HUA YI SECONDARY SCHOOL PRELIMINARY EXAM 2024

4-G3 / 5-G2

NAME

CLASS

INDEX NUMBER

ADDITIONAL MATHEMATICS
PAPER 1

4049/01

22 September 2024
2 hour 15 minutes

Candidates answer on the Question Paper
No Additional Materials is required.

READ THESE INSTRUCTIONS FIRST

Write your Name, Class, and Index Number in the spaces at the top of this page.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue, or correction fluid.

Answer **all** the questions.
The number of marks is given in the brackets [ ] at the end of each question or part question.

If working is needed for any question it must be shown with the answer. Omission of essential working will result in loss of marks.
The total number of marks for this paper is 90.

The use of an approved scientific calculator is expected, where appropriate. If the degree of accuracy is not specified in the question and if the answer is not exact, give the answer to three significant figures. Gives answers in degrees to one decimal place. For π, use either your calculator value or 3.142.

This document consists of **21** printed pages and **1** blank page.

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

$$\Delta = \frac{1}{2}bc\sin A$$

1 A cuboid has a base area of $(7 + 4\sqrt{5})\text{cm}^2$ and a volume of $(16 + 18\sqrt{5})\text{cm}^3$. Find, without using a calculator, the height of the cuboid, in cm, in the form $(a + b\sqrt{5})$, where a and b are integers.

[3]

2 The curve $5x - xy = 20$ and the line $x - 2y - 3 = 0$ intersects at the points A and B. Find the y-coordinate of A and of B. [3]

3 **(a)** Express $12x - 13 - 3x^2$ in the form $a(x + b)^2 + c$ and hence state the coordinates of the turning point of the curve $y = 12x - 13 - 3x^2$. [4]

(b) State the range of k such that $y = k$ will intersect the curve $y = 12x - 13 - 3x^2$. [1]

4 Integrate $\frac{4}{5x+1} - \frac{6}{x^3}$ with respect to x. [2]

5 Express $\frac{7x^2-17x+1}{(x^2+1)(2-3x)}$ in partial fractions. [5]

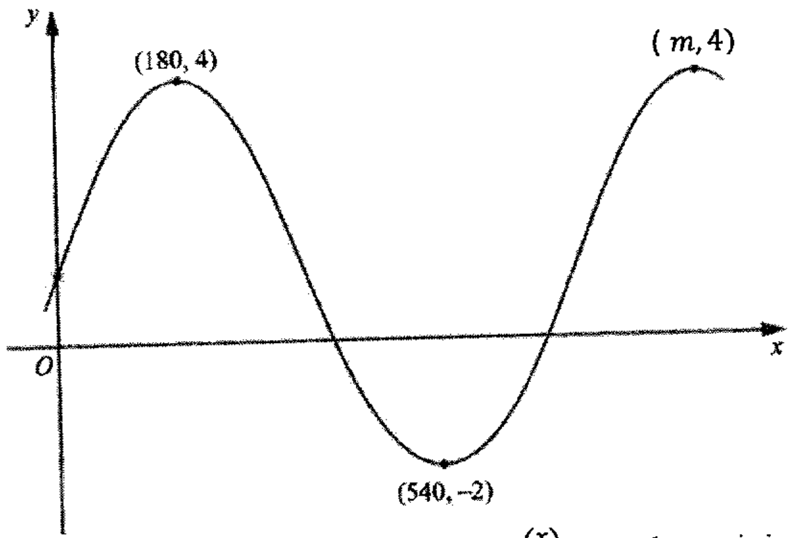
6 Given that $x^5 + ax^3 + bx^2 - 3 \equiv (x^2 - 1)\mathrm{Q}(x) - x - 2$, where $\mathrm{Q}(x)$ is a polynomial,

(a) state the degree of $\mathrm{Q}(x)$, [1]

(b) show that $a = -2$ and $b = 1$, [3]

(c) find the polynomial $\mathrm{Q}(x)$. [3]

7



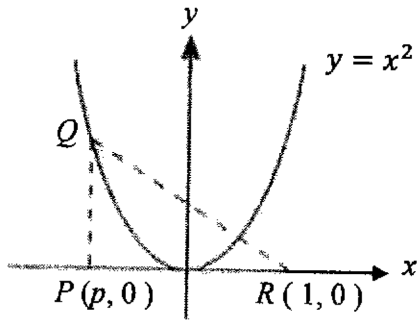
The sketch above shows part of the graph of $y = a\sin\left(\frac{x}{b}\right) + c$, where x is in degrees.

(a) Explain why $c = 1$. [2]

(b) State the value of b. [1]

(c) Find the value of m and explain how you get it. [2]

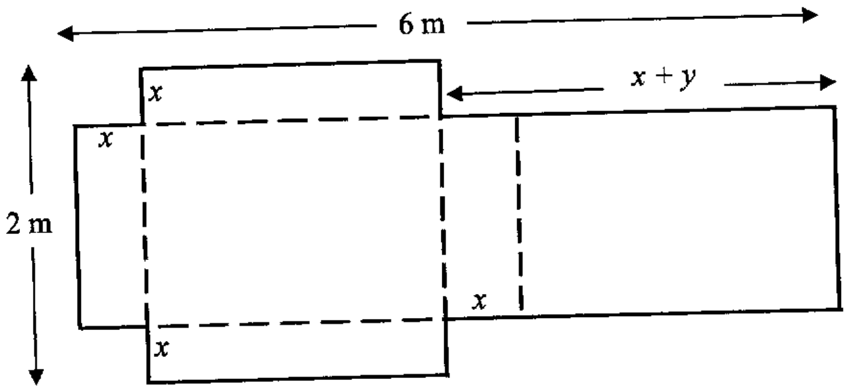
8 The figure shows the curve $y = x^2$ and the point $R(1,0)$. The variable point $P(p, 0)$ moves along the x axis and PQ is vertical. It is given that p decreases at the rate of 1.2 units per second.



(a) Show that the area of the triangle PQR, A units^2, is $A = \frac{1}{2}p^2 - \frac{1}{2}p^3$. [2]

(b) Find the rate at which A is increasing at the instant when $p = -7$. [4]

9 From a rectangular piece of metal of width 2m and length 6m, two squares of side x m and two rectangles of sides x m and $(x + y)$ m are removed as shown. The metal is then folded about the dotted lines to give a closed box with height x m.

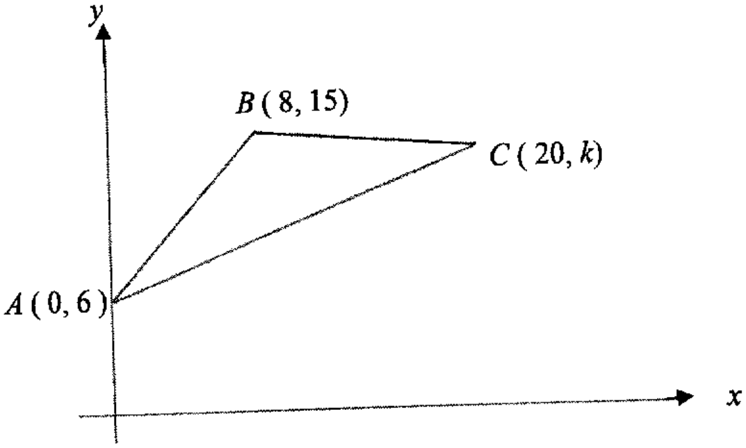


(a) Show that the volume of the box, V m^3, is given by $V = 2x^3 - 8x^2 + 6x$. [3]

(b) Given that x can vary, find the stationary value of V. [4]

(c) Show that this value of V is a maximum. [2]

10 The diagram shows a triangle ABC with vertices at $A\ (0, 6)$, $B\ (8, 15)$ and $C\ (20, k)$.



(a) Given that $AB = BC$, find the value of k. [4]

(b) D is a point on the x axis such that $ABCD$ is a kite. Find the coordinates of D. [4]

(c) Hence find the area of the kite $ABCD$. [2]

11 (a) Prove the identity $\cot\theta + \frac{\sin\theta}{1+\cos\theta} = \operatorname{cosec}\theta$. [3]

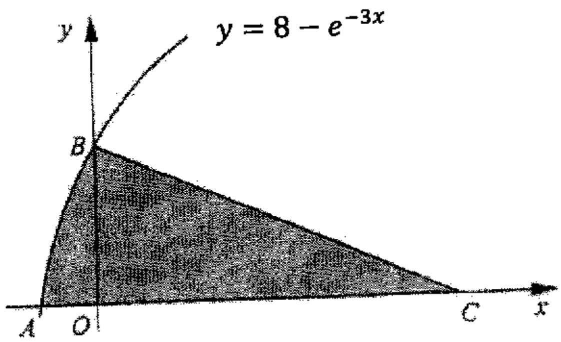
(b) Hence solve the equation $\cot 3\theta + \frac{\sin 3\theta}{1+\cos 3\theta} = -2$ for $-90° \leq \theta \leq 90°$. [4]

12 (a) Solve $6^{2x-1} = 4^x \times 5$. [4]

(b) Given that $\log_x 3 = p$, express $\log_3 \frac{27}{x}$ in terms of p. [2]

(c) Solve the equation $\lg x - 1 = \lg(x - 1)$ [3]

13 The diagram shows part of the curve $y = 8 - e^{-3x}$ which crosses the axes at A and at B.

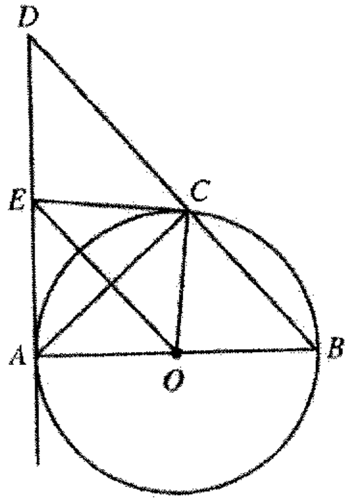


(a) Show that the x coordinate of A is $-\ln 2$. [3]

(b) The normal to the curve at B meets the x axis at C. Find the coordinates of C. [4]

(c) Find the area of the shaded region. [4]

14



The diagram shows a circle, centre O, with diameter AB. The point C lies on the circle. The tangent to the circle at A meets BC extended at D. The tangent to the circle at C meets the line AD at E.

(a) Prove that triangles EOA and EOC are congruent. [3]

(b) Prove that triangles ADC and BAC are similar. [3]

(c) Hence prove that $AC^2 = CD \times BC$. [2]

End of Paper

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1 A cuboid has a base area of $(7 + 4\sqrt{5})\text{cm}^2$ and a volume of $(16 + 18\sqrt{5})\text{cm}^3$. Find, without using a calculator, the height of the cuboid, in cm, in the form $(a + b\sqrt{5})$, where a and b are integers.

[3]

$$h = \frac{16+18\sqrt{5}}{7+4\sqrt{5}} \times \frac{7-4\sqrt{5}}{7-4\sqrt{5}}$$

$$= \frac{112-64\sqrt{5}+126\sqrt{5}-360}{-31}$$

$$= 8 - 2\sqrt{5}$$

2 The curve $5x - xy = 20$ and the line $x - 2y - 3 = 0$ intersects at the points A and B. Find the y −coordinate of A and of B. [3]

$sub\ x = 2y + 3\ into\ first\ equation,$

$5(2y + 3) - y(2y + 3) = 20$

$2y^2 - 7y + 5 = 0$

$(2y - 5)(y - 1) = 0$ or by quadratic formula

$$y = 2.5\ or\ 1$$

3 **(a)** Express $12x - 13 - 3x^2$ in the form $a(x+b)^2 + c$ and hence state the coordinates of the turning point of the curve $y = 12x - 13 - 3x^2$. [4]

$-3(x^2 - 4x) - 13$ or $-3(x^2 - 4x + \frac{13}{3})$ ------

$= -3[(x-2)^2 - 4] - 13$ or $= -3[(x-2)^2 - 4 + \frac{13}{3}]$

$= -3(x-2)^2 - 1$

Turning point $(2, -1)$

(b) State the range of k such that $y = k$ will intersect the curve $y = 12x - 13 - 3x^2$. [1]

$k \leq -1$

4 Integrate $\frac{4}{5x+1} - \frac{6}{x^3}$ with respect to x. [2]

$\int \frac{4}{5x+1} - 6x^{-3}dx = \frac{4\ln(5x+1)}{5} + \frac{3}{x^2} + c$

5 Express $\frac{7x^2-17x+1}{(x^2+1)(2-3x)}$ in partial fractions. [5]

$\frac{Ax+B}{x^2+1} + \frac{C}{2-3x}$ M1

$7x^2 - 17x + 1 = (Ax + B)(2 - 3x) + C(x^2 + 1)$

A = -4,

B = 3

C = -5

$\frac{3-4x}{x^2+1} - \frac{5}{2-3x}$-----------------

6 Given that $x^5 + ax^3 + bx^2 - 3 \equiv (x^2 - 1)Q(x) - x - 2$, where $Q(x)$ is a polynomial.

(a) State the degree of $Q(x)$. [1]

Degree of $Q(x) = 3$

(b) Show that $a = -2$ and $b = 1$. [3]

Sub $x = 1$, $a + b = -1$ ------equation

Sub $x = -1$, $-a + b = 3$ -------equation

Solve simultaneous equations,

$a = -2$ and $b = 1$ ---------------

(c) Find the polynomial $Q(x)$. [3]

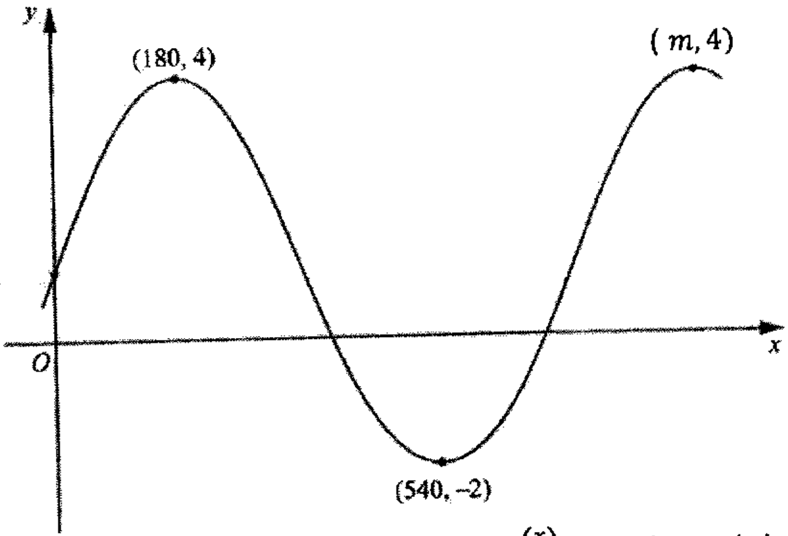
$Q(x)(x^2 - 1) = x^5 - 2x^3 + x^2 - 3 + x + 2$

$Q(x) = (x^5 - 2x^3 + x^2 - 3 + x + 2) \div (x^2 - 1)$---

By long division or comparing terms,

$$Q(x) = x^3 - x + 1$$

7



The sketch above shows part of the graph of $y = a\sin\left(\frac{x}{b}\right) + c$, where x is in degrees.

(a) Explain why $c = 1$. [2]

amplitude is 3.
So the maximum value of y is 4, 3+c =4, means that c = 1. for showing how to get c

(b) State the value of b. [1]

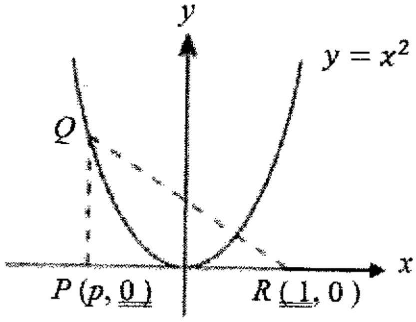
b = 2.

(c) Find the value of m and explain how you get it. [2]

The period is 720°. ---------------------

Hence m = 180 + 720 = 900

8 The figure shows the curve $y = x^2$ and the point $R(1,0)$. The variable point $P(p, 0)$ moves along the x axis and PQ is vertical. p is decreasing at the rate of 1.2 units per second.



(a) Show that he area of the triangle PQR, A units^2, is $A = \frac{1}{2}p^2 - \frac{1}{2}p^3$. [2]

getting y coordinate of Q = p^2 ----

A = $\frac{1}{2}(1 - p)(p^2)$
= $\frac{1}{2}p^2 - \frac{1}{2}p^3$(Shown)

(b) Find the rate at which A is increasing at the instant when $p = -7$ units. [4]

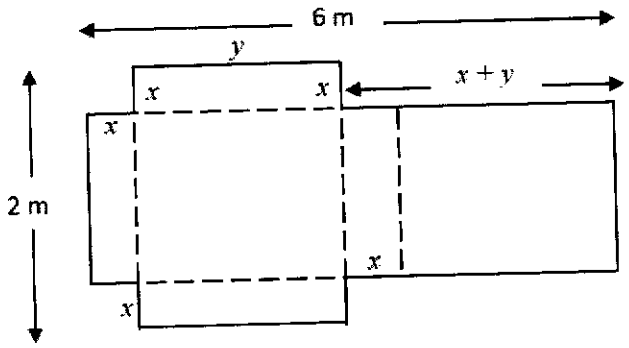
$$\frac{dA}{dt} = \frac{dA}{dp} \times \frac{dp}{dt}$$

$\frac{dA}{dt} = p - \frac{3}{2}p^2$------

$\frac{dA}{dt} = \frac{dA}{dp} \times \frac{dp}{dt}$-----

When p = -7,
$\frac{dA}{dt}$ = -80.5×-1.2
= 96.6 unit^2 per second

9 From a rectangular piece of metal of width 2m and length 6m, two squares of side x m and two rectangles of sides x m and $(x + y)$ m are removed as shown. The metal is then folded about the dotted lines. To give a closed box with height x m.



(a) Show that the volume of the box, V m^3, is given by $V = 2x^3 - 8x^2 + 6x$. [3]

Length = y , breadth = 2-2x , height = x

$x + y + x + y = 6$---→ $y = 3 - x$

V = $xy(2 - 2x)$
= $x(3 - x)(2 - 2x)$
= $2x^3 - 8x^2 + 6x$ (*shown*) – – – –

(b) Given that x can vary, find the stationary value of V. [4]

$\frac{dv}{dx} = 6x^2 - 16x + 6$ --

$$\frac{dv}{dx} = 0 \rightarrow 6x^2 - 16x + 6 = 0$$

Solve using quadratic formula (you need to show working)

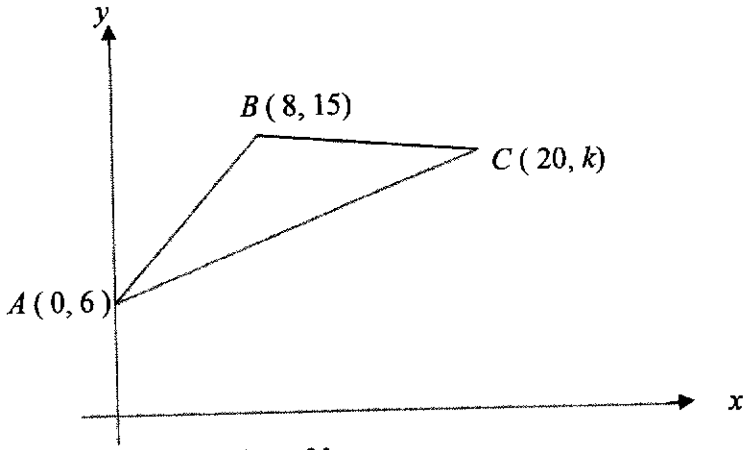
$$x = 2.215\ (reject)\ or \quad 0.4514$$

Stationary value of V = 1.26 m^3

(c) Shows that this value of V is the maximum. [2]

Show either by first or second derivative test

10 The diagram shows a triangle ABC with vertices at A (0, 6) , B (8, 15) and C (20, k).



(a) Given that $AB = BC$, find the value of k. [4]

Since AB = BC, it means $8^2 + 9^2 = 12^2 + (15 - k)^2$----

$(15 - k)^2 = 1$

(15 – k) = 1 or (k – 15) = -1

$k = 14$ -----

(b) D is a point on the x axis such that $ABCD$ is a kite. Find the coordinates of D. [4]

$M_{AC} = \frac{2}{5}$, so $M_{BD} = -\frac{5}{2}$

Midpoint of AC = (10, 10) ------

Find Equation of BD : $y = -\frac{5}{2}x + 35$

Get D (14, 0)

(c) Hence, find the area of the kite $ABCD$. [2]

Shoelace method or sum of area of triangles -----

Area of kite ABCD = 174 units^2------------

[3]

11 (a) Prove the identity $\cot\theta + \frac{\sin\theta}{1+\cos\theta} = \operatorname{cosec}\theta$.

$LHS = \frac{\cos\theta}{\sin\theta} + \frac{\sin\theta}{1+\cos\theta}$

$= \frac{\cos\theta + \square\, cos^2\theta + sin^2\theta}{\sin\theta(1+\cos\theta)}$ ----------

$= \frac{cos\theta + 1}{\sin\theta(1+\cos\theta)}$--------------

$= \frac{1}{\sin\theta}$

$= cosec\ \theta$

(b) Hence solve the equation $\cot 3\theta + \frac{\sin 3\theta}{1+\cos 3\theta} = -2$ for $-90° \leq \theta \leq 90°$. [4]

$cosec\ 3\theta = -2$------M1
$\sin 3\theta = -0.5$
Basic angle = 30

$3\theta = -30, -150, 210$
$\theta = -10, -50, 70$ ------------

12 (a) Solve $6^{2x-1} = 4^x \times 5$ [4]

$\dfrac{6^{2x}}{6} = 4^x \times 5$

$\frac{36^x}{4^x} = 30$

$9^x = 30$

$x = \log_9 30$ *or* 1.548

(b) Given that $\log_x 3 = p$, express $\log_3 \frac{27}{x}$ in terms of p. [2]

$\log_3 27 - \log_3 x$----------

$=3 - \frac{1}{p}$ ----------

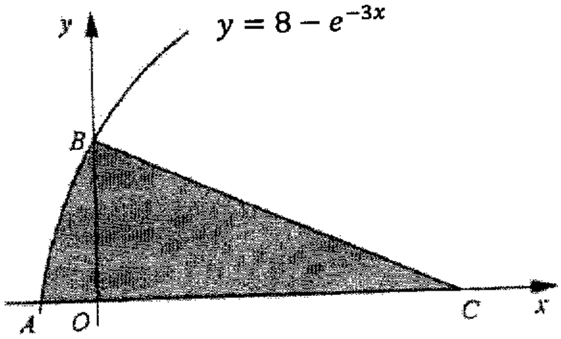
(c) Solve the equation $\lg x - 1 = \lg(x - 1)$ [3]

$$\lg x - \lg 10 = \lg(x - 1)$$

$$\frac{x}{10} = x - 1$$

$$x = \frac{10}{9}$$

13 The diagram shows part of the curve $y = 8 - e^{-3x}$ which crosses the axes at A and at B.



(a) Show that the x coordinate of A is $-\ln 2$. [3]

$when\ y = 0, e^{-3x} = 8$---
$-3x = \ln 8$
$x = -\frac{1}{3}\ln 8$----
$x = -\ln 8^{1/3}$
$x = -\ln 2$---------

(b) The normal to the curve at B meets the x axis at C. Find the coordinates of C. [4]

$\frac{dy}{dx} = 3e^{-3x}$---

At B, x = 0, so dy/dx = 3-----
Find Gradient of normal at B = -1/3

Find B (0, 7) -----------

Equation of normal : $y = -\frac{1}{3}x + 7$

So C (21, 0) ---------------

(c) Find the area of the shaded region. [4]

Area of OAB = $\int_{-\ln 2}^{0} 8 - e^{-3x} dx$

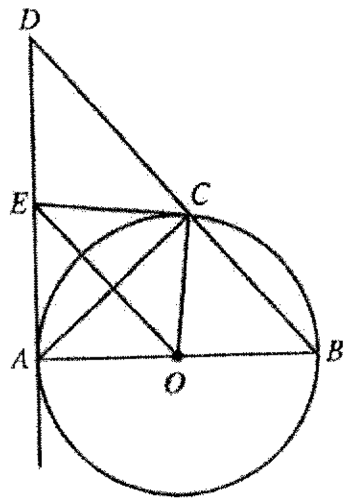
$= \left[8x + \frac{e^{-3x}}{3}\right]_{-\ln 2}^{0}$ ---------------

$= 8\ln 2 - \frac{7}{3}$-----------------

Area of triangle OBC = 0.5 x 21 x 7 = 73.5 -----

Area of shaded region = 76.7 units^2 ----------------

14



The diagram shows a circle, centre O, with diameter AB. The point C lies on the circle. The tangent to the circle at A meets BC extended at D. The tangent to the circle at C meets the line AD at E.

(a) Prove that triangles EOA and EOC are congruent. [3]

AE = CE (tangents from external point are equal)
AO = OC (radius)
OE = OE (common side) --------------------------

Hence triangle EOA is congruent to triangle EOC (SSS) -----------

(b) Prove that triangles ADC and BAC are similar. [3]

Angle ACB = 90 (angle in a semi circle)

Angle DCA = 90 (sum of angles on a straight line)

Hence angle ACB = angle DCA (A) -----

Angle CAD = Angle CBA (A)
(angles in alternate segment/ tangent chord theorem) ----

Hence triangles ADC and BAC are similar. (AA) -----------

(c) Hence prove that $AC^2 = CD \times BC$. [2]

$Since\ they\ are\ similar\ , \frac{AC}{BC} = \frac{DC}{AC}$----

$AB^2 = BC \times DC$-----------

- End of Paper -

HUA YI SECONDARY SCHOOL
PRELIMINARY EXAM 2024

4-G3 / 5-G2

NAME			
CLASS		INDEX NUMBER	

ADDITIONAL MATHEMATICS 4049/02
PAPER 2

08 October 2024

2 hour 15 minutes

Candidates answer on the Question Paper

No Additional Materials is required.

READ THESE INSTRUCTIONS FIRST

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Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue, or correction fluid.

Answer all the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

The number of marks is given in the brackets [ ] at the end of each question or part question.

The total number of marks for this paper is 90.

For Examiner's Use	
	90

This document consists of **18** printed pages.

1. ALGEBRA

Quadratic Equation

For the equation $ax^2+bx+c=0$,

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

Binomial expansion

$$(a+b)^n=a^n+\binom{n}{1}a^{n-1}b+\binom{n}{2}a^{n-2}b^2+\ldots+\binom{n}{r}a^{n-r}b^r+\ldots+b^n,$$

where n is a positive integer and $\binom{n}{r}=\frac{n!}{r!(n-r)!}=\frac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A+\cos^2 A=1$$

$$\sec^2 A=1+\tan^2 A$$

$$\operatorname{cosec}^2 A=1+\cot^2 A$$

$$\sin(A\pm B)=\sin A\cos B\pm\cos A\sin B$$

$$\cos(A\pm B)=\cos A\cos B\mp\sin A\sin B$$

$$\tan(A\pm B)=\frac{\tan A\pm\tan B}{1\mp\tan A\tan B}$$

$$\sin 2A=2\sin A\cos A$$

$$\cos 2A=\cos^2 A-\sin^2 A=2\cos^2 A-1=1-2\sin^2 A$$

$$\tan 2A=\frac{2\tan A}{1-\tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A}=\frac{b}{\sin B}=\frac{c}{\sin C}$$

$$a^2=b^2+c^2-2bc\cos A$$

$$\Delta=\frac{1}{2}ab\sin C$$

1 Show that $x = \frac{1}{2}$ is a solution of the equation $2x^3 + x^2 - 3x + 1 = 0$ and hence solve the equation completely. [5]

2 **(a)** By considering the general term in the binomial expansion of $\left(px + \frac{1}{x^3}\right)^9$, where p is a constant, explain why there are no even powers of x in this expansion. [3]

(b) Given that the coefficient of x^8 is equal to the coefficient of x in the expansion of $(2x^3 + 1)\left(px + \frac{1}{x^3}\right)^9$, find the value of p. [4]

(c) Using the value of p in **(b)**, find the term independent of x in the expansion of $(2x^3 + 1)\left(px + \frac{1}{x^3}\right)^9$. [2]

3 (a) Given that $y = \frac{2x}{(3x+1)^{\frac{1}{2}}}$, show that $\frac{\mathrm{dy}}{\mathrm{dx}} = \frac{3x+2}{(3x+1)^{\frac{3}{2}}}$. [4]

(b) Hence find the value of $\int_0^2 \frac{x}{(3x+1)^{\frac{3}{2}}}\,\mathrm{dx}$. [5]

4 Show that the equation $5e^x = \frac{1}{e^x} - 4$ has only one solution and find its value correct to 2 decimal places. [4]

5 The equation of a curve is $y = -2x^2 + 3x + 5$.

(a) Find the set of values for x for which the curve lies below the line $y = 3$ and represent this set of values on a number line. [4]

The line $y = x + k$ is a tangent to the curve at the point Z.

(b) Find the value of the constant k. [3]

(c) Find the coordinates of Z. [2]

6 **(a)** The speed V m/s of a vehicle, t s after passing a fixed point O, is given for $t \geq 0$, as

$$V = 1 + pe^{qt}, \text{ where } p \text{ and } q \text{ are constants.}$$

Explain how a straight line can be drawn to represent the formula, and state how the value of p and q can be obtained from the line. [4]

(b)(i) Data of the speeds of the vehicle was collected. The table shows the corresponding values of V and t.

t	2	4	6	8	10
V	10.35	8.40	6.70	5.42	5.95

Using **(a)**, draw the straight line graph on the next page. [3]

(ii) Estimate the values of p and q. [3]

(iii) Estimate a value of V to replace one incorrect recording of V found in the straight line graph. [2]

7 A motorcyclist, travelling along a straight road, passes a lamp post X, with speed of h km/h. A while later, the motorcyclist passes a second lamp post Y, with a speed of 60 km/h.
Between the two lamp posts, the speed is given by $V = 20e^{50t} + 10\,\text{km/h}$, where t, the time after passing lamp post X is measured in hours.

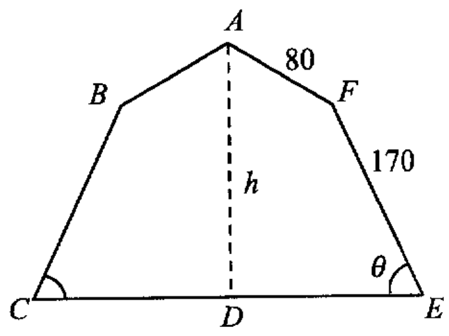
(a) Find the value of h. [2]

(b) Calculate to the nearest second, the time taken to travel from X to Y. [3]

(c) Find the acceleration of the motorcyclist as he passes Y. [3]

(d) Find the distance XY. [5]

8 The diagram shows the side view $ABCDEF$ of an ornament. The ornament rests with CE on horizontal ground and is symmetrical about the vertical AD, where D is the midpoint of CE.
Angle DEF = Angle $DAF = \theta$ radians and the lengths of AF and FE are 80 cm and 170 cm respectively. The vertical height of the ornament is h cm.



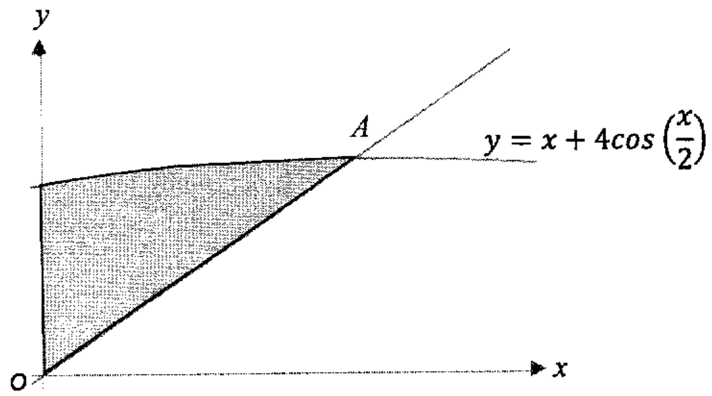
(a) Explain clearly why $h = 80cos\theta + 170sin\theta$. [2]

(b) Express h in the form $Rsin(\theta + \alpha)$, where $R > 0$ and α is an acute angle. [3]

(c) Find the greatest possible value of h and the value of θ at which this occurs. [3]

(d) Find the values of θ when $h = 180$ cm. [2]

9



The diagram shows the curve $y = x + 4cos\left(\frac{x}{2}\right)$ for $0 \leq x \leq \frac{\pi}{2}$ radians.
The point A is the stationary point of the curve and OA is a straight line.

(a) Find the exact coordinates of A. [5]

(b) Show that the area of the shaded region is $4 - \frac{\sqrt{3}}{3}\pi$ units^2. [5]

10 A tangent to a circle at the point (3,2) cuts the y-axis at 5. The line with the equation $3y = 2x + 5$ is normal to the circle.

(a) Find the equation of the circle, showing your working clearly. [7]

(b) Find the equations of tangents to the circle that are parallel to the x-axis. [2]

End of Paper

HUA YI SECONDARY SCHOOL PRELIMINARY EXAM 2024

4-G3 / 5-G2

NAME

CLASS

INDEX NUMBER

ADDITIONAL MATHEMATICS **4049/02**

PAPER 2

26 August 2024

2 hour 15 minutes

Candidates answer on the Question Paper

No Additional Materials is required.

ANSWER SCHEME

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1} a^{n-1} b + \binom{n}{2} a^{n-2} b^2 + \ldots + \binom{n}{r} a^{n-r} b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2} ab \sin C$$

1 Show that $x = \frac{1}{2}$ is a solution of the equation $2x^3 + x^2 - 3x + 1 = 0$ and hence solve the equation completely. [5]

$f(x) = 2x^3 + x^2 - 3x + 1$
$f\left(\frac{1}{2}\right) = 2\left(\frac{1}{2}\right)^3 + \left(\frac{1}{2}\right)^2 - 3\left(\frac{1}{2}\right) + 1$ ----------------------M1
$= 0$
$f(x) = (2x - 1)(x^2 + bx - 1)$ -------------------------M1

Compare coeff. of x^2, $1 = 2b - 1$
b= 1 ----------------------------M1

$f(x) = (2x - 1)(x^2 + x - 1)$-------------------------M1
$x = \frac{1}{2}, \frac{-1+\sqrt{5}}{2}, \frac{-1-\sqrt{5}}{2}$ --A1

Alternative Method : Using long division to find $(x^2 + x - 1)$.

2 (a) By considering the general term in the binomial expansion of $\left(px + \frac{1}{x^3}\right)^9$, where p is a constant, explain why there are no even powers of x in this expansion. [3]

General Term $=\binom{9}{r}(px)^{9-r}\left(\frac{1}{x^3}\right)^r$ -----------------M1

$= \binom{9}{r} p^{9-r} x^{9-4r}$ --------------------M1

Since $9 - 4r = 1 + 4(2 - r)$, one added to any even number will give an odd value, hence there are no even powers of x in this expansion. --------A1

(b) Given that the coefficient of x^8 is equal to the coefficient of x in the expansion of $(2x^3+1)\left(px+\frac{1}{x^3}\right)^9$, find the value of p. [4]

$(2x^3+1)\left(\dots\dots\binom{9}{1}(px)^8\left(\frac{1}{x^3}\right)^1+\binom{9}{2}(px)^7\left(\frac{1}{x^3}\right)^2\right)^9$ --------------------M1

$=(2x^3+1)(\dots+9p^8x^5+36p^7x+\cdots)$ --------------------------------M1

$2(9p^8)=36p^7$ --M1
$p=2$ ---A1

(c) Using the value of p in **(b)**, find the term independent of x in the expansion of $(2x^3+1)\left(px+\frac{1}{x^3}\right)^9$. [2]

Term independent of $x=(2)\binom{9}{3}(2^6)$ ---------------------M1
$=10752$ --------------------------A1

3 (a) Given that $y = \frac{2x}{(3x+1)^{\frac{1}{2}}}$, show that $\frac{dy}{dx} = \frac{3x+2}{(3x+1)^{\frac{3}{2}}}$. [4]

$\frac{dy}{dx} = \frac{2(3x+1)^{\frac{1}{2}} - 2x\left(\frac{1}{2}\right)(3)(3x+1)^{-\frac{1}{2}}}{(3x+1)^1}$ ------------------M2 (numerator and denominator)

$= \frac{2(3x+1)-3x}{(3x+1)^{\frac{3}{2}}}$ ----------------------------------M1

$= \frac{3x+2}{(3x+1)^{\frac{3}{2}}}$ --A1

(b) Hence find the value of $\int_0^2 \frac{x}{(3x+1)^{\frac{3}{2}}}\, dx$. [5]

$From\ \frac{dy}{dx} = \frac{3x+2}{(3x+1)^{\frac{3}{2}}}$,

$$\frac{dy}{dx} = \frac{3x}{(3x+1)^{\frac{3}{2}}} + \frac{2}{(3x+1)^{\frac{3}{2}}}$$

$$\frac{3x}{(3x+1)^{\frac{3}{2}}} = \frac{dy}{dx} - \frac{2}{(3x+1)^{\frac{3}{2}}}$$

$\int_0^2 \frac{3x}{(3x+1)^{\frac{3}{2}}} dx = \frac{2x}{(3x+1)^{\frac{1}{2}}} - \int_0^2 \frac{2}{(3x+1)^{\frac{3}{2}}}\, dx$ ---------------------------------M1

$$\int_0^2 \frac{x}{(3x+1)^{\frac{3}{2}}} dx = \frac{1}{3}\left[\frac{2x}{(3x+1)^{\frac{1}{2}}}\right]_0^2 - \frac{1}{3}\int_0^2 \frac{2}{(3x+1)^{\frac{3}{2}}}\, dx$$

$= \frac{1}{3}\left[\frac{2x}{(3x+1)^{\frac{1}{2}}}\right]_0^2 - \frac{1}{3}\left[\frac{2(3x+1)^{-\frac{1}{2}}}{\left(-\frac{1}{2}\right)(3)}\right]_0^2$ ---------------------M1, M1

$= \frac{1}{3}\left(\frac{4}{\sqrt{7}} - 0\right) + \left(\frac{4}{9}\right)\left(\frac{1}{\sqrt{7}} - 1\right)$ ----------------------------M1

$= 0.227$ -----------------------------------A1

4 Show that the equation $5e^x = \frac{1}{e^x} - 4$ has only one solution and find its value correct to 2 decimal places. [4]

$$5e^x = \frac{1}{e^x} - 4$$

$5e^{2x} = 1 - 4e^x$
$5e^{2x} + 4e^x - 1 = 0$ ------------------------M1
$5(e^x)^2 + 4e^x - 1 = 0$
$(5e^x - 1)(e^x + 1) = 0$ --------------------M1
$e^x = \frac{1}{5}$ or $e^x = -1$
$x = ln\left(\frac{1}{5}\right)$ (no solution, reject)----------A1
$= -1.61$ (2 dp) ------A1

5 The equation of a curve is $y = -2x^2 + 3x + 5$.

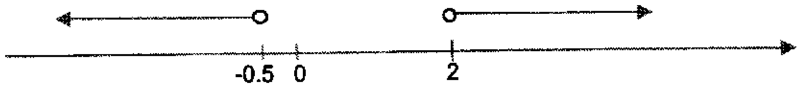
(a) Find the set of values for x for which the curve lies below the line $y = 3$ and represent this set of values on a number line. [4]

$-2x^2 + 3x + 5 < 3$ ----------------------M1

$2x^2 - 3x - 2 > 0$

$(2x + 1)(x - 2) > 0$ ---------------------M1

$x < -\frac{1}{2}$ or $x > 2$ ----------------------A1



A1

The line $y = x + k$ is a tangent to the curve at the point Z.

(b) Find the value of the constant k. [3]

$-2x^2 + 3x + 5 = x + k$
$2x^2 - 2x + k - 5 = 0$ --------------------------------M1
Line is tangent to curve => $b^2 - 4ac = 0$

$(-2)^2 - 4(2)(k - 5) = 0$ ----------------------------M1
$4 - 8k + 40 = 0$
$k = \frac{11}{2}$ or $5\frac{1}{2}$ --A1

(c) Find the coordinates of Z. [2]

$y = x + \frac{11}{2}$-------------eqn 1
Sub eqn 1 into eqn of curve,
$-2x^2 + 3x + 5 = x + \frac{11}{2}$ ----------------M1
$-4x^2 + 6x + 10 = 2x + 11$
$-4x^2 + 4x - 1 = 0$
$4x^2 - 4x + 1 = 0$
$(2x - 1)^2 = 0$
$x = \frac{1}{2}, y = 6$
$Z = \left(\frac{1}{2},\ 6\right)$ -----------------------------A1

6 **(a)** The speed V m/s of a vehicle, t s after passing a fixed point O, is given for $t \geq 0, V = 1 + pe^{qt}$, where p and q are constants.
Explain how a straight line can be drawn to represent the formula, and state how the value of p and q can be obtained from the line. [4]

$V - 1 = pe^{qt}$
$\ln(V-1) = \ln(pe^{qt})$
$\ln(V-1) = \ln p + qt$ --------------------M1
Draw $\ln(V-1)$ against t ---------------M1

y- intercept = $\ln p$ --------------------A1
gradient = q -------------------------- A1

(b)(i) Data of the speeds of the vehicle was collected. The table below shows the corresponding values of V and t.

t	2	4	6	8	10
V	10.35	8.40	6.70	5.42	5.95

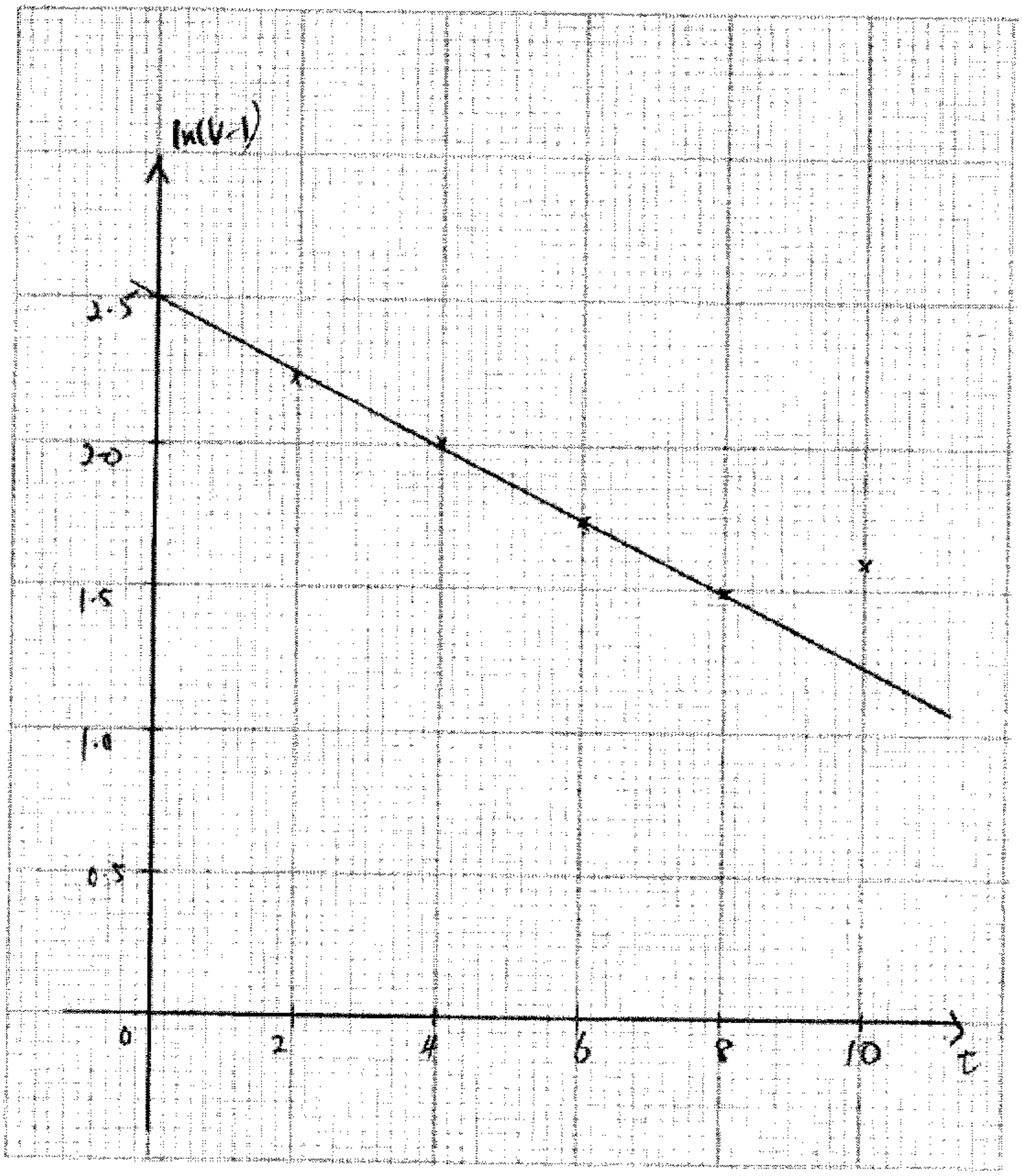
Using **(a)**, draw the straight line graph on the next page. [3]

t	2	4	6	8	10
$\ln(V\text{-}1)$	2.24	2.00	1.74	1.49	1.60

M1 - Calculate $\ln(v-1)$, M1 – correct points plotted, M1 – Best fit line

(ii) Estimate the values of p and q. [3]

$q = \frac{2.5-1.5}{0-8}$ --------------------M1
$= -0.125$ ---------------------A1
$lnp = 2.5$
$p = 12.18$ ----------------A1



(iii) Estimate a value of V to replace one incorrect recording of V found in the straight line graph. [2]

Incorrect value of V occurs at $x = 10$.
$\ln(V-1) = 1.25$ ------------------------------M1
$V = 4.49$ ------------------------------------A1

7 A motorcyclist, travelling along a straight road, passes a lamp post X, with speed of h km/h. A while later, the motorcyclist passes a second lamp post Y, with a speed of 60 km/h.
Between the two lamp posts, the speed is given by $V = 20e^{50t} + 10$ km/h where t, the time after passing lamp post X is measured in hours.

(a) Find the value of h. [2]

$t = 0,\ h = 20e^0 + 10$ ---------------------------M1

$h = 30\ km/h$ -------------------------------------A1

(b) Calculate to the nearest second, the time taken to travel from X to Y. [3]

$60 = 20e^{50t} + 10$ --------------M1

$ln(2.5) = 50t$

$t = 0.0183258\ h$ ----------------M1

$= 66$ seconds ----------------A1

(c) Find the acceleration of the motorcyclist as he passes Y. [3]

Acceleration $= \frac{dv}{dt}$

$= 20(50)e^{50t}$ -----------------M1

$= 20\ (50)e^{50(0.0183258)}$------------M1

$= 2500\ km/h^2$ -----------------A1

(d) Find the distance XY. [5]

Distance XY

$= \int_0^{\frac{ln2.5}{50}} v dt$

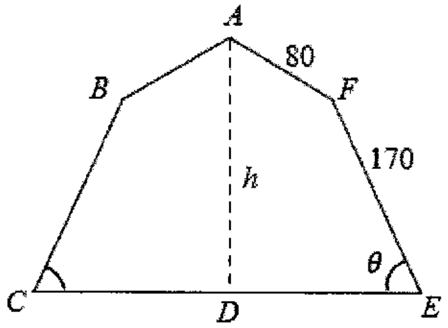
$= \int_0^{\frac{ln2.5}{50}} 20e^{50t} + 10dt$ ---------------------M1

$= \left[\frac{20}{50}e^{50t} + 10t\right]_0^{\frac{ln2.5}{50}}$ ----------------------M1, M1

$= \left(\frac{2}{5}e^{ln2.5} + 10\left(\frac{ln2.5}{50}\right)\right) - \frac{2}{5}$------------------M1

$= 0.783km$ --------------------------------A1

8 The diagram shows the side view *ABCDEF* of an ornament. The ornament rests with *CE* on horizontal ground and is symmetrical about the vertical *AD*, where *D* is the midpoint of *CE*.
Angle *DEF* = Angle *DAF* = θ radians and the lengths of *AF* and *FE* are 80 cm and 170 cm respectively. The vertical height of the ornament is h cm.



(a) Explain clearly why $h = 80cos\theta + 170sin\theta$ [2]

$cos\theta = \frac{AM}{80}$ $sin\theta = \frac{FN}{170}$ -----------------M1

$h = AM + MD$
$= 80cos\theta + 170sin\theta$------------------------A1

(b) Express h in the form $Rsin(\theta + \alpha)$, where $R > 0$ and α is an acute angle. [3]
$h = 170sin\theta + 80cos\theta$
$R = \sqrt{80^2 + 170^2} = \sqrt{35300}$ or 187.88 ------------------------M1
$\alpha = tan^{-1}\left(\frac{80}{170}\right) = 0.44\ rad$------------------------------------M1
$\sqrt{35300}\ sin(\theta + 0.44)$ --A1

(c) Find the greatest possible value of h and the value of θ at which this occurs. [3]

Greatest value of h =187.88 ----------------A1
$sin(\theta + 0.44) = 1$ --------------------------M1
$\theta + 0.44 = \frac{\pi}{2}$
$\theta = 1.13\ rad$ ------------------------------A1

(d) Find the value of θ when h = 180 cm. [2]

$\sqrt{35300}sin(\theta + 0.44) = 180$
$\theta + 0.44 = 1.280$, 1.862 --------------------------------M1
$\theta = 0.84$ rad , 1.42 rad------------------------------------A1

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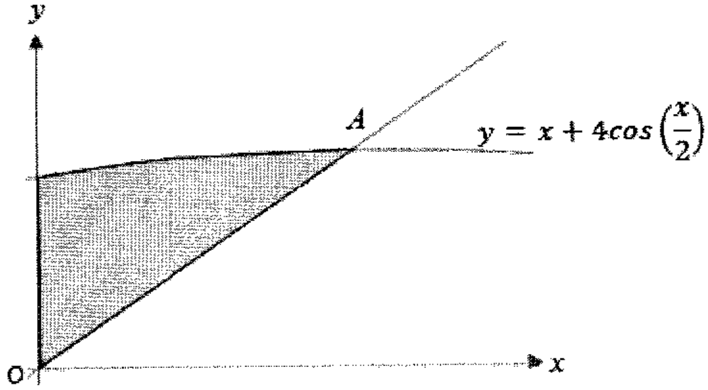

The diagram shows the curve $y = x + 4cos\left(\frac{x}{2}\right)$ for $0 \leq x \leq \pi$ radians. The point A is the stationary point of the curve and OA is a straight line.

(a) Find the coordinates of A. [5]

$\frac{dy}{dx} = 1 - 2sin\left(\frac{x}{2}\right)$ ----------------------M1, M1

At the stationary point A, $\frac{dy}{dx} = 0$

$1 - 2sin\left(\frac{x}{2}\right) = 0$ ------------M1

$sin\left(\frac{x}{2}\right) = 0.5$

$x = \frac{\pi}{3}$ ------------------------M1

$y = \frac{\pi}{3} + 2\sqrt{3}$

$A = \left(\frac{\pi}{3}, \frac{\pi}{3} + 2\sqrt{3}\right)$ ---------------------------A1

(b) Show that the area of the shaded region is $4 - \frac{\sqrt{3}}{3}\pi$ units^2. [5]

Shaded area

$= \int_0^{\frac{\pi}{3}} x + 4cos\left(\frac{x}{2}\right) dx - \frac{1}{2}\left(\frac{\pi}{3}\right)\left(\frac{\pi}{3} + 2\sqrt{3}\right)$ ------------------- M1 (Area of Δ)

$= \left[\frac{x^2}{2} + \frac{4sin\left(\frac{x}{2}\right)}{\frac{1}{2}}\right]_0^{\frac{\pi}{3}} - \frac{\pi^2}{18} - \frac{\pi\sqrt{3}}{3}$ -------------M2 (Integration of the two terms)

$= \frac{\pi^2}{18} + 8sin\left(\frac{\pi}{2}\right) - 0 - \frac{\pi^2}{18} - \frac{\pi\sqrt{3}}{3}$ -------------------M1

$= 4 - \frac{\sqrt{3}}{3}\pi$ --------------------------A1

10 A tangent to a circle at the point (3,2) cuts the y-axis at 5. The line with the equation $3y = 2x + 5$ is normal to the circle.

(a) Show all your workings, find the equation of the circle. [7]

Gradient of tangent $= \frac{5-2}{0-3} = -1$ ---------------------M1
Gradient of normal =1 -------------------------------M1
Eqn of normal : $y = x - 1$ --------------------------M1

Solve simultaneous eqns of the two normals to get the centre of circle.
----M1
$y = x - 1$ -------------eqn 1
$3y = 2x + 5$ -----------eqn 2
Sub eqn (1) into eqn 2, $3(x-1) = 2x + 5$
Centre : $x = 8, y = 7$ -----------------M1
Radius of circle = $\sqrt{(8-3)^2 + (7-2)^2} = \sqrt{50}$ -----------M1
Equation of Circle : $(x-8)^2 + (y-7)^2 = 50$ ----------A1

(b) Find the tangents to the circle that are parallel to the x-axis. [2]

$y = 7 + \sqrt{50}$ and $y = 7 - \sqrt{50}$-------------B2