NAME: ______________________________ (   )   CLASS: 4 (   )

ANGLICAN HIGH SCHOOL
SECONDARY FOUR
PRELIMINARY EXAMINATIONS 2025

S4

ADDITIONAL MATHEMATICS

4049/01

Paper 1

22 August 2025
2 hours 15 minutes

Candidates answer on the Question Paper.
Additional Material: Graph Paper (1 sheet)

READ THESE INSTRUCTIONS FIRST
Write your name, index number and class in the spaces at the top of this page.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.

The total number of marks for this paper is 90.

For Examiners' Use

Question	Marks	Question	Marks		
1		8		Clarity / Logic	
2		9		Precision / Accuracy	
3		10			
4		11		Units	
5		12			
6		13		Total:	
7		14			
Parent's Name & Signature:					90
Date:					

This paper consists of **18** printed pages.

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

$$\text{Area of } \Delta = \frac{1}{2}ab\sin C$$

1 **(a)** Find the range of values of p for which $y = x^2 - x - 2$ lies entirely above the line $y = p(x+2)$. [4]

(b) Hence, deduce, without finding the discriminant, the number of intersection points when $p = 3$. [1]

2 By using substitution or otherwise, find the values of x for which $2^{2x-1} = 2^{x+3} - 24$, giving your answer where appropriate, to one decimal place. [5]

3 A closed circular cylinder has a volume of $\left(66\sqrt{2}+2\sqrt{3}\right)\pi$ cm^3, a radius of $\left(\sqrt{2}+2\sqrt{3}\right)$cm and a height h cm. Express h in the form of $p\sqrt{2}+q\sqrt{3}$ where p and q are integers. [4]

4 Express $\dfrac{2x^3-2x^2+7x-12}{x^3+4x}$ in partial fractions. [6]

5 Given the curve $y = kx^2 + 2kx - 3$, where k is a constant.

(a) By expressing y in the form $k(x+b)^2 - 1$, determine the value of k and of b. [3]

(b) Hence, show that the curve is always below the x-axis. [2]

(iii) State the turning point of the curve and determine the nature of this point. [2]

6 The binomial expansion of $(1+px)^n$ where $n > 0$, in ascending powers of x is

$$1-12x+28p^2x^2+qx^3+...$$

Find the value of p, of n and of q. [6]

7 **(a)** The surface area of a solid cube is increasing at $0.2\,\text{cm}^2/\text{s}$. Find the rate of increase of the volume when the length of a side is 1 cm. [4]

(b) A curve has the equation $y = e^{1-8x}(\cos 2x)$, where $0 \le x \le \frac{\pi}{4}$. Find the gradient of the curve at the point where $y = 0$, leaving your answer in exact form. [4]

8 The equation of a curve is $y = -\frac{5}{8}x + \frac{6}{x}$, where $x > 0$. Given that the gradient of the normal at point P is 1, find

(a) the coordinates of P, [4]

(b) the equation of the normal at the curve at P. [2]

9 **(a)** Solve the equation $\log_2(x+2)-\log_{\sqrt{2}}(x-1)=1$. [4]

(b) It is given that $\lg a-\lg b=\lg(a+b)$.

(i) Express a in terms of b. [2]

(ii) State the range of b. [1]

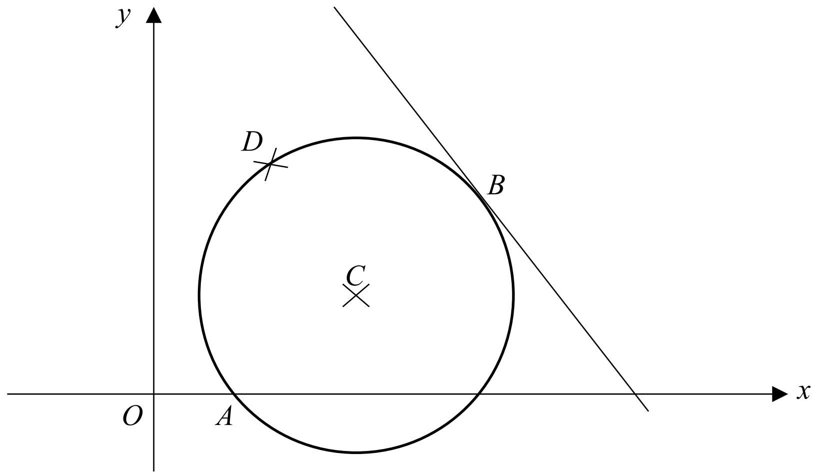
10 Given that $\sin A$ and $\cos B$ are in the same quadrant such that $\sin A = \frac{3}{5}$ and $\cos B = -\frac{5}{13}$.

Find, without using a calculator, the value of

(a) $\cos(A-B)$, [2]

(b) $\cos\frac{A}{2}$. [3]

11 Points A, $B(5,4)$ and D lie on a circle with centre C and AB is the diameter of the circle.
The line $y=-x+9$ is a tangent at point $B(5,4)$ on the circle.



(a) Given that point P lies on BA extended and the x-coordinate of point P is -1, find the coordinates of P. [2]

It is given that the distance BP is three times the distance BC.

(b) **(i)** Show that centre C is $(3,2)$. [1]

(ii) Hence, find the equation of the circle. [2]

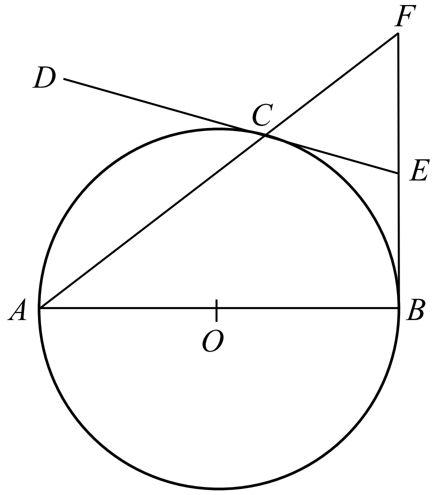
(c) Given that triangle *BCD* is a right-angled triangle, find the coordinates of *D*. [3]

12 **(a)** Given that $y = \ln\left(\sin^2 x\right)$, show that $\frac{\mathrm{d}y}{\mathrm{d}x} = 2\cot x$. [2]

(b) Hence, show that the value of $\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \left(\cot x - \cos^2 2x\right) \mathrm{d}x = a + b\ln 2$, where a and b are constants.

[4]

13 In the diagram, AB is a diameter of the circle with centre O. DE and BF are tangents to the circle at C and B respectively. DCE and BEF are straight lines. Prove that



(a) ΔABC and ΔBFC are similar, [3]

(b) hence or otherwise, show that ΔCFE is an isosceles triangle. [4]

14 The mass, m grams, of the decomposition of a radio-active substance is given by the formula $m = ae^{-bt}$, where a and b are constants, and t is the number of weeks after the initial measurement of the mass. Experimental values of t and m are given in the table below.

t	1	2	3	4	5
m	1.51	0.454	0.137	0.0183	0.0124

(i) What does the constant a refer to? [1]

(ii) By drawing a suitable straight line graph using the data provided, estimate the value of a and of b. [7]

(iii) It is discovered that one of the readings of the mass, m, is incorrect. Using your graph, estimate the correct value for this incorrect reading. [2]

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7		14			90
Parent's Name & Signature:					
Date:					

NOTE: *If the marks allocation is italicised, it has been modified from version 2 according to the suggestions given by the tester.*

1 **(a)** Find the range of values of p for which $y = x^2 - x - 2$ lies entirely above the line $y = p(x+2)$.[4]

	$x^2 - x - 2 = px + 2p$ $x^2 + (-1-p)x - 2 - 2p = 0$ $\Rightarrow$ discriminant < 0 $\Rightarrow (-1-p)^2 - 4(1)(-2-2p) < 0$ $\Rightarrow 1 + 2p + p^2 + 8 + 8p < 0$ $\Rightarrow p^2 + 10p + 9 < 0$ $\Rightarrow (p+9)(p+1) < 0$ $\Rightarrow -9 < p < -1$	M1 M1 for discriminant < 0 M1 A1

(b) Hence, deduce, without finding the discriminant, the number of intersection points when $p = 3$. [1]

	Line $y = p(x+2)$ does not intersect curve $y = (x+1)(x-2)$ when p is in the range $-9 < p < -1$. For $p = 3$, line intersects curve at 2 points.	B1

2 **(a)** By using substitution or otherwise, find the values of x for which $2^{2x-1} = 2^{x+3} - 24$, giving your answer where appropriate, to one decimal place. [5]

$2^{2x-1} = 2^{x+3} - 24$ $\frac{1}{2}\left(2^x\right)^2 = 2^3 \times 2^x - 24$ Let $u = 2^x$ $\frac{1}{2}(u)^2 = 2^3 \times u - 24$ $\frac{1}{2}u^2 = 8u - 24$ $u^2 = 16u - 48$ $u^2 - 16u + 48 = 0$ $(u-12)(u-4) = 0$ $u = 12$ or $u = 4$ $2^x = 12$ or $2^x = 4$ $\lg 2^x = \lg 12$ or $2^x = 2^2$ $x = \frac{\lg 12}{\lg 2}$ or $x = 2$ $x \approx 3.58496$ $x \approx 3.6$	M1 applying Laws of Indices to break them into product of two numbers M1 for getting quadratic equation M1 for using lg to solve. A2 for both answers

3 A closed circular cylinder has a volume of $\left(66\sqrt{2}+2\sqrt{3}\right)\pi\,\text{cm}^3$, a radius of $\left(\sqrt{2}+2\sqrt{3}\right)$cm and a height h cm. Express h in the form of $p\sqrt{2}+q\sqrt{3}$ where p and q are integers. [4]

Volume of cylinder = $\left(66\sqrt{2}+2\sqrt{3}\right)\pi$ $\pi r^2 h=\left(66\sqrt{2}+2\sqrt{3}\right)\pi$ $\left(\sqrt{2}+2\sqrt{3}\right)^2 h=\left(66\sqrt{2}+2\sqrt{3}\right)$ $\left(2+4\sqrt{6}+12\right)h=\left(66\sqrt{2}+2\sqrt{3}\right)$	M1 – expansion of r^2
$h=\dfrac{66\sqrt{2}+2\sqrt{3}}{14+4\sqrt{6}}$ $h=\dfrac{66\sqrt{2}+2\sqrt{3}}{14+4\sqrt{6}}\times\dfrac{14-4\sqrt{6}}{14-4\sqrt{6}}$	M1 – rationalisation
$h=\dfrac{924\sqrt{2}-264\sqrt{12}+28\sqrt{3}-8\sqrt{18}}{196-96}$	M1 – multiplication and simplification
$h=\dfrac{924\sqrt{2}-528\sqrt{3}+28\sqrt{3}-24\sqrt{2}}{100}$ $h=\dfrac{900\sqrt{2}-500\sqrt{3}}{100}$ $h=9\sqrt{2}-5\sqrt{3}$	A1 for answer

4 Express $\dfrac{2x^3-2x^2+7x-12}{x^3+4x}$ in partial fractions. [6]

$$\begin{array}{r} 2 \\ x^3+4x\overline{\big)\,2x^3-2x^2+7x-12} \\ -\left(2x^3 \qquad +8x\right) \\ \hline -2x^2-x-12 \\ \hline \end{array}$$	**Marking Scheme modified from Version 2** B1 for long division working
$\therefore \dfrac{2x^3-2x^2+7x-12}{x^3+4x}=2+\dfrac{-2x^2-x-12}{x^3+4x}$	
$\dfrac{-2x^2-x-12}{x^3+4x}=\dfrac{A}{x}+\dfrac{Bx+C}{x^2+4}$ Multiply by $\left(x^3+4x\right)$ throughout, $-2x^2-x-12=A\left(x^2+4\right)+x\left(Bx+C\right)$	M1 for breaking into partial fractions
Sub $x=0$, $-12=A(4)+0$ $A=-3$	*A1 for value of A*

Sub $x=1$, $-2-1-12=-3\left(1^2+4\right)+(1)(B+C)$ $-15=-15+B+C$ $B+C=0---------(1)$ Sub $x=-1$, $-2+1-12=-3\left(1^2+4\right)+(-1)(-B+C)$ $-13=-15+B-C$ $B-C=2---------(2)$ $(1)+(2):2B=2$ $B=1$ Sub $B=1$ into (1): $1+C=0$ $C=-1$ $\frac{2x^3-2x^2+7x-12}{x^3+4x}=2+\frac{x-1}{x^2+4}-\frac{3}{x}$	*M1 for forming simultaneous equations to solve for value of B and of C.* *A1 for value of B and of C.* A1 for final answer
or	
$$\begin{array}{r} 2 \\ x^3+4x \overline{\smash{\big)}\,2x^3-2x^2+7x-12} \\ -\left(2x^3 \qquad +8x\right) \\ \hline -2x^2-x-12 \\ \hline \end{array}$$ $\therefore \frac{2x^3-2x^2+7x-12}{x^3+4x}=2+\frac{-2x^2-x-12}{x^3+4x}$ $\frac{-2x^2-x-12}{x^3+4x}=\frac{A}{x}+\frac{Bx+C}{x^2+4}$ Multiply by $\left(x^3+4x\right)$ throughout, $-2x^2-x-12=A\left(x^2+4\right)+x(Bx+C)$ $=Ax^2+4A+Bx^2+Cx$ $=(A+B)x^2+Cx+4A$ By comparing constants, $4A=-12$ $A=-3$ By comparing coefficients of x, $C=-1$	B1 for long division working M1 for breaking into partial fractions *M1 for expansion and grouping* *A1 for value of A*

By comparing coefficients of x^2, $A+B=-2$ $-3+B=-2$ $B=1$	*A1 for value of B and of C.*
$\frac{2x^3-2x^2+7x-12}{x^3+4x}=2+\frac{x-1}{x^2+4}-\frac{3}{x}$	A1 for final answer

5 Given the curve $y=kx^2+2kx-3$, where k is a constant,

(i) By expressing y in the form $k(x+b)^2-1$, determine the value of k and b. [3]

(i)	$y=kx^2+2kx-3$ $y=k(x^2+2x)-3$ $y=k(x^2+2x+1-1)-3$ $y=k(x+1)^2-k-3$	**Marking Scheme modified from Version 2** M1 for $y=k(x+1)^2-k-3$
	$b=1$	*A1 for value of b*
	$-k-3=-1$ $k=-2$	*A1 for value of k*

(ii) Hence, show that the curve is always below the x-axis. [2]

(ii)	$y=-2(x+1)^2-1$ $(x+1)^2\geq 0$ $-2(x+1)^2\leq 0$ $-2(x+1)^2-1\leq -1<0$ Hence, the graph of y is always below the x-axis.	M1 A1
	or	
	Discriminant = $(-4)^2-4(-2)(-3)=-8<0$ Since discriminant < 0, the curve does not intersect the x-axis. Coefficient of x^2 is also negative, hence it is a quadratic curve with a maximum point. Hence, the curve is always above the x-axis.	M1 discriminant A1 for negative coefficient of x^2
	or	
	The turning point is $(-1,-1)$ and is a maximum point. Since the y-coordinate is less than zero, the curve is always above the x-axis.	M1 for maximum turning point. A1 for y-coor < 0

(iii) State the turning point of the curve and determine the nature of this point. [2]

(iii)	turning point $= (-1, -1)$ and is a maximum point	B1 B1

6 The binomial expansion of $(1+px)^n$ where $n > 0$, in ascending powers of x is

$$1 - 12x + 28p^2x^2 + qx^3 + \dots$$

Find the value of p, of n and of q. [6]

$(1+px)^n$ $= 1 + \binom{n}{1}(px) + \binom{n}{2}(px)^2 + \binom{n}{3}(px)^3 + \dots$ $= 1 + npx + \frac{n(n-1)}{2}p^2x^2 + \frac{n(n-1)(n-2)}{6}p^3x^3 + \dots$ $= 1 - 12x + 28p^2x^2 + qx^3 + \dots$	**Marking Scheme modified from Version 2** *B1 for applying the formula for Binomial Coefficient*
By comparing coefficients of x^2, $\frac{n(n-1)}{2} = 28$ $n^2 - n = 56$ $n^2 - n - 56 = 0$ $(n-8)(n+7) = 0$ $n = 8$ or $n = -7$ (N.A.)	*M1 for forming equation.* $\binom{n}{2}p^2 = 28$ *is not acceptable* A1 for both answers, must reject -7.
By comparing coefficients of x, $np = -12$ $p = \frac{-12}{8}$ $p = -1\frac{1}{2}$	*M1 for forming equation.* $\binom{n}{3}p = -12$ *is not acceptable* *A1 for value of p*
$q = \frac{8(7)(6)}{6}\left(-1\frac{1}{2}\right)^3$ $q = -189$	A1 for value of q

7 **(a)** The surface area of a solid cube is increasing at $0.2\,\text{cm}^2/\text{s}$. Find the rate of increase of the volume when the length of a side is 1cm. [4]

$A = 6x^2$ $\frac{\mathrm{d}A}{\mathrm{d}x} = 12x$ $\frac{\mathrm{d}A}{\mathrm{d}t} = \frac{\mathrm{d}A}{\mathrm{d}x} \times \frac{\mathrm{d}x}{\mathrm{d}t}$ $0.2 = (12x)\frac{\mathrm{d}x}{\mathrm{d}t}$ $\frac{\mathrm{d}x}{\mathrm{d}t} = \frac{1}{60x}$ $V = x^3$ $\frac{\mathrm{d}V}{\mathrm{d}x} = 3x^2$ $\frac{\mathrm{d}V}{\mathrm{d}t} = \frac{\mathrm{d}V}{\mathrm{d}x} \times \frac{\mathrm{d}x}{\mathrm{d}t}$ $\frac{\mathrm{d}V}{\mathrm{d}t} = 3x^2\left(\frac{1}{60x}\right)$ $\frac{\mathrm{d}V}{\mathrm{d}t} = \frac{x}{20}$ Subst. $x = 1$, $\frac{\mathrm{d}V}{\mathrm{d}t} = \frac{1}{20}$ $\frac{\mathrm{d}V}{\mathrm{d}t} = 0.05\ \text{cm}^3/\text{s}$	M1 – form correct connect rates of change M1 – find $\frac{\mathrm{d}x}{\mathrm{d}t}$ M1 – form correct connect rates of change A1

(b) A curve has the equation $y = e^{1-8x}(\cos 2x)$, where $0 \le x \le \frac{\pi}{4}$. Find the gradient of the curve at the point where $y = 0$, leaving your answer in exact form. [4]

$y = e^{1-8x}(\cos 2x)$	
$\frac{dy}{dx} = e^{1-8x}(-2\sin 2x) + (\cos 2x)(-8)e^{1-8x}$	M2 – using product rule
$= -2e^{1-8x}(\sin 2x + 4\cos 2x)$	
When $y = 0$,	
$e^{1-8x}(\cos 2x) = 0$	
$e^{1-8x} = 0$ (NA) or $\cos 2x = 0$	
$2x = \frac{\pi}{2}, \frac{3\pi}{2}$	
$x = \frac{\pi}{4}, \frac{3\pi}{4}$ (NA)	M1 – finding x
$\frac{dy}{dx} = -2e^{1-8\left(\frac{\pi}{4}\right)}\left(\sin 2\left(\frac{\pi}{4}\right) + 2\cos 2\left(\frac{\pi}{4}\right)\right)$	
$\frac{dy}{dx} = -2e^{1-2\pi}$	A1

8 The equation of a curve is $y = -\frac{5}{8}x + \frac{6}{x}$, where $x > 0$. Given that the gradient of the normal at point P is 1, find

(i) the coordinates of P, [4]

$y=-\frac{5}{8}x+\frac{6}{x}$ $y=-\frac{5}{8}x+6x^{-1}$ $\frac{dy}{dx}=-\frac{5}{8}-6x^{-2}$ $-1=-\frac{5}{8}-\frac{6}{x^2}$ $-1+\frac{5}{8}=-\frac{6}{x^2}$ $-\frac{3}{8}=-\frac{6}{x^2}$ $3x^2=48$ $x^2=16$ $(x-4)(x+4)=0$ $x=4$ or $x=-4$ (N.A.) $y=-\frac{5}{8}(4)+\frac{6}{4}$ $y=-1$ P is $(4,\ -1)$	**Marking Scheme modified from Version 2** *B1 for differentiation* M1 – form equation from gradient of tangent A1 – find correct x A1 If student did not reject −4 in the previous part and find coordinates of P with −4, do not award final A1

(ii) the equation of the normal at the curve at P. [2]

$y-(-1)=1(x-4)$ $y=x-4-1$ $y=x-5$	M1 A1

9 **(a)** Solve the equation $\log_2(x+2)-\log_{\sqrt{2}}(x-1)=1$. [4]

$\log_2(x+2)-\log_{\sqrt{2}}(x-1)=1$	**Marking Scheme modified from Version 2**
$\log_2(x+2)-\dfrac{\log_2(x-1)}{\log_2\sqrt{2}}=1$	M1 for using Change-of-base Law
$\log_2(x+2)-\dfrac{\log_2(x-1)}{\log_2 2^{\frac{1}{2}}}=1$	
$\log_2(x+2)-\dfrac{\log_2(x-1)}{\frac{1}{2}\log_2 2}=1$	
$\log_2(x+2)-\dfrac{\log_2(x-1)}{\frac{1}{2}}=1$	
$\frac{1}{2}\log_2(x+2)-\log_2(x-1)=\frac{1}{2}$	
$\log_2(x+2)-2\log_2(x-1)=1$	
$\log_2\left(\dfrac{x+2}{x-1}\right)=1$	M1 for using Quotient Law
$\dfrac{x+2}{(x-1)^2}=2$	*M1 for removing log either by changing to index form or changing 1 to log₃3 and remove the log₃*
$x+2=2(x^2-2x+1)$	
$x+2=2x^2-4x+2$	
$2x^2-4x+2-x-2=0$	
$2x^2-5x=0$	
$x(2x-5)=0$	
$x=0$ or $2x-5=0$	
$x=0$ (N.A) or $x=2\frac{1}{2}$	A1 for both answers. Must reject 0.

(b) It is given that $\lg a - \lg b = \lg(a+b)$.

(i) Express a in terms of b. [2]

$\lg a - \lg b = \lg(a+b)$ $\lg \frac{a}{b} = \lg(a+b)$ $\frac{a}{b} = (a+b)$ $a = ab + b^2$ $a - ab = b^2$ $a = \frac{b^2}{1-b}$	M1 for using Quotient Law A1 for answer

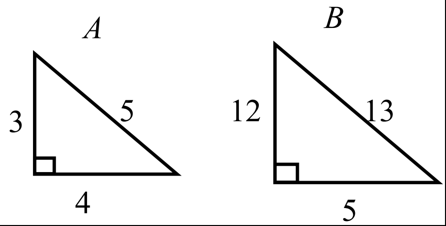
(ii) State the range of b. [1]

Since $a > 0$, $\therefore \frac{b^2}{1-b} > 0$ $\because b^2 > 0$, $\therefore 1 - b > 0$ $b < 1$ $\therefore 0 < b < 1$	B1

10 Given that $\sin A$ and $\cos B$ are in the same quadrant such that $\sin A = \frac{3}{5}$ and $\cos B = -\frac{5}{13}$.

Find, without using a calculator, the value of

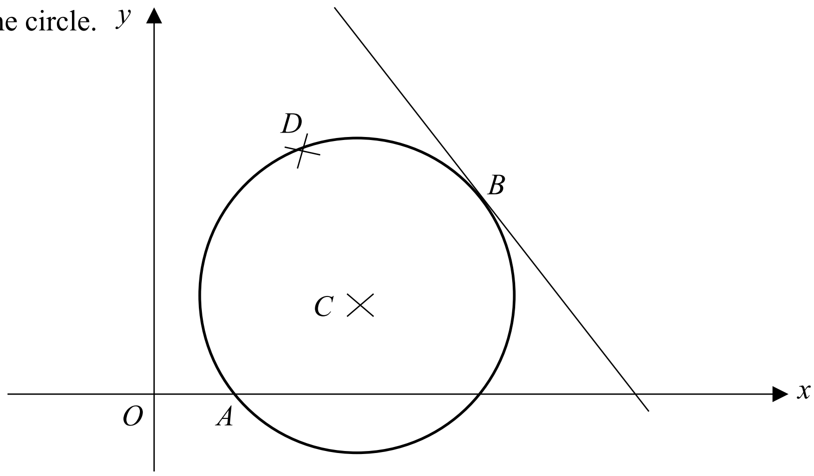
(i) $\cos(A-B)$, [2]

$\cos(A-B) = \cos A \cos B + \sin A \sin B$ $= \left(-\frac{4}{5}\right)\left(-\frac{5}{13}\right) + \left(\frac{3}{5}\right)\left(\frac{12}{13}\right)$ $= \frac{56}{65}$  	M1 A1

(ii) $\cos\frac{A}{2}$. [3]

	$\cos A = 2\cos^2\frac{A}{2} - 1$ $\cos A + 1 = 2\cos^2\frac{A}{2}$ $-\frac{4}{5} + 1 = 2\cos^2\frac{A}{2}$ $\frac{1}{5} = 2\cos^2\frac{A}{2}$ $\cos^2\frac{A}{2} = \frac{1}{10}$ $\cos\frac{A}{2} = \pm\sqrt{\frac{1}{10}}$ $90° < A < 180°$ $45° < \frac{A}{2} < 90°$ $\cos\frac{A}{2} = \frac{\sqrt{10}}{10}$ $\left(-\frac{\sqrt{10}}{10}\text{(NA)}\right)$	M1 M1 A1 with rejection

11 Points A, $B(5,4)$ and D lie on a circle with centre C and AB is the diameter of the circle. The line $y = -x + 9$ is a tangent at point $B(5,4)$ on the circle.

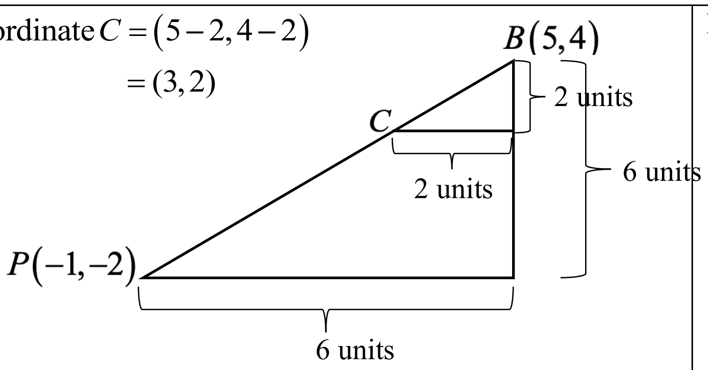


(a) Given that point P lies on BA extended and the x-coordinate of point P is -1, find the coordinates of P. [2]

Let the y-coordinate of P be a, $\frac{4-a}{5-(-1)} = 1$ $4 - a = 6$ $a = -2$ Coordinates of P is $(-1,-2)$	M1 – form equation A1

It is given that the distance BP is three times the distance BC.

(b) **(i)** Show that centre C is $(3, 2)$. [1]

Coordinate $C = (5-2, 4-2)$ $= (3, 2)$  	M1 – finding Coord C

(ii) Hence, find the equation of the circle. [2]

Distance $BC = \sqrt{(5-3)^2 + (4-2)^2}$ $= \sqrt{8}$ units Equation of circle is $(x-3)^2 + (y-2)^2 = 8$	M1 – finding distance BC A1

(c) Given that triangle BCD is right-angled at C, find the coordinates of D. [3]

Gradient of $CD = -1$ Equation of CD is $y - 2 = -1(x-3)$ $y = -x + 5$ Subst. $y = -x + 5$ into $(x-3)^2 + (y-2)^2 = 8$, $(x-3)^2 + (-x+5-2)^2 = 8$ $x^2 - 6x + 9 + x^2 - 6x + 9 = 8$ $2x^2 - 12x + 10 = 0$ $x^2 - 6x + 5 = 0$ $(x-1)(x-5) = 0$ $x = 1$ or $x = 5$ (NA) Subst. $x = 1$ into $y = -x + 5$, $y = -1 + 5$ $y = 4$ $D(1, 4)$	M1 – find equation CD M1 – solving simultaneous equations A1

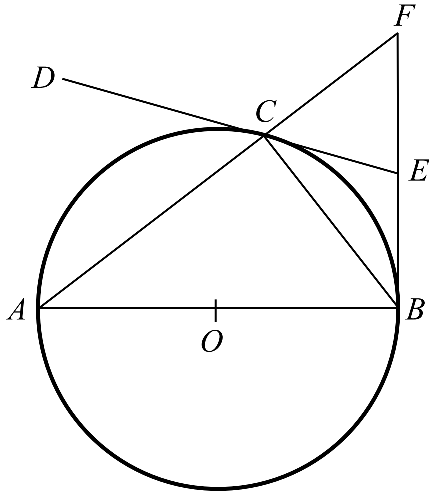
12 **(a)** Given that $y=\ln\left(\sin^2 x\right)$, show that $\dfrac{dy}{dx}=2\cot x$. [2]

$y=\ln\left(\sin^2 x\right)$ $=2\ln\left(\sin x\right)$ $\dfrac{dy}{dx}=2\left(\dfrac{1}{\sin x}\right)\left(\cos x\right)$ $=2\cot x$	M1: differentiation of ln M1: differentiation of sin x

(b) Hence show that the value of $\int_{\frac{\pi}{4}}^{\frac{\pi}{2}}\left(\cot x-\cos^2 2x\right)dx=a+b\ln 2$, where a and b are constants.[4]

$\int_{\frac{\pi}{4}}^{\frac{\pi}{2}}\cot x-\cos^2 2x\,dx=\frac{1}{2}\left[2\ln\sin x\right]_{\frac{\pi}{4}}^{\frac{\pi}{2}}-\int_{\frac{\pi}{4}}^{\frac{\pi}{2}}\frac{\cos 4x+1}{2}\,dx$	M1: for using (a) M1: for correct double angle formula
$=\frac{1}{2}\left[0-2\ln\frac{1}{\sqrt{2}}\right]-\frac{1}{2}\left[\frac{\sin 4x}{4}+x\right]_{\frac{\pi}{4}}^{\frac{\pi}{2}}$	
$=\frac{1}{2}\ln 2-\frac{1}{2}\left[\frac{\pi}{2}-\frac{\pi}{4}\right]$	M1: definite integrals
$=\frac{1}{2}\ln 2-\frac{\pi}{8}$	
$=-\frac{\pi}{8}+\frac{1}{2}\ln 2$	A1

13 In the diagram, AB is a diameter of the circle with centre O. DE and BF are tangents to the circle at C and B respectively. DCE and BEF are straight lines. Prove that



(a) ΔABC and $\Delta\ BFC$ are similar, [3]

$\angle ACB = 90°$ (rt. $\angle$ in a semi-circle)	M1
$\angle FCB = 90°$ (adj $\angle$ on straight line)	M1
$\therefore \angle ACB = \angle BCF$	
$\angle CBF = \angle CAB$ (tangent-chord theorem)	
Hence ΔABC and ΔBFC are similar.	M1

(b) Hence or otherwise, show that ΔCFE is an isosceles triangle. [4]

$\angle ABC = \angle DCA$ (tangent-chord theorem)	M1
$\angle FCE = \angle DCA$ (vertically opp. $\angle$s)	M1
$\angle ABC = \angle CFE$ (similar Δs)	M1
Since $\angle CFE = \angle FCE$, ΔCFE is isosceles (base $\angle$s are equal)	A1

14 The mass, m grams, of the decomposition of a radio-active substance is given by the formula $m = ae^{-bt}$, where a and b are constants, and t is the number of weeks after the initial measurement of the mass. Experimental values of t and m are given in the table below.

t	1	2	3	4	5
m	1.51	0.454	0.137	0.0224	0.0124

(i) What does the constant a refer to?[1]

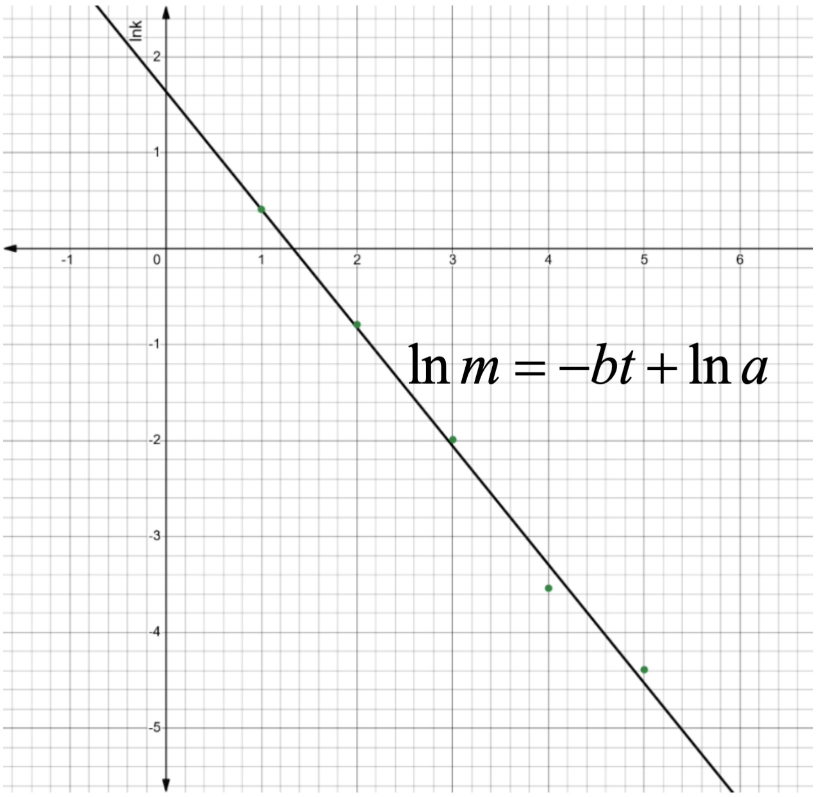
The constant a refers to the **initial mass** of the radio-active substance. Or The constant a refers to the **mass** of the radio-active substance **when t = 0**.	B1

(ii) By drawing a suitable straight line graph using the data provided, estimate the value of a and of b.[7]

$m = ae^{-bt}$ $\ln m = \ln\left(ae^{-bt}\right)$ $\ln m = \ln a - bt$ $\ln m = -bt + \ln a$	M1 A1

t	1	2	3	4	5
ln$m$	0.412	−0.790	−1.99	−4.00	−4.39

T1



G1: Plot all the points
G2: Best fit line
Penalise under Clarity if missing in labelling of axis / equation

$-b = \dfrac{-4-1}{4.7-0.5}$ $-b \approx -1.1904$ $b \approx 1.19$ $\ln a = \text{Y-intercept}$ $\ln a = 1.6$ $a \approx e^{1.6}$ $a \approx 4.9530$ $a \approx 4.95$	A2 (actual value of $a = 5 \pm 0.3$ and $b = 1.2 \pm 0.3$)

(iii) It is discovered that one of the readings of the mass, m, is incorrect. Using your graph, estimate the correct value for this incorrect reading. [2]

When $t = 4$, $\ln m = -3.15$ $m = e^{-3.15}$ $m \approx 0.042852$ $m \approx 0.0429$	M1 changing ln to exp A1

NAME: ________________________________ (   )   CLASS: 4 (   )

ANGLICAN HIGH SCHOOL
SECONDARY FOUR
PRELIMINARY EXAMINATIONS 2025

S4

ADDITIONAL MATHEMATICS **4049/02**

Paper 2 **26 August 2025**

2 hours 15 minutes

Candidates answer on the Question Paper.

READ THESE INSTRUCTIONS FIRST

Write your name, index number and class in the spaces at the top of this page.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.

The total number of marks for this paper is 90.

For Examiners' Use

Question	Marks	Question	Marks		
1		8			
2		9		Units	
3		10		Clarity / Logic	
4		11		Precision / Accuracy	
5				Total:	
6					
7					
Parent's Name & Signature:					90
Date:					

This paper consists of **18** printed pages.

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\text{Area of } \Delta = \frac{1}{2}ab \sin C$$

1 **(a)** Show that $\dfrac{4}{\cot\theta - \tan\theta} = 2\tan 2\theta$. [4]

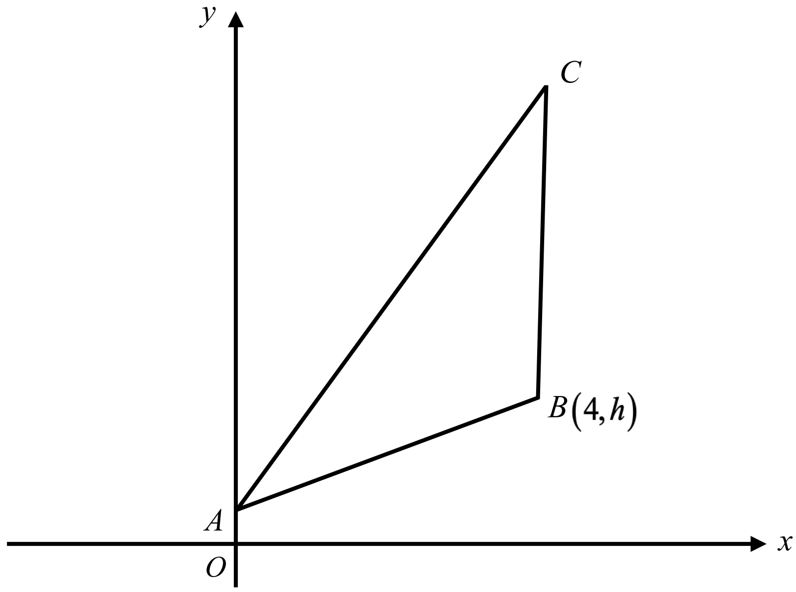
(b) Hence, solve the equation $\dfrac{4}{\cot 2x - \tan 2x} = 5$ for $0 < x < \pi$. [3]

2 The expression $4x^3 + ax + b$, where a and b are constants, has a factor of $x - 1$, and a remainder of -9 when divided by $x + 2$.

(a) Find the value of a and of b. [4]

(b) Hence solve the equation $4x^3 + ax + b = 0$ expressing the non-integer roots in the form of $\frac{c+\sqrt{d}}{2}$, where c and d are constants. [4]

3 The diagram shows an isosceles triangle ABC, where $AB = BC$ and the line segment BC is parallel to the y-axis. Point A lies on the y-axis and the equation of line AC is $y = 2x + 3$. Point B is $(4, h)$, where h is a positive constant.



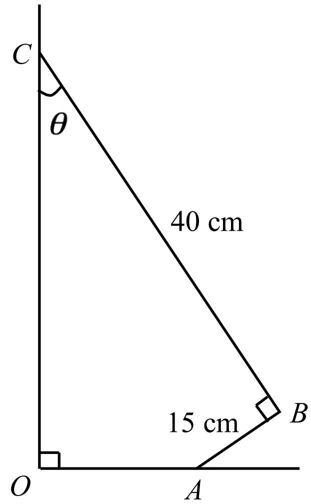
(a) Find the coordinates of A, B and C. [4]

Given that the point D is such that $ABCD$ is a kite, and the line segment CD is parallel to the x-axis.

(b) Find the coordinates of D. [4]

(c) Find the area of the kite $ABCD$. [2]

4 In the diagram, it is given that $AB = 15$ cm, $BC = 40$ cm and $\angle OCB = \theta$. The vertical line OC is perpendicular to OA and AB is perpendicular to BC.



(a) Show that $OC = 15\sin\theta + 40\cos\theta$. [2]

(b) Express OC in the form $R\sin(\theta+\alpha)$, where R is a positive constant and α is an acute angle in degrees. [3]

(c) Given that OC is at the maximum, determine the shortest distance of B to OC. [3]

5 **(a)** Show $\frac{\mathrm{d}}{\mathrm{d}x}\left[\ln\left(\tan 3x\right)\right] = \frac{k}{\sin 6x}$, stating the value of the constant k. [4]

(b) Differentiate $\ln\sqrt{\frac{7x}{x^2-1}}$ with respect to x. [3]

6 The curve $y = f(x)$ is such that $f''(x) = 2e^{x} + e^{-2x}$.

The y-axis is a normal to the curve at the point Q, where $y = 1$.

(a) Show that Q is a stationary point. [2]

(b) Hence, find an expression for $f(x)$. [6]

7 A waterwheel rotates at 1 revolution per 12 seconds. Initially, Point *A,* marked on the highest point of the rim of the wheel is 6 m above the water surface. It reaches 2 m below the water surface 6 seconds later.

The motion of point *A* can be modelled using the trigonometric equation $h = b\cos(kt) + 2$, where the height of point *A* above the water surface, *h*, is a function of time *t*, in seconds.

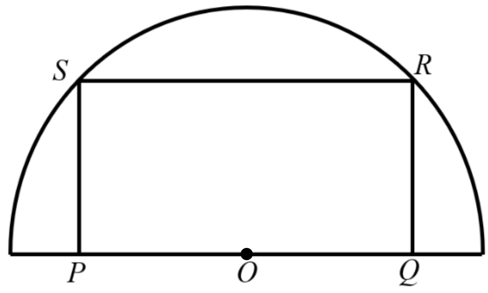
(a) Show that $b = 4$ and $k = \dfrac{\pi}{6}$. [2]

(b) On the axes below, sketch the graph of *h,* for $0 \le t \le 18$. [3]

A similar waterwheel at a different location has a point, *B,* marked on its rim. The motion of point *B* is modelled by the equation $h_B = 4\cos(\dfrac{\pi}{6}t) + 4$.

(c) By drawing a suitable straight line on the same axes above, determine the number of time(s) when point *B* touches the water surface from $0 \le t \le 18$. [2]

8 In the figure, *PQRS* is a rectangle which fits inside a semicircle of radius 8 cm and centre *O*.

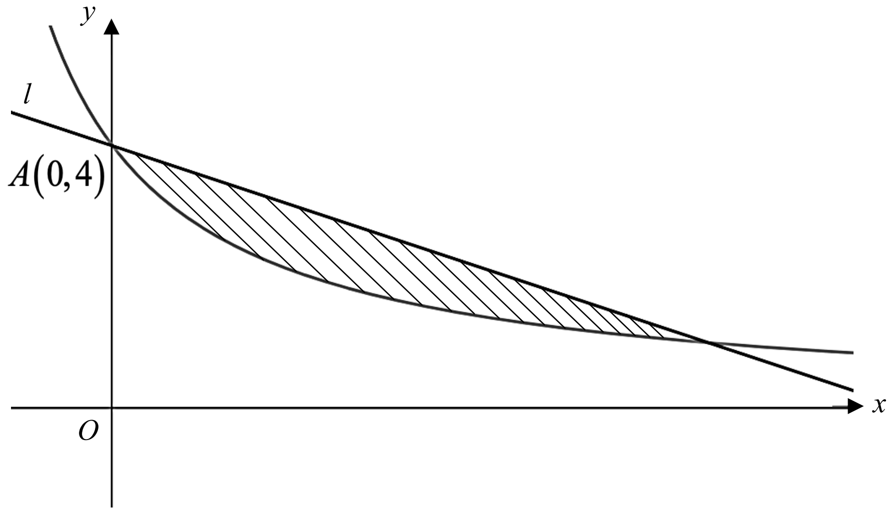


If $PQ = x$ cm and $QR = y$ cm,

(a) Show that the area of the rectangle, $A\text{ cm}^2$, is given by $A = \frac{x}{2}\sqrt{256 - x^2}$. [3]

(b) The maximum area is obtained when the value of x, which can vary, gives a stationary value of A. Find the maximum value of A. [6]

9 The diagram shows part of the curve $y = \dfrac{8}{2+3x}$. A line, l, passes through the curve at point $A(0,4)$ and is parallel to the tangent to the curve at $x = -2$.



(a) Show that the equation of line l is $y = -\dfrac{3}{2}x + 4$. [3]

(b) Find the area of the shaded region and leave your answer in the form $a+b\ln 2$. [6]

10 A particle travels in a straight line such that at time t seconds after leaving a point A, which has a displacement of 2 metres from a fixed point O, its velocity, v ms^{-1} is given by

$v = 3t^2 - 21t + 18$.

Find,

(a) the minimum velocity of the particle, [3]

(b) the time(s) at which the particle is instantaneously at rest, [1]

(c) the total distance travelled in the first 7 seconds. [6]

11 **(a)** Given the function $y = \dfrac{m-2x}{x^2-3}$, find $\dfrac{\mathrm{d}y}{\mathrm{d}x}$. [3]

(b) Hence, find the range of values of m such that the function $y = \dfrac{m-2x}{x^2-3}$ is always increasing. [4]

END OF PAPER

NAME: ______________________________ (   )   CLASS: 4 (   )

ANGLICAN HIGH SCHOOL
SECONDARY FOUR
PRELIMINARY EXAMINATIONS 2025

S4

ADDITIONAL MATHEMATICS **4049/02**

Paper 2 **MARKING SCHEME**

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The total number of marks for this paper is 90.

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5				Total:	
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Parent's Name & Signature: Date:				**90**	

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1 **(a)** Show that $\dfrac{4}{\cot\theta - \tan\theta} = 2\tan 2\theta$. [4]

(a)	$\text{LHS} = \dfrac{4}{\cot\theta - \tan\theta}$ $= \dfrac{4}{\dfrac{\cos\theta}{\sin\theta} - \dfrac{\sin\theta}{\cos\theta}}$ $= \dfrac{4}{\dfrac{\cos^2\theta - \sin^2\theta}{\cos\theta\sin\theta}}$ $= \dfrac{4\cos\theta\sin\theta}{\cos^2\theta - \sin^2\theta} \div \dfrac{\cos^2\theta}{\cos^2\theta}$ $= \dfrac{4\tan\theta}{1-\tan^2\theta}$ $= 2\tan 2\theta$	M1 for using changing tan and cot M1 M1 A1

(b) Hence, solve the equation $\dfrac{4}{\cot 2x - \tan 2x} = 5$ for $0 < x < \pi$. [3]

(b)	$2\tan 4\theta = 5$ $\tan 4\theta = \dfrac{5}{2}$ basic angle $= 1.1903$ $4\theta = 1.1903,\ \pi + 1.1903,\ 2\pi + 1.1903,\ 3\pi + 1.1903$ $\theta = 0.298,\ 1.08,\ 1.87\ ,\ 2.65$	M1 M1 A1

2 The expression $4x^3 + ax + b$, where a and b are constants, has a factor of $x - 1$, and a remainder of -9 when divided by $x + 2$.

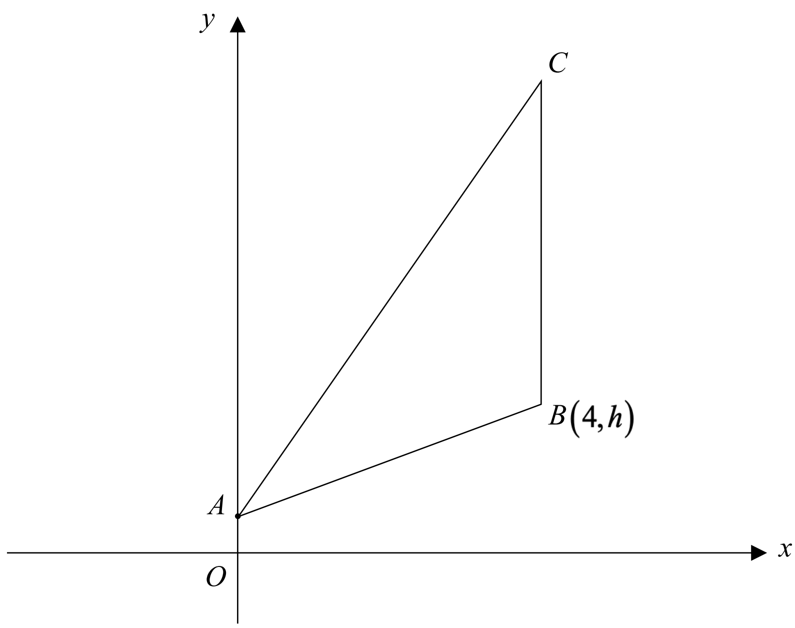
(a) Find the value of a and of b. [4]

Let $\mathrm{f}(x) = 4x^3 + ax + b$ $\mathrm{f}(1) = 0$ $4(1)^3 + a(1) + b = 0$ $4 + a + b = 0$ $a + b = -4 ------(1)$ $\mathrm{f}(-2) = -9$ $4(-2)^3 + a(-2) + b = -9$ $-32 - 2a + b = -9$ $-2a + b = 23 ------(2)$ $(1)-(2)$: $3a = -27$ $a = -9$ Sub $a = -9$ into (1) $-9 + b = -4$ $b = 5$ $\therefore a = -9$ and $b = 5$	 M1 for forming first equation M1 for forming second equation M1 for solving A1 for both answers

(b) Hence solve the equation $4x^3 + ax + b = 0$ expressing the non-integer roots in the form of $\dfrac{c+\sqrt{d}}{2}$, where c and d are constants. [4]

$\mathrm{f}(x) = 4x^3 - 9x + 5$ $\mathrm{f}(x) = (x-1)(4x^2 + cx - 5)$ By comparing coefficients of x, $-9 = -5 - c$ $c = 4$ $\mathrm{f}(x) = (x-1)(4x^2 + 4x - 5)$	M1 for getting quadratic factor
$4x^3 - 9x + 5 = 0$ $(x-1)(4x^2 + 4x - 5) = 0$ $4x^2 + 4x - 5 = 0$ or $x = 1$ $x = \dfrac{-4 \pm \sqrt{4^2 - 4(4)(-5)}}{2(4)}$ $x = \dfrac{-4 \pm \sqrt{96}}{8}$ $x = \dfrac{-4 \pm 4\sqrt{6}}{8}$ $x = \dfrac{-1 \pm \sqrt{6}}{2}$ or $x = 1$	M1 for using quadratic formula to solve the quadratic equation A2 for all answers

3 The diagram shows an isosceles triangle ABC, where $AB = BC$ and the line segment BC is parallel to the y-axis. Point A lies on the y-axis and the equation of line AC is $y = 2x + 3$. Point B is $(4, h)$, where h is a positive constant.



(a) Find the coordinates of A, B and C. [4]

(a)	Since A is on the y-axis, $A(0,3)$ Since BC is parallel to the y-axis, and x-coordinate of B is 4, x-coordinate of $C = 4$. $y = 2(4)+3$ $= 11$ $C(4,11)$ $\sqrt{4^2+(h-3)^2} = 11-h$ $16+(h-3)^2 = (11-h)^2$ $16+h^2-6h+9 = 121-22h+h^2$ $16h = 96$ $h = 6$ $B(4,6)$	B1 B1 M1: form equation using length formula A1

Given that the point D is such that $ABCD$ is a kite, and the line segment CD is parallel to the x-axis.

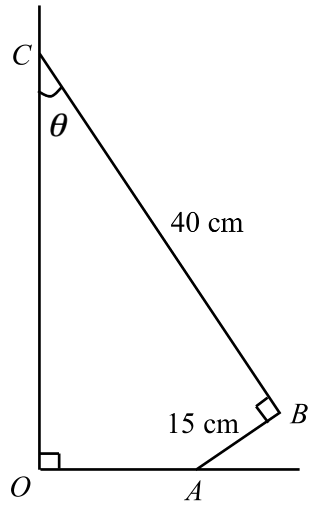
(b) Find the coordinates of D. [4]

(b)	Gradient of $BD = -\frac{1}{2}$ $y=-\frac{1}{2}x+c$ $6=-\frac{1}{2}(4)+c$ $c=8$ Equation of BD: $y=-\frac{1}{2}x+8$ Since CD is parallel to the x-axis, y-coordinate of D = 11. $11=-\frac{1}{2}x+8$ $3=-\frac{1}{2}x$ $x=-6$ $D(-6,11)$	M1: for finding gradient A1 M1: for subst $y = 11$ A1

(c) Find the area of the kite $ABCD$. [2]

(c)	Area $=\frac{1}{2}\begin{vmatrix} 0 & 4 & 4 & -6 & 0 \\ 3 & 6 & 11 & 11 & 3 \end{vmatrix}$ $=\frac{1}{2}\left[(0+44+44-18)-(12+24-66+0)\right]$ $=50\text{units}^2$	M1 in anticlockwise A1

4 In the diagram, it is given that $AB = 15$ cm, $BC = 40$ cm and $\angle OCB = \theta$. The vertical line OC is perpendicular to OA and AB is perpendicular to BC.



(a) Show that $OC = 15\sin\theta + 40\cos\theta$. [2]

(i)	$\sin\theta = \dfrac{AB}{15}$ $\cos\theta = \dfrac{BC}{40}$ $OC = AB\sin\theta + BC\cos\theta$ $= 15\sin\theta + 40\cos\theta$	B1 B1

(b) Express OC in the form $R\sin(\theta + \alpha)$, where R is a positive constant and α is an acute angle in degrees. [3]

(ii)	$R = 40^2 + 15^2 = \sqrt{1825}$ $\tan\alpha = \dfrac{40}{15}$ $\alpha = 69.444...$ $OC = 42.7\sin(\theta + 69.4°)$	M1 M1 A1

(c) Given that OC is at the maximum, determine the shortest distance of B to OC. [3]

(iii)	$\max OC = 42.7$ cm when $\theta + 69.444...° = 90°$ $\theta = 20.556°$ shortest distance $= 40\sin(20.556°)$ $= 14.0$cm	M1 M1 A1

5 **(i)** Show $\frac{\mathrm{d}}{\mathrm{d}x}\left[\ln\left(\tan 3x\right)\right]=\frac{k}{\sin 6x}$, stating the value of the constant k. [4]

(i)	$\frac{\mathrm{d}}{\mathrm{d}x}\left[\ln\left(\tan 3x\right)\right]=\frac{1}{\tan 3x}\left(\sec^2 3x\right)(3)$	M1 – differentiate correctly
	$=\frac{3\cos 3x}{\sin 3x\cos^2 3x}$	M1 – simplify trigo functions
	$=\frac{3}{\sin 3x\cos 3x}$	
	$=\frac{3(2)}{2\sin 3x\cos 3x}$	M1 – correct application of sine double angle formula
	$=\frac{6}{\sin 6x}$	
	$k=6$	A1

(ii) Differentiate $\ln\sqrt{\frac{7x}{x^2-1}}$ with respect to x. [3]

(ii)	$\frac{\mathrm{d}}{\mathrm{d}x}\left(\ln\sqrt{\frac{7x}{x^2-1}}\right)=\frac{\mathrm{d}}{\mathrm{d}x}\left(\frac{1}{2}\ln\frac{7x}{x^2-1}\right)$	M1 – use logarithm laws to simplify
	$=\frac{\mathrm{d}}{\mathrm{d}x}\left(\frac{1}{2}\left[\ln 7x-\ln\left(x^2-1\right)\right]\right)$	M1 – differentiate $\ln 7x$ and $\ln\left(x^2-1\right)$
	$=\frac{1}{2}\left(\frac{7}{7x}\right)-\frac{1}{2}\left(\frac{2x}{x^2-1}\right)$	
	$=\frac{1}{2}\left(\frac{x^2-1-2x^2}{x\left(x^2-1\right)}\right)$	
	$=-\frac{x^2+1}{2x\left(x^2-1\right)}$	A1

6 The curve $y = \mathrm{f}(x)$ is such that $\mathrm{f}''(x) = 2e^{x} + e^{-2x}$.

The y-axis is a normal to the curve at the point Q, where $y = 1$.

(a) Show that Q is a stationary point. [2]

(a)	The x-axis is perpendicular to the y-axis. Since the y-axis is a normal, the x-axis is parallel to a tangent at $y = 1$. Since the gradient of the x-axis is 0, $\frac{dy}{dx} = 0$ at $y = 1$. Therefore, there is a stationary point.	B1: finding $\frac{dy}{dx}$ B1: stating why gradient = 0

(b) Hence, find an expression for $\mathrm{f}(x)$. [6]

(b)	$\mathrm{f}'(x) = \int 2e^{x} + e^{-2x}\, dx$ $= 2e^{x} + \frac{e^{-2x}}{-2} + c$ $= 2e^{x} - \frac{1}{2e^{2x}} + c$	M1: for correct integration
	At $x = 0$, $\mathrm{f}'(x) = 0$, $2 - \frac{1}{2} + c = 0$ $c = -\frac{3}{2}$	M1
	$\mathrm{f}'(x) = 2e^{x} - \frac{1}{2e^{2x}} - \frac{3}{2}$	A1
	$\mathrm{f}(x) = \int 2e^{x} - \frac{1}{2e^{2x}} - \frac{3}{2}\, dx$ $= 2e^{x} - \frac{1}{2}\left(\frac{e^{-2x}}{-2}\right) - \frac{3}{2}x + c$ $= 2e^{x} + \frac{1}{4e^{2x}} - \frac{3}{2}x + c$	M1
	At $x = 0$, $y = 1$, $1 = 2 + \frac{1}{4} + c$ $c = -\frac{5}{4}$	M1
	$\mathrm{f}(x) = 2e^{x} + \frac{1}{4e^{2x}} - \frac{3}{2}x - \frac{5}{4}$	A1

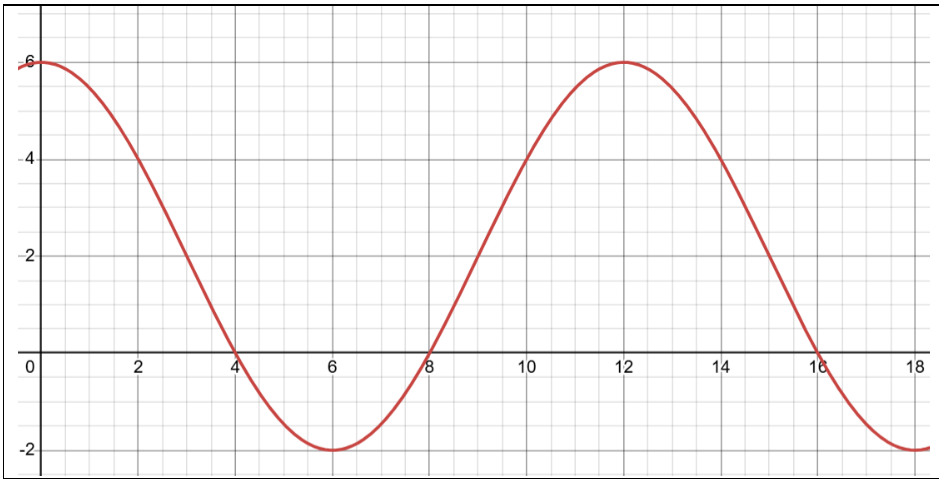
7 A waterwheel rotates at 1 revolution per 12 seconds. Initially, Point A, marked on the highest point of the rim of the wheel is 6 m above the water surface. It reaches 2 m below the water surface 6 seconds later.

The motion of point A can be modelled using the trigonometric equation $h = b\cos(kt) + 2$, where the height of point A above the water surface, h, is a function of time t, in seconds.

(i) Show that $b = 4$ and $k = \dfrac{\pi}{6}$. [2]

$b = \dfrac{6+2}{2} = 4$	B1
$k = \dfrac{2\pi}{12} = \dfrac{\pi}{6}$	B1

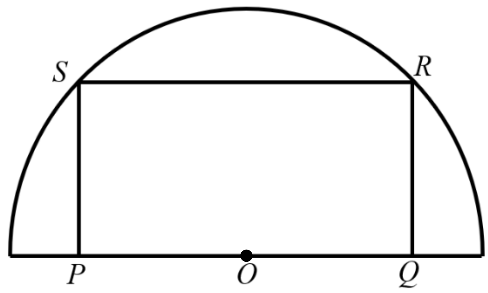
(ii) On the axes below, sketch the graph of h, for $0 \le t \le 18$. [3]

	B1 for shape (regardless no of cycles) B1 for period and correct x values (5 per cycle). Must see all 9 values. B1 for correct max, min and axis of equation Note: zero mark if student give wrong curve totally.

(iii) A similar waterwheel at a different location has a point, B, marked on its rim. The motion of point B is modelled by the equation $h_B = 4\cos(\dfrac{\pi}{6}t) + 4$. By drawing a suitable line on the same axis above, determine the number of time(s) when point B touches the water surface from $0 \le t \le 18$. [2]

2 times	M1 for drawing line of $y = -2$ A1

8 In the figure, $PQRS$ is a rectangle which fits inside a semicircle of radius 8 cm and centre O.



If $PQ = x$ cm and $QR = y$ cm,

(a) Show that the area of the rectangle, $A\,\text{cm}^2$, is given by $A = \frac{x}{2}\sqrt{256 - x^2}$. [3]

(a)	$A = xy$ $8^2 = y^2 + \left(\frac{x}{2}\right)^2$ $64 = y^2 + \frac{x^2}{4}$ $y = \sqrt{\frac{256 - x^2}{4}}$ $A = x\left(\sqrt{\frac{256 - x^2}{4}}\right)$ $A = \frac{x}{2}\sqrt{256 - x^2}$	M1 – form Pythagoras Theorem M1 – make y the subject A1

(b) The maximum area is obtained when the value of x, which can vary, gives a stationary value of A. Find the maximum value of A. [6]

<table>
<tr>
<td>(b)</td>
<td>

$A=\frac{x}{2}\sqrt{256-x^2}$

$\frac{\mathrm{d}A}{\mathrm{d}x}=\left(\frac{1}{2}x\right)\left(\frac{1}{2}\right)\left(256-x^2\right)^{-\frac{1}{2}}(-2x)+\left(256-x^2\right)^{\frac{1}{2}}\left(\frac{1}{2}\right)$

$=\frac{1}{2}\left(256-x^2\right)^{-\frac{1}{2}}\left[-x^2+256-x^2\right]$

$=\frac{-2x^2+256}{2\sqrt{256-x^2}}$

$\frac{\mathrm{d}A}{\mathrm{d}x}=0$

$\frac{-2x^2+256}{2\sqrt{256-x^2}}=0$

$-2x^2+256=0$

$x^2=128$

$x=\sqrt{128}$

$A=\frac{\sqrt{128}}{2}\sqrt{256-\left(\sqrt{128}\right)^2}$

$A=64$

<table>
<tr><td>x</td><td>$\sqrt{128}^-$</td><td>$\sqrt{128}$</td><td>$\sqrt{128}^+$</td></tr>
<tr><td>Sign of $\frac{\mathrm{d}A}{\mathrm{d}x}$</td><td>-</td><td>0</td><td>+</td></tr>
<tr><td>Sketch of tangent</td><td>/</td><td>—</td><td>\</td></tr>
</table>

By first derivative testing, A is a maximum.
</td>
<td>
M1 – differentiate correctly

M1 – equate derivative to 0

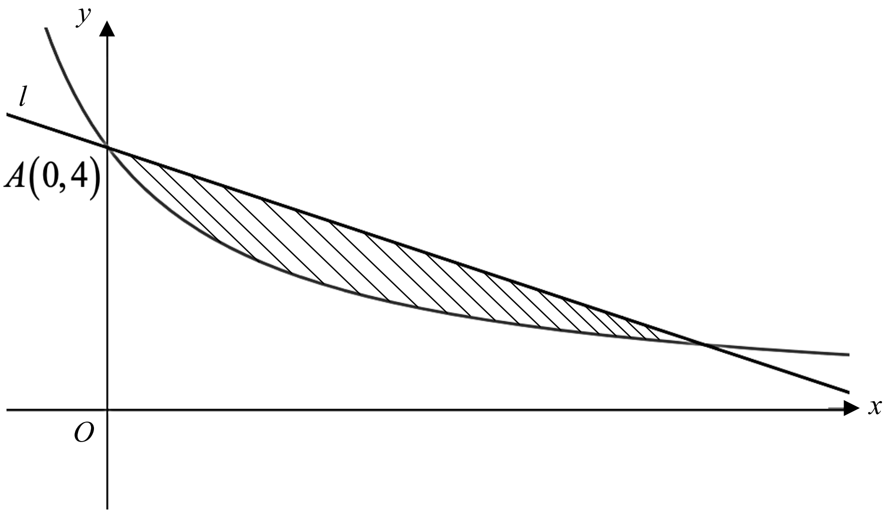
M1 – find x

A1

M1

A1
</td>
</tr>
</table>

9 The diagram shows part of the curve $y = \dfrac{8}{2+3x}$. A line, l, passes through the curve at point $A(0,4)$ and is parallel to the tangent to the curve at $x = -2$.



(a) Show that the equation of line l is $y = -\dfrac{3}{2}x + 4$. [3]

(a)	$\dfrac{dy}{dx} = 8(-1)(2+3x)^{-2}(3)$ $= -\dfrac{24}{(2+3x)^2}$ At x = -2, Gradient $= -\dfrac{24}{(2-6)^2}$ $= -\dfrac{3}{2}$ Equation of l: $y = -\dfrac{3}{2}x + 4$	M1 M1 A1

(b) Find the area of the shaded region. Leave your answer in the form $a+b\ln 2$. [6]

(b)	$\dfrac{8}{2+3x}=-\dfrac{3}{2}x+4$ $\dfrac{16}{2+3x}=-3x+8$ $16=-6x-9x^2+16+24x$ $9x^2-18x=0$ $x(x-2)=0$ $x=0 \quad or \quad 2$ At $x=2$, $y=-\dfrac{3}{2}(2)+4$ $=1$ Area $=\dfrac{1}{2}(4+1)(2)-\displaystyle\int_0^2\dfrac{8}{2+3x}\,dx$ $=5-\dfrac{8}{3}\left[\ln(2+3x)\right]_0^2$ $=5-\dfrac{8}{3}\left[\ln 8-\ln 2\right]$ $=5-\dfrac{16}{3}\ln 2$	M1: for equating the equations A1: for coordinates of other intersection point M1: for area of trapezium / integrating line minus integrating curve M1: for integration M1: for definite integrals A1

10 A particle travels in a straight line such that at time t seconds after leaving a point A, which has a displacement of 2 metres from a fixed point O, its velocity, v ms^{-1} is given by

$v = 3t^2 - 21t + 18$.

Find,

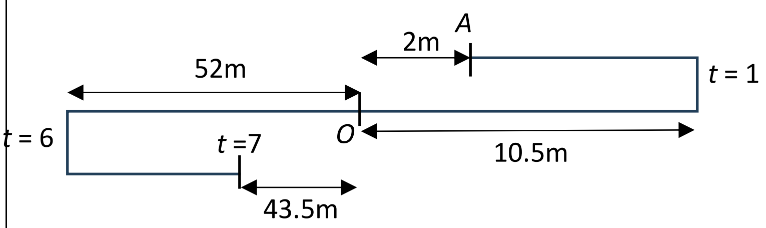
(a) the minimum velocity of the particle, [3]

(a)	$\frac{dv}{dt} = 6t - 21$ $6t - 21 = 0$ $t = \frac{21}{6} s$ $v = 3\left(\frac{21}{6}\right)^2 - 21\left(\frac{21}{6}\right) + 18$ $= -\frac{75}{4} ms^{-1}$ $\frac{d^2v}{dt^2} = 6 > 0$ Therefore, $v = -\frac{75}{4} ms^{-1}$ is the minimum velocity.	M1: for finding t (Award even if have careless mistake) A1 B1: for checking max/min (Award even if value of v is wrong)

(b) the time(s) at which the particle is instantaneously at rest, [1]

(b)	$3t^2 - 21t + 18 = 0$ $t^2 - 7t + 6 = 0$ $(t-6)(t-1) = 0$ $t = 1 \text{ or } 6$	B1

(c) the total distance travelled in the first 7 seconds. [6]

$s=\int 3t^2-21t+18\,dt$ $=t^3-\frac{21}{2}t^2+18t+c$ At $t=0$, $s=2$, therefore, $c=2$ $s=t^3-\frac{21}{2}t^2+18t+2$	B1: for getting s
At $t=1$, $s=1^3-\frac{21}{2}1^2+18(1)+2$ $=10.5m$	M1
At $t=6$, $s=6^3-\frac{21}{2}(6)^2+18(6)+2$ $=-52m$	M1
At $t=7$, $s=7^3-\frac{21}{2}(7)^2+18(7)+2$ $=-43.5m$	M1
	
Total Distance $=(10.5-2)+10.5+52+(52-43.5)$ $=79.5m$	M1 A1

11 **(a)** Given the function $y = \dfrac{m-2x}{x^2-3}$, find $\dfrac{\mathrm{d}y}{\mathrm{d}x}$. [3]

$y = \dfrac{m-2x}{x^2-3}$ $\dfrac{dy}{dx} = \dfrac{(x^2-3)(-2)-(m-2x)(2x)}{(x^2-3)^2}$ $= \dfrac{-2x^2+6-2mx+4x^2}{(x^2-3)^2}$ $= \dfrac{2x^2+6-2mx}{(x^2-3)^2}$	M2 – finding numerator and denominator $\dfrac{\mathrm{d}y}{\mathrm{d}x}$ A1

(b) Hence, find the range of values of m such that the function $y = \dfrac{m-2x}{x^2-3}$ is always increasing. [4]

Since $\dfrac{2x^2+6-2mx}{(x^2-3)^2} > 0$ and $(x^2-3)^2 > 0$, $\therefore\ 2x^2-2mx+6>0$ $2x^2-2mx+6 = 2(x^2-mx+3)$ $= 2\left[\left(x-\dfrac{m}{2}\right)^2-\left(\dfrac{m}{2}\right)^2+3\right]$ $= 2\left(x-\dfrac{m}{2}\right)^2-\dfrac{m^2}{2}+6$ For $2x^2-2mx+6>0$, $-\dfrac{m^2}{2}+6>0$ $\dfrac{m^2}{2}<6$ $m^2<12$ $m^2-12<0$ $(m+\sqrt{12})(m-\sqrt{12})<0$ $-2\sqrt{3}<m<2\sqrt{3}$	M1 – stating $\dfrac{\mathrm{d}y}{dx} > 0$ or wrong expression > 0 M1 – completing the square method M1 – stating $-\dfrac{m^2}{2}+6>0$ A1

Alternative Solution Since $\frac{2x^2+6-2mx}{\left(x^2-3\right)^2}>0$ and $\left(x^2-3\right)^2>0$, $\therefore\ 2x^2-2mx+6>0$ $2x^2-2mx+6=0$ discriminant <0 $(-2m)^2-4(2)(6)<0$ $4m^2-48<0$ $m^2-12<0$ $\left(m+\sqrt{12}\right)\left(m-\sqrt{12}\right)<0$ $-2\sqrt{3}<m<2\sqrt{3}$	M1 – stating $\frac{dy}{dx}>0$ or wrong expression >0 M1 – finding discriminant M1 – stating discriminant <0 A1

END OF PAPER