General Certificate of Education Ordinary Level
JUYING SECONDARY SCHOOL, SINGAPORE
Secondary Four Express/Five Normal Academic
Preliminary Examination

CANDIDATE NAME	

CENTRE NUMBER	S					INDEX NUMBER					

ADDITIONAL MATHEMATICS 4049/01

Paper 1

22 August 2024

2 hours 15 minutes

Candidates answer on the Question Paper.

READ THESE INSTRUCTIONS FIRST

Write your Centre number, index number and name on all the work you hand in.
Write in dark blue or black pen.
You may use a pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** questions.
The number of marks is given in brackets [ ] at the end of each question or part question.

If working is needed in any question it must be shown with the answer.
Omission of essential working will result in loss of marks.
The total number of marks for this paper is 90.

The use of an approved scientific calculator is expected, where appropriate.
If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.
For π, use either your calculator value or 3.142.

This document consists of **18** printed pages.

Set by: Mr Albert Lui
Vetted by: Mdm Norhafiani Bte Abdul Majid

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a + b)^n = a^n + \binom{n}{1} a^{n-1} b + \binom{n}{2} a^{n-2} b^2 + \ldots + \binom{n}{r} a^{n-r} b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \dfrac{n!}{r!(n-r)!} = \dfrac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2} ab \sin C$$

Answer **ALL** the questions

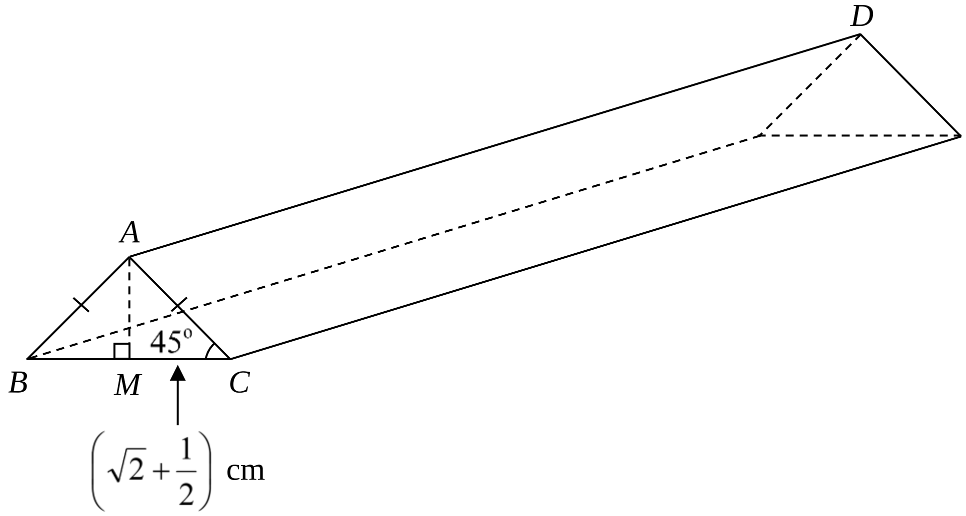
1 **(a)** The function f is defined, for all values of x, by $f(x) = (2x - x^2)e^x$.

Find the range of values of x such that $f(x)$ is a decreasing function. [4]

(b) The gradient function of the curve is $2(p+1)x + 2$, where p is a constant.

Given that the tangent to the curve at $(2\,, -2)$ is parallel to $y + 2x - 5 = 0$, find the value of p. [3]

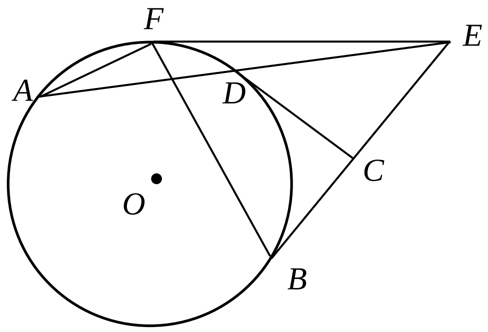
2 The diagram shows a chocolate bar in the form of a triangular prism and the cross-section of the chocolate bar is an isosceles triangle with $AB = AC$.
$MC = \left(\sqrt{2} + \frac{1}{2}\right)$ cm and $\angle ACB = 45°$.



(a) Find the exact length of AC. [3]

(b) Given that the volume of the chocolate bar is $(25 + 22\sqrt{2})\text{cm}^3$, find the length of AD in the form $(a + b\sqrt{2})$ cm, where a and b are integers. [4]

3



The diagram shows a circle, centre O, with diameter AB. The points D and F lie on the circle. The point E is such that EB and EF are tangents to the circle.

(a) Given that the points C and D are midpoints of BE and AE respectively, prove that angle $DCE = 90°$. [3]

(b) Given that triangle BEF is equilateral, prove that $\angle BEF = \angle BAF$. [2]

4 **(a)** Find the remainder when $6x^3 - 13x^2 + 17x - 6$ is divided by $2x - 1$. [2]

(b) Show that there is only one real root of the equation

$6x^3 - 13x^2 + 17x - 6 = 0$. [3]

5 Solve the following equations.

(a) $5^x - 5^{\frac{x}{2}+1} = 6,$ [3]

(b) $2\lg(x-3) - \lg(x+7) = \frac{1}{\log_{100} 10}.$ [4]

6 **(a)** State the values between which the principal value of $\sin^{-1} x$ must lie. [1]

(b) Find the principal value of $\tan^{-1} 1$ in radian in exact form. [1]

7 Given that $\cot\theta = -\frac{3}{4}$ and that $\tan\theta$ and $\cos\theta$ have opposite signs, without evaluating θ, find the exact values of each of the following.

(a) $\cos(-\theta)$, [2]

(b) $\sin 2\theta$ [2]

8. The approximate mean distance ***x*** (in millions of kilometres) from the centre of the Sun and the period of the orbit ***T*** (in Earth years) are recorded in the table.

	Mercury	**Venus**	**Mars**	**Uranus**
x	58	108	228	2871
T	0.24	0.62	1.88	84.11

It is believed that the planets orbiting around the Sun obey a law of the form $T = kx^n$, where k and n are constants.

(a) Express the equation in a form suitable for drawing a straight line graph and draw the graph using appropriate scaling on both axes. [4]

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(c) Using the graph, find the orbital period of the Earth, if the distance between the Earth and the Sun is about $149.6 \times 10^6\, km$. Give your answer correct to the nearest integer. [2]

(d) If the orbital period of the Jupiter is 11.86 Earth years, estimate the distance of the Jupiter from the Sun in km using your graph. [2]

9. **(a)** Express $\frac{2x^3+2x^2-7x+4}{x(x-1)^2}$ in partial fractions. [5]

(b) Hence evaluate $\int_2^4 \frac{4x^3+4x^2-14x+8}{3x(x-1)^2}\,dx$. [4]

10. **(a)** Find the range of values of k for which the line $2x - y = 5$ intersects the curve $xy = kx - 2$ at two distinct points. [4]

(b) Find the smallest integer value of h for which the graph $y = 2x^2 - 4x + h$ lies entirely above the line $y = 3$ for all values of x. [3]

11. **(a)** Prove the identity $\frac{1+\cos\theta}{\sin\theta}+\frac{\sin\theta}{1+\cos\theta}=2\operatorname{cosec}\theta$. [4]

(b) Hence, find all the angles from $0° \leq \theta \leq 360°$ which satisfy the equation $\frac{1+\cos 2\theta}{\sin 2\theta}+\frac{\sin 2\theta}{1+\cos 2\theta}=\tan 75°$. [3]

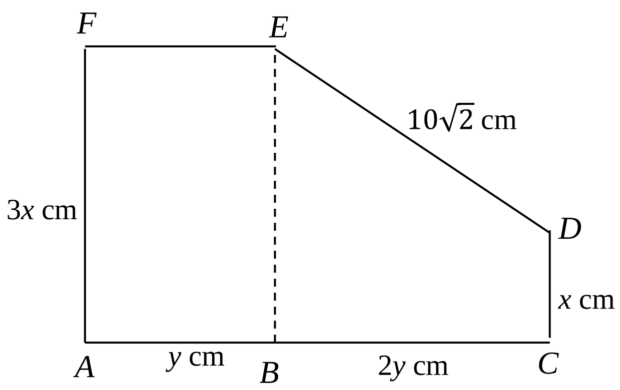
12. Find the derivatives of each of the following, simplifying your answer.

(a) $y = 3\left(1 - \frac{x}{3}\right)^4$ [1]

(b) $f(x) = (2 - 3x)\left(\sqrt{1 - 4x}\right)$ [3]

(c) $\frac{dy}{dx} = \frac{2(3x-2)}{4+x}$ [2]

13.



The diagram shows a glass window $ABCDEF$, consisting of a rectangle $ABEF$ of height $3x$ cm and width y cm and a trapezium $BCDE$ in which $CD = x$ cm and $BC = 2y$ cm. ABC is a straight line and $DE = 10\sqrt{2}$ cm.

Given that x can vary,

(a) show that the area of the glass window $S = 7x\left(\sqrt{50 - x^2}\right)$, [3]

(b) find the value of x for which S has a stationary value and determine whether this value of S is a maximum or a minimum. [5]

14. It is given that $f(x)$ is such that $f'(x) = \cos 4x - \sin 2x$. Given also that $f\left(\frac{\pi}{2}\right) = \frac{1}{4}$, show that $f''(x) + 4f(x) = 3 - 3\sin 4x$. [5]

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Find the range of values of x such that $f(x)$ is a decreasing function. [4]

$f'(x) = (2 - 2x)e^x + (2x - x^2)e^x$

$= e^x(2 - x^2)$ B1

Decreasing Function:

$f'(x) < 0$

$e^x(2 - x^2) < 0$ M1

Since $e^x > 0$,

$2 - x^2 < 0$

$x^2 - 2 > 0$

$(x + \sqrt{2})(x - \sqrt{2}) > 0$ M1

$x < -\sqrt{2}$ $x > \sqrt{2}$ A1

(b) The gradient function of the curve is $2(p + 1)x + 2$, where p is a constant.

Given that the tangent to the curve at $(2, -2)$ is parallel to $y + 2x - 5 = 0$, find the value of p. [3]

$\frac{dy}{dx} = 2(p + 1)x + 2$

$2(p + 1)x + 2 = -2$ M1

$(p + 1)x = -2$ M1

When $x = 2$,

$2p + 2 = -2$

$p = -2$ A1

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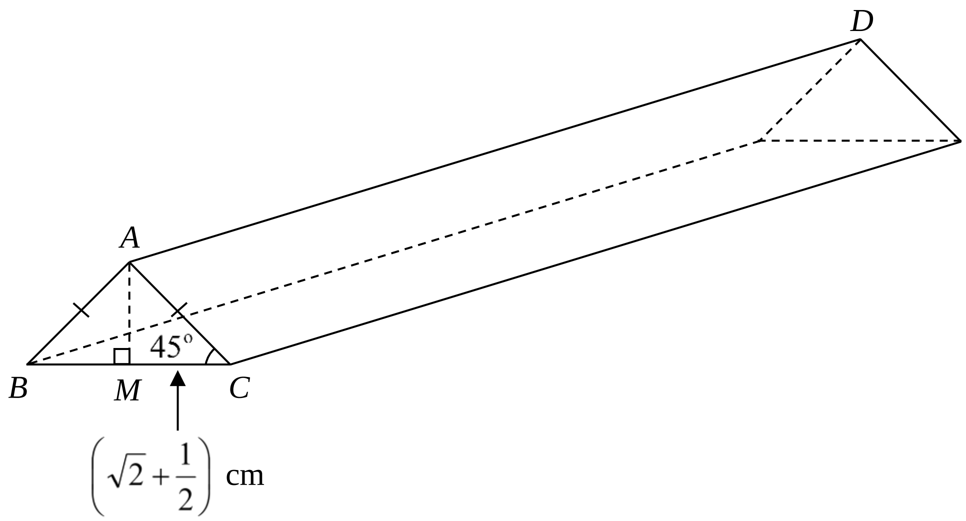
$MC = \left(\sqrt{2} + \frac{1}{2}\right)$ cm and $\angle ACB = 45°$.



(a) Find the exact length of $\boldsymbol{AC}$. [3]

$\cos 45° = \frac{\sqrt{2}+\frac{1}{2}}{AC}$ M1

$AC = \frac{2\left(\sqrt{2}+\frac{1}{2}\right)}{\sqrt{2}}$

$= \frac{2\sqrt{2}+1}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}}$ M1

$= 2 + \frac{\sqrt{2}}{2}$ or $\frac{4+\sqrt{2}}{2}$ A1

(b) Given that the volume of the chocolate bar is $(25 + 22\sqrt{2})\text{cm}^3$, find the length of $\boldsymbol{AD}$ in the form $(a + b\sqrt{2})$ cm, where a and b are integers. [4]

Vol $= \frac{1}{2} \times \left(\frac{4+\sqrt{2}}{2}\right) \times \left(\frac{4+\sqrt{2}}{2}\right) \times AD$ M1

$25 + 22\sqrt{2} = \frac{9+4\sqrt{2}}{4} AD$

$AD = \frac{25+22\sqrt{2}}{\frac{9+4\sqrt{2}}{4}}$

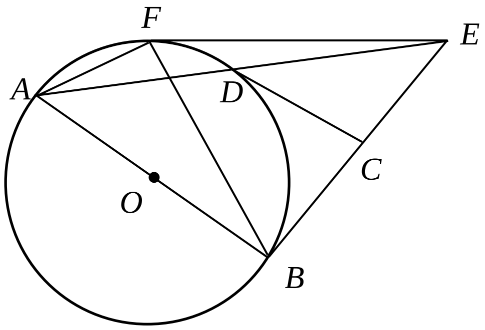
$= \frac{100+88\sqrt{2}}{9+4\sqrt{2}} \times \frac{9-4\sqrt{2}}{9-4\sqrt{2}}$ M1

$= \frac{900-400\sqrt{2}+792\sqrt{2}-704}{49}$

$= \frac{196+392\sqrt{2}}{49}$ M1

$= 4 + 8\sqrt{2}$ A1

3



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(a) Given that the points C and D are midpoints of BE and AE respectively, prove that angle $DCE = 90°$. [3]

$\angle ABC = 90°$ (tangent perpendicular radius) M1

DC parallel AB (mid point theorem) M1

Angle $DCE = 90°$ (corresponding angles) A1

(b) Given that triangle *BEF* is equilateral, prove that $\angle BEF = \angle BAF$. [2]

$\angle EBF = \angle BAF$ (alternate segment theorem) M1

Since $\angle EBF = \angle BEF$,

$\angle BEF = \angle BAF$ (shown) A1

4 **(a)** Find the remainder when $6x^3 - 13x^2 + 17x - 6$ is divided by $2x - 1$. [2]

When $x = \frac{1}{2}$,

Remainder $= 6\left(\frac{1}{2}\right)^3 - 13\left(\frac{1}{2}\right)^2 + 17\left(\frac{1}{2}\right) - 6$ M1

$= 0$ A1

(b) Show that there is only one real root of the equation

$6x^3 - 13x^2 + 17x - 6 = 0.$ [3]

$(2x - 1)(6x^2 - 10x + 12) = 0$ B1

$x = \frac{1}{2}$ $6x^2 - 10x + 12 = 0$

$3x^2 - 5x + 6 = 0$

Discriminant:

$b^2 - 4ac = 25 - 4(3)(6)$

$= -47$ B1

Since $-47 < 0$,

$3x^2 - 5x + 6 = 0$ has no real roots, hence equation has only 1 real root which is $x = \frac{1}{2}$. B1

5 Solve the following equations.

(a) $5^x - 5^{\frac{x}{2}+1} = 6,$ [3]

Let $y = 5^{\frac{x}{2}}$

$y^2 - 5y - 6 = 0$ M1

$y = 6$ $y = -1$ (reject) A1

$\frac{x}{2}\lg 5 = \lg 6$

$x = 2.23$ B1

(b) $2\lg(x-3)-\lg(x+7)=\frac{1}{\log_{100}10}$. [4]

$\lg\frac{(x-3)^2}{x+7}=\frac{\lg 100}{\lg 10}$ M2

$100=\frac{(x-3)^2}{x+7}$ M1

$x^2-106x-691=0$

$x=112$ or $x=-6.16$ (rej) A1

6 **(a)** State the values between which the principal value of $\sin^{-1}x$ must lie. [1]

$-90° \leq \sin^{-1}x \leq 90°$

$-\frac{\pi}{2} \leq \sin^{-1}x \leq \frac{\pi}{2}$

(b) Find the principal value of $\tan^{-1}1$ in radian in exact form. [1]

Principal value $=\frac{\pi}{4}$

7 Given that $\cot\theta=-\frac{3}{4}$ and that $\tan\theta$ and $\cos\theta$ have opposite signs, without evaluating θ, find the exact values of each of the following.

(a) $\cos(-\theta)$, [2]

$\tan\theta=-\frac{4}{3}$, lies in 4th quadrant M1

$\cos(-\theta)=\cos\theta$

$=\frac{3}{5}$ A1

(b) $\sin 2\theta$ [2]

$=2\sin\theta\cos\theta$

$=2\left(-\frac{4}{5}\right)\left(\frac{3}{5}\right)$ M1

$=-\frac{24}{25}$ A1

8. The approximate mean distance ***x*** (in millions of kilometres) from the centre of the Sun and the period of the orbit ***T*** (in Earth years) are recorded in the table.

	Mercury	**Venus**	**Mars**	**Uranus**
x	58	108	228	2871
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It is believed that the planets orbiting around the Sun obey a law of the form $T = kx^n$, where k and n are constants.

(a) Express the equation in a form suitable for drawing a straight line graph and draw the graph using appropriate scaling on both axes. [4]

$$\lg T = lgk + nlgx$$

(b) Use your graph to estimate the value of k and of n, to two significant figures. [3]

Lg k = -3.2

k = 0.00063 (0.00063 to 0.00079)

n = $\frac{2-9-3.2)}{3.5-0} = 1.49 = 1.5$ (1.4 to 1.6)

(c) Using the graph, find the orbital period of the Earth, if the distance between the Earth and the Sun is about $149.6 \times 10^6\,km$. Give your answer correct to the nearest integer. [2]

lg 149.6 = 2.17 = lg x

lg T = 0 ⇒ T = 1 (0.79 to 1.25)

(d) If the orbital period of the Jupiter is 11.86 Earth years, estimate the distance of the Jupiter from the Sun in km using your graph. [2]

lg 11.86 = 1.07

lg x = 2.9 = 794000000 km (631000000 to 1000000000 km)

9. **(a)** Express $\frac{2x^3+2x^2-7x+4}{x(x-1)^2}$ in partial fractions. [5]

$= 2 + \frac{6x^2-9x+4}{x(x-1)^2}$ B1

Let $\frac{6x^2-9x+4}{x(x-1)^2} = \frac{A}{x} + \frac{B}{x-1} + \frac{C}{(x-1)^2}$

$6x^2 - 9x + 4 = A(x-1)^2 + Bx(x-1) + Cx$

When $x = 0$,

$4 = A$ B1

When $x = 1$,

$6 - 9 + 4 = C$

$C = 1$ B1

When $x = -1$,

$6 + 9 + 4 = 4(4) + 2B - 1$

$2B = 4$

$B = 2$ B1

$\frac{2x^3+2x^2-7x+4}{x(x-1)^2} = 2 + \frac{4}{x} + \frac{2}{x-1} + \frac{1}{(x-1)^2}$ B1

(b) Hence evaluate $\int_2^4 \frac{4x^3+4x^2-14x+8}{3x(x-1)^2}\,dx$. [4]

$$\int_2^4 \frac{4x^3+4x^2-14x+8}{3x(x-1)^2}\,dx = \frac{2}{3}\int_2^4 \frac{2x^3+2x^2-7x+4}{x(x-1)^2}\,dx \qquad \text{M1}$$

$$= \frac{2}{3}\int_2^4 \left[2+\frac{4}{x}+\frac{2}{x-1}+\frac{1}{(x-1)^2}\right] dx$$

$$= \frac{2}{3}\left[2x+4\ln x+2\ln(x-1)-\frac{1}{x-1}\right]_2^4 \qquad \text{M1}$$

$$= \frac{2}{3}\left[8+4\ln 4+2\ln 3-\frac{1}{3}-4-4\ln 2-2\ln 1+1\right]$$

$$= \frac{2}{3}\left[\frac{14}{3}+4\ln 2+2\ln 3\right] \qquad \text{M1}$$

$$= 6.42 \qquad \text{A1}$$

10. **(a)** Find the range of values of k for which the line $2x-y=5$ intersects the curve $xy=kx-2$ at two distinct points. [4]

$x(2x-5)=kx-2$

$2x^2-5x-kx+2=0$ B1

Intersects at 2 distinct points:

$(-5-k)^2-4(2)(2)>0$ M1

$k^2+10k++9>0$

$(k+1)(k+9)>0$

$k<-9$ $\qquad$ $k>-1$ A2

(b) Find the smallest integer value of h for which the graph $y = 2x^2 - 4x + h$ lies entirely above the line $y = 3$ for all values of x. [3]

$2x^2 - 4x + h - 3 > 0$

Curve lies above line:

$b^2 - 4ac < 0$ M1

$(-4)^2 - 4(2)(h-3) < 0$

$8h > 40$

$h > 5$ A1

smallest integer value of $h = 6$ B1

11. **(a)** Prove the identity $\frac{1+\cos\theta}{\sin\theta} + \frac{\sin\theta}{1+\cos\theta} = 2\text{cosec}\,\theta$. [4]

$\text{LHS} = \frac{1+\cos\theta}{\sin\theta} + \frac{\sin\theta}{1+\cos\theta}$

$= \frac{1+2\cos\theta+\cos^2\theta+\sin^2\theta}{\sin\theta(1+\cos\theta)}$ M2

$= \frac{2(1+\cos\theta)}{\sin\theta(1+\cos\theta)}$ M1

$= 2\text{cosec}\,\theta$ A1

$= RHS$ (shown)

(b) Hence, find all the angles from $0° \leq \theta \leq 360°$ which satisfy the equation

$\frac{1+\cos 2\theta}{\sin 2\theta} + \frac{\sin 2\theta}{1+\cos 2\theta} = \tan 75°$. [3]

$2\text{cosec}\,2\theta = \tan 75°$

$\sin 2\theta = \frac{2}{\tan 75°}$ M1

$basic\ angle = \sin^{-1}\frac{2}{\tan 75°}$

$= 32.404858°$ M1

$2\theta = 32.404858°, 180° - 32.404858°, 32.404858° + 360°, 540° - 32.404858°$

$\theta = 16.2°, 73.8°, 196.2°, 253.8°$ A1

12. Find the derivatives of each of the following, simplifying your answer.

(a) $y = 3\left(1 - \frac{x}{3}\right)^4$ [1]

$$\frac{dy}{dx} = 12\left(-\frac{1}{3}\right)\left(1 - \frac{x}{3}\right)^3$$

$$= -4\left(1 - \frac{x}{3}\right)^3$$

(b) $f(x) = (2 - 3x)\left(\sqrt{1 - 4x}\right)$ [3]

$$f'(x) = -3\left(\sqrt{1 - 4x}\right) + \frac{1}{2}(-4)(2 - 3x)(1 - 4x)^{-\frac{1}{2}}$$ M1

$$= (1 - 4x)^{-\frac{1}{2}}\left[-3(1 - 4x) - 4 + 6x\right]$$ M1

$$= \frac{18x - 7}{\sqrt{1 - 4x}}$$ A1

(c) $\frac{dy}{dx} = \frac{2(3x - 2)}{4 + x}$ [2]

$$\frac{d^2y}{dx^2} = \frac{6(4 + x) - (6x - 4)}{(4 + x)^2}$$ M1

$$= \frac{28}{(4 + x)^2}$$ A1

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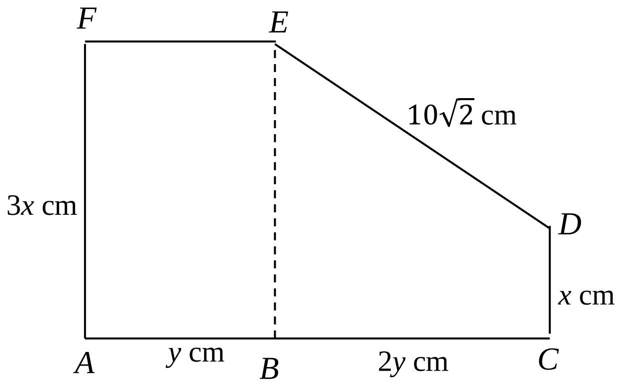

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Given that x can vary,

(a) show that the area of the glass window $S = 7x\left(\sqrt{50 - x^2}\right)$, [3]

Looking at triangle,

$4y^2 + 4x^2 = 200$

$y^2 + x^2 = 50$ B1

Total area $A = 3xy + \frac{1}{2}(x + 3x)(2y)$ M1

$= 7x\left(\sqrt{50 - x^2}\right)$ A1

(b) find the value of x for which S has a stationary value and determine whether this value of A is a maximum or a minimum. [5]

$\frac{dS}{dx} = 7\left(\sqrt{50 - x^2}\right) + \frac{1}{2}(-2x)(7x)(50 - x^2)^{-\frac{1}{2}}$

$= (50 - x^2)^{-\frac{1}{2}}[7(50 - x^2) - 7x^2]$

$= \frac{350 - 14x^2}{\sqrt{50 - x^2}}$ B1

Stationary value of S:

$\frac{dS}{dx} = 0$ M1

$350 - 14x^2 = 0$

$x = 5$ $\qquad\qquad$ $x = -5$ (rej) $\qquad\qquad$ A1

	$x = 4.9$	$x = 5$	$x = 5.1$
$\frac{dS}{dx}$	2.72	0	-2.89
shape	/	—	\

Proof B1 (can be 1st or 2nd derivative)

When $x = 5$, S is a maximum $\qquad\qquad$ B1

14. It is given that $f(x)$ is such that $f'(x) = \cos 4x - \sin 2x$. Given also that $f\left(\frac{\pi}{2}\right) = \frac{1}{4}$, show that $f''(x) + 4f(x) = 3 - 3\sin 4x$. [5]

$f''(x) = -4\sin 4x - 2\cos 2x$ B1

$f(x) = \int(\cos 4x - \sin 2x)\ dx$

$= \frac{\sin 4x}{4} + \frac{\cos 2x}{2} + c$ B1

When $x = \frac{\pi}{2}$,

$\frac{1}{4} = -\frac{1}{2} + c$

$c = \frac{3}{4}$

$f(x) = \frac{\sin 4x}{4} + \frac{\cos 2x}{2} + \frac{3}{4}$ B1

$f''(x) + 4f(x) = -4\sin 4x - 2\cos 2x + 4\left[\frac{\sin 4x}{4} + \frac{\cos 2x}{2} + \frac{3}{4}\right]$ M1

$= -4\sin 4x - 2\cos 2x + \sin 4x + 2\cos 2x + 3$

$= 3 - 3\sin 4x$ (shown) A1

General Certificate of Education Ordinary Level
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ADDITIONAL MATHEMATICS — 4049/02

Paper 2

27 August 2024

2 hours 15 minutes

Candidates answer on the Question Paper.

READ THESE INSTRUCTIONS FIRST

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For π, use either your calculator value or 3.142.

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Set by: Mr Albert Lui
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Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

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$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

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Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

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$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

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$$\sin 2A = 2\sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

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$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

$$\Delta = \frac{1}{2}ab\sin C$$

Answer **ALL** the questions

1 **(a)** Express $3x^2 - 8x - 3$ in the form $a(x+b)^2 + c$, where a, b and c are constants to be determined. [3]

(b) Hence, find the turning point and the value of x where it occurs. [2]

2 **(a)** Write down and simplify the first three terms in the expansion, in ascending powers of x, of $\left(2+\frac{x}{2}\right)^n$, where n is a positive integer. [3]

(b) The first two terms in the expansion, in ascending powers of x, of $\left(\frac{4}{3}-3x\right)\left(2+\frac{x}{2}\right)^n$ are $a+bx^2$. Find the value of n. [3]

(c) Find the term independent of x in the expansion of $\left(x-\frac{1}{2x^2}\right)^9$. [3]

3 Radiocarbon dating, or carbon-14 dating, is a scientific method that can accurately determine the age of organic materials as old as approximately 60,000 years. The technique is based on the decay of the carbon-14 isotope. The time in which half of the original number of atom decay is defined as the half-life. It is modelled by the equation $N = N_0 e^{-kt}$, where k is a constant and t is the time in years. N_0 is the initial amount of material in an object.

Carbon-14 has a half life of 5730 years, meaning that 5730 years after an organism dies, half of its carbon-14 atoms have decayed.

(a) Find the value of k. [2]

(b) A mummified body was found to have 8% of its original atoms left. How many years ago did the person die? Leave your answer to the nearest whole number. [2]

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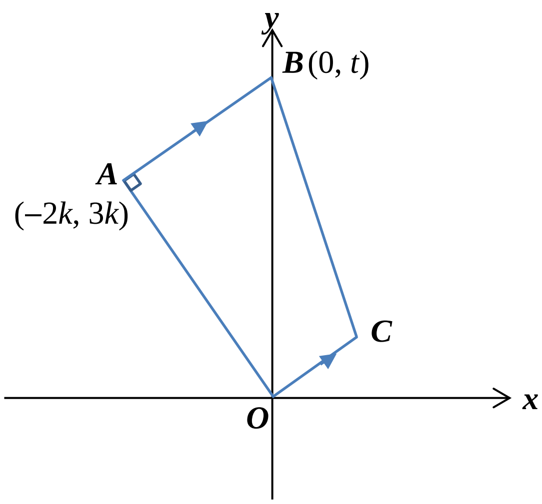


$OABC$ is a trapezium with right angle OAB. The coordinates of A and B are $(-2k, 3k)$ and $(0, t)$ respectively, where $k > 0$, and $OA = \sqrt{117}$ units. Find

(a) the value of k and show that A is $(-6, 9)$, [2]

(b) the equation of AB, [3]

(c) the coordinates of C if $OC = \frac{1}{2}AB$, [1]

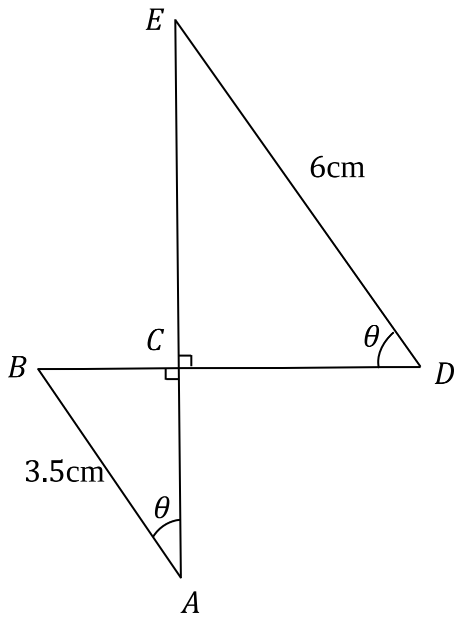
(d) the area of $OABC$. [2]

Triangle OAB lies in a circle C.

(e) Explain if OB is the diameter of the circle. [1]

(f) Find the equation of the circle C. [3]

5 The diagram shows two right-angled triangles, ABC and DEC. $\angle BAC = \angle CDE = \theta$, AB = 3.5 cm and DE = 6 cm.



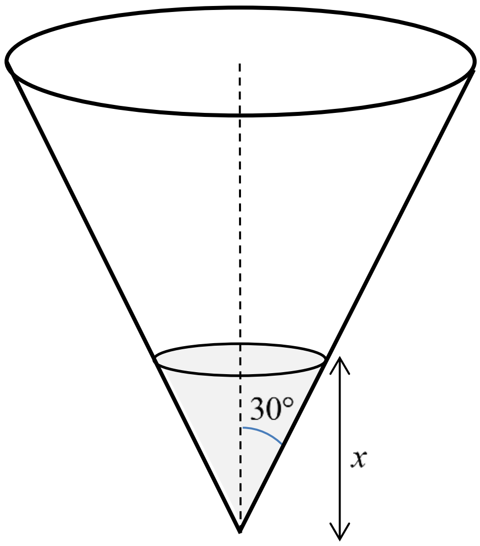
(a) Show that $BD = 6\cos\theta + 3.5\sin\theta$. [2]

(b) Express BD in the form $R\cos(\theta - \alpha)$, where $R > 0$ and α is an acute angle. [3]

(c) State the maximum length of BD and the corresponding value of θ. [2]

(d) Find the value of θ when $BD = 6$ cm. [2]

6 A conical vessel, whose vertical angle is $30°$ as shown in the diagram, has water poured into it at such a rate that after t seconds, the depth of the water is x cm and the radius of the water surface is r cm.



(a) Show that the volume of water, $V = \frac{\pi x^3}{9}$ cm^3. [3]

(b) Given further that the volume of water, $V = 8t$, find the rate of which the water level is increasing after 3 seconds. [5]

7 Show that $2^{n+5} + 2^{n+3} + 81(2^n)$ is a multiple of 11. [3]

8. Evaluate $\int_0^{\frac{\pi}{3}} \left(\frac{1}{2}\sin 3x - \cos^2 x\right)\, dx$, correct to 2 decimal places. [4]

9. A particle moves in a straight line and its initial velocity is 21 cm/s. After t seconds, its acceleration, a cm/s^2, is given by $a = 2t - 10$. When $t = 0$, its displacement, s cm, from a fixed point O is 4 cm.

Find the distance travelled by the particle during the first 7 seconds. [8]

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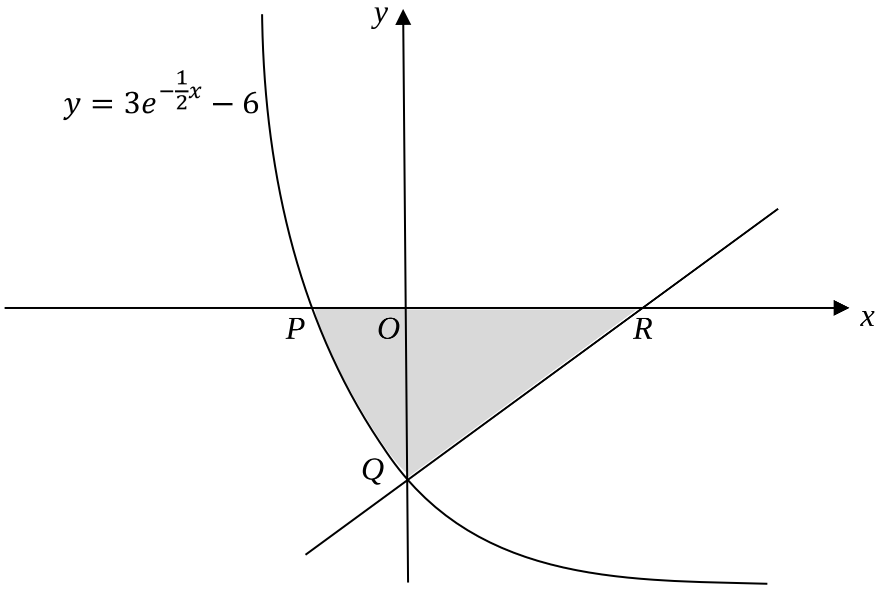
10.



The diagram shows part of the curve $y = 3e^{-\frac{1}{2}x} - 6$ which crosses the x-axis at P and the y-axis at Q. The normal to the curve at Q meets the x-axis at R. Find

(a) the equation of the normal, [4]

(b) the area of the shaded region. [6]

11. The function f is defined, for $0 \leq x \leq 2\pi$, by $f(x) = 1 - 2\cos 2x$.

(a) State the period and amplitude of f. [2]

(b) Sketch the graph of $y = f(x)$ for $0 \leq x \leq 2\pi$. [3]

(c) On the same diagram in part **(b)**, sketch the graph of $y = \frac{4x}{\pi} - 2$. [1]

(d) Hence, state the number of solutions, $0 \leq x \leq 2\pi$, to the equation

$2\cos 2x - 3 + \frac{4x}{\pi} = 0$. [1]

12. The gradient function of a curve is given by $\frac{dy}{dx} = -\frac{1}{x^2} + 6\cos 3x$. The coordinates of the point P, which lies on the curve, is $\left(\frac{\pi}{2}, \pi\right)$. A point Q exists such that the mid-point of PQ is $\left(\frac{3}{4}\pi, 2\pi\right)$. Find

(a) the coordinates of the point Q, [3]

(b) the equation of the curve. [2]

13. Without using the calculator, simplify the following trigonometric functions.

(a) $\sin\left(x + \frac{4\pi}{3}\right)$ [2]

(b) $\cos 75°$ [2]

(c) $\tan(\theta - 45°)$ [2]

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$$\Delta = \frac{1}{2} ab \sin C$$

<u>Answer **ALL** the questions</u>

1 **(a)** Express $3x^2 - 8x - 3$ in the form $a(x+b)^2 + c$, where a, b and c are constants to be determined. [3]

$3\left(x^2 - (2)\left(\frac{4}{3}\right)x + \left(\frac{4}{3}\right)^2 - \left(\frac{4}{3}\right)^2\right) \quad - 3$ M1

$= 3\left(x - \frac{4}{3}\right)^2 - 3\left(\frac{16}{9}\right) - 3$

$= 3\left(x - \frac{4}{3}\right)^2 - \frac{25}{3}$ A1

$a = 3 \qquad b = -\frac{4}{3} \qquad c = -\frac{25}{3}$ B1

(b) Hence, find the turning point and the value of x where it occurs. [2]

Turning Point $= \left(\frac{4}{3}, -\frac{25}{3}\right)$

Value of $x = \frac{4}{3}$

2 **(a)** Write down and simplify the first three terms in the expansion, in ascending powers of x, of $\left(2+\frac{x}{2}\right)^n$, where n is a positive integer. [3]

$$\left(2+\frac{x}{2}\right)^n = 2^n + n(2)^{n-1}\left(\frac{x}{2}\right) + \frac{n(n-1)}{2}(2)^{n-2}\left(\frac{x}{2}\right)^2 + \cdots \qquad \text{M2}$$

$$= 2^n + \frac{xn(2)^n}{4} + \frac{n(n-1)x^2(2)^n}{32} + \cdots \qquad \text{A1}$$

(b) The first two terms in the expansion, in ascending powers of x, of $\left(\frac{4}{3}-3x\right)\left(2+\frac{x}{2}\right)^n$ are $a + bx^2$. Find the value of n. [3]

$$\left(\frac{4}{3}-3x\right)\left(2+\frac{x}{2}\right)^n = \left(\frac{4}{3}-3x\right)\left[2^n + \frac{xn(2)^n}{4} + \frac{n(n-1)x^2(2)^n}{32} + \cdots\right]$$

$$= \frac{4}{3}\times 2^n + \frac{xn(2)^n}{3} + \frac{n(n-1)x^2(2)^n}{24} - 3x2^n - \frac{3x^2n(2)^n}{4} + \cdots$$

B1

No x term:

$$\frac{n(2)^n}{3} - 3(2)^n = 0 \qquad \text{M1}$$

$$n = 9 \qquad \text{A1}$$

(c) Find the term independent of x in the expansion of $\left(x-\frac{1}{2x^2}\right)^9$. [3]

$$\boldsymbol{T_{r+1} = \binom{9}{r}(x)^{9-r}\left(-\frac{1}{2x^2}\right)^r}$$

Term independent of $\boldsymbol{x}$:

$\boldsymbol{9 - r - 2r = 0}$ M1

$\boldsymbol{r = 3}$ A1

Term independent of $\boldsymbol{x}$

$$= \boldsymbol{\binom{9}{3}(x)^{9-3}\left(-\frac{1}{2x^2}\right)^3}$$

$$= -\boldsymbol{\frac{21}{2}} \qquad \text{B1}$$

3 Radiocarbon dating, or carbon-14 dating, is a scientific method that can accurately determine the age of organic materials as old as approximately 60,000 years. The technique is based on the decay of the carbon-14 isotope. The time in which half of the original number of atom decay is defined as the half-life. It is modelled by the equation $N = N_0 e^{-kt}$, where k is a constant and t is the time in years. N_0 is the initial amount of material in an object.

Carbon-14 has a half life of 5730 years, meaning that 5730 years after an organism dies, half of its carbon-14 atoms have decayed.

(a) Find the value of k. [2]

$\frac{1}{2} = e^{-k(5730)}$ M1

$k = \frac{\ln\left(\frac{1}{2}\right)}{-5730}$ A1

$= 0.0001209681$

$= 0.000121$

(b) A mummified body was found to have 8% of its original atoms left. How many years ago did the person die? Leave your answer to the nearest whole number. [2]

$\frac{8}{100} = e^{-0.0001209681t}$ M1

$t = \frac{\ln\left(\frac{8}{100}\right)}{-0.0001209681}$

$= 20879$ years A1

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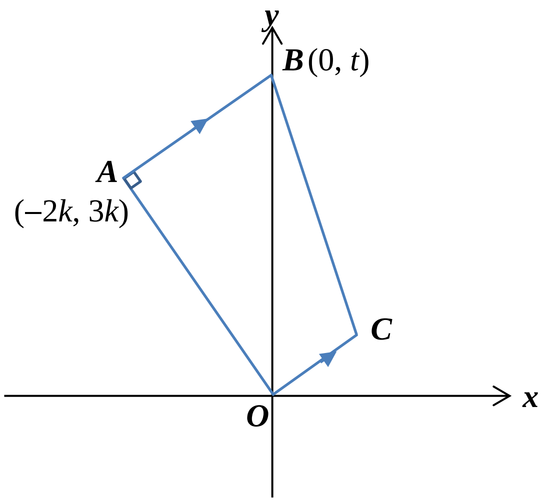


OABC is a trapezium with right angle *OAB*. The coordinates of *A* and *B* are (–2*k*, 3*k*) and (0, *t*) respectively, where $k > 0$, and $OA = \sqrt{117}$ units. Find

(a) the value of *k* and show that *A* is (–6, 9), [2]

length $OA = \sqrt{4k^2 + 9k^2}$ M1

$117 = 13k^2$

$k = 3 \quad k = -3$ (rej) A1

Hence, *A* is $(-2 \times 3\,, 3 \times 3) = (-6, 9)$

(b) the equation of *AB*, [3]\

Gradient of $OA = \frac{9}{-6}$

$= -\frac{3}{2}$ M1

Gradient of $AB = \frac{2}{3}$ M1

Equation of AB:

$y - 9 = \frac{2}{3}(x + 6)$

$y = \frac{2}{3}x + 13$ A1

(c) the coordinates of C if $OC = \frac{1}{2}AB$, [1]

Coordinates of $B = (0, 13)$

Coordinates of $C = \left(\frac{6}{2}, \frac{4}{2}\right)$

$= (3, 2)$

(d) the area of $OABC$. [2]

Area $= \frac{1}{2}\begin{vmatrix} 0 & 3 & 0 & -6 & 0 \\ 0 & 2 & 13 & 9 & 0 \end{vmatrix}$ M1

$= \frac{1}{2}|(39) - (-78)|$

$= \frac{117}{2}$ units^2 A1

Triangle OAB lies in a circle C.

(e) Explain why OB is the diameter of the circle. [1]

By property of angles in a semicircle, since angle $OAB = 90°$,

OB is the diameter of the circle C_1.

(f) Find the equation of the circle C. [3]

Radius= 6.5 B1

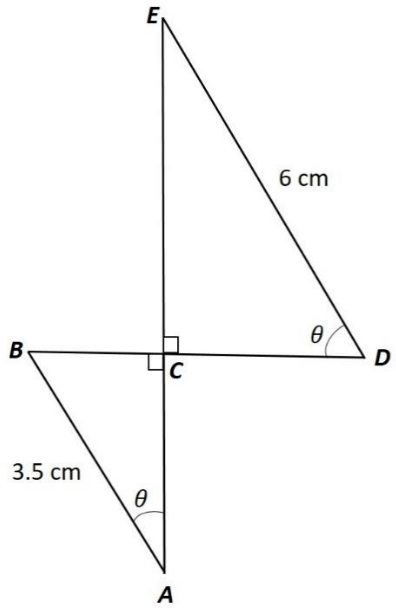
Center of circle $= \left(0, \frac{13}{2}\right)$ B1

Equation of C_1:

$(x-0)^2 + \left(y - \frac{13}{2}\right)^2 = \left(\frac{13}{2}\right)^2$

$x^2 + \left(y - \frac{11}{2}\right)^2 = \frac{169}{4}$ B1

5 The diagram shows two right-angled triangles, *ABC* and *DEC*. $\angle BAC = \angle CDE = \theta$, *AB* = 3.5 cm and *DE* = 6 cm.



(a) Show that *BD* = $6\cos\theta + 3.5\sin\theta$. [2]

$\sin\theta = \frac{BC}{3.5}$ $\quad\quad$ $\cos\theta = \frac{CD}{6}$

$BC = 3.5\sin\theta$ B1 $\quad\quad$ $CD = 6\cos\theta$ B1

$BD = BC + CD$

$= 6\cos\theta + 3.5\sin\theta$ (shown)

(b) Express *BD* in the form $R\cos(\theta - \alpha)$, where $R > 0$ and α is an acute angle. [3]

$R = \sqrt{3.5^2 + 36}$

$= \frac{\sqrt{193}}{2}$ M1

$\alpha = \tan^{-1}\left(\frac{3.5}{6}\right)$

$= 30.25643$

$= 30.3°$ M1

$BD = \frac{\sqrt{193}}{2}\cos(\theta - 30.3°)$ A1

(c) State the maximum length of *BD* and the corresponding value of θ. [2]

$Max\ BD = \frac{\sqrt{193}}{2}$

$= 6.94622$

$= 6.95$ cm B1

Occurs when $\cos(\theta - 30.256°) = 1$

$(\theta - 30.256°) = 0°$

$\theta = 30.3°$ B1

(d) Find the value of θ when $BD = 6$ cm. [2]

$6 = \frac{\sqrt{193}}{2}\cos(\theta - 30.3°)$

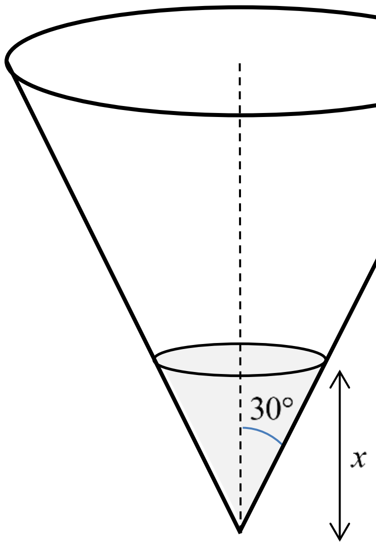
$\cos(\theta - 30.3°) = 0.8637789$ M1

$\theta - 30.25643 = 30.25643$

$\theta = 60.5°$ A1

\

6 A conical vessel, whose vertical angle is $30°$ as shown in the diagram, has water poured into it at such a rate that after t seconds, the depth of the water is x cm and the radius of the water surface is r cm.



(a) Show that the volume of water, $V = \frac{\pi x^3}{9}$ cm^3. [3]

$\tan 30° = \frac{r}{x}$

$r = \frac{\sqrt{3}}{3}x$ B1

$V = \frac{1}{3}\pi r^2 h$

$= \frac{1}{3}\pi\left(\frac{\sqrt{3}}{3}x\right)^2 x$ M1

$= \frac{\pi x^3}{9}$ cm^3 A1

(b) Given further that the volume of water, $V = 8t$, find the rate of which the water level is increasing after 3 seconds. [5]

When $t = 3$,

$V = 24$

$24 = \frac{\pi x^3}{9}$

$x = \frac{6}{\sqrt[3]{\pi}}$ M1

$\frac{dV}{dt} = 8$ M1

$\frac{dV}{dx} = \frac{\pi x^2}{3}$ M1

$\frac{dV}{dx} \times \frac{dx}{dt} = \frac{dV}{dt}$ M1

Sub $x = \frac{6}{\sqrt[3]{\pi}}$,

$\frac{dx}{dt} = \frac{24}{\pi x^2}$

$= 0.455$ cm/s A1

7 Show that $2^{n+5} + 2^{n+3} + 81(2^n)$ is a multiple of 11. [3]

$2^{n+5} + 2^{n+3} + 81(2^n)$

$= 2^5(2^n) + 2^3(2^n) + 81(2^n)$ M1

$= (2^n)(32 + 8 + 81)$

$= (2^n)(121)$ M1

$= (11^2)(2^n)$ A1

8. Evaluate $\int_0^{\frac{\pi}{3}} \left(\frac{1}{2}\sin 3x - \cos^2 x\right)\, dx$, correct to 2 decimal places. [4]

$\int_0^{\frac{\pi}{3}} \left(\frac{1}{2}\sin 3x - \cos^2 x\right)\, dx$

$= \int_0^{\frac{\pi}{3}} \left(\frac{1}{2}\sin 3x - \left(\frac{\cos 2x + 1}{2}\right)\right)\, dx$ M1

$= \left[-\frac{\cos 3x}{6} - \frac{\sin 2x}{4} - \frac{x}{2}\right]_0^{\frac{\pi}{3}}$ M2

$= \frac{1}{6} - \frac{\sqrt{3}}{8} - \frac{\pi}{6} - \left(-\frac{1}{6}\right)$

$= -0.41$ A1

9. A particle moves in a straight line and its initial velocity is 21 cm/s. After t seconds, its acceleration, a cm/s^2, is given by $a = 2t - 10$. When $t = 0$, its displacement, s cm, from a fixed point O is 4 cm.

Find the distance travelled by the particle during the first 7 seconds. [8]

$v = \int a\, dt$

$v = t^2 - 10t + c$ M1

When $t = 0, v = 21$

$c = 21$

$v = t^2 - 10t + 21$ M1

$s = \int v\, dt$

$s = \frac{t^3}{3} - 5t^2 + 21t + c_1$ M1

When $t = 0, s = 4$

$c_1 = 4$

$s = \frac{t^3}{3} - 5t^2 + 21t + 4$ M1

Turning point

$v = 0$

$t^2 - 10t + 21 = 0$ M1

$t = 3\ s$ $t = 7\ s$ A1

When $t = 3\ s$

$s = 9 - 45 + 63 + 4$

$s = 31\ cm$

When $t = 7\ s$

$s = \frac{343}{3} - 245 + 147 + 4$

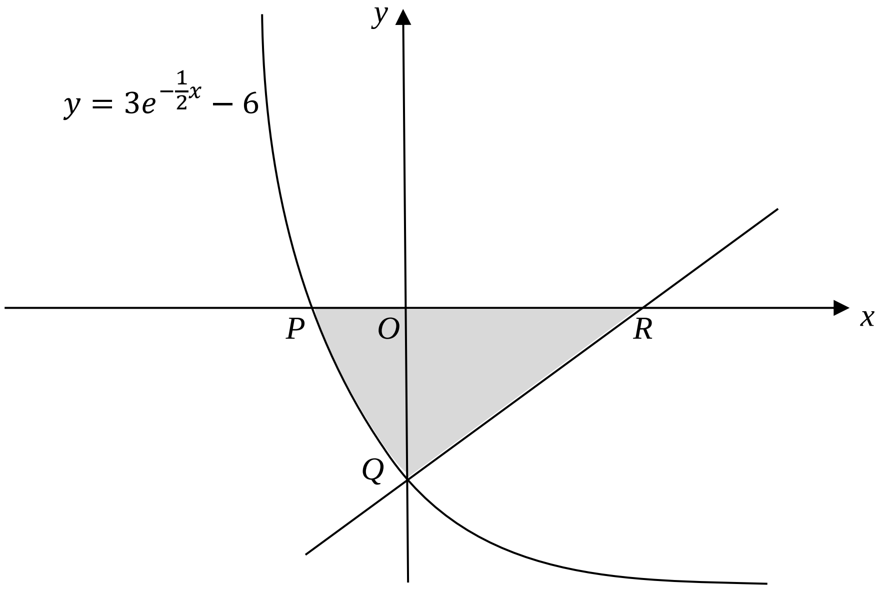
$s = \frac{61}{3}\ cm$

Distance travelled in the first 7 seconds

$= (31 - 4) + \left(31 - \frac{61}{3}\right)$ M1

$= \frac{113}{3}\ cm$ A1

10.



The diagram shows part of the curve $y = 3e^{-\frac{1}{2}x} - 6$ which crosses the x-axis at P and the y-axis at Q. The normal to the curve at Q meets the x-axis at R. Find

(a) the equation of the normal, [4]

When $x = 0$,

$y = -3$ M1

$\frac{dy}{dx} = -\frac{3}{2}e^{-\frac{1}{2}x}$

$\frac{dy}{dx} = -\frac{3}{2}$ M1

Gradient of normal $= \frac{2}{3}$ A1

Equation of normal:

$y + 3 = \frac{2}{3}(x - 0)$

$y = \frac{2}{3}x - 3$ A1

(b) the area of the shaded region. [6]

when $y = 0$,

$3e^{-\frac{1}{2}x} - 6 = 0$

$e^{-\frac{1}{2}x} = 2$

$-\frac{1}{2}x = \ln 2$

$x = -\ln 4$

$P = (-\ln 4\ , 0)$ M1

$\frac{2}{3}x - 3 = 0$

$x = \frac{9}{2}$

$R = \left(\frac{9}{2}, 0\right)$ M1

$Q = (0, -3)$ M1

Area of shaded region

$= \left|\int_{-\ln 4}^{0} \left(3e^{-\frac{1}{2}x} - 6\right)\ dx\right| + \frac{1}{2}\left(\frac{9}{2}\right)(3)$ M1

$= \left|\left[\frac{3e^{-\frac{1}{2}x}}{-\frac{1}{2}} - 6x\right]_{-\ln 4}^{0}\right| + \frac{27}{4}$

$= \left|\left[-6e^{-\frac{1}{2}x} - 6x\right]_{-\ln 4}^{0}\right| + \frac{27}{4}$ M1

$= |[-6 + 12 - 6\ln 4]| + \frac{27}{4}$

$= 9.07$ A1

11. The function f is defined, for $0 \leq x \leq 2\pi$, by $f(x) = 1 - 2\cos 2x$.

(a) State the period and amplitude of f. [2]

Period $= \pi$

Amplitude $= 2$

(b) Sketch the graph of $y = f(x)$ for $0 \leq x \leq 2\pi$. [3]

(c) On the same diagram in part **(b)**, sketch the graph of $y = \frac{4x}{\pi} - 2$. [1]

(d) Hence, state the number of solutions, $0 \leq x \leq 2\pi$, to the equation

$2\cos 2x - 3 + \frac{4x}{\pi} = 0$. [1]

12. The gradient function of a curve is given by $\frac{dy}{dx} = -\frac{1}{x^2} + 6\cos 3x$. The coordinates of the point P, which lies on the curve, is $\left(\frac{\pi}{2}, \pi\right)$. A point Q exists such that the mid-point of PQ is $\left(\frac{3}{4}\pi, 2\pi\right)$. Find

(a) the coordinates of the point Q, [3]

$\left(\frac{3}{4}\pi, 2\pi\right) = \left(\frac{\frac{\pi}{2}+x}{2}, \frac{\pi+y}{2}\right)$ M1

$\frac{3}{4}\pi = \frac{\frac{\pi}{2}+x}{2}$ $\quad$ $2\pi = \frac{\pi+y}{2}$

$\frac{3\pi}{2} = \frac{\pi}{2} + x$ $\quad$ $y = 3\pi$

$x = \pi$ A1

Coordiantes of $Q = (\pi, 3\pi)$ A1

(b) the equation of the curve. [2]

$y = \int \left(-\frac{1}{x^2} + 6\cos 3x\right) dx$

$\mathrm{y} = \frac{1}{x} + 2\sin 3x + c$ M1

When $x = \frac{\pi}{2}, y = \pi$

$\pi = \pi - 2 + c$

$c = 2$

Equation of curve:

$y = \frac{1}{x} + 2\sin 3x + 2$ A1

13. Without using the calculator, evaluate the following trigonometric functions.

(a) $\sin\left(x + \frac{4\pi}{3}\right)$ [2]

$= \sin x \cos\frac{4\pi}{3} + \cos x \sin\frac{4\pi}{3}$ M1

$= -\frac{1}{2}\sin x + \frac{\sqrt{3}}{2}\cos x$ A1

(b) $\cos 75°$ [2]

$= \cos(30° + 45°)$

$= \cos 30° \cos 45° - \sin 30° \sin 45°$

$= \frac{\sqrt{3}}{2} \times \frac{\sqrt{2}}{2} - \frac{1}{2} \times \frac{\sqrt{2}}{2}$ M1

$= \frac{\sqrt{2}(\sqrt{3}-1)}{4}$ A1

(c) $\tan(\theta - 45°)$ [2]

$= \frac{\tan\theta - \tan 45°}{1+\tan\theta\tan 45°}$ M1

$= \frac{\tan\theta - 1}{1+\tan\theta}$ A1