BP-507

Calculator Model: ___________

NAME:	CLASS:	INDEX NO:

QUEENSWAY SECONDARY SCHOOL
PRELIMINARY EXAMINATION 2024
SECONDARY 4 EXP / 5NA

Parent's Signature:

ADDITIONAL MATHEMATICS

Paper 1

4049/01
21 August 2024
2 hour 15 minutes

Candidates answer on the Question Paper.

READ THESE INSTRUCTIONS FIRST

Write your name, class and index number in the spaces at the top of this page.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction tape.

Answer **all** the questions.
Give non-exact numerical values correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.

The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

The number of marks is given in bracket [ ] at the end of each question or part question.

The total number of marks for this paper is 90.

This document consists of **15** printed pages and **1** blank page.

[Turn over

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1} a^{n-1} b + \binom{n}{2} a^{n-2} b^2 + \cdots + \binom{n}{r} a^{n-r} b^r + \cdots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{(n-r)!\,r!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = cos^2 A - sin^2 A = 2cos^2 A - 1 = 1 - 2sin^2 A$$

$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

Formulae for $\Delta\, ABC$

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2} bc \sin A$$

1 Given that $f(x) = 3x^5 - 11x^3 + 30x^2 + 39 = (x-1)(x+3)Q(x) + ax + b$ for all values of x and that $Q(x)$ is a polynomial, find

(a) the values of a and of b. [5]

(b) the remainder when $f(x) - 3$ is divided by $x^2 + 2x - 3$. [2]

2 **(a)** Water is poured into a jar at a rate of $35\ \text{cm}^3/\text{s}$. The volume of water in the jar is $V\ \text{cm}^3$, where $V = 3\left(\frac{h^2}{4} + \frac{8\pi}{h^3}\right)$ and h is the height of water in the jar. Find the rate at which the height of the water is increasing when $h = 4\ cm$. [3]

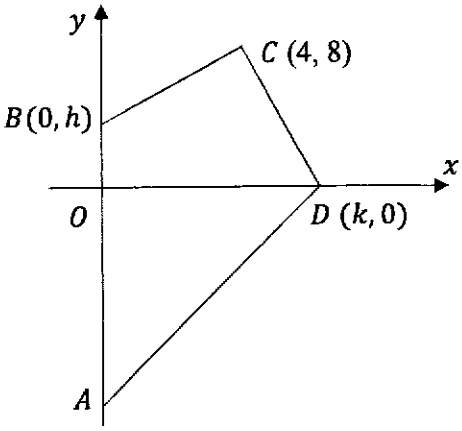
(b) **(i)** Find the set of values of x for which $y = \frac{2-5x}{e^x}$ is a decreasing function of x. [3]

(ii) Find the gradient of the curve $y = \frac{2-5x}{e^x}$ at the point where it cuts the x- axis. [2]

3 **(a)** Express $\frac{1-3x-3x^2}{x(x+1)^2}$ in partial fractions. [5]

3 **(b)** Hence, find $\int \frac{1-3x-3x^2}{2x(x+1)^2}\,dx$. [4]

4



The diagram shows a kite $ABCD$ in which $AB = AD$ and $BC = CD$, Point A and B lie on the y-axis and D lies on the x-axis. The coordinates of C is $(4, 8)$, B is $(0, h)$ and D is $(k, 0)$, where h and k are positive constants.

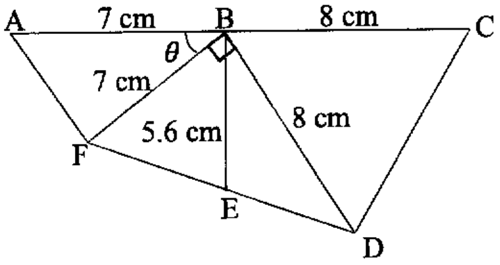
(a) Show that $h^2 - k^2 = 16h - 8k$. [2]

(b) It is now given that $h = 1$.

(i) Find the coordinates of A. [4]

(ii) Find the area of the kite. [2]

5



In the diagram, angle $ABF = \theta$, angle $FBD = 90°$. ABC is a straight line and BE is perpendicular to AC. $AB = BF = 7$ cm, $BC = BD = 8$ cm and $BE = 5.6$ cm.

(a) Show that the area Q cm^2 of the quadrilateral $ACDF$ is given by $Q = 51.6\cos\theta + 46.9\sin\theta$. [3]

(b) Express $Q = 51.6\cos\theta + 46.9\sin\theta$ in the form $R\cos(\theta - \alpha)$, where R is a positive constant and α is acute. [2]

(c) State the maximum value of Q and find the corresponding value of θ. [2]

(d) State the maximum value of $\frac{1}{Q^2+3}$. [1]

6 An insect leaves its nest and flies along in a straight path. The velocity, v m/s, that it flies in time, t s, after it leaves the nest is given by $v = 4e^{-t} - \frac{1}{2}e^{2t}$.

(a) Find the acceleration when $t = 0.5$. [2]

(b) Find the value of t when the acceleration is maximum. [3]

(c) Show that the insect is instantaneously at rest when $t = \ln k$ where k is an integer. [3]

(d) Find the total distance travelled by the insect between $t = 0$ and $t = 3$. [4]

7 **(a)** Prove the identity $\frac{\cot A - \tan A}{\cot A + \tan A} = 2cos^2A - 1$. [4]

(b) Hence, solve $\frac{\cot A - \tan A}{\cot A + \tan A} = \cos A$ for $-\pi < A < \pi$. [4]

8 A curve is given by $y = \tan x$.

(a) Find $\frac{dy}{dx}$. [1]

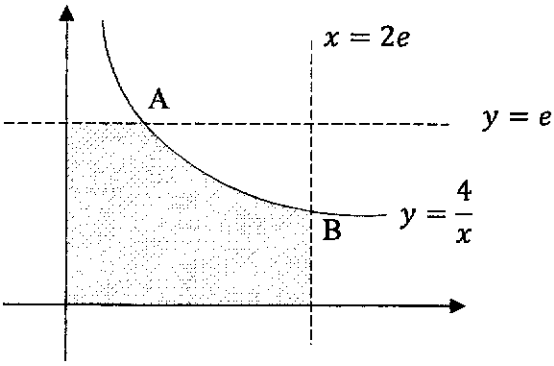
(b) Hence, find the equation of the tangent to the curve $y = \tan x$ at the point where $x = \frac{\pi}{6}$, leaving your answer in terms of π. [3]

(c) Find the value of $\frac{d^2y}{dx^2}$ when $x = \frac{\pi}{6}$. [2]

(d) A student concludes from part **(c)** that since the value of $\frac{d^2y}{dx^2} > 0$ at $x = \frac{\pi}{6}$, hence there is a minimum point at $x = \frac{\pi}{6}$. State with reason and mathematical working whether the above conclusion is correct. [2]

9 Evaluate $\int_0^{\frac{4}{3}}(3x^2 - 16x + 16)\,dx + \int_{\frac{4}{3}}^{4}(3x^2 - 16x + 16)\,dx$. Hence, state what could be deduced about the graph $y = 3x^2 - 16x + 16$ between $x = 0$ and $x = 4$? [4]

10 The diagram shows part of the curve $y = \frac{4}{x}$. The curve $y = \frac{4}{x}$ cuts the line $y = e$ at A and the line $x = 2e$ at B.



(a) Find the coordinates of A and of B. [2]

(b) Find the area bounded by the curve $y = \frac{4}{x}, y = e, x = 2e$, the y-axis and the x-axis, leaving your answer in the form $a + \ln b$, where a and b are integers. [4]

(c) Explain why the area of the shaded region is between 4 units^2 and $2e^2$ units^2. [2]

11 The table below shows experimental values of two variables x and y.

x	1	2	3	4	5	6
y	14.1	26.0	40.0	55.9	73.5	92.6

It is known that x and y are related by an equation of the form $y = A(1 + x)^n$, where A and n are constants.

(a) Plot $\lg y$ against $\lg(1 + x)$and draw a straight line graph on the grid given in the next page. [3]

(b) Use your graph to estimate the values of A and of n. [3]

(c) On the same diagram, draw a straight line representing the equation $y = \frac{16}{1+x}$. [2]

(d) Hence, find the value of x for which $A(1 + x)^{n+1} = 16$. [2]

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2024 4E5N Prelim P1 Solutions

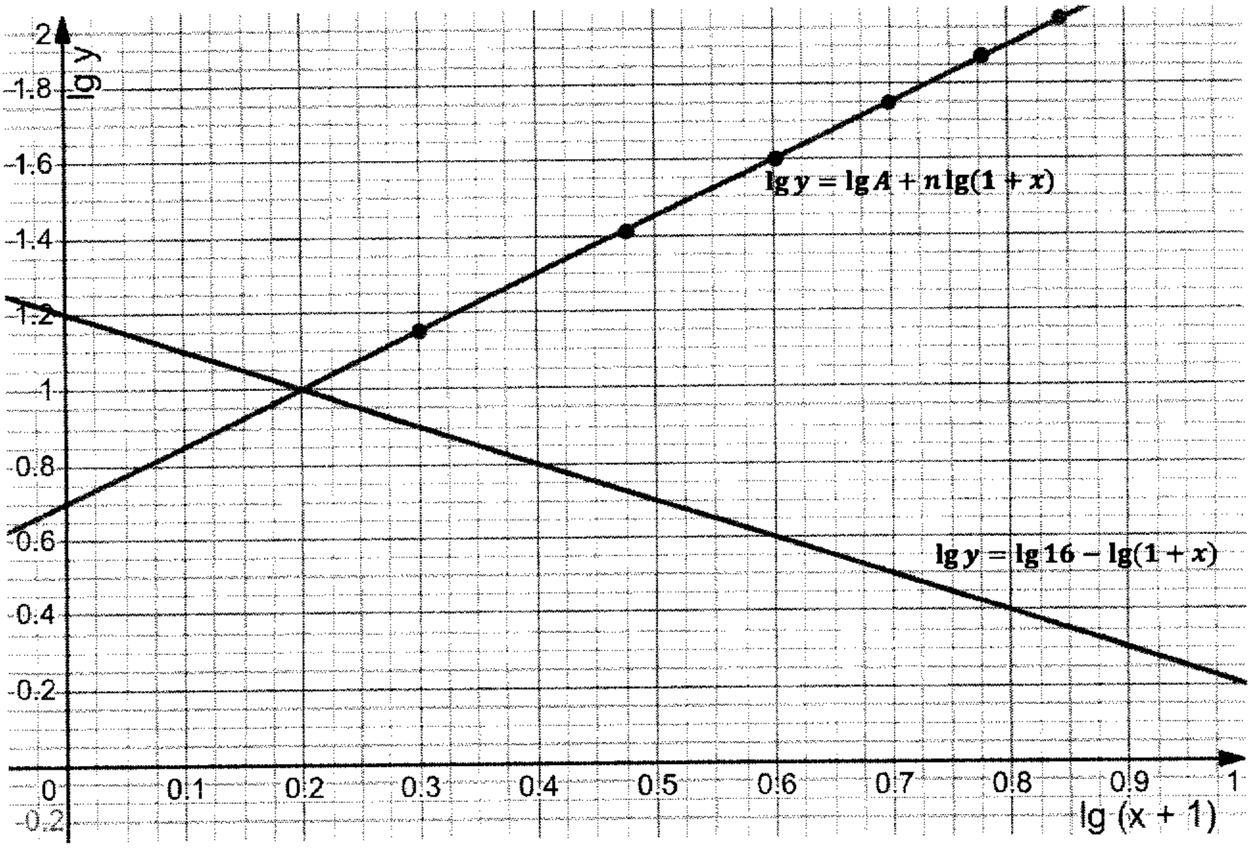
No.	**Solution**
1(a)	$f(x) = 3x^5 - 11x^3 + 30x^2 + 39 = (x-1)(x+3)Q(x) + ax + b$ When $x = 1$, $61 = a + b$ ----- eqn (1) When $x = -3$, $-123 = -3a + b$ ----- eqn (2) (1) – (2) $184 = 4a$ $a = 46$ $b = 15$
1(b)	$f(x) = (x-1)(x+3)Q(x) + 46x + 15$ $f(x) - 3 = (x-1)(x+3)Q(x) + 46x + 15 - 3$ Remainder $= 46x + 15 - 3$ $= 46x + 12$
2(a)	$V = 3\left(\frac{h^2}{4} + \frac{8\pi}{h^3}\right)$ $\frac{dV}{dh} = \frac{3}{2}h - \frac{72\pi}{h^4}$ When $h = 4$, $\frac{dV}{dh} = 6 - \frac{9\pi}{32}$ $= \frac{192-9\pi}{32}$ $(or\ \ 5.1164)$ $\frac{dh}{dt} = \frac{dh}{dV} \times \frac{dV}{dt}$ $\frac{dh}{dt} = \frac{32}{192-9\pi} \times 35$ $= \frac{1120}{192-9\pi}\ cm/s$ $(or\ \ 6.84\ cm/s)$
2(bi)	$y = \frac{2-5x}{e^x}$ $\frac{dy}{dx} = \frac{e^x(-5)-(2-5x)e^x}{(e^x)^2}$ $= \frac{5x-7}{e^x}$ For decreasing function, $\frac{5x-7}{e^x} < 0$ $x < \frac{7}{5}$ $(or\ \ 1.4)$
2(bii)	When $y = 0$, $2 - 5x = 0$ $x = 0.4$ $\frac{dy}{dx} = -3.35\ (3s.f)$
3(a)	$\frac{1-3x-3x^2}{x(x+1)^2} = \frac{A}{x} + \frac{B}{x+1} + \frac{C}{(x+1)^2}$ $1 - 3x - 3x^2 = A(x+1)^2 + Bx(x+1) + Cx$ When $x = -1$, $C = -1$ When $x = 0$, $A = 1$ When $x = 1$, $B = -4$ $\frac{1-3x-3x^2}{x(x+1)^2} = \frac{1}{x} - \frac{4}{x+1} - \frac{1}{(x+1)^2}$

3(b)	$\int \frac{1-3x-3x^2}{2x(x+1)^2}dx = \frac{1}{2}\int \frac{1-3x-3x^2}{x(x+1)^2}dx$ $= \frac{1}{2}\int \frac{1}{x} - \frac{4}{x+1} - \frac{1}{(x+1)^2}\ dx$ $= \frac{1}{2}\ln x - 2\ln(x+1) + \frac{1}{2(x+1)} + c$ [Accept $\ln\sqrt{x} - \ln(x+1)^2 + \frac{1}{2(x+1)} + c$]
4(a)	$4^2 + (h-8)^2 = (k-4)^2 + 8^2$ $16 + h^2 - 16h + 64 = k^2 - 8k + 16 + 64$ $h^2 - 16h = k^2 - 8k$ $h^2 - k^2 = 16h - 8k$ (shown)
4(bi)	When $h = 1$, $1 - k^2 = 16 - 8k$ $k^2 - 8k + 15 = 0$ $(k-5)(k-3) = 0$ $k = 5 \quad or\ k = 3\ (rejected\ based\ on\ diagram)$ Let $A(0, y)$ $(1-y)^2 = 5^2 + y^2$ $1 - 2y + y^2 = 25 + y^2$ $y = -12$ $\therefore A(0, -12)$
4(bii)	Area $= \frac{1}{2}\left\lvert \begin{matrix} 0 & 5 & 4 & 0 & 0 \\ -12 & 0 & 8 & 1 & -12 \end{matrix} \right\rvert$ $= \frac{1}{2}\lvert 44 - (-60) \rvert$ $= 52\ units^2$
5(a)	Area $= \frac{1}{2}(7)(7)\sin\theta + \frac{1}{2}(7)(5.6)\sin(90-\theta)$ $+ \frac{1}{2}(5.6)(8)\sin\theta + \frac{1}{2}(8)(8)\sin(90-\theta)$ $= \frac{49}{2}\sin\theta + \frac{98}{5}\cos\theta + \frac{112}{5}\sin\theta + 32\cos\theta$ $= 51.6\cos\theta + 46.9\sin\theta$ (shown)
5(b)	$Q = 51.6\cos\theta + 46.9\sin\theta = R\cos(\theta - \alpha)$ $R = \sqrt{51.6^2 + 46.9^2}$ $= 69.729$ $\tan\alpha = \frac{46.9}{51.6}$ $\alpha = 42.268$ $\therefore 51.6\cos\theta + 46.9\sin\theta = 69.7\cos(\theta - 42.3°)$
5(c)	Max value of Q = 69.7 $\cos(\theta - 42.268°) = 1$ $\theta = 42.268$ Corresponding value= 42.3°
5(d)	maximum value of $\frac{1}{Q^2+3} = \frac{1}{0+3}$ $= \frac{1}{3}$
	:

6(a)	$v = 4e^{-t} - \frac{1}{2}e^{2t}$ $a = -4e^{-t} - e^{2t}$ When $t = 0.5, a = -5.14\, m/s^2$	
6(b)	$\frac{da}{dt} = 4e^{-t} - 2e^{2t}$ When $\frac{da}{dt} = 0, \qquad 4e^{-t} = 2e^{2t}$ $e^{3t} = 2$ $t = \frac{1}{3}\ln 2$ $\frac{d^2a}{dt^2} = -4e^{-t} - 4e^{2t}$ When $t = \frac{1}{3}\ln 2, \quad \frac{d^2a}{dt^2} < 0$ (max)	
6(c)	When $v = 0$, $4e^{-t} = \frac{1}{2}e^{2t}$ $e^{3t} = 8$ $t = \frac{1}{3}\ln 8$ $= \ln 8^{\frac{1}{3}}$ $= \ln 2$ (shown)	
6(d)	$s = -4e^{-t} - \frac{1}{4}e^{2t} + c$ When $t = 0, s = 0, \therefore c = \frac{17}{4}$ $s = -4e^{-t} - \frac{1}{4}e^{2t} + \frac{17}{4}$ When $t = 0, s = 0$ When $t = \ln 2, s = 1.25$ When $t = 3, s = -96.806$ Total distance travelled $= 1.25 + (96.806 + 1.25)$ $= 99.3m$	
7(a)	$\frac{\cot A - \tan A}{\cot A + \tan A} = 2cos^2 A - 1$ $LHS = \frac{\cot A - \tan A}{\cot A + \tan A}$ $= \frac{\frac{\cos A}{\sin A} - \frac{\sin A}{\cos A}}{\frac{\cos A}{\sin A} + \frac{\sin A}{\cos A}}$ $= \frac{cos^2 A - sin^2 A}{cos^2 A + sin^2 A}$ $= cos^2 A - sin^2 A$ $= cos^2 A + cos^2 A - 1$ $= 2cos^2 A - 1$ $= RHS$	Alternative $LHS = \frac{\cot A - \tan A}{\cot A + \tan A}$ $= \frac{\frac{1}{\tan A} - \tan A}{\frac{1}{\tan A} + \tan A}$ $= \frac{1 - tan^2 A}{1 + tan^2 A}$ $= \frac{1 - tan^2 A}{sec^2 A}$ $= cos^2 A - sin^2 A$ $= 2cos^2 A - 1$ $= RHS$

7(b)	$\frac{\cot A-\tan A}{\cot A+\tan A}=\cos A \qquad -\pi<A<\pi$ $2cos^2A-1=\cos A$ $(2\cos A+1)(\cos A-1)=0$ $\cos A=-\frac{1}{2} \qquad or \quad \cos A=1$ $ref\ angle=\frac{\pi}{3} \quad or\ ref\ angle=0$ $A=\frac{2\pi}{3},\frac{-2\pi}{3} \qquad or \qquad A=0$ $A=\frac{-2\pi}{3},0,\frac{2\pi}{3}$
8(a)	$y=\tan x$ $\frac{dy}{dx}=sec^2x$
8(b)	When $x=\frac{\pi}{6},\quad y=\frac{\sqrt{3}}{3},\quad \frac{dy}{dx}=\frac{4}{3}$ $y=mx+c$ $\frac{\sqrt{3}}{3}=\frac{4}{3}\left(\frac{\pi}{6}\right)+c$ $c=\frac{\sqrt{3}}{3}-\frac{2\pi}{9}$ $y=\frac{4}{3}x+\frac{\sqrt{3}}{3}-\frac{2\pi}{9}$ Accept $\left(y=\frac{4}{3}x+\frac{3\sqrt{3}-2\pi}{9}\right)$ or $\left(9y=12x+3\sqrt{3}-2\pi\right)$
8(c)	$\frac{d^2y}{dx^2}=-2cos^{-3}x(-\sin x)$ $=\frac{2\sin x}{cos^3x}$ At $x=\frac{\pi}{6},\frac{d^2y}{dx^2}=1.5396$ $=1.54\ (3s.f)$
8(d)	$\frac{dy}{dx}=sec^2x$ When $\frac{dy}{dx}=0,\quad \frac{1}{cos^2x}=0$ Since $sec^2x=0$ is not defined, $\therefore$ the above conclusion is wrong.
9	$\int_0^{\frac{4}{3}}(3x^2-16x+16)\,dx+\int_{\frac{4}{3}}^{4}(3x^2-16x+16)\,dx$ $=[x^3-8x^2+16x]_0^{\frac{4}{3}}+[x^3-8x^2+16x]_{\frac{4}{3}}^{4}$ $=\frac{256}{27}+\left(0-\frac{256}{27}\right)$ $=0$ The **area above the x-axis**, bounded from $x=0$ to $x=\frac{4}{3}$ is the **same** as the **area below the x-axis,** bounded from $x=\frac{4}{3}$ to $x=4$.

<table>
<tr><td>10(a)</td><td>When $y = e,\ e = \frac{4}{x}$
$x = \frac{4}{e}$
$\therefore A\left(\frac{4}{e}, e\right)$
When x= $2e,\ y = \frac{4}{2e}$
$y = \frac{2}{e}$
$\therefore B\left(2e, \frac{2}{e}\right)$</td></tr>
<tr><td>10(b)</td><td>$Area = \int_{\frac{4}{e}}^{2e} \frac{4}{x}\, dx\ + \left(\frac{4}{e}\right)(e)$
$= 4[\ln x]_{\frac{4}{e}}^{2e} + 4$
$= 4\left[\ln 2e - \ln\frac{4}{e}\right] + 4$
$= 4[\ln 2 + \ln e - \ln 4 + \ln e] + 4$
$= 4[\ln 2 - \ln 4] + 12$
$= 4[-\ln 2] + 12$
$= 12 - \ln 16$</td></tr>
<tr><td></td><td></td></tr>
<tr><td>10(c)</td><td>Area of rect from from y-axis to A = $e \times \frac{4}{e}$
$= 4\ units^2$
Area of whole rect= $2e \times e$
$= 2e^2\ units^2$
$\therefore 4 < area\ of\ shaded\ region < 2e^2$ (explained)</td></tr>
<tr><td>11(a)</td><td>Refer to graph.
$y = A(1+x)^n$
$\lg y = \lg A + n \lg(1+x)$
<table>
<tr><td>lg(1+x)</td><td>0.301</td><td>0.477</td><td>0.602</td><td>0.699</td><td>0.778</td><td>0.845</td></tr>
<tr><td>lg y</td><td>1.15</td><td>1.41</td><td>1.60</td><td>1.75</td><td>1.87</td><td>1.97</td></tr>
</table>
Correct points plotted
Straight line
Plot table</td></tr>
<tr><td>11(b)</td><td>From the graph,
$\lg A = 0.7$ [Accept $0.68 \leq \lg A \leq 0.72$]
$A = 5.01$
$n = grad = 1.51$ [Accept $1.49 \leq n \leq 1.51$]</td></tr>
<tr><td>11(c)</td><td>$y = \frac{16}{1+x}$
$\lg y = \lg 16 - \lg(1+x)$
Draw the line $\lg y = \lg 16 - \lg(1+x)$</td></tr>
<tr><td>11(d)</td><td>$A(1+x)^{n+1} = 16$
$A(1+x)^n = \frac{16}{1+x}$
$\therefore \lg(x+1) = 0.2$
$x = 0.585$</td></tr>
<tr><td></td><td></td></tr>
</table>


lg y
2
1.8
1.6
1.4
1.2
1
0.8
0.6
0.4
0.2
0
-0.2
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
lg (x + 1)
lg y = lg A + n lg(1 + x)
lg y = lg 16 − lg(1 + x)

END OF PAPER

BP-523

Calculator Model: ______________

NAME:	CLASS:	INDEX NO:

QUEENSWAY SECONDARY SCHOOL
PRELIMINARY EXAMINATION 2024
SECONDARY 4 EXPRESS /
SECONDARY 5 NORMAL ACADEMIC

Parent's Signature:

ADDITIONAL MATHEMATICS

Paper 2

4049/02
27 August 2024

2 hour 15 minutes

Candidates answer on the Question Paper.

READ THESE INSTRUCTIONS FIRST

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Answer **all** questions.

The number of marks is given in brackets [ ] at the end of each question or part question.

If working is needed for any question, it must be shown with the answer.
Omission of essential working will result in loss of marks.
The total of the marks for this paper is 90.
The use of an approved scientific calculator is expected, where appropriate.
If the degree of accuracy is not specified in the question and if the answer is not exact, give the answer to three significant figures. Gives answers in degrees to one decimal place.
For π, use either your calculator value or 3.142.

This document consists of **18** printed pages.

[Turn over

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1} a^{n-1}b + \binom{n}{2} a^{n-2}b^2 + \cdots + \binom{n}{r} a^{n-r}b^r + \cdots + b^n,$$

where n is a positive integer and $\begin{pmatrix} n \\ r \end{pmatrix} = \frac{n!}{(n-r)!r!} = \frac{n(n-1)...(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\text{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = cos^2A - sin^2A = 2cos^2A - 1 = 1 - 2sin^2A$$

$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2} bc \sin A$$

1 The Singapore government issued a savings bond in January 2024 with a yield of 2.75% per year. Mr Tan invested \$15 000 in the bond. The total amount he will receive, after t years, is given by $A = 15000(1.0275)^t$.

(a) Calculate the total amount he will receive in January 2030, correct to the nearest dollar. [2]

(b) In which year will the amount first exceed \$22 000? [2]

2 (a) Without using a calculator, evaluate the value of 6^x given that $2^{2x+6} \times 3^{5x-1} = 27^{x+1}$. [5]

(b) Solve the equation $log_x 9 = 5 log_3 x$, giving your answers correct to 2 significant figures.

[4]

3 (a) A curve has the equation $y = (p - 1)x^2 + 2(p - 3)$, where p is a constant. A line has the equation $y = 6x + 3$. Find the range of values of p if the curve lies completely above the line.

[5]

(b) By expressing $y = -2x^2 + 10x - 5$ in the form $y = a(x + b)^2 + c$, where a, b and c are constants, find the maximum value of y.

[3]

4 (a) In the binomial expansion of $\left(1-\frac{2}{7}x\right)^{n}$, the sum of the coefficients of the second and third terms is zero. Calculate the value of n and hence, find the sixth term.

[4]

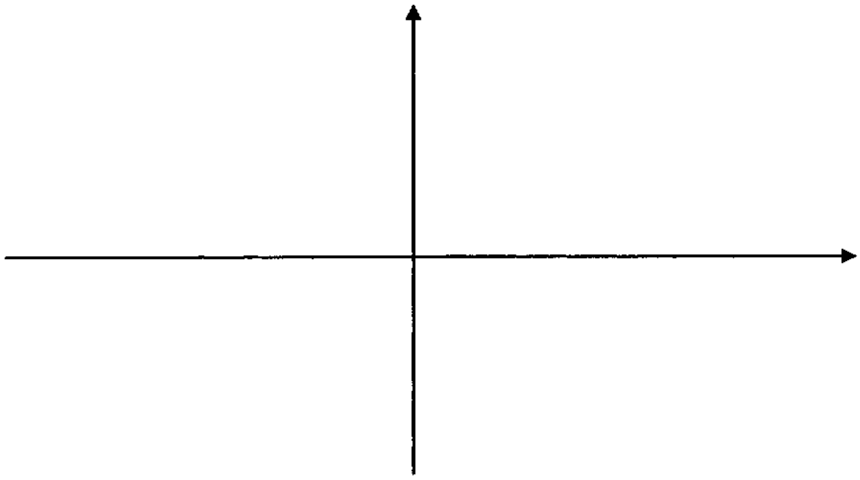
(b) Write down the general term in the binomial expansion of $\left(\frac{1}{x^3} - 2x\right)^8$.
Hence, find the value of the constant term in the expansion of
$\left(3 + \frac{x^2}{2}\right)^2 \left(\frac{1}{x^3} - 2x\right)^8$.

[6]

5 (a) The function f is defined as $f(x) = p - q\sin(rx)$, for $-\pi \leq x \leq \pi$, where p, q and r are positive integers. Given that the amplitude of the function is 6, the period is π and the maximum value is 9.

(i) State the values of p, q and r. [3]

(ii) Hence, sketch the graph of $f(x)$. [2]



(b) The acute angles A and B are such that $\cot(A - B) = \frac{1}{3}$ and $\cot A = \frac{1}{5}$.
Without using a calculator, find the exact value of $\cos B$. [5]

6 A circle passes through the points $(-5, 12)$ and $(9, 14)$. The centre of the circle lies on the line $2y + x = 15$.

(a) Find the equation of the circle. [7]

(b) Explain why the line $y = mx + 6$ intersects the circle at 2 distinct points for all values of m.

[4]

7 (a) A curve has the equation $y = \frac{3x-5}{4x+1}$ for $x > 0$. Explain, with working, why the curve has no stationary points.

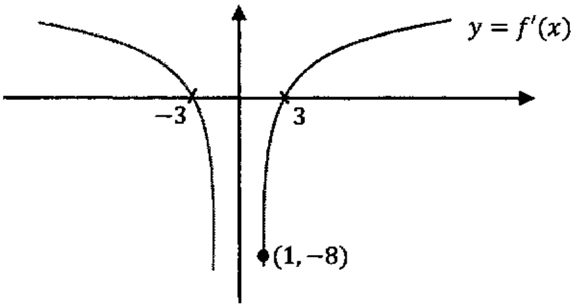
[3]

(b) Given that $f(x)$ is such that $f'(x) = \cos 4x - 3\sin 2x$ and $f(\pi) = 0$, show that $f''(x) + 4f(x) = -3(\sin 4x + 2)$.

[4]

(c) Given that $\frac{d}{dx}\left(\frac{2-x}{\sqrt{1-2x}}\right) = \frac{ax+b}{\sqrt{(1-2x)^3}}$, find the value of a and b. [4]

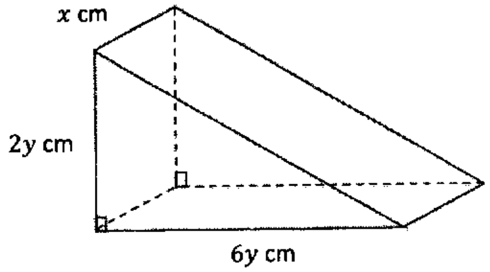
8 A curve $y = f(x)$ passes through the point $(1, 10)$.
The graph of $y = f'(x)$ is shown below.



(a) State the x-coordinates of the stationary points of the curve $y = f(x)$ and hence, determine their nature. [3]

(b) Find the equation of the normal to the curve at the point $(1, 10)$. [2]

9 The diagram below shows a wooden door stopper in the shape of a right prism with a volume of 60 cm^3. The cross-section of the prism is a triangle, with side lengths of $2y$ cm and $6y$ cm respectively, with a width of x cm.



(a) Express x in terms of y and show that the total surface area of the door stopper A, is given as $A = 12y^2 + \frac{20}{y}\left(\sqrt{10} + 4\right)$ cm^2.

[3]

(b) Given that y can vary, find the value of y for which A has a minimum value. [3]

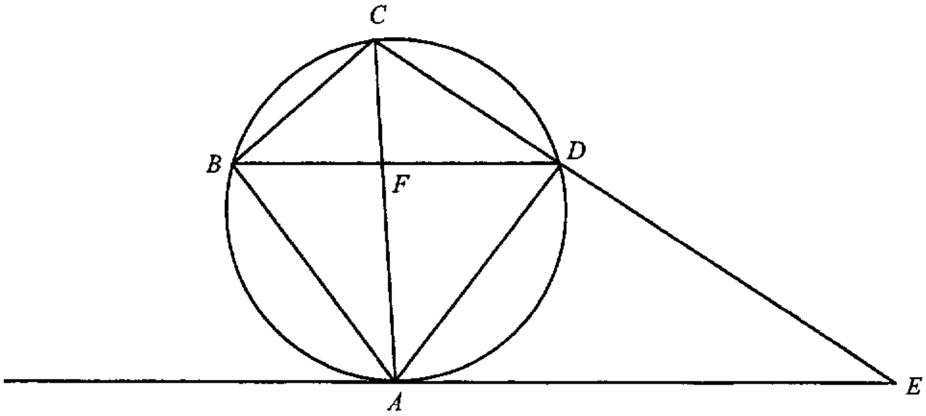
10 (a) Find all angles between 0 and 2π which satisfy $3\cos 2x + 4\sin x = 3$. [4]

(b) Without using a calculator,

(i) show that $\cos\frac{7\pi}{12} = \frac{\sqrt{2}-\sqrt{6}}{4}$. [3]

(ii) and hence, find the exact value of $\sin^2\frac{7\pi}{12}$. [3]

11



Points A, B, C and D are inscribed in a circle such that $AB = AD$. A tangent to the circle at point A meets the line CD produced at E. The lines AC and BD intersects at point F.

(a) Prove that the line BFD is parallel to line AE. [3]

(b) Show that triangle *ABC* is similar to triangle *EDA*. [3]

--- End of Paper ---

4E AM Prelim 2024 P2 Solutions

1.		The Singapore government issued a savings bond in January 2024 with a yield of 2.75% per year. Mr Tan invested $15 000 in the bond. The total amount he will receive, after t years, is given by $A = 15000(1.0275)^t$.
	(a)	Calculate the total amount he will receive in January 2030, correct to the nearest dollar.
		$A = 15000(1.0275)^6$ $= 17651.52$ $Amount = \$\ 17652$
	(b)	In which year will the amount first exceed $22 000?
		$22000 = 15000(1.0275)^t$ $1.0275^t = \frac{22}{15}$ $t = \ln\left(\frac{22}{15}\right) / \ln 1.0275$ $t = 14.117$ Year 2039.

2.	(a)	Without using a calculator, evaluate the value of 6^x given that $2^{2x+6} \times 3^{5x-1} = 27^{x+1}$
		$2^{2x+6} \times 3^{5x-1} = 3^{3x+3}$ $2^{2x} \times 2^6 \times 3^{5x} \times 3^{-1} = 3^{3x} \times 3^3$ $\frac{2^{2x} \times 3^{5x}}{3^{3x}} = \frac{3^3}{2^6 \times 3^{-1}}$ $2^{2x} \times 3^{2x} = \frac{3^4}{2^6}$ $6^{2x} = \frac{81}{64}$ $6^x = \frac{9}{8}$
	(b)	Solve the equation $log_x 9 = 5log_3 x$, giving your answers correct to 2 significant figures.
		$log_x 9 = 5log_3 x$ $\frac{log_3 9}{log_3 x} = 5log_3 x$ $2log_3 3 = 5(log_3 x)^2$ $(log_3 x)^2 = \frac{2}{5}$ $log_3 x = \pm\sqrt{\frac{2}{5}}$ $x = 3^{\sqrt{\frac{2}{5}}}\ or\ 3^{-\sqrt{\frac{2}{5}}}$ $x = 2.0\ or\ 0.50$

3.	(a)	A curve has the equation $y = (p-1)x^2 + 2(p-3)$, where p is a constant. A line has the equation $y = 6x + 3$. Find the range of values of p if the curve lies completely above the line.
		$(p-1)x^2 + 2p - 6 = 6x + 3$ $(p-1)x^2 - 6x + 2p - 9 = 0$ $b^2 - 4ac < 0$ $(-6)^2 - 4(p-1)(2p-9) < 0$ $36 - (4p-4)(2p-9) < 0$ $36 - 8p^2 + 8p + 36p - 36 < 0$ $-8p^2 + 44p < 0$ $8p^2 - 44p > 0$ $4p(2p-11) > 0$  $p < 0\ or\ p > 5.5$ (rejected)
	(b)	By expressing $y = -2x^2 + 10x - 5$ in the form $y = a(x+b)^2 + c$, where a, b and c are constants, find the maximum value of y.
		$y = -2(x^2 - 5x) - 5$ $= -2\left[x^2 - 5x + \left(-\frac{5}{2}\right)^2 - \left(-\frac{5}{2}\right)^2\right] - 5$ $= -2[(x-2.5)^2 - 2.5^2] - 5$ $= -2(x-2.5)^2 + 7.5$ Max value of $y = 7.5$

4.	(a)	In the binomial expansion of $\left(1-\frac{2}{7}x\right)^n$, the sum of the coefficient of the second and third term is zero. Calculate the value of n and hence, find the sixth term.
		$T_2=\binom{n}{1}(1)^{n-1}\left(-\frac{2}{7}x\right)^1=-\frac{2}{7}nx$ $T_{3=}\binom{n}{2}(1)^{n-2}\left(-\frac{2}{7}x\right)^2=\frac{n(n-1)}{2}\left(\frac{4}{49}x^2\right)$ $-\frac{2}{7}n+\frac{2n(n-1)}{49}=0$ $-14n+2n^2-2n=0$ $2n^2-16n=0$ $2n(n-8)=0$ $n=0\ \ or\ n=8$ (rejected) $T_6=\binom{8}{5}(1)^{8-5}\left(-\frac{2}{7}x\right)^5=-\frac{256}{2401}x^5$

(b) Write down the general term in the binomial expansion of $\left(\frac{1}{x^3} - 2x\right)^8$.

Hence, find the value of the constant term in the expansion of

$\left(3 + \frac{x^2}{2}\right)^2 \left(\frac{1}{x^3} - 2x\right)^8$.

$$T_{r+1} = \binom{8}{r}\left(\frac{1}{x^3}\right)^{8-r} (-2x)^r$$

$$= \binom{8}{r} x^{-24+3r}(-2)^r x^r$$

$$= \binom{8}{r} (-2)^r x^{4r-24}$$

$$\left(3 + \frac{x^2}{2}\right)^2 = 9 + 3x^2 + \frac{x^4}{4}$$

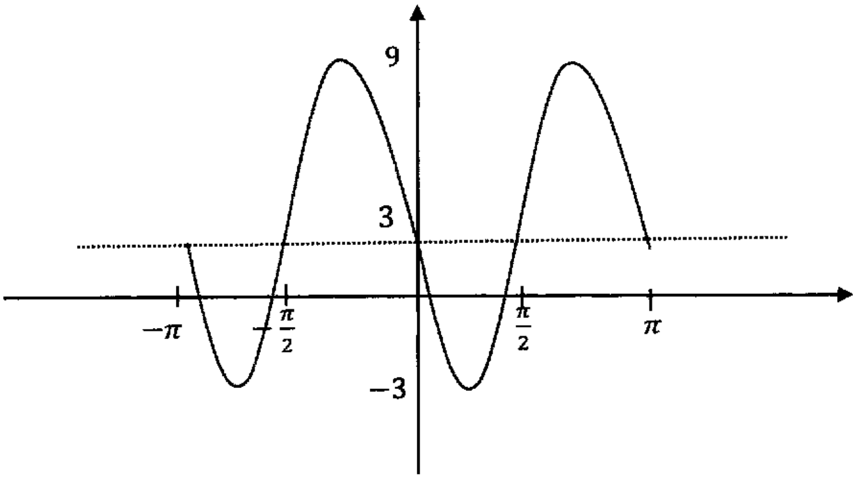
$$x^{4r-24} = x^0 \quad or\ x^{4r-24} = x^{-2} \qquad or\ x^{4r-24} = x^{-4}$$

$$r = 6 \quad or\ no\ integer\ value \quad or \quad r = 5$$

$$T_{7=} \binom{8}{6} (-2)^6 = 1792$$

$$T_{6=} \binom{8}{5} (-2)^5 = -1792$$

Constant term = $(1792 \times 9) + \left(\frac{1}{4} \times -1792\right) = 15680$

5.	(a)	The function f is defined as $f(x) = p - q\sin(rx)$, for $-\pi \leq x \leq \pi$, where p, q and r are positive integers. Given that the amplitude of the function is 6, the period is π and the maximum value of is 9.
		(i) State the values of p, q and r.
		$p = 3$ $q = 6$ $r = 2$
		(ii) Hence, sketch the graph of $f(x)$.
		 - Shape of curve and range - Points plotted correctly

(b) The acute angles A and B are such that $\cot(A - B) = \frac{1}{3}$ and $\cot A = \frac{1}{5}$.

Without using a calculator, find the exact value of $\cos B$.

$$\cot A = \frac{1}{5}$$

$$\tan A = 5$$

$$\cot(A - B) = \frac{1}{3}$$

$$\tan(A - B) = 3$$

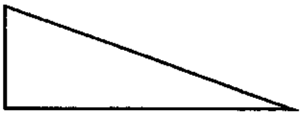
$$\frac{\tan A - \tan B}{1 + \tan A \tan B} = 3$$

$$\tan A - \tan B = 3 + 3\tan A \tan B$$

$$5 - \tan B = 3 + 15\tan B$$

$$16\tan B = 2$$

$$\tan B = \frac{1}{8}$$



$$\cos B = \frac{8}{\sqrt{65}} = \frac{8\sqrt{65}}{65}$$

6. A circle passes through the points $(-5, 12)$ and $(9, 14)$. The centre of the circle lies on the line $2y + x = 15$.

(a) Find the equation of the circle.

$$m = \frac{14 - 12}{9 - (-5)} = \frac{1}{7}$$

$$m_{\perp} = -7$$

$$midpoint = \left(\frac{-5 + 9}{2}, \frac{12 + 14}{2}\right) = (2, 13)$$

$$13 = -7(2) + c$$

$$c = 27$$

Eqn of perpendicular bisector $y = -7x + 27$

$$2(-7x + 27) + x = 15$$

$$-14x + 54 + x = 15$$

$$-13x = -39$$

$$x = 3$$

$$y = 6$$

Centre of circle (3,6)

Radius $= \sqrt{(9 - 3)^2 + (14 - 6)^2} = 10\ units$

Equation of circle

$$(x - 3)^2 + (y - 6)^2 = 100$$

Or $x^2 - 6x + y^2 - 12y - 55 = 0$

	(b)	Explain why the line $y = mx + 6$ intersects the circle at 2 distinct points for all values of m.
		$(x-3)^2 + (mx+6-6)^2 = 100$ $x^2 - 6x + 9 + m^2x^2 - 100 = 0$ $(1+m^2)x^2 - 6x - 91 = 0$ $b^2 - 4ac = (-6)^2 - 4(1+m^2)(-91)$ $= 400 + 364m^2$ $400 + 364m^2 > 0$ for all values of m $\therefore$line cuts circle at 2 distinct points
7.	(a)	A curve has the equation $y = \frac{3x-5}{4x+1}$ for $x > 0$. Explain, with working, why the curve has no stationary points.
		$\frac{dy}{dx} = \frac{3(4x+1) - 4(3x-5)}{(4x+1)^2}$ $= \frac{12x + 3 - 12x + 20}{(4x+1)^2}$ $= \frac{23}{(4x+1)^2}$ Since $(4x+1)^2 \geq 0$ for all x, $\frac{dy}{dx} > 0$ Curve has no stationary point as $\frac{dy}{dx} \neq 0$.

(b) It is given that $f(x)$ is such that $f'(x) = \cos 4x - 3\sin 2x$. Given also that $f(\pi) = 0$, show that $f''(x) + 4f(x) = -3(\sin 4x + 2)$.

$$f(x) = \frac{1}{4}sin4x + \frac{3}{2}cos2x + c$$

$$f(\pi) = 0$$

$$\frac{3}{2} + c = 0$$

$$c = -\frac{3}{2}$$

$$f(x) = \frac{1}{4}\sin 4x + \frac{3}{2}\cos 2x - \frac{3}{2}$$

$$f''(x) = -4\sin 4x - 6\cos 2x$$

$$f''(x) + 4f(x) = -4\sin 4x - 6\cos 2x + 4\left(\frac{1}{4}\sin 4x + \frac{3}{2}\cos 2x - \frac{3}{2}\right)$$

$$= -4\sin 4x - 6\cos 2x + \sin 4x + 6\cos 2x - 6$$

$$= -3\sin 4x - 6$$

$$= -3(\sin 4x + 2)\ (shown)$$

(c) Given that $\frac{d}{dx}\left(\frac{2-x}{\sqrt{1-2x}}\right) = \frac{ax+b}{\sqrt{(1-2x)^3}}$, find the value of a and b.

$$\frac{d}{dx}\left(\frac{2-x}{\sqrt{1-2x}}\right) = \frac{(1-2x)^{\frac{1}{2}}(-1) - (2-x)\left(\frac{1}{2}\right)(1-2x)^{-\frac{1}{2}}(-2)}{(1-2x)}$$

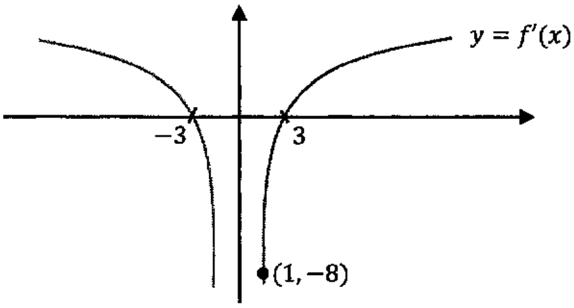
$$= \frac{(1-2x)^{-\frac{1}{2}}[(-1)(1-2x) - (2-x)\left(\frac{1}{2}\right)(-2)}{(1-2x)}$$

$$= \frac{(1-2x)^{-\frac{1}{2}}[-1 + 2x + 2 - x]}{(1-2x)}$$

$$= \frac{1+x}{(1-2x)^{\frac{3}{2}}}$$

$$a = 1,\ \ b = 1$$

8. A curve $y = f(x)$ passes through the point $(1, 10)$.



The graph shown above is $y = f'(x)$.

(a) State the x-coordinates of the stationary points of the curve $y = f(x)$ and hence, determine their nature.

$x = -3$ *and* $x = 3$

x	-3.1	-3	-2.9		2.9	3	3.1
$f'(x)$	$+$	0	$-$		$-$	0	$+$

Maximum point   Minimum point

(b) Find the equation of the normal to the curve at the point $(1, 10)$.

$$x = 1, f'(x) = -8$$

Gradient of tangent = -8

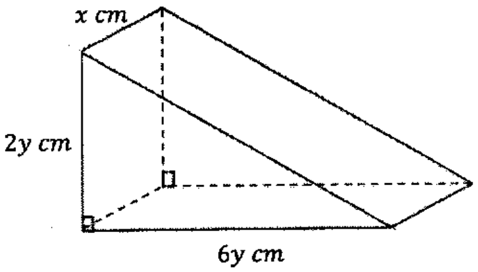
Gradient of normal = $\frac{1}{8}$

$$10 = \frac{1}{8}(1) + c$$

$$c = \frac{79}{8}$$

$$y = \frac{1}{8}x + \frac{79}{8} \quad or\ 8y = x + 79$$

9. The diagram below shows a wooden door stopper in the shape of a right prism with a volume of 60 cm^3. The cross-section of the prism is a triangle, with side lengths of 2*y* and 6*y* cm respectively, with a width of *x* cm.



(a) Express *x* in terms of *y* and show that the total surface area of the door stopper *A*, is given as $A = 12y^2 + \frac{20}{y}(\sqrt{10} + 4)\ cm^2$.

$$\frac{1}{2} \times 2y \times 6y \times x = 60$$

$$x = \frac{10}{y^2}$$

Total S.A $= 2\left(\frac{1}{2} \times 2y \times 6y\right) + 2xy + 6xy + \left(\sqrt{(2y)^2 + (6y)^2}\right)x$

$$= 12y^2 + 8xy + \left(\sqrt{40y^2}\right)x$$

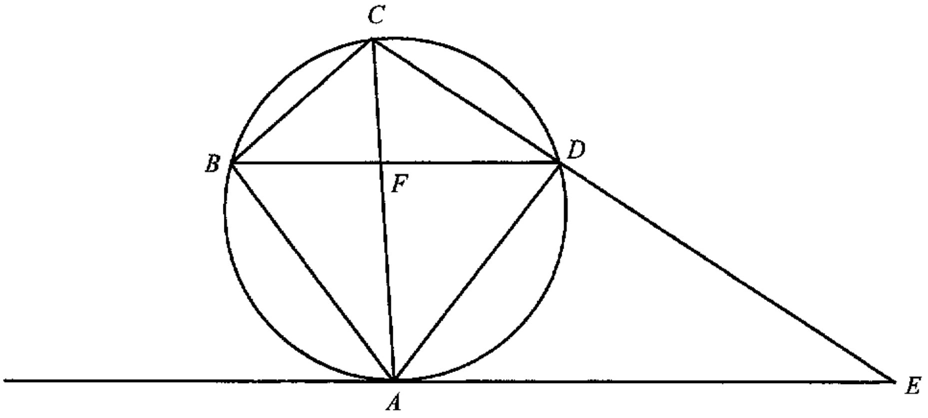
$$= 12y^2 + 8y\left(\frac{10}{y^2}\right) + 2y\sqrt{10}\left(\frac{10}{y^2}\right)$$

$$= 12y^2 + \frac{80}{y} + \frac{20\sqrt{10}}{y}$$

$$= 12y^2 + \frac{20}{y}(\sqrt{10} + 4)\ cm^2$$

	(b)	Given that y can vary, find the value of y for which A has a minimum value.
		$\frac{dA}{dy} = 24y - \left(\frac{20}{y^2}\right)(\sqrt{10} + 4)$ $24y - \left(\frac{20}{y^2}\right)(\sqrt{10} + 4) = 0$ $24y^3 = 20(\sqrt{10} + 4)$ $y^3 = \frac{5}{6}(\sqrt{10} + 4)$ $y = 1.8139$ $\frac{d^2A}{dy^2} = 24 + \frac{40}{y^3}(\sqrt{10} + 4)$ $= 72 > 0$ Value of A is at a minimum.
10.	(a)	Find all angles between 0 and 2π which satisfy $3\cos 2x + 4\sin x = 3$.
		$3\cos 2x + 4\sin x = 3$ $3(1 - 2\sin^2 x) + 4sinx - 3 = 0$ $3 - 6\sin^2 x + 4sinx - 3 = 0$ $-6\sin^2 x + 4sinx = 0$ $-2\sin x(3\sin x - 2) = 0$ $-2\sin x = 0 \quad or \quad 3\sin x - 2 = 0$ $sinx = 0 \quad or \quad sinx = \frac{2}{3}$ $Basic\ angle = 0 \quad or\ 0.72972$ $x = \pi \quad or \quad x = 0.72972\ or\ 2.4118$ $x = 0.730, 2.41, \pi$ (minus 1 mark for each wrong value)

	(b)	Without using a calculator,
	(i)	Show that $\cos\frac{7\pi}{12} = \frac{\sqrt{2}-\sqrt{6}}{4}$.
		$\cos\frac{7\pi}{12} = \cos\left(\frac{3\pi}{12}+\frac{4\pi}{12}\right)$ $= \cos\left(\frac{\pi}{4}+\frac{\pi}{3}\right)$ $= \left(\cos\frac{\pi}{4}\right)\left(\cos\frac{\pi}{3}\right) - \left(\sin\frac{\pi}{4}\right)\left(\sin\frac{\pi}{3}\right)$ $= \left(\frac{1}{\sqrt{2}}\right)\left(\frac{1}{2}\right) - \left(\frac{1}{\sqrt{2}}\right)\left(\frac{\sqrt{3}}{2}\right)$ $= \frac{1-\sqrt{3}}{2\sqrt{2}}$ $= \frac{\sqrt{2}-\sqrt{6}}{4} \quad (shown)$
	(ii)	Hence, find the exact value of $\sin^2\frac{7\pi}{12}$.
		$\sin^2\frac{7\pi}{12} = 1 - \cos^2\frac{7\pi}{12}$ $= 1 - \left(\frac{\sqrt{2}-\sqrt{6}}{4}\right)^2$ $= 1 - \frac{2 - 2\sqrt{2}\sqrt{6} + 6}{16}$ $= \frac{16 - (2 - 2\sqrt{12} + 6)}{16}$ $= \frac{16 - 2 + 4\sqrt{3} - 6}{16}$ $= \frac{8 + 4\sqrt{3}}{16}$ $= \frac{2+\sqrt{3}}{4}$

12.		
		Points *A, B, C* and *D* is inscribed in a circle such that *AB* = *AD*. A tangent to the circle at point *A* meets the line *CD* produced at E. The lines *AC* and *BD* intersects at point *F*.
	(i)	Prove that the line *BFD* is parallel to line *AE*.
		$\angle ABD = \angle DAE (alt.seg.thm)$ $\angle ABD = \angle ADB\ (base\ angle\ of\ iso\ \Delta)$ $\angle ABD = \angle DAE$ BFD is parallel to AE as $\angle ABD\ and\ \angle DAE$ are equal, alternate angles in parallel lines.
	(ii)	Show that triangle *ABC* is similar to triangle *EDA*.
		$\angle BCA = \angle BDA\ (\measuredangle in\ same\ segment)$ $\angle BCA = \angle DAE\ (part\ (i))$ $\angle ABC + \angle CDA = 180\ (\measuredangle in\ opp\ segment)$ $\angle CDA + \angle EDA = 180\ (adj. \measuredangle on\ a\ str.line)$ $\angle ABC = \angle EDA$ $\Delta ABC\ is\ similar\ to\ \Delta EDA$ (angle-angle theorem).