Name:		Index Number:		Class:	

CATHOLIC HIGH SCHOOL
Preliminary Examination
Secondary 4 (O-Level Programme)

Additional Mathematics — 4049/01

Paper 1 — **28 August 2025**

2 hours 15 minutes

Additional Materials: Answer Booklets A, B and C.

READ THESE INSTRUCTIONS FIRST

Write your name, index number and class on all the work you hand in.
Write in dark blue or black pen.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** questions in the space provided.
If working is needed for any question, it must be shown with the answer.
Omission of essential working will result in loss of marks.
The use of an approved scientific calculator is expected, where appropriate.
If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to **three significant figures**.
Give answers in **degrees to one decimal place**.

The number of marks is given in brackets [ ] at the end of each question or part question.
The total of the marks for this paper is **90**.

For examiner's use:

/ 90

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the quadratic equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial Expansion

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n \ ,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{(n-r)!r!} = \frac{n(n-1)...(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin (A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos (A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan (A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

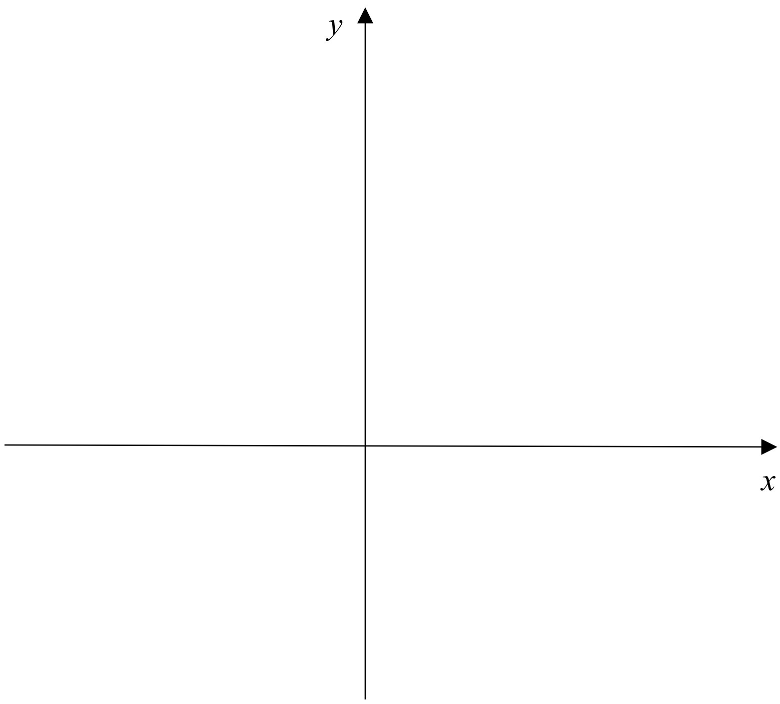
$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \tfrac{1}{2} ab \sin C$$

1 **(i)** Express $5-4x-2x^2$ in the form $c-a\left(x+b\right)^2$, where a, b and c are constants and $a>0$. [2]

(ii) Sketch the graph of $y=5-4x-2x^2$, indicating clearly he turning point and the y-intercept. [2]



(iii) Hence find the range of values of k for which the equation $5-4x-2x^2=k$ has at most one root.

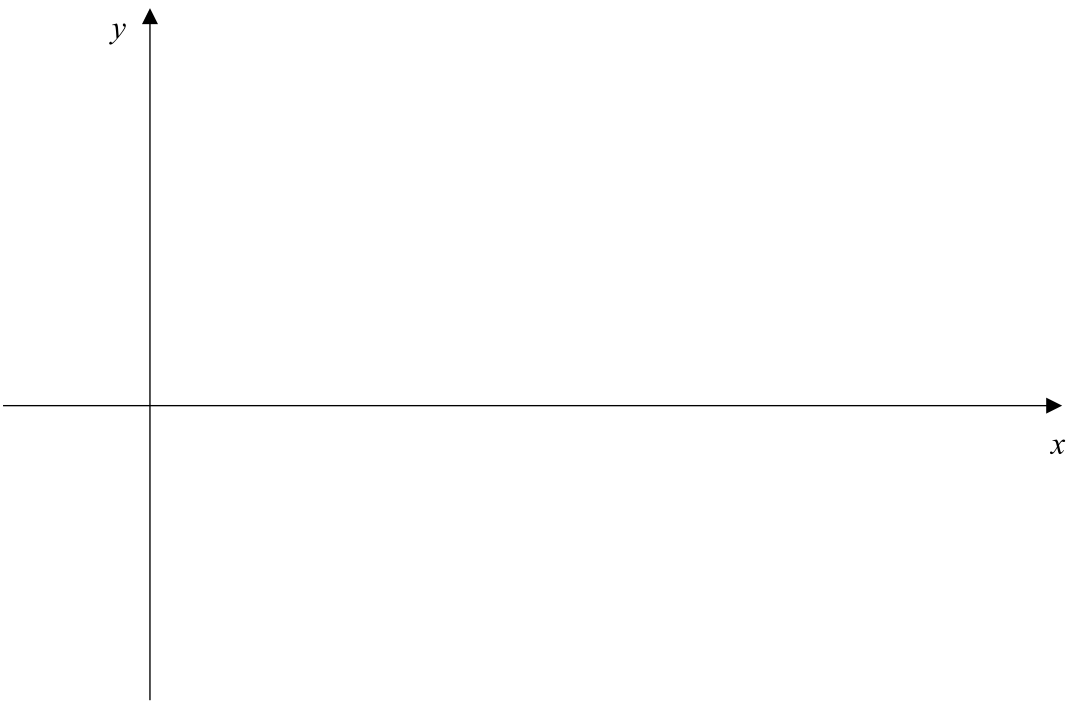
[1]

2 The function f is defined by $f(x) = 4\cos ax + b$ for $0 \le x \le 3\pi$, where a and b are constants. The maximum value of f is 6 and the period of f is 4π.

(i) State the amplitude of f. [1]

(ii) Write down the value of a and of b. [2]

(iii) Sketch the graph of $y = f(x)$ for $0 \le x \le 3\pi$. [3]



3 **(i)** Find the range of values of k for which the line $y = kx + 2$ intersects the curve $xy - y = 2x$ at two distinct points. [4]

(ii) Hence state the possible value(s) of k for which the line $y = kx + 2$ is a tangent to the curve $xy - y = 2x$. [1]

4 Solve the equation $8-\sqrt{11-2x}=-x$, giving your answer in the form $a+b\sqrt{c}$, where a, b and c are integers. [4]

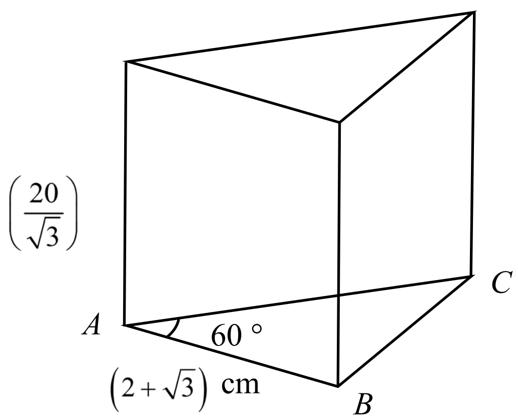
5 Water is being poured, at a constant rate of 42 m^3/s, into an empty container. After t seconds, the depth of water is h m and the volume, V m^3, of the water in the container is given by

$$V = \frac{(5+2h)^3}{3\pi}.$$

(i) Find the rate of change of the depth of water in terms of h and π. [3]

(ii) As the depth of water in the container rises, determine with a reason, whether the rate of change of h will increase or decrease. [1]

6



The diagram shows a solid prism with a triangular base, ABC, with angle $BAC = 60°$ and $AB = \left(2+\sqrt{3}\right)$ cm. The height of the prism is $\left(\frac{20}{\sqrt{3}}\right)$ cm and its volume is $\left(25+10\sqrt{3}\right)$ cm^3.

Without using a calculator, express the length of AC in the form $\left(a+b\sqrt{3}\right)$ where a and b are constants. [5]

7 **(i)** Express $\dfrac{8-3x}{(2x+3)\left(x^2+4\right)}$ in partial fractions. [4]

(ii) Differentiate $\ln\left(x^2+4\right)$ with respect to x. [1]

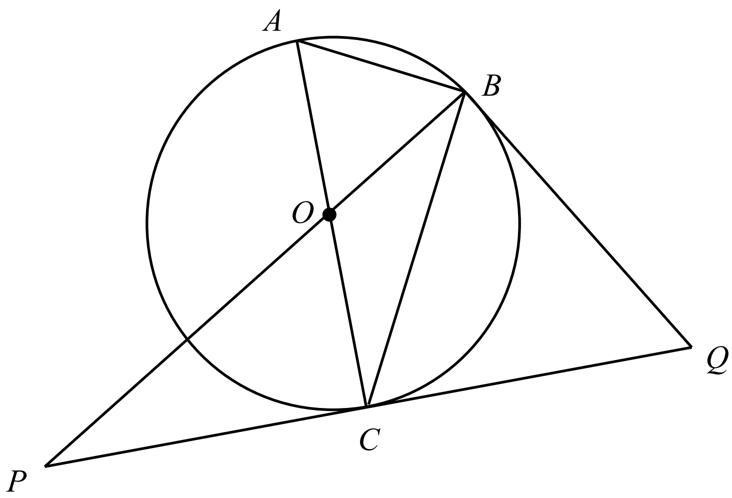
(iii) Use your answers to **parts (i)** and **(ii)** to find

$$\int_0^1 \frac{8-3x}{(2x+3)\left(x^2+4\right)}\,\mathrm{d}x\,.$$ [5]

8 Prove that $\dfrac{\tan 2\theta}{\sec 2\theta - 1} = \cot\theta$. [4]

9 The equation of a curve is $y = 3x\sin 2x + 1$. The point $A\ (\pi,\ 1)$ lies on the curve. The normal to the curve at A meets the x-axis at P and the y-axis at Q. Find length of PQ. [9]

10



In the diagram, A, B and C lie on a circle where O is the centre and AC is a diameter. The tangents to the circle at B and C meet at Q. The line BO extended meets the line QC at P.

(i) Show that angle $BOC = 2 \times$ angle BCQ. [2]

(ii) If C is the midpoint of PQ,

(a) prove that triangle POC is similar to triangle PQB, [2]

(b) show that $PO \times PB = \frac{1}{2} PQ^2$. [2]

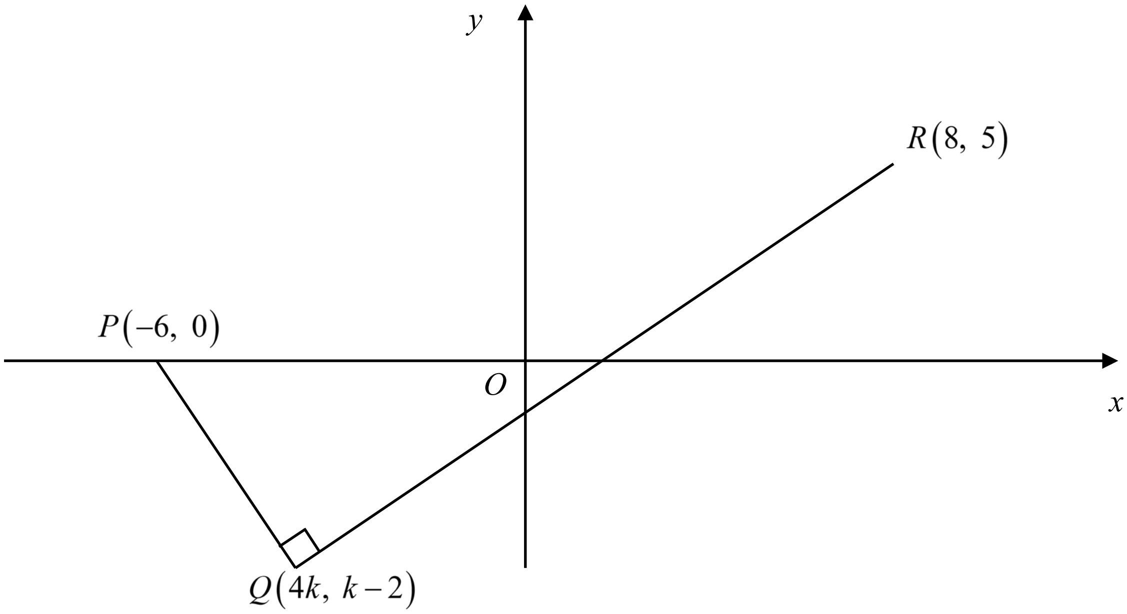
11 Solve the equation $2\cot^2 2x = 1 - \dfrac{5}{\sin 2x}$ for $-180° \le x \le 180°$. [6]

12 **(i)** Write down the general term in the binomial expansion of $\left(px-\frac{q}{x}\right)^n$, where p and q are positive integers. [1]

(ii) If the fifth term in the binomial expansion of $\left(px-\frac{q}{x}\right)^n$ is independent of x, show that $n = 8$. [2]

(iii) Hence find the value of p and of q given that the fifth term is 1120 and $p-q=1$. [5]

13 The diagram shows three points $P(-6,\ 0)$, $Q(4k,\ k-2)$ and $R(8,\ 5)$ where PQ is perpendicular to QR and $k<0$.

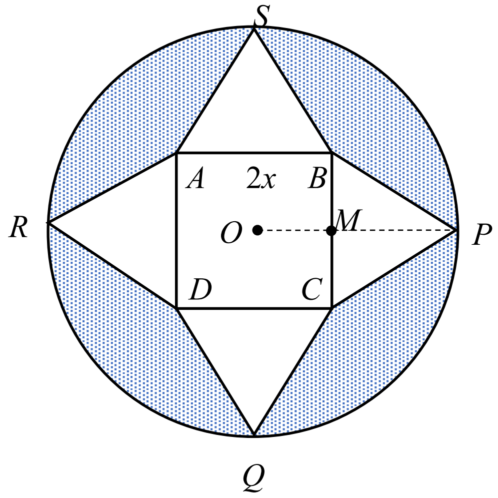


(i) Show that $k=-1$. [3]

(ii) The perpendicular bisector of PR cuts the y-axis at S. Find the coordinates of S. [4]

(iii) Find the area of the quadrilateral $PQRS$. [2]

14 [The volume of a pyramid of height h is $\frac{1}{3} \times (\text{Base Area}) \times h$.]



The diagram shows a piece of paper in the shape of a circle centre O and radius, $OP = 6$ cm. $ABCD$ is a square of side $2x$ cm with the same centre O. P, Q, R and S are points on the circumference of the circle such that $AS = BS = BP = CP = CQ = DQ = DR = AR$.

The shaded regions of the paper are to be cut off and the remaining paper can be folded to form a right pyramid with base $ABCD$.

(i) M is the midpoint of BC such that OMP is a straight line. Write down an expression for MP in terms of x and hence show that the height of the pyramid is $2\sqrt{9-3x}$ cm. [2]

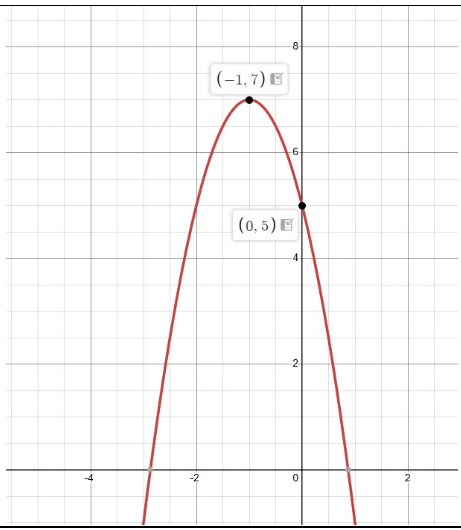
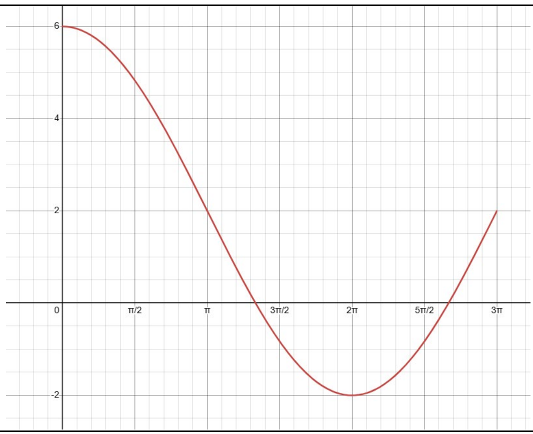
(ii) Show that the volume of the pyramid, V cm^3, is given by $V = \frac{8}{3}x^2\sqrt{9-3x}$. [1]

(iii) Given that x can vary, find the value of x that gives the stationary value of V. [4]

(iv) Explain whether this value of x gives a maximum or minimum V. [2]

--- End of Paper ---

Answer Key

1.	**(i)**	$-2\left(x+1\right)^2+7$
	(ii)	
	(iii)	$k \geq 7$
2.	**(i)**	4
	(ii)	$a=\frac{1}{2}, b=2$
	(iii)	
3.	**(i)**	$k<-8$ or $k>0$
	(ii)	$k=-8$
4.		$-9+2\sqrt{7}$
5.	**(i)**	$\frac{\mathrm{d}h}{\mathrm{d}t}=\frac{21\pi}{\left(5+2h\right)^2}$
	(ii)	decrease
6.		$4-\sqrt{3}$
7.	**(i)**	$\frac{2}{2x+3}-\frac{x}{x^2+4}$

	(ii)	$\frac{\mathrm{d}}{\mathrm{d}x}\left[\ln\left(x^2+4\right)\right]=\frac{2x}{x^2+4}$
	(iii)	$\frac{1}{2}\ln\frac{20}{9}$ or 0.399
9.		22.0 units
11.		$99.7°,\ 170.3°, -9.7°, -80.3°$
12.	**(i)**	$\binom{n}{r}\left(p^{n-r}\right)(-q)^r\ x^{n-2r}$
	(iii)	$p=2,\ q=1$
13.	**(ii)**	$S\left(0,\ \frac{53}{10}\right)$
	(iii)	48.1 units^2
14.	**(i)**	$MP=6-x$
	(iii)	2.4
	(iv)	Max V

Name:		Index Number:		Class:	

CATHOLIC HIGH SCHOOL
Preliminary Examination
Secondary 4 (O-Level Programme)

Additional Mathematics **SOLUTIONS** **4049/01**

Paper 1

28 August 2025

2 hours 15 minutes

Additional Materials: Answer Booklets A, B and C.

READ THESE INSTRUCTIONS FIRST

Write your name, index number and class on all the work you hand in.
Write in dark blue or black pen.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** questions in the space provided.
If working is needed for any question, it must be shown with the answer.
Omission of essential working will result in loss of marks.
The use of an approved scientific calculator is expected, where appropriate.
If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to **three significant figures**.
Give answers in **degrees to one decimal place**.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total of the marks for this paper is **90**.

For examiner's use:

/ 90

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the quadratic equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial Expansion

$$(a+b)^n = a^n + \binom{n}{1} a^{n-1} b + \binom{n}{2} a^{n-2} b^2 + \ldots + \binom{n}{r} a^{n-r} b^r + \ldots + b^n \ ,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{(n-r)!\,r!} = \frac{n(n-1)...(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

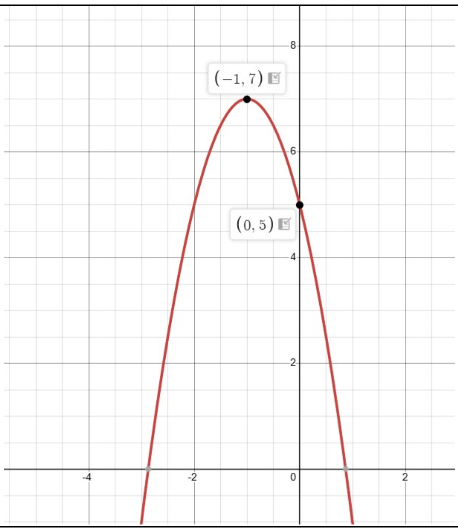
$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \tfrac{1}{2} ab \sin C$$

1 **(i)** Express $5-4x-2x^2$ in the form $c-a(x+b)^2$, where *a, b* and *c* are constants and $a>0$. [2]

$$
\begin{aligned}
5-4x-2x^2 &= -2\left[x^2+2x\right]+5 \\
&= -2(x+1)^2+7
\end{aligned}
$$

(ii) Sketch the graph of $y=5-4x-2x^2$, indicating clearly the turning point and the *y*-intercept. [2]



(iii) Hence find the range of values of *k* for which the equation $5-4x-2x^2=k$ has at most one root. [1]

$$k \geq 7$$

2 The function f is defined by $f(x) = 4\cos ax + b$ for $0 \le x \le 3\pi$, where a and b are constants. The maximum value of f is 6 and the period of f is 4π.

(i) State the amplitude of f. [1]

Amplitude = 4

(ii) Write down the value of a and of b. [2]

$f(x) = 4\cos ax + b$

Period $= 4\pi$

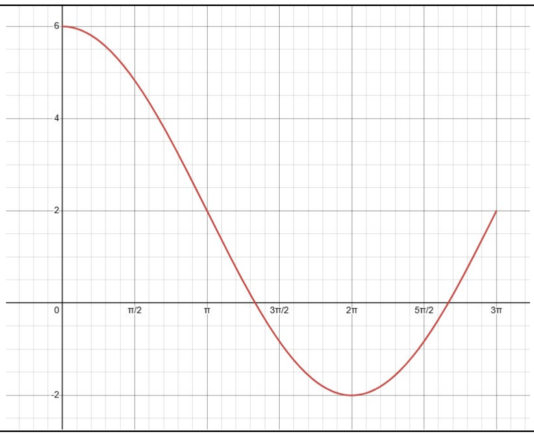
$$\frac{2\pi}{a} = 4\pi$$

$$a = \frac{1}{2}$$

$$4 + b = 6$$

$$b = 2$$

(iii) Sketch the graph of $y = f(x)$ for $0 \le x \le 3\pi$. [3]



3 **(i)** Find the range of values of k for which the line $y = kx + 2$ intersects the curve $xy - y = 2x$ at two distinct points. [4]

$$x(kx+2)-(kx+2)=2x$$
$$kx^2+2x-kx-2=2x$$
$$kx^2-kx-2=0$$

$$b^2-4ac=(-k)^2-4k(-2)$$
$$>0$$

$$k^2+8k>0$$
$$k(k+8)>0$$
$$k<-8 \quad \text{or} \quad k>0$$

(ii) Hence state the possible value(s) of k for which the line $y = kx + 2$ is a tangent to the curve $xy - y = 2x$. [1]

$$k=-8$$

4 Solve the equation $8-\sqrt{11-2x}=-x$, giving your answer in the form $a+b\sqrt{c}$, where a, b and c are integers. [4]

$$8-\sqrt{11-2x}=-x$$
$$8+x=\sqrt{11-2x}$$
$$64+16x+x^2=11-2x$$
$$x^2+18x+53=0$$
$$x=\frac{-18\pm\sqrt{18^2-4(1)(53)}}{2(1)}$$
$$=\frac{-18\pm\sqrt{112}}{2}$$
$$=-9\pm\frac{4\sqrt{7}}{2}$$
$$=-9+2\sqrt{7} \quad \text{or} \quad -9-2\sqrt{7} \text{ (rej.)}$$

5 Water is being poured, at a constant rate of 42 m^3/s, into an empty container. After t seconds, the depth of water is h m and the volume, V m^3, of the water in the container is given by

$$V = \frac{(5+2h)^3}{3\pi}.$$

(i) Find the rate of change of the depth of water in terms of h and π. [3]

$$\frac{dV}{dh} = \frac{2(5+2h)^2}{\pi}$$

$$\frac{dh}{dt} = \frac{dh}{dV} \times \frac{dV}{dt}$$

$$= \frac{\pi}{2(5+2h)^2} \times 42$$

$$= \frac{21\pi}{(5+2h)^2}$$

(ii) As the depth of water in the container rises, determine with a reason, whether the rate of change of h will increase or decrease. [1]

Mtd 1:

As h increases, $(5+2h)^2$ increases.

Since 21π is a constant, $\frac{dh}{dt} = \frac{21\pi}{(5+2h)^2}$ will decrease.

Mtd 2:

$$\frac{d^2h}{dt^2} = -\frac{84\pi}{(5+2h)^3}$$

$$h > 0$$

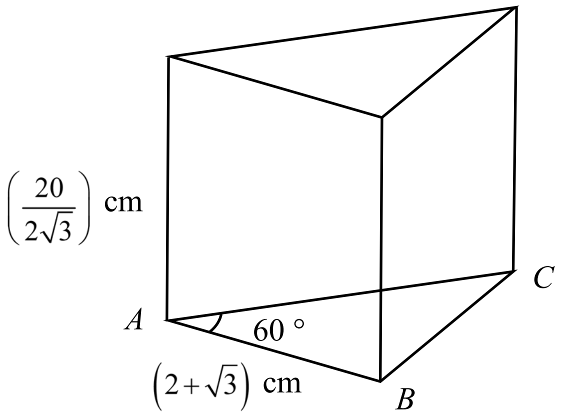
$$\Rightarrow (5+2h)^3 > 0$$

$$\Rightarrow -(5+2h)^3 < 0$$

$$\Rightarrow \frac{d^2h}{dt^2} < 0$$

$$\Rightarrow \frac{dh}{dt} \text{ will decrease}$$

6



The diagram shows a solid prism with a triangular base, ABC, with angle $BAC = 60°$ and $AB = \left(2+\sqrt{3}\right)$ cm. The height of the prism is $\left(\frac{20}{\sqrt{3}}\right)$ cm and its volume is $\left(25+10\sqrt{3}\right)$ cm^3.

Without using a calculator, express the length of AC in the form $\left(a+b\sqrt{3}\right)$ where a and b are constants. [5]

$$\frac{1}{2}\left(2+\sqrt{3}\right)(AC)\sin(60°)\times\left(\frac{20}{\sqrt{3}}\right) = 25+10\sqrt{3}$$

$$\frac{1}{2}\left(2+\sqrt{3}\right)(AC)\left(\frac{\sqrt{3}}{2}\right)\times\left(\frac{20}{\sqrt{3}}\right) = 25+10\sqrt{3}$$

$$(AC)\left(10+5\sqrt{3}\right) = 25+10\sqrt{3}$$

$$AC = \frac{25+10\sqrt{3}}{10+5\sqrt{3}}$$

$$= \frac{25+10\sqrt{3}}{10+5\sqrt{3}}\times\frac{10-5\sqrt{3}}{10-5\sqrt{3}}$$

$$= 4-\sqrt{3}$$

7 **(i)** Express $\dfrac{8-3x}{(2x+3)(x^2+4)}$ in partial fractions. [4]

Let $\dfrac{8-3x}{(2x+3)(x^2+4)} = \dfrac{A}{2x+3} + \dfrac{Bx+C}{x^2+4}$

$$8-3x = A(x^2+4) + (Bx+C)(2x+3)$$

Let $x = -\dfrac{3}{2}$: $\dfrac{25}{2} = A\left(\dfrac{25}{4}\right) \Rightarrow A = 2$

Let $x = 0$: $8 = 2(4) + C(3) \Rightarrow C = 0$

Let $x = 1$: $5 = 2(5) + B(5) \Rightarrow B = -1$

$$\therefore \quad \frac{2}{2x+3} - \frac{x}{x^2+4}$$

(ii) Differentiate $\ln(x^2+4)$ with respect to x. [1]

$$\frac{\mathrm{d}}{\mathrm{d}x}\left[\ln(x^2+4)\right] = \frac{2x}{x^2+4}$$

(iii) Use your answers to **parts (i)** and **(ii)** to find

$$\int_0^1 \frac{8-3x}{(2x+3)(x^2+4)}\,\mathrm{d}x.$$ [5]

$$\begin{aligned}
\int_0^1 \frac{8-3x}{(2x+3)(x^2+4)}\,\mathrm{d}x &= \int_0^1 \frac{2}{2x+3} - \frac{x}{x^2+4}\,\mathrm{d}x \\
&= \int_0^1 \frac{2}{2x+3}\,\mathrm{d}x - \int_0^1 \frac{x}{x^2+4}\,\mathrm{d}x \\
&= \int_0^1 \frac{2}{2x+3}\,\mathrm{d}x - \frac{1}{2}\int_0^1 \left(\frac{2x}{x^2+4}\right)\mathrm{d}x \\
&= \left[\ln(2x+3)\right]_0^1 - \frac{1}{2}\left[\ln(x^2+4)\right]_0^1 \\
&= [\ln 5 - \ln 3] - \frac{1}{2}[\ln 5 - \ln 4] \\
&= \frac{1}{2}\ln\frac{20}{9}
\end{aligned}$$

or 0.399 (3 s.f.)

8 Prove that $\dfrac{\tan 2\theta}{\sec 2\theta - 1} = \cot\theta$. [4]

$$
\begin{aligned}
\frac{\tan 2\theta}{\sec 2\theta - 1} &= \frac{\sin 2\theta}{\cos 2\theta} \times \frac{1}{\dfrac{1}{\cos 2\theta} - 1} \\
&= \frac{\sin 2\theta}{\cos 2\theta} \times \frac{\cos 2\theta}{1 - \cos 2\theta} \\
&= \frac{\sin 2\theta}{1 - \cos 2\theta} \\
&= \frac{2\sin\theta\cos\theta}{1 - \left(1 - 2\sin^2\theta\right)} \\
&= \frac{2\sin\theta\cos\theta}{2\sin^2\theta} \\
&= \frac{\cos\theta}{\sin\theta} \\
&= \cot\theta
\end{aligned}
$$

9 The equation of a curve is $y = 3x\sin 2x + 1$. The point $A\ (\pi,\ 1)$ lies on the curve. The normal to the curve at A meets the x-axis at P and the y-axis at Q. Find length of PQ. [9]

$$y = 3x\sin 2x + 1$$

$$\frac{\mathrm{d}y}{\mathrm{d}x} = 3\sin 2x + 3x \bullet 2\cos 2x$$

$$= 3\sin 2x + 6x\cos 2x$$

$$\left.\frac{\mathrm{d}y}{\mathrm{d}x}\right|_{x=\pi} = 3\sin 2\pi + 6\pi\cos 2\pi$$

$$= 6\pi$$

Grad. of normal at $A = -\dfrac{1}{6\pi}$

Eqn. of normal at A:

$$y - 1 = -\frac{1}{6\pi}(x - \pi)$$

$$y = -\frac{1}{6\pi}x + \frac{7}{6}$$

$$Q\left(0,\ \frac{7}{6}\right)$$

At P : $y = 0$

$$-\frac{1}{6\pi}x + \frac{7}{6} = 0$$

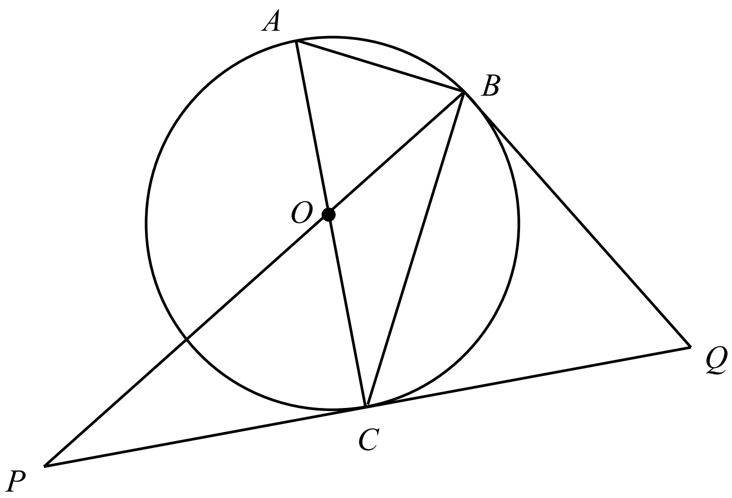
$$x = 7\pi$$

$$P(7\pi,\ 0)$$

$$PQ = \sqrt{(7\pi - 0)^2 + \left(0 - \frac{7}{6}\right)^2}$$

$= 22.0$ units (3 s.f.)

10



In the diagram, A, B and C lie on a circle where O is the centre and AC is a diameter. The tangents to the circle at B and C meet at Q. The line BO extended meets the line QC at P.

(i) Show that angle $BOC = 2 \times$ angle BCQ. [2]

Mtd 1:

$\measuredangle BAC = \measuredangle BCQ$ (Alt. Seg. Thm.)

$\measuredangle BOC = 2(\measuredangle BAC)$ ($\measuredangle$ at centre = 2$\measuredangle$s at circumference)

$= 2\measuredangle BCQ$

Mtd 2:

Let $\measuredangle BCQ = x$.

$\measuredangle OCB = 90° - x$ (rad. $\perp$ tan)

$\measuredangle BOC = 180° - 2(90° - x)$ (isosceles ΔOBC)

$= 180° - 180° + 2x$

$= 2x$

$= 2\measuredangle BCQ$

(ii) If C is the midpoint of PQ,

(a) prove that triangle POC is similar to triangle PQB, [2]

$\measuredangle OPC = \measuredangle QPB$ (common pt P)

$\measuredangle PCO = \measuredangle PBQ = 90°$ (rad. $\perp$ tan)

Since 2 pairs of corresponding angles are equal, triangle POC is similar to triangle PQB.

(b) show that $PO \times PB = \frac{1}{2}PQ^2$. [2]

Since triangle *POC* is similar to triangle *PQB*,

$$\frac{PO}{PQ} = \frac{PC}{PB}$$

Since *C* is the midpoint of *PQ*, $\frac{PC}{PQ} = \frac{1}{2} \Rightarrow PC = \frac{1}{2}PQ$

$$\frac{PO}{PQ} = \frac{\frac{1}{2}PQ}{PB}$$

$$PO \times PB = \frac{1}{2}PQ \times PQ$$

$$= \frac{1}{2}PQ^2$$

11 Solve the equation $2\cot^2 2x = 1 - \dfrac{5}{\sin 2x}$ for $-180° \le x \le 180°$. [6]

$$2\cot^2 2x = 1 - \frac{5}{\sin 2x}$$

$$2\cot^2 2x = 1 - 5\ \text{cosec}\ 2x$$

$$2\left(\text{cosec}^2\ 2x - 1\right) = 1 - 5\ \text{cosec}\ 2x$$

$$2\ \text{cosec}^2\ 2x + 5\ \text{cosec}\ 2x - 3 = 0$$

Let $y = \text{cosec}\ 2x$.

$$2y^2 + 5y - 3 = 0$$

$$(2y-1)(y+3) = 0$$

$$y = -3 \quad \text{or} \quad y = \frac{1}{2}$$

$$\text{cosec}\ 2x = -3 \quad \text{or} \quad \text{cosec}\ 2x = \frac{1}{2}$$

$$\sin 2x = -\frac{1}{3} \quad \text{or} \quad \sin 2x = 2\ \text{(rej.)}$$

$$\alpha = \sin^{-1}\frac{1}{3}$$
$$= 19.471°$$

$$2x = 199.471°,\quad 340.529°$$
$$x = 99.7°,\quad 170.3° \quad \text{(1 d.p)}$$

$$2x = -19.471°,\quad -160.529°$$
$$x = -9.7°,\quad -80.3° \quad \text{(1 d.p)}$$

12 **(i)** Write down the general term in the binomial expansion of $\left(px-\frac{q}{x}\right)^n$, where p and q are positive integers. [1]

$$\left(px-\frac{q}{x}\right)^n$$

$$T_{r+1}=\binom{n}{r}(px)^{n-r}\left(-\frac{q}{x}\right)^r$$

$$=\binom{n}{r}\left(p^{n-r}\right)(-q)^r\,x^{n-2r}$$

(ii) If the fifth term in the binomial expansion of $\left(px-\frac{q}{x}\right)^n$ is independent of x, show that $n = 8$. [2]

5th term $\Rightarrow r=4$

$$T_5=\binom{n}{4}\left(p^{n-4}\right)(-q)^4\,x^{n-8}$$

$$n-8=0$$

$$n=8$$

(iii) Hence find the value of p and of q given that the fifth term is 1120 and $p-q=1$. [5]

$$\binom{8}{4}\left(p^4\right)(-q)^4=1120$$

$$70p^4q^4=1120$$

$$(pq)^4=16$$

$$pq=2$$

$$q=\frac{2}{p}$$

Sub $q=\frac{2}{p}$ into $p-q=1$:

$$p-\frac{2}{p}=1$$

$$p^2-2=p$$

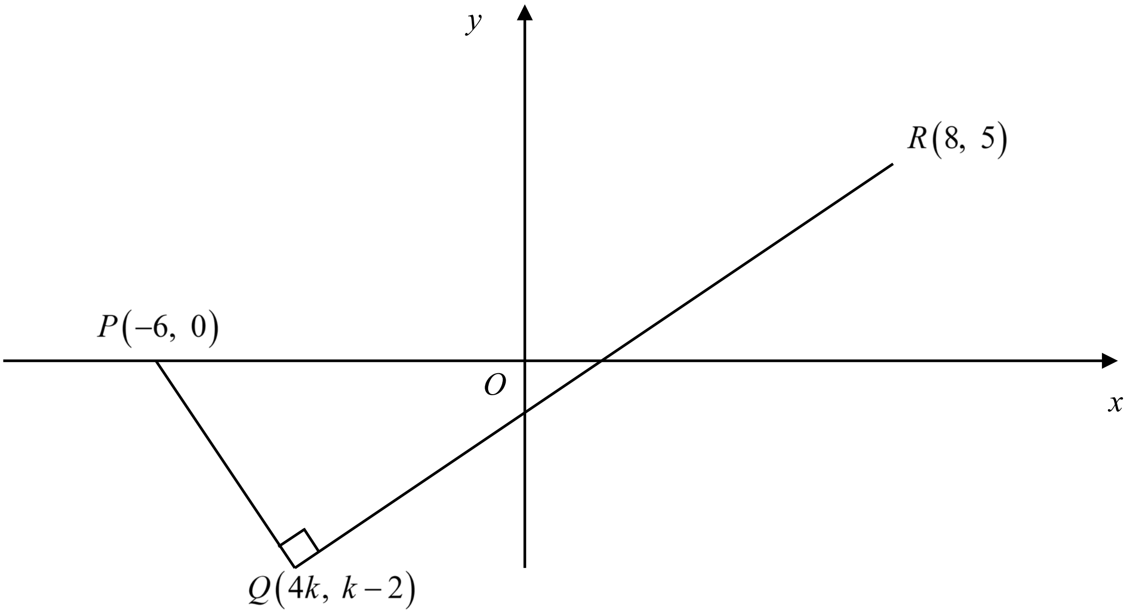
$$p^2-p-2=0$$

$$(p+1)(p-2)=0$$

$p=-1$ (rej.) or $p=2$

$\therefore$ $q=1$

13 The diagram shows three points $P(-6,\ 0)$, $Q(4k,\ k-2)$ and $R(8,\ 5)$ where PQ is perpendicular to QR and $k<0$.



(i) Show that $k=-1$. [3]

$$Grad_{PQ} = -\frac{1}{Grad_{QR}}$$

$$\frac{(k-2)-0}{4k-(-6)} = -\frac{4k-8}{(k-2)-5}$$

$$\frac{k-2}{4k+6} = \frac{-4k+8}{k-7}$$

$$(k-2)(k-7) = (-4k+8)(4k+6)$$

$$k^2-9k+14 = -16k^2+8k+48$$

$$17k^2-17k-34=0$$

$$k^2-k-2=0$$

$$(k-2)(k+1)=0$$

$$k=2 \quad \text{(N.A.) or} \quad k=-1$$

(ii) The perpendicular bisector of PR cuts the y-axis at S. Find the coordinates of S. [4]

Midpoint of $PR = \left(\frac{-6+8}{2},\ \frac{0+5}{2}\right) = \left(1,\ \frac{5}{2}\right)$

$$Grad_{PR} = \frac{0-5}{-6-8} = \frac{5}{14}$$

Eqn. of $\perp$ bisector of PR:

$$y - \frac{5}{2} = -\frac{14}{5}(x-1)$$

$$y = -\frac{14}{5}x + \frac{53}{10}$$

$$S\left(0,\ \frac{53}{10}\right)$$

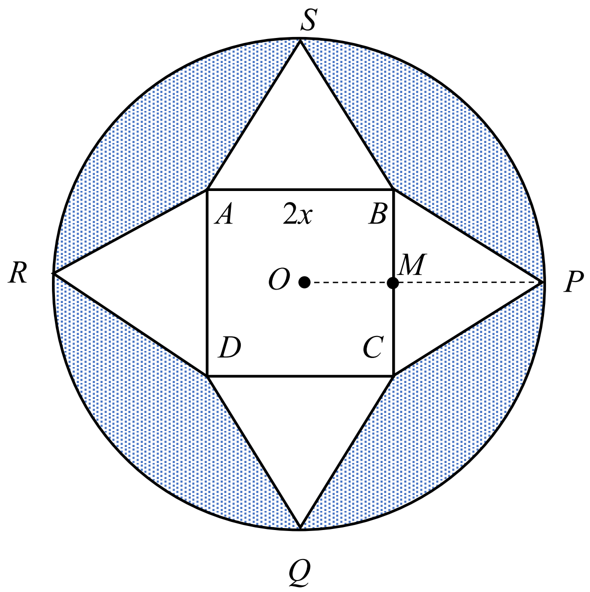
(iii) Find the area of the quadrilateral $PQRS$. [2]

$$\text{Area of } PQRS = \frac{1}{2}\begin{vmatrix} -6 & -4 & 8 & 0 & -6 \\ 0 & -3 & 5 & 5.3 & 0 \end{vmatrix}$$

$$= \frac{1}{2}\left|(18-20+42.4)-(-24-31.8)\right|$$

$$= 48.1 \text{ units}^2$$

14 [The volume of a pyramid of height h is $\frac{1}{3}\times(\text{Base Area})\times h$.]



The diagram shows a piece of paper in the shape of a circle centre O and radius 6 cm. $ABCD$ is a square of side $2x$ cm with the same centre O. P, Q, R and S are points on the circumference of the circle such that $AS = BS = BP = CP = CQ = DQ = DR = AR$.

The shaded regions of the paper are to be cut off and the remaining paper can be folded to form a right pyramid with base $ABCD$.

(i) M is the midpoint of BC such that OMP is a straight line. Write down an expression for MP in terms of x and hence show that the height of the pyramid is $2\sqrt{9-3x}$ cm. [2]

$$OM = x$$
$$MP = 6 - x$$
$$\text{Height of pyramid } = \sqrt{(6-x)^2 + x^2}$$
$$= \sqrt{36-12x}$$
$$= 2\sqrt{9-3x}$$

(ii) Show that the volume of the pyramid, V cm^3, is given by $V = \frac{8}{3}x^2\sqrt{9-3x}$. [1]

$$V = \frac{1}{3}(2x)^2\left(2\sqrt{9-3x}\right)$$
$$= \frac{8}{3}x^2\sqrt{9-3x}$$

(iii) Given that x can vary, find the value of x that gives the stationary value of V. [4]

$$\frac{dV}{dx} = \frac{16}{3}x \bullet (9-3x)^{\frac{1}{2}} + \frac{8}{3}x^2 \bullet \frac{1}{2}(9-3x)^{-\frac{1}{2}}(-3)$$

$$= \frac{16}{3}x(9-3x)^{\frac{1}{2}} - 4x^2(9-3x)^{-\frac{1}{2}}$$

$$= \frac{4x(12-5x)}{\sqrt{9-3x}}$$

$$\frac{dV}{dx} = 0$$

$$12 - 5x = 0$$

$$x = 2.4$$

(iv) Explain whether this value of x gives a maximum or minimum V. [2]

Mtd 1:

x	2.39	2.4	2.41
$\frac{dV}{dx}$	0.3533	0	– 0.3622
Tangent	/	–	\

V is a maximum.

Mtd 2:

$$\frac{d^2V}{dx^2} = \frac{2\sqrt{3}\left(5x^2 - 24x + 24\right)}{(3-x)^{\frac{3}{2}}}$$

$$\left.\frac{d^2V}{dx^2}\right|_{x=2.4} = -35.8 < 0$$

V is a maximum.

--- End of Paper ---

Name:		Index Number:		Class:	

CATHOLIC HIGH SCHOOL
2025 Preliminary Examination
Secondary 4 (O-Level Programme)

A

Additional Mathematics

4049/02

Paper 2

1 September 2025

2 hours 15 minutes

Booklet A

Additional materials: Question Booklets B and C

READ THESE INSTRUCTIONS FIRST

Write your name, index number and class on all the work you hand in.
Write in dark blue or black pen.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** questions in the space provided.
If working is needed for any question, it must be shown with the answer.
Omission of essential working will result in loss of marks.
Calculators should be used where appropriate.

If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer answer to **three significant figures**.
Give answers in **degrees to one decimal place**.
For π, use either your calculator value or 3.142, unless the question requires the answer in terms of π.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total mark for this paper is **90**.

For examiners' use

30

This booklet consists of **8** printed pages.

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the quadratic equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial Expansion

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n \ ,$$

where n is a positive integer and $\binom{n}{r} = \dfrac{n!}{(n-r)!\,r!} = \dfrac{n(n-1)...(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \tfrac{1}{2} ab \sin C$$

1 Jane bought an electric car in January 2025. After purchase, the value of the car, $\$V$, diminishes over time and can be modelled by $V = 38000e^{-kt}$, where k is a positive constant and t is time measured in years. The value of the car is expected to be \$29000 after 3 years of driving. Jane intends to sell her car when its value drops to half of its original value.

Showing clear mathematical calculations, find the year that Jane is likely to sell her electric car. [5]

2 **(i)** Given that the curve $y = ax^2 + 2bx - 1$ lies entirely below the x-axis, determine the conditions that must be applied to the constants a and b. [2]

(ii) If a and b are both integers, state an example of the values of a and b which satisfy the conditions found in **(i)**. [2]

---

3 **(a)** The variables x and y are defined such that $\log_2 x - 3\log_4 y = \log_9 3$.

(i) Give a reason why x and y must be positive numbers. [1]

(ii) Express x in terms of y. [5]

(b) Solve the equation $5^{x+1}+2\left(5^{-x}\right)=11$, leaving non-exact value(s) of x in the form $\log_a b$. [4]

4 **(a)** **(i)** The polynomials $\mathrm{P}(x) = ax^3 + x^2 + bx + 10$ and $\mathrm{Q}(x) = x^3 + ax^2 - 5x + b$ leave the same remainder when divided by $x + 2$.

Show that $4a + b = 4$. [2]

(ii) If $\mathrm{P}'(x)$ has a factor of $3x - 2$, find the values of *a* and *b*. [4]

(b) Solve the equation $2x^3 + 4x^2 - 5x - 7 = 0$, expressing non-exact solutions in the form $a \pm \frac{1}{2}\sqrt{b}$ where a and b are constants to be determined. [5]

END OF BOOKLET A

Name:		Index Number:		Class:	

CATHOLIC HIGH SCHOOL
2025 Preliminary Examination
Secondary 4 (O-Level Programme)

B

Additional Mathematics

4049/02

Paper 2

1 September 2025

Booklet B

READ THESE INSTRUCTIONS FIRST

Write your name, index number and class on all the work you hand in.
Write in dark blue or black pen.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

For examiners' use

27

This booklet consists of **6** printed pages and **2** blank pages.

BLANK PAGE

5 The depth of water, h metres, in a shallow port on a particular day is modelled by the formula $h = 7 + 2\sin\left(\frac{\pi}{6}t\right)$, where t is the number of hours after midnight.

(a) State the period of h. [1]

(b) Use the model to predict the time when the depth of water is at its lowest. [2]

A supply boat docks at the port at 5.30 am and it takes the workers 3 hours to unload its cargo immediately after it docks. To leave the port safely, the supply boat requires the depth of water to be at least 5.6 metres.

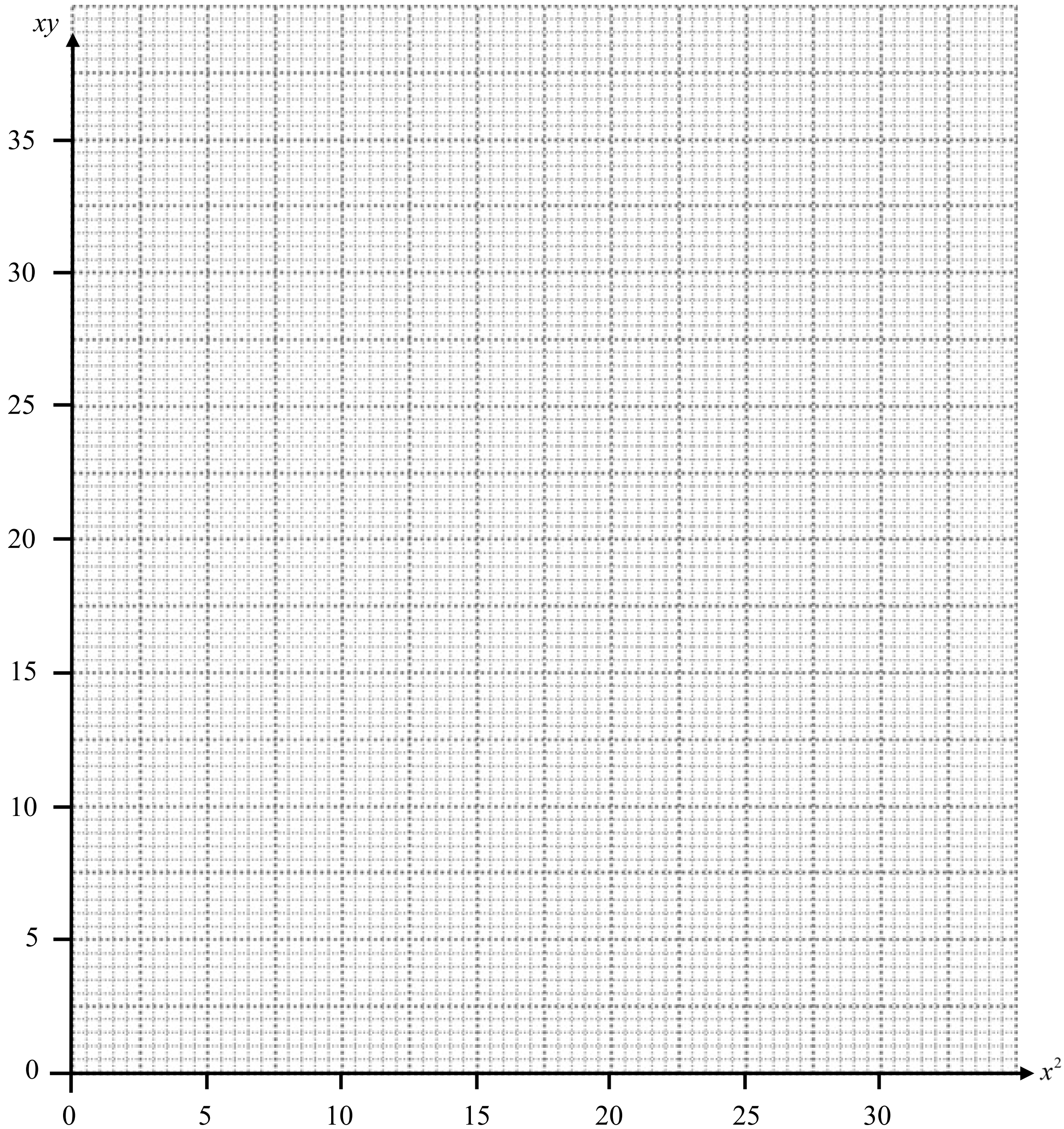
(c) Determine the earliest time, to the nearest minute, that the supply boat can leave the port. [4]

6 The variables x and y are known to be related by the formula $y = px + \frac{q}{x}$ where p and q are constants.

(a) The table below shows some experimental values of x and y.

x	1	2	3	4	5
y	8.55	6.20	5.32	5.72	6.00

(i) On the grid below, plot xy against x^2 and draw a straight line graph. [2]



(ii) Use your graph to estimate

(a) the value of p and of q. [3]

(b) the positive value of x that satisfies the equation $px + \frac{q}{x} = \frac{20}{x}$. [2]

(b) Explain how another straight line graph can be obtained from $y = px + \frac{q}{x}$ by plotting $\frac{x}{y}$ against $\frac{1}{xy}$, expressing clearly the gradient and vertical intercept in terms of p and/or q. You need not draw this straight line graph. [3]

7 The equation of a curve is $y=\dfrac{\ln 2x}{x}$.

(i) Find an expression for $\dfrac{\mathrm{d}^2y}{\mathrm{d}x^2}$ and show that $x\dfrac{\mathrm{d}^2y}{\mathrm{d}x^2}+2\dfrac{\mathrm{d}y}{\mathrm{d}x}+\dfrac{1}{x^2}=0$. [5]

(ii) Find the **exact** coordinates of the stationary point of the curve and use the second derivative test to determine if the stationary point is a maximum or minimum. [5]

END OF BOOKLET B

BLANK PAGE

Name:		Index Number:		Class:	

CATHOLIC HIGH SCHOOL
2025 Preliminary Examination
Secondary 4 (O-Level Programme)

C

Additional Mathematics — 4049/02

Paper 2

1 September 2025

Booklet C

READ THESE INSTRUCTIONS FIRST

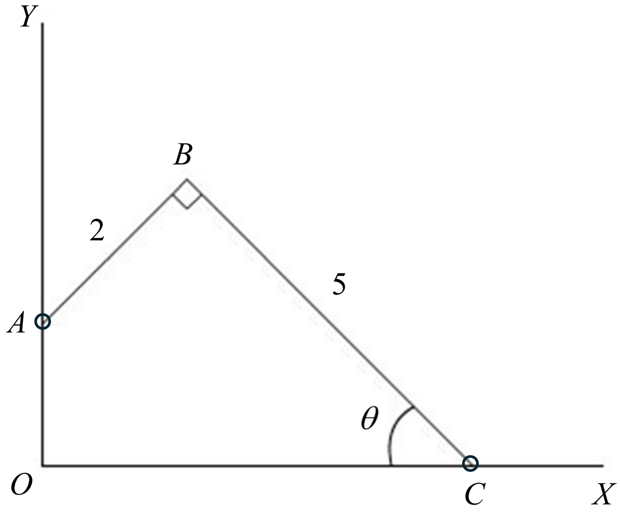
Write your name, index number and class on all the work you hand in.
Write in dark blue or black pen.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

For examiners' use

	33

This booklet consists of **8** printed pages.

8 The diagram shows two rods AB and BC of length 2 m and 5 m respectively, joined together at B such that angle $ABC = 90°$. Small rings are attached to A and C so that A can move along the vertical pole OY and C can move along the horizontal pole OX. The rod BC makes an angle θ with OX.



(i) Show that $OA = 5\sin\theta - 2\cos\theta$. [2]

(ii) Express OA in the form $R\sin(\theta - \alpha)$ where $R > 0$ and $0° < \alpha < 90°$. [3]

(iii) Given that A can be no higher than 3.5 m above O, calculate the greatest possible value of θ. [2]

---

9 A particle moves in a straight line with a fixed point O. The velocity of the particle, v m/s, is given by $v = \frac{25}{2t+1} + 10$ where t is time in seconds after passing O.

(a) Explain why the particle never change its direction of motion and will eventually travel at a speed close to 10 m/s. [2]

(b) Find the acceleration of the particle at $t = 2$. [2]

(c) Calculate the distance travelled by the particle in the fifth second of its motion. [4]

---

10 $A(1\ ,\ 7)$ and $B(9\ ,\ 7)$ lie on a circle C_1 with centre *P*. The line $4y+3x=31$ is a normal to the circle and passes through *A*.

(i) Use a geometrical property of circle to explain why the *x*-coordinate of *P* is 5. [2]

(ii) Find the equation of the circle C_1. [3]

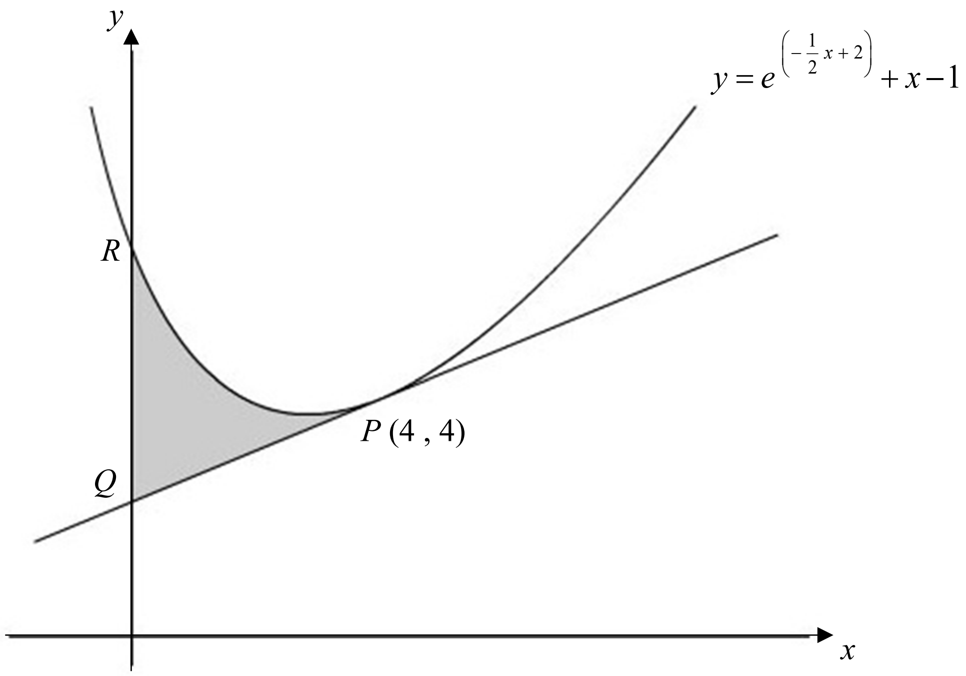
(iii) The tangent to the circle at A intersects the y-axis at D.

Find the coordinates of D. [3]

A larger circle C_2 has the same centre P and radius three times that of C_1. T is a point on C_2 such that P, A and T are collinear and the x-coordinate of T is negative.

(iv) Determine the coordinates of T. [2]

11 The diagram shows part of the curve $y = e^{\left(-\frac{1}{2}x+2\right)} + x - 1$ that intersects the y-axis at R. The tangent to the curve at $P(4, 4)$ intersects the y-axis at Q.



Show that the area of the shaded region can be expressed as $ae^2 + b$ where a and b are constants to be determined. [8]

(More working space on the next page.)

(Continuation of working space for question 11.)

END OF PAPER

Answer key

1. $t \approx 7.69$ years ; 2032

2(i) $b^2 + a < 0$ $\left(\text{or } a < -b^2\right)$ (ii) $a = -5,\ b = 1$

3(a)(i) x and y must be positive for $\log_2 x$ and $\log_2 y$ to be defined / exist / calculable / computed.

3(a)(ii) $x = \sqrt{2y^3}$

3(b) $x = -1$ or $\log_5 2$

4(a)(ii) $a = 2,\ b = -4$

4(b) $x = -1,\ -\frac{1}{2} \pm \frac{1}{2}\sqrt{15}$

5(a) $h = 12$ hours (b) 9 am (or 0900 hrs)

5(c) 10.31am or 10.32am (24-hr format acceptable)

6(a)(ii)(a) $p \approx 0.895$ $\left(\text{acceptable } 0.8 \le p \le 1.1\right)$, $q = 8$ $\left(\text{acceptable } 7.5 \le q \le 9\right)$

6(a)(ii)(b) $x = 3.64$ $\left(\text{acceptable } 3.60 \le x \le 3.75\right)$

6(b) $\frac{x}{y} = \left(-\frac{q}{p}\right)\frac{1}{xy} + \frac{1}{p}$; gradient $= -\frac{q}{p}$, vertical intercept $= \frac{1}{p}$

7(i) $\frac{\mathrm{d}^2 y}{\mathrm{d}x^2} = \frac{2\ln 2x - 3}{x^3}$ (ii) $\left(\frac{e}{2}\ ,\ \frac{2}{e}\right)$; maximum

8(ii) $OA = \sqrt{29}\sin\left(\theta - 21.8°\right)$ (iii) $62.3°$

9(b) $-2\,\text{m/s}^2$ (c) 12.5m

10(i) Perpendicular bisector of chord passes through the centre of circle.

Hence, since AB is horizontal chord, P is vertically above/below the midpoint of A.

$\therefore x_p = \frac{1+9}{2} = 5$

10(ii) $(x-5)^2 + (y-4)^2 = 25$ (iii) $D\left(0\ ,\ \frac{17}{3}\right)$

10(iv) $T\left(-7\ ,\ 13\right)$

11. $2e^2 - 10$

1 Jane bought an electric car in January 2025. After purchase, the value of the car, $\$V$, diminishes over time and can be modelled by $V = 38000e^{-kt}$, where k is a positive constant and t is time measured in years. The value of the car is expected to be \$29000 after 3 years of driving. Jane intends to sell her car when its value drops to half of its original value.

Showing clear mathematical calculations, find the year that Jane is likely to sell her electric car. [5]

$$29000 = 38000e^{-3k}$$
$$e^{-3k} = \frac{29}{38}$$
$$-3k = \ln\frac{29}{38}$$
$$k = -\frac{1}{3}\ln\frac{29}{38} \approx 0.090096$$

$$19000 = 38000e^{-0.090096t}$$
$$e^{-0.090096t} = \frac{1}{2}$$
$$-0.090096t = \ln\frac{1}{2}$$
$$t \approx 7.69 \text{ years}$$

The year is likely to be 2032.

---

2 **(i)** Given that the curve $y = ax^2 + 2bx - 1$ lies entirely below the x-axis, determine the conditions that must be applied to the constants a and b. [2]

$a < 0$ for maximum curve (not required)

$(2b)^2 - 4a(-1) < 0$ for no roots

$4b^2 + 4a < 0$

$b^2 + a < 0$ $\left(\text{or } a < -b^2\right)$

(ii) If a and b are both integers, state an example of the values of a and b which satisfy the conditions found in **(i)**. [2]

$a = -5$

$b = 1$

---

3 **(a)** The variables x and y are defined such that $\log_2 x - 3\log_4 y = \log_9 3$.

(i) Give a reason why x and y must be positive numbers. [1]

x and y must be positive for
$\log_2 x$ and $\log_2 y$ to be defined / exist / calculable / computed.

(ii) Express x in terms of y. [5]

$$\log_2 x - 3\left(\frac{\log_2 y}{\log_2 4}\right) = \frac{1}{2}$$

$$\log_2 x - \frac{3\log_2 y}{2} = \frac{1}{2}$$
$$2\log_2 x - 3\log_2 y = 1$$
$$\log_2 x^2 - \log_2 y^3 = 1$$
$$\log_2 \frac{x^2}{y^3} = 1$$
$$\frac{x^2}{y^3} = 2$$
$$x^2 = 2y^3$$
$$x = \sqrt{2y^3}$$

(b) Solve the equation $5^{x+1} + 2\left(5^{-x}\right) = 11$, leaving non-exact value(s) of x in the form $\log_a b$. [4]

$$5\left(5^x\right) + 2\left(\frac{1}{5^x}\right) = 11$$

Let $w = 5^x$

$$5w + \frac{2}{w} = 11$$
$$5w^2 - 11w + 2 = 0$$
$$(5w-1)(w-2) = 0$$
$$w = \frac{1}{5} \quad \text{or} \quad 2$$
$$5^x = 5^{-1} \quad \text{or} \quad 2$$
$$x = -1 \quad \text{or} \quad \log_5 2$$

A1

4 **(a)** **(i)** The polynomials $\mathrm{P}(x)=ax^3+x^2+bx+10$ and $\mathrm{Q}(x)=x^3+ax^2-5x+b$ leave the same remainder when divided by $x+2$.

Show that $4a+b=4$. [2]

$$a(-2)^3+(-2)^2+b(-2)+10=(-2)^3+a(-2)^2-5(-2)+b$$
$$-8a-2b+14=-8+4a+10+b$$
$$-12a-3b=-12$$
$$4a+b=4$$

(ii) If $\mathrm{P}'(x)$ has a factor of $3x-2$, find the values of a and b. [4]

$$\mathrm{P}'(x)=3ax^2+2x+b$$

$$\mathrm{P}'\left(\frac{2}{3}\right)=0$$
$$3a\left(\frac{2}{3}\right)^2+2\left(\frac{2}{3}\right)+b=0$$
$$\frac{4}{3}a+b=-\frac{4}{3}$$

$$4a+3b=-4$$
$4a+b=4$ from (i)

$$2b=-8$$
$$\left.\begin{array}{l} b=-4 \\ a=2 \end{array}\right\}$$

(b) Solve the equation $2x^3+4x^2-5x-7=0$, expressing non-exact solutions in the form $a\pm\frac{1}{2}\sqrt{b}$ where a and b are constants to be determined. [5]

$$\mathrm{P}(x)=2x^3+4x^2-5x-7$$
$$\mathrm{P}(-1)=2(-1)^3+4(-1)^2-5(-1)-7=0$$
$x+1$ is a factor of $\mathrm{P}(x)$

$$\mathrm{P}(x)=(x+1)(2x^2+Bx-7)$$

Term in x^2 :
$$4x^2=Bx^2+2x^2$$
$$B=2$$

$$\mathrm{P}(x)=(x+1)(2x^2+2x-7)$$

For $2x^2+2x-7=0$,

$$\begin{aligned}x&=\frac{-2\pm\sqrt{2^2-4(2)(-7)}}{2(2)}\\&=\frac{-2\pm\sqrt{60}}{4}\\&=-\frac{1}{2}\pm\frac{1}{2}\sqrt{15}\end{aligned}$$

$\therefore x=-1,\ -\frac{1}{2}\pm\frac{1}{2}\sqrt{15}$

---

5 The depth of water, h metres, in a shallow port on a particular day is modelled by the formula $h=7+2\sin\left(\frac{\pi}{6}t\right)$, where t is the number of hours after midnight.

(a) State the period of h. [1]

$$h=\frac{2\pi}{\pi/6}=12 \text{ hours}$$

(b) Use the model to predict the time when the depth of water is at its lowest. [2]

h is lowest when $\sin\left(\frac{\pi}{6}t\right)=-1$

$$\frac{\pi}{6}t=\frac{3\pi}{2}$$

$$t=9$$

Time is 9 am (or 0900 hrs)

A supply boat docks at the port at 5.30 am and it takes the workers 3 hours to unload its cargo immediately after it docks. To leave the port safely, the supply boat requires the depth of water to be at least 5.6 metres.

(c) Determine the earliest time, to the nearest minute, that the supply boat can leave the port. [4]

$$7+2\sin\left(\frac{\pi}{6}t\right)=5.6$$

$$\sin\left(\frac{\pi}{6}t\right)=-\frac{7}{10}$$

$$\text{basic angle}=\sin^{-1}\left(\frac{7}{10}\right)\approx 0.77539$$

$$\frac{\pi}{6}t=\pi+0.77539\ (\approx 3.9169),\ 2\pi-0.77539\ (\approx 5.5077)$$

$$t=7.4808\ ,10.519$$

Two critical timings are 7.29 am (rejected since boat is still unloading) and 10.31am. Hence, earliest time boat can leave the port is 10.31am or 10.32am (or 24hr format).

---

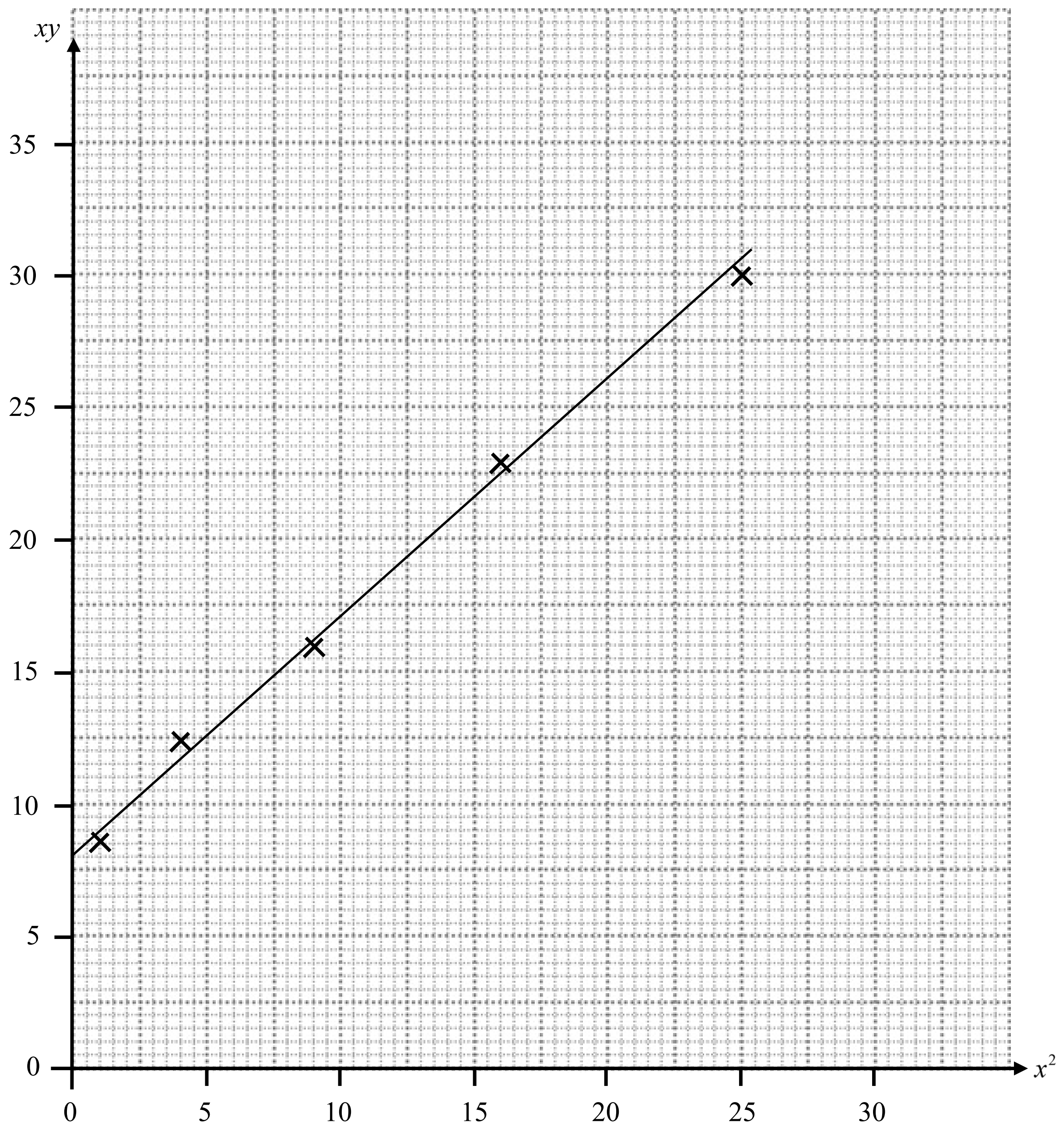
6 The variables x and y are known to be related by the formula $y = px + \frac{q}{x}$ where p and q are constants.

(a) The table below shows some experimental values of x and y.

x	1	2	3	4	5
y	8.55	6.20	5.32	5.72	6.00

x^2	1	4	9	16	25
xy	8.55	12.40	15.96	22.88	30.00

(i) On the grid below, plot xy against x^2 and draw a straight line graph. [2]



(ii) Use your graph to estimate

(a) the value of p and of q. [3]

$$xy = px^2 + q$$

$$p = \frac{26-17.5}{20-10.5} \qquad q = 8 \ \left(\text{acceptable } 7.5 \le q \le 9\right)$$

$$\approx 0.895 \ \left(\text{acceptable } 0.8 \le p \le 1.1\right)$$

(b) the positive value of x that satisfies the equation $px + \frac{q}{x} = \frac{20}{x}$. [2]

$$y = \frac{20}{x}$$

$$xy = 20$$

from graph, $x^2 = 13.25$

$$x = 3.64 \ \left(\text{acceptable } 3.60 \le x \le 3.75\right)$$

(b) Explain how another straight line graph can be obtained from $y = px + \frac{q}{x}$ by plotting $\frac{x}{y}$ against $\frac{1}{xy}$, expressing clearly the gradient and vertical intercept in terms of p and/or q.
You need not draw this straight line graph. [3]

$$xy = px^2 + q$$

$$1 = \frac{px}{y} + \frac{q}{xy}$$

$$\frac{px}{y} = -\frac{q}{xy} + 1$$

$$\frac{x}{y} = \left(-\frac{q}{p}\right)\frac{1}{xy} + \frac{1}{p}$$

gradient $= -\frac{q}{p}$

vertical intercept $= \frac{1}{p}$

7 The equation of a curve is $y = \dfrac{\ln 2x}{x}$.

(i) Find an expression for $\dfrac{d^2y}{dx^2}$ and show that $x\dfrac{d^2y}{dx^2} + 2\dfrac{dy}{dx} + \dfrac{1}{x^2} = 0$. [5]

$$\frac{dy}{dx} = \frac{x\left(\frac{1}{2x} \times 2\right) - \ln 2x}{x^2}$$
$$= \frac{1 - \ln 2x}{x^2}$$

$$\frac{d^2y}{dx^2} = \frac{x^2\left(-\frac{1}{2x} \times 2\right) - (1 - \ln 2x)(2x)}{x^4}$$
$$= \frac{-x - 2x + 2x \ln 2x}{x^4}$$
$$= \frac{2\ln 2x - 3}{x^3}$$

$$x\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + \frac{1}{x^2} = x\left(\frac{2\ln 2x - 3}{x^3}\right) + 2\left(\frac{1 - \ln 2x}{x^2}\right) + \frac{1}{x^2}$$
$$= \frac{2\ln 2x - 3 + 2 - 2\ln 2x + 1}{x^2}$$
$$= 0$$

(ii) Find the **exact** coordinates of the stationary point of the curve and use the second derivative test to determine if the stationary point is a maximum or minimum. [5]

$$\frac{dy}{dx} = \frac{1 - \ln 2x}{x^2} = 0$$

$$\ln 2x = 1$$
$$2x = e$$
$$x = \frac{e}{2}$$
$$y = \frac{\ln e}{e/2} = \frac{2}{e}$$

Stationary point is $\left(\dfrac{e}{2}, \dfrac{2}{e}\right)$ or $\left(\dfrac{e}{2}, 2e^{-1}\right)$

$$\left.\frac{\mathrm{d}^2 y}{\mathrm{d}x^2}\right|_{x=\frac{e}{2}} = \frac{2\ln e - 3}{\left(e/2\right)^3}$$ **M1 -** substitute *their* x value into *their* $\frac{\mathrm{d}^2 y}{\mathrm{d}x^2}$

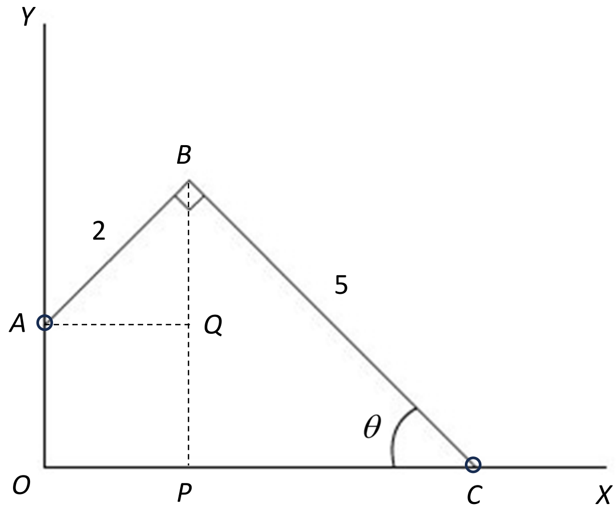
$$= \frac{-8}{e^3} \left(\text{or} \quad \frac{-1}{\left(e/2\right)^3} \quad \text{or} \quad -0.398 \right)$$

$$< 0$$

Hence, $\left(\frac{e}{2}, \frac{2}{e}\right)$ is a maximum point.

---

8 The diagram shows two rods AB and BC of length 2 m and 5 m respectively, joined together at B such that angle $ABC = 90°$. Small rings are attached to A and C so that A can move along the vertical pole OY and C can move along the horizontal pole OX. The rod BC makes an angle θ with OX.



(i) Show that $OA = 5\sin\theta - 2\cos\theta$. [2]

$$\frac{BP}{5} = \sin\theta \qquad\qquad \frac{BQ}{2} = \cos\theta$$

$$BP = 5\sin\theta \qquad\qquad BQ = 2\cos\theta$$

$$OA = BP - BQ = 5\sin\theta - 2\cos\theta$$

(ii) Express OA in the form $R\sin(\theta - \alpha)$ where $R > 0$ and $0° < \alpha < 90°$. [3]

$$OA = 5\sin\theta - 2\cos\theta = R(\sin\theta\cos\alpha - \cos\theta\sin\alpha)$$

$$\left.\begin{aligned} R\cos\alpha &= 5 \\ R\sin\alpha &= 2 \end{aligned}\right\} \quad \begin{aligned} \tan\alpha &= \frac{2}{5} \\ \alpha &\approx 21.801° \end{aligned} \qquad \begin{aligned} R^2 &= 2^2 + 5^2 = 29 \\ R &= \sqrt{29} \end{aligned}$$

$$OA = \sqrt{29}\sin(\theta - 21.8°)$$

(iii) Given that A can be no higher than 3.5 m above O, calculate the greatest possible value of θ. [2]

$$\sqrt{29}\sin\left(\theta-21.801°\right)=3.5$$
$$\sin\left(\theta-21.801°\right)=\frac{3.5}{\sqrt{29}}$$
$$\theta-21.801°=\sin^{-1}\frac{3.5}{\sqrt{29}}\approx 40.536°$$

greatest $\theta=62.3°$

---

9 A particle moves in a straight line with a fixed point O. The velocity of the particle, v m/s, is given by $v=\dfrac{25}{2t+1}+10$ where t is time in seconds after passing O.

(a) Explain why the particle never change its direction of motion and will eventually travel at a speed close to 10 m/s. [2]

Since $t>0$, $\dfrac{25}{2t+1}>0$ and hence $v>0$. Therefore particle will never change its direction.

As t increases, $\dfrac{25}{2t+1}$ **decreases and approaches 0 / becomes very small**.

Therefore particle will eventually travel at a speed close to 10 m/s.

(b) Find the acceleration of the particle at $t=2$. [2]

$$v=25\left(2t+1\right)^{-1}+10$$

$$a=-25\left(2t+1\right)^{-2}\left(2\right)$$
$$=-\frac{50}{\left(2t+1\right)^{2}}$$

$$a_2=-\frac{50}{25}=-2\text{ m/s}^2$$

(c) Calculate the distance travelled by the particle in the fifth second of its motion. [4]

$$s=\frac{25\ln(2t+1)}{2}+10t+c$$

When $t=0$, $s=0$, $c=0$

$$s=\frac{25\ln(2t+1)}{2}+10t$$

$$s_4=\frac{25\ln 9}{2}+40\approx 67.465$$

$$s_5=\frac{25\ln 11}{2}+50\approx 79.973$$

Distance travelled $=79.973-67.465$

$=12.5\text{m}$

---

10 $A(1\ ,\ 7)$ and $B(9\ ,\ 7)$ lie on a circle C_1 with centre P. The line $4y+3x=31$ is a normal to the circle and passes through A.

(i) Use a geometrical property of circle to explain why the x-coordinate of P is 5. [2]

Perpendicular bisector of chord passes through the centre of circle.

Hence, since AB is horizontal chord,
P is vertically above/below the midpoint of AB.

$$\therefore x_p=\frac{1+9}{2}=5$$

(ii) Find the equation of the circle C_1. [3]

$$4y+3(5)=31$$
$$4y=16$$
$$y=4$$

P is $(5\ ,\ 4)$

$$\begin{aligned}r&=\sqrt{(9-5)^2+(7-4)^2}\\&=\sqrt{25}\\&=5\end{aligned}$$

Equation is circle is $(x-5)^2+(y-4)^2=25$

(iii) The tangent to the circle at A intersects the y-axis at D.

Find the coordinates of D. [3]

Gradient of normal to A is $-\dfrac{3}{4}$, hence gradient of tangent at A is $\dfrac{4}{3}$.

Tangent : $y=\dfrac{4}{3}x+c$

At $A(1\ ,\ 7)$,

$$7=\frac{4}{3}+c$$

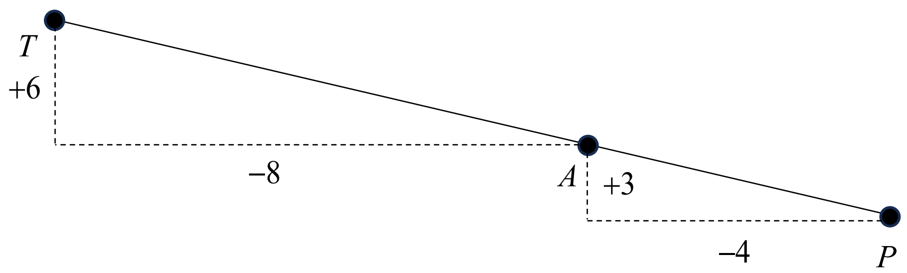
$$c=\frac{17}{3}$$

$$y=\frac{4}{3}x+\frac{17}{3}$$

D is $\left(0\ ,\ \dfrac{17}{3}\right)$

A larger circle C_2 has the same centre P and radius three times that of C_1. T is a point on C_2 such that P, A and T are collinear and the x-coordinate of T is negative.

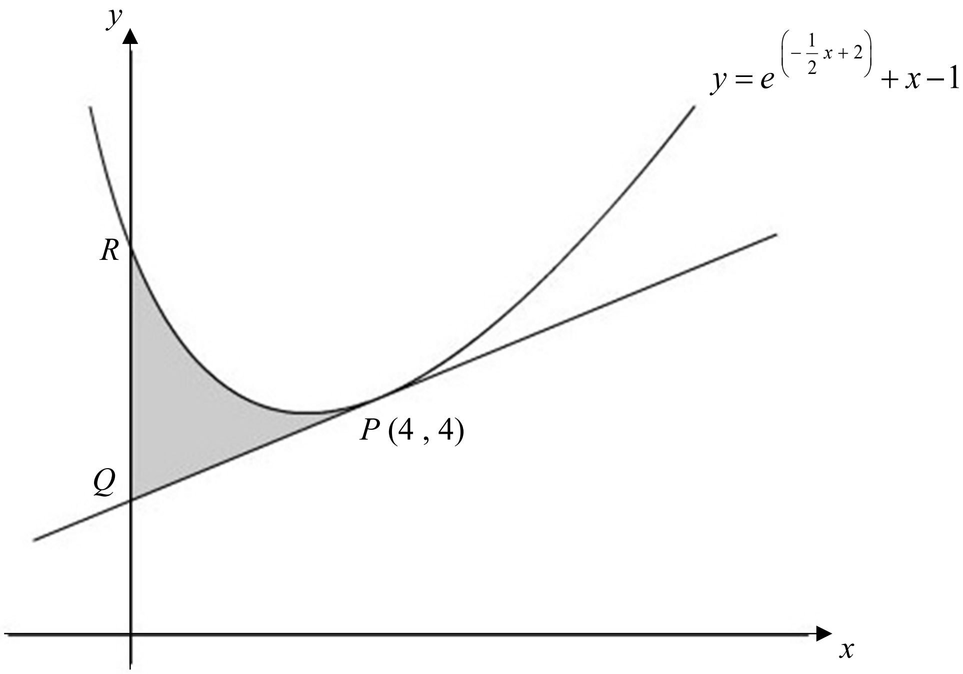
(iv) Determine the coordinates of T. [2]



$T(1-8\ ,\ 7+6)$

$T(-7\ ,\ 13)$

11 The diagram shows part of the curve $y = e^{\left(-\frac{1}{2}x+2\right)} + x - 1$ that intersects the y-axis at R. The tangent to the curve at $P(4,4)$ intersects the y-axis at Q.



Show that the area of the shaded region can be expressed as $ae^2 + b$ where a and b are constants to be determined. [8]

$$\frac{\mathrm{d}y}{\mathrm{d}x} = -\frac{1}{2}e^{\left(-\frac{1}{2}x+2\right)} + 1$$

$$\left.\frac{\mathrm{d}y}{\mathrm{d}x}\right|_{x=4} = -\frac{1}{2}e^0 + 1 = \frac{1}{2}$$

Tangent is $y = \frac{1}{2}x + c$

At P,

$$4 = \frac{1}{2}(4) + c$$
$$c = 2$$

$\therefore y = \frac{1}{2}x + 2$ and $y_Q = 2$

$$\text{Area of shaded} = \int_0^4 e^{\left(-\frac{1}{2}x+2\right)} + x - 1 \, dx - \frac{1}{2}(4)(2+4)$$

$$= \left[\frac{e^{\left(-\frac{1}{2}x+2\right)}}{-\frac{1}{2}} + \frac{x^2}{2} - x \right]_0^4 - 12$$

$$= \left(-2e^0 + \frac{4^2}{2} - 4 \right) - \left(-2e^2 \right) - 12$$

$$= 2 + 2e^2 - 12$$

$$= 2e^2 - 10$$
