1 Find the range of values of the constant m for which the curve $y = (m - 6)x^2 - 8x + m$ lies completely above the x-axis. [4]

2 Show that the line $x + y = m$ will intersect the curve $x^2 + 2y^2 = 2x + 3$ if $m^2 \leq 2m + 5$. [4]

3 It is given that $f(x) = (b - 3x)e^{2-3x}$. Find the value of the constant b if $f(x)$ is a decreasing function when $x < \frac{4}{3}$. [4]

4 Prove that $\frac{2-\operatorname{cosec}^2\theta}{\operatorname{cosec}^2\theta+2\cot\theta}=\frac{1-\cot\theta}{1+\cot\theta}$. [4]

5 Express $\frac{4x^2+5x-32}{(x+2)(x^2-9x-22)}$ in partial fractions. [5]

6 **(a)** Show that $3\cos x = 2\,\mathrm{cosec}\,x\cot x$ can be written as $\cos x\,(3\sin^2 x - 2) = 0$. [3]

(b) Hence, solve the equation $3\cos(0.6y - 1.4) = 2\,\mathrm{cosec}(0.6y - 1.4)\cot(0.6y - 1.4)$ for values of y between -3 and 4. [5]

7 **(a)** Given that $y = \frac{1+\sin x}{\cos x}$, find $\frac{dy}{dx}$. [2]

(b) Hence, without using a calculator, find the value of each of the constants p and q for which $\int_0^{\frac{\pi}{3}} \frac{3 + 3\sin x - 10\cos^3 x}{5\cos^2 x}\, dx = p + q\sqrt{3}$. [6]

8 The height of Jeremiah above the surface of the water, h metres, can be modelled by the equation $h = -4.9t^2 + 8t + 5$, where t is the time in seconds after he leaves the diving board.

(a) State the height of the diving board above the surface of the water. [1]

(b) Express h in the form $k - a(x - b)^2$, where k, a and b are constants to be determined. [3]

(c) State the greatest height reached by Jeremiah and the corresponding time when the greatest height occurs. [2]

(d) Using your answer obtained in **(b)**, calculate the duration which Jeremiah stay in the air. [2]

9 Solve the simultaneous equations.

$$\frac{5^p}{25} = 125^q$$

$$\log_3 7 = 1 + \log_3(11q - 2p)$$ [5]

10 The equation of a curve is $y = x^3 \ln x$.

(a) Find the exact coordinates of the stationary point(s) of the curve. [4]

(b) Determine the nature of the stationary point(s) of the curve. [2]

A particle moves along the curve $y = x^3 \ln x$. At point *M*, the *x*-coordinate of the particle is increasing at a rate of 0.06 units/s and the *y*-coordinate is increasing at a rate of $0.3x^2$ units/s.

(c) Find the exact coordinate of *M*. [4]

11 A circle, C_1, has equation $x^2 + y^2 + kx - 6y = h$, where k and h are constants.

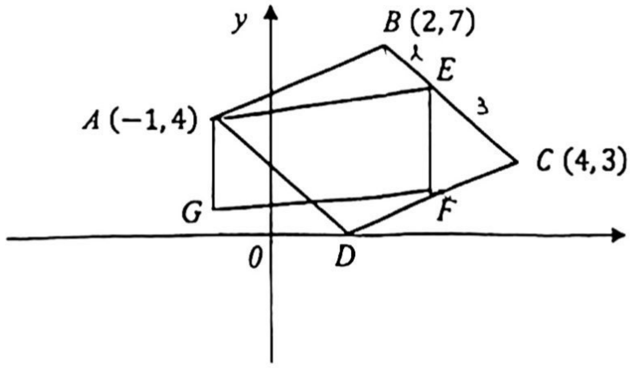
(a) Given that the radius C_1 of is 7 units and the coordinates of the centre is $(-2, m)$, find the values of k, m and h. [4]

(b) Another circle, C_2, has diameter PQ. The point P is $(3, n)$ and Q is $(-5, 5)$. The equation PQ is $4y + 3x = 5$.

(i) Find the equation of C_2. [4]

(ii) Explain, with appropriate working, why the point $S\ (4, 5)$ only lies inside the circle C_1 but not C_2. [2]

12 The diagram shows a parallelogram $ABCD$ in which D lies on the x-axis. Point A is $(-1, 4)$, $B\ (2, 7)$ and $C\ (4, 3)$. The point E lies on BC such that $5BE = 2BC$.



(a) Find the coordinates of D, E and F. [8]

Continuation of working space for question **12(a)**.

(b) *AG* and *EF* are two vertical lines. The y-coordinate of G is $\frac{2}{5}$. E and F lie on BC and DC respectively. Explain, with an appropriate working, what is the name of the special quadrilateral $AEFG$. [2]

13 A container in the shape of a right pyramid, has a height of 45 cm and a square base of side 20 cm, was initially empty. Sand is then allowed to flow into the container through a small hole at the top. After t seconds, the height of the sand in the container is $(45 - x)$ cm and the volume of the sand in the container is V cm^3.

(a) Show that $V = 6000 - \frac{16}{243}x^3$. [3]

$45 - x$

(b) Given that the rate of flow of the sand into the container is bx^2 cm^3/s, where b is a constant. Find the **numerical** value of the rate of change of x if the height of the sand in the container is 36 cm after 24 seconds. [7]

End of paper

Qn	Solutions
1	$m-6>0$ and $b^2-4ac<0$ $m>6$ $(-8)^2-4(m-6)(m)<0$ $64-4m^2+24m<0$ $m^2-6m-16>0$ $(m+2)(m-8)>0$ $m<-2$ or $m>8$ Hence, $m>8$
2	$y=m-x$ ....(1) Sub (1) into $x^2+2y^2=2x+3$, $x^2+2(m-x)^2-2x-3=0$ $3x^2-x(4m+2)+2m^2-3=0$ Discriminant $=[-(4m+2)]^2-4(3)(2m^2-3)$ $=16m^2+16m+4-24m^2+36$ $=-8(m^2-2m-5)$ Given $m^2\le 2m+5 \Rightarrow m^2-2m-5\le 0$ Hence $-8(m^2-2m-5)\ge 0$ Since, discriminant ≥ 0, hence the line will intersect the curve.
3	$f(x)=(b-3x)\,e^{2-3x}$ $f'(x)=-3e^{2-3x}+(b-3x)(-3e^{2-3x})$ $=-3e^{2-3x}(1+b-3x)$ $=3e^{2-3x}(3x-1-b)$ For decreasing function, $3e^{2-3x}(3x-1-b)<0$ Since $3e^{2-3x}>0 \Rightarrow 3x-1-b<0$ $x<\frac{1+b}{3}$ Given $x<\frac{4}{3}$, $\therefore 1+b=4$ $b=3$

Suggested Solutions for 2024 NCHS Prelim Exam Add Math Paper 1 (4049/1)

4	LHS $=\frac{2-\operatorname{cosec}^2\theta}{\operatorname{cosec}^2\theta+2\cot\theta}$ $=\frac{2-(1+\cot^2\theta)}{1+\cot^2\theta+2\cot\theta}$ $=\frac{1-\cot^2\theta}{(1+\cot\theta)^2}$ $=\frac{(1+\cot\theta)(1-\cot\theta)}{(1+\cot\theta)^2}$ $=\frac{1-\cot\theta}{1+\cot\theta}$
5	$x^2-9x-22=(x-11)(x+2)$ $\frac{4x^2+5x-32}{(x-11)(x+2)^2}=\frac{A}{x-11}+\frac{B}{x+2}+\frac{C}{(x+2)^2}$ $4x^2+5x-3=A(x+2)^2+B(x-11)(x+2)+C(x-11)$ Let $x=11$, $4(11)^2+5(11)-32=A(13)^2$ $A=3$ Let $x=-2$, $4(-2)^2+5(-2)-32=C(-13)$ $C=2$ Let $x=0$, $-32=A(4)+B(-11)(2)+C(-11)$ $-32=12-22B-22$ $B=1$ $\frac{3}{x-11}+\frac{1}{x+2}+\frac{2}{(x+2)^2}$
6(a)	$3\cos x=2(\frac{1}{\sin x})(\frac{\cos x}{\sin x})$ $3\cos x\sin^2 x=2\cos x$ $3\cos x\sin^2 x-2\cos x=0$ $\cos x(3\sin^2 x-2)=0$ (Shown)

(b)	From (a), $\cos x(3\sin^2 x - 2) = 0$ $3\sin^2 x - 2 = 0$ or $\cos x = 0$ $\sin^2 x = \frac{2}{3}$ $\sin x = \pm\sqrt{\frac{2}{3}}$ Basic angle, $\alpha = \sin^{-1}\sqrt{\frac{2}{3}}$ or Basic angle, $\alpha = \frac{\pi}{2}$ $= 0.95532$ New range: $-3.2 < 0.6y - 1.4 < 1$ Replaced x by $(0.6y - 1.4)$. $0.6y - 1.4 = 0.95532,$ or $0.6y - 1.4 = -\frac{\pi}{2}$ $-0.95532, -(\pi - 0.95532)$ $\quad \therefore y = -0.284$ (3sf) $\therefore y = -1.31, 0.741, 3.93$ (3sf) Answer: $-1.31, 0.741, 3.93$
7(a)	$\frac{dy}{dx} = \frac{(\cos x)(\cos x) - (1+\sin x)(-\sin x)}{\cos^2 x}$ $= \frac{\cos^2 x + \sin x + \sin^2 x}{\cos^2 x}$ $= \frac{1+\sin x}{\cos^2 x}$
(b)	From (a), $\int_0^{\frac{\pi}{3}} \frac{1+\sin x}{\cos^2 x}\,dx = \left[\frac{1+\sin x}{\cos x}\right]_0^{\frac{\pi}{3}}$ $= \frac{1+\sin(\frac{\pi}{3})}{\cos(\frac{\pi}{3})} - \frac{1+\sin 0}{\cos 0}$ $= \frac{1+\frac{\sqrt{3}}{2}}{\frac{1}{2}} - 1$ $= 1 + \sqrt{3}$

	$\int_0^{\frac{\pi}{3}} \frac{3 + 3\sin x - 10\cos^3 x}{5\cos^2 x}\,dx$ $= \int_0^{\frac{\pi}{3}} \frac{3+3\sin x}{5\cos^2 x} - 2\cos x\,dx$ $= \frac{3}{5}\int_0^{\frac{\pi}{3}} \frac{1+\sin x}{\cos^2 x}\,dx - \int_0^{\frac{\pi}{3}} 2\cos x\,dx$ $= \frac{3}{5}(1+\sqrt{3}) - [2\sin x]_0^{\frac{\pi}{3}}$ $= \frac{3}{5} + \frac{3}{5}\sqrt{3} - (\sqrt{3} - 0)$ $= \frac{3}{5} - \frac{2}{5}\sqrt{3}$ $p = \frac{3}{5}, q = -\frac{2}{5}$
8(a)	Sub $t = 0, h = 5$ m
(b)	$h = -4.9\left(t^2 - \frac{80}{49}t\right) + 5$ $= -4.9\left[\left(t - \frac{40}{49}\right)^2 - \left(-\frac{40}{49}\right)^2\right] + 5$ $= -4.9\left(t - \frac{40}{49}\right)^2 + \frac{160}{49} + 5$ $= 8\frac{13}{49} - 4.9\left(t - \frac{40}{49}\right)^2$
(c)	Greatest height $= 8\frac{13}{[illegible]}$ m; $t = \frac{40}{49}$ s
(d)	Sub $h = 0$. $8\frac{13}{49} - 4.9\left(t - \frac{40}{49}\right)^2 = 0$ $\left(t - \frac{40}{49}\right)^2 = 1.68680$ $t = 2.12$ s or -0.482 (rejected)
9	$5^p = 5^2 \times 5^{3q}$ $p = 2 + 3q$ ....(1) $\log_3 7 - \log_3(11q - 2p) = 1$ $\log_3 \frac{7}{11q - 2p} = 1$

	$\frac{7}{11q-2p}=3^1$ $7=33q-6p$ ....(2) Sub (1) into (2), $7=33q-6(2+3q)$ $q=\frac{19}{15}$ Sub $q=\frac{19}{15}$ into (1), $p=5\frac{4}{5}$
10 (a)	$\frac{dy}{dx}=x^3\left(\frac{1}{x}\right)+(\ln x)(3x^2)$ $=3x^2\ln x+x^2$ $=x^2(3\ln x+1)$ At stationary point, $\frac{dy}{dx}=0$ $x^2(3\ln x+1)=0$ $x^2=0$ or $3\ln x+1=0$ $x=0$ (rejected) $\ln x=-\frac{1}{3}$ $x=e^{-\frac{1}{3}}$ Sub $x=e^{-\frac{1}{3}}$ into $y=x^3\ ln\ x$, $y=e^{-1}\times\ln e^{-\frac{1}{3}}$ $=-\frac{1}{3e}$ Coordinates $=\left(e^{-\frac{1}{3}},-\frac{1}{3e}\right)$
(b)	$\frac{d^2y}{dx^2}=2x(3\ln x+1)+x^2$ ($=6x\ ln\ x+5x$ Sub $x=e^{-\frac{1}{3}}$, $\frac{d^2y}{dx^2}=6e^{-\frac{1}{3}}\ln e^{-\frac{1}{3}}+5e^{-\frac{1}{3}}=2.15>0$ Hence, $\left(e^{-\frac{1}{3}},-\frac{1}{3e}\right)$ is a minimum point.
(c)	$\frac{dy}{dt}=\frac{dy}{dx}\times\frac{dx}{dt}$ $0.3x^2=\frac{dy}{dx}\times0.06$ $\frac{dy}{dx}=5x^2$ $3x^2\ln x+x^2=5x^2$

	$x^2(3\ln x-4)=0$ $x^2=0$ or $3\ln x-4=0$ $x=0$ (rejected) or $x=e^{\frac{4}{3}}$ Sub $x=e^{\frac{4}{3}}$ into y, $y=e^4\ln e^{\frac{4}{3}}$ $=\frac{4}{3}e^4$ $M=(e^{\frac{4}{3}},\frac{4}{3}e^4)$
11 (a)	Centre $=(\frac{k}{-2},\frac{-6}{-2})$ $(-2,m)=(\frac{k}{-2},3)$ By comparing, $k=4$ $m=3$ Radius $=\sqrt{(-2)^2+(3)^2-(-h)}$ $7=\sqrt{13+h}$ $h=36$ <u>Alternative method:</u> $(x+2)^2+(y-m)^2=7^2$ $x^2+y^2+4x-2my=49-4-m^2$ Compare with $x^2+y^2+kx-6y=h$ $\therefore k=4$ $-2m=-6\Rightarrow m=3$ $h=49-4-m^2$ $=49-4-3^2$ $=36$
(bi)	Sub $x=3$, $4y+9=5$ $y=-1$ $P=(3,-1)$ Centre = midpoint of PQ $=\left(\frac{3+(-5)}{2},\frac{-1+5}{2}\right)$ $=(-1,2)$ Radius $=\frac{1}{2}\sqrt{(3+5)^2+(-1-5)^2}$ $=5$ Equation of C_2: $(x+1)^2+(y-2)^2=25$

(bii) Distance of S from centre of C_1 (-2, 3)

$= \sqrt{(4+2)^2 + (5-3)^2}$

$= 6.32 < 7$ (radius of C_1)

Distance of S from centre of C_2 (-1, 2)

$= \sqrt{(4+1)^2 + (5-2)^2}$

$= 5.83 > 5$ (radius of C_2)

Hence S only lies inside C_1 but not C_2.

12 (a) Let D be $(x, 0)$

Midpoint of AC = Midpoint of BD

$\left(\frac{x+2}{2}, \frac{0+7}{2}\right) = \left(\frac{-1+4}{2}, \frac{4+3}{2}\right)$

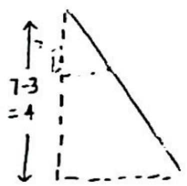
$x = 1 \Rightarrow D = (1, 0)$

By similar triangles,

$\frac{x-2}{2} = \frac{2}{5}$

$5x - 10 = 4$

$x = 2\frac{4}{5}$



$\frac{7-y}{4} = \frac{2}{5}$

$35 - 5y = 8$

$y = 5\frac{2}{5}$

$E = (2\frac{4}{5}, 5\frac{2}{5})$

Equation of DC:

$m_{DC} = m_{AB} = \frac{7-4}{2-(-1)} = 1$

Sub (4, 3), m = 1,

$y - 3 = 1(x-4)$

$y = x - 1$……….(1)

Sub $x = 2\frac{4}{5}$ into (1).

$y = 1\frac{4}{5}$

$F = (2\frac{4}{5}, 1\frac{4}{5})$

(b) $G = (-1, \frac{2}{5})$

$m_{AE} = \frac{5.4-4}{2.8+1} = \frac{7}{19}$

$m_{GF} = \frac{1.8-\frac{2}{5}}{2.8+1} = \frac{7}{19}$

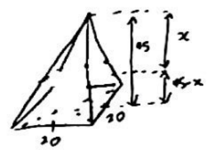
Hence $m_{AE} = m_{GE}$ and $m_{AG} = m_{EF}$ (Vertical lines)

Since there are 2 pairs of opposite parallel lines, $AEFG$ is a parallelogram.

13 (a) By similar triangles,

$\frac{s}{20} = \frac{x}{45}$

$s = \frac{4}{9}x$



Volume of sand, V

= Volume of big pyramid − volume of small pyramid

$= \frac{1}{3}(20)^2(45) - \frac{1}{3}\left(\frac{4}{9}x\right)^2(x)$

$= 6000 - \frac{16}{243}x^3$ (Shown)

(b) $\frac{dV}{dx} = -\frac{16}{81}x^2$

$\frac{dV}{dt} = \frac{dV}{dx} \times \frac{dx}{dt}$

$b\,x^2 = -\frac{16}{81}x^2 \times \frac{dx}{dt}$

$\frac{dx}{dt} = -\frac{81b}{16}$

$x = \int -\frac{81b}{16}\,dt$

$= -\frac{81b}{16}t + c$

Sub $t = 0, x = 45$

$c = 45$

Hence $x = -\frac{81b}{16}t + 45$

Given $t = 24$, $45 - x = 36 \Rightarrow x = 9$

Sub $t = 24, x = 9$,

$9 = -\frac{81b}{16} \times 24 + 45$

$b = \frac{8}{27}$

$\therefore \frac{dx}{dt} = -\frac{81b}{16}$

$= -\frac{81}{16} \times \frac{8}{27}$

$= -1.5$ cm/s

r^0

1 **(a)** Given that there is a term that ~~is in~~dependent of x in the expansion of $\left(5x^2 - \frac{1}{\sqrt{x}}\right)^n$, where n is a positive integer, find the smallest possible value of n. [3]

(b) Using the value of n found in part **(a)**, explain if there is any term independent of x in the expansion of $\left(1-\frac{1}{50x^5}\right)\left(5x^2-\frac{1}{\sqrt{x}}\right)^n$. [4]

2 The expression $10\,\mathrm{f}(x) + 3\,\mathrm{f}'(x) - \mathrm{f}''(x) + 7\sin 2x + 3\cos 2x$, may be written as $10x + 43$, when $\mathrm{f}'(x) = e^{5x} + 2\sin^2 x$. Find $\mathrm{f}(x)$.

[6]

3 **Do not use a calculator in this question.**

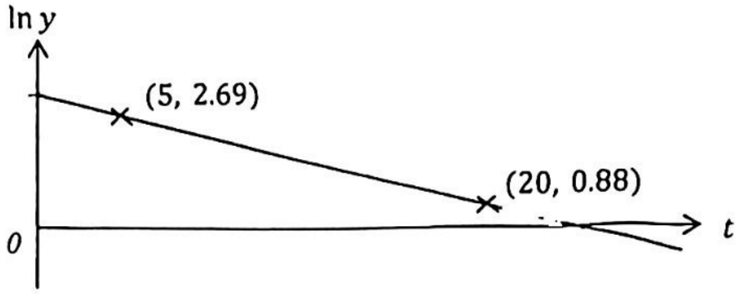
It is given that $\tan A = \frac{5}{12}$ and that $\frac{\pi}{2} < A < \frac{3\pi}{2}$.

(a) By expressing $\cos 3A = \cos(2A + A)$, find the exact value of $\cos 3A$. [4]

(b) Find the exact value of $\tan\frac{A}{2}$. [4]

4 Coffee is poured into an empty cup. At time t minutes after the coffee is poured, its temperature exceeds room temperature by y°C. The room temperature is 25°C.

(a)



The variables t and y are related by the equation $y = e^{kt+c}$, where k and c are constants. The diagram above shows the straight line graph obtained by plotting $\ln y$ against t. The line passes through the points (5, 2.69) and (20, 0.88).
Find the value of k and of c. [3]

(b) Calculate the time which the temperature of the coffee would drop to half of its initial temperature. [3]

(c) At the same time, when the coffee was poured into the cup, coffee of the same volume is also poured into an empty tumbler.

Similarly, at time t minutes after the coffee is poured into the tumbler, its temperature exceeds room temperature by y°C and is modelled by another equation.

The solution to the equation $e^{(k+0.2)t} = e^{5-c}$ is the timing where the temperature of the coffee in both the cup and tumbler are equivalent. By using the diagram in part **(a)**, outline the steps to find this timing. [4]

5

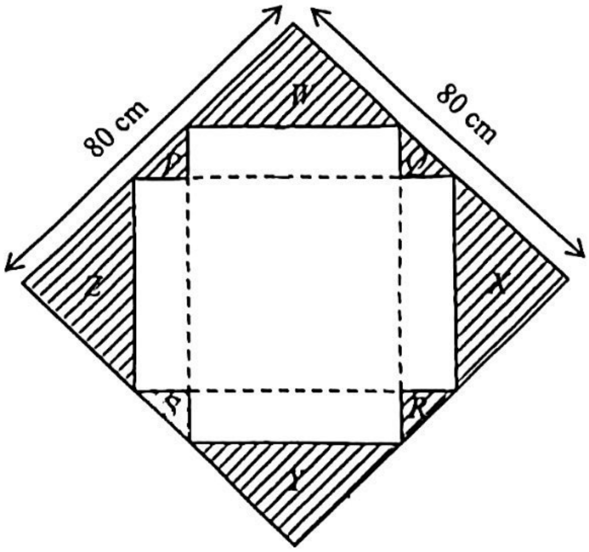


Diagram 1

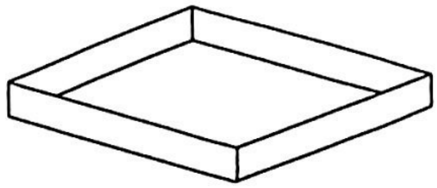


Diagram 2

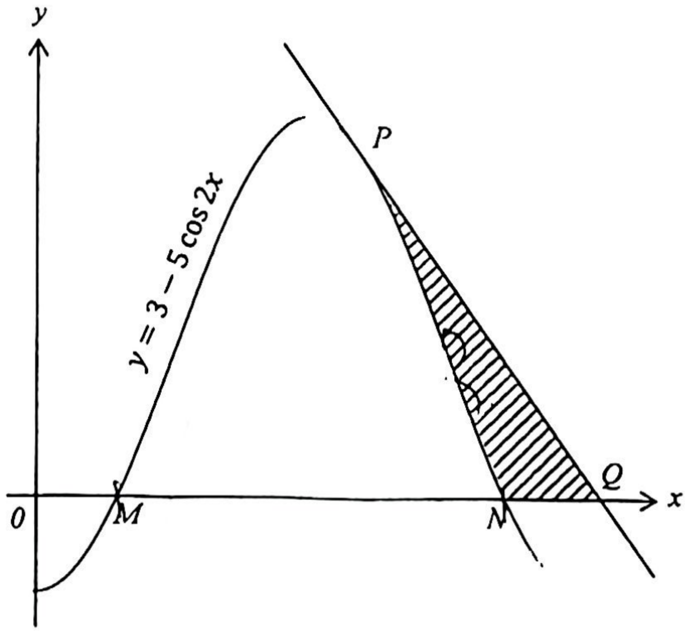
Diagram 1 shows a piece of square cardboard of side 80 cm.
Four small identical isosceles triangles, P, Q, R and S and four big identical isosceles triangles, W, X, Y and Z are removed. The remaining cardboard is folded along the dotted lines to form an open container as shown in Diagram 2.

(a) Let the height of the open container be h cm, show that the total exterior area, A cm^2, of the open container is $\cancel{3200} - 80\sqrt{2}h + h^2$. [4]

(b) Find the value of h for which the total exterior area of the open container is a maximum. [3]

(c) Hence, find the maximum total exterior area of the open container that can be obtained from the piece of square cardboard. [2]

6



The diagram shows part of the curve $y = 3 - 5\cos 2x$, which cuts the x-axis at M and N. The tangent to the curve at P is -5 and this tangent cuts the x-axis at Q. Find the area of the shaded region PNQ. [9]

Continuation of working space for Question 6.

7 **(a)** Show that $x + 2y$ is a factor of $4x^3 + x^2y - 11xy^2 + 6y^3$ and hence factorise $4x^3 + x^2y - 11xy^2 + 6y^3$ completely. [3]

(b) Hence, solve the equation $4^{p+1} + 2^{p+1} = 44 - 48(2^{-p})$. [5]

8 **(a)** The graph of $y = a\sin bx + c$ has one minimum point at $\left(\frac{\pi}{6}, 1\right)$ and the next maximum point after this has coordinates $\left(\frac{\pi}{2}, 9\right)$. Find the values of the constants a, b and c. [3]

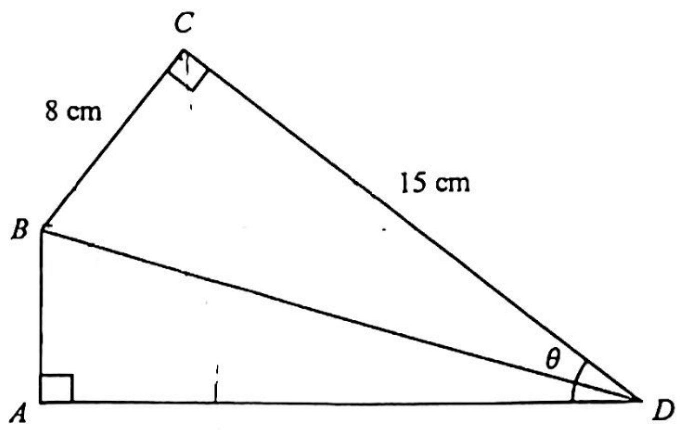
(b) A particle, travelling in a straight line, passes through a fixed point O. The velocity, v m/s, at time t seconds, is given by $v = 3t^2 - 5t + 7$ for $0 \leq t \leq 5$.

(i) Find the acceleration of the particle when $t = 4$. [2]

After $t > 5$ seconds, the particle travels at a velocity, in m/s, where $v = -4t + 77$.

(ii) Find the total distance travelled by the particle in the first 30 seconds. [7]

9



The diagram shows a metal structure $ABCD$ consisting of five metal rods of different lengths. T~~he length of BC and CD are~~ 15 m and ~~8 m respectively~~. Angle $ADC = \theta$ for $0° < \theta < 90°$.

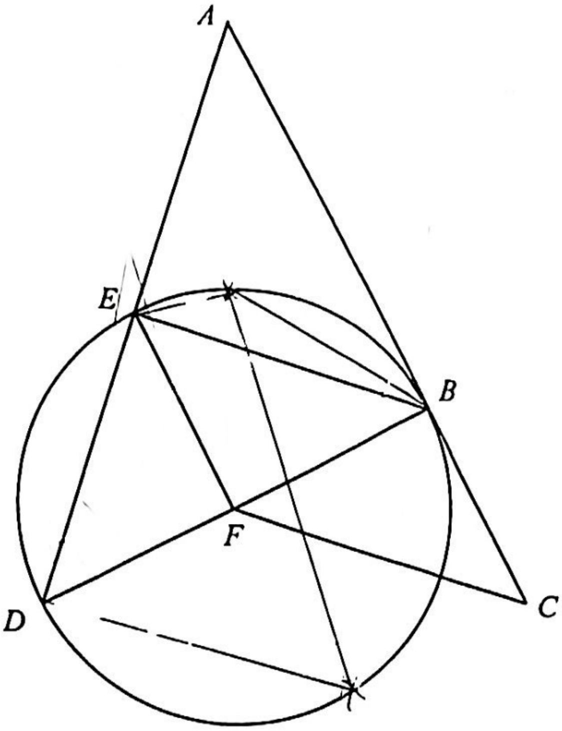
(a) Show that the total lengths, P m, of the five metal rods used is $40 + 23\sin\theta + 7\cos\theta$. [3]

(b) Express P in the form $40 + R\sin(\theta + \alpha)$, where $R > 0$ and $0° < \alpha < 90°$. [2]

(c) Find the value of θ if the total length of the five metal rods is 60 m. [3]

(d) State the minimum value of $\frac{1}{40+(R\sin(\theta+\alpha))^2}$ and the corresponding value of θ for which it occurs. [3]

10



The diagram shows a circle, centre F, with BD as diameter. The point E lies on the circle. The tangent at a point B on the circle meets DE extended at the point A. Point C lies on AB extended and $ED = AE$.

(a) State the relationship between the length of DF and AB. Give reasons to support your answer. [2]

(b) Prove that

(i) triangle ABE is similar to triangle EDF, [2]

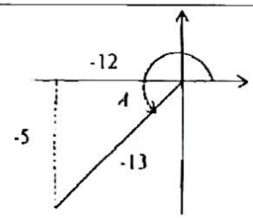
(ii) $AD^2 - BD^2 = 2 \times BE \times ED$. [3]

Point G is on minor arc EB such that BG bisects angle EBA.
Point H is on the circle such that $EGHD$ is a cyclic quadrilateral.

(c) Prove that angle $GHD = 90° -$ angle GBE. [3]

-End of Paper-

2024 NCHS Prelim AM Paper 2
Student Solutions

Qn	Solution
1(a)	$T_{r+1} = \binom{n}{r}(5x^2)^{n-r}\left(-\frac{1}{\sqrt{x}}\right)^r$ $= \binom{n}{r}(5)^{n-r}(-1)^r x^{2(n-r)}x^{-\frac{1}{2}r}$ $= \binom{n}{r}(5)^{n-r}(-1)^r x^{2n-\frac{5}{2}r}$ Since there is a term independent of x, $x^{2n-\frac{5}{2}r} = x^0$ $2n - \frac{5}{2}r = 0$ $n = \frac{5}{4}r$ Since both n and r are positive integers, Smallest $n = 5$, when $r = 4$.
1(b)	For the term $\left(5x^2 - \frac{1}{\sqrt{x}}\right)^n$, $T_{r+1} = \binom{n}{r}(5)^{n-r}(-1)^r x^{2n-\frac{5}{2}r}$ For constant, $r = 4$, Constant $= \binom{5}{4}(5)^{5-4}(-1)^4$ $= 25$ For x^5, $x^{2n-\frac{5}{2}r} = x^5$ $2n - \frac{5}{2}r = 5$ $r = 2$ Coefficient of $x^5 = \binom{5}{2}(5)^{5-2}(-1)^2$ $= 1250$ Term independent of x $= (1)(125) + \left(-\frac{1}{50}\right)(1250)$ $= 0$ Hence, there are no term independent of x.
2	<u>Method 1</u> $f'(x) = e^{5x} + 2\sin^2 x$ $= e^{5x} + 2(\sin x)^2$ $= e^{5x} + 1 - \cos 2x$ $f''(x) = \frac{d}{dx}(f'(x))$ $= \frac{d}{dx}(e^{5x} + 2(\sin x)^2)$ $= 5e^{5x} + 4\sin x\cos x$ $= 5e^{5x} + 2\sin 2x$ $f(x) = \int f'(x)\,dx$ $= \int e^{5x} + 1 - \cos 2x\,dx$ $= \frac{e^{5x}}{5} + x - \frac{\sin 2x}{2} + c$ Given $10f(x) + 3f'(x) - f''(x) + 7\sin 2x + 3\cos 2x = 10x + 43$ $10\left(\frac{e^{5x}}{5} + x - \frac{\sin 2x}{2} + c\right) + 3(e^{5x} + 2\sin^2 x) - (5e^{5x} + 2\sin 2x) + 7\sin 2x + 3\cos 2x = 10x + 43$ $2e^{5x} + 10x - 5\sin 2x + 10c + 3e^{5x} + 6\sin^2 x - 5e^{5x} - 2\sin 2x + 7\sin 2x + 3\cos 2x = 10x + 43$ $10c + 6\sin^2 x + 3\cos 2x = 43$ $10c + 3(1 - \cos 2x) + 3\cos 2x = 43$ $c = 4$ $\therefore f(x) = \frac{e^{5x}}{5} + x - \frac{\sin 2x}{2} + 4$ <u>Method 2</u> $f'(x) = e^{5x} + 2\sin^2 x$ $f''(x) = \frac{d}{dx}(f'(x))$ $= \frac{d}{dx}(e^{5x} + 2(\sin x)^2)$ $= 5e^{5x} + 4\sin x\cos x$ $= 5e^{5x} + 2\sin 2x$ Given $10f(x) + 3f'(x) - f''(x) + 7\sin 2x + 3\cos 2x = 10x + 43$ $10f(x) + 3(e^{5x} + 2\sin^2 x) - (5e^{5x} + 2\sin 2x) + 7\sin 2x + 3\cos 2x = 10x + 43$ $10f(x) + 3e^{5x} + 6\sin^2 x - 5e^{5x} - 2\sin 2x + 7\sin 2x + 3\cos 2x = 10x + 43$ $10f(x) - 2e^{5x} + 5\sin 2x + 3(1 - \cos 2x) + \cos 2x = 10x + 43$ $10f(x) - 2e^{5x} + 5\sin 2x + 3 - 3\cos 2x + \cos 2x = 10x + 43$ $10f(x) = 2e^{5x} - 5\sin 2x + 10x + 40$ $f(x) = \frac{e^{5x}}{5} + x - \frac{\sin 2x}{2} + 4$
3(a)	 $\sin A = -\frac{5}{13}$ $\cos A = -\frac{12}{13}$ <u>Method 1</u> $\cos 3A = \cos(2A + A)$ $= \cos 2A\cos A - \sin 2A\sin A$ $= (1 - 2\sin^2 A)\cos A - (2\sin A\cos A)\sin A$ $= \left(1 - 2\left(-\frac{5}{13}\right)^2\right)\left(-\frac{12}{13}\right) - \left(2\left(-\frac{5}{13}\right)\left(-\frac{12}{13}\right)\right)\left(-\frac{5}{13}\right)$ $= -\frac{828}{2197}$ <u>Method 2</u> $\cos 3A = \cos(2A + A)$ $= \cos 2A\cos A - \sin 2A\sin A$ $= (2\cos^2 A - 1)\cos A - (2\sin A\cos A)\sin A$ $= \left(2\left(-\frac{12}{13}\right)^2 - 1\right)\left(-\frac{12}{13}\right) - \left(2\left(-\frac{5}{13}\right)\left(-\frac{12}{13}\right)\right)\left(-\frac{5}{13}\right)$ $= -\frac{828}{2197}$
3(b)	$\tan A = \frac{2\tan\frac{A}{2}}{1 - \tan^2\frac{A}{2}}$ $\frac{5}{12} = \frac{2\tan\frac{A}{2}}{1 - \tan^2\frac{A}{2}}$ $5 - 5\tan^2\frac{A}{2} = 24\tan\frac{A}{2}$ $0 = 5\tan^2\frac{A}{2} + 24\tan\frac{A}{2} - 5$ $\tan\frac{A}{2} = \frac{-(24) \pm \sqrt{(24)^2 - 4(5)(-5)}}{2(5)}$ $= -5$ or $\frac{1}{5}$ Since $\pi < A < \frac{3\pi}{2}$, $\frac{\pi}{2} < \frac{A}{2} < \frac{3\pi}{4}$ (2nd Quadrant), $\therefore \tan\frac{A}{2} = -5$

4(a)

$$\text{Gradient} = \frac{0.88 - 2.69}{20 - 5} = -\frac{181}{1500}$$

$$Y - 2.69 = -\frac{181}{1500}(X - 5)$$

$$Y = -\frac{181}{1500}X + \frac{247}{75}$$

$$\ln y = -\frac{181}{1500}t + \frac{247}{75}$$

$$y = e^{-\frac{181}{1500}t + \frac{247}{75}}$$

$$\therefore k = -\frac{181}{1500}, c = \frac{247}{75}$$

4(b)

<u>Initial Temperature</u>

When $t = 0$,

$y = e^{\frac{247}{75}}$

$$\text{Initial Temperature} = 25 + e^{\frac{247}{75}} = 51.9325°C$$

<u>Temperature to drop to half</u>

$$y = \frac{51.9325}{2} - 25 = 0.966245$$

When $y = 0.966245$,

$$\ln(0.966245) = -\frac{181}{1500}t + \frac{247}{75}$$

$\therefore t = 27.577$

≈ 27.6 min (to3sf)

4(c)

$$e^{(k+0.2)t} = e^{5-c}$$

$$e^{kt}e^{0.2t} = \frac{e^5}{e^c}$$

$$e^{kt}e^{c} = \frac{e^5}{e^{0.2t}}$$

$$e^{kt+c} = e^{5-0.2t}$$

<u>Tumbler Model</u>

$y = e^{5-0.2t}$

$\ln y = 5 - 0.2t$

Step 1:

Draw a straight-line graph in (a), where the gradient of the straight line is -0.2 and the ln y-intercept is 5.

Step 2:

The intersection between the 2 straight line graphs will be the timing where the temperatures are equivalent.

5(a)

Consider ΔP,

$\text{hyp} = \sqrt{h^2 + h^2} = \sqrt{2}h$

Consider ΔZ,

$$\text{Side} = \frac{80 - \sqrt{2}h}{2}$$

$$= 2\left(\frac{80 - \sqrt{2}h}{2}\right)^2$$

$$= \frac{(80 - \sqrt{2}h)^2}{2}$$

$$\text{base} = \text{hyp} = \frac{80 - \sqrt{2}h}{\sqrt{2}}$$

$\therefore$ Total exterior area, A

$= 4 \times \text{base} \times \text{height} + \text{base} \times \text{base}$

$$= 4h\left(\frac{80 - \sqrt{2}h}{\sqrt{2}}\right) + \left(\frac{80 - \sqrt{2}h}{\sqrt{2}}\right)^2$$

$$= \frac{320}{\sqrt{2}}h - 4h^2 + \frac{6400 - 160\sqrt{2}h + 2h^2}{2}$$

$$= \frac{320\sqrt{2}}{2}h - 4h^2 + 3200 - 80\sqrt{2}h + h^2$$

$$= 3200 + 80\sqrt{2}h - 3h^2$$

<u>Method 2</u>

Consider ΔZ,

$$\text{Side} = \frac{80 - \sqrt{2}h}{2}$$

$\therefore$ Total exterior area, A

$= 80 \times 80 - 4 \times \text{small triangle} - 4 \times \text{big triangle}$

$$= 6400 - 4 \times \left(\frac{1}{2}\right)(\sqrt{2}h)^2 - 4 \times \left(\frac{1}{2}\right)\left(\frac{80 - \sqrt{2}h}{2}\right)^2$$

$$= 6400 - 4h^2 - \frac{6400 - 160\sqrt{2}h + 2h^2}{2}$$

$$= 6400 - 4h^2 - 3200 + 80\sqrt{2}h - h^2$$

$$= 3200 + 80\sqrt{2}h - 3h^2$$

5(b)

When the total exterior area is a maximum, $\frac{dA}{dh} = 0$.

$$A = 3200 + 80\sqrt{2}h - 3h^2$$

$$\frac{dA}{dh} = 80\sqrt{2} - 6h$$

$$0 = 80\sqrt{2} - 6h$$

$$h = \frac{80\sqrt{2}}{6} = \frac{40\sqrt{2}}{3} \text{ cm}$$

$$\frac{d^2A}{dh^2} = -6 \ (< 0)$$

$\therefore A$ is maximum.

5(c)

When $h = \frac{40\sqrt{2}}{3}$,

$$A = 3200 + 80\sqrt{2}\left(\frac{40\sqrt{2}}{3}\right) - 3\left(\frac{40\sqrt{2}}{3}\right)^2$$

$$= 4266\frac{2}{3} \text{ cm}^2$$

6

$y = 3 - 5\cos 2x$

<u>Point N</u>

When $y = 0$,

$3 - 5\cos 2x = 0$

$$\cos 2x = \frac{3}{5}$$

$$\alpha = \cos^{-1}\frac{3}{5} = 0.927295$$

$2x$ lies in 1^{st} (rej)/ 4^{th} quad.

$2x = 5.35589$

$x = 2.67795$

<u>Point P</u>

$y = 3 - 5\cos 2x$

$$\frac{dy}{dx} = -5(-\sin 2x)(2) = 10\sin 2x$$

When $\frac{dy}{dx} = -5$,

$10\sin 2x = -5$

$$\sin 2x = -\frac{1}{2}$$

$$\alpha = \sin^{-1}\frac{1}{2} = \frac{\pi}{6}$$

$2x$ lies in 3^{rd}/ 4^{th} (rej) quad.

$$2x = \frac{7\pi}{6}$$

$$x = \frac{7\pi}{12}$$

When $x = \frac{7\pi}{12}$,

$$y = 3 - 5\cos 2\left(\frac{7\pi}{12}\right) = 7.330127$$

$$\therefore P\left(\frac{7\pi}{12}, 7.330127\right)$$

<u>Equation PQ, Point Q</u>

$$y - 7.330127 = -5\left(x - \frac{7\pi}{12}\right)$$

$$y = -5x + 16.4931$$

When $y = 0$,

$-5x + 16.4931 = 0$

$x = 3.29862$

$\therefore Q(3.29862, 0)$

<u>Area below the curve from x_p to x_n</u>

$$\text{area} = \int_{\frac{7\pi}{12}}^{2.67795} 3 - 5\cos 2x \, dx$$

$$= \left[3x - \frac{5\sin 2x}{2}\right]_{\frac{7\pi}{12}}^{2.67795}$$

$= 10.0338 - 6.74779$

$= 3.28601$ units^2

$\therefore$ shaded area

$=$ area of triangle $-$ area below curve

$$= \frac{1}{2}(7.330127)\left(3.29862 - \frac{7\pi}{12}\right) - 3.28601$$

$= 5.37307 - 3.28601$

$= 2.0871$

$= 2.09$ units^2

7(a) To prove $(x + 2y)$ is a factor:

Method 1

Let $f(x) = 4x^3 + x^2y - 11xy^2 + 6y^3$.

$f(-2y) = 4(-2y)^3 + (-2y)^2y - 11(-2y)y^2 + 6y^3$
$= -32y^3 + 4y^3 + 22y^3 + 6y^3$
$= 0$

Since $f(-2y)$, by factor theorem, $(x + 2y)$ is a factor.

Method 2

$$\begin{array}{r|l} & 4x^2 - 7yx + 3y^2 \\ \hline x + 2y & 4x^3 + x^2y - 11xy^2 + 6y^3 \\ & [illegible] \\ \hline & [illegible] \\ & [illegible] \\ \hline & 3y^2x + 6y^3 \\ & -(3y^2x + 6y^3) \\ \hline & 0 \end{array}$$

Since the remainder is 0, $(x + 2y)$ is a factor.

Hence factorise:

Method 1 – Compare coefficients

$f(x) = (x + 2y)(4x^2 + bx$ [illegible]$)$

Comparing coefficients of,

x [illegible] [illegible] $-7y$

Method [illegible] Division

As shown earlier

$\therefore f(x) = (x + 2y)(4x^2 - 7yx + 3y^2)$
$= (x + 2y)(x - y)(4x - 3y)$

7(b) $4^{p+1} + 2^{p+1} = 44 - 48(2^{-p})$

$2^{2(p+1)} + 2^{p+1} = 44 - \frac{48}{2^p}$

$2^{2p}\,2^2 + 2^p\,2 = 44 - \frac{48}{2^p}$

Let $u = 2^p$,

$4u^2 + 2u = 44 - \frac{48}{u}$

$(\times u)$

$4u^3 + 2u^2 - 44u + 48 = 0$

From (a),

$48 = 6y^3$

$y^3 = 8$

$y = 2$

From (a), $x = u$

From (a),

$f(x) = (x + 2y)(x - y)(4x - 3y)$

$f(x) = (x + 4)(x - 2)(4x - 6) = 0$

$x = -4$, $2^p = -4$ (rej)

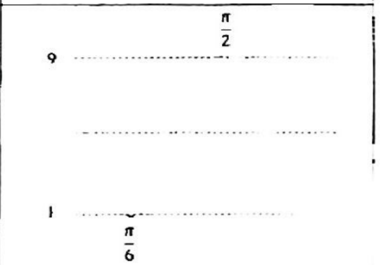
$x = 2$, $2^p = 2$, $\therefore p = 1$

$x = \frac{3}{2}$, $2^p = \frac{3}{2}$

$\therefore p = \frac{\lg\frac{3}{2}}{\lg 2}$

$= 0.585$ (to3sf

8(a)



$y = a\sin bx + c$		
$c = \frac{9+1}{2} = 5$	$\frac{1}{2}$ period $= \frac{\pi}{2} - \frac{\pi}{6}$ period $= \frac{2\pi}{3}$ $\frac{2\pi}{b} = \frac{2\pi}{3}$ $b = 3$	Amplitude $= 9 - 5 = 4$ $a = -4$

8(b)(i) $a = \frac{dv}{dt}$
$= 6t - 5$

When $t = 4$,
$a = 6(4) - 5$
$= 19\ m/s^2$

8(b)(ii) For $0 \le t \le 5$

$v = 3t^2 - 5t + 7$

Displacement expression:

$s = \int 3t^2 - 5t + 7\ dt$
$= \frac{3t^3}{3} - \frac{5t^2}{2} + 7t + c$

When $t = 0, s = 0$,
$c = 0$

$\therefore s = \frac{3t^3}{3} - \frac{5t^2}{2} + 7t$

Turning point:

When $v = 0$,

$3t^2 - 5t + 7 = 0$

$b^2 - 4ac = (-5)^2 - 4(3)(7)$
$= -59\ (< 0)$

Since $b^2 - 4ac < 0$, there is no turning point.

When $t = 0$, $s = 0$ m

$t = 5$, $s = 97.5$ m

For [illegible] 5,

$v = -4t + 77$

Displacement expression:

$s = \int -4t + 77\ dt$
$= -\frac{4t^2}{2} + 77t + d$

When $t = 5, s = 97.5$,
$97.5 = -2(5)^2 + 77(5) + d$
$d = -\frac{475}{2}$

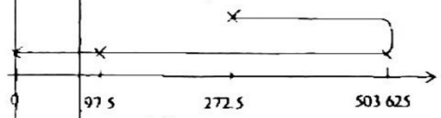
$\therefore s = -2t^2 + 77t - \frac{475}{2}$

Turning point:

When $v = 0$,
$-4t + 77 = 0$
$t = \frac{77}{4} = 19\frac{1}{4}$

When $t = 19\frac{1}{4}$, $s = 503.625$ m

$t = 30$, $s = 272.5$ m



$\therefore$ total distance
$= 503.625 + (503.625 - 272.5)$
$= 734.75$ m

9(a) Consider ΔBCD, using Pythagoras theorem,

$BD = \sqrt{8^2 + 15^2} = 17$

Consider ΔBXC,
$CX = 8\cos\theta$
$BX = 8\sin\theta$

Consider ΔCYD,
$CY = 15\sin\theta$
$YD = 15\cos\theta$

Total lengths
$= AB + 8 + 15 + AD + BD$
$= (CY - CX) + 8 + 15 + (BX + YD) + BD$
$= 15\sin\theta - 8\cos\theta + 8 + 15 + 8\sin\theta + 15\cos\theta + 17$
$= 40 + 23\sin\theta + 7\cos\theta$ (shown)

9(b) $P = 40 + 23\sin\theta + 7\cos\theta$
$= 40 + R\sin(\theta + \alpha)$

$R = \sqrt{23^2 + 7^2}$
$= \sqrt{578}$
$= 24.04163 = 24.0$ (to 3sf)

	$\alpha = \tan^{-1}\frac{7}{23} = 16.9275°$ $= 16.9°$ (to 1 dp) $\therefore P = 40 + 24.0\sin(\theta + 16.9°)$
9(c)	When $P = 60$, $40 + 24.04163\sin(\theta + 16.9275°)$ $= 60$ $\sin(\theta + 16.9275°)$ $= 0.83189$ $\alpha = \sin^{-1} 0.83189$ $= 56.2934°$ $(\theta + 16.9275°)$ lies in 1^{st} / 2^{nd} (rej) quad. $\theta + 16.9275° = 56.2934°$ $\theta = 39.3659°$ $= 39.4°$ (to 1 dp)
9(d)	$\max(R\sin(\theta + \alpha))^2 = \left(\sqrt{578}\right)^2 = 578$ $\therefore \min \frac{1}{40 + (R\sin(\theta + \alpha))^2}$ $= \frac{1}{40 + 578} = \frac{1}{618}$ $\theta + 16.9275° = 90°$ $\theta = 73.0725°$ $= 73.1°$ (to 1 dp)
10(a)	Since E is the midpoint of AD (given), F is the midpoint of DB (given), By midpoint theorem, $FE = \frac{1}{2}AB$
10(b) (i)	$\angle ABE$ $= \angle EDF$ (alternate segment theorem) From (a), by midpoint theorem, $EF \parallel AB$ $\angle DEF$ $= \angle BAE$ (corresponding angles, $EF \parallel AB$)

	$\therefore \Delta ABE$ is similar to ΔEDF (AA similari
10(b) (ii)	From (i), $\frac{AB}{ED}$ $= \frac{BE}{DF}$ (corresponding sides of similar Δ $AB \times DF = BE \times ED$ $AB \times \frac{1}{2}AB = BE \times ED$ $AB^2 = 2 \times BE \times ED$ Since $\angle DBA = 90°$ (tangent $\perp$ radius), Using Pythagoras' theorem, $AD^2 - BD^2 = 2 \times BE \times ED$
10(c)	Method 1 Let $\angle GBE$ be α, $\angle GBA = \alpha$ (BG bisects $\angle EBA$) $\angle GEB$ $= \alpha$ (alternate segment theorem) $\angle DEB = 90°$ (rt $\angle$ in a semicircle) $\angle GHD$ $= 180°$ $- (90° + \alpha)$ ($\angle$ in opp segment) $= 90° - \alpha$ $= 90° - \angle GBE$ Method 2 Let $\angle GBE$ be α, $\angle GBA = \alpha$ (BG bisects $\angle EBA$) $\angle DBA = 90°$ (tangent $\perp$ radius) $\angle GBD = 90° - \alpha$ $\angle GHD$ $= \angle GBD$ ($\angle$ in the same segment) $= 90° - \alpha$ $= 90° - \angle GBE$