BEDOK SOUTH SECONDARY SCHOOL
PRELIMINARY EXAMINATION 2025

4E5N

CANDIDATE NAME		CLASS		REGISTER NUMBER		

ADDITIONAL MATHEMATICS
Paper 1

4049 / 01
2 hours 15 minutes

90

Answer **all** the questions.

1 Prove that $\sin 2x - \cos 2x \tan x = \tan x$. [4]

2 Show that the solution of $2^{3x+4} \times 5^{2x+1} = 5^{3(x+1)} \times 16^{x}$ is $2\lg\frac{4}{5}$. [4]

3 Giving your answer in the form $\frac{c+d\sqrt{2}}{3}$, solve, without using a calculator,

$$x\sqrt{18} = 3x + \sqrt{32}.$$ [5]

4 A square of area $(11+\sqrt{120})\ \text{cm}^2$ has a length of $(\sqrt{a}+\sqrt{b})$ cm, where $a < b$.
Without using a calculator, find the values of *a* and *b*. [5]

5 The line $3x + 4y = 13$ intersects the curve $y = \frac{6x-10}{3x-1}$ at points *P* and *Q*.
Calculate the exact length of the line segment *PQ*. [5]

6 The coefficient of x^3 in the expansion of $(10+kx)\left(2-\frac{1}{2}x\right)^8$ is zero.
Find the value of the constant *k*. [5]

7 Solve the equation $\frac{5\cos x + \sin 2x}{\cos 2x + 5\sin x} = \cot x$ for $0 \le x \le 180°$. [6]

8 **(a)** Show that the derivative of $x(x-3)^4$ with respect to *x* is $(5x-3)(x-3)^3$. [2]

(b) Hence, given that $y = \frac{x(x-3)^4}{5x-3}$ and y is decreasing at a constant rate of 70 units/s, calculate the rate of change of *x* when $x = 1$. [4]

9 **(a)** Express $y = 5 - 12x - 3x^2$ in the form $y = a(x+b)^2 + c$ and hence show that y can never be greater than 20. [3]

(b) Explain why there are no values of k for which the curve $y = (k-1)x^2 + 2(k+2)x + k + 3$ is always positive. [4]

10 A rectangular field has sides $(3x-5)$ m and $(x-10)$ m. Its area is at most 200 m^2.

(a) Find the range of values of x that satisfies the above sides and area conditions. [4]

(b) Justify whether a fence of 98 m is enough to enclose the field. [3]

11 **(a)** State the range of values of x for which the equation below is valid.

$$2\log_3(4-x) - \log_9(x-4)^2 = 2$$ [1]

(b) Express $2\log_3(4-x) - \log_9(x-4)^2 = 2$ as a quadratic equation $x^2 + bx + c = 0$ and explain why there is only one real solution. [6]

12 A quadratic curve is given by $y = hx^2 - 2x - 9h + 6$, where h is a constant.

(a) Show that the equation $y = 0$ has real roots for all values of h except $h = 0$. [3]

(b) State the value of h in the case where $y = 0$ has two real and equal roots. [1]

(c) Given that the line $y = 2x - h - 12$ meets the curve $y = hx^2 - 2x - 9h + 6$, find the range of values of h. [5]

13 It is given that $f(x) = x^2 e^{x+2}$.

(a) Show that the range of values of x for which $f(x)$ is a decreasing function is $-2 < x < 0$. [4]

(b) The gradient with the least value is in the range $-2 < x < 0$.
Find the value of this gradient, giving your answer in exact form. [5]

14 It is given that $f(x) = 3x^3 + 2x^2 + 16$. The remainder when $f(x)$ is divided by $x - 3a$, where a is a constant, is the same as the remainder when it is divided by $3x + 2$.

(a) Find the possible values of a. [3]

(b) Show, with clear working, that $x = -2$ is a solution of $f(x) = 0$. [1]

(c) Explain why $f(x) = 0$ has only one real root. [5]

(d) Hence use your answers to parts (b) and (c) to solve the equation

$$16y^3 + 2y - 3 = 0.$$ [2]

END OF PAPER

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{(b^2 - 4ac)}}{2a}$$

Binomial Expansion

$$(a+b)^n = a^n + \binom{n}{1} a^{n-1} b + \binom{n}{2} a^{n-2} b^2 + \ldots \binom{n}{r} a^{n-r} b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{(n-r)!\,r!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\cos ec^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

Formulae for $\Delta\, ABC$

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2} ab \sin C$$

ANSWERS
(Bedok South) AM4 P1 (2025) Students

1. **Hint:** Convert to $\sin x$ and $\cos x$ by apply double angle formulae.

2. $2\lg\left(\frac{4}{5}\right)$

3. $\frac{8+4\sqrt{2}}{3}$

4. $a = 5,\ \ b = 6$

5. $PQ = 5$ units

6. $k = 5$

7. $30°,\ \ 90°,\ \ 150°$

8. (a) **Hint:** Apply Product Rule
$\frac{\mathrm{d}(uv)}{\mathrm{d}x} = \frac{\mathrm{d}u}{\mathrm{d}x}v + u\frac{\mathrm{d}v}{\mathrm{d}x}$

(b) 2.5 units/s

9. (a) $y = -3(x+2)^2 + 17$

(b) **Hint:** Apply the two conditions:
(i) coefficient of $\underline{x^2 > 0}$
(ii) discriminant < 0

10.(a) $10 < x \leq 15$

(b) It is enough to enclose the field.

11.(a) $x < 4$

(b) $x = -5$

12.(a) **Hint:** Apply completing the square to the discriminant.

(b) $h = \frac{1}{3}$

(c) $h \leq \frac{1}{4}$ or $h \geq 2$ and $h \neq 0$

13.(a) **Hint:** For decreasing function, $\mathrm{f}'(x) < 0$

(b) $(2 - 2\sqrt{2})e^{\sqrt{2}}$

14.(a) $a = 0$ or $a = -\frac{2}{9}$

(b) **Hint:** Apply Factor Theorem

(c) **Hint:** Show that discriminant < 0

(d) $y = \frac{1}{2}$

MARKING SCHEME

(Bedok South) 4E5N AM P1 (Prelim 2025) (w Ans)

Qn	Solution	Marks	Testing
1	**Trigonometric Identities**		
	LHS		
	$= \sin 2x - \cos 2x \tan x$		
	$= 2\sin x \cos x - (2\cos^2 x - 1)\dfrac{\sin x}{\cos x}$	**M2**	Apply double angle for sine & cosine
	$= 2\sin x \cos x - 2\sin x \cos x + \dfrac{\sin x}{\cos x}$	**M1**	Expand & simplify
	$= \tan x$	**A1**	
	$= RHS$		
2	**Exponential Equations**		
	$2^{2(x+2)} \times 5 = 5^{x+3} \times 8^x$		
	$2^4(2^{2x}) \times 5 = 5^3(5^x) \times 2^{3x}$	**M1**	Split the powers
	$\dfrac{2^{2x}}{(2^{3x})(5^x)} = \dfrac{5^3}{2^4 \times 5}$	**M1**	Separate factors with x and non-x
	$\dfrac{1}{(2^x)\,(5^x)} = \dfrac{25}{16}$		
	$\dfrac{1}{10^x} = \left(\dfrac{5}{4}\right)^2$		Combine common index
	$10^x = \left(\dfrac{4}{5}\right)^2$		
	$x = \lg\left(\dfrac{4}{5}\right)^2$	**M1**	Express in logarithm
	$\boldsymbol{x = 2\lg\left(\dfrac{4}{5}\right)}$	**A1**	Power law
	OR		
	$2^{2(x+2)} \times 5 = 5^{x+3} \times 8^x$		
	$2^{2x+4} \times 5 = 5^{x+3} \times 2^{3x}$	**M1**	Convert to prime bases
	$\dfrac{2^{2x+4}}{2^{3x}} = \dfrac{5^{x+3}}{5}$	**M1**	Separate factors of different bases
	$\dfrac{2^4}{2^x} = 5^x 5^2$		
	$10^x = \dfrac{2^4}{5^2}$		
	$10^x = \dfrac{4^2}{5^2}$		Combine common index
	$x = \lg\left(\dfrac{4}{5}\right)^2$	**M1**	Express in logarithm
	$\boldsymbol{x = 2\lg\left(\dfrac{4}{5}\right)}$	**A1**	Power law
3	**Surds Expressions**		
	$x\sqrt{18} = 3x + \sqrt{32}$		
	$3x\sqrt{2} - 3x = 4\sqrt{2}$		
	$3x(\sqrt{2} - 1) = 4\sqrt{2}$	**M1**	Grouping
	$3x = \dfrac{4\sqrt{2}}{\sqrt{2} - 1}$		
	$= \dfrac{4\sqrt{2}\,(\sqrt{2}+1)}{(\sqrt{2}-1)(\sqrt{2}+1)}$	**M1**	Rationalise denominator with conjugate
	$= \dfrac{8 + 4\sqrt{2}}{(\sqrt{2})^2 - (1)^2}$	**M1**	Expansion
		M1	Difference of squares
	$\boldsymbol{x = \dfrac{8 + 4\sqrt{2}}{3}}$	**A1**	
	OR		
	$x\sqrt{18} = 3x + \sqrt{32}$		
	$3\sqrt{2}x = 3x + 4\sqrt{2}$		
	$(3\sqrt{2} - 3)x = 4\sqrt{2}$	**M1**	Grouping
	$x = \dfrac{4\sqrt{2}}{3\sqrt{2} - 3}$		
	$= \dfrac{4\sqrt{2}\,(3\sqrt{2}+3)}{(3\sqrt{2}-3)(3\sqrt{2}+3)}$	**M1**	Rationalise denominator with conjugate
	$= \dfrac{24 + 12\sqrt{2}}{(3\sqrt{2})^2 - (3)^2}$	**M1**	Expansion
		M1	Difference of squares
	$= \dfrac{24 + 12\sqrt{2}}{9}$		
	$\boldsymbol{= \dfrac{8 + 4\sqrt{2}}{3}}$	**A1**	
4	**Surds (Application)**		
(a)	Area of square $= 11 + \sqrt{120}$		
	$(\sqrt{a} + \sqrt{b})^2 = 11 + 2\sqrt{30}$		
	$a + b + 2\sqrt{ab} = 11 + 2\sqrt{30}$	**M1**	Expand
	Equating the <u>rational</u> part,	**M1**	Equating rational & irrational parts separately
	$a + b = 11$		
	$b = 11 - a \ldots\ldots(1)$		
	Equating the <u>irrational</u> part,		
	$2\sqrt{ab} = 2\sqrt{30}$		
	$ab = 30 \ldots\ldots(2)$		
	Subst. (1) into (2),		
	$a(11 - a) = 30$	**M1**	Substitution method
	$11a - a^2 = 30$		
	$a^2 - 11a + 30 = 0$		
	$(a - 5)(a - 6) = 0$		
	$a = 5 \quad \text{or} \quad a = 6$	**M1**	Solve quadratic equation
	Corresponding b values:		
	$b = 6 \quad \text{or} \quad b = 5$		
	Since $a < b$		
	$\boldsymbol{a = 5, \quad b = 6}$	**A1**	Correct set of values

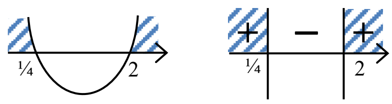
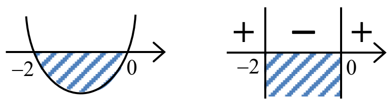
Qn	Solution	Marks	Testing
5	**Simultaneous Equations**		
	$4x - y = 5 \quad \dots\dots [1]$		
	$y = 2x^2 - 6x + 7 \quad \dots\dots [2]$		
	Subst (2) into (1)		
	$4x - (2x^2 - 6x + 7) = 5$	M1	Substitution or Elimination Method
	$4x - 2x^2 + 6x - 7 = 5$		
	$2x^2 - 10x + 12 = 0$		
	$x^2 - 5x + 6 = 0$		
	$(x-2)(x-3) = 0$	M1	Factorisation or Quadratic Formula
	$x = 2 \quad or \quad x = 3$		
	Subst respective x values into [1],		
	$y = 4(2) - 5$, $y = 3$ | $y = 4(3) - 5$, $y = 7$	M1	Find values of 2nd variable
	$P(2,3)$ and $Q(3,7)$		
	$PQ = \sqrt{(3-2)^2 + (7-3)^2}$	M1	Apply Distance formula
	$PQ = \sqrt{17}$	A1	
6	**Binomial Theorem**		
	General term of expansion $\left(2 - \frac{1}{2}x\right)^8$ is		
	$T_{r+1} = \binom{8}{r}(2)^{8-r}\left(-\frac{x}{2}\right)^r$	M1	General term of binomial expansion
	$= \binom{8}{r}(-1)^r(2)^{8-2r}x^r$		
	$T_3 = \binom{8}{2}(-1)^2(2)^{8-2(2)}x^2$		
	$= 448x^2$	A1	Extract specific terms
	$T_4 = \binom{8}{3}(-1)^3(2)^{8-2(3)}x^3$		
	$= -224x^3$	A1	
	$(10 + kx)\left(2 - \frac{1}{2}x\right)^8$		
	$= (10 + kx)(\dots 448x^2 - 224x^3 \dots)$		
	Coeff. of $x^3 = 0$		
	$(10)(-224) + k(448) = 0$	M1	Equating coefficient
	$448k = 2240$		
	$k = 5$	A1	
7	**Trigonometric Equations**		
	$\frac{5\cos x + \sin 2x}{\cos 2x + 5\sin x} = \cot x$		
	$\frac{5\cos x + \sin 2x}{\cos 2x + 5\sin x} = \frac{\cos x}{\sin x}$	M1	Convert to $\sin x$ & $\cos x$
	$\sin x(5\cos x + \sin 2x)$ $= \cos x(\cos 2x + 5\sin x)$		

Qn	Solution	Marks	Testing
	$\sin x(5\cos x + 2\sin x\cos x)$ $= \cos x(\cos^2 x - \sin^2 x + 5\sin x)$	M1	Apply double angle formula $\sin 2x$ & $\cos 2x$
	$5\sin x\cos x + 2\sin^2 x\cos x$ $= \cos^3 x - \sin^2 x\cos x + 5\sin x\cos x$		
	$2\sin^2 x\cos x = \cos^3 x - \sin^2 x\cos x$	M1	Expand & simplify
	$3\sin^2 x\cos x - \cos^3 x = 0$		
	$\cos x(3\sin^2 x - \cos^2 x) = 0$	M1	Zero product rule
	$\cos x = 0$, $x = 90°$ | $3\sin^2 x - \cos^2 x = 0$, $3\sin^2 x = \cos^2 x$, $3\tan^2 x = 1$, $\tan^2 x = \frac{1}{3}$, $\tan x = \pm\frac{1}{\sqrt{3}}$, $x = 30°,\ 150°$	A1	Either solution correct
	$\therefore x = 30°,\ 90°,\ 150°$	A1	Final solution
8	**Differentiation (Product Rule)**		
(a)	$\frac{d}{dx}[x(x-3)^4]$		
	$= (x-3)^4 + 4x(x-3)^3$	M1	Apply Product Rule
	$= (x-3)^3(x - 3 + 4x)$		
	$= (5x-3)(x-3)^3$	A1	
	Differentiation (Quotient Rule)		
(b)	$y = \frac{x(x-3)^4}{5x-3}$		
	$\frac{dy}{dx} = \frac{(5x-3)(x-3)^3(5x-3) - 5x(x-3)^4}{(5x-3)^2}$	M1	Apply Quotient Rule
	$= \frac{(x-3)^3\left((5x-3)^2 - 5x(x-3)\right)}{(5x-3)^2}$		
	$= \frac{(x-3)^3(25x^2 - 30x + 9 - 5x^2 + 15x)}{(5x-3)^2}$		
	$= \frac{(x-3)^3(20x^2 - 15x + 9)}{(5x-3)^2}$	M1	Factorise & simplify
	When $x = 1$,		
	$\frac{dy}{dt} = \frac{dy}{dx} \times \frac{dx}{dt}$		
	$-70 = \frac{(x-3)^3(20x^2 - 15x + 9)}{(5x-3)^2} \times \frac{dx}{dt}$		
	$= \frac{(1-3)^3(20 - 15 + 9)}{(5-3)^2} \times \frac{dx}{dt}$	M1	Rate of change
	$-70 = -28 \times \frac{dx}{dt}$		
	$= 2.5$ units/s	A1	

Qn	Solution	Marks	Testing
9	**Quadratic Functions**		
(a)	$y = 5 - 12x - 3x^2$		
	$= -3x^2 - 12x + 5$		
	$= -3(x^2 + 4x) + 5$		
	$= -3\{(x+2)^2 - (2)^2\} + 5$	**M1**	Completing the Square
	$= -3(x+2)^2 + 12 + 5$		
	$\boldsymbol{y = -3(x+2)^2 + 17}$	**A1**	
	Since $(x+2)^2 \geq 0$ $-3(x+2)^2 \leq 0$ $-3(x+2)^2 + 17 \leq 17$ max $y = 17 < 20$, $\therefore$ for all values of y, it can never be greater than 20.	**B1**	
(b)	Given $y = (k-1)x^2 + 2(k+2)x + k + 3$		
	For y to be always positive, (i) Coeff of $x^2 > 0$, and (ii) Discriminant < 0.		2 conditions for $y > 0$
	From (i), $k - 1 > 0$ $k > 1$	**D1**	
	From (ii), $\{2(k+2)\}^2 - 4(k-1)(k+3) < 0$	**M1**	
	$4(k+2)^2 - 4(k-1)(k+3) < 0$ $(k+2)^2 - (k-1)(k+3) < 0$ $(k^2 + 4k + 4) - (k^2 + 2k - 3) < 0$ $2k + 7 < 0$		$D = b^2 - 4ac$
	$k < -\frac{7}{2}$	**D1**	
	From the two inequalities, $k > 1$ and $k < -\frac{7}{2}$, there are no values of k for which $y > 0$.	**A1**	
10	**Quadratic Inequalities (Word Problem)**		
(a)	Since $Side > 0$, $3x - 5 > 0$ and $x - 10 > 0$ $x > \frac{5}{3}$ and $x > 10$ ……(1)	**M1**	Form Side inequalities
	$Area\ of\ rectangle \leq 200$ $(3x-5)(x-10) \leq 200$	**M1**	Form Area inequality
	$3x^2 - 35x + 50 \leq 200$ $3x^2 - 35x - 150 \leq 0$		
	$(3x+10)(x-15) \leq 0$ $-\frac{10}{3} \leq x \leq 15$ ……(2)	**M1**	Factorisation
	From (1) & (2), $\boldsymbol{10 < x \leq 15}$	**A1**	

Qn	Solution	Marks	Testing
(b)	To enclose the field, $Perimeter \leq 98$ $2(3x-5) + 2(x-10) \leq 98$	**M1**	Form Perimeter inequality
	$2(4x - 15) \leq 98$ $4x - 15 \leq 49$ $x \leq 16$ ……(3)	**A1**	
	From part (a) $10 < x \leq 15$ and (3) $x \leq 16$, it is **enough** to enclose the field.	**B1**	
11	**Logarihemic Equations**		
(a)	For a valid logarithm, $4 - x > 0$ $\therefore\ x < 4$	**B1**	
(b)	$2\log_3(4-x) - \log_9(x-4)^2 = 2$		
	$2\log_3(4-x) - \dfrac{\log_3(x-4)^2}{\log_3 9} = 2$	**M1**	Change-base law
	$\log_3(4-x)^2 - \dfrac{\log_3(x-4)^2}{\log_3 3^2} = 2$	**M1**	Power law
	$\log_3(4-x)^2 - \dfrac{\log_3(x-4)^2}{2} = 2$		Identity property
	$2\log_3(4-x)^2 - \log_3(x-4)^2 = 4$ $\log_3(4-x)^4 - \log_3(x-4)^2 = 4$ $\log_3\dfrac{(4-x)^4}{(x-4)^2} = 4$	**M1**	Quotient Law
	$\dfrac{(4-x)^4}{(x-4)^2} = 3^4$	**M1**	Convert Logarithmic form to Exponential form
	$(4-x)^4 = 81(x-4)^2$ $(x-4)^4 - 81(x-4)^2 = 0$ $(x-4)^2((x-4)^2 - 81) = 0$ $(x-4)^2 = 0$ or $(x-4)^2 = 81$ $x - 4 = 0$ or $x - 4 = \pm 9$		Solve factorised equation
	$x = 4$ or $x = -5$ or $x = 13$	**A1**	
	From (a), since $x < 4$, $\therefore x = -5$ is the only one real solution	**B1**	
	OR		
	$2\log_3(4-x) - \log_9(x-4)^2 = 2$ $2\log_3(4-x) - \dfrac{\log_3(x-4)^2}{\log_3 9} = 2$	**M1**	Change-base law
	$\log_3(4-x)^2 - \dfrac{\log_3(x-4)^2}{\log_3 3^2} = 2$	**M1**	Power law
	$\log_3(4-x)^2 - \dfrac{\log_3(x-4)^2}{2} = 2$		Identity property
	$\dfrac{\log_3(4-x)^2}{2} = 2$ $\log_3(4-x)^2 = 4$ $(4-x)^2 = 3^4$	**M1**	Convert Logarithmic form to

Qn	Solution	Marks	Testing
	$x^2-8x+16=81$		Exponential form
	$x^2-8x-65=0$	M1	
	$(x+5)(x-13)=0$		Writing $\log_3(\boldsymbol{x-4})$ is not correct as $4-x>0$, i.e. $x-4<0$
	$x=-5$ or $x=13$	A1	
	From (a), since $x<4$,		
	$\therefore x=-5$ is the only one real solution	B1	
	OR		
	$2\log_3(4-x)-\log_9(x-4)^2=2$	M1	Change-base law
	$2\log_3(4-x)-\dfrac{\log_3(x-4)^2}{\log_3 9}=2$	M1	Power law
	$\log_3(4-x)^2-\dfrac{\log_3(x-4)^2}{\log_3 3^2}=2$		Identity property
	$\log_3(4-x)^2-\dfrac{\log_3(x-4)^2}{2}=2$		
	$\log_3(4-x)^2-\log_3(4-x)=2$		
	$\log_3\dfrac{(4-x)^2}{(4-x)}=2$	M1	Quotient law
	$\dfrac{(4-x)^2}{(4-x)}=3^2$	M1	Convert Logarithmic form to Exponential form
	$x^2-8x+16=9(4-x)$		
	$x^2-8x+16=36-9x$		
	$x^2+x-20=0$		Writing $\log_3(\boldsymbol{x-4})$ is not correct as $4-x>0$, i.e. $x-4<0$
	$(x+5)(x-4)=0$		
	$x=-5$ or $x=4$	A1	
	From (a), since $x<4$,		
	$\therefore x=-5$ is the only one real solution	B1	
12	**Quadratic Equations**		
(a)	When $y=0$		
	$hx^2-2x-9h+6=0$		
	$hx^2-2x+(6-9h)=0$		
	For a quadratic curve,		
	coeff of $x^2\neq 0$,		
	$h\neq 0$ … [1]	M1	
	$D=(-2)^2-4h(6-9h)$	M1	$D=b^2-4ac$
	$=4-4h(6-9h)$		
	$=4\big(1-3h(2-3h)\big)$		
	$=4(9h^2-6h+1)$		
	$=4(3h-1)^2$ or $(6h-2)^2$		
	$D\geq 0$ …[2]	M1	
	$\therefore$ from [1] & [2], $y=0$ has real roots for all values of h except $h=0$.		

Qn	Solution	Marks	Testing
(b)	For 2 real and equal roots,		
	$D=0$		
	$4(3h-1)^2=0$		
	$h=\boldsymbol{\dfrac{1}{3}}$	A1	
(c)	For line meeting the curve,		
	$hx^2-2x-9h+6=2x-h-12$		
	$hx^2-4x+18-8h=0$	M1	
	$D\geq 0$		
	$(-4)^2-4h(18-8h)\geq 0$	M1	
	$16-8h(9-4h)\geq 0$		
	$2-h(9-4h)\geq 0$		
	$4h^2-9h+2\geq 0$		
	$(4h-1)(h-2)\geq 0$	M1	
			
	$h\leq\frac{1}{4}$ or $h\geq 2$ ……[3]	A1	
	From [1] & [3],		
	$\boldsymbol{h\leq\frac{1}{4}}$ **or** $\boldsymbol{h\geq 2}$ **and** $\boldsymbol{h\neq 0}$	A1	
13	**Increasing / Decreasing Fns**		
(a)	$f(x)=x^2e^{(x+2)}$		
	$f'(x)=2xe^{(x+2)}+x^2e^{(x+2)}$	M1	Apply Product Rule
	$=xe^{(x+2)}(2+x)$		
	$=x(x+2)e^{(x+2)}$	A1	
	For decreasing function,		
	$f'(x)<0$		
	$x(x+2)e^{(x+2)}<0$	M1	Decreasing function
	since $e^{(x+2)}>0$		Must explain exponential function is <u>positive</u> for all values of x
	$x(x+2)<0$	M1	
			
	$\boldsymbol{-2<x<0}$		
	Maxima & Minima		
(b)	$f'(x)=(x^2+2x)e^{(x+2)}$		
	$f''(x)$ $=(2x+2)e^{(x+2)}+(x^2+2x)e^{(x+2)}$	M1	Apply Product Rule
	$f''(x)=(x^2+4x+2)e^{(x+2)}$		
	For the least gradient,		
	$f''(x)=0$		

Qn	Solution	Marks	Testing
	$(x^2+4x+2)e^{(x+2)}=0$	M1	
	$x^2+4x+2=0$ $x=\dfrac{-4\pm\sqrt{4^2-4(1)(2)}}{2(1)}$	M1	Quadratic formula or Completing the Square
	$=\dfrac{-4\pm\sqrt{8}}{2}$ $=\dfrac{-4\pm2\sqrt{2}}{2}$ $=-2\pm\sqrt{2}$	A1	
	Since the least gradient lies in $-2<x<0$, $x=-2+\sqrt{2}$ Least gradient $f'(x)=x(x+2)e^{(x+2)}$ $f'(-2+\sqrt{2})$ $=(-2+\sqrt{2})(-2+\sqrt{2}+2)e^{(-2+\sqrt{2}+2)}$ $=(-2+\sqrt{2})\sqrt{2}e^{\sqrt{2}}$ $=(2-2\sqrt{2})e^{\sqrt{2}}$	A1	
14	**Remainder & Factor Thm**		
(a)	Given $f(x)=3x^3+2x^2+16$. $f(3a)=f\left(-\frac{2}{3}\right)$ $3(3a)^3+2(3a)^2+16$ $=3\left(-\frac{2}{3}\right)^3+2\left(-\frac{2}{3}\right)^2+16$	M1	Substitute values correctly
	$3(27a^3)+2(9a^2)$ $=3\left(-\frac{8}{27}\right)+2\left(\frac{4}{9}\right)$ $81a^3+18a^2=-\frac{8}{9}+\frac{8}{9}$ $9a^2(9a+2)=0$	M1	Factorisation
	$\boldsymbol{a=0}$ **or** $\boldsymbol{a=-\frac{2}{9}}$	A1	
(b)	Subst $x=-2$ into $f(x)$, $f(-2)=3(-2)^3+2(-2)^2+16$ $=-24+8+16$ $=0$	A1	Apply Factor Theorem
	$\therefore\ x=-2$ is a solution of $f(x)$.		

Qn	Solution	Marks	Testing
(c)	From (b), $x+2$ is a factor of $f(x)$. By inspection, $f(x)=3x^3+2x^2+16$ $=(x+2)(3x^2+kx+8)$	M1	
	Equating coefficient of x^2 : $k+6=2$ $k=-4$ **or** x : $2k+8=0$ $k=-4$	M1	Comparing coefficients or Long Division
	$f(x)=3x^3+2x^2+16$ $=(x+2)(3x^2-4x+8)$	M1	Factorisation
	For $f(x)=0$ $(x+2)(3x^2-4x+8)=0$ $x+2=0$ $x=-2$		Solution for linear equation
	or $3x^2-4x+8=0$ $D=(4)^2-4(3)(8)$ $D=-80$	M1	Use of discriminant or Quadratic formula
	Since $D<0$, there is no real roots for the quadratic equation	B1	
	$\therefore f(x)=0$ has only one real root $x=-2$.		
(d)	$16y^3+2y-3=0$ $16+\frac{2}{y^2}-\frac{3}{y^3}=0$ $3\left(-\frac{1}{y}\right)^3+2\left(-\frac{1}{y}\right)^2+16=0$	M1	Transform into the required format
	Let $x=-\frac{1}{y}$ $3x^3+2x^2+16=0$ $x=-2$ $-\frac{1}{y}=-2$ $\boldsymbol{y=\frac{1}{2}}$	A1	

BEDOK SOUTH SECONDARY SCHOOL
PRELIMINARY EXAMINATION 2025

4E5N

CANDIDATE NAME		CLASS		REGISTER NUMBER		

ADDITIONAL MATHEMATICS
Paper 2

4049 / 02
2 hours 15 minutes

90

Answer **all** the questions.

1 **(a)** Solve the equation $2(4^x)-8=11(2^x)-2^{2x+3}$, giving your answer correct to 3 significant figures. [5]

(b) Show that the solution from **part (a)** may be written in the form $m-\log_n p$ where m, n and p are integers to be determined. [2]

2 The mass, M grams, of a radioactive substance, present at time t years after first being observed, is given by the formula $M=200e^{-kt}$, where k is a constant.
The mass of the substance was 123.7 g after being observed for 2 years.

(a) **(i)** State the initial mass of the substance. [1]

(ii) Show that k is approximately 0.240, correct to 3 significant figures. [1]

(iii) Find the mass of the substance when $t=5$, [1]

(iv) Find the value of t when the mass of the substance is 15% of its initial mass.
Give your answers correct to three significant figures. [2]

(b) Explain, with clear working, why the mass of the substance can never be more than 200 grams. [1]

(c) Sketch the graph of M against t. [2]

3 **(a)** It is given that $y=x^2\sqrt{2x-5}$, where $x \geq h$.

Show that $\dfrac{dy}{dx}=\dfrac{kx(x-2)}{\sqrt{2x-5}}$, where k is an integer and determine the value of h and of k. [4]

(b) Hence evaluate $\displaystyle\int_3^7 \left\{\frac{10x(x-2)}{\sqrt{2x-5}}+6\right\} dx.$ [4]

4 **(a)** **(i)** Factorise completely x^3+64. [2]

(ii) **Hence**, express $\dfrac{3x^2}{x^3+64}$ in partial fractions. [5]

(b) Using the results in **part (a)**, or otherwise, find

$$\int \frac{2x^3+3x^2+128}{x^3+64}\, dx.$$ [3]

5 A particle moves in a straight line so that, at time t seconds after passing a fixed point O, its velocity is v m/s, where $v = 4 + 8\cos 2t$. Find

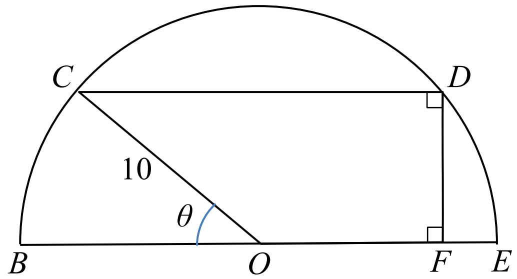
(a) the velocity of the particle at the instant it passes O, [1]

(b) the least value of the particle's acceleration, [1]

(c) the values of t, in terms of π, when the particle is at rest for $0 \le t \le 3$, [4]

(d) the distance travelled in the first 2 seconds. [4]

6



The diagram shows right angled trapezium $OCDF$ inside a semicircle with centre O and radius 10 cm such that angle BOC is θ radians, and angle CDF and angle OFD are right angles.

(a) Show that the perimeter, P cm, of trapezium $OCDF$ is given by

$$P = 10 + 30\cos\theta + 10\sin\theta$$ [2]

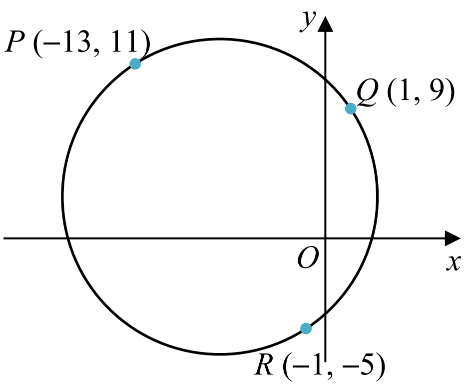
(b) Find the value of R when $10\sin\theta + 30\cos\theta$ is expressed as $R\cos(\theta - \alpha)$, where R and α are constants, and hence state the maximum perimeter of the trapezium. [3]

(c) Show that the area, A cm^2, of trapezium $OCDF$ is given by

$$A = 75\sin 2\theta$$ [2]

(d) The area of the trapezium varies with the value of θ. Find the value of θ for which the area has a stationary value and determine whether this area is a maximum or a minimum. [4]

7 **Solutions to this question by accurate drawing will not be accepted.**



In the diagram, P, Q and R are points on the circle.

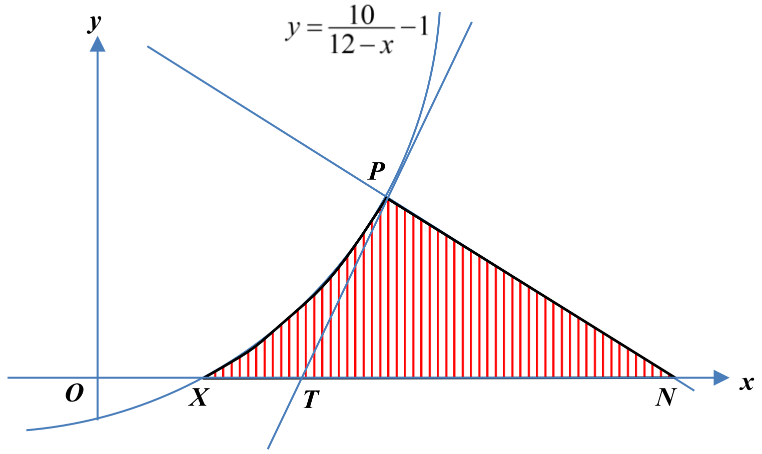
(a) Explain, with geometrical reason, why PR is the diameter of the circle. [3]

(b) Find the equation of the circle in the form $x^2 + y^2 + ax + by + c = 0$, where a, b and c are integers. [3]

(c) Find the equation of the perpendicular bisector of PQ. [3]

(d) The point S lies on the circle such that it is furthest from the point Q.
Show that the coordinates of S are $(-15, -3)$ and hence calculate the area of the quadrilateral $PQRS$. [3]

8



The diagram shows part of the curve $y = \dfrac{10}{12-x} - 1$ passing through the point $P(2k, k-1)$, where k is a constant. The curve meets the x-axis at the point X. The tangent and normal at P meet the x-axis at the points T and N respectively.

(a) Find the equation of the normal at P. [6]

(b) Find the **exact** area of the shaded region. [6]

9 The table below shows experimental values of two variables x and y.

x	1	2	3	4	5	6
y	2.20	1.74	1.71	1.83	1.87	1.96

The variables x and y are related by the equation $\dfrac{y}{a} = \dfrac{1}{x} + b\sqrt{x}$, where a and b are constants.
One value of y has been recorded incorrectly.

(a) Show how $\dfrac{y}{a} = \dfrac{1}{x} + b\sqrt{x}$ is transformed to plot a graph of xy against $x\sqrt{x}$. [1]

(b) Draw a straight line graph of xy against $x\sqrt{x}$ for the given data. [2]

(c) Using your graph,

(i) find an approximate value of y to replace the incorrect value, [2]

(ii) estimate the value of a and of b, [3]

(iii) find the value of y when $x = 2.52$. [2]

(d) A pair of values of x and y is considered acceptable only if the xy value is within 2% vertical difference from the straight line. A student recorded a pair of values such that $x = 4.50$ and $y = 1.86$.
Verify whether the recorded values by the student are acceptable. [2]

-

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{(b^2 - 4ac)}}{2a}$$

Binomial Expansion

$$(a+b)^n = a^n + \binom{n}{1} a^{n-1} b + \binom{n}{2} a^{n-2} b^2 + \ldots \binom{n}{r} a^{n-r} b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{(n-r)!\,r!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}.$$

Formulae for $\Delta\, ABC$

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2} ab \sin C$$

ANSWERS

(Bedok South) 4E5N AM P2 (Prelim 2025) (w Ans)

1. (a) $x = 0.678$ (to 3 sig. fig.)

(b) $m = 3,\ n = 2,\ p = 5$

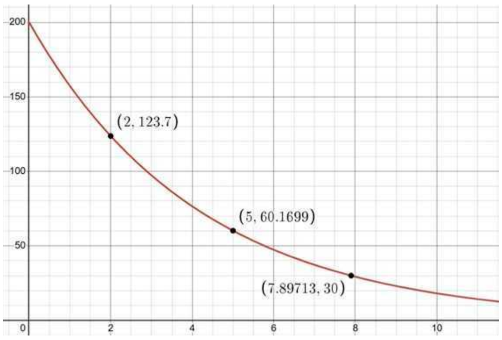
2. (a) **(i)** $M_0 = 200$ g

(ii) $k = 0.240$ (to 3 sig. fig.)

(iii) $M = 60.2$ g (to 3 sig. fig.)

(iv) $t = 7.90$ years (to 3 sig. fig.)

(c)



3. (a) $h = \frac{5}{2},\ k = 5$

(b) 300

4. (a) **(i)** $(x+4)(x^2-4x+16)$

(ii) $\dfrac{1}{x+4} + \dfrac{2x-4}{x^2-4x+16}$

(b) $2x + \ln(x^3+64) + c$

5. (a) $v = 12$ m/s

(b) Least $a = -16$ m/s^2

(c) $t = \frac{1}{3}\pi,\ \frac{2}{3}\pi$

(d) 10.3 m (to 3 sig. fig.)

6. (a) **Hint:** Draw a line $CG \perp BE$

(b) $R = 10\sqrt{10}$ or 31.6 (to 3 sig. fig.)
Max $P = 41.6$ cm (to 3 sig. fig.)

(c) **Hint:** Apply Double Angle formula

(d) $\theta = \frac{\pi}{4}$

7. (a) **Hint:** Right angle in semicircle

(b) $x^2 + y^2 + 14x - 6y - 42 = 0$

(c) $y = 7x + 52$

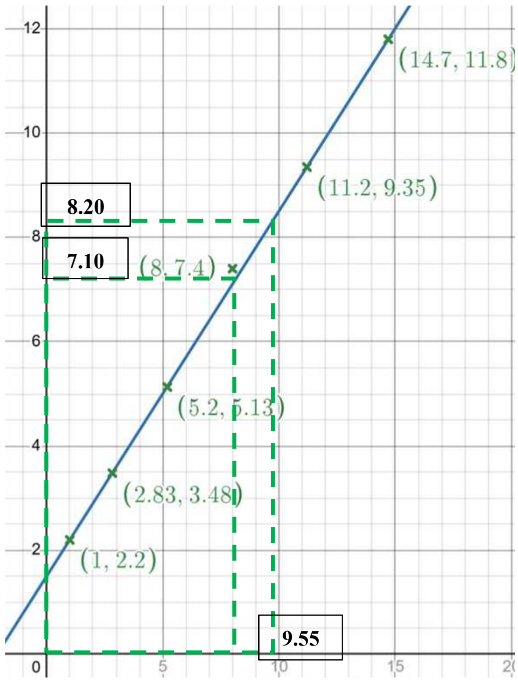
(d) **Hint:** The furthest two points on a circle is its diameter.
$Area\ of\ PQRS = 200$ units^2

8. (a) $y = -\frac{2}{5}x + 8$

(b) $10\ln 5 + 12$

9. (a) **Hint:** Multiply equation with ax to get $xy = ab\left(x\sqrt{x}\right) + a$

(b)



(c) **(i)** $y = 1.78$ (to 3 sig. fig.)

(ii) $a = 1.50$ (to 3 sig. fig.)

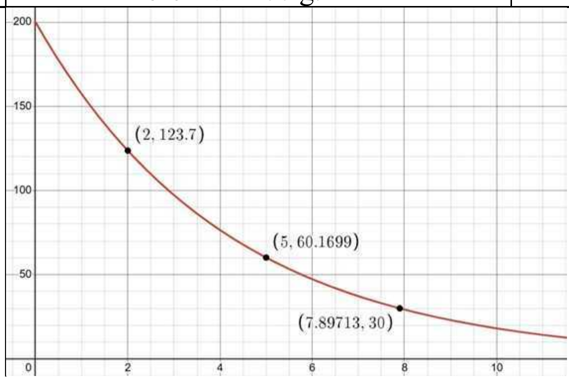
$b = 0.467$ (to 3 sig. fig.)

(iii) $y = 1.71$ (to 3 sig. fig.)

(d) Unacceptable,
Vertical difference $= 2.073\%$

MARKING SCHEME

(Bedok South) 4E5N AM P2 (Prelim 2023) (w Ans)

Qn	Solution	Testing
1.	**Exponential Equations**	
(a)	$2(4^x) - 8 = 11(2^x) - 2^{2x+3}$ $2(2^{2x}) - 8 = 11(2^x) - (2^{2x})2^3$ $2(2^x)^2 - 8 = 11(2^x) - 8(2^x)^2$	Convert to base 2
	Let $u = 2^x$ $2u^2 - 8 = 11u - 8u^2$ $10u^2 - 11u - 8 = 0$ $(2u+1)(5u-8) = 0$	Apply substitution correctly
	$u = -\frac{1}{2}$ or $u = \frac{8}{5}$ $2^x = -\frac{1}{2}$ (reject) $2^x = \frac{8}{5}$	Solving for u
	$\lg 2^x = \lg \frac{8}{5}$ $x = \frac{\lg \frac{8}{5}}{\lg 2}$ $= 0.678072$ $= \mathbf{0.678}$ (to 3 sig. fig.)	Apply logarithm appropriately
(b)	From (a), $2^x = \frac{8}{5}$ $\log_2 2^x = \log_2 \frac{8}{5}$ $x \log_2 2 = \log_2 8 - \log_2 5$	Apply $\log_2$ onto equation
	$x = \log_2 2^3 - \log_2 5$ $= 3\log_2 2 - \log_2 5$	Power law Quotient law
	$= 3 - \log_2 5$ $= m - \log_n p$ $\mathbf{m = 3, \ n = 2, \ p = 5}$	Identity property
2.	**Exponential Functions (Word Problems)**	
(a) (i)	When $t = 0$, $M = 200e^{-kt}$ $M_0 = 200e^{-k(0)}$ $= \mathbf{200\ g}$	
(a) (ii)	When $t = 2$, $M = 123.7$ $M = 200e^{-kt}$ $123.7 = 200e^{-k(2)}$ $\frac{123.7}{200} = e^{-2k}$ $\ln\left(\frac{123.7}{200}\right) = -2k$ $k = -\frac{1}{2}\ln\left(\frac{123.7}{200}\right)$ $= 0.240229$ $= \mathbf{0.240}$ (to 3 sig. fig.)	Take natural logarithm on both sides of equation
(a) (iii)	When $t = 5$, For $k = 0.24$: $M = 200e^{-kt}$, $= 200e^{-0.24(5)}$, $= 60.23884$ For $k = 0.240229$: $M = 200e^{-kt}$, $= 200e^{-0.240229(5)}$, $= 60.16991$ $= \mathbf{60.2\ g}$ (to 3 sig. fig.)	
(a) (iv)	$M = M_0 e^{-kt}$ $\frac{M}{M_0} = e^{-kt}$ $15\% = e^{-kt}$ $0.15 = e^{-kt}$ For $k = 0.24$: $0.15 = e^{-0.24t}$, $\ln 0.15 = -0.24t$, $t = -\frac{\ln 0.15}{0.24}$, $= 7.90467$ For $k = 0.240229$: $0.15 = e^{-0.240229t}$, $\ln 0.15 = -0.240229t$, $t = -\frac{\ln 0.15}{0.240229}$, $= 7.89713$ $t = \mathbf{7.90\ years}$ (to 3 sig. fig.)	Take natural logarithm on both sides of equation
(b)	Since for $t \geq 0$, $e^{kt} \geq 1$ $\frac{1}{e^{kt}} \leq 1$ $e^{-kt} \leq 1$ $200e^{-kt} \leq 200$ $M \leq 200$ $\therefore$ mass of substance can never be more than 200 g.	
(c)		

G1 – Initial state (0, 200)
G1 – Smooth curve decreasing towards asymptote t-axis

Qn	Solution	Testing
3.	**Differentiation (Product Rule)**	
(a)	For square root to be defined, $2x-5 \geq 0 \;\Rightarrow\; x \geq \frac{5}{2}$	Square root criterion
	$\therefore\ \boldsymbol{h} = \mathbf{\frac{5}{2}}$	
	$y = x^2\sqrt{2x-5}$	
	$y = x^2\,(2x-5)^{\frac{1}{2}}$	
	$\frac{dy}{dx} = 2x(2x-5)^{\frac{1}{2}}$ $+x^2(2)\frac{1}{2}(2x-5)^{-\frac{1}{2}}$	Apply Product rule
	$\frac{dy}{dx} = 2x\sqrt{2x-5} + \frac{x^2}{\sqrt{2x-5}}$	
	$= \frac{2x(2x-5)+x^2}{\sqrt{2x-5}}$	Combine to single fraction
	$= \frac{2x(2x-5)+x^2}{\sqrt{2x-5}}$	
	$= \frac{5x^2-10x}{\sqrt{2x-5}}$	
	$\frac{dy}{dx} = \frac{5x(x-2)}{\sqrt{2x-5}}$	
	$\therefore\ \boldsymbol{k} = \mathbf{5}$	
	Integration (Reverse of Differentiation)	
(b)	$\int_3^7 \left\{\frac{10x(x-2)}{\sqrt{2x-5}} + 6\right\} dx$	
	$= 2\int_3^7 \frac{5x(x-2)}{\sqrt{2x-5}}\,dx + \int_3^7 6\,dx$	Split into 2 integrals
	$= 2\left[x^2\sqrt{2x-5}\right]_3^7 + 6[x]_3^7$	Integration as reversed differentiation
	$= 2\left\{7^2\sqrt{2(7)-5} - 3^2\sqrt{2(3)-5}\right\}$ $+ 6(7-3)$	Substitution of boundaries
	$= 2\{49\sqrt{9} - 9\sqrt{1}\} + 24$	
	$= 276 + 24$	
	$= \mathbf{300}$	
	OR	
	$\frac{dy}{dx} = \frac{5x(x-2)}{\sqrt{2x-5}}$	
	$\int_3^7 \frac{5x(x-2)}{\sqrt{2x-5}}\,dx = \left[x^2\sqrt{2x-5}\right]_3^7$	Integration as reversed differentiation

Qn	Solution	Testing
	$\int_3^7 \frac{10x(x-2)}{\sqrt{2x-5}}\,dx = 2\left[x^2\sqrt{2x-5}\right]_3^7$	
	$\int_3^7 \frac{10x(x-2)}{\sqrt{2x-5}}\,dx + \int_3^7 6\,dx$	Transform to required integral
	$= 2\left\{7^2\sqrt{2(7)-5} - 3^2\sqrt{2(3)-5}\right\}$ $+ \int_3^7 6\,dx$	Substitution of boundaries
	$\int_3^7 \left\{\frac{10x(x-2)}{\sqrt{2x-5}} + 6\right\} dx$	
	$= 2\{49\sqrt{9} - 9\sqrt{1}\} + 6[x]_3^7$	
	$= 276 + 6(7-3)$	
	$= \mathbf{300}$	
4.	**Cubic Identities**	
(a) (i)	$x^3 + 64$	
	$= (x)^3 + (4)^3$	Apply sum of cubes identity
	$= (x+4)((x)^2 - (x)(4) + (4)^2)$	
	$= \boldsymbol{(x+4)(x^2-4x+16)}$	
	Partial Fractions	
(a) (ii)	$\frac{3x^2}{x^3+64}$	
	$= \frac{3x^2}{(x+4)(x^2-4x+16)}$	Express proper fraction in partial fractions with unknown coefficients
	$= \frac{A}{x+4} + \frac{Bx+C}{x^2-4x+16}$	
	$A(x^2-4x+16)$ $+ (Bx+C)(x+4) = 3x^2$	
	Let $x = -4$,	Use Substitution and/or Equating coefficients methods to find unknowns
	$A((-4)^2 - 4(-4) + 16) = 3(-4)^2$	
	$A(16+16+16) = 3(16)$	
	$A = 1$	
	Let $x = 0$,	
	$A(16) + C(4) = 0$	Award 1 mark for at least 2 correct values
	$1(16) + 4C = 0$	
	$16 + 4C = 0$	
	$C = -4$	
	Let $x = 1$,	
	$A(1-4+16)$ $+ (B+C)(1+4) = 3$	
	$1(13) + (B-4)(5) = 3$	
	$13 + 5B - 20 = 3$	

Qn	Solution	Testing
	$B = 2$	
	OR Equating coefficient of x^2 : $A + B = 3$, $B = 3 - A \cdots (1)$ x : $-4A + 4B + C = 0 \cdots (2)$ x^0 : $16A + 4C = 0$, $C = -4A \cdots (3)$ Subst (1) & (3) into (2), $-4A + 4(3 - A) + (-4A) = 0$ $-4A + 12 - 4A - 4A = 0$ $12A = 12$ $A = 1$ $B = 3 - 1 = 2$ $C = -4(1) = -4$	
	$\therefore \frac{3x^2}{x^3 + 64}$ $= \mathbf{\frac{1}{x+4} + \frac{2x-4}{x^2-4x+16}}$	
	Indefinite Integral	
(b)	$\int \frac{2x^3 + 3x^2 + 128}{x^3 + 64} \, dx$ $= \int \frac{2(x^3 + 64)}{x^3 + 64} \, dx + \int \frac{3x^2}{x^3 + 64} \, dx$ $= \int 2 \, dx + \int \frac{3x^2}{x^3 + 64} \, dx$ $= \mathbf{2x + \ln(x^3 + 64) + c}$	Split into 2 integrals Correct integration
5.	**Kinematics (Trigonometric Functions)**	
(a)	When particle passes O, $t = 0$, Hence, $v = 4 + 8\cos 2(0)$ $= \mathbf{12 \text{ m/s}}$	
(b)	$v = 4 + 8\cos 2t$ $\frac{dv}{dt} = -(2)8\sin 2t$ $a = -16\sin 2t$ Least acceleration $= \mathbf{-16 \text{ m/s}^2}$	
(c)	$0 \le t \le 3$ $0 \le 2t \le 6$ When particle is at rest, $v = 0$ m/s,	

Qn	Solution	Testing
	$4 + 8\cos 2t = 0$ $\cos 2t = -\frac{1}{2}$ Basic angle $\alpha = \cos^{-1}\left(\frac{1}{2}\right) = \frac{1}{3}\pi$ For $\cos 2t < 0$, $2t$ lies in Q2, Q3 $2t = \pi - \frac{1}{3}\pi, \ \pi + \frac{1}{3}\pi$ $= \frac{2}{3}\pi, \ \frac{4}{3}\pi$ $t = \frac{1}{3}\pi, \ \frac{2}{3}\pi$	Correct Substitution Basic Angle Angles in correct quadrants
(d)	$s = \int_0^{\frac{\pi}{3}} (4 + 8\cos 2t) \, dt$ $s = 4t + 4\sin 2t + c$ Table: t / s: 0 / 0; $\frac{1}{3}\pi$ / 7.65289; 2 / 4.97279 Distance travelled in the first 2 sec $= (7.65289 - 0)$ $+(7.65289 - 4.97279)$ $= 10.33299$ $= \mathbf{10.3 \text{ m}}$ (to 3 sig. fig.)	Correct Integration
(d)	Distance travelled in the first 2 sec $= \left\lvert \int_0^{\frac{\pi}{3}} (4 + 8\cos 2t) \, dt \right\rvert$ $+ \left\lvert \int_{\frac{\pi}{3}}^{2} (4 + 8\cos 2t) \, dt \right\rvert$ $= \left\lvert [4t + 4\sin 2t]_0^{\frac{\pi}{3}} \right\rvert$ $+ \left\lvert [4t + 4\sin 2t]_{\frac{\pi}{3}}^{2} \right\rvert$ $= \left\lvert 4\left(\frac{\pi}{3}\right) + 4\left(\sin\frac{2\pi}{3}\right) \right\rvert$ $+ \left\lvert 4\left(2 - \frac{\pi}{3}\right) + 4\left(\sin 4 - \sin\frac{2\pi}{3}\right) \right\rvert$ $= 7.65289 + 2.6801$ $= 10.33299$ $= \mathbf{10.3 \text{ m}}$ (to 3 sig. fig.)	Correct Expressions Correct Integration Correct Substitution

t	s
0	0
$\frac{1}{3}\pi$	7.65289
2	4.97279

Qn	Solution	Testing
6.	**R-Formulae**	
(a)	Draw a line $CG \perp BE$. In ΔCGO, $\angle CGO = \frac{\pi}{2}, \quad OC = 10$ $CG = 10\sin\theta$ $GO = 10\cos\theta$ $DF = CG = 10\sin\theta$ $CD = GF = 2GO$ $= 20\cos\theta$ $FO = GO = 10\cos\theta$ $P = OC + CD + DF + FO$ $= 10 + 20\cos\theta + 10\sin\theta + 10\cos\theta$ $P = 10 + 30\cos\theta + 10\sin\theta$	
(b)	$30\cos\theta + 10\sin\theta$ $= R\cos(\theta - \alpha)$ $= \sqrt{30^2 + 10^2}\cos(\theta - \alpha)$ $= 10\sqrt{10}\cos(\theta - \alpha)$ where $\mathbf{R = 10\sqrt{10}}$ $= 31.6227$ $= \mathbf{31.6}$ (to 3 s.f.) $Max\ P = 10 + R$ $= \mathbf{41.6\ cm}$ (to 3 s.f.)	
(c)	$A = \frac{1}{2}DF(CD + FO)$ $= \frac{1}{2}(10\sin\theta)(20\cos\theta + 10\cos\theta)$ $A = 5\sin\theta\,(30\cos\theta)$ $A = 150\sin\theta\cos\theta$ $A = 75\sin 2\theta$	Area $= \frac{1}{2} \times base \times ht$ Double angle formula
	Stationary Values	
(d)	$\frac{dA}{d\theta} = 150\cos 2\theta$ $\frac{d^2A}{d\theta^2} = -300\sin 2\theta$ For stationary value, $\frac{dA}{d\theta} = 0$ $150\cos 2\theta = 0$ $2\theta = \frac{\pi}{2}$ $\theta = \frac{\pi}{4}$ $\frac{d^2A}{d\theta^2} = -300\sin 2\theta$ $= -300\sin\frac{\pi}{2}$ $= -300$ $\frac{d^2A}{d\theta^2} < 0$ $\boxed{A = 75\sin 2\theta = 75\sin\frac{\pi}{2} = 75}$ A has a maximum area (of $75\ \text{cm}^2$) when $\theta = \frac{\pi}{4}$.	Value of A is not required
7.	**Circles & Coordinate Geometry**	
(a)	$P(-13, 11)$ $Q(\ 1,\ 9)$ $R(-1, -5)$ Gradient of PQ: $m_{PQ} = \frac{11-9}{-13-1} = -\frac{1}{7}$ Gradient of QR: $m_{QR} = \frac{9-(-5)}{1-(-1)} = 7$ Since $m_{PQ} \times m_{QR} = -\frac{1}{7} \times 7 = -1$ $\Rightarrow PQ$ is perpendicular to QR $\Rightarrow \angle PQR = 90°$ (**right angle in semicircle**) $\Rightarrow PR$ is the diameter of circle.	Applying $m_1 \times m_2 = -1$
	OR $PQ^2 = (-13-1)^2 + (11-9)^2$ $= 200$ $QR^2 = (1-(-1))^2 + (9-(-5))^2$ $= 200$ $PR^2 = (-13+1)^2 + (11+5)^2$ $= 400$ Since $PR^2 = PQ^2 + QR^2 = 400$, by converse of Pythagoras' Theorem, $\angle PQR = 90°$. (**right angle in semicircle**) $\Rightarrow PR$ is the diameter of circle.	

Qn	Solution	Testing
	OR Midpoint of PR, $M = \left(\frac{-13-1}{2}, \frac{11-5}{2}\right)$ $= (-7,\ 3) \ldots [1]$ $PM = \sqrt{(-13+1)^2 + (11-3)^2}$ $= 10$ $QM = \sqrt{(1+7)^2 + (9-3)^2}$ $= 10$ $RM = (-1+7)^2 + (-5-3)^2 = 10$ $\therefore PM = QM = RM = 10 \ldots [2]$ From [1] and [2], **midpoint of chord $\boldsymbol{PR}$ is equidistant to $\boldsymbol{P}$, $\boldsymbol{Q}$ & $\boldsymbol{R}$** $\Rightarrow PR$ is the diameter of circle.	
(b)	As centre is midpoint of diameter, centre $= \left(\frac{-13-1}{2}, \frac{11-5}{2}\right)$ $= (-7, 3)$ radius $= \frac{1}{2}PR$ $= \frac{1}{2}\sqrt{\left(-13-(-1)\right)^2 + \left(11-(-5)\right)^2}$ $= \frac{1}{2}\sqrt{400}$ $= 10$ Equation of circle $(x-(-7))^2 + (y-3)^2 = 10^2$ $x^2 + 14x + 49 + y^2 - 6y + 9 = 100$ $\boldsymbol{x^2 + y^2 + 14x - 6y - 42 = 0}$	Award 1 mark for either centre or radius correct Correct substitution
(c)	Let $T(x, y)$ be a general point on the perpendicular bisector of PQ. $PT = QT$ $\sqrt{(x-(-13))^2 + (y-11)^2}$ $= \sqrt{(x-1)^2 + (y-9)^2}$ $(x+13)^2 + (y-11)^2$ $= (x-1)^2 + (y-9)^2$ $x^2 + 26x + 169 + y^2 - 22y + 121$ $= x^2 - 2x + 1 + y^2 - 18y + 81$ $4y = 28x + 208$ $\boldsymbol{y = 7x + 52}$	
	OR Perpendicular bisector passes through the centre. Using centre $(-7,\ 3)$ and $m_{\perp PQ} = 7$	

Qn	Solution	Testing
	Equation of perpendicular bisector of QR is $y - 3 = 7\left(x - (-7)\right)$ $\boldsymbol{y = 7x + 52}$	
	OR Perpendicular bisector passes through the midpoint of chord. Midpoint of PQ $= \left(\frac{-13+1}{2}, \frac{11+9}{2}\right)$ $= (-6,\ 10)$ $m_{\perp PQ} = 7$ Equation of perpendicular bisector of QR is $y - 10 = 7\left(x - (-6)\right)$ $\boldsymbol{y = 7x + 52}$	
(d)	The furthest two points on a circle is the diameter of the circle. $\therefore$ centre $(-7, 3)$ is midpoint of diameter QS, $\frac{x_Q + x_S}{2} = -7$; $\frac{y_Q + y_S}{2} = 3$ $1 + x_S = -14$; $9 + y_S = 6$ $x_S = -15$; $y_S = -3$ $\boldsymbol{\therefore S(-15, -3)}$	Diameter is the longest chord Mid-point
	Area of quadrilateral $PQRS$ $= \frac{1}{2}\begin{vmatrix} \overset{P}{-13} & \overset{Q}{1} & \overset{R}{-1} & \overset{S}{-15} & \overset{P}{-13} \\ 11 & 9 & -5 & -3 & 11 \end{vmatrix}$ $= \frac{1}{2}\begin{vmatrix} (-117 - 5 + 3 - 165) \\ -(11 - 9 + 75 + 39) \end{vmatrix}$ $\boldsymbol{= 200\ \text{units}^2}$	Shoelace method
8.	**Tangent & Normal**	
(a)	$y = \frac{10}{12-x} - 1 \ldots\ldots [1]$ Subst $P(2k, 4)$ into [1] $4 = \frac{10}{12-2k} - 1$ $5 = \frac{5}{6-k}$ $6 - k = 1$ $k = 5$ $P(10, 4)$ $\frac{dy}{dx} = (-10)(12-x)^{-2}(-1)$ $\frac{dy}{dx} = \frac{10}{(12-x)^2} \ldots\ldots [2]$	Chain rule

Qn	Solution	Testing
	Subst $P(10,4)$ into [2] $\frac{dy}{dx}=\frac{10}{(12-10)^2}=\frac{5}{2}$ $m_{PT}=\frac{5}{2}$ and $m_{PN}=-\frac{2}{5}$ Using $P(10,4)$ and $m_{PN}=-\frac{2}{5}$, **Equation of normal at *P*** $y-4=-\frac{2}{5}(x-10)$ $y-4=-\frac{2}{5}x+4$ $\boldsymbol{y=-\frac{2}{5}x+8}$ ...[3]	
	Area under Curve	
(b)	Subst $X(x_X,0)$ into [1] $0=\frac{10}{12-x_X}-1$ $\frac{10}{12-x_X}=1$ $10=12-x_X$ $x_X=2$ Subst $N(x_N,0)$ into [3] $0=-\frac{2}{5}x_N+8$ $\frac{2}{5}x_N=8$ $x_N=20$ Area of shaded region $\int_2^{10}\left(\frac{10}{12-x}-1\right)dx+\frac{1}{2}(20-10)(4)$	
	$=[-10\ln(12-x)-x]_2^{10}+20$ $=-[10\ln(12-x)+x]_2^{10}+20$ $=[10\ln(12-x)+x]_{10}^{2}+20$	Integration
	$=10\ln\left(\frac{12-2}{12-10}\right)+\{2-10\}+20$ $=10\ln 5+12$	Substitution of boundaries
	OR Area of shaded region $\int_2^{10}\left(\frac{10}{12-x}-1\right)dx+\int_{10}^{20}\left(-\frac{2}{5}x+8\right)dx$ $=[-10\ln(12-x)-x]_2^{10}+$ $\left[-\frac{1}{5}x^2+8x\right]_{10}^{20}$	Integration

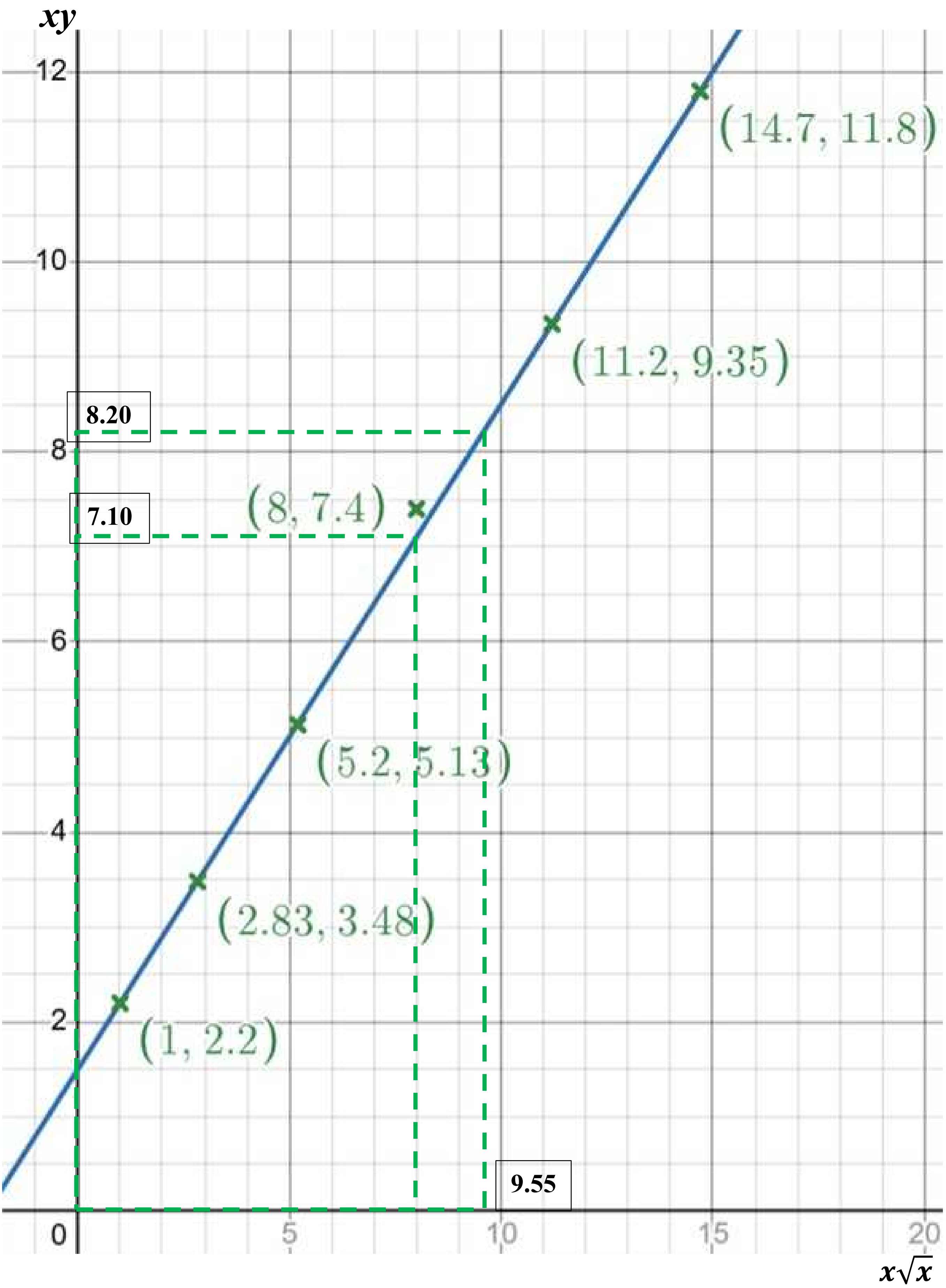
Qn	Solution	Testing
	$=-10\ln\left(\frac{12-10}{12-2}\right)-\{10-2\}-$ $\frac{1}{5}\{20^2-10^2\}+8\{20-10\}$ $=10\ln 5+12$	Substitution of boundaries
9.	**Linear Law**	
(a)	$\frac{y}{a}=\frac{1}{x}+b\sqrt{x}$ Multiply equation with ax, $xy=a+abx\sqrt{x}$ $xy=ab\left(x\sqrt{x}\right)+a$	Linearise equation $Y=mX+C$
(b)	see table below	Tabulate values for Y vs. X

x	1	2	3	4	5	6
y	2.20	1.74	1.71	1.85	1.87	1.96
$x\sqrt{x}$	1.00	2.83	5.20	8.00	11.2	14.7
xy	2.20	3.48	5.13	7.40	9.35	11.8

G1 – Plotted points
G1 – Best-fit line

Qn	Solution	Testing
(c) (i)	From the graph, the incorrect point is (8, 7.40). Corresponding to $x=4$ (in the table), For $x\sqrt{x}=8$ Correct $xy=7.05\ to\ 7.15$ $4y=7.05\ to\ 7.15$ $y=1.7625\ to\ 1.7875$ $y=\mathbf{1.76}\ to\ \mathbf{1.79}$ (to 3 sig. fig.)	Read from graph
(c) (ii)	From the graph, xy intercept $=1.5$ $a=\mathbf{1.50}$ (to 3 sig. fig.) Gradient $=\frac{11.8-2.20}{14.7-1.00}$ $ab=0.7007$ $1.50\,b=0.7007$ $b=0.467153$ $b=\mathbf{0.467}$ (to 3 sig. fig.)	Read from graph
(c) (iii)	When $x=2.52$, for $x\sqrt{x}=2.52\sqrt{2.52}$ $=4.00037$ From the graph, for $x\sqrt{x}=4.00$ $xy=4.3$ $2.52\,y=4.3$ $y=1.70634$ $y=\mathbf{1.71}$ (to 3 sig. fig.)	Read from graph

Qn	Solution	Testing
(d)	From readings of experiment, $x = 4.50$ and $y = 1.86.$ $x\sqrt{x} = 9.5459$ $xy = (4.50)(1.86)$ $= 8.37$	
	From the graph, for $x\sqrt{x} = 9.5459$ $xy = 8.20$	Read from graph
	Vertical difference $= \dfrac{8.37 - 8.20}{8.20} \times 100\%$ $= 2.073\% > 2\%$ Since vertical difference is **more than 2%**, the experimental readings are **unacceptable**.	
	OR Acceptable range $= 8.20 \times (100 \pm 2)\%$ $= 8.036 \ to \ 8.364$ Since $xy = 8.37$ is **out of** the acceptable **range**, the experimental readings are **unacceptable**.	


xy
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8
6
4
2
0
8.20
7.10
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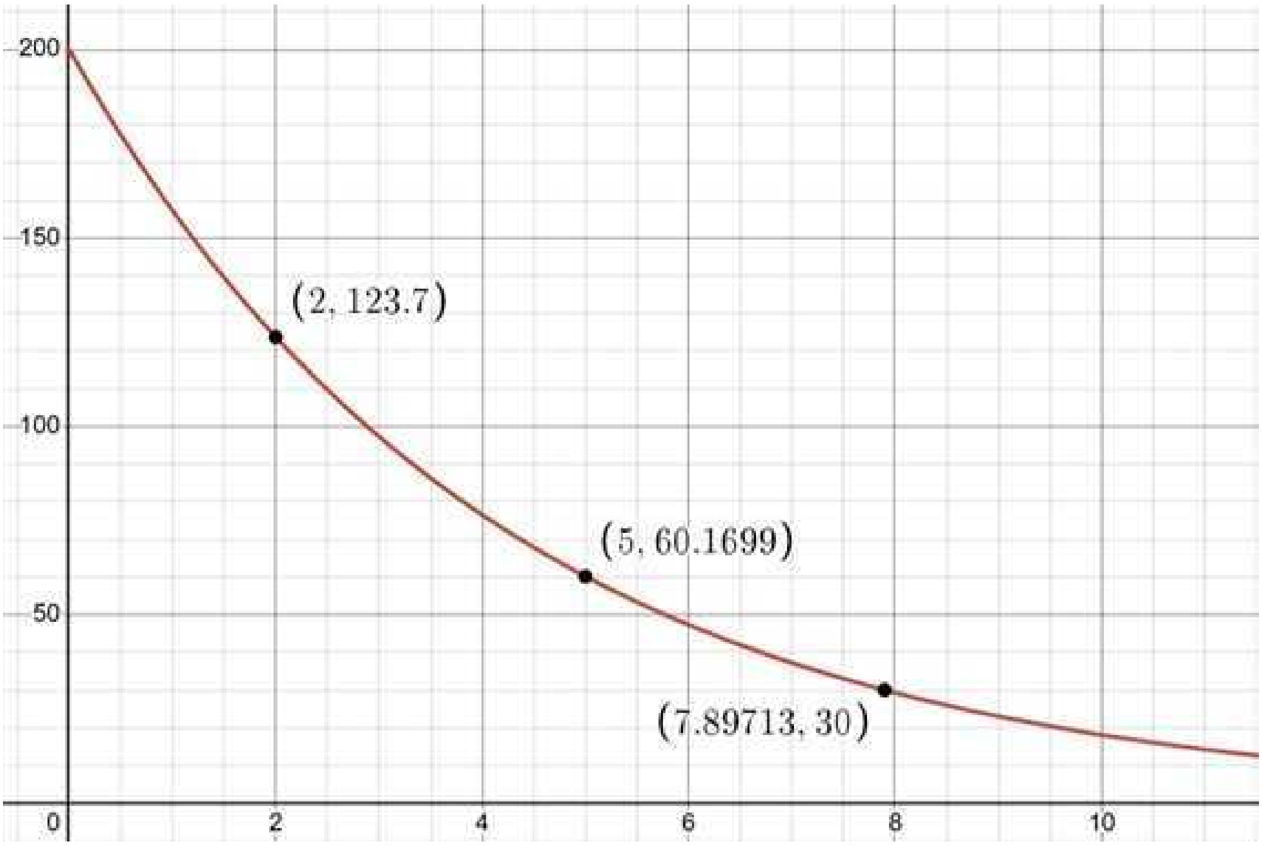
(11.2, 9.35)
(8, 7.4)
(5.2, 5.13)
(2.83, 3.48)
(1, 2.2)
9.55
5
10
15
20
$x\sqrt{x}$


200
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100
50
0
2
4
6
8
10
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(5, 60.1699)
(7.89713, 30)