AHMAD IBRAHIM SECONDARY SCHOOL
GCE O-LEVEL PRELIMINARY EXAMINATION 2025

SECONDARY 4 EXPRESS

Name:	Class:	Register No.:

ADDITIONAL MATHEMATICS **4049/01**
Paper 1 **12 August 2025**

Candidates answer on the Question Paper. **2 hours 15 minutes**

READ THESE INSTRUCTIONS FIRST

Write your name, class and index number on all the work you hand in.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** questions.
Give non-exact numerical answers to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 90.

For Examiner's Use
/90

This document consists of **19** printed pages.

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1} a^{n-1} b + \binom{n}{2} a^{n-2} b^2 + \ldots + \binom{n}{r} a^{n-r} b^r + \ldots + b^n$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$
$$\sec^2 A = 1 + \tan^2 A$$
$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$
$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$
$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$
$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$
$$\sin 2A = 2 \sin A \cos A$$
$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$
$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$
$$\Delta = \frac{1}{2} bc \sin A$$

1 It is given that P, Q and R are the angles of a triangle.

(a) Show that $\cos P = -\cos(Q + R)$. [2]

(b) Given that $Q = 45°$ and $R = 60°$, find $\cos P$ in the form $\frac{1}{4}(\sqrt{a} - \sqrt{b})$, where a and b are integers. [3]

2 Baking powder is poured onto a flat surface at a constant rate of 2π cm^{3}s^{-1} and formed a right circular cone. The radius of the cone is always $\dfrac{1}{18}$ of its height. Find the rate of change of the radius of the cone after 3 seconds of pouring. [5]

3 **(a)** Determine the set of values of m for which the equation $2x^2+4x+2m=6mx-2$ has real roots. [4]

(b) Hence state what can be deduced about the curve $y=2(x+1)^2$ and the line $y=6x-2$. Justify your statement. [2]

4 **(a)** Show that $\frac{\mathrm{d}}{\mathrm{d}x}\left(\ln\left(\cos x\right)\right) = -\tan x$. [2]

(b) Differentiate $x\tan x$ with respect to x. [2]

(c) Using the results from part **(a)** and **(b)**, find $\int x\sec^2 x \, \mathrm{d}x$ and hence show that $\int_0^{\frac{\pi}{4}} x\sec^2 x \, \mathrm{d}x = \frac{\pi}{4} - \frac{1}{2}\ln 2$. [4]

5 **(a)** In the expansion of $(2+x)^n$, where n is a positive integer, the coefficient of x^2 is twice the coefficient of x. Find the value of n. [3]

(b) Find the value of the term that is independent of x in the expansion of $\left(2x-\frac{1}{4x^4}\right)^{15}$. [4]

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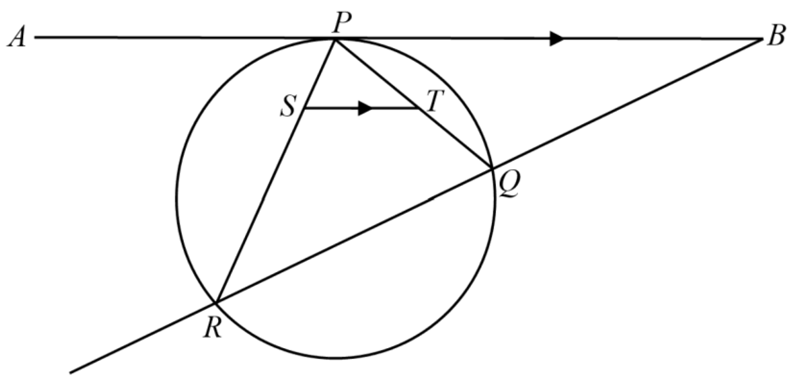

The diagram shows a circle passing through the points P, Q and R. The point Q lies on the line RB. AB is a tangent to the circle at P. The points S and T lie on PR and PQ respectively. Given that AB is parallel to ST, prove that

(a) triangle PST is similar to triangle PQR, [3]

(b) $PQ \times PT = PR \times PS$, [2]

(c) Determine if *STQR* is a cyclic quadrilateral. [4]

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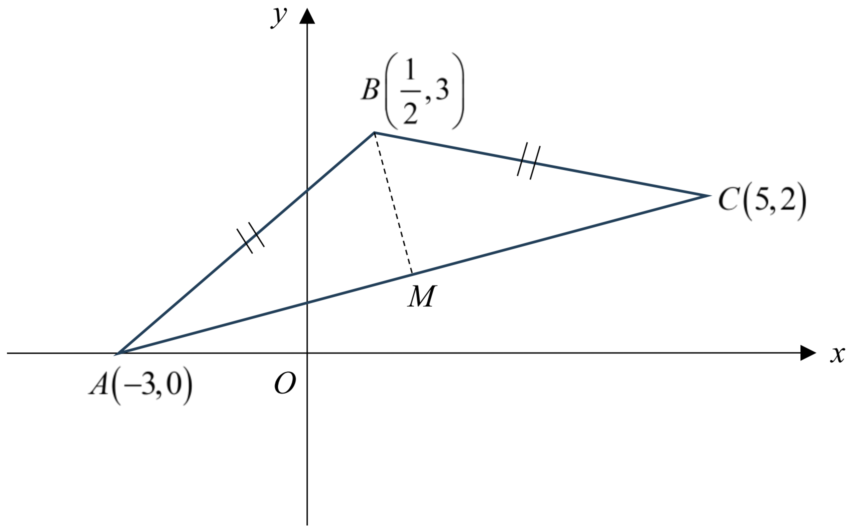

The diagram shows an isosceles triangle *ABC* in which $A(-3,0)$, $B\left(\frac{1}{2},3\right)$ and $C(5,2)$. *M* is the foot of perpendicular from *B* to *AC*.

(a) Find the coordinates of *M*. [1]

(b) Find the equation of the perpendicular bisector of *AC*. [2]

(c) Given that $ABCD$ is a kite with $BM = \frac{2}{7}BD$, find the coordinates of D. [3]

(d) Find the area of the kite $ABCD$. [2]

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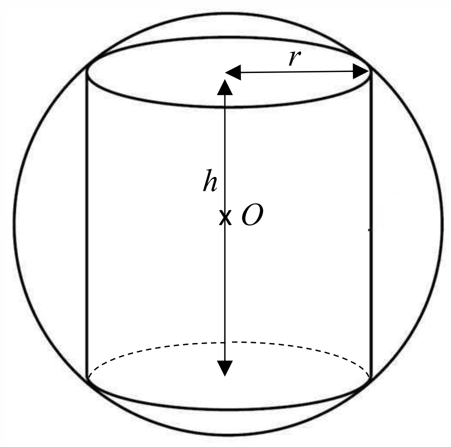

A prototype consists of a cylindrical container of height h cm and radius r cm inscribed in a hollow sphere with centre O.

The sphere has a surface area of 6400π cm^2 and both the sphere and container have negligible thickness.

(a) Show that the volume of the cylinder container, V cm^3, is given by [3]

$$V = 2\pi r^2\sqrt{1600 - r^2}\ .$$

(b) Given that r can vary, find the value of r for which the volume V is stationary. [5]

(c) A scientist plans to launch this prototype into outer space carrying as much fuel as possible. Explain whether the prototype can satisfy the scientist's requirement. [2]

9 It is given that $\mathrm{f}(x) = 4 + \cos\left(\frac{x}{2}\right)$ and $\mathrm{g}(x) = -2\sin x$.

(a) State the period and amplitude of $\mathrm{f}(x)$. [2]

(b) State the period and amplitude of $\mathrm{g}(x)$. [1]

(c) Sketch, on the same axes, the graphs of $y = \mathrm{f}(x)$ and $y = \mathrm{g}(x)$ for $0° \leq x \leq 360°$. [3]

10 **(a)** Express $\dfrac{1-3x-3x^2}{x(x+1)^2}$ in partial fractions. [5]

(b) Hence find $\displaystyle\int \frac{1-3x-3x^2}{2x(x+1)^2}\,dx$. [4]

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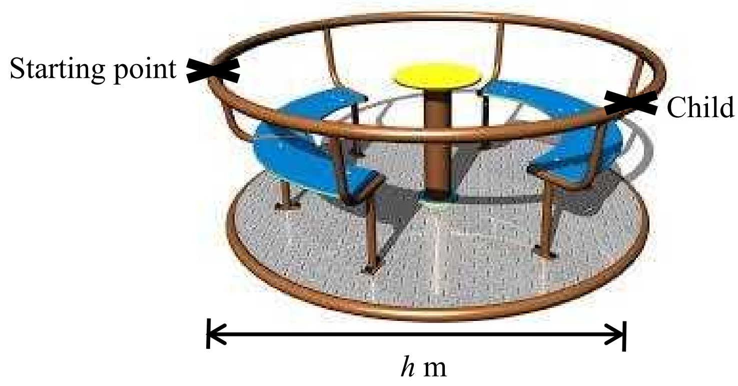

The horizontal distance of a child on a carousel, h m, from the starting point is modelled by the equation, $h = 2(1 - \cos kt)$, where k is a constant and t is the time in seconds after the child leaves the starting point. The time to complete one revolution is 20 seconds.

(a) Explain why this model suggests that the diameter of the carousel is 4 m. [1]

(b) Show that the value of k is $\dfrac{\pi}{10}$ radians per second. [2]

(c) As the carousel turns, it is possible for the child on the carousel to view a landmark, provided that the horizontal distance of the child is within 1 m from the starting point.

Find the duration of time for which the child will not be able to view the landmark during one revolution. [5]

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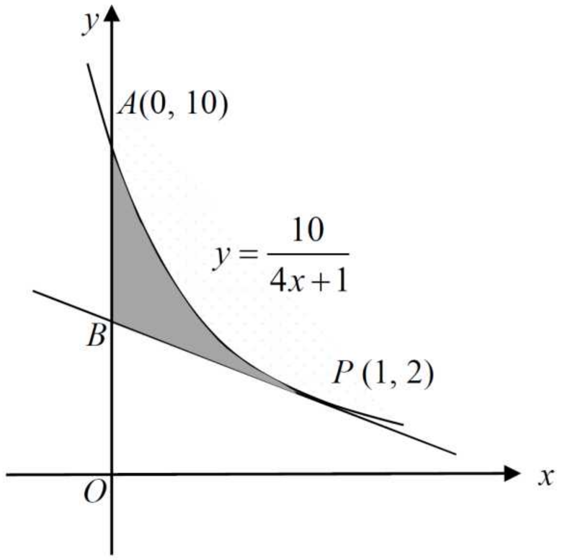

The diagram shows part of the curve $y = \dfrac{10}{4x+1}$ intersecting the y-axis at $A(0, 10)$. The tangent to the curve at the point $P(1, 2)$ intersects the y-axis at B.

(a) Show that the coordinates of B is (0, 3.6). [4]

(b) Find the **exact** area of the shaded region. [5]

END OF PAPER

1 It is given that P, Q and R are the angles of a triangle.

(a) Show that $\cos P = -\cos(Q + R)$. [2]

$\cos P$	$= \cos[180° - (Q + R)]$	[M1 – for replacing]
	$= \cos 180° \cos(Q + R) + \sin 180° \sin(Q + R)$	
	$= -\cos(Q + R) + 0$	[A1 – apply addition formula &
	$= -\cos(Q + R)$	[evaluate to arrive at result]

Accept supplementary angles:
$\cos P = -\cos(180° - P)$ M1
$= -\cos(Q + R)$ A1

(b) Given that $Q = 45°$ and $R = 60°$, find $\cos P$ in the form $\frac{1}{4}(\sqrt{a} - \sqrt{b})$, where a and b are integers. [3]

$\cos P$	$= -\cos(45° + 60°)$	
	$= -\cos 45° \cos 60° + \sin 45° \sin 60°$	[M1 – correct use of formula expansion]
	$= -\frac{\sqrt{2}}{2}(\frac{1}{2}) + \frac{\sqrt{2}}{2}(\frac{\sqrt{3}}{2})$	[M1 – correct special angles trigo ratios]
	$= \frac{1}{4}(\sqrt{6} - \sqrt{2})$	[A1]

2 Baking powder is poured onto a flat surface at a constant rate of 2π cm^{3}s^{-1} and formed a right circular cone. The radius of the cone is always $\frac{1}{18}$ of its height. Find the rate of change of the radius of the cone after 3 seconds of pouring. [5]

Vol. of cone, $V = \frac{1}{3}\pi r^2 (18r)$

$= 6\pi r^3$

$\frac{\mathrm{d}V}{\mathrm{d}r} = 18\pi r^2$ [B1]

After 3 seconds, $V = 6\pi$

$6\pi r^3 = 6\pi$ [M1 - finding corresponding r]

$r = 1$ [A1]

$\frac{\mathrm{d}V}{\mathrm{d}t} = \left.\frac{\mathrm{d}V}{\mathrm{d}r}\right|_{r=1} \times \left.\frac{\mathrm{d}r}{\mathrm{d}t}\right|_{r=1}$ [M1 - connected rate of change]

$2\pi = 18\pi(1)^2 \times \left.\frac{\mathrm{d}r}{\mathrm{d}t}\right|_{r=1}$

$\left.\frac{\mathrm{d}r}{\mathrm{d}t}\right|_{r=1} = \frac{1}{9}$

The rate of change required is $= \frac{1}{9}$ cm/s. [A1 o.e.]

3 **(a)** Determine the set of values of m for which the equation $2x^2 + 4x + 2m = 6mx - 2$ has real roots. [4]

$$2x^2 + (4-6m)x + 2m + 2 = 0$$

$$b^2 - 4ac \geq 0$$ [M1 - correct Discriminant]

$$(4-6m)^2 - 4(2)(2m+2) \geq 0$$

$$16 - 48m + 36m^2 - 8(2m+2) \geq 0$$

$$36m^2 - 64m \geq 0$$ [M1 – simplification in factors]

$$m(9m-16) \geq 0$$

$$m \leq 0 \quad or \quad m \geq \frac{16}{9}$$ [A2, minus 1 mark if inequality sign is wrong due to earlier wrong D sign]

(b) Hence state what can be deduced about the curve $y = 2(x+1)^2$ and the line $y = 6x - 2$. Justify your statement. [2]

$2x^2 + 4x + 2 = 6x - 2$

By comparing with (a), $m = 1$ [B1 – correct m value]

<u>When $m = 1$, it is not within the set of values of m for which there will be real roots</u>, <u>hence the curve will not meet the line/ the curve will not cut the line</u>. [B1]

4 **(a)** Show that $\frac{\mathrm{d}}{\mathrm{d}x}\left(\ln\left(\cos x\right)\right) = -\tan x$. [2]

$\frac{\mathrm{d}}{\mathrm{d}x}\left(\ln\left(\cos x\right)\right) = \frac{1}{\cos x} \times \frac{\mathrm{d}}{\mathrm{d}x}\left(\cos x\right)$	[M1 – show working]
$= \frac{-\sin x}{\cos x} = -\tan x$	[A1 – show fraction]

(b) Differentiate $x\tan x$ with respect to x. [2]

$\frac{\mathrm{d}}{\mathrm{d}x}x\tan x = x\sec^2 x + \tan x$	[M1 – show product rule]
	[A1 – correct ans for both]

(c) Using the results from part **(a)** and **(b)**, find $\int x\sec^2 x \ \mathrm{d}x$ and hence show that $\int_0^{\frac{\pi}{4}} x\sec^2 x \ \mathrm{d}x = \frac{\pi}{4} - \frac{1}{2}\ln 2$. [4]

From (b), $\int\left(x\sec^2 x + \tan x\right) \ \mathrm{d}x = x\tan x + C$	[M1 – use part (b), ‘C’ must be seen]
$\int x\sec^2 x \ \mathrm{d}x + \int \tan x \ \mathrm{d}x = x\tan x + C$	
$\int x\sec^2 x \ \mathrm{d}x = x\tan x - \int \tan x \ \mathrm{d}x + C$	[M1 – proper integration and final correct ans, ‘C’ must be seen]
$= x\tan x + \ln\left(\cos x\right) + C$	
$\int_0^{\frac{\pi}{4}} x\sec^2 x \ \mathrm{d}x = \left[x\tan x + \ln\left(\cos x\right)\right]_0^{\frac{\pi}{4}}$	[minus 1 mark if “C” is not seen]
$= \frac{\pi}{4}\tan\frac{\pi}{4} + \ln\left(\cos\frac{\pi}{4}\right) - 0 - \ln 1$	
$= \frac{\pi}{4} + \ln\left(\frac{1}{\sqrt{2}}\right)$	[M1 – proper evaluation]
$= \frac{\pi}{4} + \ln 2^{-\frac{1}{2}}$	
$= \frac{\pi}{4} - \frac{1}{2}\ln 2$	[A1]

5 **(a)** In the expansion of $(2+x)^n$, where n is a positive integer, the coefficient of x^2 is twice the coefficient of x. Find the value of n. [3]

$$(2+x)^n = 2^n + \binom{n}{1}2^{n-1}x + \binom{n}{2}2^{n-2}x^2 + \ldots$$

$$\binom{n}{2}2^{n-2} = 2\binom{n}{1}2^{n-1}$$ [M1 – correct coeff]

$$\frac{n(n-1)}{2}\left(2^{n-2}\right) = 2n\left(2^{n-1}\right)$$

$$\frac{n(n-1)}{2}\left(2^n \times 2^{-2}\right) = 2n\left(2^n \times 2^{-1}\right) \quad (n \neq 0)$$ [M1 – simplify]

$$\frac{n-1}{2(4)} = 1$$

$$n-1 = 8$$

$$n = 9$$ [A1 – with rejection]

(b) Find the value of the term that is independent of x in the expansion of $\left(2x - \frac{1}{4x^4}\right)^{15}$. [4]

general term

$$= \binom{15}{r}(2x)^{15-r}\left(-\frac{1}{4x^4}\right)^r$$

$$= \binom{15}{r}2^{15-r}x^{15-r}\left(-\frac{1}{4}\right)^r x^{-4r}$$ [M1]

$$= \binom{15}{r}2^{15-r}\left(-\frac{1}{4}\right)^r x^{15-5r}$$ [M1 - gather x and let power = 0]

$$15 - 5r = 0$$

$$r = 3$$ [M1 for r value]

$$\text{value} = \binom{15}{3}2^{12}\left(-\frac{1}{4}\right)^3 = -29120$$ [A1]

Max 2 marks if able to gather 15 – 5r for exponent of 'x' and equate to zero with correct r value.

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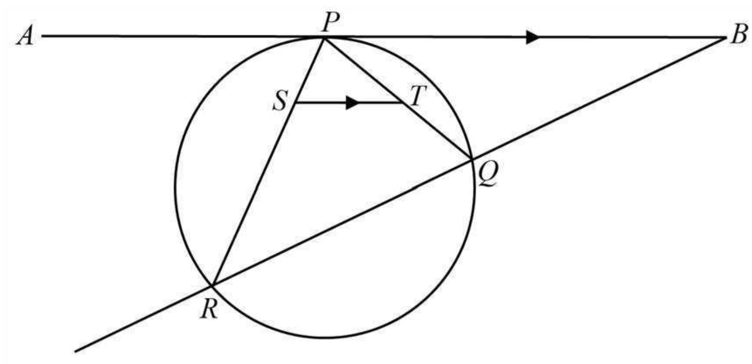

The diagram shows a circle passing through the points P, Q and R. The point Q lies on the line RB. AB is a tangent to the circle at P. The points S and T lie on PR and PQ respectively. Given that AB is parallel to ST, prove that

(a) triangle PST is similar to triangle PQR, [3]

$\angle SPT = \angle QPR$ (common angle) [M1]

$\angle PST = \angle SPA$ (alt. $\angle$s, AB // ST) **OR** $\angle PTS = \angle TPB$ (alt. $\angle$s, AB // ST)

$= \angle PQR$ (Alt. Segment Thm) $= \angle PRQ$ (Alt. Segment Thm) [M1]

$\therefore$ triangle PST is similar to triangle PQR. [A1]

(2 pairs of corresponding angles are equal)

(b) $PQ \times PT = PR \times PS$, [2]

From above result, $\dfrac{PQ}{PS} = \dfrac{PR}{PT}$ [M1]

$\Rightarrow$ $PQ \times PT = PR \times PS$ [A1]

(c) Determine if $STQR$ is a cyclic quadrilateral. [4]

$QTS = 180° - \angle PTS$ (adj. $\angle$s on a st. line)

$= 180° - \angle QRS$ (from (a) result) [M1]

So $\angle QTS + \angle QRS = 180°$ [M1]

$\angle RST + \angle RQT = 360° - (\angle QTS + \angle QRS)$ ($\angle$ sum of a quadrilateral)

$= 360° - 180°$

$= 180°$ [M1]

By converse of angles in opposite segment, $STQR$ is a cyclic quadrilateral and all four vertices lie on the circumference of a circle.

[A1 – with correct reason, accept even if no mention of four vertices]

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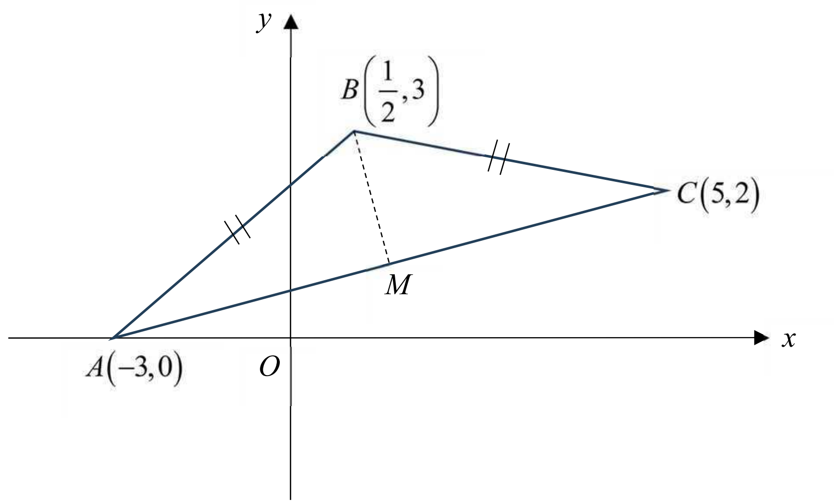


The diagram shows an isosceles triangle ABC in which $A(-3,0)$, $B\left(\frac{1}{2},3\right)$ and $C(5,2)$. M is the foot of perpendicular from B to AC.

(a) Find the coordinates of M. [1]

$M = \left(\frac{-3+5}{2}, \frac{0+2}{2}\right) = (1, 1)$ [B1]

(b) Find the equation of the perpendicular bisector of AC. [2]

Gradient of $AC = \frac{2-0}{5-(-3)} = \frac{1}{4}$

Gradient of perpendicular BM = – 4 [M1 – use $m \times m_{\perp} = -1$]

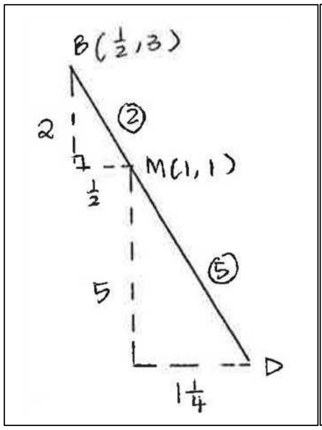
or $m_{BM} = \frac{3-1}{\frac{1}{2}-1} = -4$

Equation of perpendicular bisector:

$y-1=-4(x-1)$

$y=-4x+5$ [A1]

(c) Given that $ABCD$ is a kite with $BM = \frac{2}{7}BD$, find the coordinates of D. [3]



Correct ratio: 2 : 5

$\frac{1}{2} : 1\frac{1}{4}$ M1

Coordinates of D

$= \left(1+\frac{1}{4}, 1-5\right)$ M1

$= \left(2\frac{1}{4}, -4\right)$ A1

Alternative: using vectors

$\overrightarrow{BM} = \frac{2}{7}\overrightarrow{BD}$

$\begin{pmatrix} 0.5 \\ -2 \end{pmatrix} = \frac{2}{7}\left(\overrightarrow{OD} - \overrightarrow{OB}\right)$ [M1 - position vectors]

$\overrightarrow{OD} = 3.5\begin{pmatrix} 0.5 \\ -2 \end{pmatrix} + \begin{pmatrix} 0.5 \\ 3 \end{pmatrix} = \begin{pmatrix} 2.25 \\ -4 \end{pmatrix}$ [M1]

D$=\left(2\frac{1}{4}, -4\right)$ [A1]

(d) Find the area of the kite $ABCD$. [2]

Area of $ABCD = \frac{1}{2}\begin{vmatrix} -3 & 2\frac{1}{4} & 5 & \frac{1}{2} & -3 \\ 0 & -4 & 2 & 3 & 0 \end{vmatrix}$

M1 (ecf 1 – correct method)

$= \frac{1}{2}(31.5 + 28)$ (anti-clockwise)

$= 29.75$ units^2 (or $\frac{119}{4}$) A1

(accept 29.7 with 3 s.f. if using other method such as Pythagoras'

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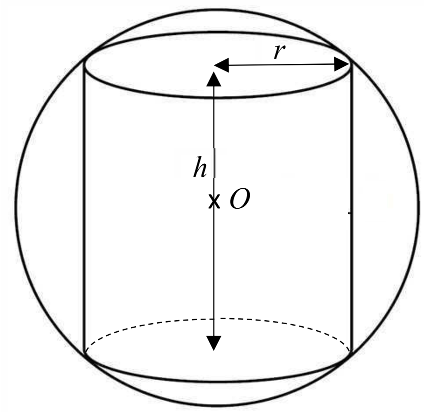


A prototype consists of a cylindrical container of height h cm and radius r cm inscribed in a hollow sphere with centre O.

The sphere has a surface area of 6400π cm^2 and both the sphere and container have negligible thickness.

(a) Show that the volume of the cylinder container, V cm^3, is given by [3]

$$V = 2\pi r^2\sqrt{1600 - r^2}\ .$$

$4\pi R^2 = 6400\pi$ [M1 – find radius R]

$R = 40$ (radius R of sphere)

By Pythagoras' Theorem,

$r^2 + \left(\dfrac{h}{2}\right)^2 = 40^2$ [M1]

$h^2 = 4\left(1600 - r^2\right)$

$h = \sqrt{4\left(1600 - r^2\right)}$

$h = 2\sqrt{1600 - r^2}$

$V = \pi r^2 h$

$\quad = 2\pi r^2\sqrt{1600 - r^2}$ [A1]

(b) Given that r can vary, find the value of r for which the volume V is stationary. [5]

$$\frac{dV}{dr}=(2\pi r^2)\times\frac{1}{2}\left(1600-r^2\right)^{-\frac{1}{2}}\times(-2r)+\sqrt{1600-r^2}\times 2\times 2\pi r$$ [M2 – correct differentiation using product rule, deduct M1 if either term is incorrect]

For stationary values, $\frac{dV}{dr}=0$

$$\boxed{\frac{dV}{dr}=4\pi r\sqrt{1600-r^2}-\frac{2\pi r^3}{\sqrt{1600-r^2}}}$$

$$4\pi r\sqrt{1600-r^2}-\frac{2\pi r^3}{\sqrt{1600-r^2}}=0$$ [M1 – equate to zero]

$$\frac{2\pi r}{\sqrt{1600-r^2}}\left[2\left(1600-r^2\right)-r^2\right]=0$$ [M1 – simplify and show this or the next line working]

$$\frac{2\pi r}{\sqrt{1600-r^2}}\left[3200-3r^2\right]=0$$

$r=0$ or $3200-r^2=0$

(reject) $r=\frac{40\sqrt{6}}{3}=32.7\text{ cm}$ [A1]

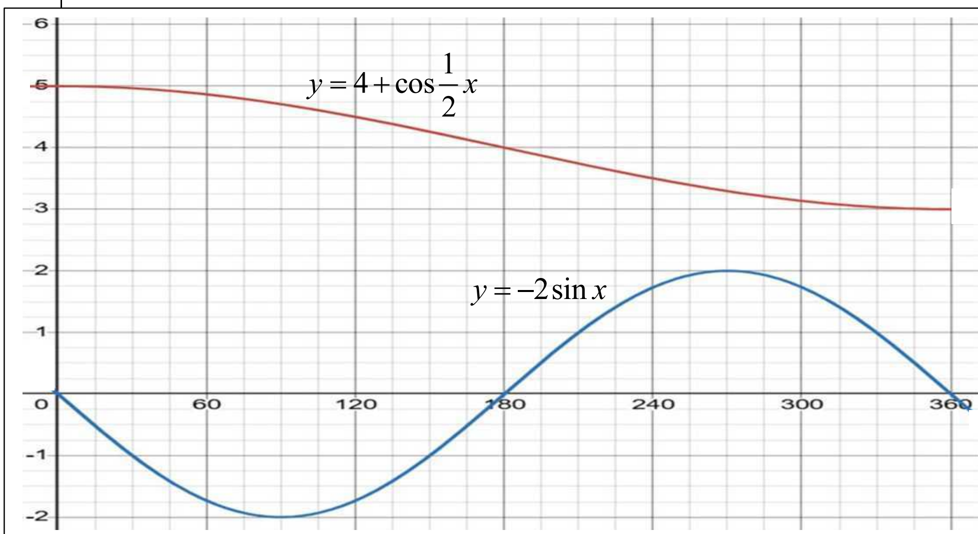
(c) A scientist plans to launch this prototype into outer space carrying as much fuel as possible. Explain whether the prototype can satisfy the scientist's requirement. [2]

x	$\left(\frac{40\sqrt{6}}{3}\right)^-$	$\frac{40\sqrt{6}}{3}$	$\left(\frac{40\sqrt{6}}{3}\right)^+$
$\frac{dV}{dx}$	+	0	–
sketch of tangent	/	—	\

[M1 – first or 2nd derivative, ecf 1 for method]

Since this value of r give maximum V value, the prototype satisfies the scientist's requirement as he could maximise the fuel to be stored in the prototype. [B1]

9	It is given that $f(x) = 4 + \cos\left(\frac{x}{2}\right)$ and $g(x) = -2\sin x$.		
	(a)	State the period and amplitude of $f(x)$.	[2]
		Period = 720° or 4π [B1] Amplitude = 1 [B1]	
	(b)	State the period and amplitude of $g(x)$.	[1]
		Period = 360° or 2π Amplitude = 2 [B1 for both]	
	(c)	Sketch, on the same axes, the graphs of $y = f(x)$ and $y = g(x)$ for $0° \le x \le 360°$.	[3]



	f(x)	g(x)
Shape	cosine	negative sine
Amplitude	1	2
No. of cycle	Half	1
Shift	4	-

[B1 – for starts and ends at ‘zero’ for $g(x)$]

[B1 – for starts ‘5’ and ends at ‘3’ for $f(x)$]

[B1 – fully correct graphs]

10 **(a)** Express $\dfrac{1-3x-3x^2}{x(x+1)^2}$ in partial fractions. [5]

$$\frac{1-3x-3x^2}{x(x+1)^2}=\frac{A}{x}+\frac{B}{x+1}+\frac{C}{(x+1)^2}$$

$1-3x-3x^2=A(x+1)^2+Bx(x+1)+Cx$ [M1]

When $x = -1$, $C=-1$ [M1]

When $x=0$, $A=1$ [M1]

When $x=1$, $B=-4$ [M1]

$$\frac{1-3x-3x^2}{x(x+1)^2}=\frac{1}{x}-\frac{4}{x+1}-\frac{1}{(x+1)^2}$$ [A1]

(b) Hence find $\displaystyle\int\frac{1-3x-3x^2}{2x(x+1)^2}\,dx$. [4]

$$\int\frac{1-3x-3x^2}{2x(x+1)^2}\,dx=\frac{1}{2}\int\frac{1-3x-3x^2}{x(x+1)^2}\,dx$$

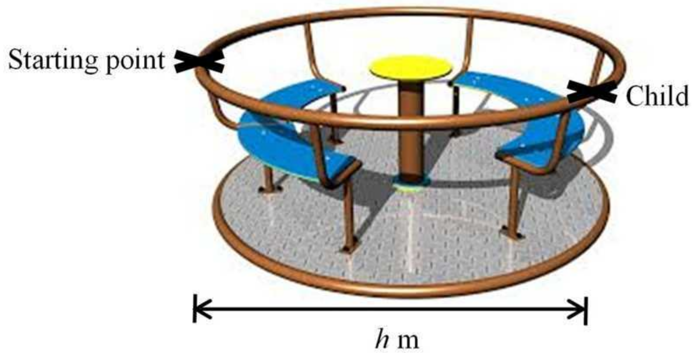
$=\dfrac{1}{2}\displaystyle\int\frac{1}{x}-\frac{4}{x+1}-\frac{1}{(x+1)^2}\,dx$ [M1 – take out 0.5]

$=\dfrac{1}{2}\ln x-2\ln(x+1)+\dfrac{1}{2(x+1)}+C$ [A1for each term, must have constant C]

Accept $=\ln\sqrt{x}-\ln(x+1)^2+\dfrac{1}{2(x+1)}+C$

or $\ln\dfrac{\sqrt{x}}{(x+1)^2}+\dfrac{1}{2(x+1)}+C$

11



The horizontal distance of a child on a carousel, h m, from the starting point is modelled by the equation, $h = 2(1 - \cos kt)$, where k is a constant and t is the time in seconds after the child leaves the starting point. The time to complete one revolution is 20 seconds.

(a) Explain why this model suggests that the diameter of the carousel is 4 m. [1]

$h = 2(1 - \cos kt)$.

Since the diameter of the carousel = max value of h, $h = 2\ (1 - (-1)) = 4$ m

[B1 – must relate diameter to *h, do not accept amplitude method as question ask on the model equation*]

(b) Show that the value of k is $\frac{\pi}{10}$ radians per second. [2]

Period = 20 s

$\frac{2\pi}{k} = 20$ [M1]

$k = \frac{\pi}{10} rad / s$ (*shown*) [A1]

accept
2π rad in 20 s
$\frac{2\pi}{20}$ rad in 1 s
$\frac{2\pi}{20}t$ rad in t s
$k = \frac{\pi}{10}$

accept
$h = 4$ when $t = 10$
to find k value

(c) As the carousel turns, it is possible for the child on the carousel to view a landmark, provided that the horizontal distance of the child is within 1 m from the starting point.

Find the duration of time for which the child will not be able to view the landmark during one revolution. [5]

$1 = 2(1-\cos\frac{\pi}{10}t)$

accept $2(1-\cos\frac{\pi}{10}t) > 1$

[M1]

$\frac{1}{2} = \cos\frac{\pi}{10}t$

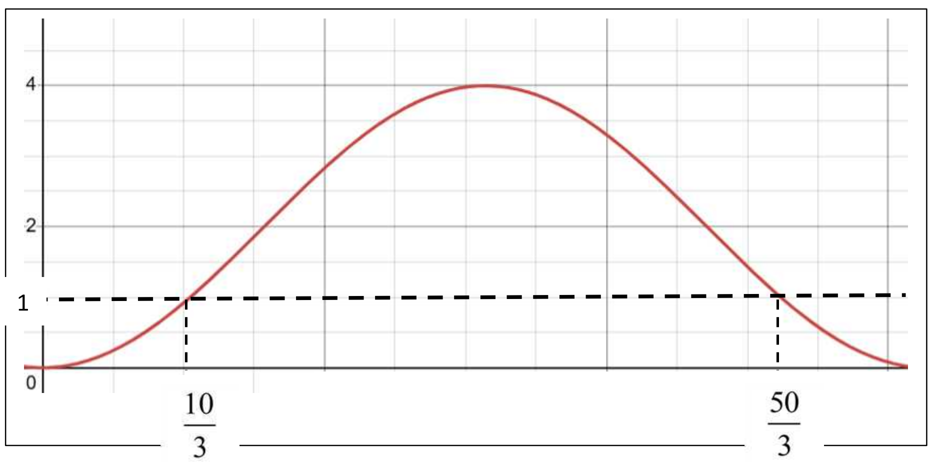
$basic\angle = \frac{\pi}{3}$ (solution in 1st and 4th quad)

$\frac{\pi}{10}t = \frac{\pi}{3}, \frac{5\pi}{3}$ [M1]

$t = \frac{10}{3}, \frac{50}{3}$ [M1]

accept $t > \frac{10}{3}$ $t < \frac{50}{3}$

Not able to view : $\frac{50}{3} - \frac{10}{3} = \frac{40}{3} s$ *or* $13.3\ s\ (3s.f.)$ [M1, A1]



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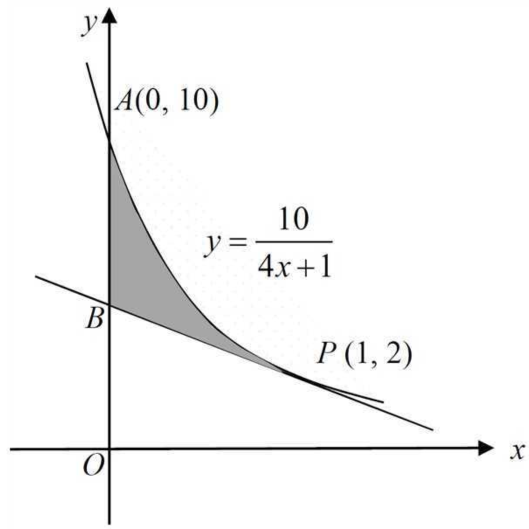


The diagram shows part of the curve $y = \frac{10}{4x+1}$ intersecting the y-axis at $A(0, 10)$. The tangent to the curve at the point $P(1, 2)$ intersects the y-axis at B.

(a) Show that the coordinates of B is (0, 3.6). [4]

$y = \frac{10}{4x+1} = 10(4x+1)^{-1}$

$\frac{dy}{dx} = -10(4x+1)^{-2}(4)$ [M1]

$\frac{dy}{dx} = -40(4x+1)^{-2}$

When $x = 1$ $\frac{dy}{dx} = -40(4(1)+1)^{-2}$

$\frac{dy}{dx} = -1.6$ [M1]

$\frac{y-2}{0-1} = -1.6$ [M1]

$y - 2 = 1.6$

$y = 3.6$

Coordinate of B is (0, 3.6) [A1]

(b) Find the **exact** area of the shaded region. [5]

$Area = \int_0^1 \frac{10}{4x+1}\,dx - \frac{1}{2}(3.6+2)(1)$	[M1], [M1]
$Area = \left[\frac{10\ln(4x+1)}{4}\right]_0^1 - 2.8$	[M1]
$Area = \left[\frac{10\ln(4+1)}{4} - \frac{10\ln(1)}{4}\right] - 2.8$	[M1]
$Area = \frac{5}{2}\ln 5 - 2.8 \text{ unit}^2$	[A1]

END OF PAPER

AHMAD IBRAHIM SECONDARY SCHOOL
GCE O-LEVEL PRELIMINARY EXAMINATION 2025

SECONDARY 4 EXPRESS

Name:	Class:	Register No.:

ADDITIONAL MATHEMATICS **4049/01**
Paper 2 **13 August 2025**

Candidates answer on the Question Paper. **2 hours 15 minutes**

READ THESE INSTRUCTIONS FIRST

Write your name, class and index number on all the work you hand in.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** questions.
Give non-exact numerical answers to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 90.

For Examiner's Use
/90

This document consists of **19** printed pages.

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1} a^{n-1} b + \binom{n}{2} a^{n-2} b^2 + \ldots + \binom{n}{r} a^{n-r} b^r + \ldots + b^n$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$
$$\sec^2 A = 1 + \tan^2 A$$
$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$
$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$
$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$
$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$
$$\sin 2A = 2 \sin A \cos A$$
$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$
$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$
$$\Delta = \frac{1}{2} bc \sin A$$

1 A curve has the equation $y = \dfrac{\sin 2x}{2 - \cos 2x}$.

(a) Show that the gradient function can be expressed in the form $\dfrac{k\cos 2x - 2}{\left(2 - \cos 2x\right)^2}$, where k is a constant. [3]

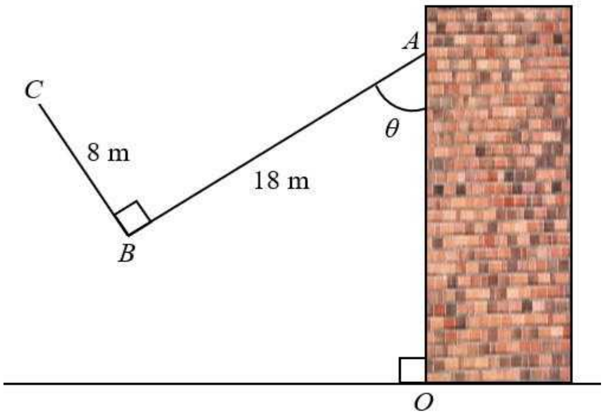
(b) Find the acute angle between the tangent to the curve at $x = \dfrac{\pi}{12}$ and the line $y = 0$. [3]

2 **(a)** Factorise $x^3 + 27k^3$ as a product of a linear and a quadratic factor. [2]

(b) Hence solve $x^3 + 27 = (x+3)(x+10)$, expressing non-integer roots in surd form. [3]

(c) Find the value of k given that $x^3 + 27k^3$ leaves a remainder of 351 when divided by $x - 2$. [2]

3 The diagram shows a L-shaped rod ABC where AB and BC have length 18 m and 8 m respectively and angle ABC is 90°. The rod is hinged to a wall at A so as to rotate in a vertical plane. The rod AB makes an acute angle θ with the vertical wall surface OA.



(a) Given that G is a point directly below C, show that $OG = p\cos\theta + q\sin\theta$, where p and q are constants to be found. [2]

(b) Express OG in the form $R\cos(\theta - \alpha)$ where $R > 0$ and $0° \le \alpha \le 90°$. [3]

(c) Find the length of OG and the corresponding value of θ if G is at maximum displacement from O. [3]

4 A circle C_1 has equation $x^2 + y^2 - 6x + 4y = 12$.

(a) Find the radius and the coordinates of the centre of C_1. [3]

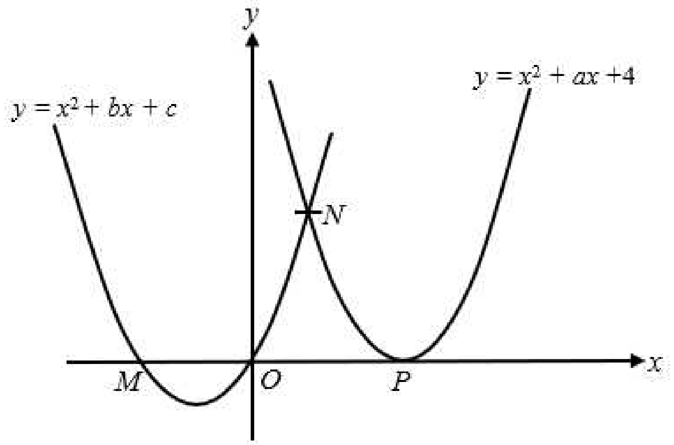
(b) Find the equation of the tangent to the circle at the point $P(7, -5)$. [3]

(c) Another circle C_2 has centre $(-8, 4)$ and radius 7 cm. Find the shortest distance between the 2 circles. [2]

5 **(a)** Prove the identity $\dfrac{(\sin A - \cos A)(1 + \sin A \cos A)}{\cos^3 A} = \tan^3 A - 1$. [4]

(b) Hence solve $(\sin A - \cos A)(1 + \sin A \cos A) + 2\cos^3 A = 0$ exactly, for $-\pi \le A \le \pi$ radians. [4]

6 The diagram shows the graph of $y = x^2 + ax + 4$ and $y = x^2 + bx + c$. The graph of $y = x^2 + ax + 4$ touches the x-axis at P. Points M and O are the x-intercepts of the graph of $y = x^2 + bx + c$. The origin O is the mid-point of MP.



(a) Find the values of a, b and c. [4]

(b) The graph of $y = x^2 + ax + 4$ and $y = x^2 + bx + c$ intersects at N. Find the coordinates of N. [2]

(c) The graph $y = px^2 + qx + r$ has its turning point at N and passes through point P. Find the values of p, q and r, where $r > 0$. [3]

7 A particle P, travels in a straight line, so that its displacement, s m, from O at time t seconds, is modelled by $s=\frac{1}{3}t^3-5t^2-3$.

(a) Find the value of t when particle P returns to its initial position. [2]

(b) Find the minimum velocity of particle P. [3]

(c) Another particle, Q, travels in a straight line from O such that its velocity, v m/s, at time t seconds after passing O is given by $v=24\left(e^{-\frac{t}{6}}-e^{-1}\right)$.

Find the value of t at which the particle Q is instantaneously at rest. [2]

(d) Find the total distance travelled by particle Q for the first 9 seconds. [4]

8 The height, h cm, of a plant is modelled by $h = \dfrac{80}{1+10\mathrm{e}^{-0.4t}}$, where t is the number of months after the first observation.

(a) Show that h is an increasing function. [3]

(b) Find the value of t when the height of the plant first exceeds four times its initial observation. [3]

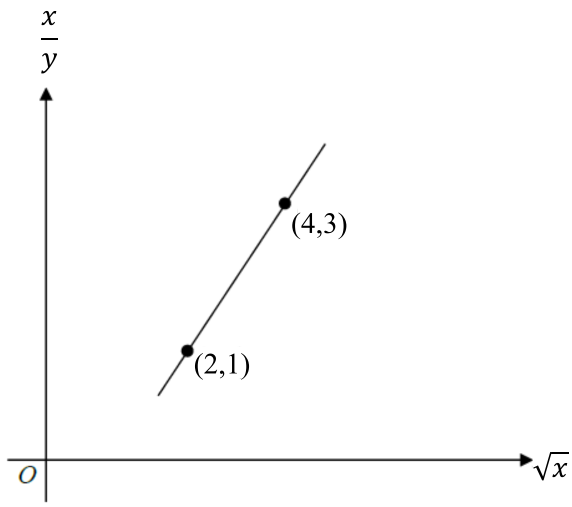
(c) The height, y cm, of another species of plant, t months after the first observation is given by $y = \dfrac{1}{a + b\mathrm{e}^{-t}}$, where a and b are constants. Explain clearly how a straight line graph can be drawn to represent this relationship. You should state which variables should be plotted on each axis and explain how the values of a and b can be calculated. [4]

9 (a) Without using a calculator, solve the equation $x\sqrt{15}+\sqrt{5}=x\sqrt{2}+\sqrt{6}$. Leave your answer in the form $p\sqrt{10}+q\sqrt{3}$, where p and q are fractions. [4]

(b) Without using a calculator, solve the equation $\log_2 x-\log_x 16=0$. [4]

(c) Solve the equation $3^{x+2} - 2\left(3^{-x}\right) = 17$. [4]

10 (a) The diagram shows part of a straight line graph which passes through $(2,1)$ and $(4,3)$.



Find the equation of the straight line in the form $y = \dfrac{x}{a + b\sqrt{x}}$, where a and b are constants. [3]

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(b) The table below shows the experimental values of two variables x and y.

x	1	2	3	4	5	6
y	63	127	258	510	1000	2100

It is known that x and y are related by an equation of the form $y = \frac{b^x}{10^a}$, where a and b are constants.

(i) On the grid next page, plot $\lg y$ against x and draw a straight line graph. [3]

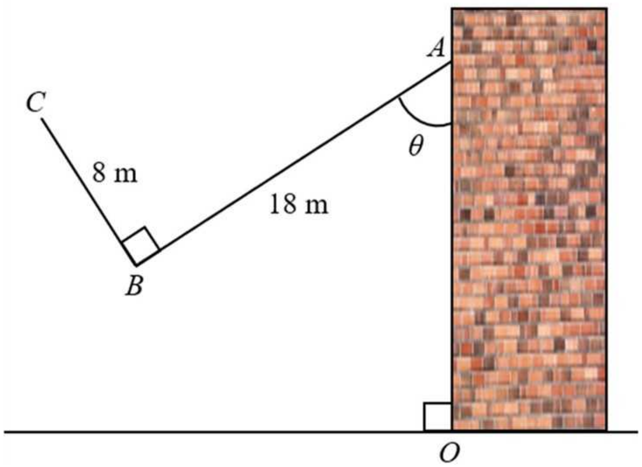
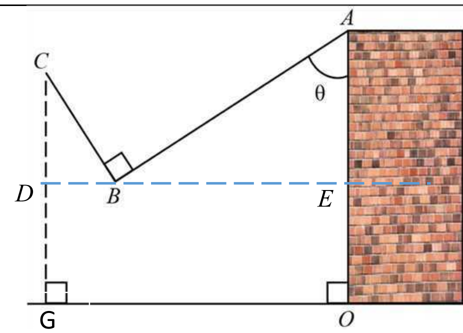
(ii) Use your graph to estimate the value of a and of b. [3]

(iii) Explain how would you use the graph to find the value of x for which $(10b)^x = 10^{a+1}$. [2]

END OF PAPER

1	A curve has the equation $y=\dfrac{\sin 2x}{2-\cos 2x}$.
	(a) Show that the gradient function can be expressed in the form $\dfrac{k\cos 2x-2}{(2-\cos 2x)^2}$, where k is a constant. [3]
	$y=\dfrac{\sin 2x}{2-\cos 2x}$ $\dfrac{dy}{dx}=\dfrac{(2-\cos 2x)2\cos 2x-\sin 2x(2\sin 2x)}{(2-\cos 2x)^2}$ M1: correct quotient/pdt rule $=\dfrac{4\cos 2x-2\cos^2 2x-2\sin^2 2x}{(2-\cos 2x)^2}$ M1: differentiate sin 2x and cos 2x correctly $=\dfrac{4\cos 2x-2(\cos^2 2x+\sin^2 2x)}{(2-\cos 2x)^2}$ A1: use of identity to reach answer $=\dfrac{4\cos 2x-2}{(2-\cos 2x)^2}$
	(b) Find the acute angle between the tangent to the curve at $x=\dfrac{\pi}{12}$ and the line $y=0$. [3]
	gradient of tangent $=\dfrac{4\cos\frac{\pi}{6}-2}{\left(2-\cos\frac{\pi}{6}\right)^2}=1.1386$ M1: correct gradient value Angle required $=\tan^{-1}(1.1386)=48.7^\circ$ (1 dp) M1, A1 (accepts 0.850 rad)

2	**(a)**	Factorise x^3+27k^3 as a product of a linear and a quadratic factor. [2]
		$x^3+27k^3=(x+3k)(x^2-3kx+9k^2)$ M1: $(x+3k)$ A1
	(b)	Hence solve $x^3+27=(x+3)(x+10)$, expressing non-integer roots in surd form. [3]
		$k=1: x^3+27=(x+3)(x^2-3x+9)$ $(x+3)(x^2-3x+9)=(x+3)(x+10)$ $x+3=0$ or $x^2-3x+9=x+10$ $x=-3$ or $x^2-4x-1=0$ $x=\dfrac{4\pm\sqrt{16-4(-1)}}{2}$ $=2\pm\sqrt{5}$ B1: identify x=-3 as a root from Hence M1: apply quad formula correctly A1
	(c)	Find the value of k given that x^3+27k^3 leaves a remainder of 351 when divided by $x-2$. [2]
		Let $f(x)=x^3+27k^3$ $f(2)=2^3+27k^3=351$ $k=\sqrt[3]{\dfrac{351-8}{27}}=\dfrac{7}{3}$ M1: applying remainder thm correctly A1

3	The diagram shows a *L*-shaped rod *ABC* where *AB* and *BC* have length 18 m and 8 m respectively and angle *ABC* is 90°. The rod is hinged to a wall at *A* so as to rotate in a vertical plane. The rod *AB* makes an acute angle θ with the vertical wall surface *OA*. 
	(a) Given that *G* is a point directly below *C*, show that $OG = p\cos\theta + q\sin\theta$, where *p* and *q* are constants to be found. [2]
	 Using triangle *BCD*: Angle $DBC = \theta$ $\cos\theta = \dfrac{BD}{8}$ $BD = 8\cos\theta$ B1: either correctly establishing BD or BE Using triangle *ABE*: Angle $BAE = \theta$ $\sin\theta = \dfrac{BE}{18}$ $BE = 18\sin\theta$ $OG = BD + BE$ $= 8\cos\theta + 18\sin\theta$, $p = 8$, $q = 18$ B1: showing the other component and clear indication that OG is a sum of the 2 values
	(b) Express *OG* in the form $R\cos(\theta - \alpha)$ where $R > 0$ and $0° \le \alpha \le 90°$. [3]

	$OG = 8\cos\theta + 18\sin\theta$ $= \sqrt{8^2+18^2}\cos\left(\theta - \tan^{-1}\dfrac{18}{8}\right)$ $= \sqrt{388}\cos\left(\theta - \tan^{-1}\dfrac{9}{4}\right)$ $= 19.7\cos\left(\theta - 66.0^\circ\right)$	M1: $\sqrt{8^2+18^2}$ M1: $\tan^{-1}\dfrac{18}{8}$ A1: correct evaluation and form, accept $\sqrt{388}$
	(c) Find the length of OG and the corresponding value of θ if G is at maximum displacement from O. [3]	
	max $OG = \sqrt{388}$ $= 19.7$ m when $\cos\left(\theta - \tan^{-1}\dfrac{9}{4}\right) = 1$ $\theta - \tan^{-1}\dfrac{9}{4} = 0$ $\theta = \tan^{-1}\dfrac{9}{4} = 66.0^\circ$	A1: $\sqrt{388}$ or $19.7m$ M1: $\theta - \tan^{-1}\dfrac{9}{4} = 0$ A1: 66.0°

4 A circle C_1 has equation $x^2+y^2-6x+4y=12$.

(a) Find the radius and the coordinates of the centre of C_1. [3]

$x^2-6x+y^2+4y=12$

$(x-3)^2+(y+2)^2=12+3^3+2^2$

$(x-3)^2+(y+2)^2=5^2$

M1: completing the square or using formula

Radius = 5 units
Centre = (3, -2)

A1 B1

(b) Find the equation of the tangent to the circle at the point $P(7,-5)$. [3]

Gradient of normal = $\dfrac{-2-(-5)}{3-7}=-\dfrac{3}{4}$

M1

Gradient of tangent = $\dfrac{4}{3}$

Eqn of tangent: $y+5=\dfrac{4}{3}(x-7)$

M1award for correct pts and their -1/m used

$y=\dfrac{4}{3}x-\dfrac{43}{3}$

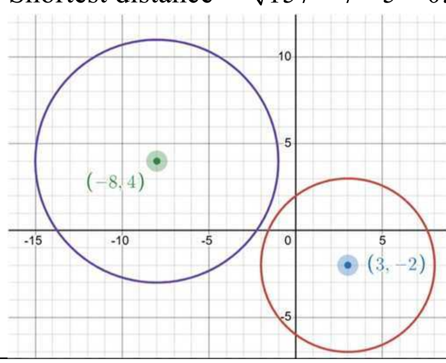
A1, or $3y=4x-43$

(c) Another circle C_2 has centre $(-8,4)$ and radius 7 cm. Find the shortest distance between the 2 circles. [2]

Distance between centres of circle = $\sqrt{(3+8)^2+(-2-4)^2}=\sqrt{157}$

Shortest distance = $\sqrt{157}-7-5=0.530$ cm (3 s.f.)

M1 A1



[Turn over

5	**(a)** Prove the identity $\dfrac{(\sin A-\cos A)(1+\sin A\cos A)}{\cos^3 A}=\tan^3 A-1$. [4]

$$LHS=\frac{(\sin A-\cos A)(1+\sin A\cos A)}{\cos^3 A}$$

$$=\frac{\sin A-\cos A+\sin^2 A\cos A-\sin A\cos^2 A}{\cos^3 A}$$

M1: correct expansion of terms

$$=\frac{\sin A-\cos A+\left(1-\cos^2 A\right)\cos A-\sin A\left(1-\sin^2 A\right)}{\cos^3 A}$$

M1: applying identity

$$=\frac{\sin A-\cos A+\cos A-\cos^3 A-\sin A+\sin^3 A}{\cos^3 A}$$

$$=\frac{\sin^3 A-\cos^3 A}{\cos^3 A}$$

M1: expand & simplify

$$=\frac{\sin^3 A}{\cos^3 A}-\frac{\cos^3 A}{\cos^3 A}$$

A1: manipulation to RHS

$$=\tan^3 A-1$$

OR

$$LHS=\frac{(\sin A-\cos A)(1+\sin A\cos A)}{\cos^3 A}$$

$$=\frac{(\sin A-\cos A)\left(\sin^2 A+\cos^2 A+\sin A\cos A\right)}{\cos^3 A}$$

M1: applying identity

$$=\frac{\sin^3 A+\sin A\cos^2 A+\sin^2 A\cos A-\sin^2 A\cos A-\cos^3 A-\sin A\cos^2 A}{\cos^3 A}$$

M1: correct expansion of terms

$$=\frac{\sin^3 A-\cos^3 A}{\cos^3 A}$$

M1: simplify

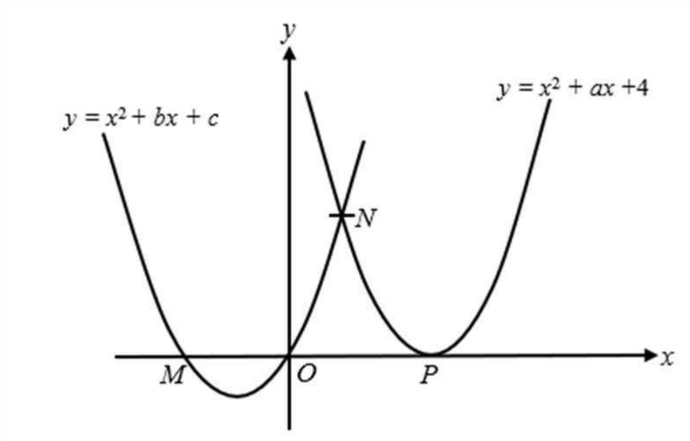
$$=\frac{\sin^3 A}{\cos^3 A}-\frac{\cos^3 A}{\cos^3 A}$$

A1: manipulation to RHS

$$=\tan^3 A-1$$

	(b) Hence solve $(\sin A - \cos A)(1+\sin A\cos A)+2\cos^3 A=0$ exactly, for $-\pi \le A \le \pi$ radians. [4]
	$(\sin A - \cos A)(1+\sin A\cos A)=-2\cos^3 A$ $\dfrac{(\sin A - \cos A)(1+\sin A\cos A)}{\cos^3 A}=-2$ $\tan^3 A - 1 = -2$ M1: simplification to single trigo equation $\tan^3 A = -1$ $\tan A = -1$ Basic angle = $\dfrac{\pi}{4}$ M1: correct basic angle – must be acute Quad: Q2, Q4 $A=\dfrac{3\pi}{4},-\dfrac{\pi}{4}$ A1, A1

6 The diagram shows the graph of $y = x^2 + ax + 4$ and $y = x^2 + bx + c$. The graph of $y = x^2 + ax + 4$ touches the x-axis at P. Points M and O are the x-intercepts of the graph of $y = x^2 + bx + c$. The origin O is the mid-point of MP.



(a) Find the values of a, b and c. [4]

Since origin (0, 0) is on $y = x^2 + bx + c$, $c = 0$ B1

Discriminant of $y = x^2 + ax + 4 = 0$ since curve intersects x-axis once only:

$a^2 - 4(1)(4) = 0$

$a = \pm 4$

Since P is on positive x-axis, $a < 0$: $a = -4$

M1: using discriminant or the equation must be a perfect square

A1

OR

$$y = x^2 + ax + 4 = \left(x + \frac{a}{2}\right)^2 - \frac{a^2}{4} + 4.$$

Since curve intersects x-axis once only:

$$4 - \frac{a^2}{4} = 0$$

$a = \pm 4$

Since P is on positive x-axis, $a < 0$: $a = -4$

M1: using discriminant or the equation must be a perfect square

A1

Coor of P = $\left(-\frac{a}{2}, 0\right) = (2, 0)$

Coor of M = $(-b, 0) = (-2, 0)$ B1

$b = 2$

	(b) The graph of $y = x^2 + ax + 4$ and $y = x^2 + bx + c$ intersects at N. Find the coordinates of N. [2]
	$x^2 - 4x + 4 = x^2 + 2x$ $6x = 4$ $x = \frac{2}{3}$ M1: equating and solving correct x (allows ecf) $y = \left(\frac{2}{3}\right)^2 + 2\left(\frac{2}{3}\right) = \frac{16}{9}$ Coordinates of $N = \left(\frac{2}{3}, \frac{16}{9}\right)$ A1
	(c) The graph $y = px^2 + qx + r$ has its turning point at N and passes through point P. Find the values of p, q and r, where $r > 0$. [3]
	Graph with turning point at N and passes P (downward opening): $y = -p\left(x - \frac{2}{3}\right)^2 + \frac{16}{9}$, $p < 0$ M1: correct completed square form At P $(2,0)$, $0 = -p\left(2 - \frac{2}{3}\right)^2 + \frac{16}{9}$ $p = -1$ $y = -\left(x - \frac{2}{3}\right)^2 + \frac{16}{9}$ $= -\left(x^2 - \frac{4}{3}x + \frac{4}{9}\right) + \frac{16}{9}$ $= -x^2 + \frac{4}{3}x + \frac{4}{3}$ A1: correct p $p = -1$, $q = r = \frac{4}{3}$ A1: correct q & r OR (longer method): form 3 equations with coor of N, P and either 2nd x-intercept or derivative and solve $y = px^2 + qx + r$ $\frac{dy}{dx} = 2px + q$ Turning point at $\left(\frac{2}{3}, \frac{16}{9}\right)$: $2p\left(\frac{2}{3}\right) + q = 0$ $4p = -3q$ ---- (1) M1: establishing 3 equations correctly Graph passes through P: $0 = p(2)^2 + q(2) + r$ $4p + 2q + r = 0$

	Subst (1): $q = r$ ----- (2) Graph passes through N: $\frac{16}{9} = p\left(\frac{2}{3}\right)^2 + q\left(\frac{2}{3}\right) + r$ $4p + 6q + 9r = 16$ Subst (1) and (2): $-3q + 6q + 9q = 16$ $12q = 16$ $q = \frac{4}{3}$ A1: correct p A1: correct q & r
7	A particle P, travels in a straight line, so that its displacement, s m, from O at time t seconds, is modelled by $s = \frac{1}{3}t^3 - 5t^2 - 3$.
	(a) Find the value of t when particle P return to its initial position. [2]
	Initial position when t = 0, s = -3 $\frac{1}{3}t^3 - 5t^2 - 3 = -3$ M1: setting s = -3 $\frac{1}{3}t^3 - 5t^2 = 0$ $t^2\left(\frac{1}{3}t - 5\right) = 0$ $t = 0$ or $t = 15s$ Particle returns to initial position at t = 15s. A1
	(b) Find the minimum velocity of particle P. [3]
	$v = t^2 - 10t$ $= (t-5)^2 - 25$ Minimum velocity = -25 m/s^2 M1: $v = t^2 - 10t$ M1: completing the square A1 OR $v = t^2 - 10t$ At stationary v, $\frac{dv}{dt} = 2t - 10 = 0$ $t = 5$ $v = (5)^2 - 10(5) = -25$ m/s M1: $v = t^2 - 10t$ M1: solving for t = 5 $\frac{d^2v}{dt^2} = 2 > 0$, minimum v at -25m/s A1: includes check for minimum

(c) Another particle, Q, travels in a straight line from O such that its velocity, v m/s, at time t seconds after passing O is given by $v = 24\left(e^{-\frac{t}{6}} - e^{-1}\right)$.

Find the value of t at which the particle Q is instantaneously at rest. [2]

$v = 24\left(e^{-\frac{t}{6}} - e^{-1}\right) = 0$

M1: Setting $v = 0$

$e^{-\frac{t}{6}} = e^{-1}$

A1

$t = 6s$

(d) Find the total distance travelled by particle Q for the first 9 seconds. [4]

$s = 24\left(\dfrac{e^{-\frac{t}{6}}}{-\frac{1}{6}} - te^{-1}\right) = -24\left(6e^{-\frac{t}{6}} + te^{-1}\right) + C$

M1: correct integration (award even if without +C)

When t=0, s=0: $s = -24(6) + C = 0$

$C = 144$

$s = 144 - 24\left(6e^{-\frac{t}{6}} + te^{-1}\right)$

M1: correct S

When $t = 6$, $s = 144 - 24\left(6e^{-1} + 6e^{-1}\right) = 38.051m$ (5 sf)

When $t = 9$, $s = 144 - 24\left(6e^{-\frac{9}{6}} + 9e^{-1}\right) = 32.407m$ (5 sf)

M1: sub t= 6 and t=9

Total distance travelled = $s = 38.051 + (38.051 - 32.407) = 43.7m$ (3 sf)

A1

OR

Displacement covered for 1st 6 sec =

$\int_0^6 24\left(e^{-\frac{t}{6}} - e^{-1}\right) \mathrm{d}t = 24\left[\dfrac{e^{-\frac{t}{6}}}{-\frac{1}{6}} - te^{-1}\right]_0^6$

M1: correct integration

$= -24\left[6e^{-\frac{t}{6}} + te^{-1}\right]_0^6$

$= -24\left[6e^{-\frac{6}{6}} + 6e^{-1} - 6e^{-\frac{0}{6}} - 0\right]$

$= 38.051m$ (3sf)

M1

Displacement covered for next 3 sec =

	$\int_6^9 24\left(e^{-\frac{t}{6}}-e^{-1}\right)\mathrm{d}t=-24\left[6e^{-\frac{t}{6}}+te^{-1}\right]_6^9$ $=-24\left[6e^{-\frac{9}{6}}+9e^{-1}-6e^{-\frac{6}{6}}-6e^{-\frac{6}{6}}\right]$ $=-5.6434m$ (5sf) M1 Total distance $= s=38.051+(38.051-32.407)=43.7m$ A1

8	The height, h cm, of a plant is modelled by $h=\dfrac{80}{1+10\mathrm{e}^{-0.4t}}$, where t is the number of months after the first observation.
	(a) Show that h is an increasing function. [3]
	$\dfrac{\mathrm{d}h}{\mathrm{d}t}=80\dfrac{\mathrm{d}}{\mathrm{d}t}\left(1+10\mathrm{e}^{-0.4t}\right)^{-1}$ $=80(-1)\left(1+10\mathrm{e}^{-0.4t}\right)^{-2}\left(10(-0.4)\mathrm{e}^{-0.4t}\right)$ M1: differentiate $1+10\mathrm{e}^{-0.4t}$ correctly $=\dfrac{320\mathrm{e}^{-0.4t}}{\left(1+10\mathrm{e}^{-0.4t}\right)^2}$ M1: simplification Since $\mathrm{e}^{-0.4t}>0$ for $t>0$, $320\mathrm{e}^{-0.4t}>0$ and $\left(1+10\mathrm{e}^{-0.4t}\right)^2>0$ $\dfrac{\mathrm{d}h}{\mathrm{d}t}=\dfrac{320\mathrm{e}^{-0.4t}}{\left(1+10\mathrm{e}^{-0.4t}\right)^2}>0$ Therefore, h is an increasing function. A1: Clear explanation on why derivative is positive and therefore h is increasing
	(b) Find the value of t when the height of the plant first exceeds four times its initial observation. [3]
	Since h is increasing, $\dfrac{80}{1+10\mathrm{e}^{-0.4t}}>\dfrac{4\times80}{1+10\mathrm{e}^{0}}$ M1: correct set up of equation (accepts equal if students round up at the end, preferably with explanation that h is an increasing function) $1+10\mathrm{e}^{-0.4t}<\dfrac{11}{4}$ $\mathrm{e}^{-0.4t}<\dfrac{\frac{11}{4}-1}{10}=0.175$ M1: simplification to exponential eqn $-0.4t<\ln0.175$ $t>4.3574$ (5 s.f.) $t=4.36$ (3 s.f.) A1 (accepts 5, though context need not be integer)

	(c)	The height, y cm, of another species of plant, t months after the first observation is given by $y=\frac{1}{a+be^{-t}}$, where a and b are constants. Explain clearly how a straight line graph can be drawn to represent this relationship. You should state which variables should be plotted on each axis and explain how the values of a and b can be calculated. [4]

$$y=\frac{1}{a+be^{-t}}$$

$$\frac{1}{y}=a+be^{-t}$$

M1: simplification to Y=mX + c form

Plot $\frac{1}{y}$ on the vertical axis against e^{-t} on the horizontal axis to obtain a straight line graph. [A1] The gradient of the graph would give the value of b [A1] and the y-intercept will give the value of a [A1].

Note: no award of full A1s if M1 not awarded.

9	**(a)**	Without using a calculator, solve the equation $x\sqrt{15}+\sqrt{5}=x\sqrt{2}+\sqrt{6}$. Leave your answer in the form $p\sqrt{10}+q\sqrt{3}$, where p and q are fractions. [4]

$$x\sqrt{15}+\sqrt{5}=x\sqrt{2}+\sqrt{6}$$

$$x\left(\sqrt{15}-\sqrt{2}\right)=\sqrt{6}-\sqrt{5}$$

$$x=\frac{\sqrt{6}-\sqrt{5}}{\sqrt{15}-\sqrt{2}}$$

M1: $x=\frac{\sqrt{6}-\sqrt{5}}{\sqrt{15}-\sqrt{2}}$

$$=\frac{\sqrt{6}-\sqrt{5}}{\sqrt{15}-\sqrt{2}}\times\frac{\sqrt{15}+\sqrt{2}}{\sqrt{15}+\sqrt{2}}$$

M1: correct rationalisation fraction

$$=\frac{\sqrt{90}-\sqrt{10}-\sqrt{75}+\sqrt{12}}{15-2}$$

M1: expansion of numerator and denominator

$$=\frac{3\sqrt{10}-\sqrt{10}-5\sqrt{3}+2\sqrt{3}}{13}$$

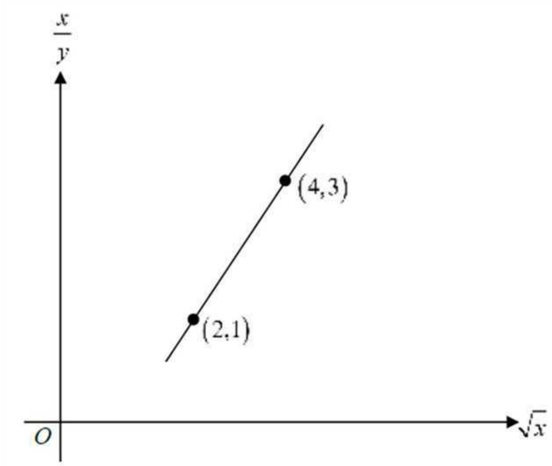
$$=\frac{2\sqrt{10}-3\sqrt{3}}{13}$$

$$=\frac{2}{13}\sqrt{10}-\frac{3}{13}\sqrt{3}$$

A1: in $p\sqrt{10}+q\sqrt{3}$, where p and q are fractions

	(b) Without using a calculator, solve the equation $\log_2 x - \log_x 16 = 0$. [4]
	$\log_2 x - \log_x 16 = 0$ $\log_2 x - \dfrac{\log_2 16}{\log_2 x} = 0$ — M1: correct change of base $(\log_2 x)^2 = 4$ $\log_2 x = \pm 2$ — M1: simplification of $\log_2 16$ and reduced to a solvable quadratic equation $x = 2^2$ or $x = 2^{-2}$ $x = 4$ or $x = \dfrac{1}{4}$ — A1, A1
	(c) Solve the equation $3^{x+2} - 2(3^{-x}) = 17$. [4]
	$3^{x+2} - 2(3^{-x}) = 17$ $9(3^x) - \dfrac{2}{3^x} = 17$ $9(3^x)^2 - 17(3^x) - 2 = 0$ — M1: correct application of indices rules and rewriting into quadratic eqn in 3^x Let $u = 3^x$ $9u^2 - 17u - 2 = 0$ $(u-2)(9u+1) = 0$ — M1: factoring or formula to solve quadratic eqn $u = 2$ or $u = -\dfrac{1}{9}$ (rej since $3^x > 0$) — A1: Solving of 3^x with rejection $3^x = 2$ $x = \dfrac{\ln 2}{\ln 3} = 0.631$ (3 s.f.) — A1: do not accept $x = \log_3 2$ as no way of computing

10 **(a)** The diagram shows part of a straight line graph which passes through $(2,1)$ and $(4,3)$.



Find the equation of the straight line in the form $y = \dfrac{x}{a + b\sqrt{x}}$, where a and b are constants. [3]

gradient of straight line: $\dfrac{3-1}{4-2} = 1$

equation:

$$Y - 1 = 1(X - 2)$$

$$Y = X - 1$$

$$\frac{x}{y} = \sqrt{x} - 1$$

$$y = \frac{x}{\sqrt{x} - 1}$$

M1: gradient of line

M1: eqn of line

A1

(b) The table below shows the experimental values of two variables x and y.

x	1	2	3	4	5	6
y	63	127	258	510	1000	2100

It is known that x and y are related by an equation of the form $y = \dfrac{b^x}{10^a}$, where a and b are constants.

(i) On the grid next page, plot $\lg y$ against x and draw a straight line graph. [3]

(ii) Use your graph to estimate the value of a and of b. [3]

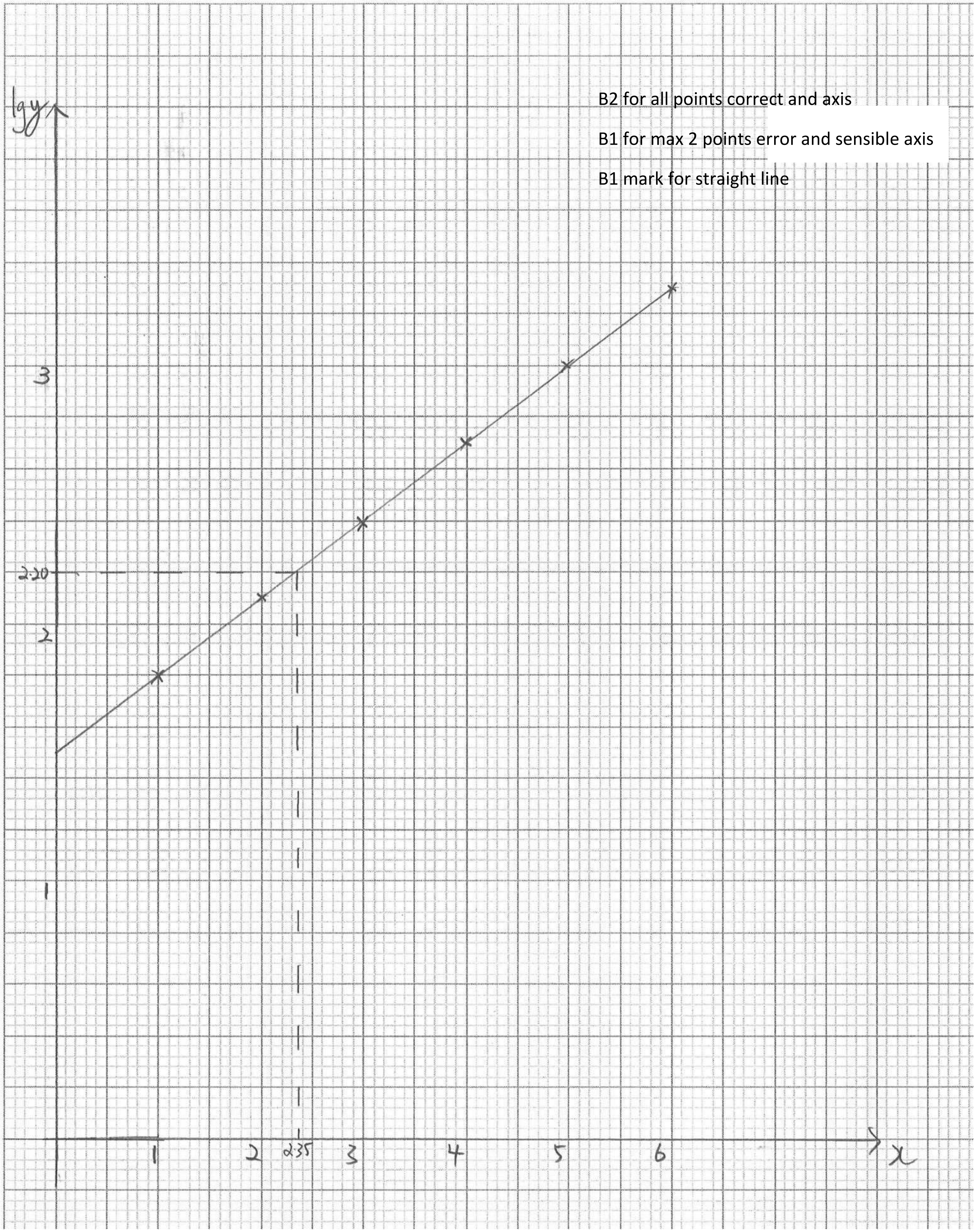
$y = \dfrac{b^x}{10^a} \Rightarrow$

	$\lg y = \lg\left(\frac{b^x}{10^a}\right)$ $\lg y = \lg b^x - \lg 10^a$ $\lg y = x\lg b - a$ M1 $-a = 1.5$ $a = -1.5$ A1 (accept -1.54 to -1.46) $\lg b = 0.3$ $b = 2.00$ A1 (accept 1.82 to 2.19)
	(iii) Explain how you would use the graph to find the value of x for which $(10b)^x = 10^{a+1}$. [2]
	$(10b)^x = 10^{a+1}$ $10^x b^x = 10^a \times 10$ $\frac{b^x}{10^a} = \frac{10}{10^x}$ M1 $y = 10^{1-x}$ $\lg y = \lg\left(10^{1-x}\right)$ $\lg y = 1 - x$ Draw the line $\lg y = 1 - x$ and find the x-coordinate of the point of intersection. A1 OR $(10b)^x = 10^{a+1}$ $x\lg(10b) = (a+1)\lg(10)$ $x\lg(10) + x\lg(b) = a+1$ $x\lg(b) - a = x+1$ $\lg y = x+1$

B2 for all points correct and axis

B1 for max 2 points error and sensible axis

B1 mark for straight line



[Turn over