DUNMAN SECONDARY SCHOOL

CANDIDATE NAME			
CLASS		**INDEX NUMBER**	

PRELIMINARY EXAMINATION 2025
SECONDARY 4 EXPRESS

ADDITIONAL MATHEMATICS 4049/01

Paper 1

20 August 2025

2 hours 15 minutes

Candidates answer on the Question Paper.

READ THESE INSTRUCTIONS FIRST

Calculator Model

Write your name, class and index number on all the work you hand in.
Write in dark blue or black pen.
You may use a pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

The number of marks is given in brackets [ ] at the end of each question or part question.

The total marks for this paper is 90.

Total Marks

1. ALGEBRA

Quadratic Equation

For the equation $ax^2+bx+c=0$,

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

Binomial expansion

$$(a+b)^n=a^n+\binom{n}{1}a^{n-1}b+\binom{n}{2}a^{n-2}b^2+\ldots+\binom{n}{r}a^{n-r}b^r+\ldots+b^n,$$

where n is a positive integer and $\binom{n}{r}=\frac{n!}{r!(n-r)!}=\frac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A+\cos^2 A=1$$

$$\sec^2 A=1+\tan^2 A$$

$$\operatorname{cosec}^2 A=1+\cot^2 A$$

$$\sin(A\pm B)=\sin A\cos B\pm\cos A\sin B$$

$$\cos(A\pm B)=\cos A\cos B\mp\sin A\sin B$$

$$\tan(A\pm B)=\frac{\tan A\pm\tan B}{1\mp\tan A\tan B}$$

$$\sin 2A=2\sin A\cos A$$

$$\cos 2A=\cos^2 A-\sin^2 A=2\cos^2 A-1=1-2\sin^2 A$$

$$\tan 2A=\frac{2\tan A}{1-\tan^2 A}$$

Formula for ΔABC

$$\frac{a}{\sin A}=\frac{b}{\sin B}=\frac{c}{\sin C}$$

$$a^2=b^2+c^2-2bc\cos A$$

$$\Delta=\frac{1}{2}bc\sin A$$

1 The line $y = x - 4$ and the curve $y = \dfrac{2}{x-3}$ intersect at the points A and B.

Find the length of AB. [6]

2 **(a)** State the range of $\tan^{-1} p$. [1]

(b) Given that $\cos^{-1} k = x$, where $0 \le x \le \dfrac{\pi}{2}$, find the principal value of $\cos^{-1}(-k)$. [1]

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3 Explain why $3x^2 + 15x + 20$ cannot be smaller than 1. [3]

4 **(a)** Express $\dfrac{x^2+3x+2}{x^2(2x+1)}$ in partial fractions. [5]

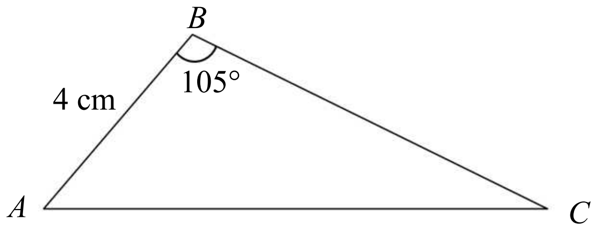
(b) Hence integrate $\dfrac{x^2+3x+2}{x^2(2x+1)}$ with respect to x. [3]

5 Solve the equation $3\log_5 y - \log_y 5 = 2$. [4]

6 **(a)** Show that $\sin 105° = \dfrac{\sqrt{6}+\sqrt{2}}{4}$. [3]

(b) The diagram below shows triangle ABC, such that $AB = 4$ cm, angle $ABC = 105°$ and the area of triangle ABC is $10+2\sqrt{3}$ cm^2.

Find BC in the form $\sqrt{2}\left(a\sqrt{3}+b\right)$, where a and b are integers. [3]



7 **(a)** Given that $u = \left(\frac{3}{2}\right)^x$, express $9^x + 6^x = 6\left(4^x\right)$ as a quadratic equation in u. [3]

(b) Hence, or otherwise, solve $9^x + 6^x = 6\left(4^x\right)$. [3]

8 The expression $2x^3 + ax^2 + bx + 10$, where a and b are constants, has a factor of $x + 2$ and leaves a remainder of 10 when divided by $x - 3$.
Find the values of a and of b. [4]

9 A cyclist starts from rest from a point A and travels in a straight line until she comes to a rest at a point B. During the motion, her velocity, v m/s, is given by $v = 12t - \frac{3}{2}t^2$, where t is the time in seconds after leaving A.

(a) Find the time taken for the cyclist to travel from A to B. [1]

(b) Find the distance AB. [3]

(c) Find the acceleration of the cyclist when $t = 5$. [2]

10 The equation of a curve is $y = xe^{-3x}$.
Find the stationary point(s) and determine the nature of the stationary point(s). [6]

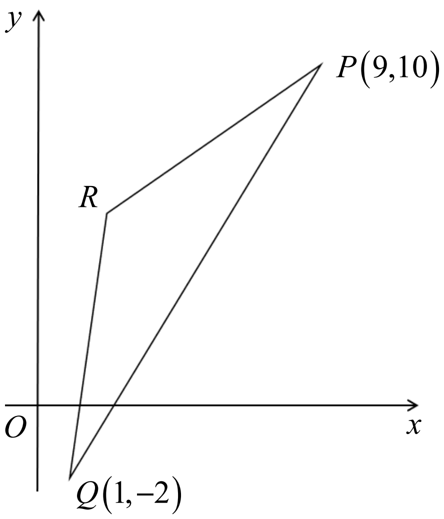
11 **(a)** Sketch the graphs of $y=-2\cos 3x+1$ and $y=-\frac{2}{\pi}x+1$ on the same axes, for $0 \le x \le \pi$.

[5]

(b) Explain how the number of solutions for $2\cos 3x=\frac{2}{\pi}x$ can be found using the graphs.

[1]

12



The diagram shows a triangle *PQR* with vertices $P(9,10)$ and $Q(1,-2)$ .The point *R* lies on the perpendicular bisector of *PQ* and the equation of the line *QR* is $y = 8x - 10$.

(a) Find the equation of the perpendicular bisector of *PQ*. [4]

(b) Find the coordinates of *R*. [2]

(c) Find the area of triangle *PQR*. [2]

13 The mass, m mg, of a radioactive substance decreases with time, t hours.
It is known that m and t are related by the equation $m = Me^{-kt}$, where M and k are constants.
The table below shows measured values of m and t.

t (hours)	1	2	3	4	5
m (mg)	6.55	5.36	4.40	3.60	2.94

(a) Plot ln m against t and draw a straight line graph. [2]

Use your graph to estimate

(b) the initial mass of the substance, [2]

(c) the value of k. [2]

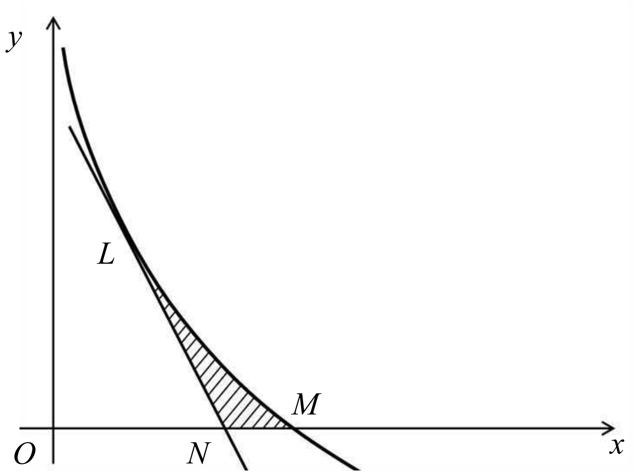
14 A curve is such that $\frac{\mathrm{d}^2 y}{\mathrm{d}x^2} = -4\sin(2x+\pi)$ and the point $P\left(\frac{\pi}{2}, \pi\right)$ lies on the curve.

The gradient of the curve at P is 3.

(a) Find the equation of the curve. [5]

(b) Justify whether y is increasing when $x = 3$. [2]

15



The diagram shows part of the curve $y = \dfrac{16}{(x+1)^2} - 1$, cutting the x-axis at M.

The tangent at the point L on the curve cuts the x-axis at N.
The gradient of this tangent is -4.
Calculate the area of the shaded region LMN. [12]

Continuation of working space for Question 15.

END OF PAPER

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DUNMAN SECONDARY SCHOOL

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CLASS		**INDEX NUMBER**	

PRELIMINARY EXAMINATION 2025
SECONDARY 4 EXPRESS / 5 NORMAL ACADEMIC

MATHEMATICS **4052/01**

Paper 1

20 August 2025

2 hours 15 minutes

Solutions

(Updated 1 Dec 2022 by HOD/Math aligned to Marking Scheme for Specimen Papers 4049)

Question	Answer
1	$x-4=\dfrac{2}{x-3}$
	$(x-4)(x-3)=2$
	$x^2-7x+12=2$
	$x^2-7x+10=0$
	$(x-2)(x-5)=0$
	$x=2$ or 5
	When $x = 2$, $y = -2$
	When $x = 5$, $y = 1$
	Length = $\sqrt{(2-5)^2+(-2-1)^2}$ = 4.24 unit
2a	$-90° < \tan^{-1} p < 90°$ or $-\dfrac{\pi}{2} < \tan^{-1} p < \dfrac{\pi}{2}$
2b	$\pi - x$
3	$3x^2+15x+20 = 3(x^2+5x)+20$
	$= 3\left[(x+2.5)^2-(2.5)^2\right]+20$
	$= 3(x+2.5)^2+1.25$
	Since $3(x+2.5)^2+1.25 \geq 1.25$, $3x^2+15x+20$ cannot be smaller than 1.

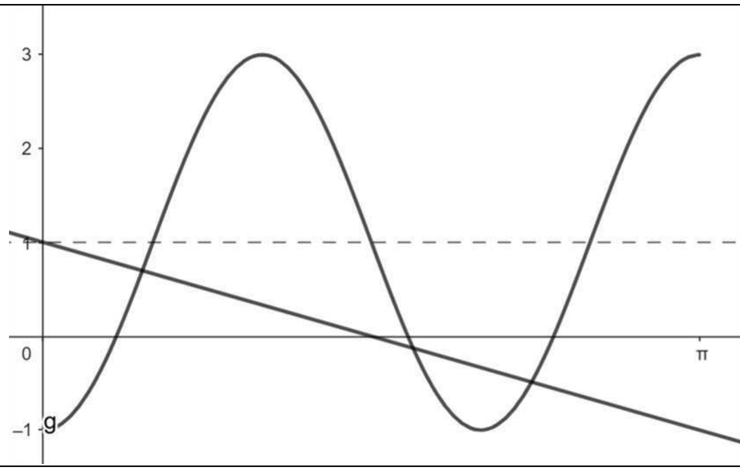
Question	Answer
4a	$\frac{x^2+3x+2}{x^2(2x+1)} = \frac{A}{x}+\frac{B}{x^2}+\frac{C}{2x+1}$
	$x^2+3x+2 = Ax(2x+1)+B(2x+1)+Cx^2$
	When $x=-\frac{1}{2}$, $\frac{1}{4}-\frac{3}{2}+2=C\left(\frac{1}{4}\right)$ $C=3$
	When $x = 0$, $B = 2$
	Comparing coefficients of x^2, $A = -1$
	$\frac{x^2+3x+2}{x^2(2x+1)} = -\frac{1}{x}+\frac{2}{x^2}+\frac{3}{2x+1}$
4b	$\int \frac{x^2+3x+2}{x^2(2x+1)}\,\mathrm{d}x = \int -\frac{1}{x}+\frac{2}{x^2}+\frac{3}{2x+1}\,\mathrm{d}x$ $= -\ln x-\frac{2}{x}+\frac{3}{2}\ln(2x+1)+c$
5	$3\log_5 y-\log_y 5=2$
	$3\log_5 y-\frac{\log_5 5}{\log_5 y}=2$
	$3\log_5 y-\frac{1}{\log_5 y}=2$
	$3(\log_5 y)^2-2\log_5 y-1=0$
	$(3\log_5 y+1)(\log_5 y-1)=0$
	$\log_5 y=1$ or $-\frac{1}{3}$
	$y=5$ or $y=5^{-\frac{1}{3}}=0.585$ (3s.f.)
Question	**Answer**
6a	$\sin(105°)=\sin(60°+45°)$
	$= \sin 60°\cos 45°+\cos 60°\sin 45°$
	$= \frac{\sqrt{3}}{2}\cdot\frac{1}{\sqrt{2}}+\frac{1}{2}\cdot\frac{1}{\sqrt{2}}$
	$= \frac{\sqrt{3}+1}{2\sqrt{2}}\times\frac{\sqrt{2}}{\sqrt{2}}$
	$= \frac{\sqrt{6}+\sqrt{2}}{4}$
6b	$\frac{1}{2}(4)(BC)\sin 105°=10+2\sqrt{3}$

	$\frac{1}{2}(4)(BC)\left(\frac{\sqrt{6}+\sqrt{2}}{4}\right)=10+2\sqrt{3}$
	$BC=\frac{2\left(10+2\sqrt{3}\right)}{\sqrt{6}+\sqrt{2}}\times\frac{\sqrt{6}-\sqrt{2}}{\sqrt{6}-\sqrt{2}}$
	$=\frac{4\left(5+\sqrt{3}\right)\left(\sqrt{6}-\sqrt{2}\right)}{6-2}$
	$=5\sqrt{6}+\sqrt{18}-5\sqrt{2}-\sqrt{6}$
	$=4\sqrt{6}+3\sqrt{2}-5\sqrt{2}$
	$=4\sqrt{6}-2\sqrt{2}$
	$=\sqrt{2}\left(4\sqrt{3}-2\right)$

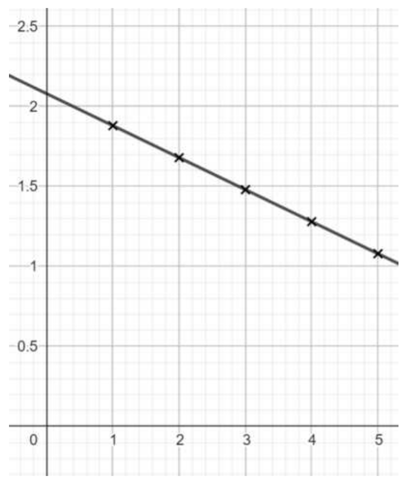
Question	Answer
7a	$9^x + 6^x = 6\left(4^x\right)$
	$\dfrac{9^x}{4^x} + \dfrac{6^x}{4^x} = 6$
	$\dfrac{3^{2x}}{2^{2x}} + \left(\dfrac{6}{4}\right)^x - 6 = 0$
	$\left(\dfrac{3}{2}\right)^{2x} + \left(\dfrac{3}{2}\right)^x - 6 = 0$
	When $u = \left(\dfrac{3}{2}\right)^x$, $u^2 + u - 6 = 0$
7b	$(u+3)(u-2) = 0$
	$u = 2$ or -3 (n.a.)
	$\left(\dfrac{3}{2}\right)^x = 2$
	$x = \dfrac{\ln 2}{\ln \frac{3}{2}}$ or $\dfrac{\lg 2}{\lg \frac{3}{2}}$ $= 1.71$
8	When $x = -2$, $2(-2)^3 + a(-2)^2 + b(-2) + 10 = 0$
	$4a - 2b = 6$
	$2a - b = 3$ - eqn 1
	When $x = 3$, $2(3)^3 + a(3)^2 + b(3) + 10 = 10$
	$9a + 3b = -54$
	$3a + b = -18$ - eqn 2
	eqn 1 + eqn 2: $5a = -15$
	$a = -3$
	$\therefore b = -9$
Question	**Answer**
9a	$12t - \dfrac{3}{2}t^2 = 0$
	$t\left(12 - \dfrac{3}{2}t\right) = 0$
	$t = 0$ or 8
	$\therefore 8$ s
9b	distance $= \int_0^8 12t - \dfrac{3}{2}t^2 \ \mathrm{d}x$
	$= \left[6t^2 - \dfrac{1}{2}t^3\right]_0^8$

	$=[128]-[0]$
	= 128 m
9c	$a=12-3t$
	When t = 5, a = -3 m/s^2

Question	**Answer**
10	$y = xe^{-3x}$
	$\dfrac{\mathrm{d}y}{\mathrm{d}x} = e^{-3x}(1) + x\left(-3e^{-3x}\right)$
	$= e^{-3x}(1-3x)$
	$e^{-3x}(1-3x) = 0$
	$x = \dfrac{1}{3}$
	$y = \dfrac{1}{3e}$ or 0.123
	$\dfrac{\mathrm{d}^2y}{\mathrm{d}x^2} = (1-3x)\left(-3e^{-3x}\right) + e^{-3x}(-3)$
	When $x = \dfrac{1}{3}$, $\dfrac{\mathrm{d}^2y}{\mathrm{d}x^2} < 0$
	$\left(\dfrac{1}{3}, -\dfrac{1}{3e}\right)$ is a maximum point.

Question	Answer
11a	
11b	The $2\cos 3x = \frac{2}{\pi}x$ is equivalent to $-2\cos 3x + 1 = -\frac{2}{\pi}x + 1$ and so its solutions can be found from the interception points of the two graphs.

Question	Answer
12a	Midpoint of $PQ = \left(\frac{1+9}{2}, \frac{-2+10}{2}\right) = (5,4)$
	Gradient of $PQ = \frac{-2-10}{1-9} = \frac{3}{2}$
	Gradient of perpendicular bisector = $-\frac{2}{3}$
	$y-4=-\frac{2}{3}(x-5)$
	$y=-\frac{2}{3}x+\frac{22}{3}$
12b	$-\frac{2}{3}x+\frac{22}{3}=8x-10$
	$-\frac{26}{3}x=-\frac{52}{3}$
	$x=2$
	$R(2,6)$
12c	Area = $\frac{1}{2}\begin{vmatrix} 9 & 2 & 1 & 9 \\ 10 & 6 & -2 & 10 \end{vmatrix}$
	$=\frac{1}{2}\left[(54-4+10)-(20+6-18)\right]$
	= 26 unit^2

<table>
<tr><th>Question</th><th>Answer</th></tr>
<tr><td>13a</td><td>
<table>
<tr><td>t</td><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td></tr>
<tr><td>m</td><td>6.55</td><td>5.36</td><td>4.40</td><td>3.60</td><td>2.94</td></tr>
<tr><td>$\ln m$</td><td>1.88</td><td>1.68</td><td>1.48</td><td>1.28</td><td>1.08</td></tr>
</table>


</td></tr>
<tr><td rowspan="2">13b</td><td>$\ln M = 2.08$</td></tr>
<tr><td>$M = 8.00$</td></tr>
<tr><td rowspan="2">13c</td><td>Gradient = -0.2</td></tr>
<tr><td>$k = 0.2$</td></tr>
</table>

Question	Answer
14a	$\dfrac{d^2y}{dx^2} = -4\sin(2x+\pi)$
	$\dfrac{dy}{dx} = 2\cos(2x+\pi) + c_1$
	$3 = 2\cos\left[2\left(\dfrac{\pi}{2}\right)+\pi\right] + c_1$
	$3 = 2(1) + c_1$
	$c_1 = 1$
	$\dfrac{dy}{dx} = 2\cos(2x+\pi) + 1$
	$y = \sin(2x+\pi) + x + c_2$
	$\pi = \sin\left[2\left(\dfrac{\pi}{2}\right)+\pi\right] + \dfrac{\pi}{2} + c_2$
	$c_2 = \dfrac{\pi}{2}$
	$y = \sin(2x+\pi) + x + \dfrac{\pi}{2}$
14b	When $x = 3$, $\dfrac{dy}{dx} = 2\cos\left[2(3)+\pi\right] + 1 = -0.920$
	y is decreasing when $x = 3$.

Question	**Answer**
15	$y=\dfrac{16}{(x+1)^2}-1=16(x+1)^{-2}-1$
	$\dfrac{dy}{dx}=-32(x+1)^{-3}$
	$-4=-32(x+1)^{-3}$
	$(x+1)^3=8$
	$x=1$
	At L, $y=\dfrac{16}{(1+1)^2}-1=3$
	$y-3=-4(x-1)$
	$y=-4x+7$
	At N, $0=-4x+7$
	$x=1.75$
	At M, $0=\dfrac{16}{(x+1)^2}-1$
	$x=3$ or -5 (n.a.)
	Area $=\left[\int_1^3 16(x+1)^{-2}-1\ dx\right]-\int_1^{1.75}-4x+7\ dx$
	$=\left[-16(x+1)^{-1}-x\right]_1^3-\left[-2x^2+7x\right]_1^{1.75}$
	$=2-1.125$
	$=0.875$ unit^2

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ADDITIONAL MATHEMATICS — 4049/02

Paper 2

25 August 2025

2 hours 15 minutes

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Total Marks

1. ALGEBRA

Quadratic Equation

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$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1} a^{n-1} b + \binom{n}{2} a^{n-2} b^2 + \ldots + \binom{n}{r} a^{n-r} b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

Formula for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2} bc \sin A$$

1 **(a)** A ball is thrown vertically upwards. Its height h m, above the ground at time t seconds after being thrown is given by the formula $h = 7 + 20t - 5t^2$.
Find the maximum height attained by the ball and the time at which this occurs. [4]

(b) Find the set of values of the constant k for which the curve $y = x^2 + 6x + k$ lies entirely above the line $y = k(x-1)$. [4]

2 A and B are acute angles, such that $\cos A = \frac{3}{5}$ and $\tan B = \frac{12}{5}$.

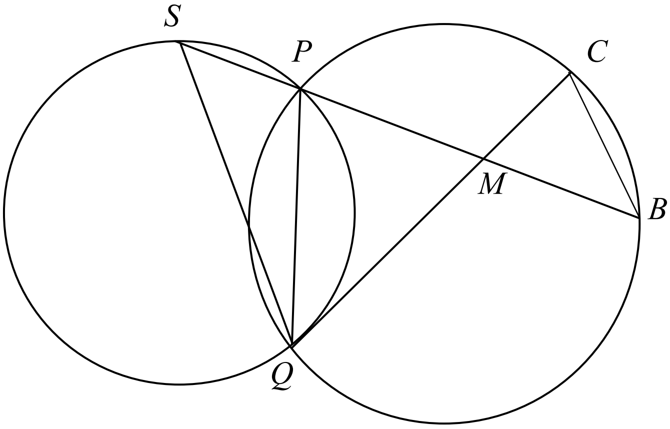
Find the exact values of

(a) $\tan 2A$, [2]

(b) $\sin\left(B + \frac{3\pi}{2}\right)$, [3]

(c) $\cos\frac{B}{2}$. [3]

3 In the diagram, PQS lies on a circle and $PCBQ$ lies on another circle.
PQ is a common chord of the two circles. BPS is a straight line.
QC is a tangent to the circle PQS at Q. QC and BP intersect at M.



(a) Prove that BC is parallel to QS. [3]

(b) Prove that triangle PQM is similar to triangle QSM. [2]

(c) Hence show that $QM^2 = PM \times SM$. [2]

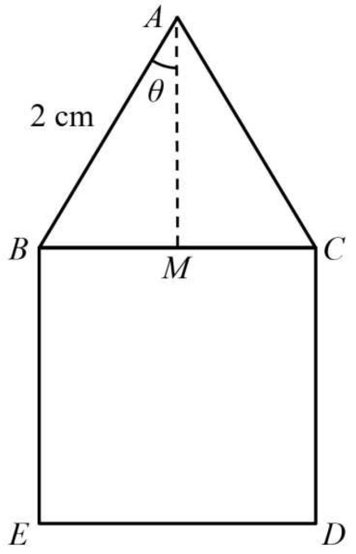
4 **(a)** Show that $\frac{\mathrm{d}}{\mathrm{d}x}(\cot x) = -\operatorname{cosec}^2 x$. [4]

(b) Hence find the value of $\int_{\frac{\pi}{4}}^{\frac{\pi}{3}} -\cot^2 x \, \mathrm{d}x$. [4]

5 **(a)** Given that the constant term in the binomial expansion of $\left(x-\frac{k}{x^{3}}\right)^{8}$ is 7, find the value of the positive constant k. [4]

(b) Using the value of k found in part **(a)**, show that there is no constant term in the expansion of $\left(1+x^{4}\right)\left(x-\frac{k}{x^{3}}\right)^{8}$. [4]

6 The diagram below shows an isosceles triangle ABC, where $AB = AC = 2$ cm, and a square $BCDE$. M is the midpoint of BC.
Angle $BAM = \theta$ and can vary.



(a) Show that the total area of $ABEDC$ is $16\sin^2\theta + 4\sin\theta\cos\theta$ cm^2. [3]

(b) Show that $16\sin^2\theta + 4\sin\theta\cos\theta = 2\sin 2\theta - 8\cos 2\theta + 8$. [2]

(c) Express $2 \sin 2\theta - 8 \cos 2\theta$ in the form $R \sin(2\theta - \alpha)$, where $R > 0$ and $0° < \alpha < 90°$. [3]

7 **(a)** Prove the identity $\dfrac{\cos^2 x - \sin^2 x}{1 + 2\sin x \cos x} = \dfrac{1 - \tan x}{1 + \tan x}$. [4]

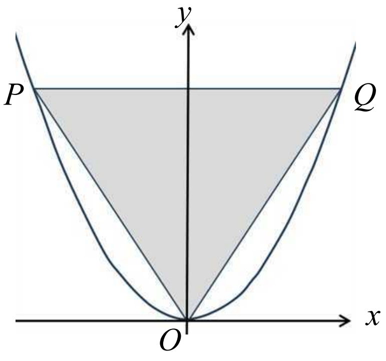
(b) Solve the equation $\dfrac{\cos^2 x - \sin^2 x}{1 + 2\sin x \cos x} = \dfrac{2}{3}\tan x$ for $0 \leq x \leq 2\pi$. [4]

8 **(a)** Using long division, divide $2x^3 - 7x^2 + 2x + 3$ by $2x - 4$. [2]

(b) Solve $2x^3 - 7x^2 + 2x + 3 = 0$. [4]

(c) Kun Ye claims that the solutions to the equation $2x^3 - 7x^2 + 2x + 3 = 0$ can be used to solve $3y^6 + 2y^4 - 7y^2 + 2 = 0$. Explain how he is correct. [2]

9 The diagram shows part of the curve $y = 3x^2$.
The points P and Q lie on the curve, and have the same y-coordinates.



(a) Show that the area of the triangle OPQ, A cm^2, is given by $A = 3x^3$. [2]

The points P and Q move along the curve such that their y-coordinates are changing at a rate of 3 units per second.

Find, at the point where $x = 4$,

(b) the rate of change of PQ, [4]

(c) the rate of change of A. [3]

10 The lines $y = 9$, $y = -1$ and $3y = 4x + 9$ are tangents to a circle.

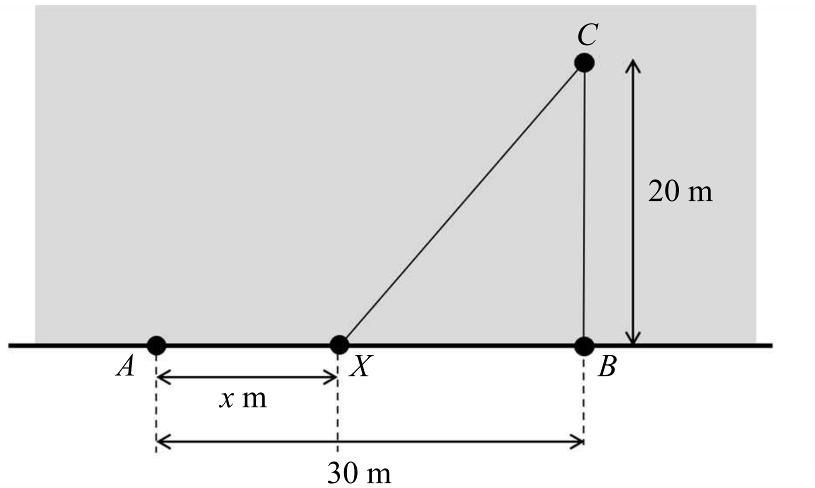
The x-coordinate of the centre of the circle is positive.

(a) Explain why the y-coordinate of the centre of the circle is 4. [1]

(b) By considering the discriminant, or otherwise, show that the x-coordinate of the centre of the circle is 7. [7]

(c) Determine if the point $(3, 0)$ lies within the circle. [2]

11 The diagram shows a lake (shaded) bordered by a straight level road AXB.
Point C is on the surface of the lake and B is the point closest to C.
The distance AB is 30 m and BC is 20 m.



Natalie is at point A on the road and wants to get to point C as quickly as possible. She can run at a speed of 4 m/s and swim at a speed of 2 m/s.
She realizes that in order to minimize the total time to get to C, it is necessary to leave the road at some point X between A and B, and then swim directly to C.

Let AX be x m and T be the total time, in seconds, taken by Natalie to get from A to X and then from X to C.

(a) Show that $T = \frac{x}{4} + \frac{1}{2}\sqrt{x^2 - 60x + 1300}$. [2]

(b) Find an expression for $\frac{dT}{dx}$. [2]

(c) Find the value of x for which the total time taken is minimum. You are **not** required to justify that this value of x leads to the minimum total time. [4]

END OF PAPER

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DUNMAN SECONDARY SCHOOL

CANDIDATE NAME			
CLASS		INDEX NUMBER	

PRELIMINARY EXAMINATION 2025
SECONDARY 4 EXPRESS / 5 NORMAL ACADEMIC

MATHEMATICS **4052/02**

Paper 2

25 August 2025

2 hours 15 minutes

Marking Scheme

(Updated 1 Dec 2022 by HOD/Math aligned to Marking Scheme for Specimen Papers 4049)

Question	Answer
1a	$h = 7 + 20t - 5t^2$
	$= -5\left(t^2 - 4t\right) + 7$
	$= -5\left[\left(t-2\right)^2 - 4\right] + 7$
	$= -5\left(t-2\right)^2 + 27$
	maximum height = 27
	OR
	$\frac{\mathrm{d}h}{\mathrm{d}t} = -10t + 20$
	$-10t + 20 = 0$
	$t = 2$
	When $t = 2$, $h = 7 + 20(2) - 5(2)^2 = 27\,\mathrm{m}$
1b	$x^2 + 6x + k = k(x-1)$
	$x^2 + (6-k)x + 2k = 0$
	$(6-k)^2 - 4(1)(2k) < 0$
	$36 - 12k + k^2 - 8k < 0$
	$k^2 - 20k + 32 < 0$
	$(k-2)(k-18) < 0$
	$\therefore 2 < x < 18$
2a	$\tan A = \frac{4}{3}$
	$\tan 2A = \frac{2\left(\frac{4}{3}\right)}{1 - \left(\frac{4}{3}\right)^2}$
	$= -\frac{24}{7}$
Question	**Answer**
2b	$\sin B = \frac{12}{13}$, $\cos B = \frac{5}{13}$
	$\sin\left(B + \frac{3\pi}{2}\right) = \sin B \cos\frac{3\pi}{2} + \sin\frac{3\pi}{2}\cos B$
	$= \frac{12}{13} \times 0 + (-1) \times \frac{5}{13}$
	$= -\frac{5}{13}$
	OR
	$\sin\left(B + \frac{3\pi}{2}\right) = \sin\left(2\pi - \frac{\pi}{2} + B\right)$

	$= \sin\left(-\frac{\pi}{2}+B\right) = -\sin\left(\frac{\pi}{2}-B\right)$
	$= -\cos B = -\frac{5}{13}$
2c	$\cos B = 2\cos^2\frac{B}{2}-1$
	$\frac{5}{13} = 2\cos^2\frac{B}{2}-1$
	$2\cos^2\frac{B}{2} = \frac{18}{13}$
	$\cos^2\frac{B}{2} = \frac{9}{13}$
	$\cos\frac{B}{2} = \frac{3}{\sqrt{13}}$ or $-\frac{3}{\sqrt{13}}$ (rej)

Question	Answer
3a	$\angle PQC = \angle PSQ$ (Alternate segment theorem)
	$\angle PQC = \angle PBC$ (angles in same segment)
	Hence $\angle PBC = \angle PSQ$
	By alternate angles of parallel lines, BC is parallel to QS
3b	$\angle PQC = \angle PSQ$ (Alternate segment theorem)
	$\angle PMQ = \angle QMS$ (Common angle)
	ΔPQM is similar to ΔQSM (AA Similarity Test)
3c	Since ΔPQM is similar to ΔQSM , $\frac{QM}{SM} = \frac{PM}{QM}$
	$(QM)^2 = PM \times SM$

4a	$\frac{\mathrm{d}}{\mathrm{d}x}(\cot x)=\frac{\mathrm{d}}{\mathrm{d}x}\left(\frac{\cos x}{\sin x}\right)$
	$=\frac{\sin x(-\sin x)-\cos x(\cos x)}{\sin^2 x}$
	$=\frac{-\sin^2 x-\cos^2 x}{\sin^2 x}$
	$=-\frac{1}{\sin^2 x}$
	$=-\operatorname{cosec}^2 x$ (Shown)
	OR
	$\frac{\mathrm{d}}{\mathrm{d}x}(\cot x)=\frac{\mathrm{d}}{\mathrm{d}x}(\tan x)^{-1}$
	$=-(\tan x)^{-2}(\sec^2 x)$
	$=-\left(\frac{\sin x}{\cos x}\right)^{-2}\left(\frac{1}{\cos^2 x}\right)$
	$=-\left(\frac{\cos^2 x}{\sin^2 x}\right)\left(\frac{1}{\cos^2 x}\right)$
	$=-\frac{1}{\sin^2 x}$
	$=-\operatorname{cosec}^2 x$ (Shown)
4b	$\int_{\frac{\pi}{4}}^{\frac{\pi}{3}} -\cot^2 x \ \mathrm{d}x=\int_{\frac{\pi}{4}}^{\frac{\pi}{3}} 1-\ \operatorname{cosec}^2 x \ \mathrm{d}x$
	$=\left[x+\cot x\right]_{\frac{\pi}{4}}^{\frac{\pi}{3}}$
	$=\left[\frac{\pi}{3}+\cot\frac{\pi}{3}\right]-\left[\frac{\pi}{4}+\cot\frac{\pi}{4}\right]$
	$=\frac{\pi}{3}+\frac{1}{\sqrt{3}}-\frac{\pi}{4}-1$
	$=\frac{\pi}{12}+\frac{1}{\sqrt{3}}-1$ or -0.161

Question	Answer
5a	General term $=\binom{8}{r}x^{8-r}\left(-\frac{k}{x^3}\right)^r$
	$=\binom{8}{r}x^{8-r}(-1)^r k^r\left(x^{-3}\right)^r$
	$=\binom{8}{r}(-1)^r k^r x^{8-4r}$
	When $8-4r=0$, $r=2$
	$\binom{8}{2}(-1)^2 k^2 x^{8-4(2)}=7$
	$28k^2=7$
	$k^2=\frac{1}{4}$
	$k=\frac{1}{2}$
5b	When $8-4r=-4$, $r=3$
	$T_4=\binom{8}{3}(-1)^3\left(\frac{1}{2}\right)^3 x^{-4}$
	$=-7x^{-4}$
	$\left(1+x^4\right)\left(x-\frac{k}{x^3}\right)^8=\left(1+x^4\right)\left(...+7-7x^{-4}+...\right)$
	$=...+7-7+...=...+0+...$
	There is no constant term in the expansion. (Shown)

Question	**Answer**
6a	$AM = 2\cos\theta$
	$BM = 2\sin\theta$
	$BC = 4\sin\theta$
	Total area = $(4\sin\theta)^2 + \frac{1}{2}(2\cos\theta)(4\sin\theta)$
	$= 16\sin^2\theta + 4\sin\theta\cos\theta$
6b	$16\sin^2\theta + 4\sin\theta\cos\theta$
	$= 8(2\sin^2\theta) + 2(2\sin\theta\cos\theta)$
	$= 8(1 - \cos 2\theta) + 2(\sin 2\theta)$
	$= 2\sin 2\theta - 8\cos 2\theta + 8$
6c	$R = \sqrt{2^2 + 8^2} = \sqrt{68}$
	$\tan\alpha = \frac{8}{2}$
	$\alpha = 76.0°$
	$2\sin 2\theta - 8\cos 2\theta = \sqrt{68}\sin(2\theta - 76.0°)$
7a	$LHS = \dfrac{\cos^2 x - \sin^2 x}{1 + 2\sin x\cos x}$
	$= \dfrac{\cos^2 x - \sin^2 x}{\sin^2 x + \cos^2 x + 2\sin x\cos x}$
	$= \dfrac{(\cos x + \sin x)(\cos x - \sin x)}{(\cos x + \sin x)^2}$
	$= \dfrac{\cos x - \sin x}{\cos x + \sin x}$
	$= \dfrac{1 - \tan x}{1 + \tan x} = RHS$

Question	Answer
7b	$\dfrac{1-\tan x}{1+\tan x}=\dfrac{2}{3}\tan x$
	$3(1-\tan x)=2\tan x+\tan^2 x$
	$2\tan^2 x+5\tan x-3=0$
	$(2\tan x-1)(\tan x+3)=0$
	$\tan x=\dfrac{1}{2}$ or -3
	$x=0.464$ or 3.61
	$x=1.89$ or 5.03
8a	$\begin{array}{r\|r} & x^2-\frac{3x}{2}-2 \\ \hline 2x-4 & 2x^3-7x^2+2x+3 \\ & - \\ & 2x^3-4x^2 \\ \hline & -3x^2+2x+3 \\ & - \\ & -3x^2+6x \\ \hline & 3-4x \\ & - \\ & 8-4x \\ \hline & -5 \end{array}$

Question	Answer
8b	$2x^3 - 7x^2 + 2x + 3 = 0$
	Let $\mathrm{f}(x) = 2x^3 - 7x^2 + 2x + 3$
	When $x = 3$, $\mathrm{f}(3) = 2(3)^3 - 7(3)^2 + 2(3) + 3 = 0$
	By factor theorem, $(x-3)$ is a factor of f(x).
	$(x-3)(2x^2 - x - 1) = 0$
	$(x-3)(2x+1)(x-1) = 0$
	$x = -\frac{1}{2}$, 1 or 3
8c	$3y^6 + 2y^4 - 7y^2 + 2 = 0$
	$3 + \frac{2}{y^2} - \frac{7}{y^4} + \frac{2}{y^6} = 0$
	$3 + 2\left(\frac{1}{y^2}\right) - 7\left(\frac{1}{y^4}\right) + 2\left(\frac{1}{y^6}\right) = 0$
	$3 + 2\left(\frac{1}{y^2}\right) - 7\left(\frac{1}{y^2}\right)^2 + 2\left(\frac{1}{y^2}\right)^3 = 0$
	$\therefore x = \frac{1}{y^2}$, Kun Ye is correct

Question	Answer
9a	Height = $3x^2$
	Base = $2x$
	Area = $\frac{1}{2}(2x)(3x^2) = 3x^3$
9b	$\frac{dy}{dx} = 6x$
	$\frac{dy}{dt} = \frac{dy}{dx} \times \frac{dx}{dt}$
	$3 = 6x \times \frac{dx}{dt}$
	$3 = 6x \times \frac{dx}{dt}$
	$\frac{dx}{dt} = \frac{1}{2x}$
	When $x = 4$, $\frac{dx}{dt} = \frac{1}{8}$
	Rate of change of PQ = $\frac{1}{4}$ unit/s
9c	$\frac{dA}{dx} = 9x^2$
	$\frac{dA}{dt} = \frac{dA}{dx} \times \frac{dx}{dt}$
	$\frac{dA}{dt} = 9x^2 \times \frac{1}{8}$
	When $x = 4$, $\frac{dA}{dt} = 18$ unit^2/s

Question	Answer
10a	y-coordinate of centre = $\dfrac{-1+9}{2}=4$
10b	Let the x-coordinate of the centre be a,
	$(x-a)^2+(y-4)^2=25$
	From the line, $y=\dfrac{4x+9}{3}$
	$(x-a)^2+\left(\dfrac{4x+9}{3}-4\right)^2=25$
	$(x-a)^2+\left(\dfrac{4}{3}x-1\right)^2=25$
	$x^2-2ax+a^2+\dfrac{16}{9}x^2-\dfrac{8}{3}x+1=25$
	$\dfrac{25}{9}x^2+\left(-2a-\dfrac{8}{3}\right)x+a^2-24=0$
	Since the line is a tangent, $b^2-4ac=0$
	$\left(-2a-\dfrac{8}{3}\right)^2-4\left(\dfrac{25}{9}\right)\left(a^2-24\right)=0$
	$4a^2+\dfrac{32}{3}a+\dfrac{64}{9}-\dfrac{100}{9}a^2+\dfrac{2400}{9}=0$
	$36a^2+96a+64-100a^2+2400=0$
	$-64a^2+96a+2464=0$
	$2a^2-3a-77=0$
	$(2a+11)(a-7)=0$
	$a=7$ or -5.5 (rej)

Question	**Answer**
10c	Distance between point and centre $=\sqrt{(7-3)^2+(4-0)^2}$
	$=5.66$ unit
	Since the distance is longer than the radius, the point lies outside.
11a	$CX=\sqrt{(30-x)^2+20^2}$
	$=\sqrt{x^2-60x+1300}$
	$T=\frac{x}{4}+\frac{1}{2}\sqrt{x^2-60x+1300}$
11b	$\frac{\mathrm{d}T}{\mathrm{d}x}=\frac{1}{4}+\frac{1}{2}\left[\frac{1}{2}\left(x^2-60x+1300\right)^{-\frac{1}{2}}(2x-60)\right]$
	$=\frac{1}{4}+\frac{2x-60}{4\sqrt{x^2-60x+1300}}$
11c	$\frac{1}{4}+\frac{2x-60}{4\sqrt{x^2-60x+1300}}=0$
	$\sqrt{x^2-60x+1300}+2x-60=0$
	$\sqrt{x^2-60x+1300}=60-2x$
	$x^2-60x+1300=3600-240x+4x^2$
	$3x^2-180x+2300=0$
	$x=\frac{180\pm\sqrt{180^2-4(3)(2300)}}{2(3)}$
	$x=41.5$ (rej) or 18.5