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TAMPINES SECONDARY SCHOOL

Secondary Four Express / Five Normal Academic
Preliminary Examination 2024

NAME			
CLASS		**REGISTER NUMBER**	

ADDITIONAL MATHEMATICS — **4049/01**

Paper 1 — **28 August 2024**

2 hours 15 minutes

Candidates answer on the Question Paper.

READ THESE INSTRUCTIONS FIRST

Write your name, class and register number on all the work you hand in.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

For Examiner's Use

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.

The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

The number of marks is given in brackets [ ] at the end of each question or part question.

The total number of marks for this paper is **90**.

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1} a^{n-1} b + \binom{n}{2} a^{n-2} b^2 + \ldots + \binom{n}{r} a^{n-r} b^r + \ldots + b^n ,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{(n-r)!\,r!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$.

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2} ab \sin C$$

1 The equation of a curve is $y = \frac{2x^2}{x+1}$ where $x \neq -1$. Determine the range of values of x for which y is a decreasing function. [4]

2 A missile, TP-1 was launched such that its height, h_1 metres above the ground was given by $h_1(x) = -20x^2 + 120x + 3$, where x metres was the horizontal distance of the missile from the launched position.

(a) Express $-20x^2 + 120x + 3$ in the form $a(x + b)^2 + c$, where a, b and c are constants. [2]

Another missile, TP-2 was launched from the same position as TP-1. The height of TP-2, h_2 metres above the ground was given by $h_2(x) = -\frac{49}{10}(x-6)^2 + 183$.

(b) Find $h_1(0)$ and $h_2(0)$ and hence interpret the meaning of your answers. [2]

(c) The missile that could reach a greater height and travel a distance further away from its launched position was acquired. Determine if missile TP-1 or TP-2 was acquired.
Show your working clearly. [3]

3 A cuboid with volume $(2x+1)^2$ cm^3 has a rectangular base with dimensions x^2 cm and $(2x-1)$ cm. Find an expression for the height of the cuboid, leaving your answer in partial fractions. [6]

4 (a) (i) Find in ascending powers of x, the simplified first three terms in the expansion of $(2 + qx)^6$. [3]

(ii) Given that the first two non-zero terms in the expansion of $(2 + px)(2 + qx)^6$, where $q > 0$, are 128 and $-168x^2$, find the values of p and q. [4]

(b) Find the term independent of x in $\left(x^3 - \frac{2}{x}\right)^{12}$. [3]

5 A function f(x) is defined for all real values of x such that $f''(x) = 18e^{-3x}$. The gradient of the curve $y = f(x)$ at $x = 0$ is 2 and the curve passes through $\left(1, \frac{2}{e^3}\right)$.

(a) Find the exact value of the x-coordinate of the stationary point of the curve and determine its nature. [6]

(b) Find the equation of the curve. [2]

6 A graph has the equation $y = x^2 + (6 - 2m)x + m + 5$ where m is a constant.

(a) Given that the graph cuts the x-axis at A and B and the point (4, 0) is the midpoint of AB, find the value of m. [3]

(b) If $m = 8$, find the range of values of p such that the graph of $y = x^2 + (6 - 2m)x + m + 5 + p$ lies above the x-axis. [2]

7 The number of insects present in a colony t weeks after observation began, can be modelled by the equation $m = ab^{\frac{t}{3}}$. Measurements of m and t are shown in the table below.

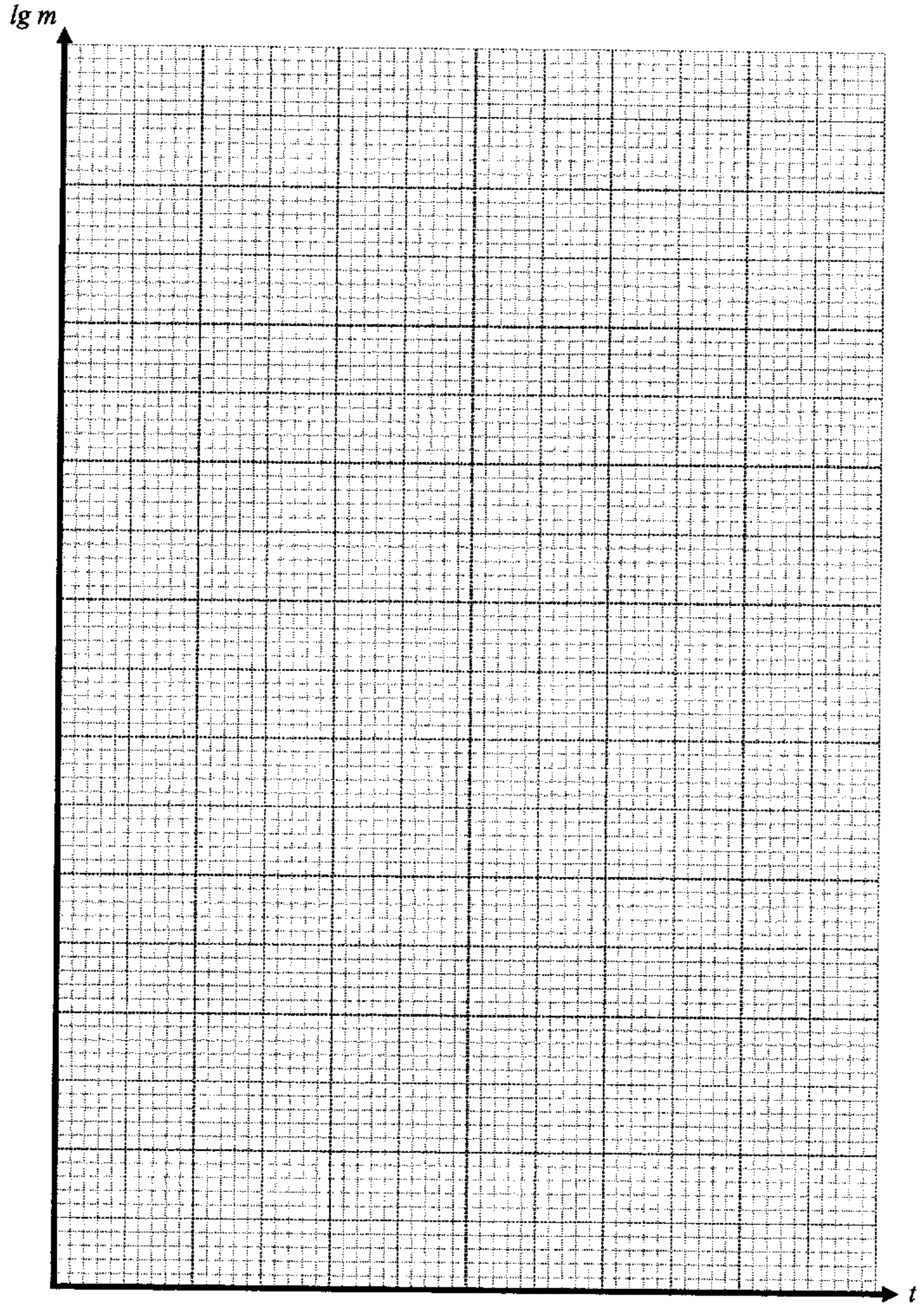
t	2	4	6	8	10
m	920	1108	1333	1605	1930

(a) Plot lg m against t and draw a straight line graph to illustrate the information. [2]

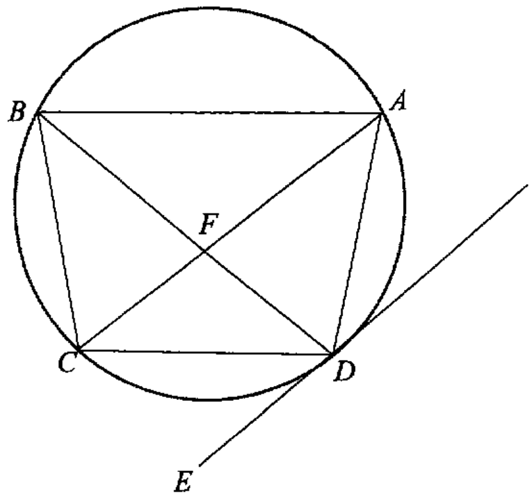
(b) Use your graph to estimate the values of a and b. [3]

The number of insects present in another colony t hours after observation began, can be modelled by the equation $\lg m^{40} = t + 120$.

(c) By drawing a suitable straight line on your graph estimate the time taken for the two colonies to have the same number of insects. [2]


lg m
t

8



In the diagram, A, B, C and D lie on the circumference of the circle such that BD is the diameter of the circle. BD and AC intersect at F. DE is a tangent to the circle at D and $AD = CD$.

(a) Show that AC is parallel to DE. [3]

(b) Prove that triangle ABD is similar to triangle CBD. [3]

9 The height, y metres of the water level near a beach can be modelled by the equation $y = a - b\cos(pt)$, where t is the number of hours after midnight. and p is the radians per hour.

A low tide is observed at midnight and the duration between successive low tides is 12 hours.

(a) Show that $p = \frac{\pi}{6}$. [1]

The greatest and least heights of the water level are 8 metres and 4 metres respectively.

(b) State the value of a and of b for $a > 0$ and $b > 0$. [2]

(c) Hence sketch the graph of $y = a - b\cos(pt)$ for $0 \leq t \leq 12$. [3]

(d) People have been advised to stay away from the beach when the height of the water level is 6 metres or higher. Determine the periods from midnight to 23 00 when the people must stay away. [2]

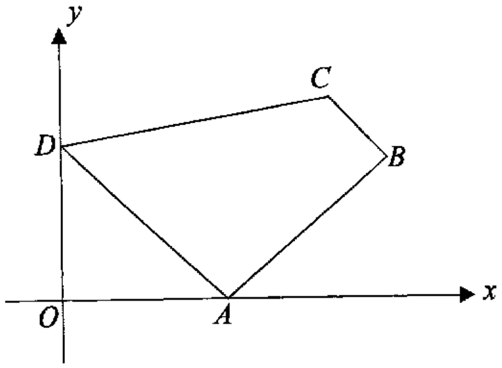
10 A particle is at 3 metres past a fixed point O. It starts to move in a straight line such that t seconds later, its velocity v m/s is given by $v = t^2 - 6t + 5$. The particle comes to an instantaneous rest first at point A then at point B.

(a) Find an expression, in terms of t, for the distance of the particle from O at t seconds. [2]

(b) Find the total distance travelled by the particle in the first 5 seconds. [5]

(c) Point C is where the particle has zero acceleration. Determine if point C is nearer to its initial starting position or point B. [3]

11 The trapezium $ABCD$ is such that AD is parallel to BC and A is $(2, 0)$. The equation of BC is $y = 11 - 3x$ and $2BC = AD$.



(a) Find the equation of AD. [2]

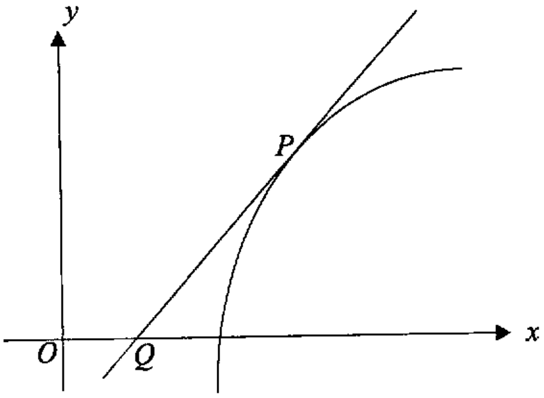
(b) Find the equation of the perpendicular bisector of AD. [3]

(c) The perpendicular bisector found in part **(b)** passes through the point C. Find the coordinates of C. [2]

(d) Show that B has coordinates $\left(\frac{7}{2}, \frac{1}{2}\right)$. [1]

(e) Hence find the area of trapezium $ABCD$. [2]

12



The diagram shows part of the curve $y = 2 - \dfrac{3}{4x-5}$ for $x > \dfrac{5}{4}$.

(a) Determine if the curve has a stationary point. [3]

The tangent to the curve at P cuts the x-axis at $Q\left(\frac{5}{4},0\right)$. The normal to the curve at P is parallel to

$y=-\frac{3}{4}x+10$.

(b) Find the coordinates of P. [3]

(c) The normal to the curve at P cuts the x-axis at R. Find the area of triangle PQR. [3]

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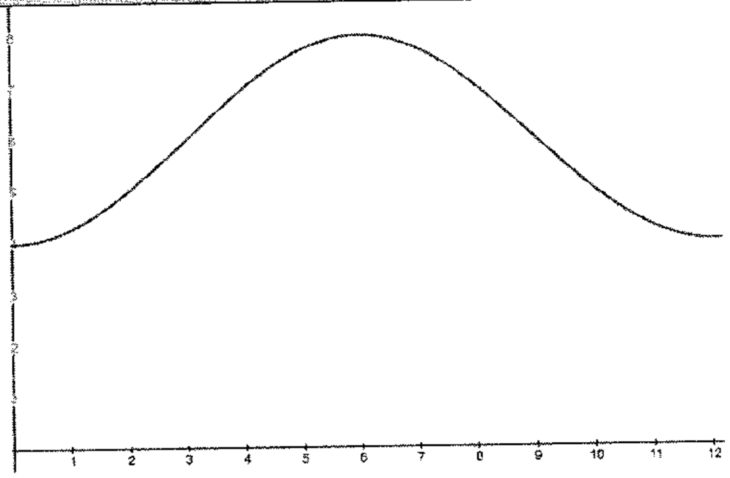
Tampines Secondary School

Sec 4&5 Express Additional Math Paper 1 2024 Marking Scheme

No.	Answers	Marks
1	$y=\dfrac{2x^2}{x+1}$ $\dfrac{dy}{dx}=\dfrac{(x+1)(4x)-(2x^2)(1)}{(x+1)^2}$ $=\dfrac{2x^2+4x}{(x+1)^2}$ Decreasing function $\rightarrow \dfrac{dy}{dx}<0$ Since $(x+1)^2>0$, $2x^2+4x<0$ $2x(x+2)<0$ $\therefore -2<x<0$	 M1 A1 M1 A1
2(a)	$-20x^2+120x+3=-20(x^2-6x)+3$ $=-20[(x-3)^2-3^2]+3$ $=-20(x-3)^2+183$	 M1 A1
(b)	$h_1(0)=3$ $\quad$ $h_2(0)=6.6$ TP-1 was fired from a height of 3 metres above ground while TP-2 was fired from a height of 6.6 metres above ground.	B1 B1
(c)	From TP-1's max pt (3, 183) and TP-2's max pt (6, 183), they both reach the <u>same height</u>. TP-1: $h=0 \rightarrow x=6.02$ m TP-2: $h=0 \rightarrow x=12.1$ m >6.02 m Since TP-2 could reach a further distance from the launched position, compared to TP-1, TP-2 should be acquired.	B1 M1 B1
3	Height $=\dfrac{(2x+1)^2}{x^2(2x-1)}$ $=\dfrac{4x^2+4x+1}{x^2(2x-1)}=\dfrac{A}{x}+\dfrac{B}{x^2}+\dfrac{C}{2x-1}$ $4x^2+4x+1=Ax(2x-1)+B(2x-1)+Cx^2$ Let $x=0$, $\quad B=-1$ Let $x=\dfrac{1}{2}$, $\quad 4=\dfrac{1}{4}C \rightarrow C=16$ Compare coeff of x^2, $\quad 4=2A+16 \rightarrow A=-6$ $\therefore$ Height $=\dfrac{16}{2x-1}-\dfrac{6}{x}-\dfrac{1}{x^2}$	M1 A1 M1: either sub mtd or compare coeff A3

No.	Answers	Marks
4a(i)	$(2+qx)^6 = 64 + 192qx + 240q^2x^2 + \ldots$	B3
(ii)	$(2+px)(2+qx)^6 = (2+px)(64 + 192qx + 240q^2x^2 + \ldots)$ Term in x: $384q + 64p = 0 \quad \rightarrow p = -6q$ Term in x^2: $480q^2x^2 + 192pqx^2 = -168$ $480q^2 + 192(-6q)q = -168$ $-480q^2 = -168$ $q = \frac{1}{2}$ *or* $q = -\frac{1}{2}$ (rej since $q > 0$) $p = -3$	 M1 M1 A1 A1
(b)	$T_{r+1} = {}^{12}C_r\,(x^3)^{12-r}(-2)^r(x^{-1})^r$ $36 - 3r - r = 0$ $r = 9$ Term $= {}^{12}C_9\,(-2)^9$ $= -112\,640$	M1 M1 A1
5(a)	$f'(x) = \frac{18e^{-3x}}{-3} + c$ $= -6e^{-3x} + c$ When $x = 0$, $f'(x) = 2$, $\quad -6e^{-3(0)} + c = 2$ $c = 8$ Stationary point, $\quad f'(x) = -6e^{-3x} + 8 = 0$ $6e^{-3x} = 8$ $e^{-3x} = \frac{4}{3}$ $x = -\frac{1}{3}\ln\frac{4}{3}$ When $x = -\frac{1}{3}\ln\frac{4}{3}$, $\quad f''(x) = 18e^{-3x}$ $= 24 > 0 \quad \rightarrow$ point is minimum	M1 M1: subt $x = 0$ & $f'(x) = 2$ M1 A1 M1 A1
(b)	$f'(x) = -6e^{-3x} + 8$ $f(x) = \frac{-6e^{-3x}}{-3} + 8x + d$ $= 2e^{-3x} + 8x + d$ Subt $\left(1, \frac{2}{e^3}\right)$, $\quad \frac{2}{e^3} = 2e^{-3} + 8 + d \quad \rightarrow d = -8$ Hence eqn of curve is $f(x) = 2e^{-3x} + 8x - 8$	 M1 A1

No.	Answers	Marks
6(a)	$\dfrac{dy}{dx}=2x+6-2m$	M1
	At $x=4$, $\dfrac{dy}{dx}=0$ → $14-2m=0$ $m=7$	M1 A1
	OR $y=(x+3-m)^2-(3-m)^2+m+5$	OR M1(complete the sq)
	Min pt is at $x=4$ → $(3-m)=4$ $m=7$	M1 A1
(b)	When $m=8$, $y=x^2-10x+13+p$ Lies above x-axis, discriminant <0 → $(-10)^2-4(13+p)<0$ $48-4p<0$ $p>12$	B1 B1
8(a)	$\angle EDC=\angle DAC$ (alt seg thm) $\angle DAC=\angle DCA$ ($AD=CD$, isos triangle) $\therefore \angle EDC=\angle DCA$ Hence AC is parallel to DE (alt angles)	B1 B1 B1
(b)	$\angle BCD=\angle BAD$ (angles in semicircle)	B1
	Let $\angle EDC=x$ $\angle EDC=\angle CBD=x$ (alt seg thm) $\angle DCA=\angle DAC=x$ (shown in part (a)) $\angle DAC=\angle ADM=x$ (alt angle, AD parallel DE) $\angle ADM=\angle ABD=x$ (alt seg thm) $\therefore \angle ABD=\angle CBD$	A1
	By AA, triangle ABD is similar to triangle CBD.	A1
9(a)	Period $=\dfrac{2\pi}{p}$ $12=\dfrac{2\pi}{p}$ $p=\dfrac{\pi}{6}$ (shown)	A1
(b)	$a=6$ $b=2$	B2

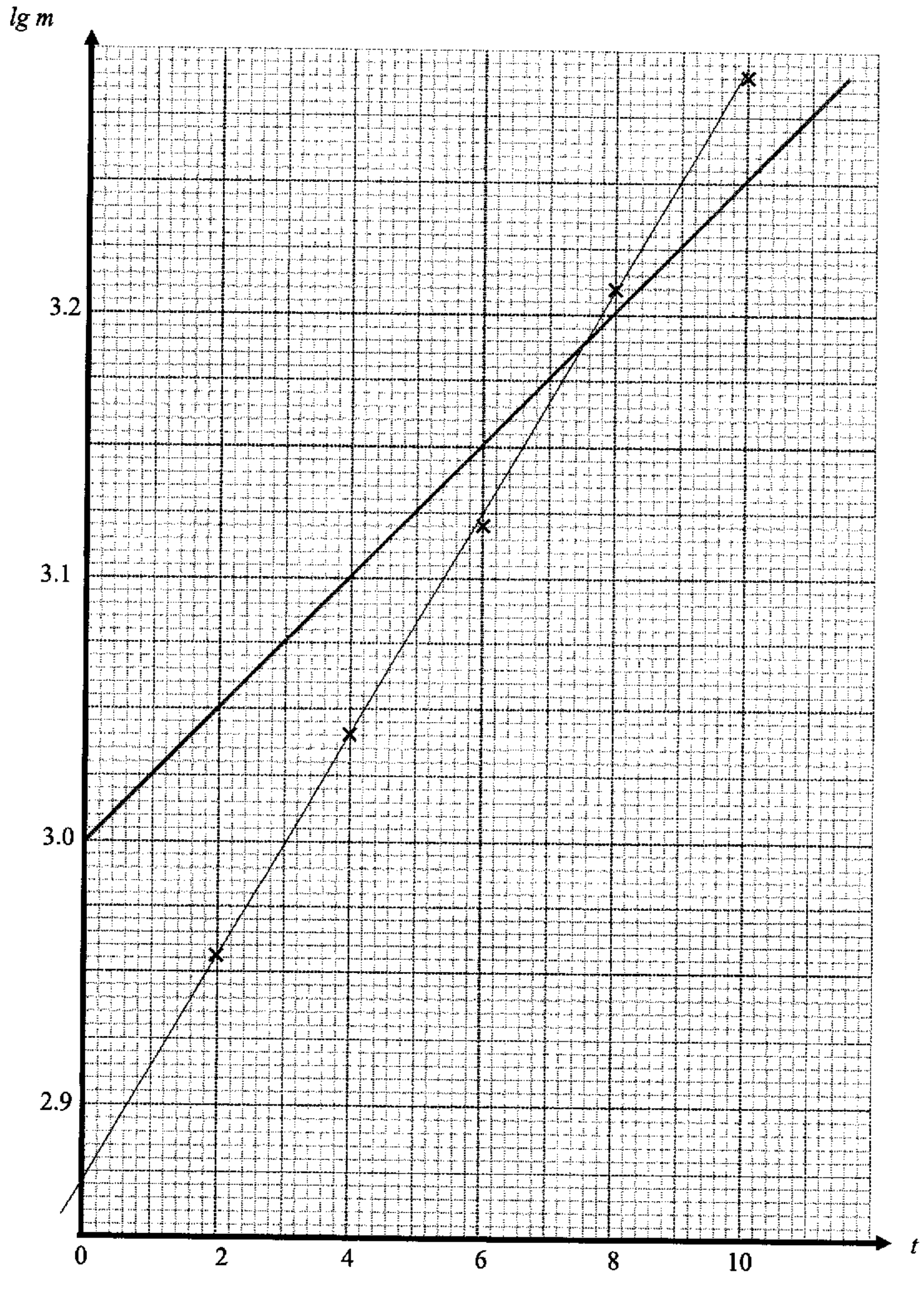
No.	Answers	Marks
(c)		G1: Shape G1: period G1: max & min points
(d)	03 00 to 09 00 or 3 a.m. to 9 a.m. 15 00 to 21 00 or 3 p.m. to 9 p.m.	B1 B1
10(a)	$s=\int t^2-6t+5\ dt$ $=\dfrac{t^3}{3}-\dfrac{6t^2}{2}+5t+c$ When $t=0, s=3$ $\therefore$ $s=\dfrac{t^3}{3}-3t^2+5t+3$	 M1 A1
(b)	When $v=0$, $t^2-6t+5=0$ $t=5$ or $t=1$ When $t=0$, $s=3$ When $t=1$, $s=\dfrac{16}{3}$ When $t=5$, $s=-\dfrac{16}{3}$ Total dist $=\left(\dfrac{16}{3}-3\right)+\dfrac{16}{3}\times 2$ $=13$ m	M1 A1 M1: for $t=1$ or 5 M1 A1
(c)	$a=2t-6$ At $a=0, t=3,$ $s=0$ Hence it is <u>nearer to its initial starting position</u> which is 3 m away compared to point B which is $\dfrac{16}{3}$ m away.	M1 M1 A1
11(a)	Grad $=-3$ Eqn: $0=-3(2)+c$ $\rightarrow$ $c=6$ Equation of AD is $y=-3x+6$	B1 B1

No.	Answers	Marks
(b)	$D(0, 6)$ Midpoint of AD is $(1, 3)$ Grad of bisector = $\frac{1}{3}$ Eqn: $3=\frac{1}{3}(1)+d \rightarrow d=\frac{8}{3}$ Equation of perpendicular bisector is $y=\frac{1}{3}x+\frac{8}{3}$	M1 B1 A1
(c)	$11-3x=\frac{1}{3}x+\frac{8}{3}$ $11-\frac{8}{3}=\frac{1}{3}x+3x$ $x=\frac{5}{2}$ $y=\frac{7}{2}$ $\quad \therefore C\left(\frac{5}{2},\frac{7}{2}\right)$	M1 A1
(d)	$BC=\frac{1}{2}AD$ $B\left(\frac{5}{2}+1,\frac{7}{2}-3\right)=B\left(\frac{7}{2},\frac{1}{2}\right)$ (shown)	 A1
(e)	Area $=\frac{1}{2}\begin{vmatrix} 2 & \frac{7}{2} & \frac{5}{2} & 0 & 2 \\ 0 & \frac{1}{2} & \frac{7}{2} & 6 & 0 \end{vmatrix}$ $= 7.5$ sq units	M1 A1
12(a)	$\frac{dy}{dx}=\frac{12}{(4x-5)^2}$ For curve to have stationary point, $\frac{dy}{dx}=0$. In this case, $\frac{dy}{dx}=\frac{12}{(4x-5)^2}\neq 0$ since $12\neq 0$. Hence this curve <u>does not have</u> a stationary point.	M1 A1 B1
(b)	$\frac{dy}{dx}=\frac{12}{(4x-5)^2}=\frac{4}{3}$ $(4x-5)^2=9$ $x=2$ *or* $\frac{1}{2}$ (rej since $x>\frac{5}{4}$) $y=1$ $\therefore P(2, 1)$	M1 M1 A1

No.	Answers	Marks
(c)	Eq of normal: $1 = -\frac{3}{4}(2) + c$ $c = \frac{5}{2}$ → $y = -\frac{3}{4}x + \frac{5}{2}$ When $y = 0, x = \frac{10}{3}$ Area $= \frac{1}{2} \times \left(\frac{10}{3} - \frac{5}{4}\right) \times 1$ $= \frac{25}{24}$ or 1.04 sq units	M1 M1 A1
7(a)	see table below	
(b)	$\lg m = \lg a + \frac{\lg b}{3} t$ $\lg a = 2.87$ → $a = 741$ [accept 724, 733, 750] grad $= \frac{\lg b}{3} = \frac{3.04 - 2.96}{4 - 2}$ $b = 1.32$	M1 A1 A1
(c)	$\lg m^{40} = t + 120$ $\lg m = 0.025t + 3$ $\therefore t = 7.5$ weeks	G1 (correct line drawn on grid) A1

7(a):

t	2	4	6	8	10
$\lg m$	2.96	3.04	3.12	3.21	3.29


lg m
3.2
3.1
3.0
2.9
0
2
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TAMPINES SECONDARY SCHOOL

Secondary Four Express / Five Normal Academic
Preliminary Examination 2024

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where n is a positive integer and $\binom{n}{r} = \dfrac{n!}{(n-r)!\,r!} = \dfrac{n(n-1)\ldots(n-r+1)}{r!}$.

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$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

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$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2} ab \sin C$$

1 The expression $2x^3+3x^2-8x+3$ leaves a remainder of p when divided by $(x+1)$.

(a) Find the value of p. [2]

(b) Solve the equation $2x^3+3x^2-8x+3=0$. [3]

(c) Hence, solve the equation $2(x-1)^3+3(x-1)^2-8x+11=0$. [2]

2 Peter buys a new car. After t months, its value $\$C$ is given by $C = 120000e^{-at}$, where a is a constant.

(a) Find the value of the car when Peter bought it. [1]

(b) The value of the car after 12 months is expected to be $\$90000$.

(i) Show that $a = 0.02397$, correct to 4 significant figures. [3]

(ii) Calculate the age of the car, to the nearest month, when its expected value will be $\$70000$. [2]

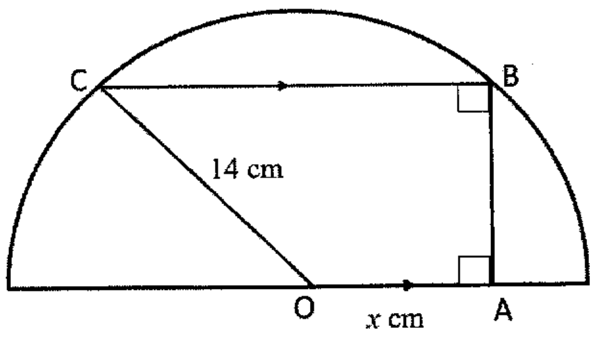
(iii) After 5 years, a car dealer offers to pay Peter $\$29000$ for your car. Based on the equation above, would you agree to sell it? Explain your answer. [2]

3 **(a)** Without using a calculator, show that $\cos 75° = \dfrac{\sqrt{6}-\sqrt{2}}{4}$. [3]

(b) Prove that $\dfrac{\tan x + \cot x}{\sec x + \operatorname{cosec} x} = \dfrac{1}{\cos x + \sin x}$. Hence solve the equation [6]

$\dfrac{\tan x + \cot x}{\sec x + \operatorname{cosec} x} = \dfrac{3}{4\cos x - 3\sin x}$ for $0 \leq x \leq 2\pi$.

4



The diagram shows a trapezium $OABC$ inscribed in a semi-circle, centre O and radius 14 cm. CB is parallel to the diameter, and AB is perpendicular to both CB and OA. Given that $OA = x$ cm,

(a) show that the area, A cm^2, of the trapezium is given by $A = \frac{3x}{2}\left(\sqrt{196 - x^2}\right)$. [2]

Given that x may vary,

(b) find the value of x for which A has a stationary value and determine whether this value of A is a maximum or a minimum, [5]

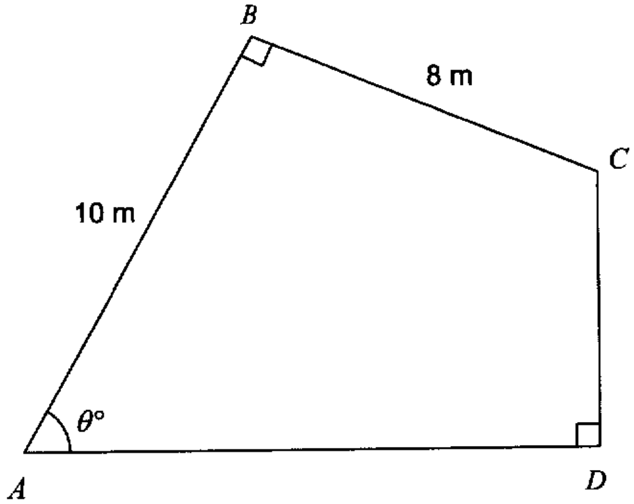
(c) find this stationary value of A. [2]

5 **(a)** By using a suitable substitution, solve the equation $2^{3+2x} + 2^{5+x} = 2^x + 4$. [5]

(b) Given that $x = \log_2 a$ and $y = \log_2 b$, express $\log_a \sqrt{ab^3}$ in terms of x and y. [3]

(c) Solve $\log_5 50 + 4\log_{25} y = \log_5(2y+4) + 2$. [4]

6 The diagram shows a plot of garden $ABCD$ in which $AB = 10$ m and $BC = 8$ m, $angle\ ABC = angle\ ADC = 90°$ and $angle\ BAD = \theta°$.
The garden will be used to grow sunflower plants.



(a) Show that the length $AD = 10\cos\theta + 8\sin\theta$. [2]

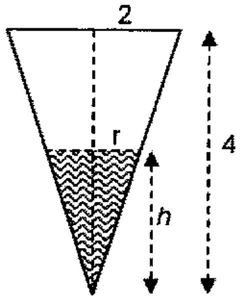
(b) Express $AD = 10\cos\theta + 8\sin\theta$ in the form $R\cos(\theta - \alpha)$, where R is a positive constant and α is acute. [3]

(c) Hence, find the values of θ such that $AD = 12$ m. [3]

7 (a) Show that $\frac{d}{dx}\{x(3x-1)^{\frac{5}{3}}\} = (8x-1)(3x-1)^{\frac{2}{3}}$. [5]

(b) Hence find $\int x(3x-1)^{\frac{2}{3}}\,dx$, giving your answer in the form $(ax+b)(3x-1)^{\frac{5}{3}}+c$, where a and b are constant to be found and c is a constant of integration which cannot be found. [5]

8



The diagram shows a vertical cross-section of an inverted conical water tank of height 4 m and base radius 2 m. When the base radius of the water surface is r m, the height of the water level is h m and the volume of the water in the tank is V m^3.

(a) Show that

(i) $r = \frac{h}{2}$, [2]

(ii) $V = \frac{\pi}{12}h^3$. [2]

(b) When $t = 0$, the tank is full and there is a leak in the water tank. If the volume of water in the tank is decreasing at a rate of 2 m^3 per minute, find the rate at which the water level is decreasing when the water is 2 m deep. Correct your answer to 3 significant figures. [5]

9 The equation of the circle, C_1 is $x^2+y^2+2x-6y-6=0$.

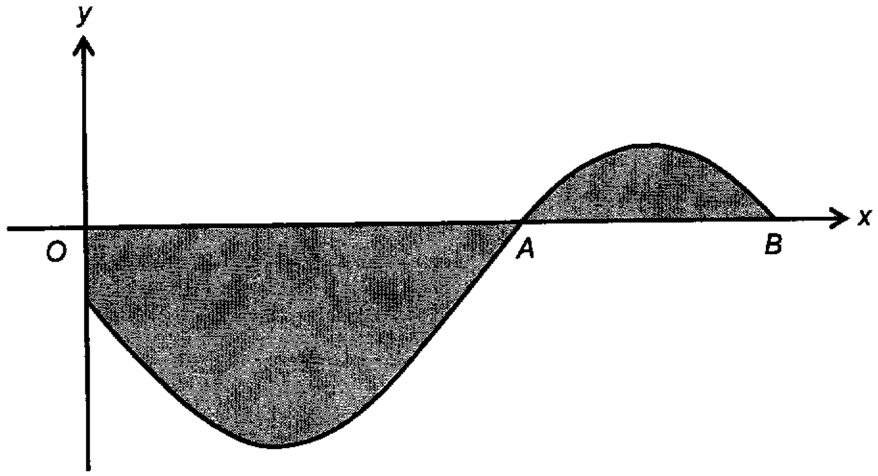
(a) Find the coordinates of the centre and radius of the circle. [4]

(b) Find the value of k, where $k>0$, if $A(-1,k)$ is a point on the circle, C_1. [3]

(c) **(i)** Given that the circle C_2 is a reflection of circle C_1 in the x-axis, state the coordinates of the centre of the circle, C_2? [1]

(ii) Determine whether the point $P(5,\ 0)$ is inside, outside or on the circle C_2. Explain. [2]

10 The diagram shows part of the curve $y = 2\sin(2x + \pi) - 1$, meeting the x-axis at the points A and B.



(a) Show the exact value of x-coordinate of A is $\dfrac{7\pi}{12}$. [3]

(b) State the exact value of x-coordinate of B. [1]

(c) Find the total area of the shaded regions. [4]

End of Paper 2

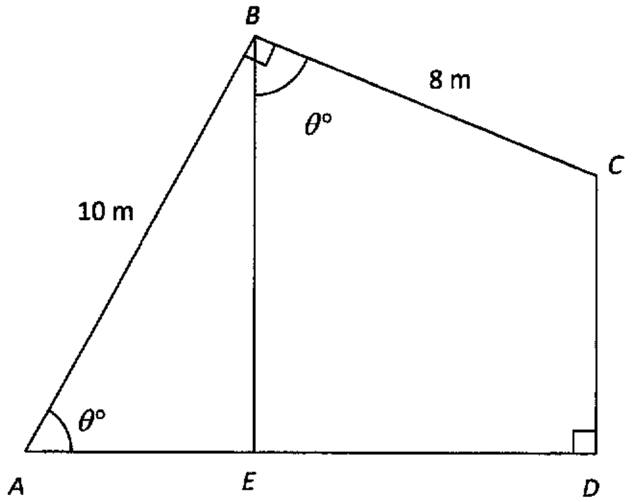
Marking Scheme for 2024 4E5N A.Math Prelim Paper 2

Qn				
1	(a)	Let $f(x) = 2x^3 + 3x^2 - 8x + 3$. By the Remainder Theorem, $f(-1) = p$ $p = 2(-1)^3 + 3(-1)^2 - 8(-1) + 3$ $p = -2 + 3 + 8 + 3$ $p = 12$	M1 A1	[2]
	(b)	$(2x-1)$ is a factor of $f(x)$ [or (x+3) or (x-1) is a factor] $2x^3 + 3x^2 - 8x + 3 = 0$ $(2x-1)(x^2 + 2x - 3) = 0$ $(2x-1)(x+3)(x-1) = 0$ $2x - 1 = 0$ xo $x + 3 = 0$ or $x - 1 = 0$ $2x = 1$ $x = -3$ $x = 1$ $x = \frac{1}{2}$ $x = -3, \frac{1}{2}$ or 1	M1 M1 [another quad expression] A1 [3 ans]	[3]
	(c)	$2(x-1)^3 + 3(x-1)^2 - 8x + 11 = 0$ $2(x-1)^3 + 3(x-1)^2 - 8(x-1) + 3 = 0$ Let $y = x - 1$. $2y^3 + 3y^2 - 8y + 3 = 0$ $y = -3, \frac{1}{2}$ or 1 $x - 1 = -3, \frac{1}{2}$ or 1 $x = -2, \frac{3}{2}$ or 2 Hence, $x = -2, \frac{3}{2}$ or 2.	M1 A1	[2]
2	(a)	\$120 000	B1	[1]

	(b)	(i) Value after 1 year $= 120000e^{-12a}$ $90000 = 120000e^{-12a}$ $e^{-12a} = 0.75$ $-12a = \ln 0.75$ $a = \dfrac{\ln 0.75}{-12} = 0.0239735$ $= 0.02397$ *shown*	M1 M1 must show 0.0239735	[2]
		(ii) $70000 = 120000e^{-at}$ $\dfrac{7}{12} = e^{-at}$ $-0.02397t = \ln\dfrac{7}{12}$ $t = 22.49$ ≈ 23 months	M1 M1 A1	[3]
		(iii) 5 years = 60 months Value after 5 years $= 120000e^{-60(0.02397)}$ $= \$28482.55$ Yes, since car dealer is paying more ($29000). or No, there is not much difference, so I would rather use the car.	M1 A1	[2]
3	(a)	$\cos 75° = \cos(45° + 30°)$ $= \cos 45° \cos 30° - \sin 45° \sin 30°$ $= \left(\dfrac{\sqrt{2}}{2}\right)\left(\dfrac{\sqrt{3}}{2}\right) - \left(\dfrac{\sqrt{2}}{2}\right)\left(\dfrac{1}{2}\right)$ $= \dfrac{\sqrt{6} - \sqrt{2}}{4}$ (Shown)	M1[use 45+30] M1 [any one -surd form]] M1 [simplify to desired ans]	[3]
	(b)	LHS $= \dfrac{\dfrac{\sin x}{\cos x} + \dfrac{\cos x}{\sin x}}{\dfrac{1}{\cos x} + \dfrac{1}{\sin x}}$ $= \dfrac{\sin^2 x + \cos^2 x}{\sin x \cos x} \times \dfrac{\sin x \cos x}{\cos x + \sin x}$ $= \dfrac{1}{\cos x + \sin x}$ $=$ RHS	M1 use identities M1 [same denominator] M1 simplify	[6]

		$\frac{1}{\cos x+\sin x}=\frac{3}{4\cos x-3\sin x}$ $3\cos x+3\sin x=4\cos x-3\sin x$ $6\sin x=\cos x$ $\tan x=\frac{1}{6}$ $\alpha=0.165148677$ $x=0.165,\ 3.31$	M1 cross multiply M1 [change to tan] A1	
4	(a)	$AB=\sqrt{196-x^2},\ BC=2x$ $A=\frac{1}{2}(x+2x)\left(\sqrt{196-x^2}\right)$ $=\frac{3x}{2}\left(\sqrt{196-x^2}\right)$	B1 [$AB=\sqrt{196-x^2}$] M1 formula with h=3x	[2]
	(b)	$\frac{dA}{dx}=\frac{3x}{2}\left(-\frac{2x}{2\sqrt{196-x^2}}\right)+\frac{3\sqrt{196-x^2}}{2}$ $=\frac{3}{2}\left(-\frac{x^2}{\sqrt{196-x^2}}+\sqrt{196-x^2}\right)$ $=\frac{3}{2}\left(\frac{196-x^2-x^2}{\sqrt{196-x^2}}\right)$ $=\frac{3(98-x^2)}{\sqrt{196-x^2}}$ $\frac{dA}{dx}=0$ $\frac{3(98-x^2)}{\sqrt{196-x^2}}=0$ $x=\sqrt{98}$ $x\approx 9.90$ $\frac{d^2A}{dx_2}=\frac{3\sqrt{196-x^2}(-2x)-3(98-x^2)(\frac{1}{2})(196-x^2)^{-\frac{1}{2}}(-2x)}{196-x^2}$ $\frac{d^2A}{dx^2}<0$, *A* is max. [or Using first derivative test]	M1 M1 A1 M1 A1	[5]

	(c)	$A=\frac{3\sqrt{98}}{2}\left(\sqrt{196-98}\right)$ $=147\text{ cm}^2$	M1 A1	[2]
5	(a)	$2^{3+2x}+2^{5+x}=2^x+4$ $2^3\times\left(2^x\right)^2+2^5\times 2^x=2^x+4$ $8\left(2^x\right)^2+32\left(2^x\right)=2^x+4$ Let $y=2^x$. $8y^2+32y=y+4$ $8y^2+31y-4=0$ $(8y-1)(y+4)=0$ $8y-1=0$ or $y+4=0$ $8y=1$ $\quad y=-4$ $y=\frac{1}{8}$ $\quad 2^x=-4$ $2^x=\frac{1}{8}$ $2^x=2^{-3}$ $x=-3$ Since $2^x>0$, $x=-3$.	M1 [$8\left(2^x\right)^2$ seen] M1 [$32\left(2^x\right)$seen] M1 factorise A1 for both y A1	[5]
	(b)	$\log_a\sqrt{ab^3}=\frac{\log_2\sqrt{ab^3}}{\log_2 a}$ $=\frac{\frac{1}{2}[\log_2(ab^3)]}{\log_2 a}$ $=\frac{\log_2(a)+3\log_2(b)}{2(\log_2 a)}$ $=\frac{x+3y}{2x}$	M1 [change base 2] M1 [$\log_2(a)+3\log_2(b)$] A1	[3]

	(c)	$\log_5 50 + 4\log_{25} y = \log_5(2y+4)+2$ $\log_5 50 + 4\dfrac{\log_5 y}{\log_5 25} = \log_5(2y+4)+\log_5 5^2$ $\log_5 50 + 2\log_5 y = \log_5 25(2y+4)$ $\log_5 50y^2 = \log_5 25(2y+4)$ $50y^2 = 50y+100$ $y^2 - y - 2 = 0$ $(y-2)(y+1)=0$ $y=2$ or -1 (reject)	M1$[\dfrac{\log_5 y}{\log_5 25}]$ M1[$2=\log_5 5^2$] M1 use product law A1	[4]
6	(a)	 $AE = 10\cos\theta$ $\sin\theta = \dfrac{ED}{8}$ $ED = 8\sin\theta$ $AD = 10\cos\theta + 8\sin\theta$ (shown)	Use trigo ratio M1 M1	[2]
	(b)	$10\cos\theta + 8\sin\theta = R\cos(\theta-\alpha)$ $R = \sqrt{8^2+10^2}$ $= \sqrt{164}$ or 12.8 $\alpha = \tan^{-1}\left(\dfrac{8}{10}\right)$ $= 38.66$ $\approx 38.7°$ $10\cos\theta + 8\sin\theta = \sqrt{164}\cos(\theta - 38.7)$	M1 M1 A1	[3]

	(c)	$\sqrt{164}\cos(\theta-38.7)=12$ $\cos(\theta-38.7)=\frac{12}{\sqrt{164}}$ basic $\angle=20.4$ $\theta-38.7=20.4,\ -20.4$ $\theta=59.1,\ 18.3$ (1 decimal place)	M1 M1 A1 both	[3]
7	(a)	Show that $\frac{d}{dx}\{x(3x-1)^{\frac{5}{3}}\}=(8x-1)(3x-1)^{\frac{2}{3}}$. $\frac{d}{dx}\{x(3x-1)^{\frac{5}{3}}\}$ $=x\left(\frac{5}{3}\right)(3x-1)^{2/3}(3)+(3x-1)^{5/3}$ $=5x(3x-1)^{2/3}+(3x-1)^{5/3}$ $=(3x-1)^{2/3}[5x+3x-1]$ $=(3x-1)^{2/3}[8x-1]$	B1 for $x\left(\frac{5}{3}\right)(3x-1)^{2/3}$ seen B1 for (3) seen B1 for $(3x-1)^{5/3}$ seen M1 for factorization A1 for showing clear working leading to the desired expression	[5]
	(b)	$\int(8x-1)(3x-1)^{\frac{2}{3}}\,dx=x(3x-1)^{\frac{5}{3}}+c$ $\int 8x(3x-1)^{\frac{2}{3}}\,dx-\int(3x-1)^{\frac{2}{3}}\,dx=x(3x-1)^{\frac{5}{3}}+c$ $\int 8x(3x-1)^{\frac{2}{3}}\,dx=x(3x-1)^{\frac{5}{3}}+\left(\frac{(3x-1)^{\frac{5}{3}}}{\frac{5}{3}(3)}\right)+c$ $\int 8x(3x-1)^{\frac{2}{3}}\,dx=x(3x-1)^{\frac{5}{3}}+\left(\frac{(3x-1)^{\frac{5}{3}}}{5}\right)+c$ $\int x(3x-1)^{\frac{2}{3}}\,dx=\frac{1}{8}x(3x-1)^{\frac{5}{3}}+\frac{1}{40}(3x-1)^{\frac{5}{3}}+c$ $=\left(\frac{1}{8}x+\frac{1}{40}\right)(3x-1)^{\frac{5}{3}}+c$	M1 M1 M1 $\left(\frac{(3x-1)^{\frac{5}{3}}}{\frac{5}{3}(3)}\right)$ A1 [1/8 seen] A1 [1/40 seen]	[5]

8	(a)	(i) From the diagram, by similar triangles, $\frac{r}{h}=\frac{2}{4}$ $4r=2h$ $r=\frac{h}{2}$ (shown)	M1 M1	[2]
		(ii) From (a)(i), $V=\frac{1}{3}\pi r^2 h$ $=\frac{1}{3}\pi\left(\frac{h}{2}\right)^2 h$ $=\frac{1}{3}\pi\left(\frac{h^2}{4}\right)h$ $=\frac{\pi}{12}h^3$ (shown)	M1 M1	[2]
	(b)	$V=\frac{\pi}{12}h^3$ $\frac{dV}{dh}=\frac{\pi}{4}h^2$ Given: $\frac{dV}{dt}=-2\text{m}^3/\text{min}$ When $h=2$, $\frac{dV}{dt}=\frac{dV}{dh}\times\frac{dh}{dt}$ $-2=\frac{\pi}{4}(2)^2\times\frac{dh}{dt}$ $-2=\pi\times\frac{dh}{dt}$ $\frac{dh}{dt}=\frac{-2}{\pi}$ $=-0.637\text{m/min}$ (correct to 3 sig. fig.)	M1 B1 M1 M1 A1	[5]
9	(a)	$x^2+y^2+2x-6y-6=0$ Radius $=\sqrt{1^2+(-3)^2-(-6)}$ $=\sqrt{16}$ $=4\text{ cm}$ $\therefore$ Centre is $(-1,3)$	M1A1 M1A1 [or A2]	[4]

	(b)	$(-1)^2+k^2+2(-1)-6k-6=0$ $k^2-6k-7=0$ $(k-7)(k+1)=0$ $k=7$ or $k=-1$ (NA)	M1 M1 A1	[3]
	(c)	(i) Coordinates of *centre* C_2 = (-1, -3).	B1	[1]
		(ii) $\sqrt{(5+1)^2+(3+0)^2}$ $=\sqrt{45}=6.71>4=radius$ P(5,0) lies outside the circle C_2.	M1 A1	[2]
10	(a)	$2\sin(2x+\pi)-1=0$ $\sin(2x+\pi)=\frac{1}{2}$ $\alpha=\sin^{-1}\frac{1}{2}$ $=\frac{\pi}{6}$ $2x+\pi=\frac{\pi}{6},\ \pi-\frac{\pi}{6},\ \frac{\pi}{6}+2\pi,\ \pi-\frac{\pi}{6}+2\pi$ $x=-\frac{5\pi}{12},\ -\frac{\pi}{12},\ \frac{7\pi}{12},\ \frac{11\pi}{12}$ x coordinate of $A=\frac{7\pi}{12}$	M1 M1 A1	[3]
	(b)	x coordinate of $B=\frac{11\pi}{12}$	B1	[1]
	(c)	Shaded area $=-\int_0^{\frac{7\pi}{12}}2\sin(2x+\pi)-1\,dx+\int_{\frac{7\pi}{12}}^{\frac{11\pi}{12}}2\sin(2x+\pi)-1\,dx$ $=-\left[-\cos(2x+\pi)-x\right]_0^{\frac{7\pi}{12}}+\left[-\cos(2x+\pi)-x\right]_{\frac{7\pi}{12}}^{\frac{11\pi}{12}}$ $=-\left[-2.69862-1\right]+\left[-2.01377-(-2.69862)\right]$ $=3.69862+0.684853$ $=4.38$ unit^2	M1 M1 M1 A1	[4]

Total = 90 m