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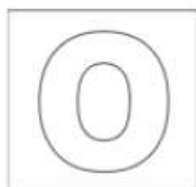
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GAN ENG SENG SCHOOL
Preliminary Examination 2024



**CANDIDATE
NAME**

CLASS

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**INDEX
NUMBER**

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ADDITIONAL MATHEMATICS

Paper 1

4049/01

27 August 2024
2 hours 15 minutes

Sec 4 Express/ 5 Normal (Academic)

Candidates answer on the Question Paper

No Additional Materials are required

READ THESE INSTRUCTIONS FIRST

Write your class, index number and name on all the work you hand in.

Write in dark blue or black pen.

You may use an HB pencil for any diagrams or graphs.

Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.

Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.

The use of a scientific calculator is expected, where appropriate.

You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.

The number of marks is given in brackets [] at the end of each question or part question.

The total of the marks for this paper is 90.

	For Examiner's Use
Total	90

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \cdots + \binom{n}{r}a^{n-r}b^r + \cdots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{(n-r)!r!} = \frac{n(n-1)\cdots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2 \cos^2 A - 1 = 1 - 2 \sin^2 A$$

$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

Formulae for $\triangle ABC$

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2} ab \sin C$$

- 1 The equation of a curve is $y = -2x^2 + 3x + 5$.
- (a) Write $y = -2x^2 + 3x + 5$ in the form $y = a(x - h)^2 + k$, where a , h and k are constants. [3]
- (b) Using your results in part (a), explain clearly why the maximum value of y is k when $x = h$. [2]

2 (a) Factorise $64 - (x+1)^3$.

[2]

(b) Hence, solve $64 - (x+1)^3 = 15(3-x)$, expressing non-integer roots in surd form. [2]

- 3 Express $\frac{x+7}{(x^2-9)(x-3)}$ as the sum of 3 partial fractions.

[6]

- 4 A curve has the equation $y = x^3 + kx^2 + kx + 8$. Find the set of values of k such that
- (a) the curve is an increasing function, [3]

- (b) the curve has exactly 1 stationary point. [2]

- 5 (a) Find the term independent of x in the expansion of $\left(2x^2 + \frac{3}{x}\right)^9$. [3]

- (b) (i) Find the first three terms of the expansion of $(2 - 3x)^5$ in ascending powers of x . [2]

- (ii) Hence, find the value of n such that the coefficient of x in the expansion of $(2 - 3x)^5 (1 - 3x)^n$ is -720 . [3]

- 6 The gradient function of a curve $y = f(x)$ is given by $\frac{36}{(2x+1)^2}$ for $x > -\frac{1}{2}$.

The curve passes through the point $\left(\frac{1}{2}, 2\right)$.

- (a) Find the equation of the curve.

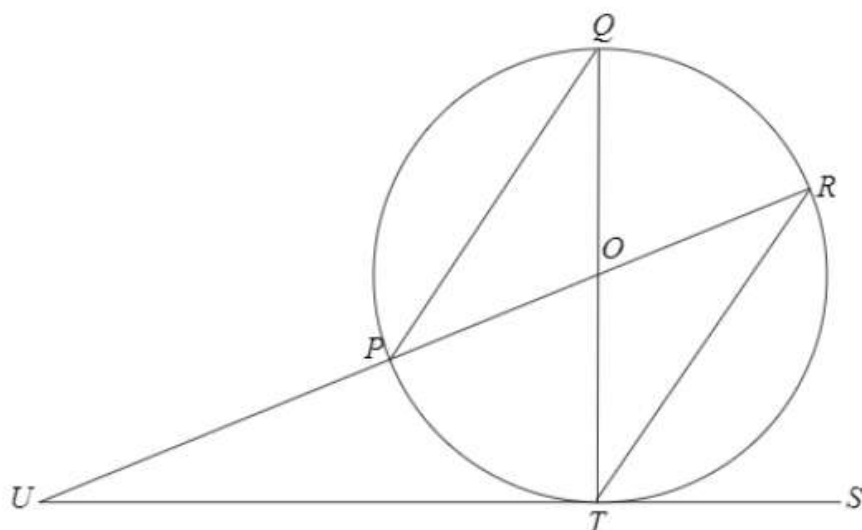
[3]

- (b) Find the angle, in degrees, that the tangent to the curve at $x = 1$ makes with the x -axis. [2]

- 7 The point P lies on the curve $y = \ln\left(\frac{x+1}{x-1}\right)$ for $x > 1$. The normal to the curve at P is parallel to the line $2y = 3x + 2$. The equation of the tangent to the curve at P cuts the y axis at Q . Find the area of the triangle POQ , where O is the origin.

[9]

8



In the diagram, TQ and PR are the diameters of the circle with centre O .
The tangent at T meets RP produced at U . Prove that

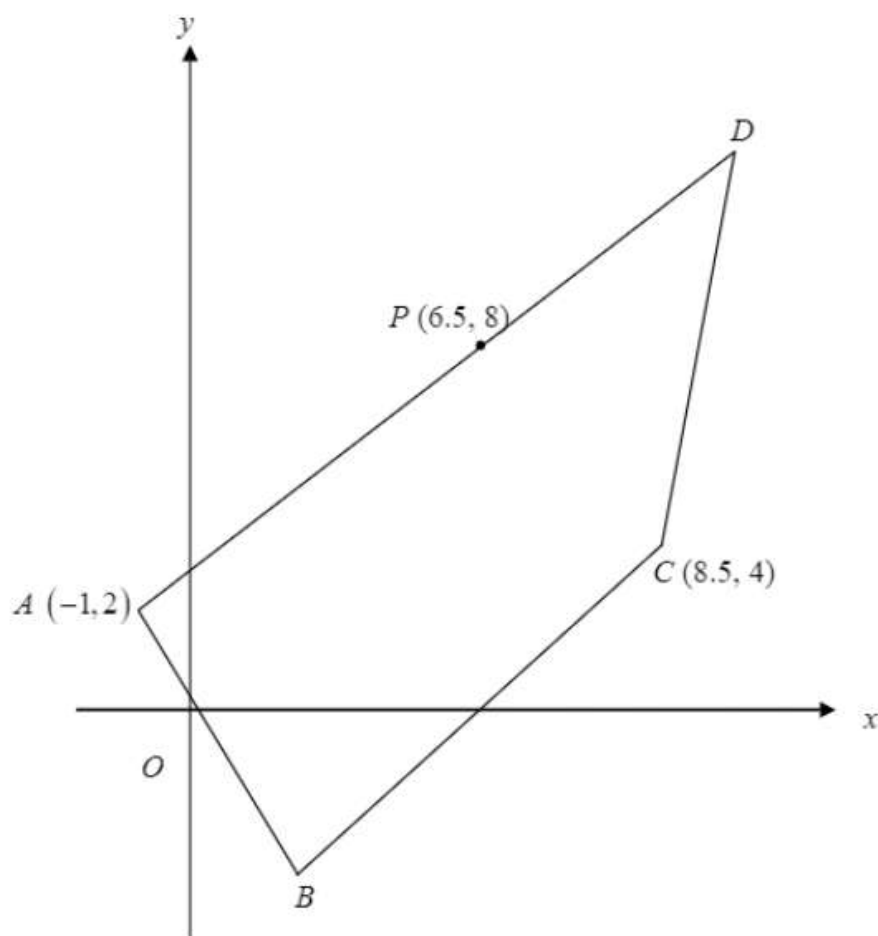
(a) $TR = PQ$,

[3]

(b) $QP \times UT = TP \times UR,$

[3]

- 9 The diagram shows the quadrilateral $ABCD$ in which point A is $(-1, 2)$ and point C is $(8.5, 4)$. The point $P(6.5, 8)$ lies on AD such that $AP : PD = 3 : 2$. The midpoint of BC , point M lies on the x -axis and directly below point P .



- (a) Find the coordinates of D , M and B .

[5]

- (b) Calculate the area of the quadrilateral $ABCD$.

[2]

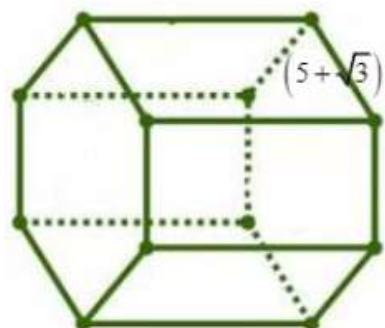
- 10 (a) Sketch, on the same diagram, the curves $y = 3 \cos x$ and $y = 1 - 2 \sin\left(\frac{2x}{3}\right)$ for $0 \leq x \leq 3\pi$. [3]

- (b) Hence, find the number of distinct values of x for $0 \leq x \leq 3\pi$, for which

(i) $3 \cos x + 2 \sin\left(\frac{2x}{3}\right) = 1$, [1]

(ii) $3 \cos 3x + 2 \sin 2x = 1$. [1]

- 11 The diagram shows a prism in which its cross-section is a regular hexagon of side $(5 + \sqrt{3})$ cm.



Side view

Front view

- (a) Find an expression for the cross-sectional area of the prism in the form $q\sqrt{3} + p$ cm², where p and q are integers.

[3]

- (b) Given that the volume of the prism is $3(32\sqrt{3}+138)\text{ cm}^3$, find the height of the [3]
prism in the form $(a\sqrt{3}+b)\text{ cm}$, where a and b are integers.

- 12 A particle passes a fixed point O and moves in a straight line such that, t s after leaving O , its velocity, v m/s, is given by $v = \frac{3}{t+2} - \frac{t+2}{3}$. The particle reaches its greatest distance from O at P .

- (a) Find the time when the particle reaches P . [2]

- (b) Find the distance travelled by the particle when $t = 2$.

[5]

- (c) Show that the velocity of the particle is decreasing at point P .

[2]

- 13 (a) Given that $\log_a 125 - 3\log_a \sqrt{b} + \log_a c = 3$, express a in terms of b and c . [3]

- (b) Show that $5^{n+1} - 4(5^n) + 5^{n-1}$ is divisible by 2 for all positive integers of n . [3]

- 14 A metal container of hot liquid is left to cool in a laboratory. The temperature of the liquid, T °C, after n minutes is given by $T = ae^{-kn} + 15$, where a and k are constants. The table below shows the measured values of T and n .

n (minutes)	10	20	30	40	50
T °C	66.9	46.5	35.1	27.2	22.4

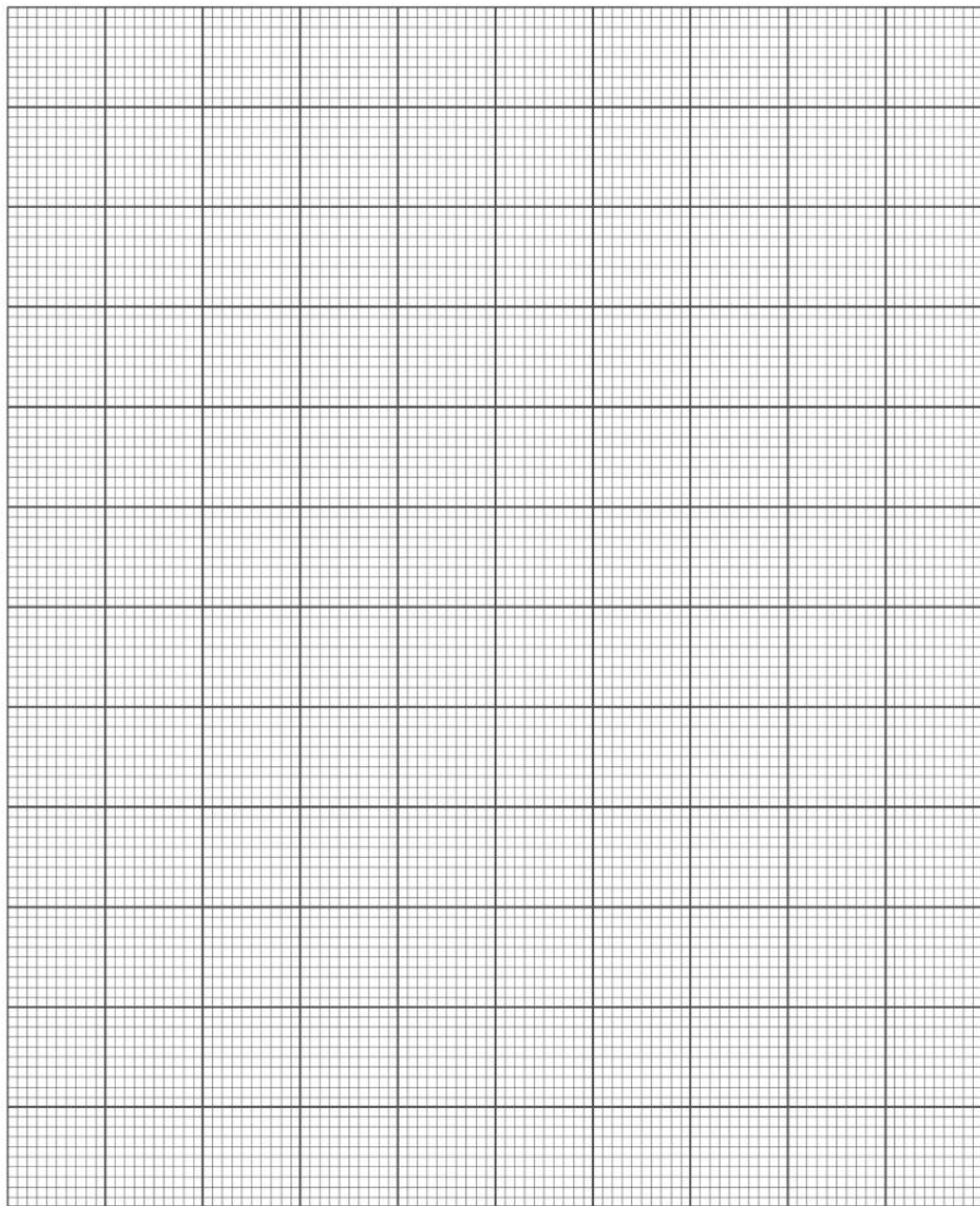
- (a) Plot $\ln(T - 15)$ against n and draw a straight line graph on page 21 to illustrate the information. [3]

- (b) Use your graph to estimate

- (i) the value of k , [2]

- (ii) the number of minutes it takes for the temperature of the liquid to drop to 40% of its initial value.

[3]



End of paper

Marking Scheme

2024 Sec 4 Express PRELIM AM Paper 1

		Marker's Comments	
1(a)	$y = -2x^2 + 3x + 5$ $= -2\left[x^2 - \frac{3}{2}x\right] + 5 \text{ --- M1}$ $= -2\left[\left(x - \frac{3}{4}\right)^2 - \frac{9}{16}\right] + 5 \text{ --- M1}$ $= -2\left(x - \frac{3}{4}\right)^2 + \frac{18}{16} + 5$ $= -2\left(x - \frac{3}{4}\right)^2 + \frac{49}{8} \text{ --- A1}$	3 marks	
(b)	For all real values of x, $\left(x - \frac{3}{4}\right)^2 \geq 0 \text{ --- M1}$ $-2\left(x - \frac{3}{4}\right)^2 \leq 0$ $-2\left(x - \frac{3}{4}\right)^2 + \frac{49}{8} \leq \frac{49}{8} \text{ --- A1}$ $\therefore x = \frac{3}{4}, y_{\max} = \frac{49}{8}$	2 marks	
	Alternative method $y = -2\left(x - \frac{3}{4}\right)^2 + k$ When $x = \frac{3}{4}, y = -2(0) + k$ $= k$ $\frac{dy}{dx} = -4\left(x - \frac{3}{4}\right)$ When $\frac{dy}{dx} = 0 \text{ --- M1}$ $x = \frac{3}{4}$ $\frac{d^2y}{dx^2} = -4(<0) \text{ --- A1}$ $\therefore y$ is a maximum value, k	2 marks	

2(a)	$64 - (x+1)^3$ $= 4^3 - (x+1)^3$ $= [4 - (x+1)][4^2 + 4(x+1) + (x+1)^2] \text{--- M1}$ $= (3-x)(16 + 4x + 4 + x^2 + 2x + 1)$ $= (3-x)(x^2 + 6x + 21) \text{--- A1}$	2 marks	
2(b)	$(3-x)(x^2 + 6x + 21) = 15(3-x)$ $(3-x)(x^2 + 6x + 21) - 15(3-x) = 0$ $(3-x)(x^2 + 6x + 21 - 15) = 0$ $(3-x)(x^2 + 6x + 6) = 0 \text{--- M1}$ $(3-x) = 0 \text{ or } (x^2 + 6x + 6) = 0$ $x = 3 \text{ or } x = \frac{-6 \pm \sqrt{36 - 4(6)}}{2}$ $x = 3 \text{ or } x = \frac{-6 \pm \sqrt{12}}{2}$ $x = 3 \text{ or } x = \frac{-6 \pm 2\sqrt{3}}{2}$ $x = 3 \text{ or } x = -3 \pm \sqrt{3} \text{--- A1, 0}$	2 marks	
3.	$\frac{x+7}{(x^2-9)(x-3)}$ $= \frac{x+7}{(x+3)(x-3)(x-3)}$ $\frac{x+7}{(x+3)(x-3)^2} = \frac{A}{x+3} + \frac{B}{x-3} + \frac{C}{(x-3)^2} \text{--- M1}$ $A(x-3)^2 + B(x+3)(x-3) + C(x+3) = x+7 \text{--- M1}$ Sub $x = 3$ $6C = 10$ $C = \frac{5}{3} \text{--- A1}$ Sub $x = -3$ $36A = 4$ $A = \frac{1}{9} \text{--- A1}$ Sub $x = 0$ $7 = 9A - 9B + 3C$ $9\left(\frac{1}{9}\right) - 9B + 3\left(\frac{5}{3}\right) = 7$ $B = -\frac{1}{9} \text{--- A1}$ $\frac{x+7}{(x-3)(x-3)(x+3)} = \frac{1}{9(x+3)} - \frac{1}{9(x-3)} + \frac{5}{3(x-3)^2} \text{--- A1}$	6 marks	

4(a)	$y = x^3 + kx^2 + kx + 8$ $\frac{dy}{dx} = 3x^2 + 2kx + k \text{ --- M1}$ Let $\frac{dy}{dx} > 0$ $3x^2 + 2kx + k > 0$ Let discriminant < 0 $(2k)^2 - 4(3)(k) < 0 \text{ --- M1}$ $4k^2 - 12k < 0$ $k^2 - 3k < 0$ $k(k - 3) < 0$ $0 < k < 3 \text{ --- A1}$	3 marks	
(b)	$\frac{dy}{dx} = 3x^2 + 2kx + k$ $\frac{dy}{dx} = 0$ $3x^2 + 2kx + k = 0$ Let discriminant $= 0$ $(2k)^2 - 4(3)(k) = 0$ $4k^2 - 12k = 0$ $k^2 - 3k = 0$ $k(k - 3) = 0$ $k = 0 \text{ or } k = 3 \text{ --- A1}$	2 marks	
5(a)	$\left(2x^2 + \frac{3}{x}\right)^9$ $T_{r-1} = \binom{9}{r} (2x^2)^{9-r} \left(\frac{3}{x}\right)^r$ $= \binom{9}{r} (2)^{9-r} (x^2)^{9-r} (3)^r x^{-r}$ $= \binom{9}{r} (2)^{9-r} (3)^r x^{18-3r} \text{ --- M1}$ Let $18 - 3r = 0$ $r = 6 \text{ --- A1}$ $T_7 = \binom{9}{6} (2)^3 (3)^6$ $= 489888 \text{ --- A1}$	3 marks	

5(b)	$(2-3x)^5$ $= 2^5 + \binom{5}{1}(2)^4(-3x) + \binom{5}{2}(2)^3(-3x)^2 + \dots$ $= 32 - 240x + 720x^2 + \dots \text{--- B2,1,0}$	2 marks	
(c)	$(2-3x)^5(1-3x)^n$ $= (32 - 240x + 720x^2 + \dots)(1-3x)^n$ $= (32 - 240x + 720x^2 + \dots) \left(1 + \binom{n}{1}(-3x) + \dots \right) \text{--- M1}$ Coefficient of $x = 32 \binom{n}{1}(-3x) - 240x \text{--- M1}$ $32 \times n \times (-3x) - 240x = -720x$ $-96nx - 240x = -720x$ $-96n = -720 + 240$ $n = 5 \text{--- A1}$	3 marks	
6(a)	$\frac{dy}{dx} = 36(2x+1)^{-2}$ $y = \int 36(2x+1)^{-2} dx$ $= 36 \times \frac{(2x+1)^{-1}}{-1(2)} + c \text{ (c : constant)}$ $= \frac{-18}{(2x+1)} + c \text{--- M1}$ Sub $\left(\frac{1}{2}, 2\right)$ $2 = \frac{-18}{(1+1)} + c \text{--- M1}$ $2 = -9 + c$ $c = 11$ $y = \frac{-18}{(2x+1)} + 11 \text{--- A1}$	3 marks	
(b)	$\frac{dy}{dx} = 36(2x+1)^{-2}$ At $x = 1$ $\frac{dy}{dx} = \frac{36}{3^2}$ $= 4 \text{--- M1}$ $\tan \theta = 4$ $\theta = 76.0^\circ \text{--- A1}$	2 marks	

7

$$y = \ln\left(\frac{x+1}{x-1}\right)$$

$$y = \ln(x+1) - \ln(x-1)$$

$$\frac{dy}{dx} = \frac{1}{x+1} - \frac{1}{x-1} \text{ --- M1}$$

$$2y = 3x + 2$$

$$y = \frac{3}{2}x + 1$$

$$\text{Gradient of normal at } A = \frac{3}{2}$$

$$\therefore \text{Gradient of tangent} = -\frac{2}{3} \text{ --- M1}$$

$$\frac{1}{x+1} - \frac{1}{x-1} = -\frac{2}{3}$$

$$\frac{x-1-(x+1)}{(x+1)(x-1)} = \frac{-2}{3}$$

$$\frac{-2}{(x+1)(x-1)} = \frac{-2}{3} \text{ --- M1}$$

$$x^2 - 1 = 3$$

$$x^2 = 4$$

$$x = 2 \text{ or } x = -2$$

$$\therefore x = 2 \text{ --- A1}$$

$$\text{At } x = 2, y = \ln\left(\frac{2+1}{2-1}\right)$$

$$y = \ln 3 \text{ --- A1}$$

$$P(2, \ln 3)$$

Equation of tangent

$$y = -\frac{2}{3}x + c$$

$$\text{Sub } (2, \ln 3)$$

$$\ln 3 = -\frac{2}{3}(2) + c \text{ --- M1}$$

$$c = \ln 3 + \frac{4}{3}$$

$$y = -\frac{2}{3}x + \ln 3 + \frac{4}{3} \text{ --- A1}$$

	At $x = 0$ $y = \ln 3 + \frac{4}{3}$ $Q(0, \ln 3 + \frac{4}{3})$ Area of triangle POQ $= \frac{1}{2} \times 2 \times \left(\ln 3 + \frac{4}{3} \right) \text{--- M1}$ $= \ln 3 + \frac{4}{3}$ $= 2.43 \text{--- A1}$	9 marks	
8(a)	$OQ = OP = OR = OT$ (radii of a circle) --- M1 $\angle QOP = \angle ROT$ (vertically opp angles) --- M1 $\triangle QOP$ is congruent to $\triangle ROT$ (SAS congruency) --- A1 Hence, $TR = PQ$ (Shown)	3 marks	
(b)	$\angle TUR = \angle PUT$ (Common angle) $\angle PTU = \angle TRU$ (Tangent-chord theorem) --- M1 $\triangle UTR$ is similar to $\triangle UPT$ (AA similarity) --- M1 $\frac{TP}{RT} = \frac{UT}{UR} = \frac{PU}{TU}$ $RT = QP$ (From (i)) $\frac{TP}{QP} = \frac{UT}{UR} \text{--- M1}$ $QP \times UT = TP \times UR$ (Shown)	3 marks	
9(a)	$D\left(6.5 + \frac{7.5}{3} \times 2, 8 + \frac{6}{3} \times 2\right) \text{--- M1}$ $= D(11.5, 12) \text{--- A1}$ $M(6.5, 0) \text{--- B1}$ $\left(\frac{x+8.5}{2}, \frac{y+4}{2}\right) = (6.5, 0) \text{--- M1}$ $x = 4.5, y = -4$ $B(4.5, -4) \text{--- A1}$	5 marks	

(b) $A(-1, 2), B(4.5, -4), C(8.5, 4), D(11.5, 12)$

$$\text{Area} = \frac{1}{2} \begin{vmatrix} -1 & 4.5 & 8.5 & 11.5 & -1 \\ 2 & -4 & 4 & 12 & 2 \end{vmatrix}$$

$$= \frac{1}{2} [4 + 4.5(4) + 8.5(12) + 2(11.5) - (2(4.5) - 4(8.5) + 4(11.5) + 12(-1))] \text{---M1}$$

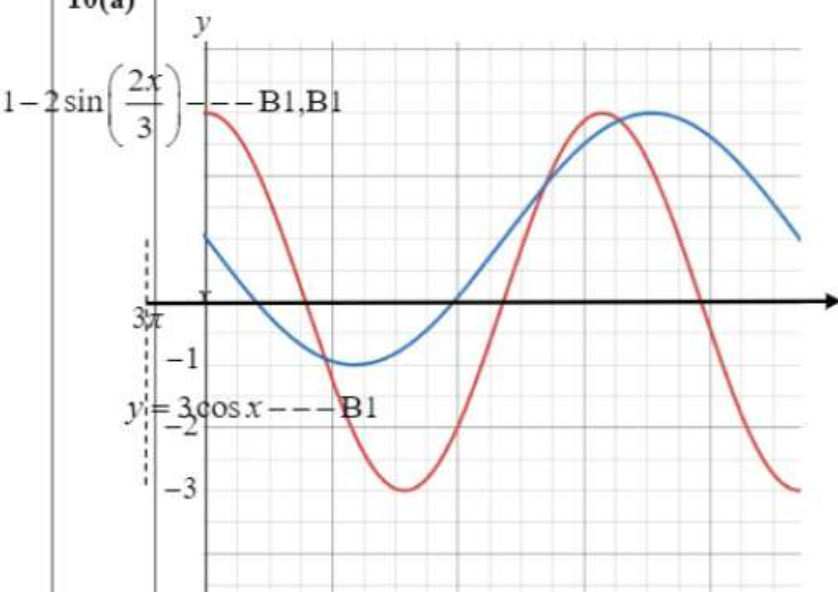
$$= \frac{1}{2} (147 - 9)$$

$$= \frac{138}{2}$$

$$= 69 \text{ units}^2 \text{---A1}$$

2 marks

10(a)



B1 Shape
B1
Shifting
curve
vertically
up by 1
unit

B1 - $y = 3 \cos x$, shape, 1.5 oscillations over 3π
3 and -3 to be marked on y axis

π , 2π and 3π to be marked on x axis

(b)(i) 3

1 mark

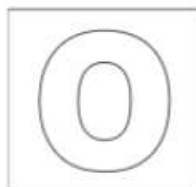
(b)(ii) 9

1 mark

11(a)	Area of cross-sectional area of prism $= 6 \times \frac{1}{2} \times (5 + \sqrt{3})^2 \sin 60^\circ \text{ --- M1}$ $= 3 \times (25 + 10\sqrt{3} + 3) \times \frac{\sqrt{3}}{2}$ $= 3(28 + 10\sqrt{3}) \times \frac{\sqrt{3}}{2} \text{ --- M1}$ $= \frac{3}{2}(28\sqrt{3} + 30)$ $= 42\sqrt{3} + 45 \text{ --- A1}$	3 marks	
(b)	Volume of the prism = $3(32\sqrt{3} + 138) \text{ cm}^3$ $3(14\sqrt{3} + 15) \times h = 3(32\sqrt{3} + 138)$ $(14\sqrt{3} + 15) \times h = (32\sqrt{3} + 138)$ $h = \frac{32\sqrt{3} + 138}{14\sqrt{3} + 15} \times \frac{14\sqrt{3} - 15}{14\sqrt{3} - 15} \text{ --- M1}$ $= \frac{32\sqrt{3}(14\sqrt{3} - 15) + 138(14\sqrt{3} - 15)}{(14\sqrt{3})^2 - 15^2}$ $= \frac{448(3) - 480\sqrt{3} + 1932\sqrt{3} - 2070}{(14\sqrt{3})^2 - 15^2} \text{ --- M1}$ $= \frac{1452\sqrt{3} - 726}{363}$ $= 4\sqrt{3} - 2 \text{ --- A1}$	3 marks	
12(a)	$v = \frac{3}{t+2} - \frac{t+2}{3}$ Let $v = 0$ $\frac{3}{t+2} - \frac{t+2}{3} = 0 \text{ --- M1}$ $(t+2)^2 = 9$ $t+2 = 3 \text{ or } t+2 = -3$ $t = 1 \text{ or } t = -5(\text{rej}) \text{ --- A1}$	2 marks	

(b)	$s = \int \frac{3}{t+2} - \frac{t+2}{3} dt$ $= 3 \ln(t+2) - \frac{t^2}{6} - \frac{2}{3}t + c \text{ --- M1}$ At $t = 0, s = 0$ $3 \ln(0+2) + c = 0$ $c = -3 \ln 2$ $s = 3 \ln(t+2) - \frac{t^2}{6} - \frac{2}{3}t - 3 \ln 2 \text{ --- A1}$	2 marks	
(b)	At $t = 1$ $s = 3 \ln 3 - \frac{1}{6} - \frac{2}{3} - 3 \ln 2$ $= 0.383 \text{ m --- A1}$ At $t = 2$ $s = 3 \ln 4 - \frac{4}{6} - \frac{4}{3} - 3 \ln 2$ $= 0.0794 \text{ --- A1}$ Distance travelled $= 0.383 + (0.383 - 0.0794)$ $= 0.687 \text{ m --- A1}$	5 marks	
(c)	$a = \frac{dv}{dt}$ $= \frac{-3}{(t+2)^2} - \frac{1}{3} \text{ --- M1}$ At $t = 1$ $a = -\frac{3}{9} - \frac{1}{3}$ $= -\frac{2}{3} < 0 \text{ --- A1}$ Since $a < 0$, the velocity of the particle is decreasing.	2 marks	
13(a)	$\log_a 125 - 3 \log_a \sqrt{b} + \log_a c = 3$ $\log_a 125 - \log_a b^{\frac{3}{2}} + \log_a c = 3$ $\log_a \frac{125c}{b^{\frac{3}{2}}} = 3 \text{ --- M1}$ $a^3 = \frac{125c}{b^{\frac{3}{2}}} \text{ --- M1}$ $a = \frac{5\sqrt[3]{c}}{\sqrt{b}} \text{ --- A1}$	3 marks	

(b)	$5^{n+1} - 4(5^n) + 5^{n-1}$ $= 5^{n-1}(5^2) - 4(5^{n-1})(5) + 5^{n-1} \text{ --- M1}$ $= 5^{n-1}(5^2 - 20 + 1) \text{ --- M1}$ $= 6(5^{n-1}) \text{ --- A1}$ Alternative Method $5 \times 5^n - 4(5^n) + \frac{5^n}{5} \text{ --- M1}$ $= \frac{1}{5}(25(5^n) - 20(5^n) + 5^n)$ $= \frac{6}{5}(5^n) \text{ --- M1}$ $= 6(5^{n-1}) \text{ --- A1}$ Since the expression has a factor of 6 which is divisible by 2, hence $5^{n+1} - 4(5^n) + 5^{n-1}$ is divisible by 2 for all positive integers of n	3 marks	
14	Refer to solution on the graph paper.	8 marks	



GAN ENG SENG SCHOOL
Preliminary Examination 2024



**CANDIDATE
NAME**

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CLASS

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**INDEX
NUMBER**

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ADDITIONAL MATHEMATICS

Paper 2

4049/02

28 August 2024
2 hours 15 minutes

Candidates answer on the Question Paper

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Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a + b)^n = a^n + \binom{n}{1} a^{n-1} b + \binom{n}{2} a^{n-2} b^2 + \dots + \binom{n}{r} a^{n-r} b^r + \dots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{(n-r)!r!} = \frac{n(n-1)\dots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2 \cos^2 A - 1 = 1 - 2 \sin^2 A$$

$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

Formulae for $\triangle ABC$

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2} ab \sin C$$

- 1 (a) Given that the expression $2x^3 + ax^2 + bx + c$ leaves the same remainder when divided by $(x - 5)$ and when divided by $(2x + 1)$, show that $9a + 2b = -91$. [3]

- (b) Given that the remainder is -6 when the expression $2x^3 + ax^2 + bx + c$ is divided by $(x - 1)$, and $(x + 1)$ is a factor, calculate the value of c . [4]

- 2 Mr Phua bought a new car. After t months, its value V is given by $V = 250000e^{-pt}$, where p is a constant. The value of the car after 36 months is \$180 000.

(a) Show that p is approximately 0.00913. [2]

(b) Calculate the age of the car, to the nearest month, when its expected value will be \$100000. [2]

- (c) 5 years after Mr Phua bought the car, a second-hand car agent offers to pay him \$130000 [2]
for his car. Would you advise Mr Phua to take up the offer. Explain your answer.

- (d) Sketch the graph of $V = 250000e^{-2t}$, for $t \geq 0$. [2]

3. The line $y = 10$ and $3y + 4x = 32$ are tangent to a circle C at the points $(-2, 10)$ and $(2, 8)$ respectively.

(a) Show that the equation of C is $(x + 2)^2 + (y - 5)^2 = 25$. [5]

(b) Explain if the x -axis is tangent to C . [2]

(c) Write down the equations of the two vertical tangents. [2]

(d) Two points P and Q lie on the circle and the length of PQ is 4 units. [2]
Calculate the shortest distance from the centre of the circle to the line PQ .

4. A , B and C are the angles of a triangle. Given that $\sin(A+B) = \frac{12}{13}$.

(a) Find the exact value of $\tan C$. Show your working clearly. [3]

(b) Evaluate $\cos\left[(A+B) + \frac{\pi}{3}\right]$. Leave your answer in surd form. [3]

5. (a) Solve the equation $15(16^x) - 4^x = 6$.

[4]

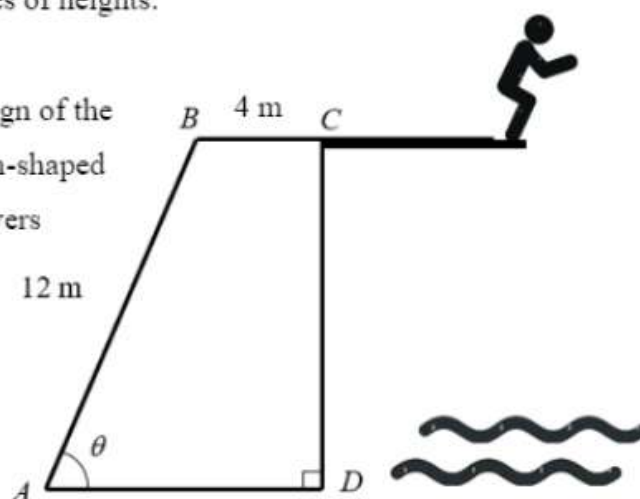
(b) Solve the equation $\lg(2y) + 2\ln(y) = 2$.

[4]

6. A competitive platform diving competition involves one or two divers performing spins, twists and somersaults into a pool from a series of heights.

The diagram shows the cross-section of a design of the diving platform, which consists of a trapezium-shaped tower $ABCD$. AB is the stairway which the divers use to reach the platform.

Given that $\angle BAD = \theta$, where θ can vary and $\angle ADC = 90^\circ$, the length of AB and BC are 12 m and 4 m respectively,



- (a) show that L , the perimeter of $ABCD$, can be expressed in the form $p + q \sin \theta + r \cos \theta$. [2]

- (b) express L in the form of $p + R \cos(\theta - \alpha)$, where $R > 0$ and $0^\circ < \alpha < 90^\circ$. [3]

(c) State the maximum value of L , and the corresponding value of θ . [2]

(d) Find the value of θ if $L = 25$ m. Explain why a platform with this value of L is a poor design. [4]

- 7 (a) Prove the identity $\operatorname{cosec} 4x + \cot 4x = \cot 2x$. [3]

- (b) Hence, find, for $-180^\circ \leq \theta \leq 180^\circ$, the values of θ for which $\operatorname{cosec} 2\theta + \cot 2\theta = -\sqrt{3}$. [4]

8. The equation of a curve is $y = \frac{x}{e^{2x}}$

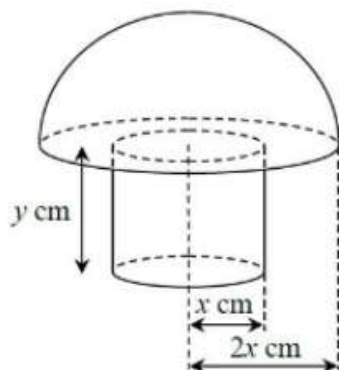
(a) Find an expression for $\frac{dy}{dx}$. [2]

(b) Given that x is changing at a constant rate of 0.18 units per second, find the rate of change of y when $x = 1$. [2]

(c) Explain if the curve has more than one stationary point. [2]

9. The diagram shows a solid object. It is made up of a cylinder with a hemisphere above it.

The cylinder has a radius of x cm and a height of y cm. The radius of the hemisphere is $2x$ cm.

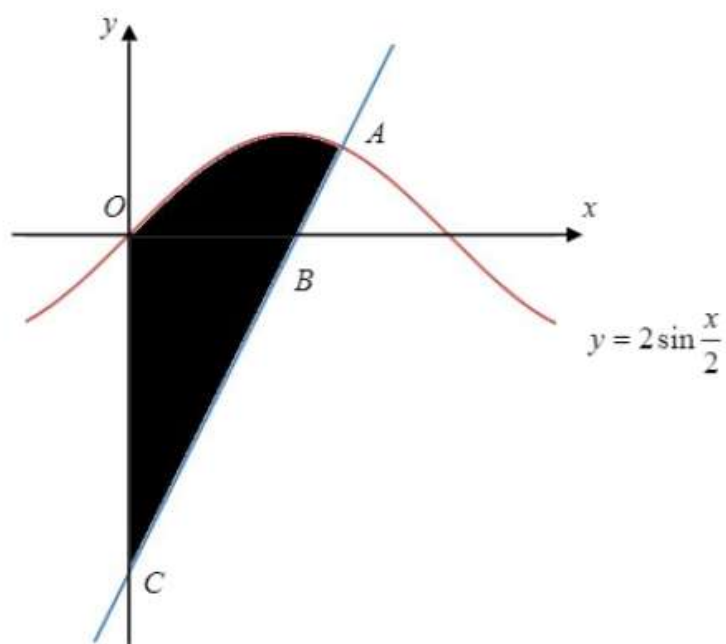


- (a) Express y in terms of x if the total volume of the solid is 72π cm³. [2]

- (b) Show that the total surface area, A cm², of the solid is $144\pi\left(\frac{1}{x} + \frac{x^2}{108}\right)$. [2]

- (c) Find the value of x for which A is a stationary value and determine the nature of the stationary value. [4]

10.



The diagram shows part of the curve $y = 2 \sin \frac{x}{2}$, where A lies on the curve.

The x -coordinate of A is $\frac{4\pi}{3}$ and the normal to the curve at A meets the x -axis at B and the y -axis at C .

- (a) Find the coordinates of B and of C , leaving your answers in its **exact form**.

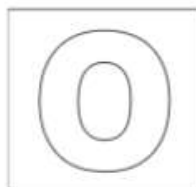
[6]

(b) Find the area of the shaded region.

[6]

11. (a) Show that $\frac{d}{dx} \left[(2x)(3-5x)^{\frac{6}{5}} \right] = (6-22x)(3-5x)^{\frac{1}{5}}.$ [3]

(b) Hence, evaluate $\int x(3-5x)^{\frac{1}{5}} dx.$ [3]



GAN ENG SENG SCHOOL
Preliminary Examination 2024



**CANDIDATE
NAME**

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CLASS

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**INDEX
NUMBER**

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ADDITIONAL MATHEMATICS

Paper 2

4049/02

28 August 2024
2 hours 15 minutes

Candidates answer on the Question Paper

No Additional Materials are required

READ THESE INSTRUCTIONS FIRST

Write your class, index number and name on all the work you hand in.

Write in dark blue or black pen.

You may use an HB pencil for any diagrams or graphs.

Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.

Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.

The use of a scientific calculator is expected, where appropriate.

You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.

The number of marks is given in brackets [] at the end of each question or part question.

The total of the marks for this paper is 90.

	For Examiner's Use
Total	90

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \dots + \binom{n}{r}a^{n-r}b^r + \dots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{(n-r)!r!} = \frac{n(n-1)\dots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2 \cos^2 A - 1 = 1 - 2 \sin^2 A$$

$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

Formulae for $\triangle ABC$

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2} ab \sin C$$

- 1 (a) Given that the expression $2x^3 + ax^2 + bx + c$ leaves the same remainder when divided by $(x - 5)$ and when divided by $(2x + 1)$, show that $9a + 2b = -91$. [3]

$$\text{Let } F(x) = 2x^3 + ax^2 + bx + c$$

$$F(5) = 2(5)^3 + a(5)^2 + b(5) + c \quad (\text{M1})$$

$$= 250 + 25a + 5b + c$$

$$F\left(-\frac{1}{2}\right) = 2\left(-\frac{1}{2}\right)^3 + a\left(-\frac{1}{2}\right)^2 + b\left(-\frac{1}{2}\right) + c$$

$$= -\frac{1}{4} + \frac{1}{4}a - \frac{1}{2}b + c$$

$$250 + 25a + 5b + c = -\frac{1}{4} + \frac{1}{4}a - \frac{1}{2}b + c \quad (\text{M1})$$

$$1000 + 100a + 20b = -1 + a - 2b$$

$$99a + 22b = -1001 \quad (\text{M1})$$

$$9a + 2b = -91 \quad (\text{Shown})$$

- (b) Given that the remainder is -6 when the expression $2x^3 + ax^2 + bx + c$ is divided by $(x - 1)$, and $(x + 1)$ is a factor, calculate the value of c . [4]

$$F(x) = 2x^3 + ax^2 + bx + c$$

$$F(1) = 2(1)^3 + a(1)^2 + b(1) + c = -6$$

$$a + b + c = -8 \quad \dots(1) \quad (\text{M1})$$

$$F(-1) = 2(-1)^3 + a(-1)^2 + b(-1) + c = 0$$

$$a - b + c = 2 \quad \dots(2) \quad (\text{M1})$$

$$(1) - (2): 2b = -10$$

$$b = -5 \quad (\text{M1})$$

$$\Rightarrow 9a + 2(-5) = -91$$

$$a = -9$$

$$\Rightarrow c = 2 + 9 - 5 = 6 \quad (\text{A1})$$

Marker's remarks:

- 2 Mr Phua bought a new car. After t months, its value V is given by $V = 250000e^{-pt}$, where p is a constant. The value of the car after 36 months is \$180 000.

- (a) Show that p is approximately 0.00913. [2]

$$180000 = 250000e^{-p(36)} \quad (\text{M1})$$

$$\frac{18}{25} = e^{-36p}$$

$$\ln(0.72) = -36p$$

$$-0.3285 = -36p$$

$$p = 0.00913 \quad (3 \text{ s.f.}) \quad (\text{A1})$$

- (b) Calculate the age of the car, to the nearest month, when its expected value will be \$100000. [2]

$$100000 = 250000e^{-0.00913t} \quad (\text{M1})$$

$$\frac{2}{5} = e^{-0.00913t}$$

$$\ln(0.4) = -0.00913t$$

$$t = 100.4 \text{ months}$$

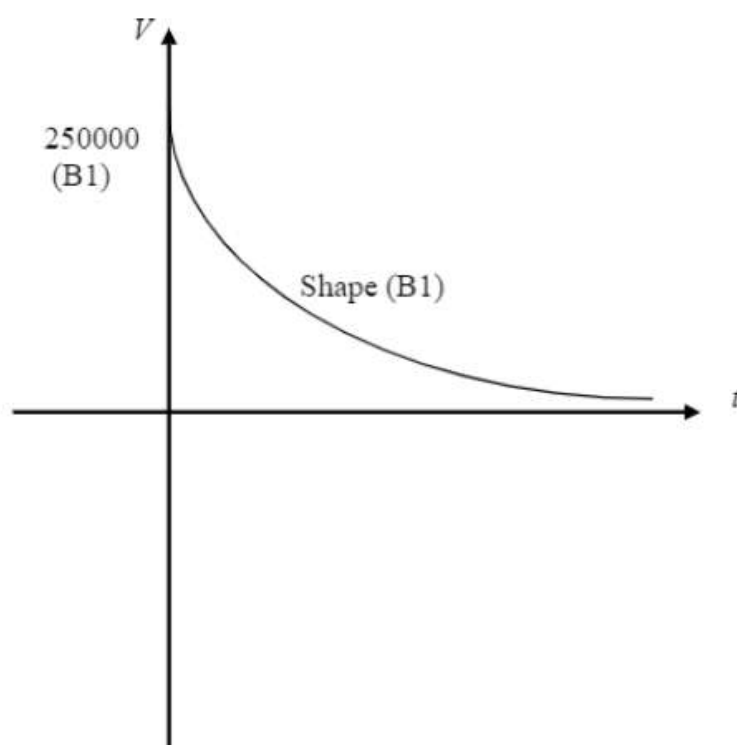
$$= 100 \text{ months (nearest month)} \quad (\text{A1})$$

- (c) 5 years after Mr Phua bought the car, a second-hand car agent offers to pay him \$130000 [2]
for his car. Would you advise Mr Phua to take up the offer. Explain your answer.

$$\begin{aligned}\text{Value of the car after 5 years} &= 250000e^{-0.00913(60)} \\ &= \$144555.12 \quad (\text{M1})\end{aligned}$$

No, he should not take up the offer as the value offered is far below the expected value. (A1)

- (d) Sketch the graph of $V = 250000e^{-kt}$, for $t \geq 0$. [2]



Marker's remarks:

3. The line $y = 10$ and $3y + 4x = 32$ are tangent to a circle C at the points $(-2, 10)$ and $(2, 8)$ respectively.

- (a) Show that the equation of C is $(x + 2)^2 + (y - 5)^2 = 25$. [5]

$$3y + 4x = 32$$

$$y = -\frac{4}{3}x + \frac{32}{3}$$

$$\text{Gradient of the } \perp \text{ line} = \frac{3}{4} \quad (\text{M1})$$

Eqn of the line passing thru' $(2, 8)$ and centre of the circle:

$$(y - 8) = \frac{3}{4}(x - 2)$$

$$y = \frac{3}{4}x + \frac{13}{2} \quad (\text{M1})$$

$$x\text{-coord of the centre of the circle} = -2 \quad (\text{B1})$$

$$\Rightarrow y = \frac{3}{4}(-2) + \frac{13}{2} = 5 \quad (\text{M1})$$

$$\text{Radius of the circle} = 10 - 5 = 5 \quad (\text{B1})$$

$$\text{Eqn of the circle: } (x + 2)^2 + (y - 5)^2 = 5^2$$

$$\text{or } (x + 2)^2 + (y - 5)^2 = 25$$

- (b) Explain if the x -axis is tangent to C . [2]

The lowest point of C is $(-2, 5 - 5) = (-2, 0)$, which is on the x -axis. (M1)

Therefore, x -axis is tangent to C . (A1)

- (c) Write down the equations of the two vertical tangents.

[2]

$$x = -7 \quad (\text{B1})$$

$$x = 3 \quad (\text{B1})$$

- (d) Two points P and Q lie on the circle and the length of PQ is 4 units.

[2]

Calculate the shortest distance from the centre of the circle to the line PQ .

The length of the centre of the circle to P or Q = radius = 5 units

Length between mid-point of PQ to P = 2 units

$$\text{Shortest distance} = \sqrt{5^2 - 2^2} = \sqrt{21} = 4.58 \text{ units (3 s.f.)} \quad (\text{A1})$$

Marker's remarks:

4. A, B and C are the angles of a triangle. Given that $\sin(A+B) = \frac{12}{13}$.

(a) Find the exact value of $\tan C$. Show your working clearly.

[3]

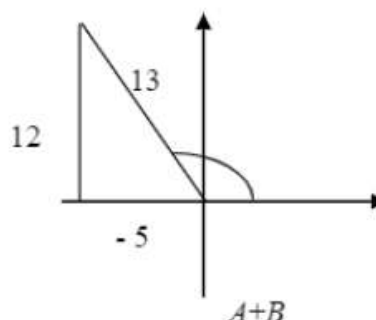
$$\sin(A+B) = \frac{12}{13}$$

$$\Rightarrow \tan(A+B) = -\frac{12}{5} \quad (\text{M1})$$

$$\tan C = \tan(180^\circ - (A+B))$$

$$\tan C = -\tan(A+B) \quad (\text{M1})$$

$$\tan C = -\frac{-12}{5} = \frac{12}{5} = 2.4 \quad (\text{A1})$$



Alternative solution: $(A+B) < 90^\circ$

$$\Rightarrow \tan(A+B) = \frac{12}{5} \quad (\text{M1})$$

$$\tan C = \tan(180^\circ - (A+B))$$

$$\tan C = -\tan(A+B) \quad (\text{M1})$$

$$\tan C = -\frac{12}{5} = -2.4 \quad (\text{A1})$$

- (b) Evaluate $\cos\left[(A+B) + \frac{\pi}{3}\right]$. Leave your answer in surd form.

[3]

$$\cos\left[(A+B) + \frac{\pi}{3}\right] = \cos(A+B)\cos\frac{\pi}{3} - \sin(A+B)\sin\frac{\pi}{3} \quad (\text{M1})$$

$$= \left(\frac{-5}{13}\right)\left(\frac{1}{2}\right) - \left(\frac{12}{13}\right)\left(\frac{\sqrt{3}}{2}\right) \quad (\text{M1})$$

$$= -\frac{5+12\sqrt{3}}{26} \quad \text{or} \quad \frac{-5-12\sqrt{3}}{26} \quad (\text{A1})$$

Alternative solution when $(A+B) < 90^\circ$

$$= \cos(A+B)\cos\frac{\pi}{3} - \sin(A+B)\sin\frac{\pi}{3} \quad (\text{M1})$$

$$= \left(\frac{5}{13}\right)\left(\frac{1}{2}\right) - \left(\frac{12}{13}\right)\left(\frac{\sqrt{3}}{2}\right) \quad (\text{M1})$$

$$= \frac{5-12\sqrt{3}}{26} \quad (\text{A1})$$

Marker's remarks:

5. (a) Solve the equation $15(16^x) - 4^x = 6$. [4]

$$15(16^x) - 4^x = 6$$

$$15(4^{2x}) - 4^x - 6 = 0 \quad (\text{M1})$$

$$15(4^x)^2 - 4^x - 6 = 0$$

$$\text{Let } a = 4^x$$

$$\Rightarrow 15a^2 - a - 6 = 0 \quad (\text{M1})$$

$$(5a + 3)(3a - 2) = 0$$

$$(5a + 3) = 0 \quad \text{or} \quad (3a - 2) = 0$$

$$a = -\frac{3}{5} \quad \text{or} \quad a = \frac{2}{3}$$

$$4^x = -\frac{3}{5} \quad (\text{no solution}) \quad \text{or} \quad 4^x = \frac{2}{3}$$

$$(A1) \quad x = \log_4\left(\frac{2}{3}\right) = -0.292 \quad (3 \text{ s.f.}) \quad (A1)$$

- (b) Solve the equation $\lg(2y) + 2\ln(y) = 2$. [4]

$$\lg(2y) + 2\ln(y) = 2$$

$$\lg 2 + \lg(y) + \frac{2\lg(y)}{\lg(e)} = 2$$

$$(M1) \quad (M1)$$

$$\lg(y) \left[1 + \frac{2}{\lg(e)} \right] = 2 - \lg 2 \quad (M1)$$

$$\lg(y) \times (5.6051) = 1.69897$$

$$\lg(y) = 0.303108$$

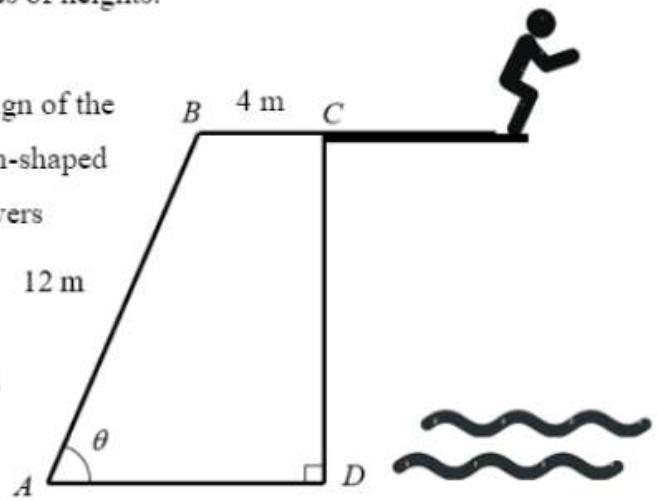
$$y = 2.00959 = 2.01 \quad (3 \text{ s.f.}) \quad (A1)$$

Marker's remarks:

6. A competitive platform diving competition involves one or two divers performing spins, twists and somersaults into a pool from a series of heights.

The diagram shows the cross-section of a design of the diving platform, which consists of a trapezium-shaped tower $ABCD$. AB is the stairway which the divers use to reach the platform.

Given that $\angle BAD = \theta$, where θ can vary and $\angle ADC = 90^\circ$, the length of AB and BC are 12 m and 4 m respectively,



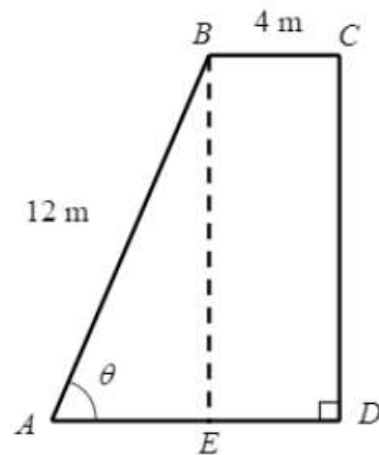
- (a) show that L , the perimeter of $ABCD$, can be expressed in the form $p + q \sin \theta + r \cos \theta$. [2]

$$L = AB + BC + CD + DE + EA$$

$$L = 12 + 4 + BE + 4 + 12 \cos \theta$$

$$L = 12 + 4 + 12 \sin \theta + 4 + 12 \cos \theta \quad (\text{M1}) \quad 12 \text{ m}$$

$$L = 20 + 12 \sin \theta + 12 \cos \theta \quad (\text{A1})$$



- (b) express L in the form of $p + R \cos(\theta - \alpha)$, where $R > 0$ and $0^\circ < \alpha < 90^\circ$. [3]

$$R \cos(\theta - \alpha) = 12 \cos \theta + 12 \sin \theta$$

$$R = \sqrt{12^2 + 12^2} = \sqrt{288} = 12\sqrt{2} \quad (\text{M1})$$

$$\alpha = \tan^{-1}\left(\frac{12}{12}\right) = 45^\circ \quad (\text{M1})$$

$$\Rightarrow L = 20 + 12\sqrt{2} \cos(\theta - 45^\circ) \quad (\text{A1})$$

- (c) State the maximum value of L , and the corresponding value of θ . [2]

$$\text{Max. } L = 20 + 12\sqrt{2} \quad \text{or} \quad 37.0 \text{ m (3 s.f.)} \quad (\text{B1})$$

$$\text{When } \cos(\theta - 45^\circ) = 1$$

$$(\theta - 45^\circ) = 0$$

$$\theta = 45^\circ \quad (\text{B1})$$

- (d) Find the value of θ if $L = 25$ m. Explain why a platform with this value of L is a poor design. [4]

$$L = 25$$

$$20 + 12\sqrt{2} \cos(\theta - 45^\circ) = 25 \quad (\text{M1})$$

$$\cos(\theta - 45^\circ) = \frac{5}{12\sqrt{2}} = 0.29463 \quad (\text{M1})$$

$$\alpha = 72.865^\circ$$

$$\theta - 45^\circ = 72.865^\circ, \quad 287.135^\circ$$

$$\theta = 117.865^\circ, \quad 332.135^\circ \quad (\text{M1})$$

θ cannot be obtuse or more than 90° as AB is a stairway. Otherwise, the divers cannot climb up to the platform. (A1)

Marker's remarks:

- 7 (a) Prove the identity $\operatorname{cosec} 4x + \cot 4x = \cot 2x$. [3]

$$\begin{aligned}
 \text{LHS} &= \frac{1}{\sin 4x} + \frac{\cos 4x}{\sin 4x} & (\text{M1}) \\
 &= \frac{1 + \cos 2(2x)}{\sin 2(2x)} \\
 &= \frac{1 + (2\cos^2 2x - 1)}{2\sin 2x \cos 2x} & (\text{M1}) \\
 &= \frac{2\cos^2 2x}{2\sin 2x \cos 2x} \\
 &= \frac{\cos 2x}{\sin 2x} & (\text{M1}) \\
 &= \cot 2x = \text{RHS}
 \end{aligned}$$

- (b) Hence, find, for $-180^\circ \leq \theta \leq 180^\circ$, the values of θ for which $\operatorname{cosec} 2\theta + \cot 2\theta = -\sqrt{3}$. [4]

$$\begin{aligned}
 \operatorname{cosec} 2\theta + \cot 2\theta &= -\sqrt{3} \\
 \Rightarrow \cot \theta &= -\sqrt{3} & (\text{M1}) \\
 \tan \theta &= -\frac{1}{\sqrt{3}} \\
 \alpha &= 30^\circ & (\text{M1}) \\
 \theta &= 180^\circ - 30^\circ, -30^\circ \\
 \theta &= 150^\circ, -30^\circ & (\text{A1, A1})
 \end{aligned}$$

Marker's remarks:

8. The equation of a curve is $y = \frac{x}{e^{2x}}$

(a) Find an expression for $\frac{dy}{dx}$. [2]

$$y = \frac{x}{e^{2x}} = xe^{-2x}$$

$$\frac{dy}{dx} = x(-2)e^{-2x} + e^{-2x} \quad \text{or} \quad = \frac{e^{2x} - x(2)e^{2x}}{e^{4x}} \quad (\text{M1})$$

$$= e^{-2x}(1-2x) = \frac{e^{2x}(1-2x)}{e^{4x}} = \frac{1-2x}{e^{2x}} \quad (\text{A1})$$

(b) Given that x is changing at a constant rate of 0.18 units per second, find the rate of change of y when $x = 1$. [2]

$$\frac{dx}{dt} = 0.18$$

$$\frac{dy}{dt} = \frac{dy}{dx} \times \frac{dx}{dt}$$

$$\frac{dy}{dt} = \frac{1-2x}{e^{2x}} \times 0.18 \quad (\text{M1})$$

$$\text{When } x = 1, \quad \frac{dy}{dt} = \frac{1-2}{e^2} \times 0.18 = -0.0244 \text{ units/sec} \quad (\text{A1})$$

(c) Explain if the curve has more than one stationary point. [2]

$$\frac{dy}{dx} = \frac{1-2x}{e^{2x}}$$

$$\text{As } e^{2x} > 0, \quad \Rightarrow \text{ when } \frac{dy}{dx} = 0, \quad 1-2x = 0$$

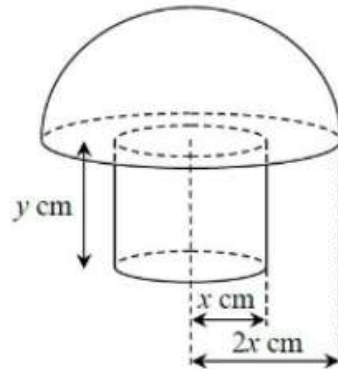
$$x = \frac{1}{2} \quad (\text{M1})$$

$$\text{Therefore there is only one stationary value at } x = \frac{1}{2} \quad (\text{A1})$$

Marker's remarks:

9. The diagram shows a solid object. It is made up of a cylinder with a hemisphere above it.

The cylinder has a radius of x cm and a height of y cm. The radius of the hemisphere is $2x$ cm.



- (a) Express y in terms of x if the total volume of the solid is $72\pi \text{ cm}^3$.

[2]

$$72\pi = \frac{1}{2} \times \frac{4}{3} \times \pi \times (2x)^3 + \pi \times (x)^2 \times y \quad (\text{M1})$$

$$72 = \frac{16}{3}x^3 + x^2y$$

$$y = \frac{72}{x^2} - \frac{16}{3}x \quad (\text{A1})$$

- (b) Show that the total surface area, $A \text{ cm}^2$, of the solid is $144\pi \left(\frac{1}{x} + \frac{x^2}{108} \right)$.

[2]

$$A = \frac{1}{2} \times 4\pi(2x)^2 + \pi(2x)^2 + 2\pi(x) \times y \quad (\text{M1})$$

$$= 12\pi x^2 + 2\pi x \left(\frac{72}{x^2} - \frac{16}{3}x \right)$$

$$= 12\pi x^2 + \frac{144\pi}{x} - \frac{32\pi x^2}{3}$$

$$= \frac{144\pi}{x} + \frac{4\pi x^2}{3} \quad (\text{A1})$$

$$= 144\pi \left(\frac{1}{x} + \frac{x^2}{108} \right)$$

- (c) Find the value of x for which A is a stationary value and determine the nature of the stationary value. [4]

$$\begin{aligned} A &= 144\pi \left(\frac{1}{x} + \frac{x^2}{108} \right) \\ \frac{dA}{dx} &= 144\pi \left(\frac{-1}{x^2} + \frac{x}{54} \right) \end{aligned} \quad (\text{M1})$$

At stationary value, $\frac{dA}{dx} = 0$

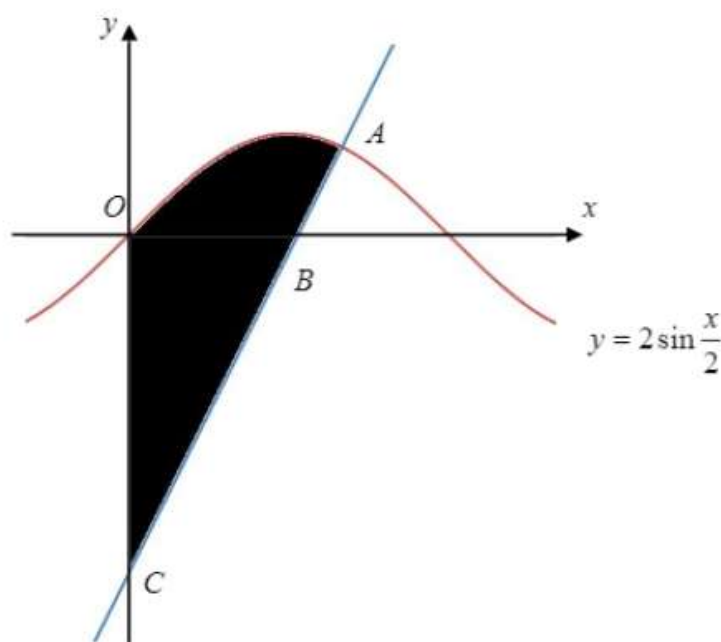
$$\begin{aligned} 144\pi \left(\frac{-1}{x^2} + \frac{x}{54} \right) &= 0 \\ \left(\frac{-1}{x^2} + \frac{x}{54} \right) &= 0 \\ x^3 &= 54 \\ x &= 3.78 \text{ cm (3 s.f.)} \end{aligned} \quad (\text{M1})$$

$$\frac{d^2A}{dx^2} = 144\pi \left(\frac{2}{x^3} + \frac{1}{54} \right) \quad (\text{M1})$$

$$\left. \begin{aligned} \text{When } x &= 3.78, \quad \frac{d^2A}{dx^2} = 25.1 (> 0) \\ \therefore \text{ the area is a minimum value.} \end{aligned} \right\} \quad (\text{A1}) - \text{value must be shown}$$

Marker's remarks:

10.



The diagram shows part of the curve $y = 2 \sin \frac{x}{2}$, where A lies on the curve.

The x -coordinate of A is $\frac{4\pi}{3}$ and the normal to the curve at A meets the x -axis at B and the y -axis at C .

- (a) Find the coordinates of B and of C , leaving your answers in its **exact form**.

[6]

$$y = 2 \sin \frac{x}{2} \quad \frac{dy}{dx} = \cos \frac{x}{2} \quad (\text{M1})$$

$$\text{When } x = \frac{4\pi}{3} \quad \frac{dy}{dx} = \cos \frac{4\pi}{6} = -\frac{1}{2}$$

$$\text{Gradient of } AC = 2 \quad (\text{M1})$$

$$y = 2 \sin \frac{4\pi}{6} = \sqrt{3} \quad (\text{M1})$$

$$\text{Equation of } AC: \quad (y - \sqrt{3}) = 2 \left(x - \frac{4\pi}{3} \right) \quad (\text{M1})$$

$$y = 2x - \frac{8\pi}{3} + \sqrt{3}$$

$$\text{When } x = 0, \quad y = -\frac{8\pi}{3} + \sqrt{3} \quad \therefore C \text{ is } \left(0, \sqrt{3} - \frac{8\pi}{3} \right)$$

(A1) - Must be in exact value as required

$$\text{When } y = 0, \quad 0 = 2x - \frac{8\pi}{3} + \sqrt{3}$$

$$x = \frac{4\pi}{3} - \frac{\sqrt{3}}{2} \quad \therefore B \text{ is } \left(\frac{4\pi}{3} - \frac{\sqrt{3}}{2}, 0 \right)$$

(A1) - Must be in exact value as required

(b) Find the area of the shaded region.

[6]

Area = area under the curve – area of the triangle ABD + area of triangle BOC

$$= \int_0^{\frac{4\pi}{3}} y \, dx - \frac{1}{2}(\sqrt{3})\left(\frac{4\pi}{3} - \left(\frac{4\pi}{3} - \frac{\sqrt{3}}{2}\right)\right) + \frac{1}{2}\left(\frac{4\pi}{3} - \frac{\sqrt{3}}{2}\right)\left(\sqrt{3} - \frac{8\pi}{3}\right)$$

(M1) (M1) (M1)

$$= \int_0^{\frac{4\pi}{3}} 2\sin \frac{x}{2} \, dx - \frac{1}{2}(\sqrt{3})\left(\frac{\sqrt{3}}{2}\right) + \frac{1}{2}(3.3228)(6.6455)$$

$$= \left[-4\cos \frac{x}{2}\right]_0^{\frac{4\pi}{3}} - \frac{3}{4} + 11.041$$

(M1)

$$= 6 - \frac{3}{4} + 11.041 \quad (M1)$$

$$= 16.291$$

$$= 16.3 \text{ units}^2 \quad (A1)$$

Marker's remarks:

11. (a) Show that $\frac{d}{dx} \left[(2x)(3-5x)^{\frac{6}{5}} \right] = (6-22x)(3-5x)^{\frac{1}{5}}.$ [3]

$$\begin{aligned} \frac{d}{dx} \left[(2x)(3-5x)^{\frac{6}{5}} \right] &= (2x) \left(\frac{6}{5} \right) (3-5x)^{\frac{1}{5}} (-5) + (3-5x)^{\frac{6}{5}} (2) \\ &\quad \text{(M1)} \qquad \qquad \qquad \text{(M1)} \\ &= (2)(3-5x)^{\frac{1}{5}} [-6x + (3-5x)] \qquad \text{(M1)} \\ &= (2)(3-5x)^{\frac{1}{5}} [-11x + 3] \\ &= (6-22x)(3-5x)^{\frac{1}{5}} \quad \text{(Shown)} \end{aligned}$$

(b) Hence, evaluate $\int x(3-5x)^{\frac{1}{5}} dx.$ [3]

From Part (i):

$$\begin{aligned} \frac{d}{dx} \left[(2x)(3-5x)^{\frac{6}{5}} \right] &= (6-22x)(3-5x)^{\frac{1}{5}} \\ \left[(2x)(3-5x)^{\frac{6}{5}} \right] &= \int (6-22x)(3-5x)^{\frac{1}{5}} dx \\ \left[(2x)(3-5x)^{\frac{6}{5}} \right] &= \int 6(3-5x)^{\frac{1}{5}} dx - \int 22x(3-5x)^{\frac{1}{5}} dx \qquad \text{(M1)} \\ \int 22x(3-5x)^{\frac{1}{5}} dx &= \frac{6(3-5x)^{\frac{6}{5}}}{(-5)\left(\frac{6}{5}\right)} - (2x)(3-5x)^{\frac{6}{5}} + c \quad (c : \text{arbitrary constant}) \\ &\quad \text{(M1)} \\ &= (3-5x)^{\frac{6}{5}} (-1-2x) + c \\ \int x(3-5x)^{\frac{1}{5}} dx &= \frac{-1}{22}(1+2x)(3-5x)^{\frac{6}{5}} + d \quad (d : \text{arbitrary constant}) \qquad \text{(A1)} \end{aligned}$$

Marker's remarks:

End of paper