SINGAPORE CHINESE GIRLS' SCHOOL
PRELIMINARY EXAMINATION 2025
SECONDARY FOUR
O-LEVEL PROGRAMME

CANDIDATE NAME

CLASS 4

REGISTER NUMBER

CENTRE NUMBER

INDEX NUMBER

ADDITIONAL MATHEMATICS 4049/01

Paper 1

Thursday **28 August 2025** **2 hours 15 minutes**

Candidates answer on the Question Paper.
No Additional Materials are required.

READ THESE INSTRUCTIONS FIRST

Write your name, class, register number, centre number and index number at the top of this page.
Write in dark blue or black pen on both sides of the paper.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved electronic scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

The number of marks is given in brackets [ ] at the end of each question or part question.

The total number of marks for this paper is 90.

FOR EXAMINERS USE

Q1		**Q5**		**Q9**			
Q2		**Q6**		**Q10**			
Q3		**Q7**		**Q11**			
Q4		**Q8**		**Q12**		90	

The Question Paper consists of **19** printed pages and **1** blank page.

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the quadratic equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial Expansion

$$(a+b)^n = a^n + \binom{n}{1} a^{n-1}b + \binom{n}{2} a^{n-2}b^2 + \ldots + \binom{n}{r} a^{n-r}b^r + \ldots + b^n \ ,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \tfrac{1}{2} ab \sin C$$

1. Oil is dripping into an empty inverted right circular cone at the rate of 16 cm^3 per second.

The height, h cm, of the cone is three-quarters its radius, r cm.

[Volume of a right circular cone = $\frac{1}{3} \times \pi \times r^2 \times h$]

(a) Show that the volume, V cm^3, of the cone is $V = \frac{1}{4}\pi r^3$. [1]

(b) Calculate, at the instant when the radius is 8 cm, the rate of change of the radius. [3]

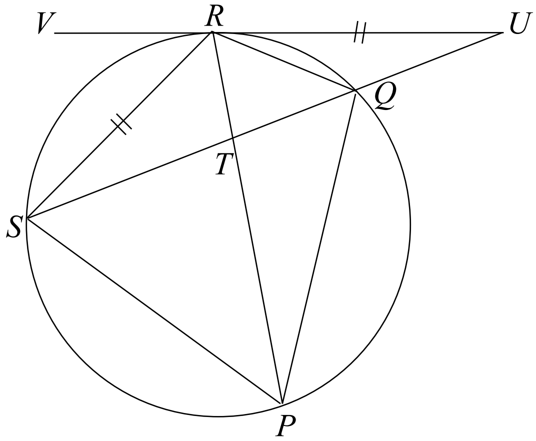
2. The straight line $2x + y = 1$ intersects the curve $x^2 - xy - y^2 + 5 = 0$ at the points *A* and *B*.

Show that the length *AB* is $5\sqrt{5}$ units. [5]

3. **(a)** Find $\frac{\mathrm{d}}{\mathrm{d}x}(2x-3)e^{2x}$. [3]

(b) Hence evaluate $\int_0^1 4xe^{2x}\,\mathrm{d}x$, giving your answers in exact form. [4]

4. The diagram shows a circle passing through the points *P, Q, R* and *S*. Chord *SQ* and *RP* intersect at *T*. The straight line *URV* is a tangent to the circle at *R*. The tangent meets the line *SQ* extended at *U* such that *RU* = *RS*.



(a) Show that triangle *RQU* is isosceles. [3]

(b) Show that angle *SPT* = 2 × angle *QPT*. [3]

5. The equation of a curve is $y=(m+1)x^2-8x+3m$, where m is a constant.

(a) In the case where $m = 1$, show that the line $y=1-4x$ is a tangent to the curve. Find the coordinates of the point of contact. [3]

(b) Find the range of values of m for which the curve is above the line $y = 5$. [5]

6. **(a)** By using long division, show that $2x^2+1$ is a factor of $2x^3-4x^2+x-2$. [2]

(b) Express $\dfrac{11x-5x^2-11}{2x^3-4x^2+x-2}$ in partial fractions. [5]

7. **(a)** State the amplitude and period of the graph of $y = 3\sin\frac{x}{2} + 1$. [2]

(b) Sketch the graph of $y = 3\sin\frac{x}{2} + 1$ for $0 \leq x \leq 4\pi$. [3]

(c) By adding another suitable curve on your sketch, determine the number of solutions of the equation $\sin\frac{x}{2} = -\frac{1}{3} - \frac{2}{3}\cos\frac{x}{2}$. [3]

8. The highest point on a circle C_1 is (2, 8). The equation of the tangent, *T*, to C_1 at the point (6, 6) is $3y+4x=42$.

(a) Find the equation of C_1. [6]

A second circle, C_2, is the reflection of C_1 in the line *T*.

(b) Find the equation of C_2. [3]

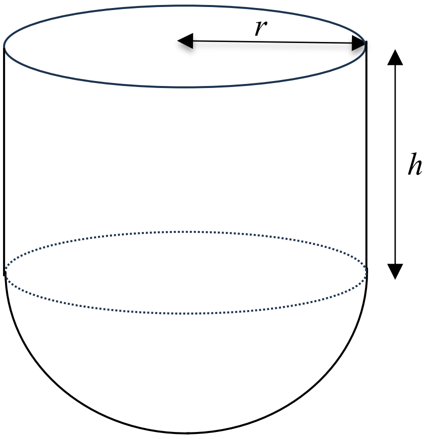
Continuation of working space for question **8(b)**.

9. **(a)** Show that the value of $\sin 105^\circ$ can be expressed as $\frac{1}{4}(p+q)$, where p and q are constants. [3]

(b) Given that $\frac{\cos(A+B)}{\cos(A-B)} = \frac{2}{7}$ and $\sin A \sin B \neq 0$, find the exact value of $\cot A \cot B$. [3]

(c) Given that B is a reflex angle and $\cot B = 3$, find the exact value of $\cos 2B$. [2]

10.



The diagram shows a container which consists of an **open** cylinder of radius r m and height h m joined to a hemispherical bottom of radius r m. The container is to be made of thin metal sheets of negligible thickness. The volume of the cylindrical part of the container is 300 m^3.
[Surface area of a sphere of radius $r = 4\pi r^2$]

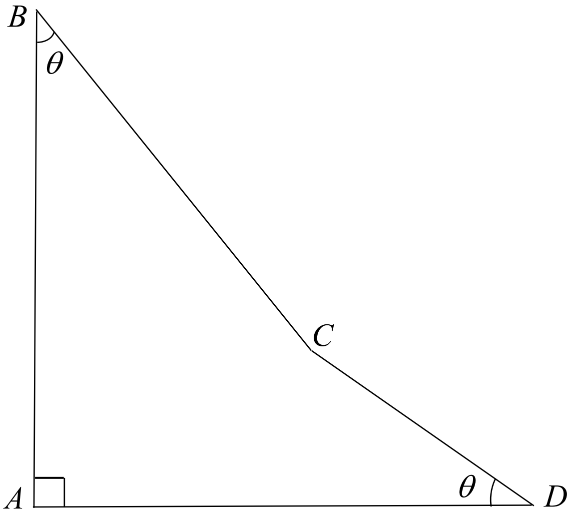
(a) Express h in terms of r. [1]

(b) The manufacturer bought metal sheets at \$60 per square metre.
Show that the total cost, \$$C$, of the metal sheets required to make the container is given by

$$C = 120\pi r^2 + \frac{36000}{r}.$$ [2]

(c) Given that r can vary, find the stationary value of C and determine its nature. [6]

11.



The diagram shows a figure *ABCD*, in which $BC = 13$ m, $CD = 6$ m, angle $BAD = 90^{\circ}$ and angle $ADC =$ angle $ABC = \theta$.

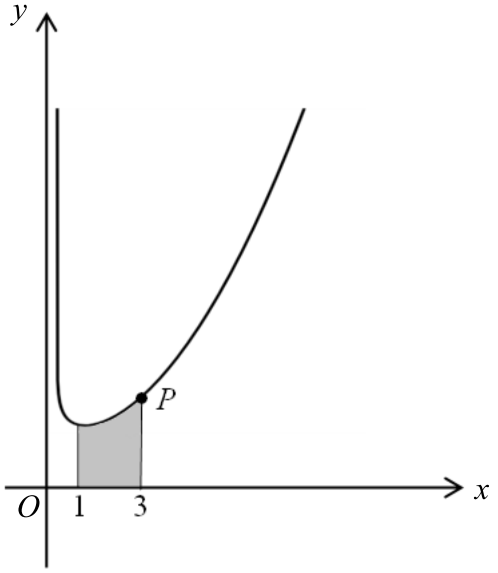
(a) Show that the perimeter, *L* metres, of *ABCD* is given by

$$L = 19 + 19\cos\theta + 19\sin\theta.$$ [2]

(b) By expressing L in the form of $19+R\cos(\theta-\alpha)$, where $R>0$ and α is acute, find the maximum possible perimeter of $ABCD$. [4]

(c) Find the values of θ for which $L = 45$ m. [3]

12. The diagram shows part of the curve of $y = \frac{1}{4}x^2 - \frac{1}{2}\ln\left(\frac{x}{3}\right)$ and $P(3,\ 2.25)$ is a point on the curve.



(a) By using calculus, determine, with full working, whether $y = \frac{4}{3}x - 1$ is the equation of the tangent to the curve at *P*. [5]

(b) Differentiate $x\ln\left(\frac{x}{3}\right)-x$ with respect to x. [2]

(c) Using the result from part **(b)**, find the exact area of the region enclosed by the curve $y=\frac{1}{4}x^2-\frac{1}{2}\ln\left(\frac{x}{3}\right)$, the x-axis, the line $x = 1$ and the line $x = 3$. [3]

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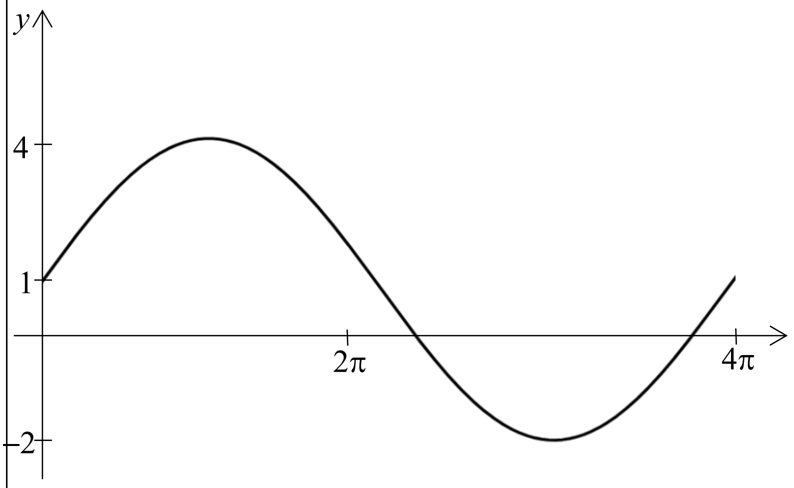
2025 A Math Prelim Paper 1 Solutions

1. (a) **(b)**	$V=\frac{1}{3}\pi r^2\left(\frac{3}{4}r\right)$ $=\frac{\pi}{4}r^3$ $\frac{\mathrm{d}v}{\mathrm{d}r}=\frac{3\pi}{4}r^2$ $\frac{\mathrm{d}v}{\mathrm{d}t}=\frac{\mathrm{d}v}{\mathrm{d}r}\times\frac{\mathrm{d}r}{\mathrm{d}t}$ $16=\frac{3\pi}{4}(8)^2\times\frac{\mathrm{d}r}{\mathrm{d}t}$ $\frac{\mathrm{d}r}{\mathrm{d}t}=\frac{1}{3\pi}=0.106\text{ cm/s}$
2.	$y=1-2x\ \ -----(1)$ $x^2-xy-y^2+5=0\ -----(2)$ Sub (1) into (2) $x^2-x(1-2x)-(1-2x)^2+5=0$ $x^2-x+2x^2-1+4x-4x^2+5=0$ $-x^2+3x+4=0$ $x^2-3x-4=0$ $(x-4)(x+1)=0$ $x=4\quad\text{or}\quad-1$ $y=-7\quad\text{or}\quad 3$ Length = $\sqrt{(4+1)^2+(-7-3)^2}$ $=\sqrt{125}$ $=5\sqrt{5}$ units

3(a)	$\frac{\mathrm{d}}{\mathrm{d}x}(2x-3)e^{2x}=2e^{2x}+(2x-3)2e^{2x}$ $=2e^{2x}(1+2x-3)$ $=4e^{2x}(x-1)$
(b)	$\int 4e^{2x}(x-1)\,\mathrm{d}x=(2x-3)e^{2x}+c$ $\int 4xe^{2x}\,\mathrm{d}x-\int 4e^{2x}\,\mathrm{d}x=(2x-3)e^{2x}+c$ $\int 4xe^{2x}\,\mathrm{d}x=(2x-3)e^{2x}+\int 4e^{2x}\,\mathrm{d}x+c$ $\int 4xe^{2x}\,\mathrm{d}x=(2x-3)e^{2x}+2e^{2x}+c$ $\int 4xe^{2x}\,\mathrm{d}x=(2x-1)e^{2x}+c$ $\int_0^1 4xe^{2x}\,\mathrm{d}x=\left[(2x-1)e^{2x}\right]_0^1$ $=e^2-(-1)$ $=e^2+1$
4(a)	$\angle RSU=\angle RUS\ (RS=RU)$ $=\angle URQ$ (alt seg thm) $\angle URQ=\angle RUQ$ $\therefore \Delta RQU$ is isosceles
(b)	$\angle RQS=2\angle QRU$ (ext $\angle$ of a triangle, ΔRQU is isosceles) $\angle RQS=\angle SPT$ (angles in the same seg) $\angle QRU=\angle QPT$ (alt seg theorem) $\therefore \angle SPT=2\angle QPT$

5(a)	$2x^2-8x+3=1-4x$ $2x^2-4x+2=0$ $2(x-1)^2=0$ $x=1$ Since there is only one intersection point between the curve and the line, the line is a tangent to the curve. Or Discriminant = $(-4)^2-4(2)(2)=0$ $\therefore$ line is a tangent to the curve Coordinates = $(1,\ -3)$
(b)	$(m+1)x^2-8x+3m>5$ $(m+1)x^2-8x+3m-5>0$ $(-8)^2-4(m+1)(3m-5)<0$ and $m+1>0$ $64-12m^2+8m+20<0$ $\qquad\qquad m>-1$ $12m^2-8m-84>0$ $3m^2-2m-21>0$ $(3m+7)(m-3)>0$ $m<-\frac{7}{3}$ or $m>3$ $\therefore\ m>3$
6(a)	$\begin{array}{rl} & \qquad x\ \ -2 \\ 2x^2+1\big) & \overline{2x^3-4x^2+x-2} \\ & \underline{-(2x^3 \qquad +x)} \\ & \qquad -4x^2 \qquad -2 \\ & \qquad \underline{-(-4x^2 \qquad -2)} \\ & \qquad\qquad\quad 0 \end{array}$ $\therefore 2x^2+1$ is a factor since remainder = 0
(b)	$\frac{11x-5x^2-11}{(x-2)(2x^2+1)}=\frac{A}{x-2}+\frac{Bx+C}{2x^2+1}$ $11x-5x^2-11=A(2x^2+1)+(Bx+C)(x-2)$ Sub $x=2$, $-9=9A \Rightarrow A=-1$ Compare x^2, $-5=2A+B \Rightarrow B=-3$ Compare constants, $-11=A-2C \Rightarrow C=5$

<table>
<tr><td></td><td>$\therefore \frac{11x-5x^2-11}{(x-2)(2x^2+1)}=-\frac{1}{x-2}+\frac{5-3x}{2x^2+1}$</td></tr>
<tr><td>7(a)</td><td>Amplitude =3
Period = 4π or 720°</td></tr>
<tr><td>(b)</td><td>



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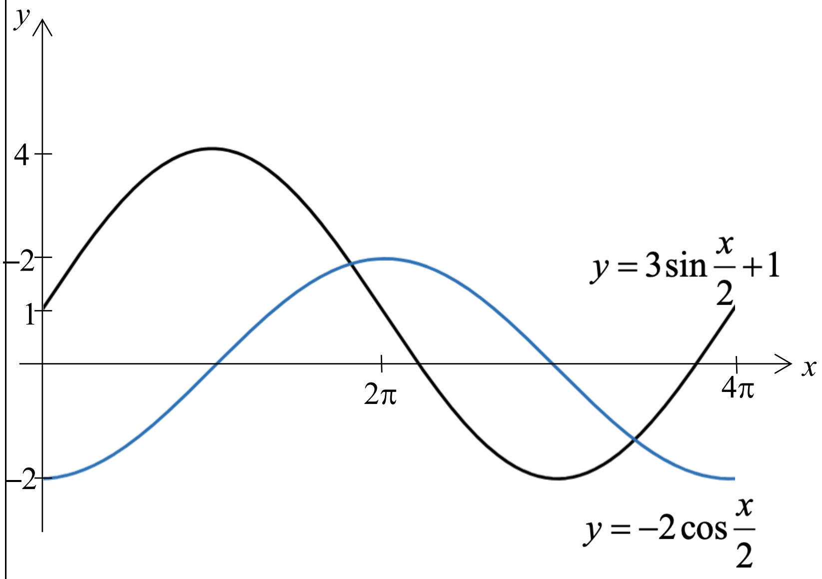
<tr><td>(c)</td><td>



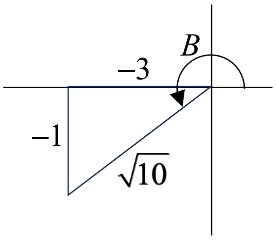
$3\sin\frac{x}{2}+1=-2\cos\frac{x}{2}$

Sketch $y=-2\cos\frac{x}{2}$

Number of solutions = 2

</td></tr>
</table>

8(a)	Centre of C_1 = (2, y) Grad of normal = $\frac{3}{4}$ $\frac{y-6}{2-6}=\frac{3}{4}$ $y=\frac{3}{4}(-4)+6$ $=3$ $\therefore$ Centre of C_1 = (2, 3) Radius = 5 units Equation of C_1: $(x-2)^2+(y-3)^2=25$
(b)	Let the centre C_2 = (x, y) $\left(\frac{x+2}{2},\frac{y+3}{2}\right)=(6,6)$ $x=10,\ y=9$ Equation of C_2: $(x-10)^2+(y-9)^2=25$
9(a)	$\sin 105^\circ=\sin\left(45^\circ+60^\circ\right)$ $=\sin 45^\circ\cos 60^\circ+\cos 45^\circ\sin 60^\circ$ $=\frac{1}{\sqrt{2}}\left(\frac{1}{2}\right)+\left(\frac{1}{\sqrt{2}}\right)\left(\frac{\sqrt{3}}{2}\right)$ $=\frac{1+\sqrt{3}}{2\sqrt{2}}$ $=\frac{1}{4}\left(\sqrt{2}+\sqrt{6}\right)$
(b)	$\frac{\cos(A+B)}{\cos(A-B)}=\frac{2}{7}$ $\frac{\cos A\cos B-\sin A\sin B}{\cos A\cos B+\sin A\sin B}=\frac{2}{7}$ $7\cos A\cos B-7\sin A\sin B=2\cos A\cos B+2\sin A\sin B$ $\frac{5\cos A\cos B}{9\sin A\sin B}=1$ $\cot A\cot B=\frac{9}{5}$

(c)	$\tan B = \frac{1}{3}$  $\cos 2B = 2\cos^2 B - 1$ $= 2\left(-\frac{3}{\sqrt{10}}\right)^2 - 1$ $= \frac{4}{5}$ or $\cos 2B = 1 - 2\sin^2 B$ $= 1 - 2\left(-\frac{1}{\sqrt{10}}\right)^2$ $= \frac{4}{5}$ or $\cos 2B = \cos^2 B - \sin^2 B$ $= \left(-\frac{3}{\sqrt{10}}\right)^2 - \left(-\frac{1}{\sqrt{10}}\right)^2$ $= \frac{4}{5}$
10(a)	$V = \pi r^2 h$ $300 = \pi r^2 h$ $h = \frac{300}{\pi r^2}$
(b)	$A = 2\pi r^2 + 2\pi r\left(\frac{300}{\pi r^2}\right)$ $= 2\pi r^2 + \frac{600}{r}$ $C = 60\left(2\pi r^2\right) + 60\left(\frac{600}{r}\right)$ $= 120\pi r^2 + \frac{36000}{r}$ (Shown)

(c)

$$C = 120\pi r^2 + \frac{36000}{r}$$

$$\frac{\mathrm{d}C}{\mathrm{d}r} = 240\pi r - \frac{36000}{r^2}$$

$$\frac{\mathrm{d}C}{\mathrm{d}r} = 0$$

$$\Rightarrow \quad 240\pi r - \frac{36000}{r^2} = 0$$

$$\Rightarrow \frac{36000}{r^2} = 240\pi r$$

$$\Rightarrow r^3 = \frac{36000}{240\pi}$$

$$\therefore r = 3.6278 = 3.63$$

$$\frac{\mathrm{d}C}{\mathrm{d}r} = 240\pi r - \frac{36000}{r^2}$$

$$\frac{\mathrm{d}^2C}{\mathrm{d}r^2} = 240\pi + \frac{72000}{r^3}$$

When $r = 3.6278$,

$$C = 14884.9 = 14900$$

$$\frac{\mathrm{d}^2C}{\mathrm{d}r^2} > 0 \Rightarrow \text{Minimum cost}$$

or

When $r = 3.6278$,

$$C = 14884.9 = 14900$$

x	3.63^{-}	3.63	3.63^{+}
$\frac{\mathrm{d}C}{\mathrm{d}r}$	$-$	0	$+$
slope	╲	—	╱

$\therefore$ minimum cost

11(a)	$L = 13 + 6 + AD + AB$ $= 19 + 6\cos\theta + 13\cos\theta + 6\sin\theta + 13\sin\theta$ $= 19 + 19\cos\theta + 19\sin\theta$
(b)	$R = \sqrt{19^2 + 19^2} = \sqrt{722}$ $\tan^{-1}(\alpha) = 1$ $\alpha = 45^\circ$ $L = 19 + \sqrt{722}\cos(\theta - 45^\circ)$ Max $L = \sqrt{722} + 19 = 45.9\,\text{m}$
(c)	$\sqrt{722}\cos(\theta - 45^\circ) + 19 = 45$ $\cos(\theta - 45^\circ) = \dfrac{26}{\sqrt{722}}$ Basic angle = $\cos^{-1}\left(\dfrac{26}{\sqrt{722}}\right)$ = 14.620° $\theta - 45^\circ = -14.620^\circ,\ 14.620^\circ$ $\therefore \theta = 30.4^\circ,\ 59.6^\circ$
12(a)	$y = \dfrac{1}{4}x^2 - \dfrac{1}{2}\ln\left(\dfrac{x}{3}\right)$ $\dfrac{\mathrm{d}y}{\mathrm{d}x} = \dfrac{x}{2} - \dfrac{1}{2}\left(\dfrac{1}{3}\right)\left(\dfrac{3}{x}\right)$ $= \dfrac{x}{2} - \dfrac{1}{2x}$ When $x = 3$, $\dfrac{\mathrm{d}y}{\mathrm{d}x} = \dfrac{4}{3}$ $y - 2.25 = \dfrac{4}{3}(x - 3)$ $y = \dfrac{4}{3}x - \dfrac{7}{4}$ Hence $y = \dfrac{4}{3}x - 1$ is not a tangent to the curve at P.
(b)	$\dfrac{\mathrm{d}}{\mathrm{d}x}\left[x\ln\dfrac{x}{3} - x\right] = x\left(\dfrac{3}{x}\right)\left(\dfrac{1}{3}\right) + \ln\left(\dfrac{x}{3}\right) - 1$ $= \ln\left(\dfrac{x}{3}\right)$
(c)	Area = $\displaystyle\int_1^3 \left[\frac{1}{4}x^2 - \frac{1}{2}\ln\left(\frac{x}{3}\right)\right]\mathrm{d}x$

$$=\left[\frac{x^3}{12}\right]_1^3-\frac{1}{2}\left[x\ln\left(\frac{x}{3}\right)-x\right]_1^3$$

$$=\left[\frac{27}{12}-\frac{1}{12}\right]-\frac{1}{2}\left[\left(3\ln 1-3\right)-\left(\ln\frac{1}{3}-1\right)\right]$$

$$=\frac{13}{6}-\frac{1}{2}\left(-2-\ln\frac{1}{3}\right)$$

$$=\left(\frac{19}{6}+\frac{1}{2}\ln\frac{1}{3}\quad\text{or}\quad\frac{19}{6}-\frac{1}{2}\ln 3\ \ \text{or}\ \ \frac{19}{6}-\ln\sqrt{3}\ \ \text{or}\ \ \frac{19}{6}+\ln\sqrt{\frac{1}{3}}\right)$$

units^2

SINGAPORE CHINESE GIRLS' SCHOOL
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CANDIDATE NAME					
CLASS	4			REGISTER NUMBER	
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ADDITIONAL MATHEMATICS **4049/02**

Paper 2

Friday **29 August 2025** **2 hours 15 minutes**

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Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved electronic scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

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FOR EXAMINERS USE

Q1		**Q5**		**Q9**				
Q2		**Q6**		**Q10**				
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Q4		**Q8**					90	

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[Turn over

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the quadratic equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial Expansion

$$\left(a+b\right)^n = a^n + \binom{n}{1} a^{n-1} b + \binom{n}{2} a^{n-2} b^2 + \ldots + \binom{n}{r} a^{n-r} b^r + \ldots + b^n \ ,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!\left(n-r\right)!} = \frac{n\left(n-1\right)\ldots\left(n-r+1\right)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin (A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos (A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan (A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2 \sin^2 A$$

$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

Formulae for Δ ABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \tfrac{1}{2} ab \sin C$$

1. In an experiment, 5 million cells were stored in a container. It was observed that 40% of the cells are dying every minute. The number of cells remaining, N, after t minutes, is given by $N = Ae^{kt}$, where A and k are constants.

(a) Explain why $k = \ln\frac{3}{5}$. [2]

(b) Find the time taken when the number of cells decreases to 2000. [3]

2. **(a)** Use the substitution $u = 7^x$, solve the equation $7^{2x+2} - 4\left(7^{x+1}\right) - 5 = 0$ and show that the solution may be written in the form $\log_a b - 1$ where a and b are integers to be determined. [4]

(b) Solve the equation $\log_3(2x-1) - \log_9(x^2+2) = \log_{25} 5$. [5]

3. A curve has the equation $y = x^3 - 3x^2 + 3x - 7$.

(a) Find the coordinates of the stationary point on the curve. [3]

(b) Observe the changes in the sign of the gradient of the curve and determine the nature of this stationary point. [2]

4. A tank is firing rounds during a target practice. The height, *h* metres, of a projectile above the ground during flight is governed by the equation $h = 2 + \frac{4}{5}t - \frac{1}{250}t^2$, where *t* is the time in seconds for the projectile in flight.

(a) Express *h* in the form $a + b(t - c)^2$, where *a*, *b* and *c* are constants to be determined. [2]

(b) Hence state the maximum height attained by the projectile and the time at which this occurs. [2]

(c) Find the length of time for which the height of the projectile is at least 32 metres above the ground. [3]

5. **(a)** When the expression $x^3 + 2kx + 2$ is divided by $x + 2$, the remainder is 3 less than the remainder obtained when the expression is divided by $x - 1$.
Find the value of k. [3]

(b) Solve the equation $2x^3 - 5x^2 - 3x + 10 = 0$, expressing non-integer roots in exact form. [4]

6. **(a)** By considering the general term in the binomial expansion of $\left(x^4 - \frac{1}{kx^2}\right)^6$,

where k is a positive constant, explain why there are only even powers of x in this expansion. [3]

(b) Given that the term independent of x in the binomial expansion of $\left(x^4 - \frac{1}{kx^2}\right)^6$ is $\frac{5}{27}$,

show that k is 3. [2]

(c) Hence find the term independent of x in $\left(2-3x^6\right)\left(x^4-\dfrac{1}{kx^2}\right)^6$. [4]

7. A particle, P, travels in a straight line so that at time t seconds, its velocity, v m/s, is given by

$$v = \cos(\frac{\pi}{4}t + \frac{\pi}{6}).$$

The initial displacement of P from a fixed point O is $\frac{1}{\pi}$ m.

(a) Find the initial velocity of P, giving your answer in exact value. [1]

(b) Find the first two values of t when P comes to instantaneous rest. [3]

(c) Find the distance travelled by P in the first 4 seconds. [4]

8. **(a)** Prove that $\sin^3 x \sec^2 x + \sin x = \tan x \sec x$. [4]

(b) Solve the equation $\sin^3 x \sec^2 x + \sin x = 5$ for $0 \leq x \leq 360^\circ$. [4]

Continuation of working space for Question 8.

9. **(a)(i)** Find the equation of the normal to the curve $y = \dfrac{2x - 6}{x - 2}$ at the point where the curve meets the y-axis. [4]

(a)(ii) Determine, with explanation, whether y is an increasing or decreasing function for $x > 2$. [2]

(b) A curve is such that $\frac{\mathrm{d}^2 y}{\mathrm{d}x^2} = \left(3x+1\right)^2$ and the point Q (2, 36) lies on the curve. The gradient of the curve at Q is 45. Find the equation of the curve. [5]

10. The number, n, of a certain type of bacteria increases with time t minutes. Recorded values of n and t are shown in the table below.

t	2	4	6	8	10
n	3215	3446	3693	3959	4243

It is known that n and t are related by the formula $n = n_0\left(2\right)^{kt}$, where n_0 and k are constants.

(a) Explain how a straight line graph can be drawn to represent the formula and draw it for the given data. [4]

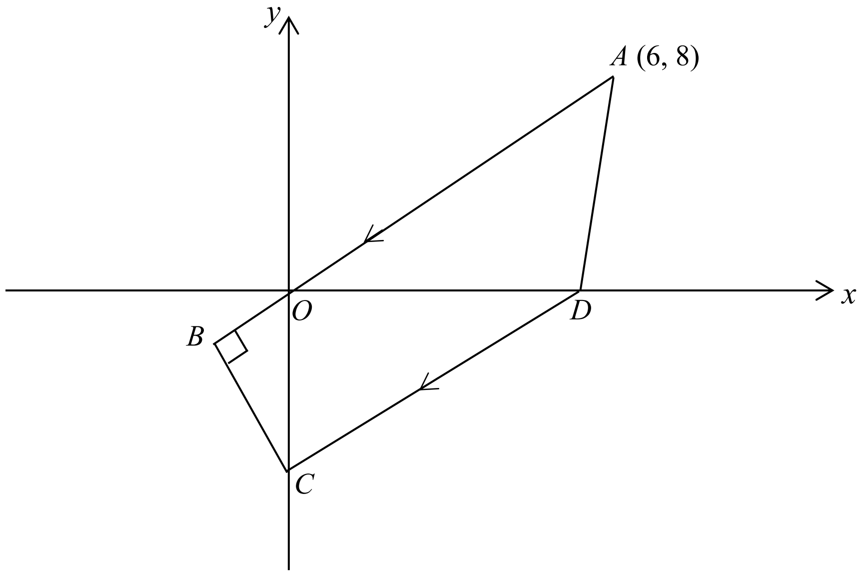
Use your graph to estimate

(b) the initial number of bacteria, [2]

(c) the value of k, [2]

(d) the time taken for the number of bacteria to increase by 25%. [2]

11



The diagram shows a trapezium with vertices $A(6, 8)$, B, C and D.
The sides AB and DC are parallel and angle ABC is $90°$.
AB passes through the origin O and the length of AB is 15 units.
The points C and D lie on the y-axis and x-axis respectively.

(a) Show that the coordinates of B are $(-3, -4)$. [4]

(b) Find the coordinates of *C* and of *D*. [5]

(c) Find the area of the trapezium *ABCD*. [2]

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2025 A Math Prelim Paper 2 Solutions

1(a)	$\frac{60}{100}A = Ae^k$ $e^k = \frac{3}{5}$ $k = \ln\frac{3}{5}$ *or* $3000000 = 5000000e^k$ $\ln 3000000 = \ln(5000000e^k)$ $\ln 3000000 = \ln 5000000 + \ln e^k$ $\ln 3000000 = \ln 5000000 + k$ $k = \ln 3000000 - \ln 5000000 = \ln\frac{3}{5}$
(b)	$2000 = 5000000e^{\left(\ln\frac{3}{5}\right)t}$ $\left(\ln\frac{3}{5}\right)t = \ln\frac{2000}{5000000}$ $t = \frac{\ln\left(\frac{1}{2500}\right)}{\ln\left(\frac{3}{5}\right)}$ $t = 15.3$ min
2(a)	$49u^2 - 28u - 5 = 0$ $(7u-5)(7u+1) = 0$ $u = \frac{5}{7}$ or $u = -\frac{1}{7}$ $7^x = \frac{5}{7}$ or $7^x = -\frac{1}{7}$ (rej) $x = \log_7\frac{5}{7}$ $= \log_7 5 - \log_7 7$ $= \log_7 5 - 1$ Or

<table>
<tr><td></td><td>$7^x = \frac{5}{7}$
$x\lg 7 = \lg\frac{5}{7}$
$x = \frac{\lg\left(\frac{5}{7}\right)}{\lg 7} = \log_7\frac{5}{7} \quad [\log_a b = \frac{\log_c b}{\log_c a}]$
$Note: \frac{\lg A}{\lg B} \neq \lg(A-B)$</td></tr>
<tr><td>2 (b)</td><td>$\log_3(2x-1) - \log_9(x^2+2) = \log_{25}5$
$\log_3(2x-1) - \frac{\log_3(x^2+2)}{\log_3 9} = \frac{1}{2}$
$2\log_3(2x-1) - \log_3(x^2+2) = 1$
$\log_3\frac{(2x-1)^2}{x^2+2} = \log_3 3$
$\frac{(2x-1)^2}{x^2+2} = 3$
$4x^2 - 4x + 1 = 3x^2 + 6$
$x^2 - 4x - 5 = 0$
$(x-5)(x+1) = 0$
$x = 5$ or $x = -1$ (reject)</td></tr>
</table>

<table>
<tr><td>3(a)</td><td>$\frac{dy}{dx} = 3x^2 - 6x + 3$
$\frac{dy}{dx} = 0$
$\Rightarrow 3x^2 - 6x + 3 = 0$
$\Rightarrow 3(x-1)^2 = 0$
$x = 1$, $y = -6$
Coordinates of the stationary point are $(1, -6)$.</td></tr>
<tr><td>(b)</td><td><table>
<tr><td>x</td><td>1^-</td><td>1</td><td>1^+</td></tr>
<tr><td>$\frac{dy}{dx}$</td><td>+</td><td>0</td><td>+</td></tr>
<tr><td></td><td>/</td><td>—</td><td>/</td></tr>
</table></td></tr>
</table>

	$(1,-6)$ is a point of inflexion.
4(a)	$h=2+\frac{4}{5}t-\frac{1}{250}t^2$ $=-\frac{1}{250}\left(t^2-200t\right)+2$ $=-\frac{1}{250}\left[\left(t-100\right)^2-100^2\right]+2$ $=-\frac{1}{250}\left(t-100\right)^2+40+2$ $=42-\frac{1}{250}\left(t-100\right)^2$
(b)	Max height = 42 m Time = 100 s
(c)	$42-\frac{1}{250}\left(t-100\right)^2=32$ $\frac{1}{250}\left(t-100\right)^2=10$ $\left(t-100\right)^2=2500$ $t=100\pm50$ t = 50 or 150 $\therefore$ length of time = 100s Or $42-\frac{1}{250}\left(t-100\right)^2\geq32$ $\left(t-100\right)^2\leq2500$ $\left[\left(t-100\right)^2-50^2\right]\leq0$ $\left[\left(t-100\right)+50\right]\left[\left(t-100\right)-50\right]\leq0$ $(t-50)(t-150)\leq0$ $50\leq t\leq150$ Duration=100s

5**(a)**	$\mathrm{f}(x)=x^3+2kx+2$ $\mathrm{f}(-2)=(-2)^3+2k(-2)+2$ $=-6-4k$ $\mathrm{f}(1)=(1)^3+2k(1)+2$ $=2k+3$ $-6-4k=2k+3-3$ $6k=-6$ $k=-1$
(b)	$\mathrm{f}(x)=2x^3-5x^2-3x+10$ $\mathrm{f}(2)=2(2)^3-5(2)^2-3(2)+10$ $=0$ $\therefore$ $(x-2)$ is a factor $2x^3-5x^2-3x+10=(x-2)(2x^2+bx-5)$ Compare x^2, $-5=-4+b$ $b=-1$ $\mathrm{f}(x)=(x-2)(2x^2-x-5)$ When f (x) = 0. $x-2=0$ or $2x^2-x-5=0$ $x=2$ or $x=\dfrac{1\pm\sqrt{1-4(2)(-5)}}{4}$ $=\dfrac{1\pm\sqrt{41}}{4}$

6(a)	$T_{r+1} = \binom{6}{r}\left(x^4\right)^{6-r}\left(-\frac{1}{kx^2}\right)^r$ $= \binom{6}{r}\left(-\frac{1}{k}\right)^r \left(x^{24-4r}\right)x^{-2r}$ $= \binom{6}{r}\left(-\frac{1}{k}\right)^r x^{24-6r}$ Powers of $x = 24 - 6r$ Since 24 and $6r$ are even numbers, subtraction between even numbers will give an even number. $\therefore$ there are only even powers of x in this expansion.
(b)	$24 - 6r = 0$ $r = 4$ $\binom{6}{4}\left(-\frac{1}{k}\right)^4 = \frac{5}{27}$ $\left(-\frac{1}{k}\right)^4 = \frac{1}{81}$ $k = 3$
(c)	$24 - 6r = -6$ $r = 5$ Coefficient of $x^{-6} = \binom{6}{5}\left(-\frac{1}{3}\right)^5 = -\frac{6}{243} = -\frac{2}{81}$ For the expansion $\left(2 - 3x^6\right)\left(x^4 - \frac{1}{kx^2}\right)^6$ Term independent of $x = 2\left(\frac{5}{27}\right) - 3\left(-\frac{6}{243}\right)$ $= \frac{10}{27} + \frac{2}{27}$ $= \frac{12}{27}$ $= \frac{4}{9}$

7(a)	When $t = 0$, $v = \cos\frac{\pi}{6} = \frac{\sqrt{3}}{2}$ m/s
(b)	When $v = 0$, $\cos(\frac{\pi}{4}t + \frac{\pi}{6}) = 0$ $\frac{\pi}{4}t + \frac{\pi}{6} = \frac{\pi}{2}$ or $\frac{\pi}{4}t + \frac{\pi}{6} = \frac{3\pi}{2}$ $t = \frac{4}{3}$ or $\frac{16}{3} = 1.33$ or 5.33 $\cos(\frac{\pi}{4}t + \frac{\pi}{6}) = 0$ $\frac{\pi}{4}t + \frac{\pi}{6} = \frac{\pi}{2}$ or $\frac{\pi}{4}t + \frac{\pi}{6} = \frac{3\pi}{2}$ $t = \frac{4}{3}$ or $\frac{16}{3} = 1.33$ or 5.33
(c)	$s = \int \cos(\frac{\pi}{4}t + \frac{\pi}{6})\ \mathrm{d}t$ $s = \frac{4}{\pi}\sin(\frac{\pi}{4}t + \frac{\pi}{6}) + c$ When $t = 0,\ s = \frac{1}{\pi}$, $\frac{1}{\pi} = \frac{4}{\pi}\left(\sin\frac{\pi}{6}\right) + c$ $\frac{1}{\pi} = \frac{4}{\pi}\left(\frac{1}{2}\right) + c$ $c = \frac{1}{\pi} - \frac{2}{\pi} = -\frac{1}{\pi}$ $\therefore s = \frac{4}{\pi}\left[\sin(\frac{\pi}{4}t + \frac{\pi}{6})\right] - \frac{1}{\pi}$ When $t = 0$ $\quad s = \frac{1}{\pi}$ m = 0.31830 m When $t = \frac{4}{3}$ $\quad s = \frac{4}{\pi}\left(\sin\frac{\pi}{2}\right) - \frac{1}{\pi} = \frac{3}{\pi}$ m or 0.95492 m When $t = 4$ $\quad s = \frac{4}{\pi}\left(\sin\frac{7\pi}{6}\right) - \frac{1}{\pi} = -\frac{3}{\pi}$ or -0.95492 m Total distance travelled = $\frac{2}{\pi} + \frac{3}{\pi} + \frac{3}{\pi} = \frac{8}{\pi}$ or 2.55 m

8(a)	$\text{LHS} = \sin^3 x \sec^2 x + \sin x$ $= \sin^3 x\left(\frac{1}{\cos^2 x}\right) + \sin x$ $= \tan^2 x \sin x + \sin x$ $= \sin x\left(\tan^2 x + 1\right)$ $= \sin x \sec^2 x$ $= \sin x\left(\frac{1}{\cos^2 x}\right)$ $= \tan x \sec x$ = RHS (shown)
(b)	$\tan x \sec x = 5$ $\frac{\sin x}{\cos x}\left(\frac{1}{\cos x}\right) = 5$ $\frac{\sin x}{\cos^2 x} = 5$ $\sin x = 5\left(1 - \sin^2 x\right)$ $5\sin^2 x + \sin x - 5 = 0$ $\sin x = \frac{-1 \pm \sqrt{(1)^2 - 4(5)(-5)}}{2(5)}$ $\sin x = 0.90498$ or -1.10498 (NA) Basic angle = 64.8215° $x = 64.8^\circ,\ 115.2^\circ$

9(a)(i)	$y=\frac{2x-6}{x-2}$ $\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{(x-2)(2)-(2x-6)(1)}{(x-2)^2}$ $=\frac{2}{(x-2)^2}$ When the curve meets the y-axis, $x = 0$ $\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{2}{(0-2)^2}=\frac{1}{2}$ Gradient of normal = -2 $x=0, y=3$ Equation of normal is $y-3=-2(x-0)$ $y=-2x+3$
9(a)(ii)	For $x>2,\ x-2>0,$ $\Rightarrow (x-2)^2>0$ Since $\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{2}{(x-2)^2}$>0 for x >2, y is an increasing function.

9b

$$\frac{d^2y}{dx^2} = (3x+1)^2$$

$$\frac{dy}{dx} = \int (3x+1)^2 \, dx$$

$$= \frac{(3x+1)^3}{(3)(3)} + C$$

$$= \frac{(3x+1)^3}{9} + C$$

$$\frac{dy}{dx} = 45, \ x = 2,$$

$$45 = \frac{343}{9} + C$$

$$C = \frac{62}{9}$$

$$\frac{dy}{dx} = \frac{(3x+1)^3}{9} + \frac{62}{9}$$

$$y = \int \left[\frac{(3x+1)^3}{9} + \frac{62}{9} \right] dx$$

$$y = \frac{(3x+1)^4}{108} + \frac{62}{9}x + D$$

$$36 = \frac{2401}{108} + \frac{124}{9} + D$$

$$D = -\frac{1}{108}$$

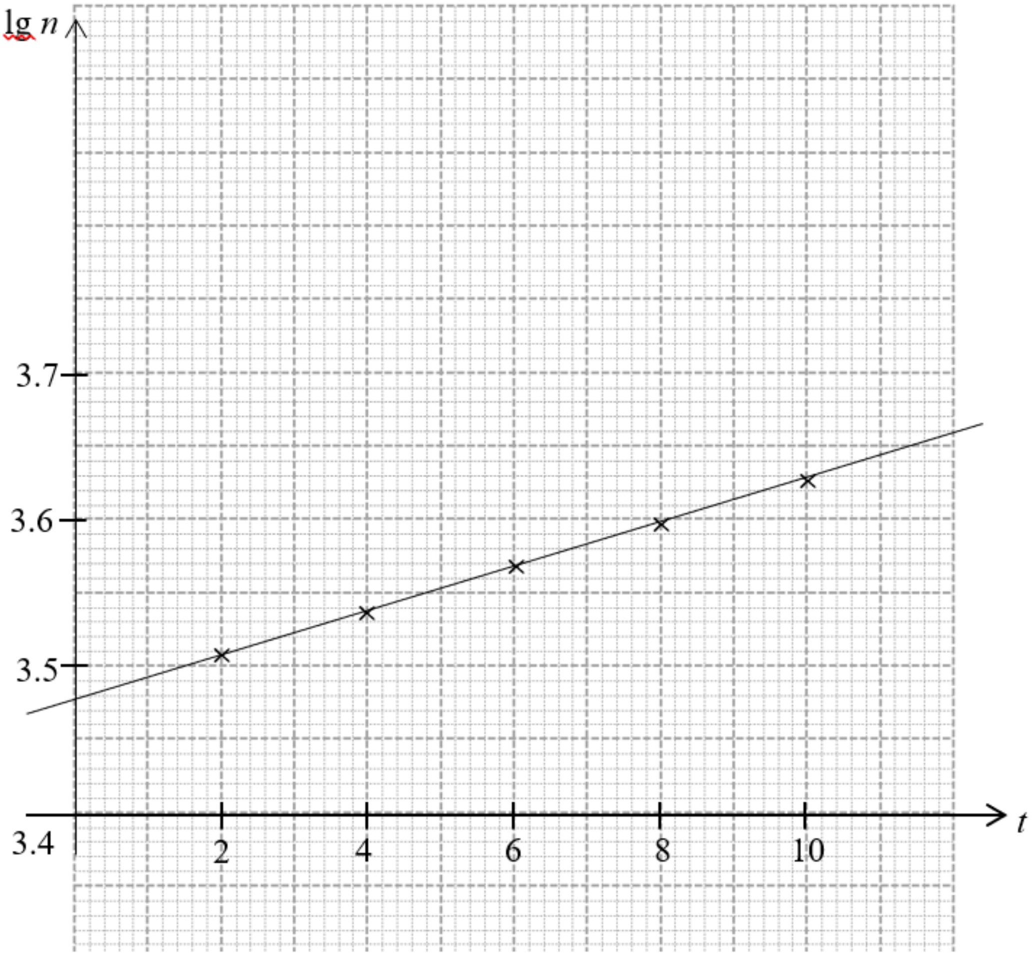
$$\therefore y = \frac{(3x+1)^4}{108} + \frac{62}{9}x - \frac{1}{108}$$

Solution 1

<table>
<tr>
<td>10(a)</td>
<td>
$n = n_0\left(2\right)^{kt}$
$\lg n = \lg n_0 + \left(k \lg 2\right) t$

Draw a straight line of $\lg n$ against <i>t</i>.
<table>
<tr><td><i>t</i></td><td>2</td><td>4</td><td>6</td><td>8</td><td>10</td></tr>
<tr><td>lg <i>n</i></td><td>3.51</td><td>3.54</td><td>3.57</td><td>3.60</td><td>3.63</td></tr>
</table>

Best fit line
</td>
</tr>
<tr>
<td>(b)</td>
<td>
$\lg n_o = 3.48$
$n_o = 10^{3.48}$
$= 3020$
</td>
</tr>
<tr>
<td>(c)</td>
<td>
$k \lg 2 = \frac{3.584 - 3.48}{7}$
$k = 0.0494$
</td>
</tr>
<tr>
<td>(d)</td>
<td>
$\lg\left[1.25\left(10^{3.48}\right)\right] = 3.58$
$t = 6.7$ (6.6 - 7.2)
</td>
</tr>
</table>


lg n
3.7
3.6
3.5
3.4
2
4
6
8
10
t

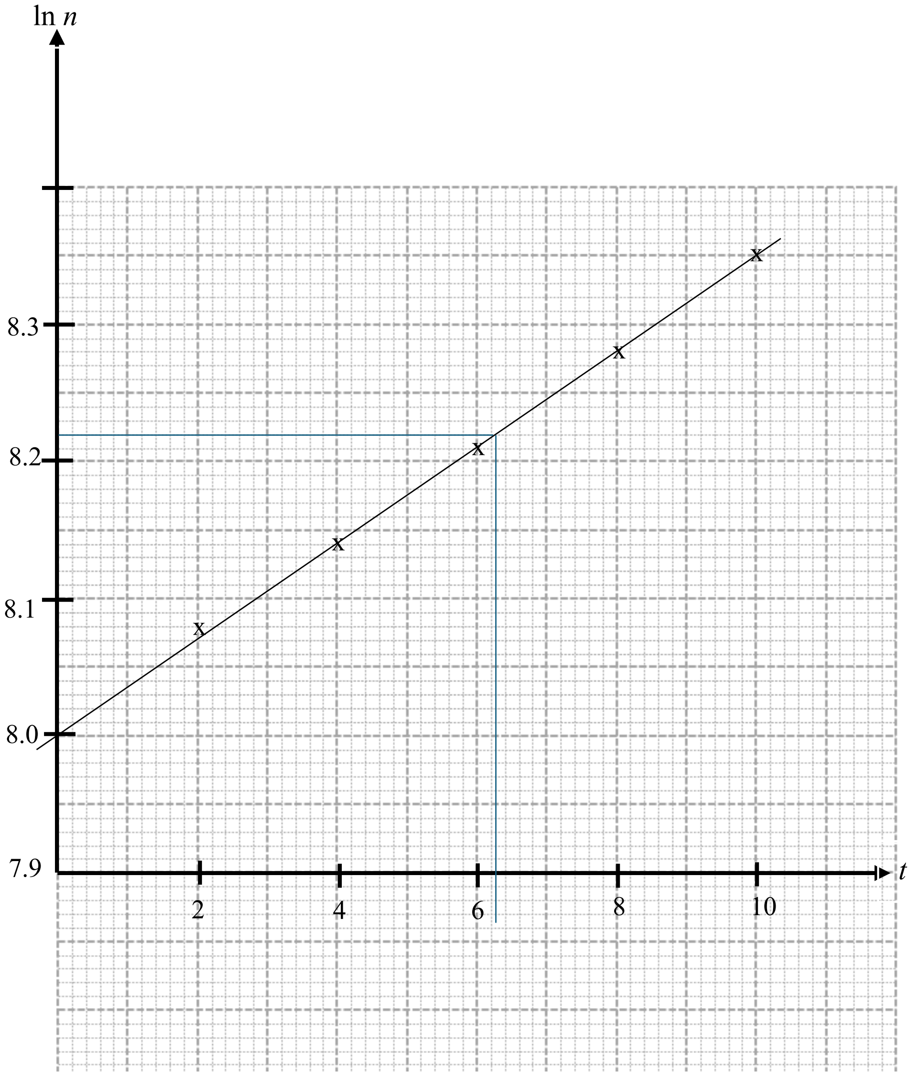
Solution 2

<table>
<tr><td>10(a)</td><td>$n = n_0 \left(2\right)^{kt}$
$\ln n = \ln(n_o) + \left(k \ln 2\right) t$

Draw a straight line of $\ln n$ against *t*.

<table><tr><td><i>t</i></td><td>2</td><td>4</td><td>6</td><td>8</td><td>10</td></tr><tr><td>ln <i>n</i></td><td>8.08</td><td>8.14</td><td>8.21</td><td>8.28</td><td>8.35</td></tr></table>
Best fit line</td></tr>
<tr><td>(b)</td><td>$\ln n_o = 8.00$
$n_o = e^8$
= 2980.957
= 2980</td></tr>
<tr><td>(c)

(d)</td><td>$k \ln 2 = \frac{8.35 - 8.0}{10}$
$k = 0.050494$
$= 0.0505$
$\ln\left[1.25\left(e^8\right)\right] = 8.223$
$t = 6.3$ (6.0 - 6.6)</td></tr>
</table>



ln n
8.3
8.2
8.1
8.0
7.9
2
4
6
8
10
t
x
x
x
x
x

11(a)	Let $B = (x, y)$ Distance $AB = \sqrt{(x-6)^2+(y-8)^2}=15$ $(x-6)^2+(y-8)^2=225$ ------ (1) Grad $AB = \frac{4}{3}$ Equation of AB: $y=\frac{4}{3}x$ ------ (2) Solving, $(x-6)^2+\left(\frac{4}{3}x-8\right)^2=225$ $\frac{25}{9}x^2-\frac{100}{3}x-125=0$ $x^2-12x-45=0$ $(x+3)(x-15)=0$ $x=-3$ or $x=15$ (rej) $y=-4$ $\therefore B=(-3, -4)$ Or Distance $OA=\sqrt{6^2+8^2}=10$ units Distance OB = 5 units Let $B = (x, y)$ By ratio theorem, $\frac{5(6)+10x}{15}=0, \quad \frac{5(8)+10y}{15}=0$ $\therefore B=(-3, -4)$
(b)	Grad $BC = -\frac{3}{4}$ $y=-\frac{3}{4}x+c$ $-4=-\frac{3}{4}(-3)+c \Rightarrow c=-\frac{25}{4}$ Coordinates of $C = \left(0,-\frac{25}{4}\right)$ Let $D = (x, 0)$ $\frac{\frac{25}{4}}{x}=\frac{4}{3}$ $x=\frac{75}{16}$ Coordinates of $D = \left(\frac{75}{16},0\right)$

(c)	Area $= \frac{1}{2}\begin{vmatrix} 6 & -3 & 0 & \frac{75}{16} & 6 \\ 8 & -4 & -\frac{25}{4} & 0 & 8 \end{vmatrix}$ $= \frac{1}{2}\left\lvert -24+\frac{75}{4}+\frac{75}{2}-\left(-24-\frac{1875}{64}\right)\right\rvert$ $= \frac{5475}{128}$ or $42\frac{99}{128}$ or 42.8 units^2 or Area$=\frac{1}{2}\left(AB+CD\right)\left(BC\right)$ $=\frac{1}{2}\left(15+\frac{125}{16}\right)\left(\frac{15}{4}\right)=\frac{5475}{128}$