| Full Name | Class Index No | Class |
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Anglo-Chinese School
(Barker Road)

**PRELIMINARY EXAMINATION 2024**
**SECONDARY FOUR EXPRESS / FIVE NORMAL (ACADEMIC)**

**ADDITIONAL MATHEMATICS**
**4049**
**PAPER 1**

**2 HOURS 15 MINUTES**

Candidates answer on the Question Paper.

**READ THESE INSTRUCTIONS FIRST**

Write your index number and name in the spaces at the top of this page.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.

Answer **all** questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

The number of marks is given in brackets [ ] at the end of each question or part question.

The total number of marks for this paper is 90.

| For Examiner's Use |
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## Mathematical Formulae

### 1. ALGEBRA

*Quadratic Equation*

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

*Binomial expansion*

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n,$$

where $n$ is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

### 2. TRIGONOMETRY

*Identities*

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2\sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

*Formulae for $\Delta ABC$*

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

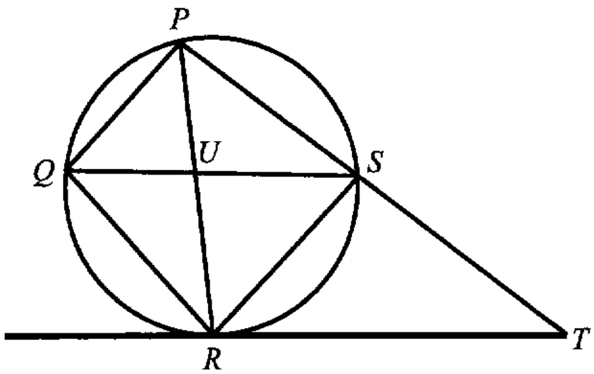
$$\Delta = \frac{1}{2}bc\sin A$$

1 The line $x - y = 3$ intersects the curve $x^2 - 3xy + y^2 + 19 = 0$ at the points $P$ and $Q$.
Find the coordinates of $P$ and of $Q$. [5]

**2** Solve the equation $2^{x+1} + 3(2^{-x}) = 7$. [5]

**3** Express the equation $\log_2 x + \log_4 (x-1) = 3$ as a cubic equation in $x$. [4]

4



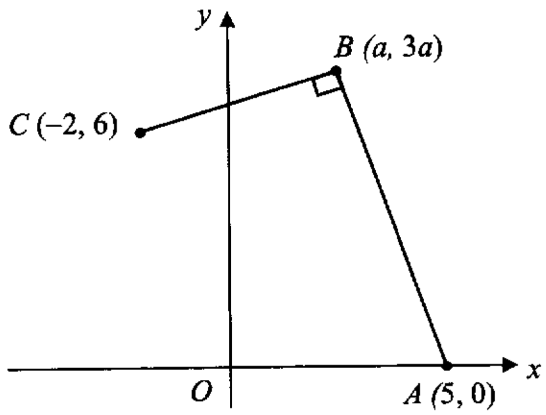

The diagonals of a cyclic quadrilateral $PQRS$ intersect at $U$. The tangent to the circle at $R$ meets $PS$ produced at $T$. If $QR = RS$, prove that

**(a)** $QS$ is parallel to $RT$, [3]

**(b)** triangles $QUR$ and $RST$ are similar. [3]

**TURN OVER FOR QUESTION 5**

5




The diagram shows points $A(5, 0)$, $B(a, 3a)$ and $C(-2, 6)$ such that the line $AB$ is perpendicular to the line $BC$.

**(a)** Show that $a = 2.5$. [3]

5 **(b)** Find the coordinates of the midpoint of $AC$. Hence find the coordinates of $D$ such that $ABCD$ is a parallelogram. [3]

**(c)** The area of triangle $AEC$ is 5 units$^2$. Find the value of $x$ given that the point $E$ is $(x, 2x)$, where $x > 1$. [2]

**6** **(a)** Express $\dfrac{13x-6}{x^2(2x-3)}$ in partial fractions. [5]

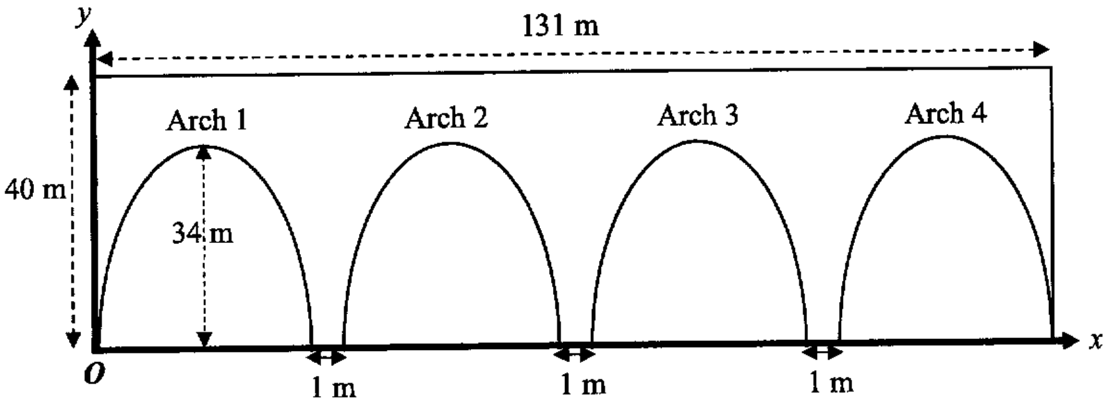
**6** **(b)** Hence $\int \frac{13x-6}{x^2(2x-3)}\,dx$. [3]

7 **(a)** **(i)** Write down and simplify, in ascending powers of $x$, the first four terms of the expansion of $(1-x)^{12}$. [2]

**(ii)** Hence find the value of $p$ given that the coefficient of $x^3$ in the expansion of $(2x^2+17x+p)(1-x)^{12}$ is 6598. [3]

7 **(b)** In the binomial expansion of $\left(x+\frac{k}{x}\right)^5$, where $k$ is a positive constant, the coefficients of $x^3$ and $x$ are the same. Find the value of $k$. [4]

8




A civil engineer is designing a bridge which is 131 metres long, 40 metres high and is to have four identical parabolic arches along its length. Each arch is 34 metres high and there is one metre between bases of each adjacent pair of arches as shown in the diagram. A set of axes is placed with the origin at the left- hand end of the base of the first arch.

**(a)** Find the $x$-intercepts of Arch 1. [2]

**(b)** The equation representing Arch 1 can be written in the form $y = a(x-p)(x-q)$. Show that $a = -\frac{17}{128}$. [2]

**8** **(c)** Explain, with working, if the point (50, 30) lies on Arch 2. [3]

9 (a) Differentiate $x\sin x+\cos x$ with respect to $x$. [2]

(b) Show that $\dfrac{\mathrm{d}}{\mathrm{d}x}\left(\dfrac{1}{3}x\sin^3 x\right)=\dfrac{1}{3}\sin^3 x-x\cos^3 x+x\cos x$. [3]

**9** **(c)** Using the results found in **part (a)** and **part (b)**, find $\int\left(\sin^3 x - 3x\cos^3 x\right)\mathrm{d}x$. [4]

**10** **(a)** Prove the identity $\dfrac{\cos x}{\operatorname{cosec} x-1}+\dfrac{\cos x}{\operatorname{cosec} x+1}=2\tan x$. [5]

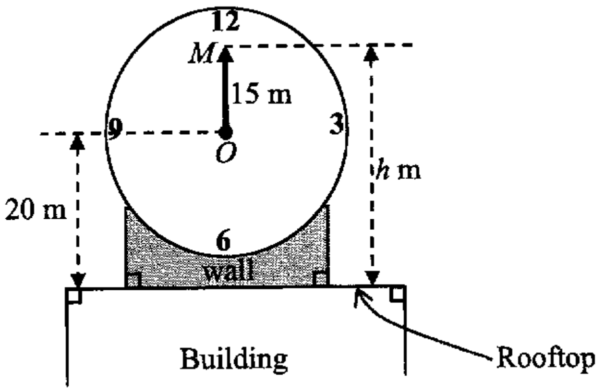
10 **(b)** Solve the equation $\tan x + 2\sec^2 x = 5$ for $0 \leq x \leq 2\pi$. [5]

**11** **(a)** **(i)** Find the range of values of $x$ for which $\ln(x^2-3)$ is defined. Leave your answer in surd form. [2]

**(ii)** Given that $y=\ln\left[e^x(x^2-3)\right]$, show that $\frac{\mathrm{d}y}{\mathrm{d}x}$ can be expressed in the form of $\frac{(x+a)(x+b)}{x^2-3}$. [3]

**11** **(b)** It is given that $y = x^3 + px^2 + qx + 10$ where $p$ and $q$ are integers. The only values of $x$ for which $y$ is a decreasing function of $x$ are those values for which $3 < x < 7$. Find the value of $p$ and of $q$. [4]

12




A clock is set on the vertical wall on the roof of a building.
The distance from the centre of the clock, $O$, to the tip, $M$, of the minute hand is 15 m.
The height, $h$ m, of $M$ above the rooftop is given by $h = a\cos kt + b$, where $t$ is the time in minutes past the hour. $O$ is 20 m above the rooftop. The rooftop is taken to be horizontal.

**(a)** Write down the value of $a$ and explain why $b = 20$. [2]

**(b)** Explain why the value of $k$ is $\frac{\pi}{30}$. [1]

**12** **(c)** Sketch the graph of $h$ for $0 \leq t \leq 45$. [3]

**(d)** Find the two timings in the first hour for which the height of $M$ is 10 m. [4]

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| Answers | | |

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## *Mathematical Formulae*

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*Binomial expansion*

General Term

$$(a+b)^n = a^n + \binom{n}{1} a^{n-1} b + \binom{n}{2} a^{n-2} b^2 + \ldots + \binom{n}{r} a^{n-r} b^r + \ldots + b^n,$$

where $n$ is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

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**1** The line $x - y = 3$ intersects the curve $x^2 - 3xy + y^2 + 19 = 0$ at the points $P$ and $Q$. Find the coordinates of $P$ and of $Q$. [5]

$x - y = 3$

$x = y + 3 \quad \cdots (1)$

sub (1) into $x^2 - 3xy + y^2 + 19 = 0$

$(y+3)^2 - 3(y+3)y + y^2 + 19 = 0$

$y^2 + 6y + 9 - 3y^2 - 9y + y^2 + 19 = 0$

$-y^2 - 3y + 28 = 0$

$y^2 + 3y - 28 = 0$

$(y-4)(y+7) = 0$

$y = 4$ or $y = -7$

$\therefore x = 4 + 3$ or $x = -7 + 3$

$= 7$ $\qquad = -4$

$\therefore P(7, 4)$ and $Q(-4, -7)$

**2** Solve the equation $2^{x+1}+3(2^{-x})=7$. [5]

use
① $a^{m+n} = a^m \times a^n$
② $a^{-n} = \frac{1}{a^n}$

$(2^x)(2)+\frac{3}{2^x}=7$

Let $u = 2^x$

$2u+\frac{3}{u}=7$

$2u^2+3=7u$

$2u^2-7u+3=0$

$(2u-1)(u-3)=0$

$u=\frac{1}{2}$ or $u=3$

$2^x=2^{-1}$ or $2^x=3$ (bring lg or ln to both sides as lg & ln are found in calculator)

$x=-1$ or $\lg 2^x = \lg 3$

$x \lg 2 = \lg 3$

$\therefore x = \frac{\lg 3}{\lg 2}$

$= 1.58$ (to 3 sf)

$\therefore x = -1$ or $1.58$

3 Express the equation $\log_2 x + \log_4(x-1) = 3$ as a cubic equation in $x$. → higher order = 3 [4]

use change of base $\log_a b = \frac{\log_c a}{\log_c b}$

$$\log_2 x + \frac{\log_2 (x-1)}{\log_2 4} = 3$$

$$\log_2 x + \frac{\log_2 (x-1)}{2} = 3$$ ) ×2 throughout

$$2\log_2 x + \log_2 (x-1) = 6$$

use $\log_a b^r = r\log_a b$

$$\log_2 x^2 + \log_2 (x-1) = 6$$

$$\log_2 x^2(x-1) = 6$$

use $\log_c a + \log_c b = \log_c ab$

$$\log_2 x^2(x-1) = 6\log_2 2$$

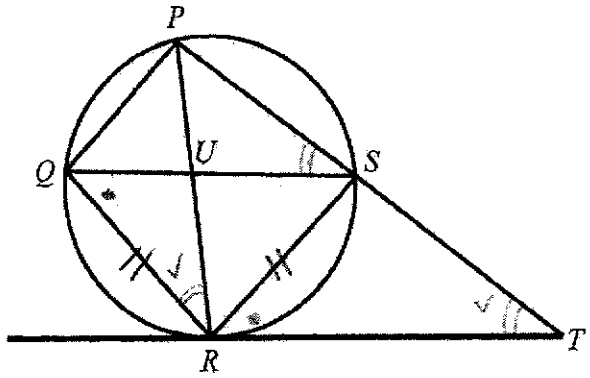
$$\log_2 x^2(x-1) = \log_2 2^6$$

use $\log_a M = \log_a N$ $\therefore M = N$

$$\therefore x^2(x-1) = 2^6$$

$$x^3 - x^2 - 64 = 0$$

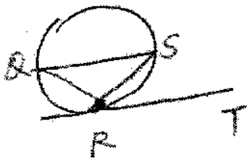
4




4 vertices of quad. on circle

The diagonals of a cyclic quadrilateral $PQRS$ intersect at $U$. The tangent to the circle at $R$ meets $PS$ produced at $T$. If $QR = RS$, prove that

**(a)** $QS$ is parallel to $RT$, [3]




$\angle SRT = \angle RQS$ (Alternate Segment Theorem)

Given $QR = RS$,
$\therefore \angle RQS = \angle QSR$ (base $\angle$s of isos. $\triangle$)

$\therefore \angle QSR = \angle SRT$

By converse of alternate angles of parallel lines, $QS$ is parallel to $RT$. (Shown)

**(b)** triangles $QUR$ and $RST$ are similar. [3]

$\angle QRP = \angle QSP$ ($\angle$s in same segments)

Since $QS$ is parallel to $RT$
$\therefore \angle QSP = \angle RTS$ (corr. $\angle$s, // lines)

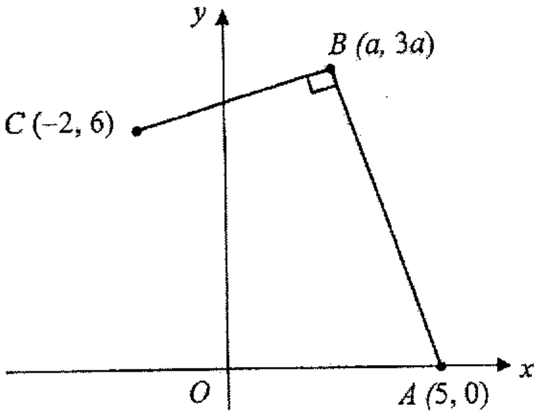
Hence $\angle QRP = \angle RTS$

Also $\angle RQU = \angle TRS$ (Alternate Segment Theorem)

Hence, by Angle-Angle Similarity Test, triangles $QUR$ and $RST$ are similar.

**TURN OVER FOR QUESTION 5**

5




The diagram shows points $A(5, 0)$, $B(a, 3a)$ and $C(-2, 6)$ such that the line $AB$ is perpendicular to the line $BC$.

**(a)** Show that $a = 2.5$. [3]

$$m_{BC} \times m_{BA} = -1$$

$$\left(\frac{3a-6}{a+2}\right) \times \left(\frac{3a-0}{a-5}\right) = -1$$

$$\frac{3a-6}{a+2} \times \frac{3a}{a-5} = -1$$

$$\frac{3a(3a-6)}{(a+2)(a-5)} = -1$$

$$9a^2 - 18a = -(a^2 - 3a - 10)$$

$$9a^2 - 18a = -a^2 + 3a + 10$$

$$10a^2 - 21a - 10 = 0$$

$$(5a+2)(2a-5) = 0$$

$$a = -\frac{2}{5} \text{ or } a = \frac{5}{2} \text{ (shown)}$$

(rejected)

5 **(b)** Find the coordinates of the midpoint of $AC$. Hence find the coordinates of $D$ such that $ABCD$ is a parallelogram. [3]

Midpoint of $AC = \left(\frac{5-2}{2}, \frac{6+6}{2}\right)$

$= \left(\frac{3}{2}, 3\right)$

$\left(\frac{3}{2}, 3\right) = \left(\frac{2.5+x}{2}, \frac{7.5+y}{2}\right)$

Compare coordinates

$\frac{3}{2} = \frac{2.5+x}{2}$

$\therefore x = 0.5$

$3 = \frac{7.5+y}{2}$

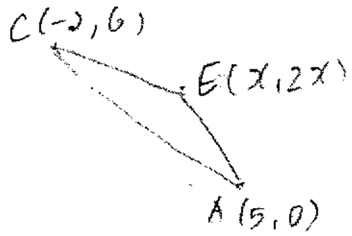
$6 = 7.5+y$

$y = -1.5$

$\therefore D(0.5, -1.5)$

**(c)** The area of triangle $AEC$ is 5 units$^2$. Find the value of $x$ given that the point $E$ is $(x, 2x)$, where $x > 1$. [2]

Anti-clockwise




$\frac{1}{2}\begin{vmatrix} 5 & x & -2 & 5 \\ 0 & 2x & 6 & 0 \end{vmatrix} = 5$

$\left|(5\times 2x + 6\times x + 0) - (5\times 6 + (-2\times 2x) + 0)\right| = 10$

$16x + 4x - 30 = 10$

$20x = 40$

$x = 2$

6 (a) Express $\dfrac{13x-6}{x^2(2x-3)}$ in partial fractions. [5]

Let $\dfrac{13x-6}{x^2(2x-3)} = \dfrac{A}{x} + \dfrac{B}{x^2} + \dfrac{C}{2x-3}$

$13x-6 = Ax(2x-3) + B(2x-3) + Cx^2$

Sub $x=0$, $-6 = -3B$

$B = 2$

Sub $x \frac{3}{2}$, $\frac{39}{2} - 6 = \frac{9}{4}C$

$\frac{27}{2} = \frac{9}{4}C$

$C = 6$

Compare coefficient of $x^2$,

$0 = 2A + C$

$0 = 2A + 6$

$2A = -6$

$A = -3$

$\therefore \dfrac{13x-6}{x^2(2x-3)} = \dfrac{-3}{x} + \dfrac{2}{x^2} + \dfrac{6}{2x-3}$

6 (b) Hence $\int \frac{13x-6}{x^2(2x-3)}\,dx$.

$\int x^{-1}\,dx = \frac{x^{-1+1}}{0}$ ← undefined

[3]

$$\int \frac{13x-6}{x^2(2x-3)}dx = \int -\frac{3}{x}\,dx + \int \frac{2}{x^2}\,dx + \int \frac{6}{2x-3}\,dx$$

$$= -3\left[\ln x\right] + 2\left[\frac{x^{-1}}{(-1)}\right] + 6\,\frac{\ln(2x-3)}{2} + c$$

$$= -3\ln x - \frac{2}{x} + 3\ln(2x-3) + C$$

use $\int x^n\,dx = \frac{x^{n+1}}{n+1} + c$

$\int \frac{1}{x}\,dx = \ln x + c$

$\int \frac{1}{ax+b}\,dx = \frac{\ln(ax+b)}{a} + c$

7 **(a)** **(i)** Write down and simplify, in ascending powers of $x$, the first four terms of the expansion of $(1-x)^{12}$. [2]

$(1-x)^{12} = 1 + \binom{12}{1}(-x)^1 + \binom{12}{2}(-x)^2 + \binom{12}{3}(-x)^3 + \cdots$

$= 1 - 12x + 66x^2 - 220x^3 + \cdots$

**(ii)** Hence find the value of $p$ given that the coefficient of $x^3$ in the expansion of $(2x^2+17x+p)(1-x)^{12}$ is 6598. [3]

$(2x^2+17x+p)(1-12x+66x^2-220x^3+\cdots)$

Term in $x^3$ $= 2x^2 \times (12x) + 17x(66x^2) + p(-220x^3)$

$= 24x^3 + 1122x^3 - 220px^3$

$= (1098 - 220p)x^3$

Compare coefficient of $x^3$, $1098 - 220p = 6598$

$-220p = 5500$

$p = -25$

7 **(b)** In the binomial expansion of $\left(x+\frac{k}{x}\right)^5$, where $k$ is a positive constant, the coefficients of $x^3$ and $x$ are the same. Find the value of $k$. [4]

$$\text{General Term} = \binom{5}{r}(x)^{5-r}\left(\frac{k}{x}\right)^r$$
$$= \binom{5}{r}(k)^r x^{5-2r}$$

Let $x^{5-2r} = x^3$

$5-2r = 3$

$2r = 2$

$r = 1$

$\therefore$ Term in $x^3 = \binom{5}{1}(k)^1(x)^3$

$= 5kx^3$ ✓

Let $x^{5-2r} = x^1$

$5-2r = 1$

$2r = 4$

$r = 2$

Term in $x = \binom{5}{2}(k)^2(x)$

$= 10k^2x$ ✓

Same coefficient

$5k = 10k^2$

$5k - 10k^2 = 0$

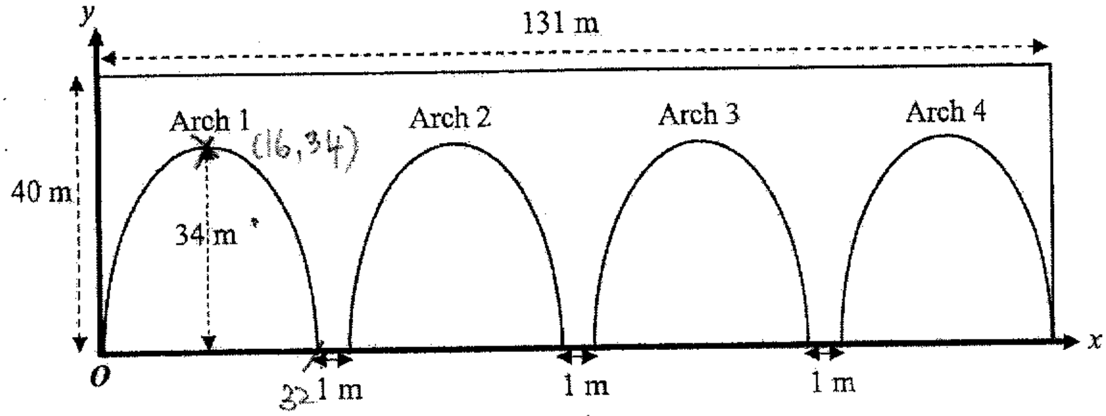
$5k(1-2k) = 0$

$5k = 0$ or $1-2k = 0$

rejected $\quad k = \frac{1}{2}$

(given k is positive constant)

**8**




A civil engineer is designing a bridge which is 131 metres long, 40 metres high and is to have four identical parabolic arches along its length. Each arch is 34 metres high and there is one metre between bases of each adjacent pair of arches as shown in the diagram. A set of axes is placed with the origin at the left- hand end of the base of the first arch.

**(a)** Find the $x$-intercepts of Arch 1. [2]

$$\frac{131-(3\times 1)}{4} = 32$$

$x$ intercepts $= 0$ and $32$

**(b)** The equation representing Arch 1 can be written in the form $y = a(x-p)(x-q)$. Show that $a = -\dfrac{17}{128}$. [2]

$y = a(x-0)(x-32)$

$y = ax(x-32)$

Turning point of equation for Arch 1 is $\left(\frac{32}{2}, 34\right)$
$= (16, 34)$

Sub $x = 16$, $y = 34$

$34 = a(16)(16-32)$

$34 = -256a$

$\therefore a = \dfrac{34}{-256} = -\dfrac{17}{128}$ (shown)

8 (c) Explain, with workings, if the point (50, 30) ~~passes through~~ lies on Arch 2. [3]

x-intercepts of Arch 2 = 33, (33 + 32)
= 33 and 65

$y = -\frac{17}{128}(x-33)(x-65)$

sub $x = 50$

$y = -\frac{17}{128}(50-33)(50-65)$

$= 33.8671$

$\neq 30$

Since (50, 30) does not satisfy the equation representing Arch 2, the point is not lying on the Arch 2.

**9** **(a)** Differentiate $x\sin x+\cos x$ with respect to $x$. [2]

$$\frac{d}{dx}(x\sin x + \cos x)$$

product rule

$u\frac{dv}{dx}+v\frac{du}{dx} \longrightarrow$

$$= \left[1\cdot\sin x + x\cdot\cos x\right] - \sin x$$

$$= x\cos x$$

**(b)** Show that $\frac{\mathrm{d}}{\mathrm{d}x}\left(\frac{1}{3}x\sin^3 x\right)=\frac{1}{3}\sin^3 x - x\cos^3 x + x\cos x$. [3]

$$\frac{d}{dx}\left(\frac{1}{3}x\sin^3 x\right) = \frac{1}{3}\sin^3 x + \frac{1}{3}x\left[3\sin^2 x\cdot\cos x\right]$$

$$= \frac{1}{3}\sin^3 x + x\sin^2 x\cos x$$

$$= \frac{1}{3}\sin^3 x + x(1-\cos^2 x)\cos x$$

$$= \frac{1}{3}\sin^3 x + x\cos x - x\cos^3 x$$

$$= \frac{1}{3}\sin^3 x - x\cos^3 x + x\cos x$$

9 (c) Using the results found in **part (a)** and **part (b)**, find $\int\left(\sin^3 x-3x\cos^3 x\right)\mathrm{d}x$. [4]

use (b) $\Rightarrow \int \frac{1}{3}\sin^3 x - x\cos^3 x + x\cos x \, dx = \frac{1}{3}x\sin^3 x + C_1$

$\int \frac{1}{3}\sin^3 x - x\cos^3 x \, dx + \int x\cos x \, dx = \frac{1}{3}x\sin^3 x + C_1$

$\int \frac{1}{3}\sin^3 x - x\cos^3 x \, dx = \frac{1}{3}x\sin^3 x + C_1 - \int x\cos x \, dx$ use (a)

$= \frac{1}{3}x\sin^3 x + C_1 - [x\sin x + \cos x + C_2]$

$\int \frac{1}{3}\sin^3 x - x\cos^3 x \, dx = \frac{1}{3}x\sin^3 x - x\sin x - \cos x + C_3$

factorise $\frac{1}{3}$ $\frac{1}{3}\int \sin^3 x - 3x\cos^3 x \, dx = \frac{1}{3}x\sin^3 x - x\sin x - \cos x + C_3$

$\times 3$ $\therefore \int \sin^3 x - 3x\cos^3 x \, dx = x\sin^3 x - 3x\sin x - 3\cos x + C$

**10** **(a)** Prove the identity $\dfrac{\cos x}{\operatorname{cosec} x - 1} + \dfrac{\cos x}{\operatorname{cosec} x + 1} = 2\tan x$. [5]

$$LHS = \frac{\cos x(\operatorname{cosec} x + 1) + \cos x(\operatorname{cosec} x - 1)}{\operatorname{cosec}^2 x - 1}$$

$*\ \operatorname{cosec}^2 x = 1 + \cot^2 x$

$$= \frac{\cos x\left[\left(\frac{1}{\sin x} + 1\right) + \left(\frac{1}{\sin x} - 1\right)\right]}{\cot^2 x}$$

$*\ \operatorname{cosec} x = \frac{1}{\sin x}$

$$= \frac{\cos x\left[\frac{2}{\sin x}\right]}{\cot^2 x}$$

$*\ \frac{\cos x}{\sin x} = \cot x$

$$= \frac{2\cot x}{\cot^2 x}$$

$*\ \frac{1}{\cot x} = \tan x$

$$= \frac{2}{\cot x}$$

$= 2\tan x$

$= RHS$ (proven)

**10** **(b)** Solve the equation $\tan x + 2\sec^2 x = 5$ for $0 \le x \le 2\pi$. [5]

$\tan x + 2(1+\tan^2 x) = 5$

$\tan x + 2 + 2\tan^2 x = 5$

$2\tan^2 x + \tan x - 3 = 0$

$(2\tan x + 3)(\tan x - 1) = 0$

or

Let $u = \tan x$

Q2, Q4 $\tan x = -\frac{3}{2}$ or $\tan x = 1$ Q1, Q4

basic $\angle = \tan^{-1}\left(\frac{3}{2}\right)$

$= 0.9827$

$\therefore x = \pi - 0.9287,\ 2\pi - 0.987$

$= 2.16$ or $5.30$ (to 3sf)

or

$\tan x = 1$

$x = \frac{\pi}{4},\ \pi + \frac{\pi}{4}$

$= \frac{\pi}{4},\ \frac{5\pi}{4}$

$\therefore x = \frac{\pi}{4},\ 2.16,\ \frac{5\pi}{6},\ 5.30$ radians

11 (a) (i) Find the range of values of $x$ for which $\ln(x^2-3)$ is defined. Leave your answer in surd form. [2]

$x^2-3>0$ above $x$-axis

$x^2-a^2$
$=(x+a)(x-a)$
$\sqrt{a^2}$

$(x+\sqrt{3})(x-\sqrt{3})>0$


$x<-\sqrt{3}$ or $x>\sqrt{3}$ //

$\therefore x^2-a$
$=(x+\sqrt{a})(x-\sqrt{a})$

$\boxed{x^2>3 \quad x>\pm\sqrt{3}}$ ✗

(ii) Given that $y=\ln\left[e^x(x^2-3)\right]$, show that $\frac{dy}{dx}$ can be expressed in the form of $\frac{(x+a)(x+b)}{x^2-3}$. [3]

$$\frac{dy}{dx}=\frac{1}{e^x(x^2-3)}\cdot\left[e^x\cdot 2x+e^x(x^2-3)\right]$$

$$=\frac{e^x\left[x^2+2x-3\right]}{e^x(x^2-3)}$$

$$=\frac{(x+3)(x-1)}{(x^2-3)} //$$

$\frac{d(\ln x)}{dx}=\ln x$

$\frac{d}{dx}(e^x)=e^x$

$\frac{d}{dx}(\ln f(x))=\frac{f'(x)}{f(x)}$

**11** **(b)** It is given that $y = x^3 + px^2 + qx + 10$ where $p$ and $q$ are integers. The only values of $x$ for which $y$ is a decreasing function of $x$ are those values for which $3 < x < 7$. Find the value of $p$ and of $q$. $\frac{dy}{dx} < 0$ [4]

$3 < x < 7$

$(x-3)(x-7) < 0$

$x^2 - 10x + 21 < 0 \quad \cdots\cdots (1)$

$y = x^3 + px^2 + qx + 10$

$\frac{dy}{dx} = 3x^2 + 2px + q$

$\frac{dy}{dx} < 0$ since $y$ is decreasing

$\therefore 3x^2 + 2px + q < 0$

$x^2 + \frac{2}{3}px + \frac{q}{3} < 0 \quad \cdots (2)$

Compare coefficients of $x$ & constants in (1) & (2)

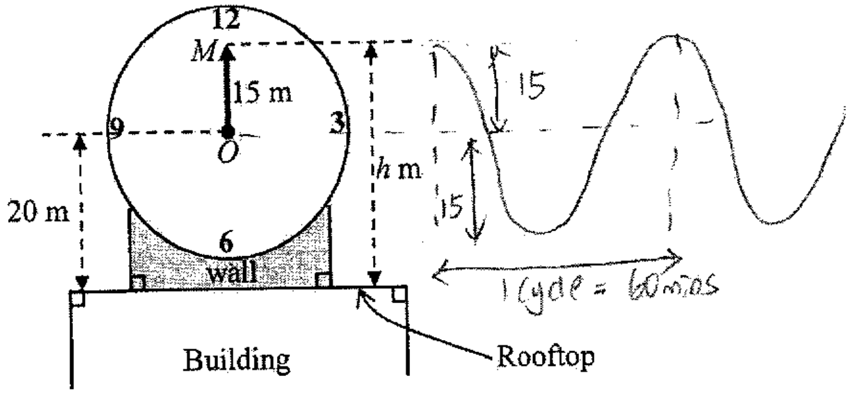
$\frac{2}{3}p = -10$

$p = -15$

$\frac{q}{3} = 21$

$q = 63$

12




A clock is set on the vertical wall on the roof of a building.
The distance from the centre of the clock, $O$, to the tip, $M$, of the minute hand is 15 m.
The height, $h$ m, of $M$ above the rooftop is given by $h = a\cos kt + b$, where $t$ is the time in minutes past the hour. $O$ is 20 m above the rooftop. The rooftop is taken to be horizontal.

↳ start from digit 12 & move towards digit 1, 2, 3, ...

**(a)** Write down the value of $a$ and explain why $b = 20$. [2]

$a = 15$

Maximum height, $h$, of $M$ = 35 m above the rooftop.
Minimum height of $M$ = 5 m

$b$ = average of the maximum and minimum values of $h$

$= \frac{35+5}{2} = 20\text{m}$ //

**(b)** Explain why the value of $k$ is $\frac{\pi}{30}$. [1]

The minute hand takes 60 minutes to complete 1 cycle, before return back to digit 12.

Period $= \frac{2\pi}{k} = 60$

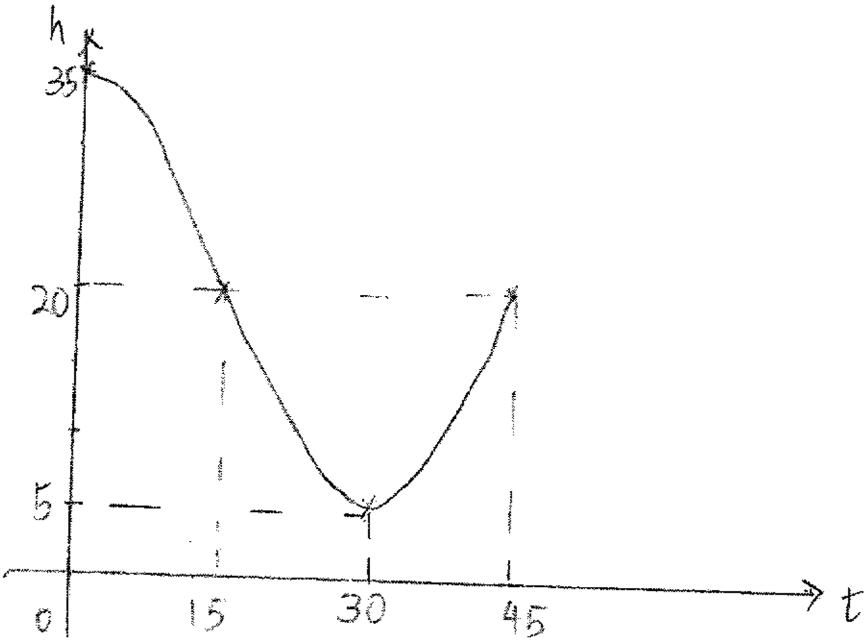
$\therefore k = \frac{2\pi}{60}$

$= \frac{\pi}{30}$ //

12 (c) Sketch the graph of $h$ for $0 \le t \le 45$. [3]

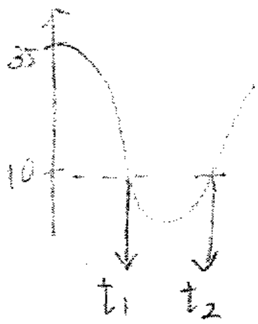
$y = 15\cos\frac{\pi}{30}t + 20$

$60 \div 4$
$= 15$




(d) Find the two timings in the first hour for which the height of $M$ is 10 m. [4]

$\rightarrow$ 1st 60 mins




$15\cos\frac{\pi}{30}t + 20 = 10$ $\quad 0 < t < 60$

$\cos\frac{\pi}{30}t = -\frac{10}{15} \rightarrow$ Q2, Q3 $\quad 0 < \frac{\pi}{30}t < 2\pi$

basic $\angle = \cos^{-1}\left(\frac{2}{3}\right)$

$= 0.84106$

$\frac{\pi}{30}t = \pi - 0.84106, \ \pi + 0.84106$

$t = \frac{2.300532}{\left(\frac{\pi}{30}\right)}, \ \frac{3.98265}{\left(\frac{\pi}{30}\right)}$

$= 21.9684, \ 38.03153$

$t \approx 22.0$ mins or $38.0$ mins

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# Anglo-Chinese School (Barker Road)

**PRELIMINARY EXAMINATION 2024**
**SECONDARY FOUR EXPRESS / FIVE NORMAL (ACADEMIC)**

**ADDITIONAL MATHEMATICS**
**4049**
**PAPER 2**

**2 HOURS 15 MINUTES**

Candidates answer on the Question Paper.

**READ THESE INSTRUCTIONS FIRST**

Write your index number and name in the spaces at the top of this page.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.

Answer **all** questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
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The number of marks is given in brackets [ ] at the end of each question or part question.

The total number of marks for this paper is 90.

| For Examiner's Use |
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| |

### *Mathematical Formulae*

## 1. ALGEBRA

*Quadratic Equation*

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

*Binomial expansion*

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n,$$

where $n$ is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

## 2. TRIGONOMETRY

*Identities*

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2\sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

*Formulae for $\Delta ABC$*

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

$$\Delta = \frac{1}{2}bc\sin A$$

1 A particle moves along the curve $y = \dfrac{3(x+6)}{x+4}$, where $x \neq -4$, in such a way that the $y$-coordinate of the particle is increasing at a constant rate of $\dfrac{4}{27}$ units per second. Find the $x$-coordinates of the particle at the instant that the $x$-coordinate of the particle is decreasing at a rate of 2 units per second. [5]

2 The number, $v$, of a certain virus present in a sample collected by a vaccine laboratory is given by $v = me^{2t} + n$, where $m$ and $n$ are constants and $t$ is measured in days. Initially, the number of virus present was 2000. It increased to 5000 after 1 day.

**(a)** Find the value of $m$ and of $n$. [4]

**(b)** Find the number of days in which the number of virus present first reach 1 million. [2]

3 **(a)** Show that the roots of the equation $ax^2 + (3a+b)x + 3b = 0$ are real for all real values of $a$ and $b$. [3]

**(b)** Find the range of values of $m$ for which the line $y = mx - 3$ will never cut the curve $y^2 = 4x - 6y - 34$. [4]

4 A tangent to a circle at the point (6, 10) passes through the point (9, 6). The centre of the circle lies on the line $3y = 4x + 13$.
Showing all your working, find the equation of the circle. [7]

**5** **Do not use a calculator in this question.**

**(a)** Express $\sin 22.5°$ in the form of $\dfrac{\sqrt{a-\sqrt{a}}}{a}$, where $a$ is an integer. [3]

**(b)** Show that $\tan 15° = 2-\sqrt{3}$. [5]

6 **(a)** **(i)** Using the substitution $u = x^3$ or otherwise, express $x^6 - 1$ as the product of two factors. [1]

**(ii)** Hence express $x^6 - 1$ as the product of four factors with integer coefficients. [1]

6 **(b)** **(i)** Find the remainder when $f(x)=3x^3-5x^2+7x-4$ is divided by $x-1$. [1]

**(ii)** Hence show that $h=-1$ for which $g(x)=f(x)+h$ is divisible by $x-1$. [2]

**(iii)** Explain why the equation $g(x)=0$ has only one real root. [4]

7 **(a)** Two variables $x$ and $y$ are related by the equation $xy^2 = ax + by$. Explain how a straight line graph can be drawn to represent the given equation. [2]

**(b)** The table shows experimental values of two variables $x$ and $y$. It is known that $x$ and $y$ are related by the equation $y = pe^{-qx}$ where $p$ and $q$ are constants.

| $x$ | 1 | 3 | 5 | 7 | 9 |
|---|---|---|---|---|---|
| $y$ | 98.2 | 65.9 | 44.1 | 29.6 | 19.8 |

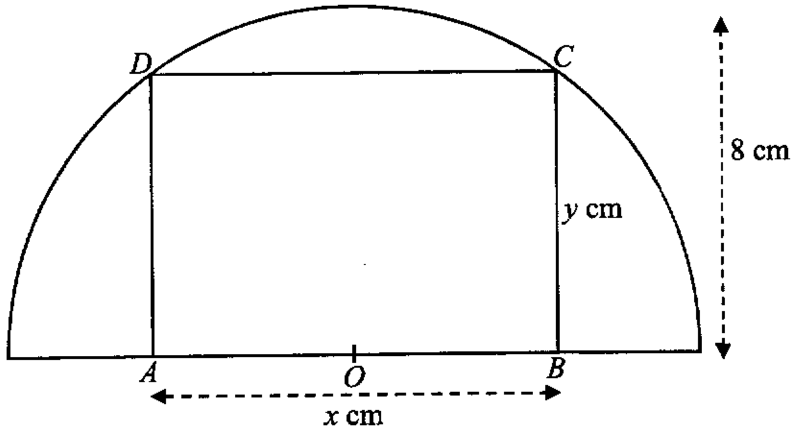
**(i)** On the grid below plot $\ln y$ against $x$ and draw a straight line graph. [2]

7 **(b)** **(ii)** Use your graph to estimate

**(a)** the value of $p$ and of $q$, [3]

**(b)** the value of $y$ when $x = 2$. [1]

**8** *ABCD* is a rectangle which fits inside a semicircle of radius 8 cm and centre *O*. It is given that $AB = x$ cm and $BC = y$ cm.




**(a)** Show that that $A$ cm$^2$, the area of the rectangle, is given by $A = \frac{x}{2}\sqrt{256 - x^2}$. [2]

8 **(b)** Given that $x$ can vary, find the value of $x$ which gives a stationary value of $A$. [4]

**(c)** By considering the sign of $\frac{\mathrm{d}A}{\mathrm{d}x}$, determine whether the stationary value of $A$ is maximum or minimum. [2]

9 A particle starts from rest from a point $O$ and moves in a straight line such that its velocity $v$ m/s, is given by $v = 24t - 6t^2$, where $t$ is the time in seconds after the start of its motion.

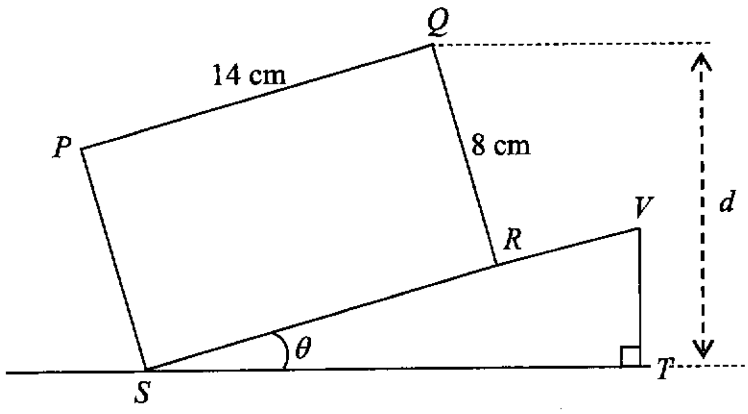
**(a)** Find the value of $t$ at which the particle is instantaneously at rest. [2]

**(b)** When will the particle return to its starting point? [3]

**9** **(c)** Determine if the particle is accelerating after 2 seconds. Explain your answer with clear workings. [3]

**(d)** Calculate the total distance travelled during the first 7 seconds. [4]

**10**



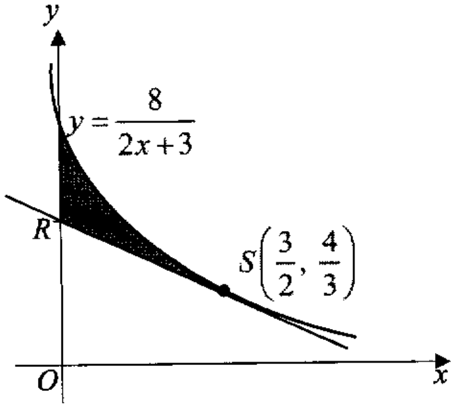

The diagram shows the side view of a 14 cm by 8 cm rectangular block *PQRS*, placed on a ramp, *VS*, tilted at an acute angle of $\theta°$.
The ramp is placed on a horizontal surface *ST* and *d* is the perpendicular distance from *Q* to *ST*. $\angle VTS = 90°$.

**(a)** Show that $d = 8\cos\theta + 14\sin\theta$. [2]

**(b)** Express *d* in the form $R\cos(\theta - \alpha)$, where $R > 0$ and $0° < \alpha < 90°$. [4]

**10** **(c)** Find the smallest value of $\theta$ such that $d = 10\sqrt{2}$. [4]

11 The diagram shows part of the curve $y = \dfrac{8}{2x+3}$. The tangent to the curve at the point $S\left(\dfrac{3}{2}, \dfrac{4}{3}\right)$ intersects the $y$-axis at $R$.




**(a)** Find the $y$-coordinate of $R$. [4]

**11** **(b)** Find the exact area of the shaded region. Express your answer in the form of $\left(\ln a - \frac{b}{c}\right)$ units$^2$, where $a$, $b$ and $c$ are integers. [6]

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| Answers | | |

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Given $\frac{dy}{dt} = \frac{4}{27}$ units/s , $\frac{dx}{dt} = -2$ units/s

$$y = \frac{3x+18}{x+4}$$

$$\frac{dy}{dx} = \frac{(x+4)(3) - (3x+18)\cdot 1}{(x+4)^2}$$

$$= \frac{3x+12-3x-18}{(x+4)^2}$$

$$= \frac{-6}{(x+4)^2}$$

$$\frac{dy}{dt} = \frac{dy}{dx} \times \frac{dx}{dt}$$

$$\frac{4}{27} = -\frac{6}{(x+4)^2} \times -2$$

$$\frac{4}{27} = \frac{12}{(x+4)^2}$$

$$(x+4)^2 = 81$$

$$x+4 = \pm 9$$

$$x = 5 \text{ or } -13$$

2 The number, $v$, of a certain virus present in a sample collected by a vaccine laboratory is given by $v = me^{2t} + n$, where $m$ and $n$ are constants and $t$ is measured in days. Initially, the number of virus present was 2000. It increased to 5000 after 1 day.

**(a)** Find the value of $m$ and of $n$. [4]

$v = me^{2t} + n$

$t = 0,\ v = 2000$

$2000 = me^{0} + n$

$m + n = 2000 \quad \cdots (1)$

$t = 1,\ v = 5000$

$5000 = me^{2} + n \quad \cdots (2)$

$(2) - (1),\quad 3000 = me^{2} - m$

$3000 = m(e^{2} - 1)$

$m = \dfrac{3000}{e^{2} - 1}$

$= 469.5529 \approx 470$ (to 3sf)

$n = 2000 - 469.5529$

$= 1530.4470$

$\approx 1530$ (to 3sf)

**(b)** Find the number of days in which the number of virus present first reach 1 million. [2]

$1 \times 10^{6} = 469.5529\, e^{2t} + 1530.4470$

$e^{2t} = \dfrac{1 \times 10^{6} - 1530.4470}{469.5529}$

$e^{2t} = 2126.426$

$2t \ln e = \ln 2126.426$

$t = \dfrac{\ln 2126.426}{2}$

$= 3.83109$

$\approx 3.83$ days (to 3sf)

$\rightarrow$ show $D \geq 0$

$\rightarrow D \geq 0$

3 (a) Show that the roots of the equation $ax^2 + (3a+b)x + 3b = 0$ are real for all real values of $a$ and $b$. [3]

$ax^2 + (3a+b)x + 3b = 0$

real & distinct roots $D > 0$

Discriminant $= (3a+b)^2 - 4a(3b)$

$= 9a^2 + 6ab + b^2 - 12ab$

$= 9a^2 - 6ab + b^2$

real roots $D \geq 0$

$= (3a-b)^2$

For all real values of $a$ and $b$, $(3a-b)^2 \geq 0$.

Since discriminant $\geq 0$, hence roots of $ax^2 + (3a+b)x + 3b = 0$ are real

(b) Find the range of values of $m$ for which the line $y = mx - 3$ will never cut the curve $y^2 = 4x - 6y - 34$. [4]

$\Rightarrow D < 0$

$(mx-3)^2 = 4x - 6(mx-3) - 34$

$m^2x^2 - 6mx + 9 = 4x - 6mx + 18 - 34$

$m^2x^2 - 4x + 25 = 0$

$b^2 - 4ac < 0$

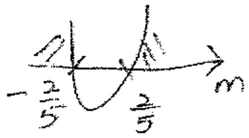
$(-4)^2 - 4m^2(25) < 0$

$16 - 100m^2 < 0$

$100m^2 - 16 > 0$

$4(25m^2 - 4) > 0$

$(5m-2)(5m+2) > 0$

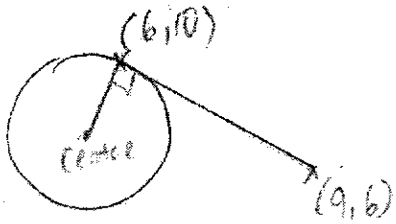


$m < -\frac{2}{5}$ or $m > \frac{2}{5}$

4 A tangent to a circle at the point (6, 10) passes through the point (9, 6). The centre of the circle lies on the line $3y = 4x + 13$.
Showing all your working, find the equation of the circle. [7]

gradient of tangent $= \frac{6-10}{9-6}$

$= -\frac{4}{3}$




gradient of radius $= \frac{3}{4}$

$(6, 10)$, $m = \frac{3}{4}$

$y = \frac{3}{4}x + C$

$10 = \frac{3}{4}(6) + C$

$C = \frac{26}{5}$

Equation of radius: $y = \frac{3}{4}x + \frac{11}{2}$ $\quad \dots (1)$

$3y = 4x + 13$ $\quad \dots (2)$

sub (1) into (2)

$3\left(\frac{3}{4}x + \frac{11}{2}\right) = 4x + 13$

$\frac{9}{4}x + \frac{33}{2} = 4x + 13$

$-\frac{7}{4}x = -\frac{7}{2}$

$\therefore x = 2$

$y = \frac{3}{4}(2) + \frac{11}{2}$

$= 7$

centre $(2, 7)$

radius $= \sqrt{(6-2)^2 + (10-7)^2}$

$= 5$ units

Equation of circle is $(x-2)^2 + (y-7)^2 = 25$ //

$\sin 2A = 2\sin A\cos A$

**5** **Do not use a calculator in this question.**

**(a)** Express $\sin 22.5°$ in the form of $\frac{\sqrt{a-\sqrt{a}}}{a}$, where $a$ is an integer. [3]

$\cos 2A = 1 - 2\sin^2 A$

$\cos 45° = 1 - 2\sin^2 22.5°$

$\frac{\sqrt{2}}{2} = 1 - 2\sin^2 22.5°$

$\frac{\sqrt{2}}{2} - 1 = -2\sin^2 22.5°$

$\frac{\sqrt{2}-2}{2} = -2\sin^2 22.5°$

$\sin^2 22.5° = \frac{2-\sqrt{2}}{4}$

$\sin 22.5° = \pm\frac{\sqrt{2-\sqrt{2}}}{\sqrt{4}}$

Since $\sin 22.5° > 0$,

$\therefore \sin 22.5 = \frac{\sqrt{2-\sqrt{2}}}{2}$ (shown)

($-\frac{\sqrt{2-\sqrt{2}}}{2}$ is rejected.)

**(b)** Show that $\tan 15° = 2-\sqrt{3}$. [5]

$\tan 15° = \tan(60° - 45°)$

$= \frac{\tan 60° - \tan 45°}{1 + \tan 60° \tan 45°}$

$= \frac{\sqrt{3}-1}{1+\sqrt{3}} \times \frac{1-\sqrt{3}}{1-\sqrt{3}}$

$= \frac{\sqrt{3}-3-1+\sqrt{3}}{1^2 - \sqrt{3}^2}$

$= \frac{-4+2\sqrt{3}}{-2}$

$= 2-\sqrt{3}$ (shown)

not! $\tan 15° \neq \tan 65° - \tan 15°$ wrong!

$\tan(45° - 30°)$

$\frac{1}{\sqrt{3}}$

$\tan 15° = \tan 45° - \tan 30°$ X

to help, not give in $u$! final answer in terms of $x$!

6 (a) (i) Using the substitution ~~$u = x^3$~~ or otherwise, express $x^6 - 1$ as the product of two factors. [1]

$$x^6 - 1 = (x^3)^2 - 1$$
$$= (x^3+1)(x^3-1)$$

or $u^2 - 1$
$= (u+1)(u-1)$
$= (x^3+1)(x^3-1)$

Do not leave answer in terms of $u$.

(ii) Hence express $x^6 - 1$ as the product of four factors with integer coefficients. [1]

$$x^3 + 1 = (x+1)(x^2 - x + 1) \leftarrow$$
$$x^3 - 1 = (x-1)(x^2 + x + 1) \leftarrow$$

$$\boxed{\begin{matrix} a^3 + b^3 \\ a^3 - b^3 \end{matrix}}$$

$$\therefore (x^3+1)(x^3-1) = (x+1)(x^2-x+1)(x-1)(x^2+x+1)$$

6 (b) (i) Find the remainder when $f(x) = 3x^3 - 5x^2 + 7x - 4$ is divided by $x - 1$. [1]

$f(1) = 3(1)^3 - 5(1)^2 + 7(1) - 4$

$= 1$

Remainder $= 1$

(ii) Hence show that $h = -1$ for which $g(x) = f(x) + h$ is divisible by $x - 1$. [2]

$g(x)$ is divisible by $x-1 \Rightarrow g(1) = 0$

$g(1) = 0$

$f(1) + h = 0$

$1 + h = 0$

from (b)(i) $f(1) = 1$

$h = -1$ (shown)

(iii) Explain why the equation $g(x) = 0$ has only one real root. [4]

$g(x) = f(x) + h = 3x^3 - 5x^2 + 7x - 4 + (-1)$

$= 3x^3 - 5x^2 + 7x - 5$

$g(x)$ is divisible by $x-1$, $\therefore x-1$ is a factor of $g(x)$

$$\begin{array}{r|l} & 3x^2 - 2x + 5 \\ x-1 & 3x^3 - 5x^2 + 7x - 5 \\ & -(3x^3 - 3x^2) \\ & -2x^2 + 7x \\ & -(-2x^2 + 2x) \\ & 5x - 5 \\ & 5x - 5 \\ & 0 \end{array}$$

$\therefore (x-1)(3x^2 - 2x + 5) = 0$

$x - 1 = 0$, $\therefore x = 1$

or $3x^2 - 2x + 5 = 0$

discriminant $= (-2)^2 - 4(3)(5)$

$= -56 < 0$

hence $3x^2 - 2x + 5 = 0$ has no real roots.

$\therefore$ when $g(x) = 0$, $x = 1$ is the only real root

$x = \dfrac{-(-2) \pm \sqrt{(-2)^2 - 4(3)(5)}}{2(3)}$

$= \dfrac{2 \pm \sqrt{-56}}{6}$

since $x$ is undefined, $3x^2 - 2x + 5 = 0$ has no real root

$x-1$ is not a root

$x-1$ is a factor.

root $\Rightarrow$ solution $\Rightarrow x = 1$

7 **(a)** Two variables $x$ and $y$ are related by the equation $xy^2 = ax + by$. Explain how a straight line graph can be drawn to represent the given equation. [2]

$xy^2 = ax + by$

$y^2 = a\left(\frac{x}{x}\right) + b\left(\frac{y}{x}\right)$

$y^2 = b\left(\frac{y}{x}\right) + a$ ← no $x$ or $y$ multiply to a!

Plot $y^2$ against $\frac{y}{x}$ to draw the straight line graph

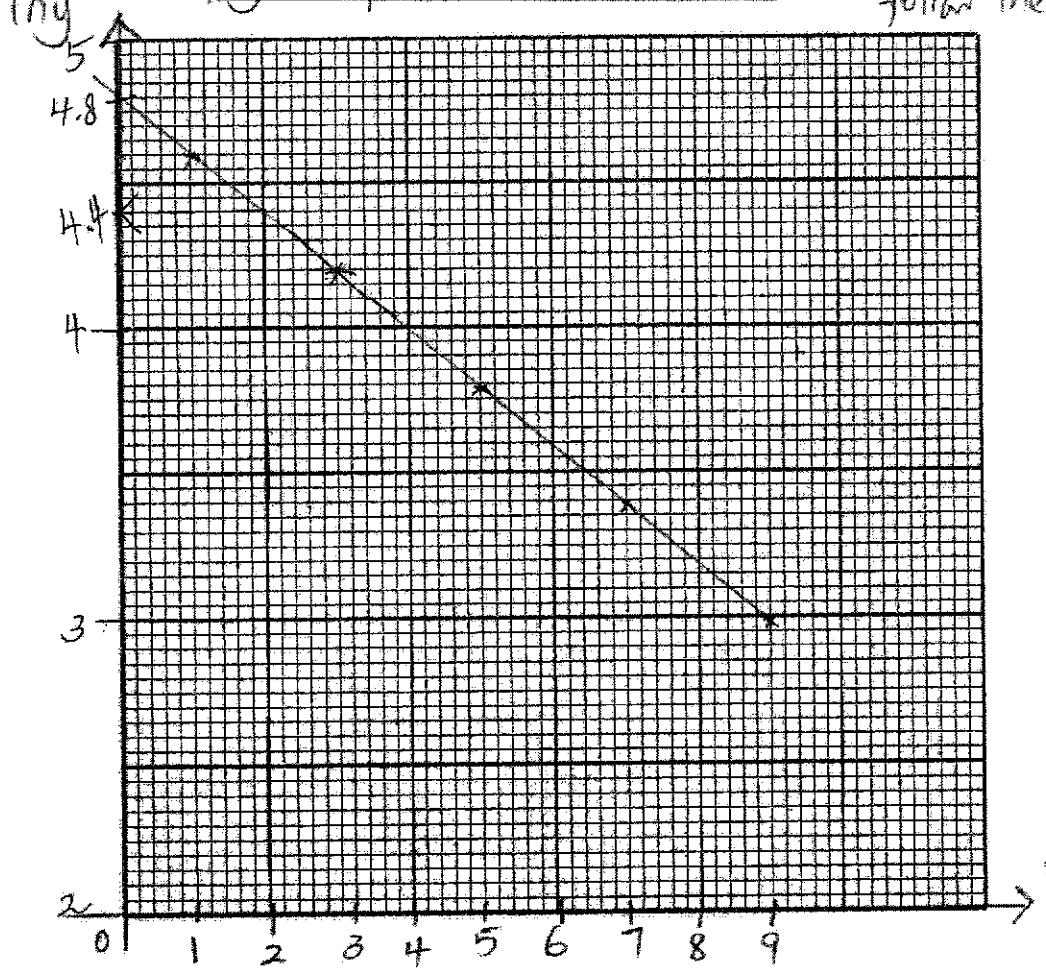
**(b)** The table shows experimental values of two variables $x$ and $y$. It is known that $x$ and $y$ are related by the equation $y = pe^{-qx}$ where $p$ and $q$ are constants.

| $x$ | 1 | 3 | 5 | 7 | 9 |
|---|---|---|---|---|---|
| $y$ | 98.2 | 65.9 | 44.1 | 29.6 | 19.8 |

**(i)** On the grid below plot $\ln y$ against $x$ and draw a straight line graph. [2]

| $x$ | 1 | 3 | 5 | 7 | 9 |
|---|---|---|---|---|---|
| $\ln y$ | 4.59 | 4.19 | 3.79 | 3.39 | 2.99 |

← give to 3 s.f. follow the table.




X-axis must start from 0 to read Y-intercept

**7** **(b)** Use your graph to estimate

**(ii)** the value of $p$ and of $q$, [3]

No sub. X & Y values into equation

$y = pe^{-qx}$

$\ln y = \ln(pe^{-qx})$

$\ln y = \ln p + (-qx)\ln e$

$\ln y = -qx + \ln p$

$-q$ = gradient

using $(2, 4.4)$ and $(8.4, 3.1)$

$-q = \frac{4.4 - 3.1}{2 - 8.4}$

$= -0.203125$

$\therefore q = 0.203125$

or $q = \frac{13}{64}$

$\ln p$ is the vertical intercept

$\therefore \ln p = 4.8$

$p = e^{4.8}$

$\approx 121.5104$

$\approx 122$

$p = 4.8$ X

$\ln p = 4.8$

$p = e^{4.8}$

**(iii)** the value of $y$ when $x = 2$. [1]

No. sub. of X into eqn.

Read from graph

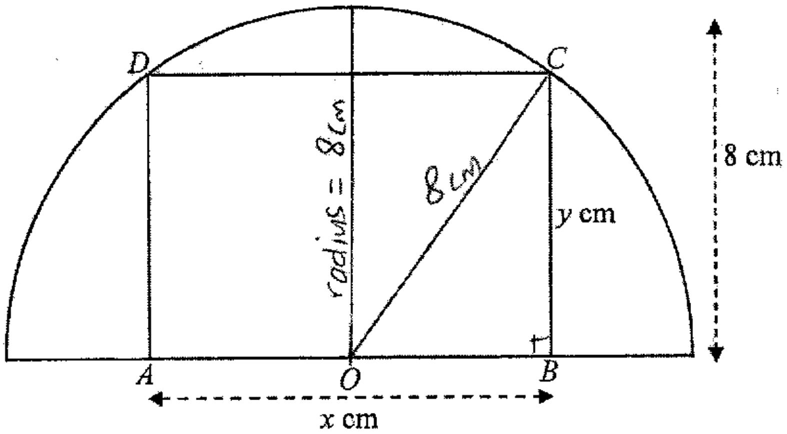
$\ln y = 4.4$

$y = e^{4.4}$

$\approx 81.4508$

$\approx 81.5$

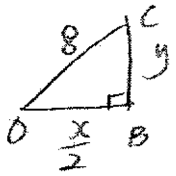
**8** $ABCD$ is a rectangle which fits inside a semicircle of radius 8 cm and centre $O$. It is given that $AB = x$ cm and $BC = y$ cm.




**(a)** Show that that $A$ cm$^2$, the area of the rectangle, is given by $A = \frac{x}{2}\sqrt{256 - x^2}$. [2]

$OB = \frac{x}{2}$

In $\triangle OBC$,




$y^2 + \left(\frac{x}{2}\right)^2 = 8^2$

$y^2 = 64 - \frac{x^2}{4}$

$y^2 = \frac{1}{4}(256 - x^2)$

$y = \frac{1}{2}\sqrt{256 - x^2}$ , $y > 0$,

$\therefore -\frac{1}{2}\sqrt{256^2 - x^2}$ is rejected

Area of rectangle

$A = x \times y$

$= x \times \frac{1}{2}\sqrt{256 - x^2}$

$= \frac{x}{2}\sqrt{256 - x^2}$ (shown)

$\frac{dA}{dx}=0$

**8** **(b)** Given that $x$ can vary, find the value of $x$ which gives a stationary value of $A$. [4]

$$A = \frac{x}{2}(256-x^2)^{\frac{1}{2}}$$

$$\frac{dA}{dx} = \frac{1}{2}(256-x^2)^{\frac{1}{2}} + \frac{x}{2}\left[\frac{1}{2}(256-x^2)^{-\frac{1}{2}}(-2x)\right]$$

$$= \frac{1}{2}\sqrt{(256-x^2)} - \frac{x^2}{2\sqrt{256-x^2}}$$

$$\frac{dA}{dx}=0$$

$$\frac{1}{\cancel{2}}\sqrt{256-x^2} = \frac{x^2}{\cancel{2}\sqrt{256-x^2}}$$

$$\sqrt{256-x^2}\times\sqrt{256-x^2} = x^2$$

$$256-x^2 = x^2$$

$$256 = 2x^2$$

$$x = \sqrt{\frac{256}{2}}, \; x>0$$

$$= 11.3137$$

$$\approx 11.3 \text{ cm} \quad (\text{to } 3 \text{ sf})$$

**(c)** By considering the sign of $\frac{dA}{dx}$, determine whether the stationary value of $A$ is maximum or minimum. → use 1st derivative test. [2]

do not use 11.31

| $x$ | 11.2 | 11.3137 | 11.4 |
|---|---|---|---|
| Value of $\frac{du}{dx}$ | 0.224 | 0 | −0.175 |
| Sketch of tangent | / | — | \ |

$$\frac{dA}{dx} = \frac{1}{2}\sqrt{256-x^2} - \frac{x^2}{2\sqrt{256-x^2}}$$

$$= \frac{256-x^2-x^2}{2\sqrt{256-x^2}}$$

$$= \frac{256-2x^2}{2\sqrt{256-x^2}}$$

$$= \frac{2(128-x^2)}{2\sqrt{256-x^2}}$$

$$= \frac{128-x^2}{\sqrt{256-x^2}}$$

The stationary value of A is maximum.

$V = 0$, $t = 0$ (Starting already at rest)

9 A particle starts from rest from a point $O$ and moves in a straight line such that its velocity $v$ m/s, is given by $v = 24t - 6t^2$, where $t$ is the time in seconds after the start of its motion.

**(a)** Find the value of $t$ at which the particle is instantaneously at rest. [2]

$v = 24t - 6t^2$ $\quad \to v = 0$

$V = 0$

$6t(4 - t) = 0$

$t = 0$ or $t = 4$

rejected

$\therefore t = 4$ when it is instantaneously at rest

**(b)** When will the particle return to its starting point? [3]

$\leftarrow s = 0$

$S = \int v \, dt$

$= \int 24t - 6t^2 \, dt$

$S = 12t^2 - 2t^3 + C$

when $t = 0$, $S = 0$, $C = 0$

$\therefore S = 12t^2 - 2t^3$

$12t^2 - 2t^3 = 0$

$2t^2(6 - t) = 0$

$t^2 = 0$ or $6 - t = 0$

$\therefore$ After 6 second, the particle will return to the starting point.

9 (c) Determine if the particle is accelerating after 2 seconds. Explain your answer with clear workings. [3]

$a = \frac{dv}{dt}$

$= 24 - 12t$

After 2 seconds, $t > 2$

$-12t < -24$

$24 - 12t < 24 - 24$

$24 - 12t < 0$

Since acceleration is negative, hence the particle is not accelerating after 2 seconds

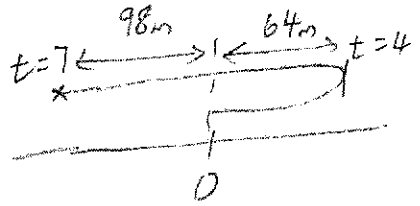
(d) Calculate the total distance travelled during the first 7 seconds. [4]

When $t = 0$, $S = 0$

When $t = 4$, $S = 12(4)^2 - 2(4)^3$

$= 64 m$

When $t = 7$, $S = 12(7)^2 - 2(7)^3$




$= -98 m$

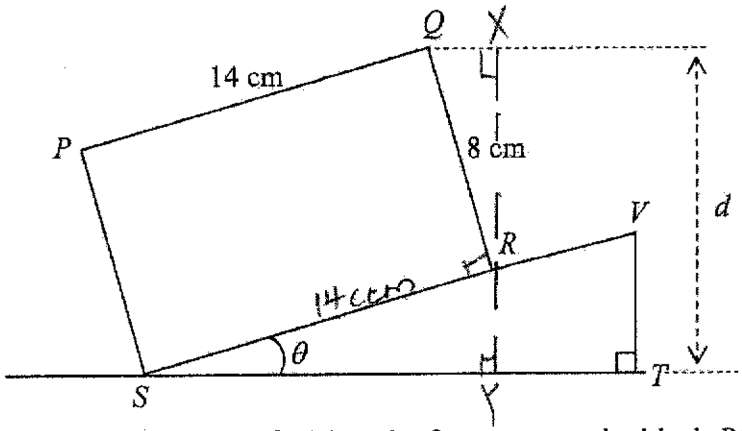
on left side of point O.

Total distance travelled

$= 64 \times 2 + 98$

$= 226 m$

**10**




The diagram shows the side view of a 14 cm by 8 cm rectangular block *PQRS*, placed on a ramp, *VS*, tilted at an acute angle of $\theta°$.
The ramp is placed on a horizontal surface *ST* and *d* is the perpendicular distance from *Q* to *ST*. $\angle VTS = 90°$.

show the trigo ratios

**(a)** Show that $d = 8\cos\theta + 14\sin\theta$. [2]

Let $\angle SYR = \angle QXR = 90°$

In $\triangle SYR$, $\sin\theta = \frac{RY}{14}$ ★

$RY = 14\sin\theta$

$\angle SRY = 180° - \theta - 90° = 90° - \theta$

$\angle QRY = 180° - 90° - (90° - \theta)$

$= \theta$

In $\triangle QXR$, $\cos\theta = \frac{XR}{8}$ ★

$XR = 8\cos\theta$

$\therefore d = XR + RY$

$= 8\cos\theta + 14\sin\theta$ //

**(b)** Express *d* in the form $R\cos(\theta - \alpha)$, where $R > 0$ and $0° < \alpha < 90°$. [4]

Let $8\cos\theta + 14\sin\theta = R\cos(\theta - \alpha)$

$R = \sqrt{8^2 + 14^2}$

$= \sqrt{260}$

$= 2\sqrt{65}$

$\alpha = \tan^{-1}\left(\frac{14}{8}\right)$

$= 60.2551°$

$\therefore d = 2\sqrt{65}\cos(\theta - 60.3°)$ //

**10** **(c)** Find the smallest value of $\theta$ such that $d = 10\sqrt{2}$. [4]

$$2\sqrt{65}\cos(\theta - 60.2551^\circ) = 10\sqrt{2}$$

$$\cos(\theta - 60.2551^\circ) = 0.877058$$

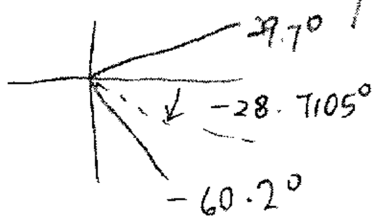
basic $\angle = 28.7105^\circ$ $\quad\quad$ $0 < \theta < 90^\circ$

$-60.2551^\circ < \theta - 60.2551^\circ < 29.7^\circ$

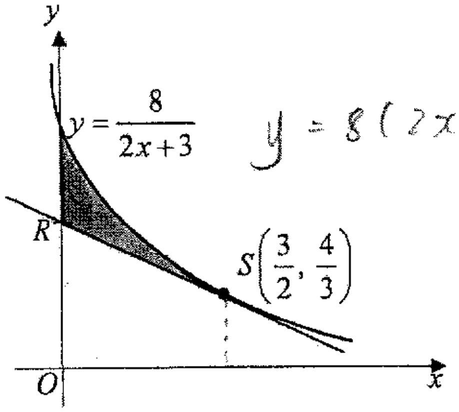
$\therefore \theta - 60.2551^\circ = -28.7105^\circ, 28.7105^\circ$

$\theta = 31.5446^\circ$ or $88.9656^\circ$

smallest value of $\theta = 31.5^\circ$ (to 1dp)



11 The diagram shows part of the curve $y = \frac{8}{2x+3}$. The tangent to the curve at the point $S\left(\frac{3}{2}, \frac{4}{3}\right)$ intersects the $y$-axis at $R$.




$y = 8(2x+3)^{-1}$

**(a)** Find the $y$-coordinate of $R$. [4]

$\frac{dy}{dx} = 8\left[-1(2x+3)^{-2} \cdot 2\right]$

$= \frac{-16}{(2x+3)^2}$

at $x = \frac{3}{2}$, gradient of RS $= \frac{-16}{(2 \times \frac{3}{2} + 3)^2}$

$= -\frac{16}{36} = -\frac{4}{9}$

$y = -\frac{4}{9}x + c$

$\frac{4}{3} = -\frac{4}{9}\left(\frac{3}{2}\right) + c$

$c = 2$

$y$-coordinate of $R = 2$

**11** **(b)** Find the exact area of the shaded region. Express your answer in the form of $\left(\ln a - \frac{b}{c}\right)$ units$^2$, where $a$, $b$ and $c$ are integers. [6]

$$8\int_0^{1.5} \frac{1}{2x+3}\,dx = 8\left[\frac{\ln(2x+3)}{2}\right]_0^{1.5}$$

$$= 4\left[\ln(2x+3)\right]_0^{1.5}$$

$$= 4\left[\ln(2\times1.5+3) - \ln(0+3)\right]$$

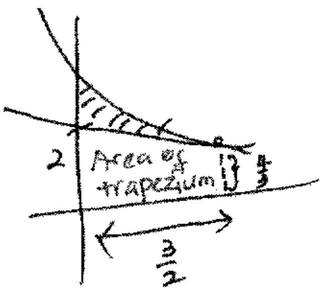
$$= 4\left[\ln 6 - \ln 3\right] \qquad \log a - \log b = \log\left(\frac{a}{b}\right)$$

$$= 4\ln\left(\frac{6}{3}\right)$$

$$= 4\ln 2 \qquad a\log b = \log b^a$$

$$= \ln 2^4 = \ln 16$$

$\therefore$ Area of shaded region




$$= \ln 16 - \frac{1}{2}\left(2 + \frac{4}{3}\right) \times \frac{3}{2}$$

$$= \ln 16 - \frac{5}{2} \text{ units}^2$$

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