ THESE INSTRUCTIONS FIRST

Write your index number and name in the spaces at the top of this page.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

The number of marks is given in brackets [ ] at the end of each question or part question.

The total number of marks for this paper is 90.

Name of Setter:

This document consists of 18 printed pages.

[Turn over]

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1} a^{n-1} b + \binom{n}{2} a^{n-2} b^2 + \ldots + \binom{n}{r} a^{n-r} b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2} bc \sin A$$

1 **(a)** **Without using a calculator,** show that $\sin 75° = \frac{\sqrt{2}}{4}\left(1+\sqrt{3}\right)$ [3]

$\sin 75° = \sin(30° + 45°)$

$= \sin 30° \cos 45° + \cos 30° \sin 45°$

$= \left(\frac{1}{2}\right)\left(\frac{\sqrt{2}}{2}\right) + \left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{2}}{2}\right)$

$= \frac{\sqrt{2}}{4} + \frac{\sqrt{3}\sqrt{2}}{4}$

$= \frac{\sqrt{2}}{4}\left(1+\sqrt{3}\right)$ (shown)

(b) Hence find the exact value of $\sin 285°$. [2]

$\sin 285° = -\sin 75°$

$= -\frac{\sqrt{2}}{4}\left(\sqrt{3}+1\right)$

2 Solve the equation $3^{y+2} = 2\left(3^{-y}\right) + 17$. [4]

$$3^{y+2} = 2\left(3^{-y}\right) + 17$$

let $u = 3^y$

$$9u = \frac{2}{u} + 17$$

$$9u^2 - 17u - 2 = 0$$

$$(9u + 1)(u - 2) = 0$$

$$u = -\frac{1}{9} \text{ or } 2$$

$$3^y = -\frac{1}{9} \text{ (n.a.) or } 3^y = 2$$

$$y = 0.631$$

3 In a town whose population is 95 000, a disease created an epidemic. The total number of people, N, infected t days after the disease has begun, is given by

$$N = \frac{95000}{2 + 498e^{-0.6t}}$$

(a) Find the number of people infected 5 days after the disease has begun. [1]

3546 people
Accept 3550 people (3sf)

(b) The authority will issue home quarantine orders when 8% of the population is infected. After how many days will the authority issue the order? [3]

$$7600 = \frac{95000}{2 + 498e^{-0.6t}}$$

$$e^{-0.6t} = \frac{7}{332}$$

$$t = \ln\frac{7}{332} \div -0.6 = 6.43$$

After 7 days

(c) Explain whether the entire town of 95 000 people will be infected with the disease after a long period of time. [1]

No. As t approaches infinity, N approaches 47500, but never 95000.

4 The expression $2x^3 + x^2 + ax + b$ is exactly divisible by $(x-2)$ and has a remainder of -4 when divided by $(x-1)$.

(a) Find the value of a and of b. [4]

Let $f(x) = 2x^3 + x^2 + ax + b$

When $x = 2$, $f(2) = 0$

$$2(2)^3 + (2)^2 + a(2) + b = 0$$

$$2a + b = -20 \text{ ------- } (1)$$

When $x = 1$, $f(1) = -4$

$$2(1)^3 + (1)^2 + a(1) + b = -4$$

$$a + b = -7 \text{ -------- } (2)$$

$1 - 2$, $(2a+b)-(a+b) = -20-(-7)$

$$\therefore \quad a = -13$$

sub a into 2, $-13 + b = -7$

$$\therefore \quad b = 6$$

(b) Hence, factorise the polynomial completely. [3]

Long division

$$f(x) = 2x^3 + x^2 - 13x + 6$$
$$= (x-2)(2x^2 + 5x - 3)$$
$$= (x-2)(2x-1)(x+3)$$

5 **(a)** Find $\frac{d}{dx}\left(\ln\sqrt{\frac{3x+1}{3x-1}}\right)$, leaving your answer as a single fraction in its simplest form. [4]

$$\frac{d}{dx}\left(\ln\sqrt{\frac{3x+1}{3x-1}}\right) = \frac{d}{dx}\left[\frac{1}{2}(\ln(3x+1) - \ln(3x-1))\right]$$

$$= \frac{1}{2}\left[\frac{3}{3x+1} - \frac{3}{3x-1}\right]$$

$$= \frac{-3}{(3x+1)(3x-1)}$$

(b) Hence, find $\int \frac{2}{9x^2-1}\,dx$. [3]

$$\int \frac{2}{9x^2-1}dx = -\frac{2}{3}\int \frac{-3}{9x^2-1}\ dx$$

$$= -\frac{2}{3}\int \frac{-3}{(3x+1)(3x-1)}\ dx$$

$$= -\frac{2}{3}\ln\sqrt{\frac{3x+1}{3x-1}}\ + c$$

6 **(a)** Prove the identity $\frac{\tan A - \cot A}{\tan A + \cot A} + 1 = 2\sin^2 A$. [4]

$$LHS = \frac{tanA - cotA}{tanA + cotA} + 1$$

$$= \frac{\left(\frac{\sin A}{\cos A} - \frac{\cos A}{\sin A}\right)}{\left(\frac{\sin A}{\cos A} + \frac{\cos A}{\sin A}\right)} + 1$$

$$= \frac{sin^2 A - cos^2 A}{sin^2 A + cos^2 A} + 1$$

$$= \frac{sin^2 A - cos^2 A + sin^2 A + cos^2 A}{sin^2 A + cos^2 A}$$

$$= (sin^2 A - cos^2 A) + (sin^2 A + cos^2 A)$$
$$= 2sin^2 A = \text{RHS}$$

(b) Hence, solve the equation $\frac{\tan A - \cot A}{\tan A + \cot A} + 1 = 5\sin A\cos A$ for $0° < A < 360°$. [5]

$$2sin^2 A = 5\sin A\cos A$$

$$\sin A(2\sin A - 5\cos A) = 0$$

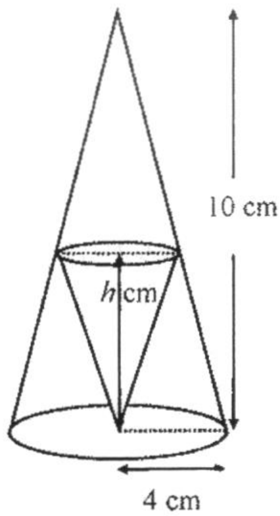
$$\sin A = 0 \quad or \quad 2\sin A = 5\cos A$$

$$A = 180°(rejected) \quad or \quad \tan A = \frac{5}{2}$$

$$basic\ angle, \alpha = tan^{-1}\frac{5}{2}$$
$$= 68.2°$$

$$A = 68.2°,\ 248.2°$$

7 The diagram shows an inverted smaller cone inside a bigger upright cone of base radius 4 cm and height 10 cm. It is given that the vertex of the smaller cone touches the centre of the base of the bigger cone. The smaller cone has radius r cm and height h cm.



(a) Express r in terms of h. [2]

$\frac{r}{4}=\frac{10-h}{10}$ (Using similar triangles)

$r=\frac{2(10-h)}{5}$

(b) Show that the volume, V cm^3, of the smaller cone is

$$V = \frac{4\pi h\left(h^2 - 20h + 100\right)}{75}$$ [2]

$$V = \frac{1}{3}\pi\left(\frac{2(10-h)}{5}\right)^2 h$$

$$= \frac{1}{3}\pi h\left(\frac{4\left(100-20h+h^2\right)}{25}\right)$$

$$V = \frac{4\pi h\left(h^2 - 20h + 100\right)}{75} \quad \text{(shown)}$$

(c) If h varies, find the value of h for which V has a maximum value. [5]

$$V = \frac{4\pi\left(h^3 - 20h^2 + 100h\right)}{75}$$

$$\frac{dV}{dh} = \frac{4\pi}{75}\left(3h^2 - 40h + 100\right)$$

When $\frac{dV}{dh} = 0$,

$$\frac{4\pi}{75}\left(3h^2 - 40h + 100\right) = 0$$

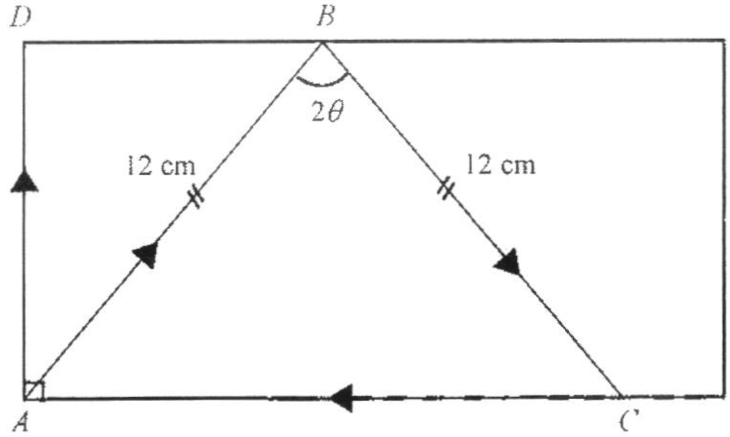
$$(3h-10)(h-10) = 0$$

$$h = \frac{10}{3} \quad \text{or} \quad h = 10 \quad \text{(rej)}$$

$$\frac{d^2V}{dh^2} = \frac{4\pi}{75}(6h - 40)$$

When $h = \frac{10}{3}$, $\frac{d^2V}{dh^2} = \frac{4\pi}{75}(20-40) < 0 \Rightarrow V$ is maximum.

8 An ant crawls on a flat rectangular piece of cardboard. It starts at point A and moves to points B, C, A and D in sequence. It stops at point D and does not move anymore at that point.



It is given $\angle ABC = 2\theta$, $\angle CAD = 90°$ and $AB = BC = 12$ cm for $0° < \theta < 90°$.

(a) Show that S cm, the total distance travelled by the ant, can be expressed as $p + q \sin \theta + r \cos \theta$, where p, q and r are constants to be found. [3]

$S = AB + BC + AC + AD$

$= 12 + 12 + 24 \sin \theta + 12 \cos \theta$

$= 24 + 24 \sin \theta + 12 \cos \theta$

$p = 24$

$q = 24$

$r = 12$

(b) Express S in the form $p + R \sin(\theta + \alpha)$, where $R > 0$ and α is an acute angle. [3]

$S = 24 + \sqrt{12^2 + 24^2} \sin(\theta + \tan^{-1} 0.5)$

$= 24 + \sqrt{720} \sin(\theta + 26.6°)$

(c) Given that the ant travelled a total distance of 50 cm, find the value(s) of θ. [3]

$50 = 24 + \sqrt{720} \sin(\theta + 26.5651°)$

$\theta = 49.1°$ or $77.7°$

9 **(a)** Given $y = \dfrac{5x}{\sqrt{4x+3}}$, find $\dfrac{dy}{dx}$. [4]

$$\frac{dy}{dx} = \frac{\left(\sqrt{4x+3}\right)(5) - (5x)(\frac{1}{2})(4x+3)^{-\frac{1}{2}}(4)}{4x+3}$$

$$= \frac{5\sqrt{4x+3} - (10x)(4x+3)^{-\frac{1}{2}}}{4x+3}$$

$$= \frac{5(4x+3) - 10x}{(4x+3)^{\frac{3}{2}}}$$

$$= \frac{10x+15}{(4x+3)^{\frac{3}{2}}}$$

(b) A particle P moves along this such that the y-coordinate of P decreases at a constant rate of 0.4 units per second when $x = 3$. Find the rate at which the x-coordinate of P is changing at that instant. [3]

$$\frac{dy}{dx} = \frac{10x+15}{(4x+3)^{\frac{3}{2}}}$$

When $x = 3$,

$$\frac{dy}{dt} = \frac{dy}{dx} \times \frac{dx}{dt}$$

$$-0.4 = \frac{45}{(15)^{\frac{3}{2}}} \times \frac{dx}{dt}$$

$$\frac{dx}{dt} = -0.516 \text{ units/sec}$$

(c) Use the result from part (a) to evaluate $\int_0^3 \frac{20x+32}{(4x+3)^{\frac{3}{2}}}\,dx$ [4]

$$\int_0^3 \frac{20x+32}{(4x+3)^{\frac{3}{2}}}\,dx$$

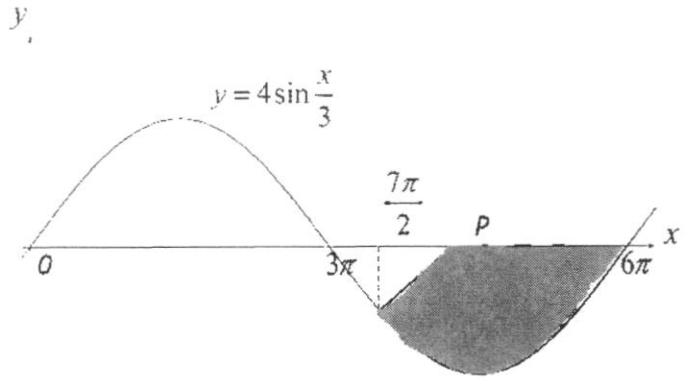
$$=\int_0^3 \frac{20x+30}{(4x+3)^{\frac{3}{2}}}\,dx+\int_0^3 \frac{2}{(4x+3)^{\frac{3}{2}}}\,dx$$

$$=2\int_0^3 \frac{10x+15}{(4x+3)^{\frac{3}{2}}}\,dx+\int_0^3 \frac{2}{(4x+3)^{\frac{3}{2}}}\,dx$$

$$=2\left[\frac{5x}{\sqrt{4x+3}}\right]_0^3+2\left[\frac{(4x+3)^{\frac{1}{2}}}{-\frac{1}{2}\times 4}\right]_0^3$$

$$=8.07$$

10 The graph shows part of the curve $y = 4\sin\frac{x}{3}$. The normal to the curve at $x = \frac{7\pi}{2}$ cuts the x-axis at point P.



(a) Find the **exact** coordinates of P. [6]

$$\frac{dy}{dx} = \frac{4}{3}\cos\frac{x}{3}$$

when $x = \frac{7\pi}{2}$, $y = -2$

$$\frac{dy}{dx} = -\frac{2\sqrt{3}}{3}$$

$$m_{\text{normal}} = \frac{3}{2\sqrt{3}}$$

equation of normal:

$$y - (-2) = \frac{3}{2\sqrt{3}}\left(x - \frac{7\pi}{2}\right)$$

$$y = \frac{3x}{2\sqrt{3}} - \frac{21\pi}{4\sqrt{3}} - 2$$

when $y = 0$,

$$x = \frac{7\pi}{2} + \frac{4\sqrt{3}}{3}$$

$$P\left(\frac{7\pi}{2} + \frac{4\sqrt{3}}{3}, 0\right)$$

(b) Hence, find the area of the shaded region. [4]

$$\text{shaded area} = \left|\int_{\frac{7\pi}{2}}^{6\pi} 4\sin\frac{x}{3}\,dx\right| - \frac{1}{2}(2)\left(\frac{4\sqrt{3}}{3}\right)$$

$$= \left|\left[-12\cos\frac{x}{3}\right]_{\frac{7\pi}{2}}^{6\pi}\right| - \frac{4\sqrt{3}}{3}$$

$$= \left|\left(-12\cos\frac{6\pi}{3}\right) - \left(-12\cos\frac{7\pi}{6}\right)\right| - \frac{4\sqrt{3}}{3}$$

$$= \left|-12 - 6\sqrt{3}\right| - \frac{4\sqrt{3}}{3}$$

$$= 20.1 \text{ units}^2$$

11 The equation of a circle C is $x^2 + y^2 - 10x + 6y + 25 = 0$.

(a) Find the radius and the coordinates of the centre of the circle C. [3]

$x^2 + y^2 - 10x + 6y + 25 = 0$ $x^2 - 10x + 25 + y^2 + 6y + 9 = -25 + 25 + 9$ $(x-5)^2 + (y+3)^2 = 3^2$ Centre of the circle is $(5, -3)$ Radius = 3 units	

(b) Find the values of k if the circle touches the line $y = k$. [2]

$k = -3 + 3 = 0 \quad or \quad k = -3 - 3 = -6$	

(c) The line L with equation $x + y = 5$ intersects the circle at the points P and Q. Find the coordinates of P and Q and hence find the shortest distance of the centre of the circle from L. [7]

$(5-y)^2 + y^2 - 10(5-y) + 6y + 25 = 0$ $2y^2 + 6y = 0$ $2y(y+3) = 0$ $y = 0$ or $y = -3$ Sub $y = 0$ $\therefore x = 5$ $\Rightarrow P(5, 0)$ Sub $y = -3$ $\therefore x = 5+3 = 8$ $\Rightarrow Q(8, -3)$ Midpoint $= \left(\frac{5+8}{2}, \frac{0-3}{2}\right) = \left(\frac{13}{2}, \frac{-3}{2}\right)$ Perpendicular distance from C to line $= \sqrt{\left(5 - \frac{13}{2}\right)^2 + \left(-3 + \frac{3}{2}\right)^2}$ $= \sqrt{4.5}$ or 2.12 units	

(d) Find the equation of the circle which is a reflection of circle C in the x-axis. [2]

Centre of the circle is $(5, 3)$ Equation of the circle: $(x-5)^2 + (y-3)^2 = 9$ or $x^2 + y^2 - 10x - 6y + 25 = 0$	

- End of Paper -