Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1} a^{n-1} b + \binom{n}{2} a^{n-2} b^2 + \cdots + \binom{n}{r} a^{n-r} b^r + \cdots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2} bc \sin A$$

1 Solve the simultaneous equations

[4]

[Turn over

$$y - x = 4,$$
$$4x^2 - 6xy - 6y = 4.$$

2 Solve the equation $9 \sin x = -2 \sec x$ for $-\pi \leq x \leq 0$. [4]

For Examiner's Use

3 **(a)** Explain why the principal value of $\cos^{-1}\left(-\frac{\sqrt{2}}{2}\right)$ cannot be $-\frac{\pi}{4}$. [1]

[Turn over

For Examiner's Use

(b) Prove that $\cot A + \tan A = \sec A \operatorname{cosec} A$. [4]

For Examiner's Use

4 Express $\dfrac{2x^3+4}{x(x+2)^2}$ in partial fractions. [5]

For Examiner's Use

5 The function f is given by $\mathrm{f}(x) = \dfrac{x+a}{x^2+3}$.

(a) Find $\mathrm{f}'(x)$. [2]

It is given that f increases for $b < x < 1$.

(b) Find the values of a and b. [4]

For Examiner's Use

6 Find the set of values of the constant k for which the curve $y = (k-6)x^2 - 8x + k$ does not intersect the x-axis and has a minimum point. [6]

[Turn over

For Examiner's Use

7 **(a)** Find $\dfrac{\mathrm{d}}{\mathrm{d}x}\left[(x-1)\mathrm{e}^{-x}\right]$. [2]

(b) Hence find $\displaystyle\int x\mathrm{e}^{-x}\,\mathrm{d}x$. [4]

For Examiner's Use

8 The radius of a circle, r cm, decreases at a rate of $\frac{2}{(t+1)^2}$ cm per minute.

(a) Given that the initial radius is 4 cm, find an expression for r in terms of t. [3]

(b) Find the rate of change of the area of the circle when the radius is 2.6 cm. [3]

For Examiner's Use

9 **(a)** Find the value of the constant k such that the line $y = kx + 6$ is a tangent to the curve $2x^2 - xy = 3$. [3]

(b) Solve $\left(4 - 2\sqrt{2}\right)x + \sqrt{2} = 3x\sqrt{2} - 1$, giving your answer in the form $a\sqrt{2} + b$, where a and b are rational numbers. [4]

For Examiner's Use

10 An isosceles triangle PQR has vertices $P(1, 0)$, $Q(0, 2)$ and $R(2, 3)$.
$PQ = QR$.

(a) Find the area of the triangle PQR. [2]

(b) If S is a point such that $PQRS$ is a kite and it also lies on the line $x - y = 5$, find the coordinates of S. [5]

11 **(a)** Solve the equation $2^{x} + 4^{x-1} = 24$. [4]

[Turn over

For Examiner's Use

(b) Solve the equation $\log_{\sqrt{3}}(x-1) = \log_3(x+5)$. [4]

For Examiner's Use

12 The gradient at any point on a curve y is given by $6x - \frac{1}{kx^3}$. The line $4x - 4y = 3$ is a normal to the curve at the point where $x = \frac{1}{2}$.

(a) Show that $k = 2$. [2]

(b) Hence find the x-coordinates of the stationary points. [2]

(c) Find $\dfrac{d^2y}{dx^2}$ and explain whether the gradient $6x - \dfrac{1}{kx^3}$ has a turning point. [4]

For Examiner's Use

13 The cost in thousands of dollars, C, of producing n hundred pairs of a certain type of running shoes is given by the formula $C = 1.2n^2 - 14.4n + 53.7$.

(a) Find the initial cost of production. [1]

(b) Express C in the form $a(x+h)^2 + k$ where a, h and k are constants to be determined. [3]

(c) Explain the meaning of the values of h and k in the formula as expressed in **part (b)**. [1]

(d) Find the maximum number of pairs of running shoes for which the cost will be [3]

[Turn over

For Examiner's Use

50 thousand dollars.

For Examiner's Use

14 The line $y = -x + 4$ is a tangent to a circle at the point A and the centre of the circle is at $B(-4, -2)$.

(a) Find coordinates of A. [3]

(b) Find the radius and the equation of the circle. [3]

[Turn over

For Examiner's Use

(c) Find the equation of another tangent to the circle which is parallel to $y = -x + 4$. [4]

For Examiner's Use

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For Examiner's Use

Bedok View Secondary School
Department of Mathematics — Mark Scheme

Year	2025	**Level & Stream**	4E5N
Type of Assessment	Preliminary Exam	**Subject & Paper**	Additional Math P1

Qns	Working
1	$4x^2-6xy-6y=4$ ----(1) $y-x=4$ ------(2) Sub (2) into (1): $4x^2-6x(4+x)-6(4+x)=4$ $4x^2-24x-6x^2-24-6x=4$ $2x^2+30x+28=0$ $x^2+15x+14=0$ $(x+1)(x+14)=0$ $x=-14$ or $x=-1$ $y=-10$ or $y=3$
2	$9\sin x=-2\sec x$ $9\sin x=-\dfrac{2}{\cos x}$ $\sin x\cos x=-\dfrac{2}{9}$ $2\sin x\cos x=-\dfrac{4}{9}$ $\sin 2x=-\dfrac{4}{9}$ Basic angle = $\sin^{-1}\left(\dfrac{4}{9}\right)$ $=0.46055$ $2x=-0.46055,\ -(\pi-0.46055)$ $=-0.46055$ or -2.68104 $x=-0.230$ or -1.34

Qns	Working
3a	Principal range of $\cos^{-1}x$ is $0 \le \cos^{-1}x \le \pi$, hence principal value of $\cos^{-1}\left(-\frac{\sqrt{2}}{2}\right)$ is $\frac{3\pi}{4}$ instead of $-\frac{\pi}{4}$.
3b	L.H.S: $\cot A + \tan A = \frac{1}{\tan A} + \tan A$ $= \frac{\cos A}{\sin A} + \frac{\sin A}{\cos A}$ $= \frac{\cos^2 A + \sin^2 A}{\sin A \cos A}$ $= \frac{1}{\sin A \cos A}$ $= \sec A \operatorname{cosec} A$ = RHS (shown) **OR** RHS: $\sec A \operatorname{cosec} A = \frac{1}{\cos A} \times \frac{1}{\sin A}$ $\frac{1}{\cos A \sin A}$ $= \frac{\sin^2 A + \cos^2 A}{\cos A \sin A}$ $= = \frac{\sin^2 A}{\cos A \sin A} + \frac{\cos^2 A}{\cos A \sin A}$ $= \frac{\sin A}{\cos A} + \frac{\cos A}{\sin A}$ $= \tan A + \cot A$ = LHS (shown)

Qns	Working
4	By long division, $$\frac{2x^3+4}{x(x+2)^2}=\frac{2x^3+4}{x^3+4x^2+4x}$$ $$=2+\frac{-8x^2-8x+4}{x(x+2)^2}$$ $$\frac{-8x^2-8x+4}{x(x+2)^2}=\frac{A}{x}+\frac{B}{x+2}+\frac{C}{(x+2)^2}$$ $-8x^2-8x+4=A(x+2)^2+Bx(x+2)+Cx$ Let $x=0$: $4=4A$ $A=1$ Let $x=-2$: $-12=-2C$ $C=6$ Let $x=1$: $-8-8+4=9+3B+6$ $B=-9$ $$\therefore \frac{2x^3+4}{x(x+2)^2}=2+\frac{1}{x}-\frac{9}{x+2}+\frac{6}{(x+2)^2}$$
5a 5b	$$\mathrm{f}'(x)=\frac{(x^2+3)-2x(x+a)}{(x^2+3)^2}$$ $$=\frac{-x^2-2ax+3}{(x^2+3)^2}$$ $$\frac{-x^2-2ax+3}{(x^2+3)^2}>0,\ \text{for } b<x<1$$ Since $(x^2+3)^2>0$, $-x^2-2ax+3>0$ $x^2+2ax-3<0$ Given that $b<x<1$ $(x-b)(x-1)<0$ $x^2-bx-x+b<0$ By comparing: $b=-3$ $2a=-b-1$ $2a=3-1$ $a=1$

Qns	Working
6	$y=(k-6)x^2-8x+k$ Discriminant $=(-8)^2-4(k-6)(k)$ $=64-4k^2+24k$ For curve that does not intersect the x-axis, Discriminant < 0 $k^2-6k-16>0$ $(k+2)(k-8)>0$ $k<-2$ or $k>8$ The curve has a minimum point: $k-6>0$ $k>6$ $\therefore k>8$
7a 7b	$\frac{\mathrm{d}}{\mathrm{d}x}\left[(x-1)\mathrm{e}^{-x}\right]=\mathrm{e}^{-x}-(x-1)\mathrm{e}^{-x}$ $=2\mathrm{e}^{-x}-x\mathrm{e}^{-x}$ $\int 2\mathrm{e}^{-x}-x\mathrm{e}^{-x}\mathrm{d}x=(x-1)\mathrm{e}^{-x}+c$ $\int 2\mathrm{e}^{-x}\mathrm{d}x-\int x\mathrm{e}^{-x}\mathrm{d}x=(x-1)\mathrm{e}^{-x}+c$ $-\int x\mathrm{e}^{-x}\mathrm{d}x=-\int 2\mathrm{e}^{-x}\mathrm{d}x+(x-1)\mathrm{e}^{-x}+c$ $\int x\mathrm{e}^{-x}\mathrm{d}x=-2\mathrm{e}^{-x}-(x-1)\mathrm{e}^{-x}+c$ $=-2\mathrm{e}^{-x}-x\mathrm{e}^{-x}+\mathrm{e}^{-x}+c$ $=-\mathrm{e}^{-x}-x\mathrm{e}^{-x}+c$

Qns	Working
8a	$\frac{\mathrm{d}r}{\mathrm{d}t} = -\frac{2}{(t+1)^2}$ $r = -\int \frac{2}{(t+1)^2}\mathrm{d}t$ $= -\int 2(t+1)^{-2}\mathrm{d}t$ $= 2(t+1)^{-1} + c$ $r = \frac{2}{t+1} + c$ At $r = 4$ and $t = 0$, $4 = 2 + c$ $c = 2$ $r = \frac{2}{t+1} + 2$
8b	$A = \pi r^2$ $\frac{\mathrm{d}A}{\mathrm{d}r} = 2\pi r$ At $r = 2.6$, $\frac{\mathrm{d}A}{\mathrm{d}r} = 5.2\pi$, $2.6 = \frac{2}{t+1} + 2$ $0.6 = \frac{2}{t+1}$ $t = \frac{7}{3}$ $\frac{\mathrm{d}r}{\mathrm{d}t} = -\frac{2}{\left(\frac{7}{3}+1\right)^2} = -0.18$ $\frac{\mathrm{d}A}{\mathrm{d}t} = 5.2\pi \times -0.18$ $\frac{\mathrm{d}A}{\mathrm{d}t} = -0.936\pi \text{ cm}^2/\text{min}$

Qns	Working
9a	$y = kx + 6$ ---- (1) $2x^2 - xy = 3$ -----(2) Sub (1) into (2): $2x^2 - x(kx+6) = 3$ $2x^2 - kx^2 - 6x - 3 = 0$ $(2-k)x^2 - 6x - 3 = 0$ Discriminant $= (-6)^2 - 4(2-k)(-3)$ $= 60 - 12k$ For tangent, $60 - 12k = 0$ $k = 5$
9b	$\left(4 - 2\sqrt{2}\right)x + \sqrt{2} = 3x\sqrt{2} - 1$ $4x - 2x\sqrt{2} + \sqrt{2} = 3x\sqrt{2} - 1$ $\sqrt{2} + 1 = 3x\sqrt{2} + 2x\sqrt{2} - 4x$ $\sqrt{2} + 1 = x\left(5\sqrt{2} - 4\right)$ $x = \dfrac{\sqrt{2}+1}{5\sqrt{2}-4} \times \dfrac{5\sqrt{2}+4}{5\sqrt{2}+4}$ $= \dfrac{5(2) + 4\sqrt{2} + 5\sqrt{2} + 4}{\left(5\sqrt{2}\right)^2 - 4^2}$ $= \dfrac{9\sqrt{2} + 14}{50 - 16}$ $= \dfrac{9}{34}\sqrt{2} + \dfrac{7}{17}$

Qns	Working
10a	Area of $\Delta PQR = \frac{1}{2}\begin{vmatrix} 1 & 2 & 0 & 1 \\ 0 & 3 & 2 & 0 \end{vmatrix}$ $= \frac{1}{2}[3+4-2]$ $= 2.5 \text{ units}^2$
10b	$m_{PR} = \frac{3-0}{2-1}$ $= 3$ $m_{QS} = -\frac{1}{3}$ Equation of QS: $y-2 = -\frac{1}{3}(x-0)$ $y = -\frac{1}{3}x + 2 --- (1)$ $x - y = 5 --- (2)$ Sub (1) into (2): $x + \frac{1}{3}x - 2 = 5$ $x = \frac{21}{4}$ $y = \frac{1}{4}$ coordinates of $S = \left(\frac{21}{4}, \frac{1}{4}\right)$

Qns	Working
11a	$2^x + 4^{x-1} = 24$ $2^x + \left(4^x\right)\left(4^{-1}\right) = 24$ $2^x + \frac{2^{2x}}{4} = 24$ Let $y = 2^x$, $y + \frac{y^2}{4} = 24$ $4y + y^2 = 96$ $y^2 + 4y - 96 = 0$ $(y-8)(y+12) = 0$ $y = 8$ or $y = -12$ $2^x = 8$ or $2^x = -12$ $\left(\text{Rejected since } 2^x > 0\right)$ $2^x = 2^3$ $x = 3$
11b	$\log_{\sqrt{3}}(x-1) = \log_3(x+5)$ $\frac{\log_3(x-1)}{\log_3\sqrt{3}} = \log_3(x+5)$ $\frac{\log_3(x-1)}{\log_3 3^{\frac{1}{2}}} = \log_3(x+5)$ $2\log_3(x-1) = \log_3(x+5)$ $\log_3(x-1)^2 = \log_3(x+5)$ $(x-1)^2 = x+5$ $x^2 - 2x + 1 - x - 5 = 0$ $x^2 - 3x - 4 = 0$ $(x+1)(x-4) = 0$ $x + 1 = 0$ $x = -1$ [N.A. as $\log_{\sqrt{3}}(x-1)$ will be undefined] or $x - 4 = 0$ $x = 4$

Qns	Working
12a	$4x-4y=3$ $y=x-\frac{3}{4}$ m of normal $=1$ m of tangent $=-1$ At $x=\frac{1}{2}$ and $\frac{\mathrm{d}y}{\mathrm{d}x}=-1$, $6\left(\frac{1}{2}\right)-\frac{1}{k\left(\frac{1}{2}\right)^3}=-1$ $3-\frac{8}{k}=-1$ $k=2$ (shown)
12b	At stat points, $6x-\frac{1}{2x^3}=0$ $6x=\frac{1}{2x^3}$ $x^4=\frac{1}{12}$ $x=\pm\sqrt[4]{\frac{1}{12}}$ $x=-0.537$ or 0.537
12c	$\frac{\mathrm{d}^2y}{\mathrm{d}x^2}=6+\frac{3}{2x^4}$ $6>0$ $\frac{3}{2x^4}>0$ $\therefore \frac{\mathrm{d}^2y}{\mathrm{d}x^2}=6+\frac{3}{2x^4}>0$ Since $\frac{\mathrm{d}^2y}{\mathrm{d}x^2}=6+\frac{3}{2x^4}>0$ the gradient has no turning point.

Qns	Working
13a	$C = 1.2n^2 - 14.4n + 53.7$ At $n = 0$, $C = \$53.7$ thousands
13b	$C = 1.2n^2 - 14.4n + 53.7$ $= 1.2\left[n^2 - 12n + 44.75\right]$ $= 1.2\left[(n-6)^2 - 6^2 + 44.75\right]$ $= 1.2(n-6)^2 + 10.5$
13c	For 600 pairs of running shoes produced, the minimum cost of 10.5 thousand dollars will be incurred.
13d	$1.2(n-6)^2 + 10.5 = 50$ $1.2(n-6)^2 = 39.5$ $(n-6)^2 = \dfrac{39.5}{1.2}$ $n - 6 = \pm 5.7373$ $n = 11.7373 \text{ or } 0.2627$ Maximum number of pairs of running shoes for which the cost is at most 50 thousand dollars = 11.7373 = 11.7 hundreds

Qns	Working
14a	$m_{AB} = 1$ Equation of AB: $y + 2 = x + 4$ $y = x + 2 ----(1)$ $y = -x + 4 ----(2)$ $(1) = (2): x + 2 = -x + 4$ $2x = 2$ $x = 1$ $y = 3$ Coordinates of A = (1, 3)
14b	$\text{Radius} = \sqrt{(1+4)^2 + (3+2)^2}$ $= \sqrt{50}$ units Equation of circle: $(x+4)^2 + (y+2)^2 = 50$
14c	Let the coordinates of the other end of the diameter passes through A = $C(x, y)$. Since AC is the diameter, B is the midpoint of AC. $\left(\frac{x+1}{2}, \frac{y+3}{2}\right) = (-4, -2)$ $\frac{x+1}{2} = -4, \quad \frac{y+3}{2} = -2$ $x = -9, \ y = -7$ Equation of another tangent which is parallel to $y = -x + 4$: $y + 7 = -1(x + 9)$ $y = -x - 16$

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1} a^{n-1} b + \binom{n}{2} a^{n-2} b^2 + \cdots + \binom{n}{r} a^{n-r} b^r + \cdots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2} bc \sin A$$

[Turn over

1 The nutrient absorption rate, N, in mg/hour, of a plant depends on the diameter x, in mm, of its root hairs and is modelled by the function $\mathrm{N}(x) = x^2 \ln\left(\frac{2}{x}\right)$, where $0 < x < 2$.

(a) Find $\mathrm{N}'(x)$. [3]

(b) Find the optimal root hair diameter that maximises nutrient absorption and show that the nutrient absorption is a maximum. [5]

For Examiner's Use

2 Hot coffee is heated to a temperature of $90\ °\mathrm{C}$. The temperature, $T\ °\mathrm{C}$, t minutes after removal from the heat source is given by $T = 22 + Ae^{kt}$, where A and k are constants.

(a) Explain why $A = 68$. [1]

(b) After 3.5 minutes, the temperature of coffee is $79\ °\mathrm{C}$. Find k. [3]

[Turn over

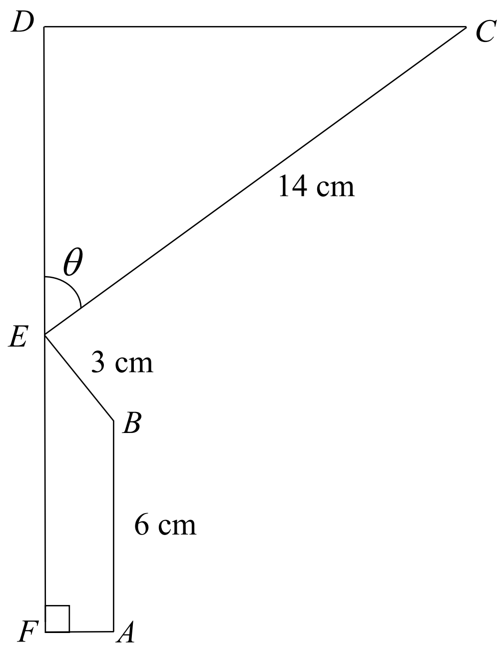
For Examiner's Use

(c) The coffee should only be served when it is between $55\ °C$ to $70\ °C$. Determine, with working, whether the coffee is suitable to be served 10 minutes after removal from the heat source. [2]

(d) After a long time, T approaches a value. Calculate this value. [1]

For Examiner's Use

3 In the diagram, angle $CED = \theta$. EC is perpendicular to BE. The lengths of AB, BE and EC are 6 cm, 3 cm and 14 cm respectively. AB is parallel to FED and CD is parallel to AF.



(a) Show that $FD = 14\cos\theta + 3\sin\theta + 6$. [3]

(b) Express FD in the form $R\cos(\theta - \alpha) + d$ where $R > 0$, $0° < \alpha < 90°$, and d is a constant. [3]

(c) Find the value of θ when $FD = 12\,\text{cm}$. [2]

(d) Determine if it is possible for FD to be 15 cm. [2]

4 In a biology experiment, a certain type of bacteria multiplies rapidly. The number of bacteria, N, after t hours is modelled by the formula $N = a10^{kt}$, where a and k are constants. The table below shows corresponding values of t and N.

t	2	4	6	8
N	900	2700	8100	24 300

(a) On the grid on the next page, plot $\lg N$ against t and draw a straight line graph to illustrate the information. [4]

[Turn over

(b) Use the graph to estimate the value of each of the constants a and k. [2]

For Examiner's Use

5 **(a)** The cubic polynomial $\mathrm{f}(x)$ is such that the coefficient of x^3 is -2 and the roots of $\mathrm{f}(x)=0$ are -1, 2 and $\frac{3}{2}$. Find the expression of $\mathrm{f}(x)$ in descending powers of x. [3]

(b) Find the quotient when x^3-3x^2-2x+5 is divided by x^2-1. [2]

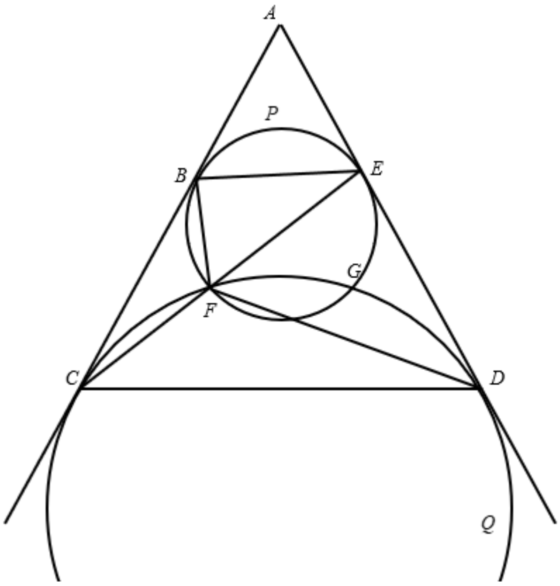
(c) The expression hx^3-7x^2+kx-2, where h and k are constants, has a factor $3x^2-5x-2$. Find the value of h and k. [5]

[Turn over

For Examiner's Use

For Examiner's Use

6 The diagram shows two circles P and Q which intersect each other at F and G. The line ABC is tangent to P and Q at B and C respectively. The line AED is tangent to P and Q at E and D respectively. EFC is a straight line.



(a) Prove that triangle ABE is similar to triangle ACD. [4]

(b) Explain why a circle can be drawn to pass through *BEDC*. [3]

(c) Prove that angle *CBF* = angle *EDF*. [3]

7 **(a)** Find the coefficient of x^3 in the expansion of $(7+2x)\left(1-\dfrac{x}{5}\right)^8$. [4]

[Turn over

(b) In the expansion of $\left(3+\frac{x}{2}\right)^n$, the coefficient of x^2 is three times the coefficient of x^3.

Find n. [4]

For Examiner's Use

8 **Do not use a calculator in this question.**

(a) Find the exact value of $\operatorname{cosec} A$ when $\cos 2A = \frac{1}{3}$ and A is acute. [4]

[Turn over

For Examiner's Use

(b) Hence, find the exact value of $\operatorname{cosec} A - \sqrt{2}\sin 105°$. [5]

For Examiner's Use

9 A particle moves in a straight line so that its acceleration, a m/s^2, is given by $a = -2\cos\frac{1}{2}t$, where t is the time in seconds after leaving a fixed point O.

(a) Sketch the acceleration-time graph of the particle for $0 \le t \le 2\pi$. [2]

(b) The initial velocity of the particle is 3 m/s. Find the earliest time when the particle first comes to rest. [4]

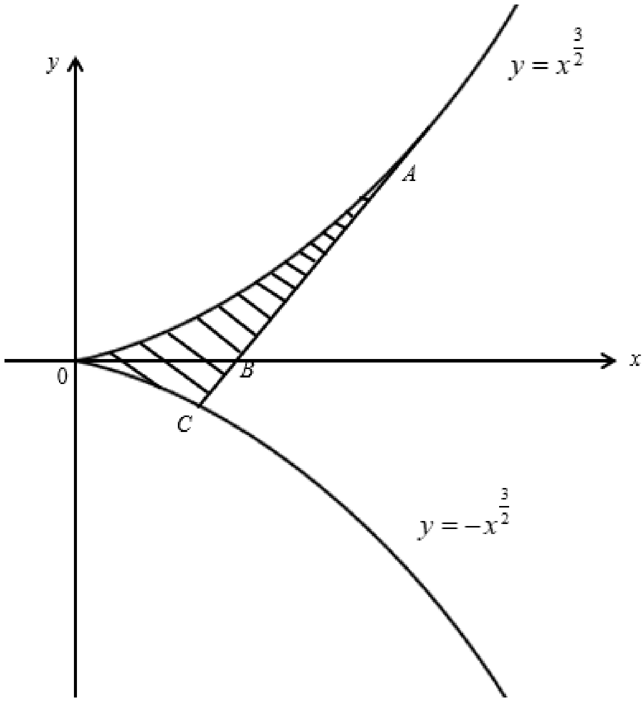
Continuation of working space for question **9(b)**.

[Turn over

For Examiner's Use

(c) Find the total distance travelled in the first 5 seconds. [5]

10 The diagram shows the graphs of $y = x^{\frac{3}{2}}$ and $y = -x^{\frac{3}{2}}$. The line $y = 2x - 1$ intersects the graph of $y = x^{\frac{3}{2}}$ at $A(1, 1)$, the x-axis at B, and the graph of $y = -x^{\frac{3}{2}}$ at C.



(a) Show that the coordinates of C are (0.38197, -0.23607), correct to 5 significant figures. [5]

For Examiner's Use

Continuation of working space for question **10(a)**.

(b) Find the area of the shaded region. [6]

For Examiner's Use

Continuation of working space for question **10(b)**.

End of Paper

Bedok View Secondary School
Department of Mathematics — Mark Scheme

Year	2025	**Level & Stream**	4E
Type of Assessment	Preliminary Exam	**Subject & Paper**	Additional Math P2

Qns	Working
1a	$\mathrm{N}(x) = x^2 \ln\left(\dfrac{2}{x}\right)$ let $u = x^2$ and $v = \ln\left(\dfrac{2}{x}\right)$ $\dfrac{\mathrm{d}u}{\mathrm{d}x} = 2x \qquad v = \ln 2 - \ln x$ $\qquad\qquad \dfrac{\mathrm{d}v}{\mathrm{d}x} = -\dfrac{1}{x}$ $\mathrm{N}'(x) = x^2.\left(-\dfrac{1}{x}\right) + \ln\left(\dfrac{2}{x}\right).2x$ $\qquad = -x + 2x\ln\left(\dfrac{2}{x}\right)$
b	stationary point $\mathrm{N}'(x) = 0$ $-x + 2x\ln\left(\dfrac{2}{x}\right) = 0$ $x\left(-1 + 2\ln\left(\dfrac{2}{x}\right)\right) = 0$ $x = 0$ (rej) or $-1 + 2\ln\left(\dfrac{2}{x}\right) = 0$ $2\ln\left(\dfrac{2}{x}\right) = 1$ $\ln\left(\dfrac{2}{x}\right) = \dfrac{1}{2}$ $e^{0.5} = \dfrac{2}{x}$ $x = 2 \div e^{0.5}$ $x = 1.21306$

x	1.1	1.21306	1.3
Sign of N'(x)	+ve	0	−ve
Sketch of tangent			

$\therefore$ max absorption rate at x = 1.21 (3 s.f.)

Qns	Working
2a	$T = 22 + Ae^{kt}$ when $t = 0,\ T = 90$ $90 = 22 + Ae^{k(0)}$ $90 = 22 + A$ $A = 68$
b	when $t = 3.5,\ T = 79$ $79 = 22 + 68e^{k(3.5)}$ $57 = 68e^{k(3.5)}$ $\frac{57}{68} = e^{k(3.5)}$ $\ln\left(\frac{57}{68}\right) = k(3.5)$ $k = \ln\left(\frac{57}{68}\right) \div 3.5$ $k = -0.050416$ $k = -0.0504$ (3 s.f.)
c	$T = 22 + 68e^{-0.050416t}$ when $t = 10$, $T = 22 + 68e^{-0.050416(10)}$ $= 63.0728$ Since 63.0728 °C is between 55 °C and 70 °C, the coffee is suitable to be served.
d	After a long time, T approaches 22 °C.

Qns	Working
3a	$\cos\theta = \frac{ED}{14}$ $ED = 14\cos\theta$ Let G be the point on FD such that $BG \perp FD$ $\sin\theta = \frac{EG}{3}$ $EG = 3\sin\theta$ $FD = 14\cos\theta + 3\sin\theta + 6$ (shown)
b	$R = \sqrt{14^2 + 3^2}$ $= \sqrt{205}$ $\tan\alpha = \frac{3}{14}$ $\alpha = 12.0947$ $FD = \sqrt{205}\cos(\theta - 12.1°) + 6$ (3 s.f.)
c	$\sqrt{205}\cos(\theta - 12.0947°) + 6 = 12$ $\cos(\theta - 12.0947°) = \frac{6}{\sqrt{205}}$ $\theta - 12.0947° = 65.2248°$ $\theta = 77.3195°$ $\theta = 77.3°$ (1 d.p.)
d	Maximum $\sqrt{205}\cos(\theta - 12.0947°) + 6$ $= \sqrt{205} + 6$ $= 20.317$ $\therefore$ it is possible for FD to be 15 cm Or $\sqrt{205}\cos(\theta - 12.0947°) + 6 = 15$ $\cos(\theta - 12.0947°) = \frac{9}{\sqrt{205}}$ $\theta - 12.0947° = 51.054°$ $\theta = 63.1°$ (1 d.p.) FD is 15 cm when $\theta = 63.1°$

Qns	Working
4a	

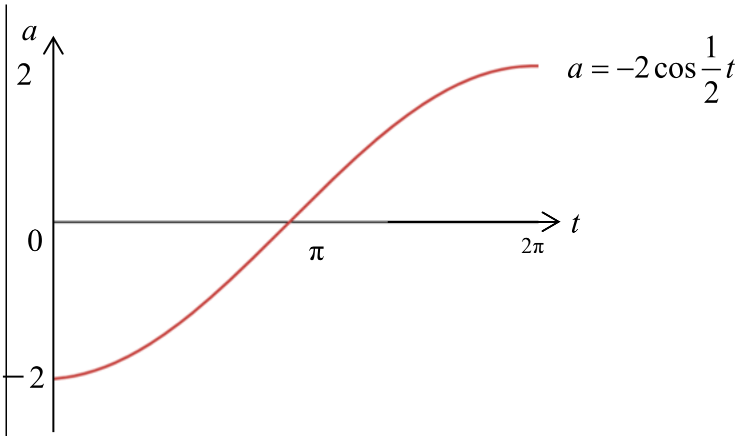
Qns	Working
b	$N = a10^{kt}$ $\lg N = \lg a10^{kt}$ $\lg N = \lg a + \lg 10^{kt}$ $\lg N = \lg a + kt$ $\lg N = kt + \lg a$ $k =$ gradient $= 0.232$ (3 s.f.) ± 0.1 $\lg a =$ vertical axis $-$ intercept $\lg a = 2.5 \pm 0.05$ $a = 10^{2.5}$ $=316$ (3 s.f.)
5a b	$-2(x+1)(x-2)(x-1.5)$ $= (x^2 - x - 2)(-2x+3)$ $= -2x^3 + 3x^2 + 2x^2 - 3x + 4x - 6$ $= -2x^3 + 5x^2 + x - 6$ $$\begin{array}{r} x-3 \\ x^2+0x-1 \overline{\smash{\big)} x^3-3x^2-2x+5} \\ \underline{-\left(x^3+0x^2-x\right)} \\ -3x^2-x+5 \\ \underline{-\left(-3x^2+0x+3\right)} \\ -x+2 \end{array}$$ $\therefore$ Quotient $= x-3$

Qns	Working
c	$hx^3 - 7x^2 + kx - 2 = \left(3x^2 - 5x - 2\right) \times \text{Quotient}$ $hx^3 - 7x^2 + kx - 2 = (3x+1)(x-2) \times \text{Quotient}$ sub $x = -\frac{1}{3}$, $h\left(-\frac{1}{3}\right)^3 - 7\left(-\frac{1}{3}\right)^2 + k\left(-\frac{1}{3}\right) - 2 = 0$ $-\frac{1}{27}h - \frac{1}{3}k = \frac{25}{9}$ ----------(1) sub $x = 2$, $h(2)^3 - 7(2)^2 + k(2) - 2 = 0$ $8h + 2k = 30$----------(2) From (2), $4h + k = 15$ $k = 15 - 4h$----------(3) sub (3) into (1), $-\frac{1}{27}h - \frac{1}{3}(15 - 4h) = \frac{25}{9}$ $-\frac{1}{27}h - 5 + \frac{4}{3}h = \frac{25}{9}$ $\frac{35}{27}h = \frac{70}{9}$ $h = 6$ sub $h = 6$ into (3), $k = 15 - 4(6)$ $k = -9$

Qns	Working
6a	$AB = AE$ (tangent from external point) $AC = AD$ (tangent from external point) $\frac{AB}{AC} = \frac{AE}{AD}$ (ratio of corresponding sides are equal) $\angle BAE = \angle CAD$ (common angle) ΔABE is similar to ΔACD (ratio of 2 pairs of corresponding sides and a pair of included $\angle$s are equal)
b	$\angle ABE = \angle ACD$ (ΔABE is similar to ΔACD) $\angle ACD = \angle ADC$ (base $\angle$ of isosceles Δ) $\angle EBC = 180° - \angle ABE$ (adjacent $\angle$s on a straight line) Since $\angle ADC + \angle EBC = 180°$, by the converse of angles in opposite segments, a circle can be drawn to pass through $BEDC$.
c	Let $\angle CBF = x$, $\angle BEF = x$ (alternate segment thm) $BE // CD$ since ΔABE is similar to ΔACD $\angle ECD = x$ (alternate angles, $BE // CD$) $\angle EDF = x$ (alternate segment thm) $\therefore \angle CBF = \angle EDF$ (proven)

Qns	Working
7a	General term of $\left(1-\frac{x}{5}\right)^8$ $=\binom{8}{r}(1)^{8-r}\left(-\frac{x}{5}\right)^r$ $=\binom{8}{r}\left(-\frac{x}{5}\right)^r$ $(7+2x)\left(1-\frac{x}{5}\right)^8$ $=(7+2x)\left(\ldots+\binom{8}{3}\left(-\frac{x}{5}\right)^3+\binom{8}{2}\left(-\frac{x}{5}\right)^2+\ldots\right)$ x^3 term $=7\binom{8}{3}\left(-\frac{x}{5}\right)^3+2x\binom{8}{2}\left(-\frac{x}{5}\right)^2$ $=7(56)\left(-\frac{x^3}{125}\right)+2x(28)\left(\frac{x^2}{25}\right)$ $=-\frac{392}{125}x^3+\frac{56}{25}x^3$ Coefficient of $x^3=-\frac{112}{125}$
b	$\binom{n}{2}3^{n-2}\left(\frac{1}{2}x\right)^2=\binom{n}{2}3^{n-2}\frac{1}{4}x^2$ $\binom{n}{3}3^{n-3}\left(\frac{1}{2}x\right)^3=\binom{n}{3}3^{n-3}\frac{1}{8}x^3$ $\binom{n}{2}3^{n-2}\frac{1}{4}=3\times\binom{n}{3}3^{n-3}\frac{1}{8}$ $\binom{n}{2}3^{n-2}.2=\binom{n}{3}3^{n-2}$ $\binom{n}{2}.2=\binom{n}{3}$ $\frac{n(n-1)}{2}.2=\frac{n(n-1)(n-2)}{2\times3}$ $6=n-2$ $n=8$

Qns	Working
8a	$\cos 2A = \frac{1}{3}$ $1 - 2\sin^2 A = \frac{1}{3}$ $\sin^2 A = \frac{1}{3}$ $\sin A = \frac{1}{\sqrt{3}}$ or $-\frac{1}{\sqrt{3}}$ (rej) $\text{cosec } A = \frac{1}{\sin A}$ $= 1 \div \frac{1}{\sqrt{3}}$ $= \sqrt{3}$
b	$\sin 105°$ $= \sin(45° + 60°)$ $= \sin 45° \cos 60° + \cos 45° \sin 60°$ $= \frac{1}{\sqrt{2}} . \frac{1}{2} + \frac{1}{\sqrt{2}} . \frac{\sqrt{3}}{2}$ $= \frac{1+\sqrt{3}}{\sqrt{2}.2}$ $\text{cosec } A - \sqrt{2} \sin 105°$ $= \sqrt{3} - \sqrt{2} . \frac{1+\sqrt{3}}{\sqrt{2}.2}$ $= \sqrt{3} - \frac{1+\sqrt{3}}{2}$ $= \frac{\sqrt{3}-1}{2}$

Qns	Working
9a	
b	$v=\int -2\cos\frac{1}{2}t\ \mathrm{d}t$ $=\dfrac{-2\sin\frac{1}{2}t}{\frac{1}{2}}+c$ $=-4\sin\frac{1}{2}t+c$ Given $t=0$ and $v=3$, find c $3=-4\sin\frac{1}{2}(0)+c$ $c=3$ $v=-4\sin\frac{1}{2}t+3$ find t when $v=0$, $-4\sin\frac{1}{2}t+3=0$ $\sin\frac{1}{2}t=\frac{3}{4}$ basic $\angle$ of $\frac{1}{2}t=0.84806$ $t=1.6961$ $t=1.70$ (3 s.f.)

Qns	Working
c	basic $\angle$ of $\frac{1}{2}t = 0.84806$ since $\sin\frac{1}{2}t$ is +ve, $\frac{1}{2}t$ is also in 2nd quad. $\frac{1}{2}t = \pi - 0.84806$ $\frac{1}{2}t = 2.2935$ $t = 4.5870$ Displacement $=\int\left(-4\sin\frac{1}{2}t+3\right)\mathrm{d}t$ $=-4.\dfrac{-\cos\frac{1}{2}t}{\frac{1}{2}}+3t+C_1$ $=8\cos\frac{1}{2}t+3t+C_1$ Distance travelled in first 5 min $=\left\|\left[8\cos\frac{1}{2}t+3t\right]_0^{1.6961}\right\|+\left\|\left[8\cos\frac{1}{2}t+3t\right]_{1.6961}^{4.5870}\right\|+\left\|\left[8\cos\frac{1}{2}t+3t\right]_{4.5870}^{5}\right\|$ $=\|10.379-8\|+\|8.4696-10.379\|+\|8.5908-8.4696\|$ $=2.3798+1.9100+0.1212$ $=4.41$ m

Qns	Working
10a	$y = 2x - 1 -----(1)$ $y = -x^{\frac{3}{2}} --------(2)$ fr (2), $y^2 = x^3 ---(3)$ sub (1) into (3), $(2x-1)^2 = x^3$ $4x^2 - 4x + 1 = x^3$ $0 = x^3 - 4x^2 + 4x - 1$ $(x-1)$ is a factor since $A(1,1)$ is intersection $0 = (x-1)(Ax^2 + Bx + C)$ $x^3 - 4x^2 + 4x - 1 = (x-1)(Ax^2 + Bx + C)$ compare x^3 : $1 = A$ compare constant: $-1 = -C$ $1 = C$ sub $x = 2$, $-1 = 2^2 + 2B + 1$ $-3 = B$ $(x-1)(x^2 - 3x + 1) = 0$ $x - 1 = 0$ or $x^2 - 3x + 1 = 0$ $x = 1$ $\quad x = \frac{-(-3) \pm \sqrt{(-3)^2 - 4(1)(1)}}{2(1)}$ $= 2.6180$ or 0.381966 sub $x = 0.381966$, $y = 2x - 1$ $y = 2(0.381966) - 1$ $= -0.236068$ $\therefore C = (0.38197, -0.23607)$ (shown)

Qns	Working
b	sub $y=0$ to find $x-$coordinate of B $y=2x-1$ $0=2x-1$ $x=0.5$ Let A be the shaded area above the x-axis Let B be the shaded area below the x-axis $A=\left\|\int_0^1 x^{\frac{3}{2}}\mathrm{d}x\right\|-\frac{1}{2}(0.5)(1)$ $=\left[\frac{x^{\frac{5}{2}}}{\frac{5}{2}}\right]_0^1-0.25$ $=\frac{2}{5}-0.25$ $=0.15$ units^2 $B=\int_0^{0.38197}-x^{\frac{3}{2}}\mathrm{d}x+\frac{1}{2}(0.5-0.38197)(0.23607)$ $=\left\|\left[-\frac{x^{\frac{5}{2}}}{\frac{5}{2}}\right]_0^{0.38197}\right\|+0.013931$ $=0.03606089+0.0139391$ $=0.049999$ units^2 Total shaded area $=0.15+0.049999$ $=0.200$ units^2 (3 s.f.)