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BP~303

Name : ______________________

Class	Index Number

METHODIST GIRLS' SCHOOL

Founded in 1887

PRELIMINARY EXAMINATION 2024
Secondary 4

Monday
12 August 2024

ADDITIONAL MATHEMATICS
Paper 1

4049/01
2 h 15 min

Candidates answer on the Question Paper.
No Additional Materials are required.

READ THESE INSTRUCTIONS FIRST
Write your class, index number and name in the spaces at the top of this page.
Write in dark blue or black pen
You may use a HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** questions.
Give non-exact numerical answers correct to 3 significant figure, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

The number of marks is given in brackets [ ] at the end of each question or part question.

The total number of marks for this paper is 90.

90

This question paper consists of 21 printed pages and 3 blank pages.

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the quadratic equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial Expansion

$$(a+b)^n = a^n + \binom{n}{1} a^{n-1} b + \binom{n}{2} a^{n-2} b^2 + \ldots + \binom{n}{r} a^{n-r} b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$.

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2} bc \sin A$$

1 **(i)** Express $y = 4x^2 + 24x + 49$ in the form $a(x+b)^2 + c$. [2]

(ii) Hence, state the maximum value of $\frac{1}{y}$ and the corresponding value of x at which $\frac{1}{y}$ is maximum. [2]

2 By using a suitable substitution, solve the equation $e^{2x} = 4(1 - e^{-2x})$. [3]

3 Show that the equation $x^2 + px + 2p = 2x + 8$ has real roots for all real values of p.

[3]

4 A and B are acute angles such that $\sin(A-B)=\frac{1}{4}$ and $\sin A\cos B=\frac{3}{5}$.

Without using a calculator, find the value of

(i) $\cos A\sin B$, [2]

(ii) $\sin(A+B)$, [1]

(iii) $2\tan A\cot B$. [2]

5 A curve has the equation $y = \dfrac{e^{2x+1}}{x+3}$, where $x \neq -3$.

(i) Find an expression for $\dfrac{dy}{dx}$. [2]

(ii) Hence, stating your reasons clearly, determine if the graph of $y = \dfrac{e^{2x+1}}{x+3}$ is decreasing or increasing for which $x > 0$. [2]

6 Find the values of k for which the curve $y=2x^2+(3k+1)x+4$ lies entirely above the line $y=2x-3k^2+5$. [5]

7 A curve is such that $f''(x) = 6x - 10$. The tangent to the curve $y = f(x)$ at the point $(1, -2)$ passes through the point $(0, 13)$. Find the equation of the curve. [6]

8 **(a)** Given that $\log_x p = 9$ and $\log_x q = 6$, find the value of $\log_{\frac{p}{q}} x^2$. [3]

(b) Solve the equation $\lg(3x+1)+\lg(2x-1)=2-\lg 2$ and explain why there is only one solution. [5]

9 Two cubic expressions are defined by $\mathrm{f}(x)=x^3+(a-3)x+2b$ and $\mathrm{g}(x)=3x^3+x^2+5ax+4b$ where a and b are constants.

(i) Given that $\mathrm{f}(x)$ and $\mathrm{g}(x)$ have a common factor $(x+3)$, show that $a=-4$ and find the value of b. [3]

(ii) Using the values of a and b, factorise $\mathrm{f}(x)$ completely. Hence, show that $\mathrm{f}(x)$ and $\mathrm{g}(x)$ have two common factors. [4]

10 It is given that $y = 4x\sin 2x$.

(i) Show that $\frac{dy}{dx}$ can be expressed in the form $ax\cos 2x + b\sin 2x$, where a and b are constants. [2]

(ii) Given that x is increasing at $\frac{2}{5}$ radians per second, find the rate of change of y with respect to time when $x = \frac{\pi}{2}$. [2]

(iii) Using your answer in part **(i)**, evaluate $\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} x\cos 2x \, \mathrm{d}x$, leaving your answer in terms of π. [4]

(iv) Explain what your answer in part **(iii)** implies about the curve $y = x\cos 2x$. [1]

11 The table shows experimental values of two variables, x and y.

x	2	4	6	8
y	2.25	0.81	0.47	0.33

(i) On the graph paper, plot xy against $\frac{1}{x}$ and draw a straight line graph. [3]

Using your graph,

(ii) estimate the values of x and y for which $x = \frac{3}{y}$, [2]

(iii) express y in terms of x. [4]

12 A particle travels in a straight line, so that t seconds after passing a fixed point O, its velocity, v m/s, is given by $v = 22 + 7t - 2t^2$. The particle comes to instantaneous rest at P. Find

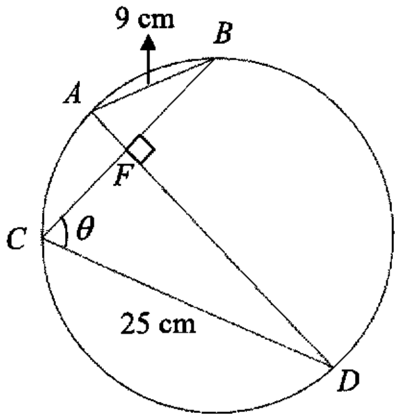
(i) the velocity of the particle when the acceleration is zero, [2]

(ii) the value of t when the particle is at P, [2]

(iii) the distance OP, [2]

(iv) the total distance travelled by the particle in the interval $t = 0$ to $t = 9$. [2]

13 In the diagram, AB and CD are chords of a circle, with $AB = 9$ cm and $CD = 25$ cm. It is given that AD and BC intersect each other at 90° at F and $\angle BCD = \theta$, where θ varies.



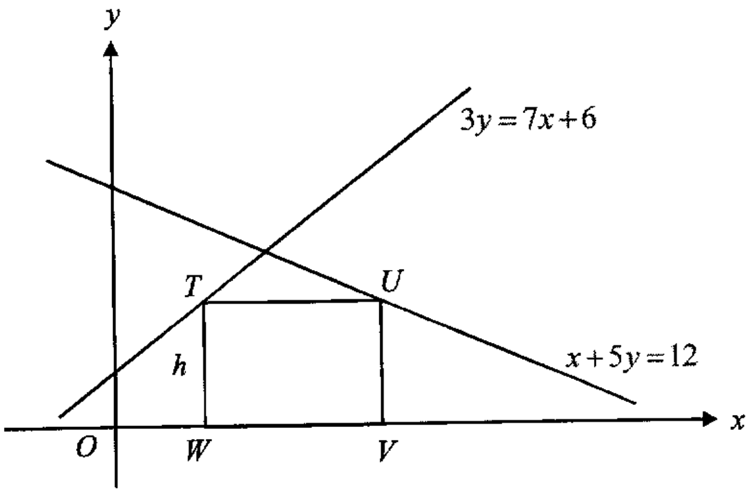
(i) Prove that $AD = 9\cos\theta + 25\sin\theta$. [2]

(ii) Express AD in the form $R\cos(\theta - \alpha)$, where $R > 0$ and $0° < \alpha < 90°$. [3]

(iii) Find the values of θ when $AD = 26$ cm. [3]

(iv) Calculate the angle θ for which AD is the diameter of the circle. [2]

14 The diagram shows parts of the graphs of $3y = 7x + 6$ and $x + 5y = 12$. T is a point on $3y = 7x + 6$ and U is a point on $x + 5y = 12$. V and W are two points on the x-axis such that $TUVW$ is a rectangle.



It is given that $TW = h$ units.

(i) Find the x-coordinates of T and U in terms of h. [2]

(ii) Hence, show that the area of the rectangle $TUVW$, represented by A square units, is given by $A = \frac{90}{7}h - \frac{38}{7}h^2$. [2]

(iii) Given that h can vary, find the stationary value of A and determine whether this value of A is a maximum or a minimum. [5]

End of Paper

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PAPER 1
SOLUTIONS

1 (i) Express $y = 4x^2 + 24x + 49$ in the form $a(x+b)^2 + c$. [2]

$4x^2 + 24x + 49$
$= 4(x^2 + 6x) + 49$
$= 4[(x+3)^2 - 9] + 49$
$= 4(x+3)^2 + 13$

(ii) Hence, state the maximum value of $\frac{1}{y}$ and the corresponding value of x at which $\frac{1}{y}$ is maximum. [2]

Maximum $\frac{1}{y} = \frac{1}{13}$

It occurs when $x = -3$

2 By using a suitable substitution, solve the equation $e^{2x} = 4(1-e^{-2x})$. [3]

$$e^{2x} = 4(1-e^{-2x})$$
$$e^{2x} = 4 - \frac{4}{e^{2x}}$$
$$y = 4 - \frac{4}{y}$$
$$y^2 = 4y - 4$$
$$y^2 - 4y + 4 = 0$$
$$(y-2)^2 = 0$$
$$y = 2$$
$$e^{2x} = 2$$
$$x = \frac{\ln 2}{2}$$
$$= 0.347$$

3 Show that the equation $x^2+px+2p=2x+8$ has real roots for all values of p. [3]

$x^2+px+2p=2x+8$
$x^2+(p-2)x+2p-8=0$
Discriminant
$=(p-2)^2-4(1)(2p-8)$
$=p^2-4p+4-8p+32$
$=p^2-12p+36$
$=(p-6)^2$

For all values of p, $(p-6)^2 \geq 0$, hence roots of the equation $x^2+px+2p=2x+8$ is always real.

4 A and B are acute angles such that $\sin(A-B)=\frac{1}{4}$ and $\sin A\cos B=\frac{3}{5}$.

Without using a calculator, find the value of

(i) $\cos A\sin B$, [2]

$$\sin(A-B)=\frac{1}{4}$$
$$\sin A\cos B-\cos A\sin B=\frac{1}{4}$$
$$\frac{3}{5}-\cos A\sin B=\frac{1}{4}$$
$$\cos A\sin B=\frac{7}{20}$$

(ii) $\sin(A+B)$, [1]

$$\sin(A+B)=\sin A\cos B+\cos A\sin B$$
$$\sin(A+B)=\frac{3}{5}+\frac{7}{20}=\frac{19}{20}$$

(iii) $2\tan A\cot B$. [2]

$$2\tan A\cot B$$
$$=2\times\frac{\sin A}{\cos A}\times\frac{\cos B}{\sin B}$$
$$=2\frac{\left(\frac{3}{5}\right)}{\left(\frac{7}{20}\right)}$$
$$=\frac{24}{7}$$

5 A curve has the equation $y = \frac{e^{2x+1}}{x+3}$, where $x \neq -3$.

(i) Find an expression for $\frac{\mathrm{d}y}{\mathrm{d}x}$. [2]

$$y = \frac{e^{2x+1}}{x+3}$$

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{(x+3)\left(2e^{2x+1}\right) - e^{2x+1}}{(x+3)^2}$$

$$= \frac{e^{2x+1}(2x+5)}{(x+3)^2}$$

(ii) Hence, stating your reasons clearly, determine if the graph of $y = \frac{e^{2x+1}}{x+3}$ is decreasing or increasing for which $x > 0$. [2]

For $x > 0$,

$(x+3)^2 > 0$,

$e^{2x+1} > 0$,

$(2x+5) > 0$

Thus $\frac{\mathrm{d}y}{\mathrm{d}x} > 0$ and y is an increasing function.

6 Find the values of k for which the curve $y=2x^2+(3k+1)x+4$ lies entirely above the line $y=2x-3k^2+5$. [5]

$$2x^2+(3k+1)x+4>2x-3k^2+5$$
$$2x^2+3kx+x+4-2x+3k^2-5>0$$
$$2x^2+(3k-1)x+3k^2-1>0$$
$$b^2-4ac<0$$
$$(3k-1)^2-4(2)(3k^2-1)<0$$
$$9k^2-6k+1-24k^2+8<0$$
$$-15k^2-6k+9<0$$
$$5k^2+2k-3>0$$
$$(5k-3)(k+1)>0$$
$$k>\frac{3}{5} \quad \text{or} \quad k<-1$$

7 A curve is such that $f''(x)=6x-10$. The tangent to the curve $y=f(x)$ at the point $(1,-2)$ passes through the point $(0,\ 13)$. Find the equation of the curve. [6]

$f''(x)=6x-10$

$f'(x)=\int 6x-10\ \mathrm{d}x$

$=3x^2-10x+c$

gradient of tangent $=\dfrac{-2-13}{1-0}=-15$

at $x=1$, $f'(x)=-15$

$-15=3(1)-10(1)+c$

$-8=c$

$f'(x)=3x^2-10x-8$

$f(x)=\int 3x^2-10x-8\ \mathrm{d}x$

$=x^3-5x^2-8x+d$

at $(1,-2)$,

$-2=1-5(1)-8+d$

$10=d$

$y=x^3-5x^2-8x+10$

8 (a) Given that $\log_x p = 9$ and $\log_x q = 6$, find the value of $\log_{\frac{p}{q}} x^2$. [3]

$$\log_{\frac{p}{q}} x^2$$
$$= \frac{\log_x x^2}{\log_x \frac{p}{q}}$$
$$= \frac{2}{\log_x p - \log_x q}$$
$$= \frac{2}{9-6}$$
$$= \frac{2}{3}$$

(b) Solve the equation $\lg(3x+1)+\lg(2x-1)=2-\lg 2$ and explain why there is only one solution. [5]

$$\lg(3x+1)+\lg(2x-1)=2-\lg 2$$
$$\lg(3x+1)(2x-1)=\lg\frac{100}{2}$$
$$(3x+1)(2x-1)=50$$
$$6x^2-3x+2x-1-50=0$$
$$6x^2-x-51=0$$
$$(6x+17)(x-3)=0$$
$$x=-\frac{17}{6} \quad \text{or} \quad 3$$

$$3x+1>0 \quad \text{and} \quad 2x-1>0$$
$$x>-\frac{1}{3} \qquad\qquad x>\frac{1}{2}$$

Since $x>\frac{1}{2}$, $x=3$ is the only solution.

9 Two cubic expressions are defined by $f(x)=x^3+(a-3)x+2b$ and $g(x)=3x^3+x^2+5ax+4b$ where a and b are constants.

(i) Given that $f(x)$ and $g(x)$ have a common factor $(x+3)$, show that $a=-4$ and find the value of b. [3]

$(-3)^3+(a-3)(-3)+2b=0$
$-27-3a+9+2b=0$
$-3a+2b=18$ ---------- (1)

$3(-3)^3+(-3)^2+5a(-3)+4b=0$
$-81+9-15a+4b=0$
$-15a+4b=72$ ---------- (2)

$(1)\times 2,\ -6a+4b=36$ ---------- (3)
(2) - (3), $-9a=36$
$a=-4$
$b=3$

(ii) Using the values of a and b, factorise $f(x)$ completely. Hence, show that $f(x)$ and $g(x)$ have two common factors. [4]

$f(x)=x^3-7x+6$
$x^3-7x+6=(x+3)(x^2+px+2)$
compare coeff of x^2, $0=p+3$
$p=-3$
$x^3-7x+6=(x+3)(x^2-3x+2)$
$=(x+3)(x-1)(x-2)$
$g(1)=3(1)^3+1^2+5(-4)(1)+4(3)=-4$
$(x-1)$ is not a factor of $g(x)$
$g(2)=3(2)^3+2^2+5(-4)(2)+4(3)=0$
$(x-2)$ is a factor of $g(x)$
Thus, $f(x)$ and $g(x)$ have 2 common factors $(x+3)$ and $(x-2)$

10 It is given that $y = 4x\sin 2x$.

(i) Show that $\frac{\mathrm{d}y}{\mathrm{d}x}$ can be expressed in the form $ax\cos 2x + b\sin 2x$, where a and b are constants. [2]

$$y = 4x\sin 2x$$
$$\frac{\mathrm{d}y}{\mathrm{d}x} = 4x(\cos 2x)(2) + 4\sin 2x$$
$$\frac{\mathrm{d}y}{\mathrm{d}x} = 8x\cos 2x + 4\sin 2x$$

(ii) Given that x is increasing at $\frac{2}{5}$ radians per second, find the rate of change of y with respect to time when $x = \frac{\pi}{2}$. [2]

$$\frac{\mathrm{d}y}{\mathrm{d}t} = \frac{\mathrm{d}y}{\mathrm{d}x} \times \frac{\mathrm{d}x}{\mathrm{d}t}$$
$$= (8 \times \left(\frac{\pi}{2}\right)\cos\pi + 4\sin\pi) \times \frac{2}{5}$$
$$= -\frac{8\pi}{5} \text{ radians/sec}$$

(iii) Using your answer in part **(i)**, evaluate $\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} x\cos 2x \, dx$, leaving your answer in terms of π. [4]

$$\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} 8x\cos 2x \, dx + \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} 4\sin 2x \, dx = \left[4x\sin 2x\right]_{\frac{\pi}{4}}^{\frac{\pi}{2}}$$

$$\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} x\cos 2x \, dx + \frac{1}{2}\left[\frac{-\cos 2x}{2}\right]_{\frac{\pi}{4}}^{\frac{\pi}{2}} = \left[\frac{1}{2}x\sin 2x\right]_{\frac{\pi}{4}}^{\frac{\pi}{2}}$$

$$\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} x\cos 2x \, dx = \left[\frac{1}{2}x\sin 2x + \frac{1}{4}\cos 2x\right]_{\frac{\pi}{4}}^{\frac{\pi}{2}}$$

$$\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} x\cos 2x \, dx = \left(\frac{\pi}{4}\sin \pi + \frac{1}{4}\cos \pi\right) - \left(\frac{\pi}{8}\sin\frac{\pi}{2} + \frac{1}{4}\cos\frac{\pi}{2}\right)$$

$$\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} x\cos 2x \, dx = -\frac{1}{4} - \frac{\pi}{8}$$

(iv) Explain what your answer in part **(iii)** implies about the curve $y = x\cos 2x$. [1]

It is the area enclosed by the curve $y = x\cos 2x$ below the x-axis between $x = \frac{\pi}{4}$ and $x = \frac{\pi}{2}$.

11 The table shows experimental values of two variables, x and y.

x	2	4	6	8
y	2.25	0.81	0.47	0.33

(i) On the graph paper, plot xy against $\frac{1}{x}$ and draw a straight line graph. [3]

$\frac{1}{x}$	0.5	0.25	0.17	0.125
xy	4.5	3.24	2.82	2.64

Table of values

Plot xy against $\frac{1}{x}$

Line cuts vertical axis

Using your graph,

(ii) estimate the value of x and y for which $x = \frac{3}{y}$, [2]

Draw horizontal line $xy = 3$

x from 4.8 to 5.2

y from 0.5 to 0.7

(iii) express y in terms of x. [4]

Finding gradient

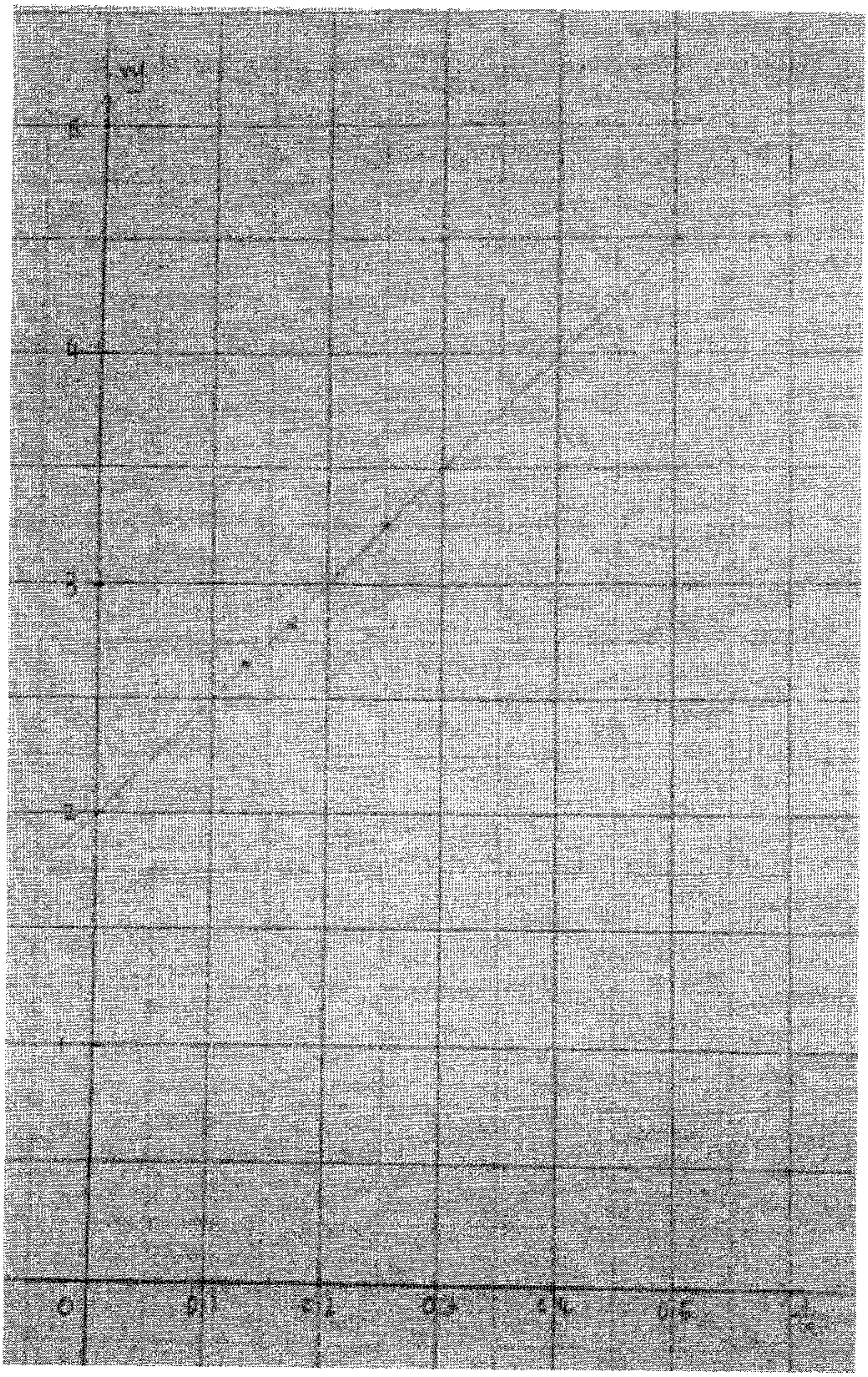
Gradient from 4.8 to 5.2

Vertical intercept from 1.9 to 2.1

Use $Y = \mathrm{m}X + \mathrm{c}$

$y = \frac{5}{x^2} + \frac{2}{x}$ (oe)

BP~359



12 A particle travels in a straight line, so that t seconds after passing a fixed point O, its velocity, v m/s, is given by $v = 22 + 7t - 2t^2$. The particle comes to instantaneous rest at P. Find

(i) the velocity of the particle when the acceleration is zero, [2]

$$a = 7 - 4t$$

$$\text{at } a = 0,\ t = \frac{7}{4}$$

$$v = 22 + 7\left(\frac{7}{4}\right) - 2\left(\frac{7}{4}\right)^2 = 28\frac{1}{8} \text{ m/s}$$

(ii) the value of t when the particle is at P, [2]

$$\text{at } v = 0,\ 22 + 7t - 2t^2 = 0$$

$$(11 - 2t)(2 + t) = 0$$

$$t = 5.5 \text{ or } t = -2 \text{ (NA)}$$

(iii) the distance OP, [2]

$$s = \int 22 + 7t - 2t^2 \ \mathrm{d}t$$

$$s = 22t + \frac{7}{2}t^2 - \frac{2}{3}t^3 + c$$

at $t = 0, s = 0$,

$c = 0$

$$s = 22t + \frac{7}{2}t^2 - \frac{2}{3}t^3$$

$$\text{at } t = 5.5,\ s = 22(5.5) + \frac{7}{2}(5.5)^2 - \frac{2}{3}(5.5)^3 = 115\frac{23}{24} \text{ m}$$

Alt Mtd :

$$s = \int_0^{5.5} (22 + 7t - 2t^2)\ dt$$

$$= \left[22t + \frac{7}{2}t^2 - \frac{2}{3}t^3\right]_0^{5.5} = 115\frac{23}{24}\ m$$

(iv) the total distance travelled by the particle in the interval $t = 0$ to $t = 9$. [2]

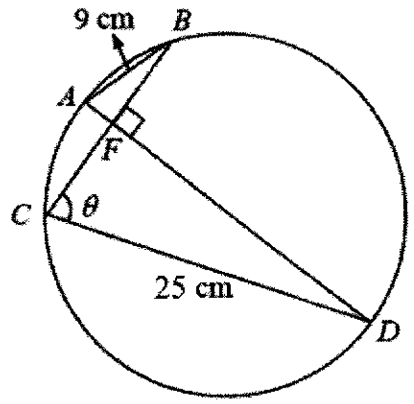
at $t = 0$, $s = 0$

at $t = 5.5$, $s = 115\frac{23}{24}$

at $t = 9$, $s = -\frac{9}{2}$

$$T_D = 115\frac{23}{24} \times 2 + \frac{9}{2} = 236\frac{5}{12}\text{ m}$$

13 In the diagram, AB and CD are chords of a circle, with $AB = 9$ cm and $CD = 25$ cm. It is given that AD and BC intersect each other each at 90° at F and $\angle BCD = \theta$, where θ varies.



(i) Prove that $AD = 9\cos\theta + 25\sin\theta$. [2]

$\angle BCD = \angle DAB = \theta$ (angle in the same segment)
$AD = AF + FD$
$AD = 9\cos\theta + 25\sin\theta$

(ii) Express AD in the form $R\cos(\theta - \alpha)$, where $R > 0$ and $0° < \alpha < 90°$. [3]

$R = \sqrt{9^2 + 25^2} = \sqrt{706}$

$\tan\alpha = \dfrac{25}{9}$

$\alpha = 70.2011...$

$AD = \sqrt{706}\cos(\theta - 70.2°)$

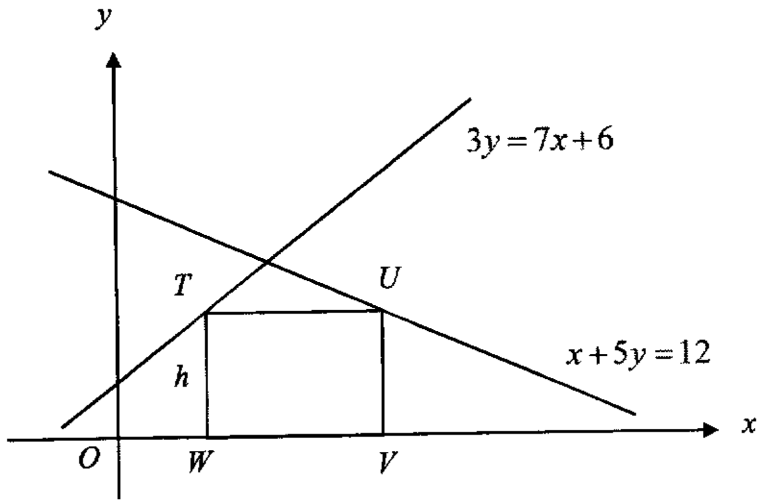
(iii) Find the values of θ when $AD = 26$ cm. [3]

$$\sqrt{706}\cos(\theta - 70.2011°) = 26$$
$$\cos(\theta - 70.2011°) = 0.9785229...$$
$$\theta - 70.2011° = -11.896...,11.896...$$
$$\theta = 58.3°,\ 82.1°$$

(iv) Calculate the angle θ for which AD is the diameter of the circle. [2]

AD is the longest. So $\cos(\theta - 70.2011°) = \cos 0°$
$\theta = 70.2°$

14 The diagram shows parts of the graphs of $3y=7x+6$ and $x+5y=12$. T is a point on $3y=7x+6$ and U is a point on $x+5y=12$. V and W are two points on the x-axis such that $TUVW$ is a rectangle.



It is given that $TW = h$ units.

(i) Find the x-coordinates of T and U in terms of h. [2]

$3h = 7x_T + 6$ $\qquad$ $x_U + 5h = 12$

$\frac{3h-6}{7} = x_T$ $\qquad$ $x_U = 12 - 5h$

(ii) Hence, show that the area of the rectangle $TUVW$, represented by A square units, is given by $A = \frac{90}{7}h - \frac{38}{7}h^2$. [2]

$$VW = x_2 - x_1 = 12 - 5h - \frac{3h-6}{7}$$

$$VW = \frac{84-35h-(3h-6)}{7} = \frac{90-38h}{7}$$

$$A = \frac{90-38h}{7} \times h = \frac{90}{7}h - \frac{38}{7}h^2$$

(ii) Given that h can vary, find the stationary value of A and determine whether this value of A is a maximum or a minimum. [5]

$$\frac{dA}{dh}=\frac{90}{7}-\frac{76}{7}h$$

$$\text{at } \frac{dA}{dh}=0,\ \frac{90}{7}=\frac{76}{7}h$$

$$h=1.18$$

$$\frac{d^2A}{dh^2}=-\frac{76}{7}<0$$

Thus area is a maximum.

$$A=\frac{90}{7}(1.18421)-\frac{38}{7}(1.18421)^2=7.61278...$$

$$A=7.61$$

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Class | Index Number

Name : ______________________________

METHODIST GIRLS' SCHOOL

Founded in 1887

PRELIMINARY EXAMINATION 2024
Secondary 4

Tuesday **ADDITIONAL MATHEMATICS** **4049/02**

13 August 2024 **PAPER 2** 2 hours 15 mins

Candidates answer on the Question Paper.
No Additional Materials are required.

READ THESE INSTRUCTIONS FIRST
Write your class, index number and name in the spaces at the top of this page.
Write in dark blue or black pen
You may use a HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** questions.
Give non-exact numerical answers correct to 3 significant figure, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.

The total number of marks for this paper is 90.

/ 90

1. ALGEBRA

Quadratic Equation

For the quadratic equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial Expansion

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$.

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2\sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

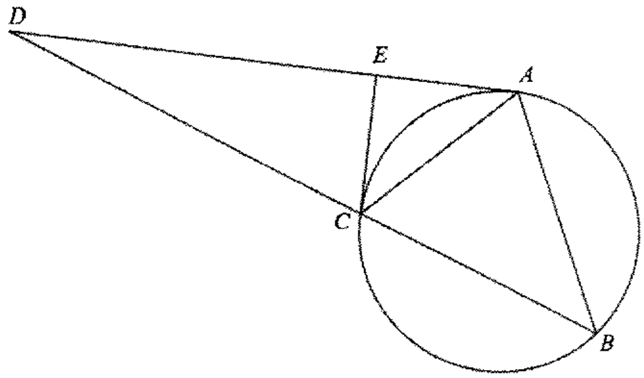
$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2} bc \sin A$$

1 **(a)** Solve the equation $\sqrt{2x+1}-\sqrt{x}=1$. [3]

(b) Express $\dfrac{\left(2\sqrt{3}-5\right)^2}{\sqrt{3}-2}$ in the form $p\sqrt{3}+q$ where p and q are integers. [4]

2 The diagram shows a circle passing through the vertices of a triangle ABC. The tangent to the circle at A meets BC extended at point D. The tangent at C meets AD at E.



(i) Prove that angle CED is twice of angle ABC. [3]

(ii) By proving a pair of similar triangles, show that $BD \times CD = AD^2$. [3]

3 The mass, m grams, of a radioactive substance detected in a piece of stone is given by the formula $m = \alpha e^{-kt}$, where $\alpha \neq 0$, k is a constant and t is the time interval in months.

(i) The mass of the substance is reduced to half its original value four months after it was first being detected, find the value of k. [2]

(ii) Find the initial mass of the substance given its mass after 1 month is 0.25 g. [2]

(iii) Calculate the time taken for the mass to reduce to 0.01 g. [2]

4 **(a)** Solve the equation $3\cos 2x = \sin x + 2$, for $0° \leq x \leq 360°$. [4]

(b) Find all the angles between 0 and 5 which satisfy the equation $\sin 2x + 3\cos^2 x = 0$. [4]

5 (a) (i) Given that m is a constant, expand $(3+mx)^4$, in ascending powers of x, simplifying each term in your expansion. [2]

(ii) Given also that the coefficient of x is equal to the coefficient of x^2, find the value of m. [1]

(b) (i) By considering the general term in the binomial expansion of $\left(3x-\dfrac{1}{2x}\right)^9$, explain why there are no even powers of x in this expansion. [3]

(ii) Using the value of m found in **part (a)(ii)**, find the term independent of x in the expansion of $(3+mx)^4\left(3x-\dfrac{1}{2x}\right)^9$. [4]

6 A circle C has equation $x^2+y^2+6x-4y=12$.

(i) Find the centre and the radius of C. [3]

The points $P\,(-8, 2)$ and $Q\,(1, -1)$ lie on the circumference of C.

(ii) Determine whether PQ is a diameter of C. [2]

(iii) Find the equation of tangent to the circle at the point R (0, 6). [3]

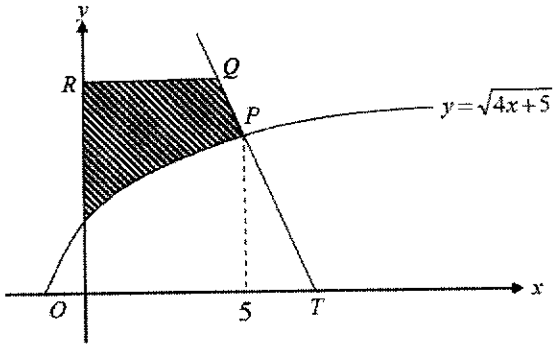
(iv) The circle C is reflected in the tangent line at R obtained in **part (iii)**. Write down the equation of the new circle. [2]

7 **(a)** Express $\dfrac{2x^3+x^2+x}{(x-1)(x^2+1)}$ in partial fractions. [5]

(b) Differentiate $\ln(x^2+1)$ with respect to x. [1]

(c) Hence, find $\int \frac{2x^3+x^2+x}{(x-1)(x^2+1)}\,dx$. [4]

8 The diagram shows part of the curve $y = \sqrt{4x+5}$. The normal of the curve at P meets the x-axis at T. The x-coordinate of P is 5. Given that PT is twice of PQ and RQ is parallel to the x-axis,



(i) find the equation of the normal at P, [3]

(ii) explain why the curve has no stationary points, [2]

(iii) find the coordinates of Q, [2]

(iv) find the area of the shaded region. [3]

9 **(a)** It is given that $y = a\sin bx + c$, for $0° \le x \le 360°$, where a, b, c are integers and $a > 0$.
The period is $120°$ and the maximum and minimum value of y is 3 and -5.

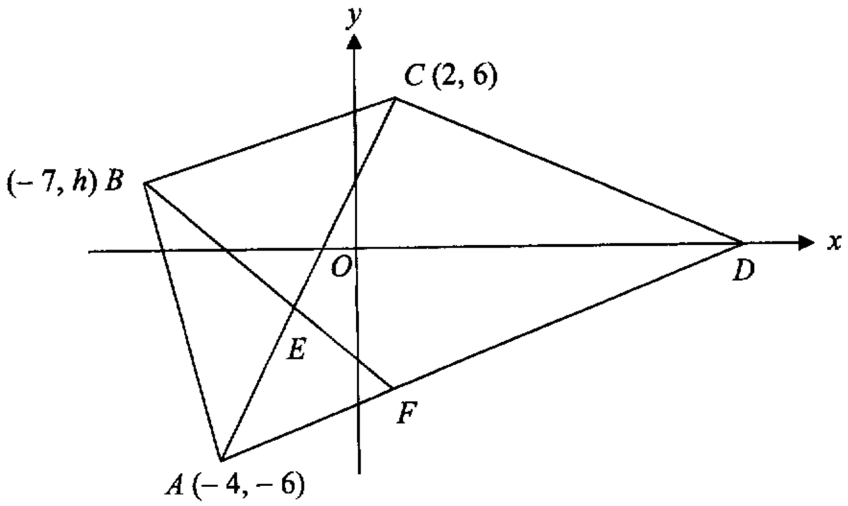
(i) State the value of a, of b and of c. [3]

(ii) Sketch the graph of $y = a\sin bx + c$. [2]

(b) Prove that $\dfrac{1+\cos x}{1-\cos x} = (\operatorname{cosec} x + \cot x)^2$. [3]

(c) Without the use of a calculator, find x such that $\cos 2x = \sin 230°$ where $0° < x < 180°$. [3]

10 **Solutions to this question by accurate drawing will not be accepted.**



The diagram, which is not drawn to scale, shows a quadrilateral $ABCD$ with vertices $A(-4,-6)$, $B(-7,\ h)$ and $C(2,\ 6)$ and D, which lies on the x-axis. AD is parallel to BC. The point E lies on AC such that $AE:AC=1:3$. The line BE produced meets the line AD at F.

(i) Given that $AB = BC$, show that $h = 3$. [2]

(ii) Show that $\angle ABC = 90°$. [2]

(iii) Find the coordinates of D. [2]

(iv) Find the coordinates of E. [2]

(v) Find the coordinates of G such that $ABCG$ is a square. [2]

(vi) Find the area of triangle ABC. [2]

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PAPER 2
SOLUTIONS

1 **(a)** Solve the equation $\sqrt{2x+1}-\sqrt{x}=1$. [3]

$\sqrt{2x+1}-\sqrt{x}=1$

$\sqrt{2x+1}=1+\sqrt{x}$

$2x+1=1+2\sqrt{x}+x$ M1

$x=2\sqrt{x}$

$x^2=4x$ M1

$x(x-4)=0$

$x=0$ or $x=4$ A1

(b) Express $\dfrac{\left(2\sqrt{3}-5\right)^2}{\sqrt{3}-2}$ in the form $p\sqrt{3}+q$ where p and q are integers. [4]

$\dfrac{\left(2\sqrt{3}-5\right)^2}{\sqrt{3}-2}$

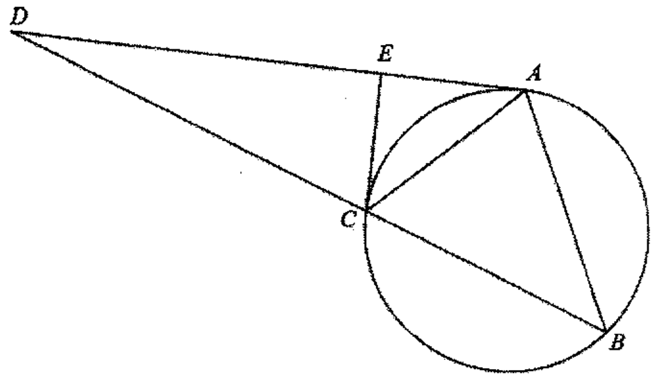
$=\dfrac{12-20\sqrt{3}+25}{\sqrt{3}-2}\times\dfrac{\sqrt{3}+2}{\sqrt{3}+2}$ M1

$=\dfrac{\left(37-20\sqrt{3}\right)\left(\sqrt{3}+2\right)}{3-4}$ M1

$=-\left(37\sqrt{3}+74-60-40\sqrt{3}\right)$ M1

$=3\sqrt{3}-14$ A1

2 The diagram shows a circle passing through the vertices of a triangle ABC. The tangent to the circle at A meets BC extended at point D. The tangent at C meets AD at E.



(i) Prove that angle CED is twice of angle ABC. [3]

Let angle $EAC = x$ and angle $ECD = y$
angle $ECA = x$ (base angles, isosceles triangle) M1
angle $ABC = x$ (alternate segment theorem) M1
angle $CED = 2x$ (exterior angle = sum of interior opposite angles) A1
Therefore, angle CED is twice of angle ABC.

(ii) By proving a pair of similar triangles, show that $BD \times CD = AD^2$. [3]

angle ABD = angle CAD (alternate segment theorem) M1
angle ADB = angle CDA (common)
Therefore, triangle ABD is similar to triangle CAD. M1

$$\frac{AD}{CD} = \frac{BD}{AD}$$

$BD \times CD = AD^2$ A1

3 The mass, m grams, of a radioactive substance detected in a piece of stone is given by the formula $m = \alpha e^{-kt}$, where $\alpha \neq 0$, k is a constant and t is the time interval in months.

(i) The mass of the substance is reduced to half its original value four months after it was first being detected, find the value of k. [2]

$m = \alpha e^{-kt}$

$\frac{1}{2}\alpha = \alpha e^{-k(4)}$

$\ln\left(\frac{1}{2}\right) = -4k$ M1

$\frac{1}{2}\alpha = \alpha e^{-k(4)}$

$k = 0.1732867...$

$k = 0.173$ A1

(ii) Find the initial mass of the substance given its mass after 1 month is 0.25 g. [2]

$0.25 = \alpha e^{-0.1732867(1)}$

$\frac{0.25}{e^{-0.1732867}} = \alpha$ M1

$\alpha = 0.29730177...$

$\alpha = 0.297$ A1

(iii) Calculate the time taken for the mass to reduce to 0.01 g. [2]

$0.01 = 0.29730177e^{-0.1732867t}$

$\frac{0.01}{0.29730177} = e^{-0.1732867t}$ M1

$\ln\left(\frac{0.01}{0.29730177}\right) = -0.1732867t$

$t = 19.575...$

$t = 19.6$ A1

4 **(a)** Solve the equation $3\cos 2x = \sin x + 2$, for $0° \le x \le 360°$. [4]

$3(1-2\sin^2 x) = \sin x + 2$ M1

$6\sin^2 x + \sin x - 1 = 0$

$(3\sin x - 1)(2\sin x + 1) = 0$ M1

$\sin x = \frac{1}{3}$ $\qquad$ $\sin x = -\frac{1}{2}$ A2

$x = 19.5°, 160.5°$ $\qquad$ $x = 210°, 330°$

(b) Find all the angles between 0 and 5 which satisfy the equation $\sin 2x + 3\cos^2 x = 0$. [4]

$\sin 2x + 3\cos^2 x = 0$

$2\sin x\cos x + 3\cos^2 x = 0$ M1

$\cos x(2\sin x + 3\cos x) = 0$ M1

$\cos x = 0$ $\quad$ $2\sin x = -3\cos x$

$x = \dfrac{\pi}{2}, \dfrac{3\pi}{2}$ or $\quad$ $\tan x = -1.5$ A2

$x = 2.16$

5 **(a)** **(i)** Given that m is a constant, expand $(3+mx)^4$, in ascending powers of x, simplifying each term in your expansion. [2]

$$(3+mx)^4$$
$$=3^4+\binom{4}{1}(3^3)mx+\binom{4}{2}(3^2)(mx)^2+\binom{4}{3}(3)(mx)^3+(mx)^4 \quad \text{M1}$$
$$=81+108mx+54m^2x^2+12m^3x^3+m^4x^4 \quad \text{A1}$$

(ii) Given also that the coefficient of x is equal to the coefficient of x^2, find the value of m. [1]

$108m=54m^2$ A1

$m=2$

(b) **(i)** By considering the general term in the binomial expansion of $\left(3x-\frac{1}{2x}\right)^9$, explain why there are no even powers of x in this expansion. [3]

$$\binom{9}{r}(3x)^{9-r}\left(-\frac{1}{2x}\right)^r \quad \text{M1}$$

Powers of $x=9-r-r=9-2r$ M1

$2r$ is always even for all integer values of r

The difference between an odd and even number is always odd. A1

Thus, there are no even powers of x in the expansion

(ii) Using the value of m found in **part (a)(ii)**, find the term independent of x in the expansion of $(3+mx)^4\left(3x-\dfrac{1}{2x}\right)^9$. [4]

$$(81+108mx+54m^2x^2+12m^3x^3+m^4x^4)\left(3x-\frac{1}{2x}\right)^9$$ M1

$$=(81+216x+216x^2+96x^3+16x^4)(\ldots\ldots\ x^{-1}\ \ \ldots\ldots x^{-3}\ \ \ldots\ldots)$$

$x^{9-2r}=x^{-1}$	$x^{9-2r}=x^{-3}$	
$9-2r=-1$	$9-2r=-3$	M1
$2r=10$	$2r=12$	
$r=5$	$r=6$	

$$\text{cofficient}=216\times\binom{9}{5}(3)^4(-\frac{1}{2})^5+96\times\binom{9}{6}(3)^3(-\frac{1}{2})^6$$ M1

$$=-\frac{130977}{2}$$ A1

6 A circle C has equation $x^2+y^2+6x-4y=12$.

(i) Find the centre and the radius of C. [3]

$x^2+y^2+6x-4y=12$ M1

$(x+3)^2+(y-2)^2=25$

centre is $(-3,2)$ A2

radius = 5 units

The points P (–8, 2) and Q (1, –1) lie on the circumference of C.

(ii) Determine whether PQ is a diameter of C. [2]

$\sqrt{(-8-1)^2+(2+1)^2}=9.4868\neq 10$ M1 A1

PQ is not a diameter of C

(iii) Find the equation of tangent to the circle at the point R (0, 6). [3]

$m=\frac{6-2}{0+3}=\frac{4}{3}$ M1

$m\perp=-\frac{3}{4}$ M1

eqn of tangent,

$y=-\frac{3}{4}x+6$ A1

(iv) The circle C is reflected in the tangent line at R obtained in **part (iii)**. Write down the equation of the new circle. [2]

new centre = (3, 10) M1

new equation : $(x-3)^2+(y-10)^2=25$ A1

7 **(a)** Express $\dfrac{2x^3+x^2+x}{(x-1)(x^2+1)}$ in partial fractions. [5]

$(x-1)(x^2+1)=x^3-x^2+x-1$ M1

$$\begin{array}{r l}
 & \quad 2 \\
x^3-x^2+x-1 \,\big) & 2x^3+x^2+x+0 \\
-) & \underline{2x^3-2x^2+2x-2} \\
 & \quad 3x^2-x+2
\end{array}$$

$\dfrac{2x^3+x^2+x}{(x-1)(x^2+1)} = 2+\dfrac{3x^2-x+2}{(x-1)(x^2+1)}$ M1

$\dfrac{3x^2-x+2}{(x-1)(x^2+1)}=\dfrac{A}{x-1}+\dfrac{Bx+C}{x^2+1}$

$3x^2-x+2=A(x^2+1)+(x-1)(Bx+C)$

at $x=1$,	at $x=0$,	$3=A+B$	M2
$4=2A$	$2=2-C$	$1=B$	
$2=A$	$C=0$		

$\dfrac{2x^3+x^2+x}{(x-1)(x^2+1)} = 2+\dfrac{2}{x-1}+\dfrac{x}{x^2+1}$ A1

(b) Differentiate $\ln(x^2+1)$ with respect to x. [1]

$\dfrac{\mathrm{d}\left[\ln(x^2+1)\right]}{\mathrm{d}x}=\dfrac{2x}{x^2+1}$

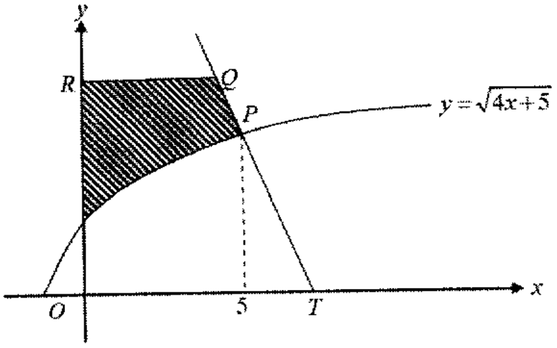
(c) Hence, find $\int \frac{2x^3+x^2+x}{(x-1)(x^2+1)}\,\mathrm{d}x$. [4]

$$\int \frac{2x^3+x^2+x}{(x-1)(x^2+1)}\,\mathrm{d}x = \int 2+\frac{2}{x-1}+\frac{x}{x^2+1}\,\mathrm{d}x$$ M1

$$\int \frac{2x^3+x^2+x}{(x-1)(x^2+1)}\,\mathrm{d}x = \left[2x+2\ln(x-1)\right]+c_1+\frac{1}{2}\int \frac{2x}{x^2+1}\,\mathrm{d}x$$ M1

$$\int \frac{2x^3+x^2+x}{(x-1)(x^2+1)}\,\mathrm{d}x = 2x+2\ln(x-1)+\frac{1}{2}\ln(x^2+1)+c_2$$ M1 A1

8 The diagram shows part of the curve $y=\sqrt{4x+5}$. The normal of the curve at P meets the x-axis at T. The x-coordinate of P is 5. Given that PT is twice of PQ and RQ is parallel to the x-axis,



(i) find the equation of the normal at P, [3]

$y=\sqrt{4x+5}$

$\frac{dy}{dx}=\frac{1}{2}(4x+5)^{-\frac{1}{2}}(4)=\frac{2}{\sqrt{4x+5}}$ M1

at $x=5$, $\frac{dy}{dx}=\frac{2}{5}$ M1

$y=5$

eqn of normal,

$y-5=-\frac{5}{2}(x-5)$

$5x+2y=35$ A1

(ii) explain why the curve has no stationary points, [2]

$\frac{dy}{dx}=\frac{2}{\sqrt{4x+5}}$

since $\sqrt{4x+5}>0$,

$2>0$, M1

$\frac{dy}{dx}>0$ A1

so, the curve has no stationary points.

(iii) find the coordinates of Q, [2]

at $y = 0, x = 7$
$T(7, 0)$ M1

Since $PT = 2\,PQ$,
$x_Q = 4$
$Q(4,\ 7.5)$ A1

(iv) find the area of the shaded region. [3]

$$\int_0^5 \sqrt{4x+5}\ \mathrm{d}x$$

$$=\left[\frac{2(4x+5)^{\frac{3}{2}}}{3(4)}\right]_0^5 = 18.9699...$$ M1

Area of shaded region $= 5(5) + \frac{1}{2}(4+5)(2.5) - 18.9699... = 17.3$ M1 A1

9 **(a)** It is given that $y = a\sin bx + c$, for $0° \le x \le 360°$, where a, b, c are integers and $a > 0$.
The period is $120°$ and the maximum and minimum value of y is 3 and -5.

(i) State the value of a, of b and of c. [3]

$3 = a(1) + c$ $\quad\quad$ $-5 = a(-1) + c$

$3 = a + c$ $\quad\quad$ $-5 = -a + c$

$-2 = 2c$

$-1 = c$ $\quad\quad$ A1

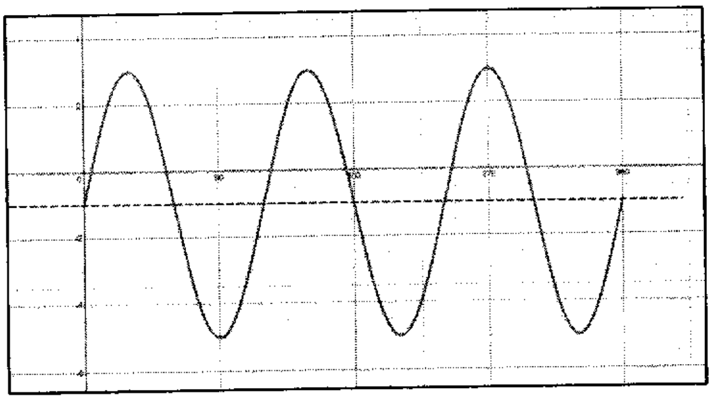
$a = 4$ $\quad\quad$ A1

$$\text{period} = \frac{360°}{b}$$

$$120° = \frac{360°}{b}$$

$b = 3$ $\quad\quad$ A1

(ii) Sketch the graph of $y = a\sin bx + c$. [2]



(b) Prove that $\dfrac{1+\cos x}{1-\cos x} = (\operatorname{cosec} x+\cot x)^2$. [3]

$(\operatorname{cosec} x+\cot x)^2$

$==\left(\dfrac{1}{\sin x}+\dfrac{\cos x}{\sin x}\right)^2$

$=\dfrac{(1+\cos x)^2}{\sin^2 x}$ M1

$=\dfrac{(1+\cos x)^2}{1-\cos^2 x}$

$=\dfrac{(1+\cos x)^2}{(1-\cos x)(1+\cos x)}$ M1

$=\dfrac{1+\cos x}{1-\cos x}$ A1

(c) Without the use of a calculator, find x such that $\cos 2x = \sin 230°$ where $0° < x < 180°$. [3]

$\cos 2x = \sin 230° = -\sin 50°$ $0° < 2x < 360°$

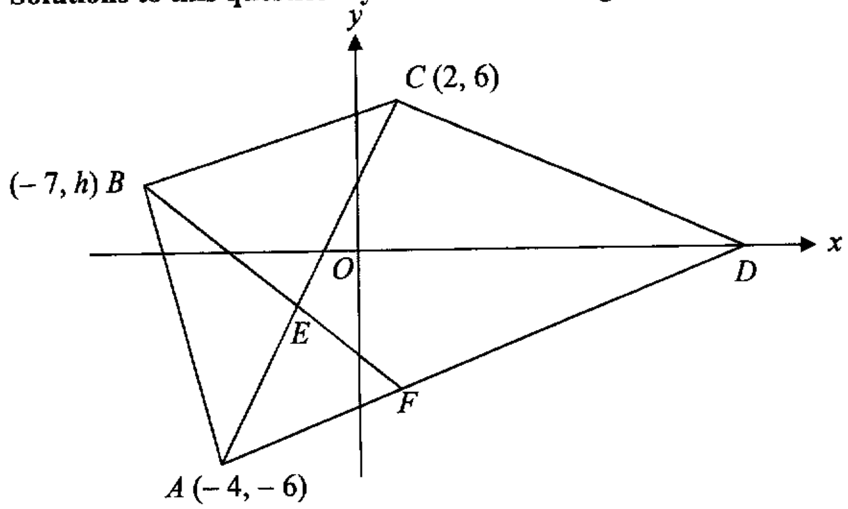
$\cos 2x = -\cos 40°$ M1

basic angle $= 40°$

$2x = 140°,\ 220°$ M1

$x = 70°,\ 110°$ A1

10 **Solutions to this question by accurate drawing will not be accepted.**



The diagram, which is not drawn to scale, shows a quadrilateral $ABCD$ with vertices $A(-4,-6)$, $B(-7,\ h)$ and $C(2,\ 6)$ and D, which lies on the x-axis. AD is parallel to BC. The point E lies on AC such that $AE:AC=1:3$. The line BE produced meets the line AD at F.

(i) Given that $AB = BC$, show that $h = 3$. [2]

$AB = BC$

$(h+6)^2+(-7+4)^2=(6-h)^2+(2+7)^2$ M1

$h^2+12h+36+9=36-12h+h^2+81$

$24h=72$

$h=3\,(\text{shown})$ A1

(ii) Show that $\angle ABC = 90°$. [2]

$m_{AB}=\dfrac{3+6}{-7+4}=-3$ M1

$m_{BC}=\dfrac{6-3}{2+7}=\dfrac{1}{3}$

Since gradient of AB × gradient of $BC=-1$, A1

$\angle ABC=90°$.

(iii) Find the coordinates of D. [2]

$m_{AD} = \frac{1}{3}$ and D lies on the x-axis

$\frac{0+6}{x+4} = \frac{1}{3}$ M1

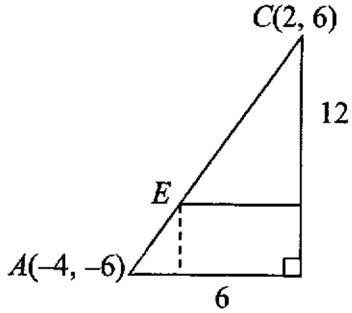
$x = 14$

$D(14, 0)$ A1

(iv) Find the coordinates of E. [2]

$E(-4+2, -6+4)$ M1

$E(-2, -2)$ A1



(v) Find the coordinates of G such that $ABCG$ is a square. [2]

Let the coordinates of G be (x, y).

Midpoint of $BG = \left(\frac{-7+x}{2}, \frac{3+y}{2}\right) = (-1, 0)$ M1

$\frac{-7+x}{2} = -1$ and $\frac{3+y}{2} = 0$

$x = 5$ $\quad$ $y = -3$

Therefore, $G(5, -3)$ A1

(vi) Find the area of triangle ABC. [2]

$$A = \frac{1}{2}\begin{vmatrix} -7 & -4 & 2 & -7 \\ 3 & -6 & 6 & 3 \end{vmatrix}$$

$A = \frac{1}{2} \times [(42-24+6)-(-12-12-42)]$ M1

$A = 45$ A1

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