

Visit
FREETESTPAPER.com
for more papers



Website: freetestpaper.com



[Facebook.com/freetestpaper](https://www.facebook.com/freetestpaper)



[Twitter.com/freetestpaper](https://www.twitter.com/freetestpaper)

Candidate Name _____ ()

Class: Sec 4 / _____



**Anglican High School
Preliminary Examination 2014
Secondary Four
Mathematics Paper 1
[4016 / 01]**

Date of Examination: 29 July 2014

Duration : 2 hours

READ THESE INSTRUCTIONS FIRST

Write your name, register number and class in the spaces at the top of this page.

Write in dark blue or black pen.

You may use a pencil for any diagrams or graphs.

Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** questions.

If working is needed for any question it must be shown with the answer.

Omission of essential working will result in loss of marks.

Calculators should be used where appropriate.

If degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.

For π , use either your calculator value or 3.142, unless the question requires the answer in terms of π .

The number of marks is given in brackets [] at the end of each question or part question.
The total of the marks for this paper is 80.

This document consists of 17 printed pages.

[Turn Over

Mathematical Formulae

Compound Interest

$$\text{Total amount} = P \left(1 + \frac{r}{100} \right)^n$$

Mensuration

$$\text{Curved surface area of a cone} = \pi r l$$

$$\text{Surface area of a sphere} = 4\pi r^2$$

$$\text{Volume of a cone} = \frac{1}{3} \pi r^2 h$$

$$\text{Volume of a sphere} = \frac{4}{3} \pi r^3$$

$$\text{Area of triangle } ABC = \frac{1}{2} ab \sin C$$

$$\text{Arc length} = r\theta, \text{ where } \theta \text{ is in radians}$$

$$\text{Sector area} = \frac{1}{2} r^2 \theta, \text{ where } \theta \text{ is in radians}$$

Trigonometry

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

Statistics

$$\text{Mean} = \frac{\sum fx}{\sum f}$$

$$\text{Standard deviation} = \sqrt{\frac{\sum fx^2}{\sum f} - \left(\frac{\sum fx}{\sum f} \right)^2}$$

Answer all the questions.

- 1 Evaluate $\frac{-(3.5)^2 + \frac{13}{15}}{\left[1\frac{1}{6} - (-4)\right]^3 - \sqrt[3]{2197}}$, giving your answer correct to 3 significant figures.

Answer [2]

- 2 From the set of numbers $\left\{-3, 0, 1.2, \sqrt{0.9}, \sqrt{2}, \sqrt{9}, \sqrt{0.64}, -\frac{1}{3}\right\}$, write down

(a) all the integers,

Answer (a) [1]

(b) all the rational numbers,

Answer (b) [1]

(c) the name of the type of numbers not listed in (a) and (b).

Answer (c) [1]

- 3 (a) A shopkeeper bought a digital camera for \$200. He then sold it at a profit of 35%. What was the selling price?

Answer (a) Selling price = \$..... [1]

(b) Mr Smith was able to buy the digital camera sold in (a), at a discount. If he paid y dollars for the camera, find, in terms of y , the percentage discount.

Answer (b)% [1]

- 4 (a) Express each of the numbers 14, 21 and 49 as a product of its prime factors.

Answer (a) [1]

.....

.....

- (b) Three train services operate from the same station. The first service leaves at 14-minute intervals, the second at 21-minute intervals and the third at 49-minute intervals. All three services leave the station together at 19 30 hours on Thursday. When will the three train services next leave the station together?

Answer (b) hours [2]

- 5 (a) A sum of money is divided in the ratio 2:4:5. Given that the difference between the largest share and the smallest share is \$72, find the sum of money.

Answer (a) \$..... [1]

- (b) Given that $A:B = \frac{1}{2}:\frac{1}{3}$ and B is twice of C . How many times is A bigger than C ?

Answer (b)times [2]

- 6 (a) Express $3.4 \times 10^7 - 5 \times 10^6$ in standard form.

Answer (a) [1]

- (b) The frequency of radio waves is measured in hertz, kilohertz and megahertz.
A radio station broadcasts on 92.4 megahertz.

- (i) Express 92.4 megahertz in hertz, leaving your answer in standard form.

Answer (b) (i) hertz [1]

- (ii) The wavelength, λ , of radio waves in metres, is calculated using the formula,

$$\lambda = \frac{3 \times 10^8}{f},$$

where f is the frequency in hertz.

Calculate the wavelength, in metres, that the radio station, which broadcasts on 92.4 megahertz, uses for its broadcast.

Answer (b) (ii) metres [1]

-
- 7 (a) Given that w is directly proportional to $\sqrt[3]{m}$. If $w = 4$ and $m = 125$, express w in terms of m .

Answer [1]

- (b) It is given that the force between two particles is inversely proportional to the square of the distance between them. If the force is F when the distance between them is D , and pF when the distance is $4D$, find the value of p .

Answer [2]

- 8 (a) Factorise $(10x + 17)^2 - 2(70x + 119) + 49$ completely.

Answer (a) [2]

- (b) Hence, without the use calculators, find the value of
 $(990 + 17)^2 - 2(6930 + 119) + 49$.

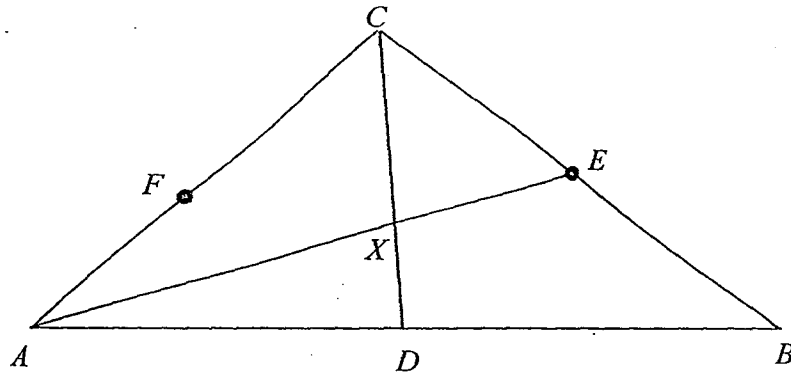
Answer (b) [1]

-
- 9 Express, as a single fraction, in its simplest form $\frac{5 - 7x - x^2}{-6x^2 + 11x - 4} - \frac{x - 1}{1 - 2x}$.

Answer [3]

7

In triangle ABC , D , E and F are midpoints of AB , BC and AC respectively. The lines AE and CD intersect at point X .



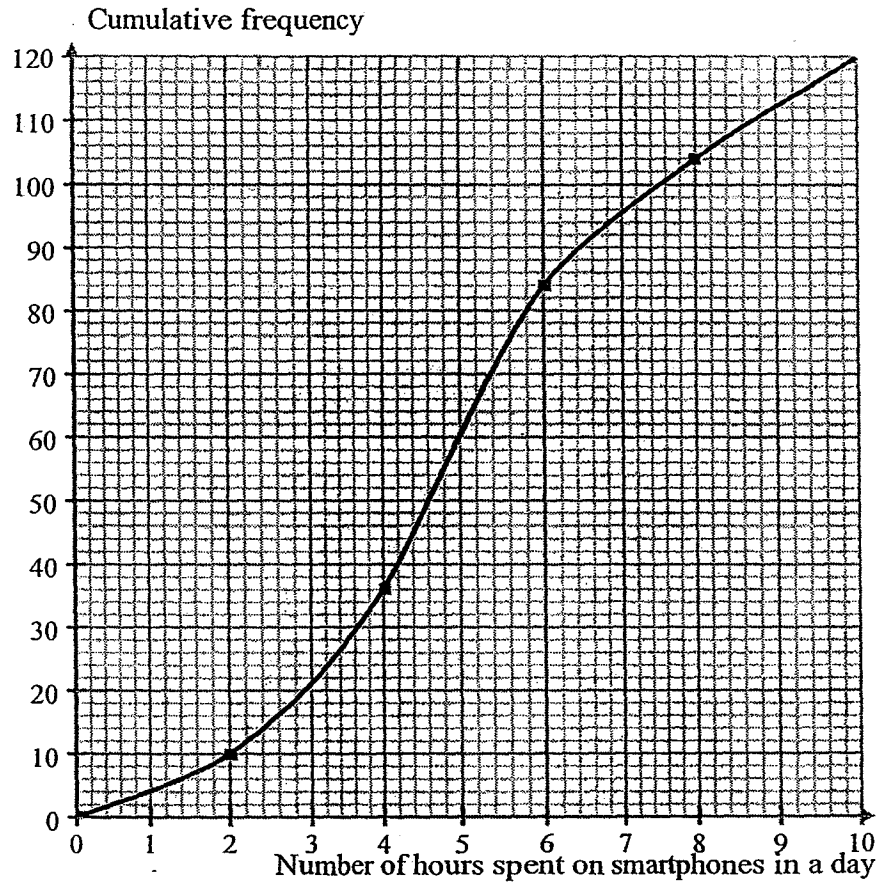
- (a) Find the ratio of area of $\triangle ABC$ to area of $\triangle AEC$. [1]
- (b) Given that $\overline{AB} = 5\mathbf{m}$, $\overline{BC} = 3\mathbf{n}$, express as simple as possible, in terms of $\mathbf{m}$ and/ or $\mathbf{n}$,
- (i) $\overline{AC}$, [1]
- (ii) $\overline{AE}$ and [1]
- (iii) $\overline{CD}$. [1]
- (c) Given that $\overline{CX} = \frac{2}{3}\overline{CD}$, express $\overline{BX}$ in terms of $\mathbf{m}$ and $\mathbf{n}$, in its simplest form. [2]
- (d) Obtain an expression for $\overline{XF}$. Hence prove that points B , X and F lie on the same straight line. [3]
- (e) It is given that C is a point $(-1, 7)$ and $\overline{CE} = \begin{pmatrix} 3 \\ -2 \end{pmatrix}$.
- (i) Calculate $|\overline{CE}|$. [1]
- (ii) Find the coordinates of E . [1]
- (iii) Find the equation of the line BC . [3]

[Turn Over]

8

A group of 120 smartphone users in Singapore were surveyed based on their usage.

The cumulative frequency curve below shows the number of hours spent by the users on their smartphones in a day.



- (a) Use the graph to find
- (i) the median number of hours spent, [1]
 - (ii) the interquartile range. [1]
- (b) Copy and complete the grouped frequency table of the number of hours spent on smartphones by each user.

Number of hours (x hours)	$0 \leq x < 2$	$2 \leq x < 4$	$4 \leq x < 6$	$6 \leq x < 8$	$8 \leq x < 10$
Frequency					

[2]

- 12 (a) Solve the inequality $\frac{x}{4} - \frac{3x-1}{2} > 5$.

Answer (a) [1]

- (b) Hence, write down the largest integer x such that $\frac{x}{4} - \frac{3x-1}{2} > 5$.

Answer (b) Largest integer $x =$ [1]

- 13 The matrices below shows the quantity of each item required to be placed in a Goodie Bag, and the number of participants on each day of a 2-day event.

Pen	$\begin{pmatrix} 4 \\ 1 \\ 2 \end{pmatrix}$
Drinking Water	
Notepad	

Mon	Tues
(14	23)

- (a) Find $\begin{pmatrix} 4 \\ 1 \\ 2 \end{pmatrix} \begin{pmatrix} 14 & 23 \end{pmatrix}$.

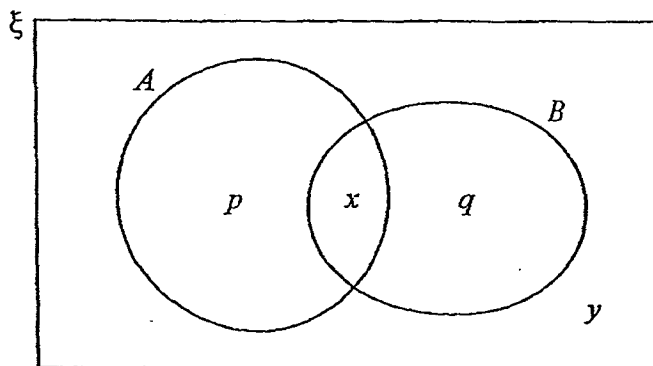
Answer (a)..... [1]

- (b) Explain what your answer to part (a) represents.

Answer (b)

[2]

14



In the Venn diagram, ξ is the set of all students from class 4A in Happy Valley School.

$A = \{\text{students going for the National Day Parade at the Padang}\}$ and

$B = \{\text{students going for the Youth Olympic Games Opening Ceremony}\}.$

The letters p, q, x and y in the diagram represent the number of students in each subset.

Given that $n(\xi) = 14$, $n(A) = 5$ and $n(B) = 8$,

(a) express p in terms of x ,

Answer (a) $p = \dots\dots\dots$ [1]

(b) find the largest possible value of y .

Answer (b) Largest possible value of $y = \dots\dots\dots$ [1]

15 The ordered stem and leaf chart shows the distribution of the heights, in cm, of 30 boys.

Stem	Leaf
14	8 9
15	1 2 2 3 4 8 8 9
16	3 5 6 7 7 9 9
17	0 1 1 3 4 8 9
18	1 2 2 x 4 4

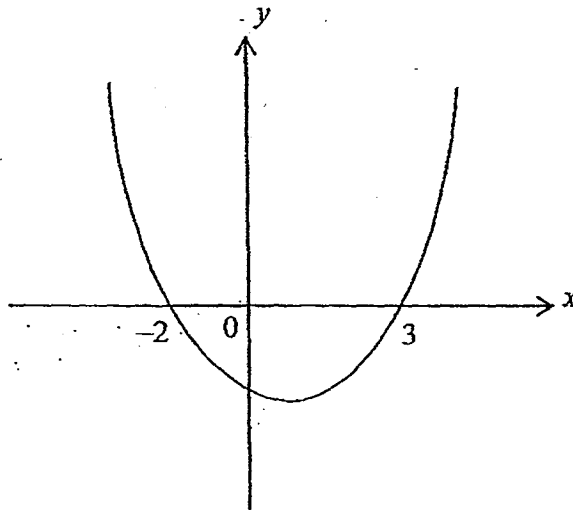
(a) What is the median of the distribution?

Answer (a) median = $\dots\dots\dots$ cm [1]

(b) State a possible value of x if the mode of the distribution is greater than 180.

Answer (b) $x = \dots\dots\dots$ [1]

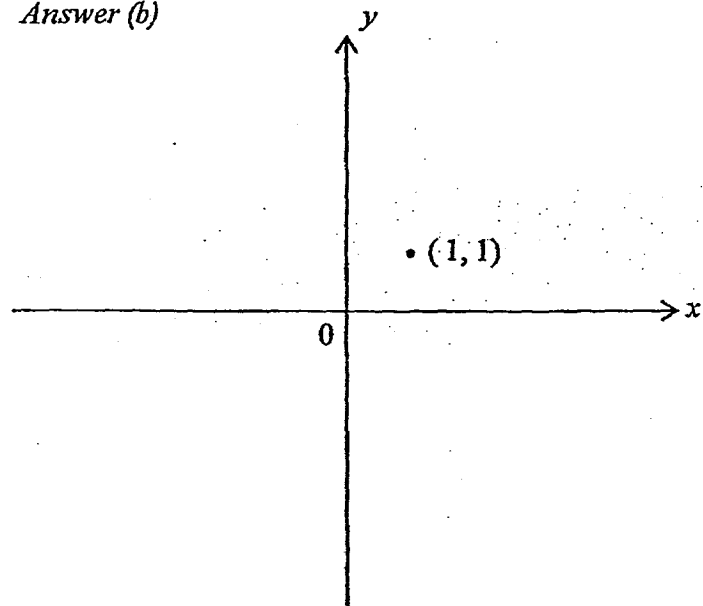
- 16 (a) The diagram shows a quadratic curve which can be expressed in the form $y = ax^2 + bx + c$.
Given that the curve cuts the x -axis at -2 and 3 , find the values of a , b and c .



Answer $a = \dots\dots\dots b = \dots\dots\dots c = \dots\dots\dots$ [3]

- (b) In the axes provided, sketch the graph of $y = 4 - (x+1)^2$, indicating all intercepts and turning point(s) clearly.

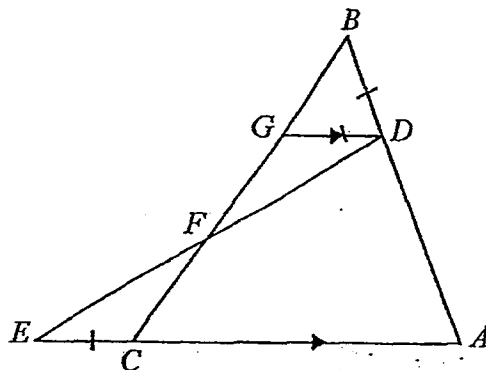
Answer (b)



[2]

17

The diagram shows an isosceles triangle ABC with $AB = AC$. D lies on AB , E lies on AC produced, and G lies on BC such that $BD = CE = DG$. The lines DE and BC intersect at F . DG is parallel to AE .



- (a) State the triangle that is similar to $\triangle ABC$.

Answer (a).....[1]

- (b) Prove that $\triangle DGF$ is congruent to $\triangle ECF$.

Answer (c)

 [2]

Given that $BC = 9$ cm, $CF = 3$ cm, and AB is 3 times of BD ,

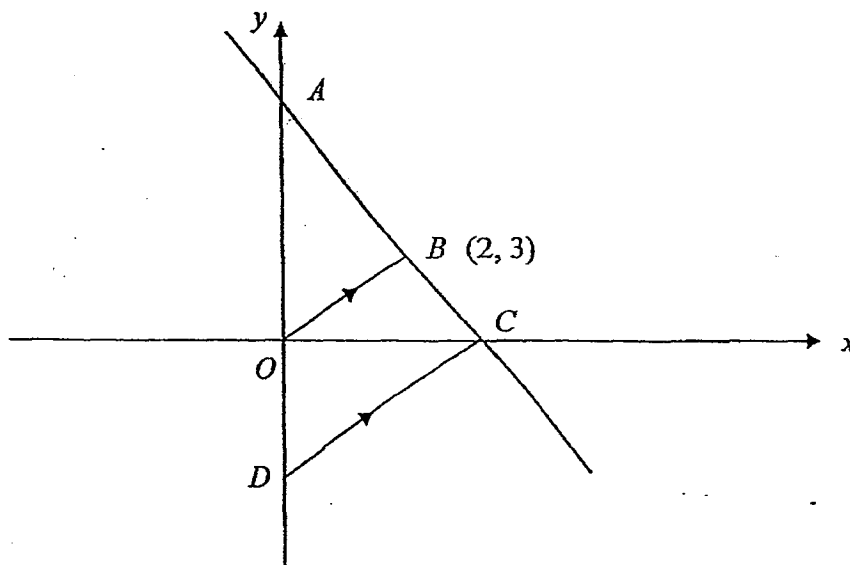
- (c) Find $\frac{\text{Area } \triangle BDG}{\text{Area } \triangle BAC}$.

Answer (c).....[1]

- (d) Find $\frac{\text{Area } \triangle CEF}{\text{Area } \triangle ABC}$.

Answer (d).....[2]

- 18 In the diagram, B has coordinates $(2, 3)$. Given that $5AB = 3AC$, and that OB is parallel to DC .



- (a) Show that the coordinates of C is $\left(3\frac{1}{3}, 0\right)$.

Answer (a) [2]

- (b) Hence, find
(i) the equation of line joining CD ,

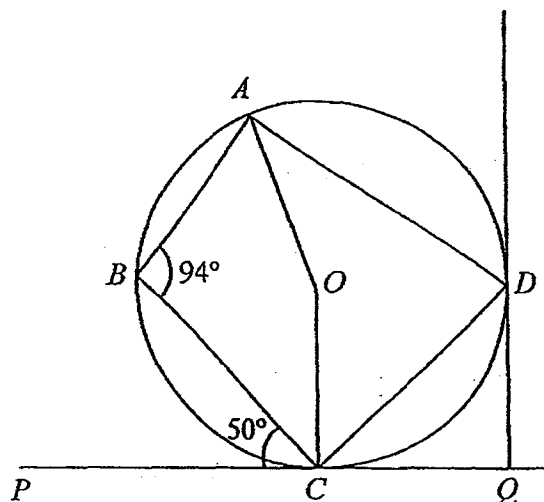
Answer (b)(i) [2]

- (ii) the length of AC .

Answer (b)(ii) $AC = \dots\dots\dots$ units [2]

19

In the diagram, O is the centre of a circle. PCQ is a tangent to the circle at C and DQ is a tangent to the circle at D . Angle $ABC = 94^\circ$ and angle $BCP = 50^\circ$.



- (a) Find
(i) reflex $\angle AOC$,

Answer (a)(i)° [1]

- (ii) $\angle BAO$.

Answer (a)(ii)° [2]

- (b) If the area of $\triangle CDQ = 8 \text{ cm}^2$, find CD^2 .

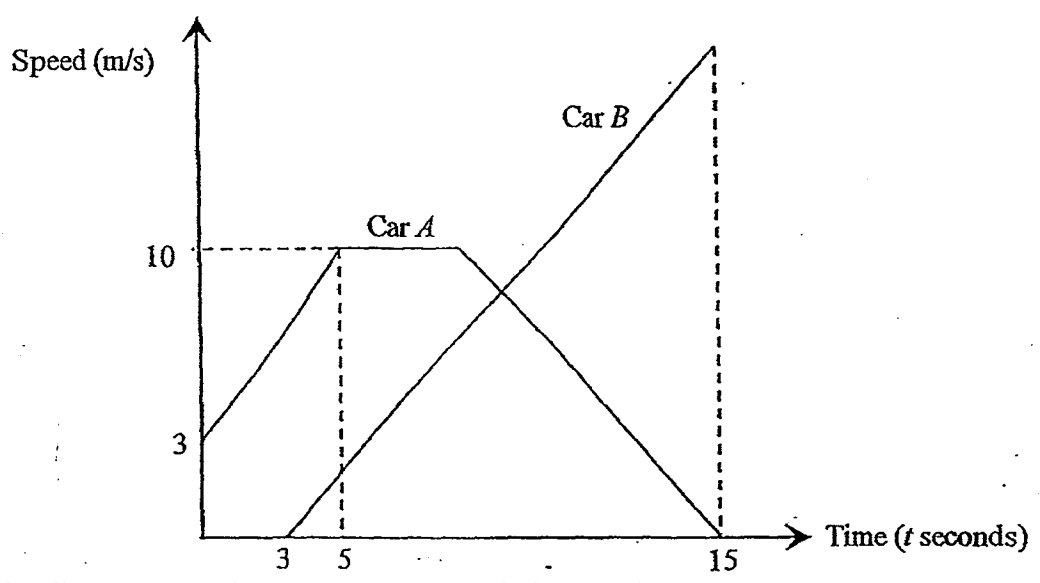
Answer (b) cm^2 [2]

For
Examiner's
Use

21

15

For
Examiner's
Use



The diagram above is the speed-time graph for cars A and B.

(a) Find the speed of Car A when $t = 3s$.

Answer (a)m/s [1]

Given that Car A travelled a total of 107 m,

(b) find the time that Car A travelled at a constant speed.

Answer (b)s [2]

Car B starts 3 s after Car A. Given that cars A and B meets when $t = 15s$,

(c) find the acceleration of car B, leaving your answer correct to 3 significant figures.

Answer (c)m/s² [2]

22

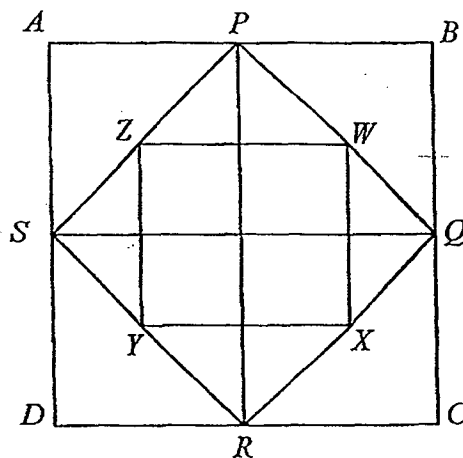
- (a) Two boxes were used to store pens. Box *A* contains 3 blue pens and 7 green pens. Box *B* contains 5 blue pens and 9 green pens. Julie took a pen from Box *A* and placed it inside Box *B*. Then, she took out a pen from Box *B*. Given that the selections are random, find the probability that the pen taken from Box *B* is green.

Answer [2]

- (b) *ABCD* is a square dartboard of sides 40 cm. *P*, *Q*, *R* and *S* are the midpoints of *AB*, *BC*, *CD* and *DA* respectively. *W*, *X*, *Y* and *Z* are midpoints of *PQ*, *QR*, *RS* and *SP* respectively.

The score for a dart landing on each region of the dartboard is shown below.

Region	Score
<i>WXYZ</i>	3
<i>PZW</i> or <i>RYX</i>	2
<i>SZY</i> or <i>QWX</i>	1
Other regions	0



- (i) A dart is thrown once. What is the probability of scoring 3 points?

Answer (b)(i) [1]

- (ii) Two darts are thrown one after another. What is the probability of scoring 3 points?

Answer (b)(ii) [2]

- 23 Mr Pica Chu is an artist who is famous for his abstract artwork. The following is part of his abstract artwork, which forms a circle. Use a ruler and compasses to complete the circle. Mark the centre of the circle with 'x' and measure its radius.

[3]



Answer Radius = cm [1]

End of Paper

Answer all the questions.

- 1 (a) Given that $\sqrt{\frac{a^2 - b}{b}} = \frac{2}{3}$, express b in terms of a . [2]
- (b) Express as a single fraction in its simplest form $\frac{4x}{4-5x} - \frac{2}{x+3}$. [3]
- (c) (i) Simplify $(p+q)(p-q) - p^2$. [1]
- (ii) Hence evaluate $10012019 \times 10012009 - 10012014^2$. [1]
- 2 (a) A company invested \$ x in a business venture and made a profit of 30% amounting to \$72,000. This profit is 20% more compared to the previous year's profit.
Calculate
(i) the value of x , [1]
(ii) the profit in the previous year. [2]
- (b) Mathew placed a \$20,000 deposit with Bank A with simple interest at 5% per annum over 3 years. Mandy placed \$20,000 with Bank B with compound interest at 4.9% per annum compounded monthly over 3 years. Calculate the individual interest earned by Mathew and Mandy after 3 years. [2]
- (c) The cash price of a 70 inches TV is \$7,000. Mr Tan bought the TV on hire purchase by putting a downpayment of \$1,000 and monthly instalments of \$150 over y months.
(i) Express the total amount payable by hire purchase, in terms of y .
Express, in terms of y , the amount Mr Tan could have saved if he had paid in full by cash. [2]
(ii) Given that the seller charged simple interest at 10% per annum, form an equation to calculate the value of y . [3]

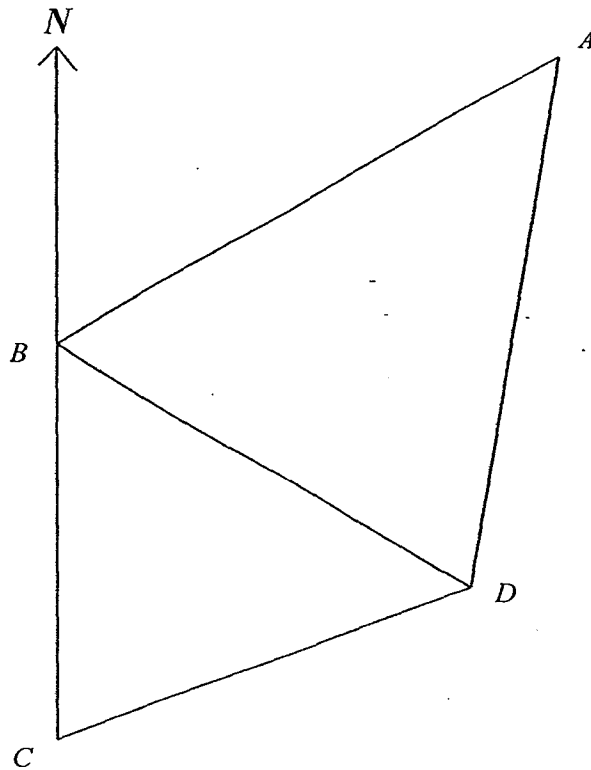
[Turn Over

- 3 (a) The scale of Map *A* is given as 1: 200 000. Find,
- (i) the actual distance, in km, of two points measuring 6 cm on the map, [1]
 - (ii) the area of a town on the map given that its actual size is 1.5 km^2 . [2]
- (b) Given that the scale of Map *B* is 1: 250 000 and the size of a reservoir on Map *B* is 3.6 cm^2 . Calculate the area of the same reservoir, in cm^2 , on Map *A*. [2]
- 4 Alice can type x number of words in one minute.
- (a) Write down an expression, in terms of x , for the number of seconds Alice takes to type 500 words. [1]
- (b) In one minute, Alice can type 10 more words than Betsy.
Write down an expression, in terms of x , for
- (i) the number of words Betsy types in one minute, [1]
 - (ii) the number of seconds Betsy takes to type 500 words. [1]
- (c) It takes Betsy 100 seconds longer to type out 500 words than it takes Alice to type out the same 500 words.
- (i) Write down an equation in x to represent this information, and show that it reduces to $x^2 - 10x - 3000 = 0$. [3]
 - (ii) Solve the equation $x^2 - 10x - 3000 = 0$. [2]
 - (iii) Find the time taken, in minutes, for Betsy to type 3500 words. [2]

5

The diagram represents two fields ABD and BCD in a horizontal plane.

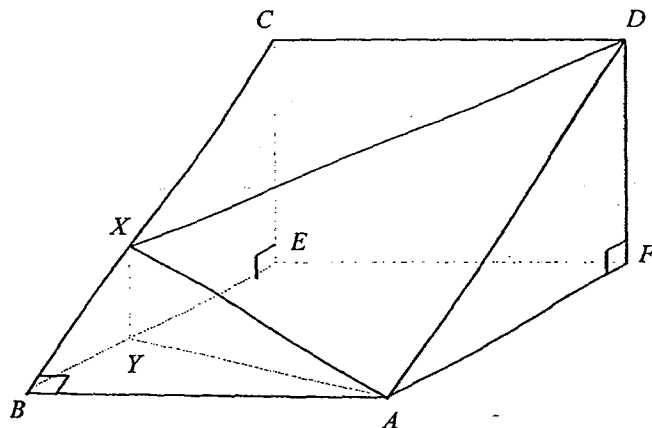
$AB = 270$ m, $BD = 250$ m, $\angle ABD = 57^\circ$ and $\angle CDB = x^\circ$. B is due north of C and D is 063° from C .



- (a) Find,
- (i) the value of x , given that the bearing of B from D is 293° , [2]
 - (ii) the bearing of A from B . [2]
- (b) Calculate, correct to the nearest metre, the length of
- (i) AD , [2]
 - (ii) BC . [2]
- (c) A man walking along AB is looking at a vertical tower directly above D . Calculate the distance from AB to D that will result in the greatest angle of elevation from the man to the top of the tower. [2]

[Turn Over

6



In the figure, $ABCD$ is a rectangular plane inclined to the horizontal plane $ABEF$. X is a point on BC and directly above Y . Y is a point on BE . Given that $AB = 5$ cm, $AF = 4.8$ cm, $AD = 7$ cm and $BX = 3$ cm.

Find

- | | | |
|-----|----------------------------------|-----|
| (a) | $\angle CBY$, | [1] |
| (b) | the length of AX , | [1] |
| (c) | the value of $\sin \angle XAY$, | [3] |
| (d) | the length of DX , | [1] |
| (e) | the area of triangle ADX . | [2] |

- 10 Study the number pattern below.

Row	X	Y	Sum of digits of Y
1	2^2	4	4
2	32^2	1024	7
3	332^2	110224	10
4	3332^2	11102224	13
$\vdots$	$\vdots$	$\vdots$	$\vdots$
7	3333332^2	P	22

- (a) State the value of P .

Answer (a) [1]

- (b) Find the sum of digits of Y in the n^{th} row, in terms of n .

Answer (b) [1]

- (c) A student found the sum of digits of Y in a particular row to be 110.
Give a reason why his answer is not acceptable.

Answer (c) [1]

- 11 (a) Simplify the following, leaving your answers in positive indices.

(i) $\left(\frac{2x^{-3}y^2}{4x^{-\frac{1}{2}}y} \right)^{-2}$,

Answer (a) (i) [2]

(ii) $\frac{(-x)^{-2}y^{-\frac{1}{4}}}{4x} \div \frac{7(-x)^{-3}}{y^{-\frac{3}{4}}}$.

Answer (a) (ii) [3]

- (b) Given $4^k \div 128 = 2^{29}$, find the value of k .

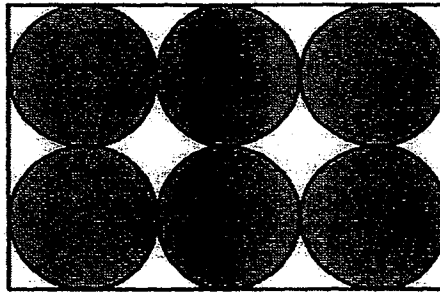
Answer (b) $k =$ [2]

- 8 (c) Using your grouped frequency table, calculate an estimate of
- (i) the mean number of hours spent on smartphone by each user, [2]
- (ii) the standard deviation. [2]
- (d) A similar survey was also conducted in Seoul and the table below shows the results of the processed data.

Mean	5.1
Standard Deviation	3.50

Compare, briefly, the results for the two cities. [1]

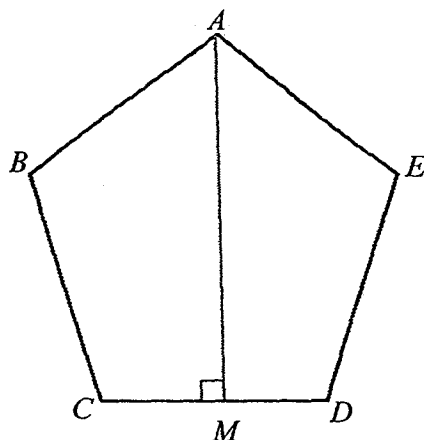
- 9 (a) A box containing 6 spherical chocolates, each of radius 1.2 cm, are packed touching each other and all the interior sides of the box. The diagram below shows the top view of the box and the chocolates.



- (i) Calculate the volume of the box in cm^3 . [2]
- (ii) Calculate the percentage of the total volume of the box not filled by the chocolates. [3]
- (iii) All the chocolates are removed from the box and each is uniformly coated with caramel 0.5 cm thick around it. How many complete boxes of spherical chocolates can be coated with 250 ml of caramel? [3]

[Turn Over

9 (b)



A regular pentagon, $ABCDE$, has sides of length 8 cm.

- (i) Find the length of AM , where M is the midpoint of CD . [4]
- (ii) Calculate the area of the pentagon. [2]
- (c) A decagon has eight angles of 134° each. The two remaining angles are $3x^\circ$ and $(x+60)^\circ$. Find x . Hence, state the largest angle in the decagon. [3]

10

Answer the whole of this question on a sheet of graph paper.

- (a) The volume of a solid is given by the equation $V = x^3 + 3x$. The table below gives some values of x and the corresponding values of V that satisfy the equation.

x	0	1	2	3	4	5
V	0	4	14	p	76	q

Calculate the value of p and of q . [2]

- (b) Using a scale of 2 cm to represent 1 unit on the horizontal axis and a scale of 2 cm to represent 20 units on the vertical axis, draw the graph of $V = x^3 + 3x$ for the values of x shown in the table. [3]
- (c) Use your graph to estimate
- (i) the volume of the solid if $x = 3.5$, [1]
- (ii) the value of x if the volume of the solid is 90 cm^3 . [1]
- (d) If this solid is melted down and made into a solid rectangular block of dimensions 3 cm, 5 cm and $(6-x)$ cm, draw a suitable straight line on the same axes as in part (b) above in order to find the value of x . [3]

=====END OF PAPER=====

1	Evaluate $\frac{-(3.5)^2 + \frac{13}{15}}{[1\frac{1}{6} - (-4)]^3 - \sqrt[3]{2197}}$, giving your answer correct to 3 significant figures.
	$\frac{-(3.5)^2 + \frac{13}{15}}{[1\frac{1}{6} - (-4)]^3 - \sqrt[3]{2197}}$ $= -0.091124041$ ≈ -0.0911
2	From the set of numbers $\left\{-3, 0, 1.2, \sqrt{0.9}, \sqrt{2}, \sqrt{9}, \sqrt{0.64}, -\frac{1}{3}\right\}$, write down (a) all the integers, (b) all the rational numbers, (c) the name for the remaining numbers.
	(a) $-3, 0, \sqrt{9}$ (b) $-3, 0, 1.2, \sqrt{9}, \sqrt{0.64}, -\frac{1}{3}$ (c) irrational numbers
3	(a) A shopkeeper bought a digital camera for \$200. He then sold it at a profit of 35%. What was the selling price? (b) Mr Smith was able to buy the digital camera sold in (a), at a discount. If he paid y dollars for the camera, find, in terms of y, the percentage discount.
	a) Selling price = $\frac{135}{100} \times 200 = \\270 b) Percentage discount = $\frac{270 - y}{270} \times 100\%$ $= \frac{2700 - 10y}{27} \%$

4	(a) Express each of the numbers 14, 21 and 49 as a product of its prime factors. (b) During the F1 period, three train services operate from the same station. The first service (North-East Line) leaves at 14-minute intervals, the second (East-West Line) at 21-minute intervals and the third (Circle Line) at 49-minute intervals. All three services leave the station together at 19 30 hours on Thursday. When will the three train services next leave the station together?
	a) $14 = 2 \times 7$ $21 = 3 \times 7$ $49 = 7 \times 7$ b) LCM of 14, 21, 49 = $2 \times 3 \times 7 \times 7 = 294$ $294 = 4 \text{ hours } 54 \text{ minutes}$ The three train services next leave the station together at 0024 hours on Friday.
5	(a) A sum of money is divided in the ratio 2:4:5. Given that the difference between the largest share and the smallest share is \$72, find the sum of money. (b) Given that $A:B = \frac{1}{2}:\frac{1}{3}$ and B is twice of C. How many times is A bigger than C?
	(a) Sum of money = $\frac{72}{5-2} \times (5+4+2) = \\264 (b) $\frac{A}{B} = \frac{\frac{1}{2}}{\frac{1}{3}}$ $= \frac{3}{2}$ $2A = 3B$ $A = \frac{3B}{2}$ $A = \frac{3(2C)}{2}$ $A = 3C$ A is three times that of C.

6	(a) Express $3.4 \times 10^7 - 5 \times 10^6$ in standard form. (b) The frequency of radio waves is measured in hertz, kilohertz and megahertz. A radio station broadcasts on 92.4 megahertz. (i) Express 92.4 megahertz in hertz. Leave your answers in standard form. (ii) The wavelength, λ, of radio waves in metres, is calculated using the formula $\lambda = \frac{3 \times 10^8}{f}$ where f is the frequency in hertz. Calculate the wavelength, in metres, that the radio station, 92.4, uses for its broadcast.
	a) $3.4 \times 10^7 - 5 \times 10^6 = 29000000 = 2.90 \times 10^7$ bi) $92.4 \text{ megahertz} = 92.4 \times 10^6 = 92400000 = 9.24 \times 10^7$ bii) $\lambda = \frac{3 \times 10^8}{9.24 \times 10^7} = 3.24675... \approx 3.25$
7	(a) Given that w is directly proportional to $\sqrt[3]{m}$. If $w = 4$ and $m = 125$, express w in terms of m. (b) It is given that the force between two particles is inversely proportional to the square of the distance between them. If the force is F when the distance between them is D, and pF when the distance is $4D$, find the value of p.
	$w = k\sqrt[3]{m}$ $4 = k\sqrt[3]{125}$ (a) $k = \frac{4}{5}$ $\therefore w = \frac{4}{5}\sqrt[3]{m}$ $F = \frac{k}{D^2}$ $pF = \frac{k}{(4D)^2}$ (b) $pF = \frac{k}{16D^2}$ $pF = \frac{1}{16} \times \frac{k}{D^2}$ $pF = \frac{1}{16}F$ $p = \frac{1}{16}$
8	a) Factorise $(10x+17)^2 - 2(70x+119) + 49$ completely.

	b) Hence, without the use calculators, find the value of $(990 + 17)^2 - 2(6930 + 119) + 49$.		
	Solution a) $ \begin{aligned} (10x + 17)^2 - 2(70x + 119) + 49 &= (10x + 17)^2 - 2(7)(10x + 17) + 49 \\ &= (10x + 17 - 7)^2 \\ &= (10x + 10)^2 \\ &= 100(x + 1)^2 \end{aligned} $ b) $ \begin{aligned} 100(99 + 1)^2 &= 100^3 \\ &= 1000000 \end{aligned} $		
9	Express, as a single fraction, in its simplest form $\frac{5 - 7x - x^2}{-6x^2 + 11x - 4} - \frac{x - 1}{1 - 2x}$.		
	Solution $ \begin{aligned} \frac{5 - 7x - x^2}{-6x^2 + 11x - 4} - \frac{x - 1}{1 - 2x} &= \frac{5 - 7x - x^2}{(3x - 4)(1 - 2x)} - \frac{x - 1}{1 - 2x} \\ &= \frac{5 - 7x - x^2 - (3x - 4)(x - 1)}{(3x - 4)(1 - 2x)} \\ &= \frac{-4x^2 + 1}{(3x - 4)(1 - 2x)} \\ &= \frac{(1 - 2x)(1 + 2x)}{(3x - 4)(1 - 2x)} \\ &= \frac{1 + 2x}{3x - 4} \end{aligned} $		
10	Study the number pattern below		
	Row	X	Y
			Sum of digits of Y

1	2^2	4	4
2	32^2	1024	7
3	332^2	110224	10
4	3332^2	11102224	13
7	3333332^2	P	22

- a) State the value of P .
 b) Find the sum of digits of Y in the n^{th} -row, in terms of n .
 c) A student found the sum of digits of Y in a particular row to be 110.
 Give a reason his answer is not acceptable.

Solution

a) $P = 11111102222224$

b) Row n : Sum of digits of $Y = 3n + 1$

c)

$$3n + 1 = 110$$

$$3n = 109$$

$$n = 36\frac{1}{3}$$

Since n is not an integer, 110 does not belong to any row.

- 11 (a) Simplify the following, leaving your answer in positive index.

$$(i) \left(\frac{2x^{-3}y^2}{4x^{-\frac{1}{2}}y} \right)^{-2},$$

$$(ii) \frac{(-x)^{-2}y^{\frac{1}{4}}}{4x} \div \frac{7(-x)^{-3}}{y^{\frac{3}{4}}}.$$

(b) Given $4^k \div 128 = 2^{29}$, find the value of k .

$$ai) \left(\frac{2x^{-3}y^2}{4x^{-\frac{1}{2}}y} \right)^{-2}$$

$$= \left(\frac{4x^{-\frac{1}{2}}y}{2x^{-3}y^2} \right)^2$$

$$= \left(\frac{16x^{-1}y^2}{4x^{-6}y^4} \right)$$

$$= \frac{4x^5}{y^2}$$

$$aii) \frac{(-x)^{-2}y^{\frac{1}{4}}}{4x} \div \frac{7(-x)^{-3}}{y^{\frac{3}{4}}}$$

$$= \frac{(-x)^{-2}y^{\frac{1}{4}}}{4x} \times \frac{y^{\frac{3}{4}}}{7(-x)^{-3}}$$

$$= \frac{1}{4 \cdot 7} \times \frac{(-x)^{-2}}{x \cdot (-x)^{-3}} \times \frac{y^{\frac{1}{4}} \cdot y^{\frac{3}{4}}}{1}$$

$$= \frac{1}{28} \times \frac{-1}{1} \times \frac{y^{-1}}{1}$$

$$= -\frac{1}{28y}$$

$$b) 4^k \div 128 = 2^{29}$$

	$2^{2k} \div 2^7 = 2^{29}$ $2k - 7 = 29$ $2k = 36$ $k = 18$
12	a) Solve the inequality $\frac{x}{4} - \frac{3x-1}{2} > 5$. b) Hence, write down the largest integer x such that $\frac{x}{4} - \frac{3x-1}{2} > 5$.
	Solution a) $\frac{x}{4} - \frac{3x-1}{2} > 5$ $\frac{x - 2(3x-1)}{4} > 5$ $-5x + 2 > 20$ $5x < -18$ $x < -3\frac{3}{5}$ b) $x = -4$
13	The matrices below shows the quantity of each item required to be placed in a Goodie Bag, and the

number of participants on each day of a 2-day event.

Pen $\begin{pmatrix} 4 \\ 1 \\ 2 \end{pmatrix}$
 Drinking Water
 Notepad

Mon Tues
 $\begin{pmatrix} 14 & 23 \end{pmatrix}$

(a) Find $\begin{pmatrix} 4 \\ 1 \\ 2 \end{pmatrix} \begin{pmatrix} 14 & 23 \end{pmatrix}$.

(b) Explain what your answer to part (a) represents.

Solution

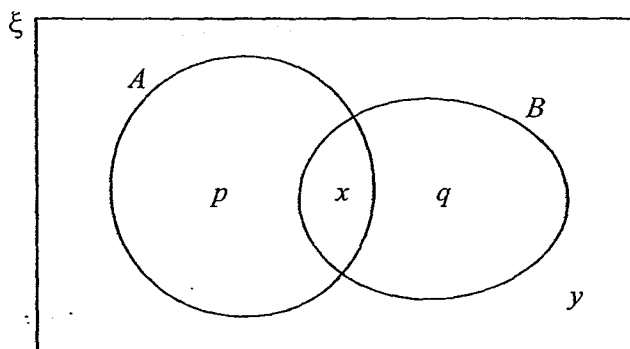
a)

$$\begin{pmatrix} 4 \\ 1 \\ 2 \end{pmatrix} \begin{pmatrix} 14 & 23 \end{pmatrix} = \begin{pmatrix} 56 & 92 \\ 14 & 23 \\ 28 & 46 \end{pmatrix}$$

b)

Total number required for
 each item
 on each day

14



In the Venn diagram, ξ is the set of all students from class 4A in Happy Valley School.
 $A = \{\text{students going for the National Day Parade at Padang}\}$ and
 $B = \{\text{students going for the Youth Olympic Games Opening Ceremony}\}.$

The letters p , q , x and y in the diagram represent the number of students in each subset.

Given that $n(\xi) = 14$, $n(A) = 5$ and $n(B) = 8$,

(a) express p in terms of x ,

(b) find the largest possible value of y .

a) $p + x = 5$

$\therefore p = 5 - x$

b) Largest possible $y = 14 - 8 = 6$

15

The ordered stem and leaf chart shows the distribution of the heights, in cm, of 30 boys.

Stem	Leaf
14	8 9
15	1 2 2 3 4 8 8 9
16	3 5 6 7 7 9 9
17	0 1 1 3 4 8 9
18	1 2 2 x 4 4

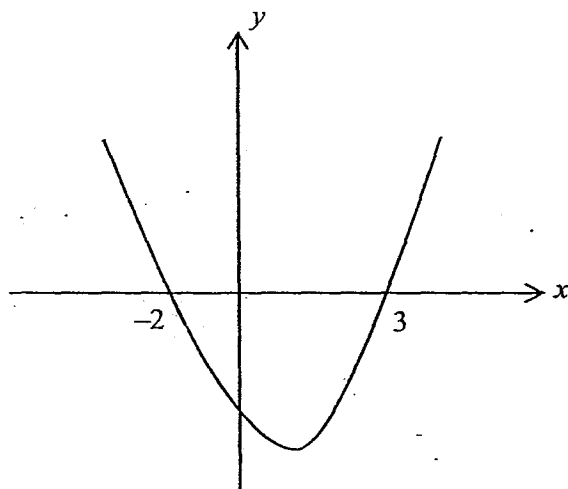
(a) What is the median of the distribution?

(b) State a possible value of x if the mode of the distribution is greater than 180.

a) Median = $\frac{167 + 169}{2} = 168$ cm

b) 2 or 4 (either answer)

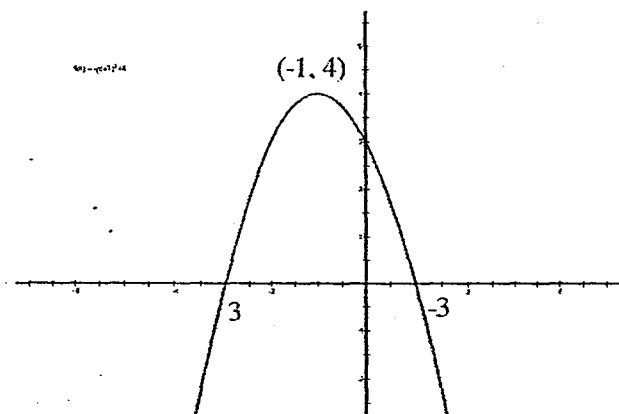
- 16 (a) The diagram shows a quadratic curve which can be expressed in the form $y = ax^2 + bx + c$. Given that the curve cuts the x -axis at -2 and 3 , find the values of a , b and c .



- (b) In the axes provided, sketch the graph of $y = 4 - (x+1)^2$, indicating **all** intercepts and turning point(s) clearly.

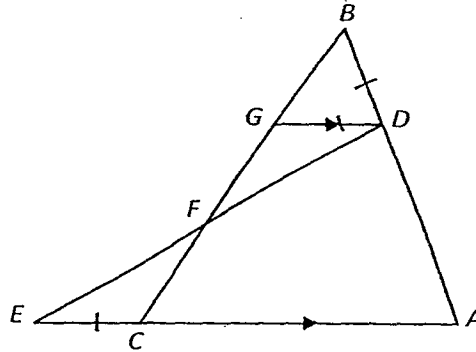
a) $a = 1$, $b = -1$, $c = -6$

b)



G1 : For labelling all intercepts
G1 : Correct shape

- 17 The diagram shows an isosceles triangle ABC with $AB = AC$. D and E lie on AB and AC produced such that $BD = CE$. The lines DE and BC intersect at the point F . DG is parallel to AE .



- a) State the triangle that is similar to $\triangle ABC$.
 b) Prove that $\triangle DGF$ is congruent to $\triangle ECF$.

Given that $BC = 9$ cm, $CF = 3$ cm, and AB is 3 times of BD ,

- c) Find $\frac{\text{Area } \triangle BDG}{\text{Area } \triangle BAC}$
 d) Find $\frac{\text{Area } \triangle CEF}{\text{Area } \triangle ABC}$

Solution

- a) $\triangle DBG$

- b)
 $\angle GDF = \angle CEF$ (alternate angle)
 $\angle DFG = \angle EFC$ (vertically opposite angle)
 $BD = CE$

By AAS, $\triangle DGF$ is congruent to $\triangle ECF$.

- c)

$$\frac{\text{Area } \triangle BDG}{\text{Area } \triangle BAC} = \left(\frac{1}{3}\right)^2 = \frac{1}{9}$$

- d)

$$\frac{\text{Area } \triangle BDG}{\text{Area } \triangle DGF} = \frac{3}{9-3-3} = 1$$

$$\frac{\text{Area } \triangle BDG}{\text{Area } \triangle CEF} = 1 \text{ (from (iii))}$$

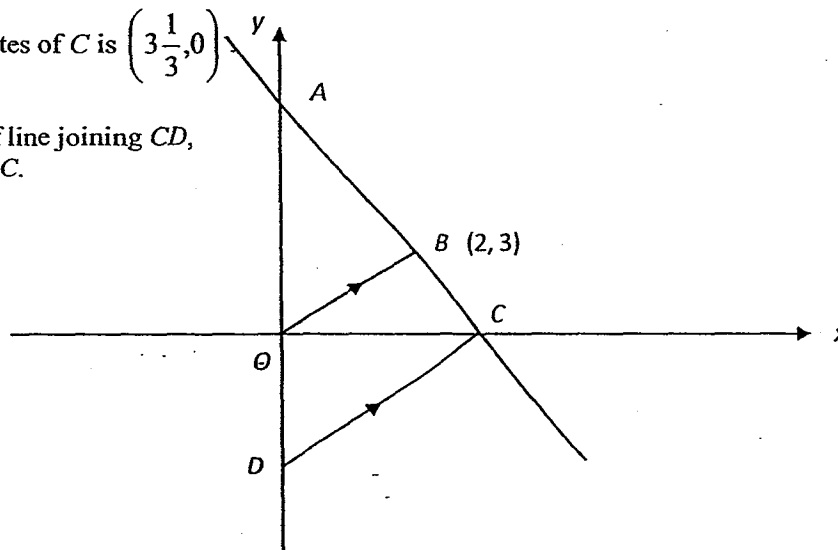
$$\frac{\text{Area } \triangle CEF}{\text{Area } \triangle ABC} = \frac{1}{9}$$

18 In the diagram, B has coordinates $(2, 3)$. Given that $5AB = 3AC$, and that OB is parallel to CD .

i) Show that the coordinates of C is $\left(3\frac{1}{3}, 0\right)$

ii) Hence, find

- the equation of line joining CD ,
- the length of AC .



Solution

i)

$$\frac{AB}{AC} = \frac{3}{5}$$

$$\frac{OC}{2} = \frac{5}{3}$$

$$OC = \frac{10}{3}$$

$$C = \left(3\frac{1}{3}, 0\right)$$

ii a)

$$\text{Gradient} = \frac{3}{2}$$

$$y = \frac{3}{2}x + c$$

$$0 = \frac{3}{2}\left(\frac{10}{3}\right) + c$$

$$c = -5$$

$$CD: y = \frac{3}{2}x - 5$$

ii b)

$$BC = \sqrt{(3-0)^2 + \left(2 - \frac{10}{3}\right)^2}$$

$$= \sqrt{\frac{97}{9}}$$

$$= \frac{\sqrt{97}}{3}$$

$$BC = \frac{2}{5} AC$$

$$AC = \frac{5}{2} \left(\frac{\sqrt{97}}{3} \right)$$

$$= 8.20738$$

$$= 8.21 \text{ units (3 s.f.)}$$

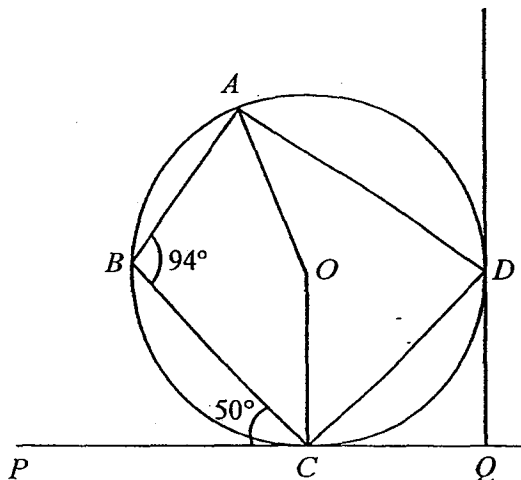
- 19 In the diagram, O is the centre of a circle. PCQ is a tangent to the circle at C and DQ is a tangent to the circle at D . Angle $ABC = 94^\circ$ and angle $BCP = 50^\circ$.

(a) Find

(i) reflex angle AOC ,

(ii) angle BAO .

(b) If the area of triangle $CDQ = 8 \text{ cm}^2$, find CD^2 .



ai) reflex $\hat{AOC} = 2 \times 94 = 188^\circ$ ($\angle$ at centre = $2 \angle$ at circumference)

aii) obtuse $\hat{AOC} = 360 - 188 = 172^\circ$ ($\angle$ s at a point).

$$\hat{PCO} = 90^\circ \text{ (tan } \perp \text{ rad)}$$

$$\hat{BAO} = 360 - 94 - 40 - 172 = 54^\circ$$

$$\text{b) } \frac{1}{2} \times CO \times OD = 8$$

$$CO^2 = 16$$

$$CO = 4 \text{ cm}$$

$$CD^2 = 4^2 + 4^2 = 32$$

20

The diagram above is the speed-time graph for cars A and B.

a) Find the speed of car A when $t = 3s$.

Given that car A travelled a total of 107 m,

b) How long did car A travel at constant speed?

Car B starts 7 s after car A. Given that cars A and B meet when $t = 15s$,

c) Find the acceleration of car B, leaving your answer correct to 3s.f.

Solution

a)

Let x be the speed of A when $t = 3$.

$$\frac{x-3}{10-3} = \frac{3}{5}$$

$$x-3 = \frac{21}{5}$$

$$x = 7.2ms^{-1}$$

b)

Let the time car A begin to decelerate be y .

$$\frac{1}{2}(3+10)(5) + 10(y-5) + \frac{1}{2}(15-y)(10) = 107$$

$$5y = 49.5$$

$$y = 9.9s$$

Time for constant speed = $9.9 - 5 = 4.9s$

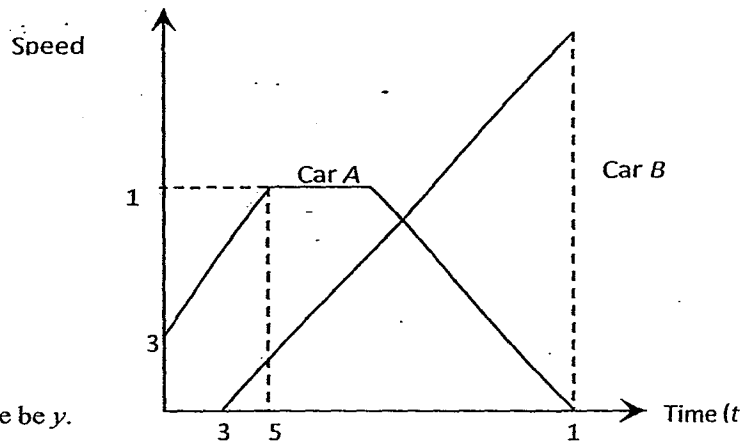
c)

Let h be the speed of B at $t=15s$

$$107 = \frac{1}{2}(15-3)(h)$$

$$h = 17.83333$$

$$\text{Acceleration} = \frac{17.83333}{15-3} = 1.49ms^{-2} \text{ (3s.f.)}$$



- 21 (a) Two boxes were used to store pens. Box A contains 3 blue pens and 7 green pens. Box B contains 5 blue pens and 9 green pens. Julie took a pen from Box A and placed it inside Box B. Then, she took out a pen from Box B. Given that the selections are random, find the probability that the pen taken from Box B is green.

- (b) $ABCD$ is a square dartboard. P , Q , R and S are the midpoints of AB , BC , CD and DA respectively. W , X , Y and Z are midpoints of PQ , QR , RS and SP respectively.

The score for a dart landing on each region of the dartboard is shown below.

Region	Score
$WXYZ$	3
PZW or RYY	2
SZY or QWX	1
Other regions	0

- (i) A dart is thrown once. What is the probability of scoring 3 points?

- (ii) Two darts are thrown one after another. What is the probability of scoring 3 points?

(a) $P(\text{pen is green from Box B}) = \left(\frac{3}{10} \times \frac{9}{15}\right) + \left(\frac{7}{10} \times \frac{10}{15}\right) = \frac{97}{150}$

(b)

By taking areas. Let AB be 4 units. That will give:

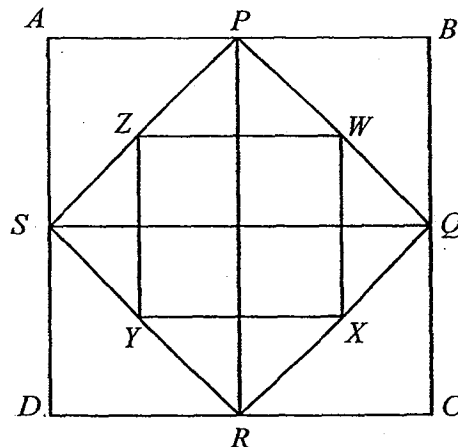
Total area = 16 units²

Area for "3" points = 4 units²

Area for "2" points = 2 units²

Area for "1" point = 2 units²

"Other" area = 8 units²



i) $P(\text{score of 3 points}) = \frac{4}{16} = \frac{1}{4}$

ii) $P(\text{score of 3 points, thrown twice}) = 2\left(\frac{4}{16} \times \frac{8}{16}\right) + 2\left(\frac{2}{16} \times \frac{2}{16}\right)$
 $= \frac{9}{32}$

- 23 Mr Pica Chu is an artist who is famous for his abstract artwork. The following is part of his abstract artwork, which forms a circle. Use a ruler and compasses to complete the circle. Mark

the centre of the circle with 'x' and measure its radius.



Draw 2 perpendicular bisectors

Bisect the chords; the intersection of the 3 chords will be the centre of the circle (x position)

Radius – 5.1 cm

Sec 4 EM Prelim 2014

Paper 2 Solutions

1 (a) Given that $\sqrt{\frac{a^2-b}{b}} = \frac{2}{3}$, express b in terms of a . [2]

(b) Express as a single fraction in its simplest form $\frac{4x}{4-5x} - \frac{2}{x+3}$. [3]

(c) (i) Simplify $(p+q)(p-q) - p^2$. [1]

(ii) Hence evaluate $10012019 \times 10012009 - 10012014^2$. [1]

1(a)

$$\begin{aligned}\sqrt{\frac{a^2-b}{b}} &= \frac{2}{3} \\ \frac{a^2-b}{b} &= \frac{4}{9} \\ 9a^2 - 9b &= 4b \\ 13b &= 9a^2 \\ b &= \frac{9a^2}{13}\end{aligned}$$

(b)

$$\begin{aligned}\frac{4x}{4-5x} - \frac{2}{x+3} &= \frac{4x(x+3) - 2(4-5x)}{(4-5x)(x+3)} \\ &= \frac{4x^2 + 12x - 8 + 10x}{(4-5x)(x+3)} \\ &= \frac{4x^2 + 22x - 8}{(4-5x)(x+3)} \\ &= \frac{2(2x^2 + 11x - 4)}{(4-5x)(x+3)}\end{aligned}$$

1(c)

$$\begin{aligned}(p+q)(p-q) - p^2 &= p^2 - q^2 - p^2 \\ &= -q^2\end{aligned}$$

$$\begin{aligned}10012019 \times 10012009 - 10012014^2 \\ &= (10012014 + 5)(10012014 - 5) - 10012014^2 \\ &= -5^2 \\ &= -25\end{aligned}$$

- 2 (a) A company invested \$ x in a business venture and made a profit of 30% amounting to \$72,000. This profit is 20% more compared to the previous year's profit.
Calculate
- (i) the value of x , [1]
- (ii) the profit of the previous year. [2]
- (b) Mathew placed a \$20,000 deposit with Bank A with interest at 5% p.a. over 3 years. Mandy placed \$20,000 with Bank B with interest at 4.9% p.a. compounded monthly over 3 years. Calculate the individual interest earned by Mathew and Mandy after 3 years. [2]
- (c) The cash price of a 70 inches TV is \$7,000. Mr Tan bought the TV on hire purchase by putting a downpayment of \$1,000 and monthly instalments of \$150 over y months.
- (i) Express the total amount payable by hire purchase, in terms of y . Express, in terms of y , the amount Mr Tan could have saved if he had paid in full by cash. [2]
- (ii) Given that the seller charge simple interest at 10% p.a., form an equation to calculate the value of y . [3]

- 2 (a) (i) $30\% \text{ of profit} = 72000$
 $x = 100\%$
 $= \frac{72000}{30} \times 100$
 $= 240000$
- (ii) Previous year's profit:
 $120\% \text{ of profit} = \72000
 $100\% \text{ of prev year's profit} = \frac{72000}{120} \times 100$
 $= \$60000$
- (b) Matthew's interest $= \frac{5}{100} \times 20000 \times 3$
 $= \$3000$
 Mandy's interest $= 20000(1 + \frac{4.9}{1200})^{36} - 20000$
 $= 3160.146$
 $\approx \$3160.15$
- (c) (i) $Total \text{ amount paid} = \$1000 + 150y$
 $Extra \text{ amount payable} = \$1000 + 150y - 7000$
 $= \$(150y - 6000)$
- (ii) $6000 \times 10\% \times \frac{y}{12} = 150y - 6000$
 $50y = 150y - 6000$

$$100y = 6000$$

$$y = 60$$

- 3 (a) The scale of Map A is given as 1:200 000. Find,
- (i) the actual distance, in km, of two points measuring 6 cm on the map. [1]
 - (ii) the area of a town on the map given that its actual size is 1.5 km^2 . [2]
- (b) Given that the scale of Map B is 1: 250 000 and the size of a reservoir on Map B is 3.6 cm^2 . Calculate the area of the same reservoir, in cm^2 , on Map A. [2]
- 3 (a) (i) 1 cm represents 2 km on Map A
Actual distance = 6×2
 $= 12 \text{ km}$
- (ii) 1 cm^2 represents 4 km^2 on Map A
Area on map = $\frac{1.5}{4} \text{ cm}^2$
 $= \frac{3}{8}$ or 0.375 cm^2
- (b) On Map B, 1 cm represents 2.5 km
1 cm^2 represents 6.25 km^2
3.6 cm^2 represent 22.5 km^2
On Map A, area = $\frac{22.5}{4}$
 $= 5.625 \text{ cm}^2$
- 4 Alice can type x number of words in one minute.
- (a) Write down an expression, in terms of x , for the number of seconds Alice takes to type 500 words. [1]
 - (b) In one minute, Alice can type 10 more words than Betsy.
Write down an expression, in terms of x , for
 - (i) the number of words Betsy types in one minute, [1]
 - (ii) the number of seconds Betsy takes to type 500 words. [1]
 - (c) It takes Betsy 100 seconds longer to type out 500 words than it takes Alice to type out the same 500 words.
 - (i) Write down an equation in x to represent this information, and show that it reduces to $x^2 - 10x - 3000 = 0$. [3]

(ii) Solve the equation $x^2 - 10x - 3000 = 0$. [2]

(iii) Find the time taken, in minutes, for Betsy to type 3500 words. [2]

4(a) Time taken by Alice to type 500 words

$$= 500 \div \frac{x}{60}$$

$$= \frac{30000}{x} \text{ s}$$

(b)(i) Number of words Betsy types in 1 min $= x - 10$

(ii) Time taken by Betsy to type 500 words $= \frac{30000}{x-10} \text{ s}$

(c)(i)

$$\frac{30000}{x-10} - \frac{30000}{x} = 100$$

$$\frac{30000x - 30000(x-10)}{x(x-10)} = 100$$

$$\frac{300000}{x^2 - 10x} = 100$$

$$100x^2 - 1000x = 300000$$

$$100x^2 - 1000x - 300000 = 0$$

$$x^2 - 10x - 3000 = 0$$

(ii)

$$x^2 - 10x - 3000 = 0$$

$$(x-60)(x+50) = 0$$

$$x = 60 \text{ or } x = -50 (\text{rejected})$$

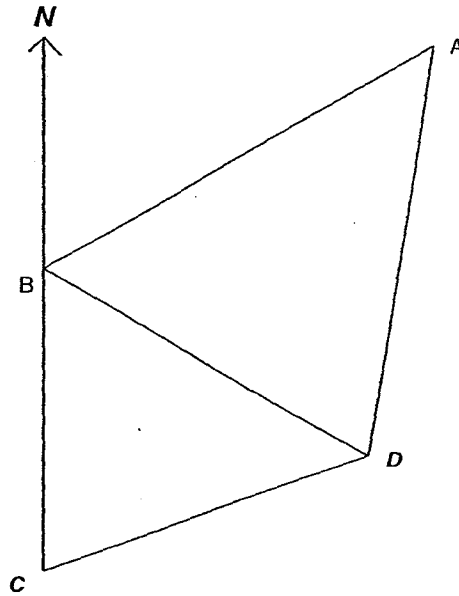
(iii)

$$\text{Time taken by Betsy} = \frac{3500}{50}$$

$$= 70 \text{ mins}$$

5

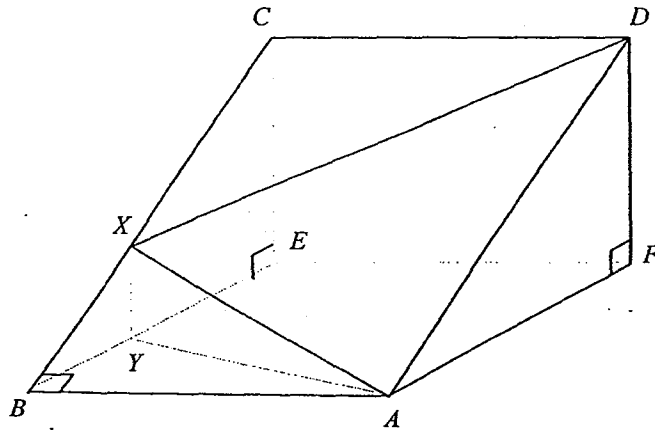
The diagram represents two fields ABD and BCD in a horizontal plane.
 $AB = 270 \text{ m}$, $BD = 250 \text{ m}$, $\angle ABD = 57^\circ$ and $\angle CDB = x^\circ$. B is due north of C and D is 063° from C .



- (a) Find,
- the value of x , given that the bearing of B from D is 293° , [2]
 - the bearing of A from B . [2]
- (b) Calculate, correct to the nearest metre, the length of
- AD , [2]
 - BC . [2]
- (c) A man walking along AB is looking at a vertical tower directly above D . Calculate the distance from AB to D that will result in the greatest angle of elevation from the man to the top of the tower. [2]
- 5 (a) (i) $180^\circ + 63^\circ + x^\circ = 293^\circ$
 $\therefore x = 293 - 180 - 63$
 $= 50$
- (ii) Exterior angle of $\triangle BDC = 63^\circ + 50^\circ$
 $= 113^\circ$
 $113^\circ - 57^\circ = 56^\circ$
- $\therefore$ Bearing of A from $B = 056^\circ$
- (b) (i) $AD^2 = 270^2 + 250^2 - 2 \times 270 \times 250 \times \cos 57^\circ$
 $= 61,873.73027$
 $AD = 248.7443$
 $\approx 249m$ (3 s.f.)
- (ii) $\frac{BC}{\sin 50^\circ} = \frac{250}{\sin 63^\circ}$
 $\therefore BC = \frac{250}{\sin 63^\circ} \times \sin 50^\circ$
 $= 214.9379$
 $\approx 215m$ (3 s.f.)

- (c) Shortest distance from AB to D $= 250 \times \sin 57^\circ$
 $= 209.6676$
 $\cong 210 \text{ m (3s.f.)}$

6



In the figure, $ABCD$ is a rectangular plane inclined to the horizontal plane $ABEF$. X is a point on BC and directly above Y . Y is a point on BE . Given that $AB = 5 \text{ cm}$, $AF = 4.8 \text{ cm}$, $AD = 7 \text{ cm}$ and $BX = 3 \text{ cm}$. Find,

- | | | |
|-----|----------------------------------|-----|
| (a) | $\angle CBY$ | [1] |
| (b) | the length of AX , | [1] |
| (c) | the value of $\sin \angle XAY$, | [3] |
| (d) | the length of DX , | [1] |
| (e) | the area of triangle ADX | [2] |

6 (a) $\cos \angle CBY = \frac{4.8}{7}$
 $\therefore \angle CBY = 46.708^\circ$
 $\approx 46.7^\circ \text{ (1 dp)}$

(b) $AX^2 = 3^2 + 5^2$
 $= 9 + 25$
 $= 34$
 $AX = 5.8309$
 $\approx 5.83 \text{ cm (3 s.f.)}$

(c) $CE^2 = DF^2 = 7^2 - 4.8^2$

$$= 25.96$$

$$CE = 5.09509 \text{ cm}$$

$$\frac{XY}{3} = \frac{5.09509}{7}$$

$$XY = 2.1836$$

$$\sin \angle XAY = \frac{2.1836}{5.8309}$$

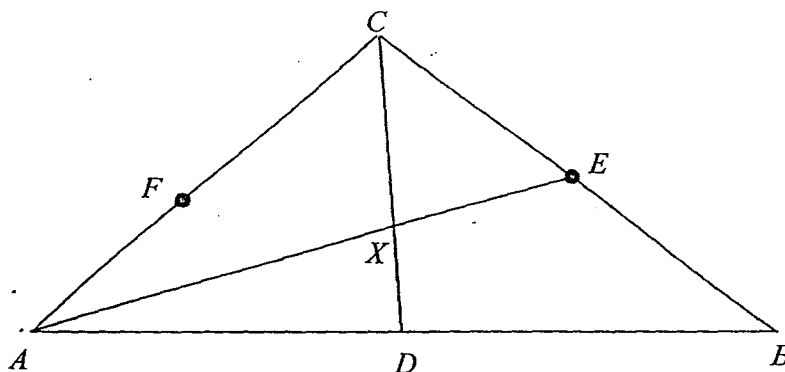
$$= 0.374489$$

$$\begin{aligned}
 &\approx 0.374 \text{ (3 s.f.)} \\
 \text{(d)} \quad CX &= 7 - 3 \\
 &= 4 \text{ cm} \\
 DX^2 &= CX^2 + CD^2 \\
 &= 16 + 25 \\
 &= 41 \\
 DX &= 6.4031 \\
 &\approx 6.40 \text{ (3 s.f.)}
 \end{aligned}$$

$$\begin{aligned}
 \text{(e)} \quad \angle XAD &= 59.036^\circ \\
 \text{Area of } \triangle ADX &= \frac{1}{2} \times 5.8309 \times 7 \times \sin 59.036^\circ \\
 &= 17.4997 \\
 &\approx 17.5 \text{ cm}^2 \text{ (3 s.f.)} \\
 \text{OR Area of } \triangle ADX &= \frac{1}{2} \times 5 \times 7 = 17.5 \text{ cm}^2
 \end{aligned}$$

7

In triangle ABC , D , E and F are midpoints of AB , BC and AC respectively. The lines AE and CD intersect at point X .



- (a) Find the ratio of area of $\triangle ABC$ to area of $\triangle AEC$. [1]
- (b) Given that $\overrightarrow{AB} = 5\mathbf{m}$ and $\overrightarrow{BC} = 3\mathbf{n}$, express as simple as possible, in terms of $\mathbf{m}$ and/or $\mathbf{n}$, [1]
- (i) $\overrightarrow{AC}$ [1]
- (ii) $\overrightarrow{AE}$ and [1]
- (iii) $\overrightarrow{CD}$ [1]
- (c) Given that $\overrightarrow{CX} = \frac{2}{3}\overrightarrow{CD}$, express $\overrightarrow{BX}$ in terms of $\mathbf{m}$ and $\mathbf{n}$, in its simplest form. [2]
- (d) Obtain an expression for $\overrightarrow{XF}$. Hence prove that points B , X and F lie on the same straight line. [3]
- (e) It is given that C is a point $(-1, 7)$ and $\overrightarrow{CE} = \begin{pmatrix} 3 \\ -2 \end{pmatrix}$. [1]

(i) Calculate $|\overrightarrow{CE}|$. [1]

(ii) Find the coordinates of E . [3]

(iii) Find the equation of the line BC .

7 (a) Using BC as base of $\triangle ABC$, let height be h .

$$\text{Area of } \triangle ABC = \frac{1}{2} \times BC \times h$$

Using EC as base of $\triangle AEC$, height = h .

$$\text{Area of } \triangle AEC = \frac{1}{2} \times EC \times h$$

$$\therefore \text{Area of } \triangle ABC : \text{Area of } \triangle AEC$$

$$= BC : EC$$

$$= 2 : 1$$

(b) (i) $\overrightarrow{AC} = \overrightarrow{AB} + \overrightarrow{BC}$
 $= 5\mathbf{m} + 3\mathbf{n}$

(ii) $\overrightarrow{AE} = \overrightarrow{AB} + \overrightarrow{BE}$
 $= 5\mathbf{m} + \frac{3}{2}\mathbf{n}$

(iii) $\overrightarrow{CD} = \overrightarrow{CA} + \overrightarrow{AD}$
 $= -5\mathbf{m} - 3\mathbf{n} + \frac{5}{2}\mathbf{m}$
 $= -\frac{5}{2}\mathbf{m} - 3\mathbf{n}$

(c) $\overrightarrow{CX} = \frac{2}{3}\overrightarrow{CD}$
 $= \frac{2}{3}(-\frac{5}{2}\mathbf{m} - 3\mathbf{n})$
 $= -\frac{5}{3}\mathbf{m} - 2\mathbf{n}$
 $\overrightarrow{BX} = \overrightarrow{BC} + \overrightarrow{CX}$
 $= 3\mathbf{n} - \frac{5}{3}\mathbf{m} - 2\mathbf{n}$
 $= -\frac{5}{3}\mathbf{m} + \mathbf{n}$

(d) $\overrightarrow{XF} = \overrightarrow{XA} + \overrightarrow{AF}$
 $= -5(\frac{2}{3})\mathbf{m} - \frac{3}{2}(\frac{2}{3})\mathbf{n} + \frac{5}{2}\mathbf{m} + \frac{3}{2}\mathbf{n}$
 $= -\frac{5}{6}\mathbf{m} + \frac{1}{2}\mathbf{n}$
 $\overrightarrow{BX} = -\frac{5}{2}(\frac{2}{3})\mathbf{m} + (3 - 3 \times \frac{2}{3})\mathbf{n}$
 $= -\frac{10}{6}\mathbf{m} + \mathbf{n}$
 $= 2(-\frac{5}{6}\mathbf{m} + \frac{1}{2}\mathbf{n})$
 $= 2\overrightarrow{XF}$

Since X is a common point, therefore B , X and F are collinear

(e) (i) $|\overrightarrow{CE}| = \sqrt{3^2 + (-2)^2}$
 $= \sqrt{13} \text{ or } \approx 3.61 \text{ units}$

(ii) Coordinates of $E = (-1 + 3, 7 - 2)$
 $= (2, 5)$

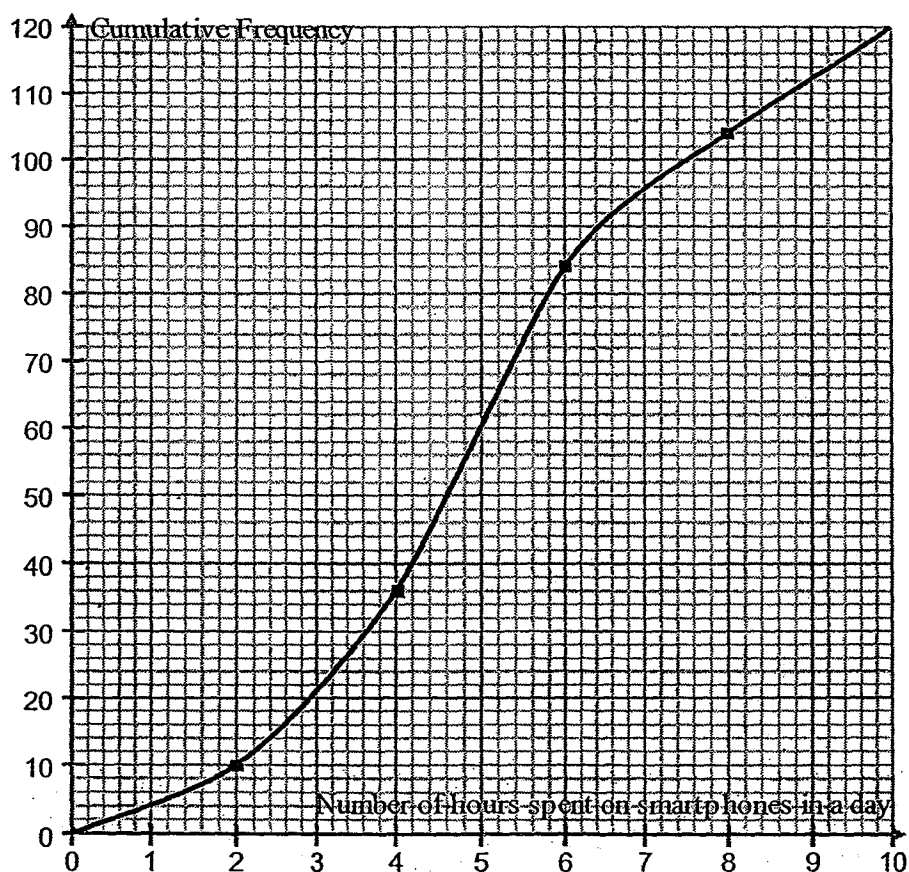
$$(iii) \quad \text{Gradient of } BC = \frac{7-5}{-1-2} = -\frac{2}{3}$$

Equation of line BC is

$$(y-7) = -\frac{2}{3}(x+1)$$

$$y = -\frac{2}{3}x + \frac{19}{3} \text{ or } 3y = -2x + 19$$

- 8 A group of 120 smartphone users in Singapore were surveyed based on their usage. The cumulative frequency curve below shows the number of hours spent by the users on their smartphones in a day.



- (a) Use the graph to find
- the median number of hours spent, [1]
 - the interquartile range. [1]
- (b) Copy and complete the grouped frequency table of the number of hours spent on smartphones by each user.

Number of hours /	$0 \leq x < 2$	$2 \leq x < 4$	$4 \leq x < 6$	$6 \leq x < 8$	$8 \leq x < 10$
-------------------	----------------	----------------	----------------	----------------	-----------------

(x hours)					
Frequency					

[2]

- (c) Using your grouped frequency table, calculate an estimate of
- (i) the mean number of hours spent on smartphone by each user, [2]
 - (ii) the standard deviation. [2]
- (d) A similar survey was also conducted in Seoul and the table below shows the results of the processed data.

Mean	5.1
Standard Deviation	3.50

Compare, briefly, the results for the two cities.

[1]

8(a)(i) Median = 5 hours

(ii) Interquartile Range = $6.4 - 3.6 = 2.8$ hours (Accept 2.8-2.9)

(b)

Number of hours (x hours)	$0 \leq x < 2$	$2 \leq x < 4$	$4 \leq x < 6$	$6 \leq x < 8$	$8 \leq x < 10$
Frequency	10	26	48	20	16

(c)(i)

$$\begin{aligned}
 \text{Mean} &= \frac{1(10) + 3(26) + 5(48) + 7(20) + 9(16)}{120} \\
 &= \frac{612}{120} \\
 &= 5.1
 \end{aligned}$$

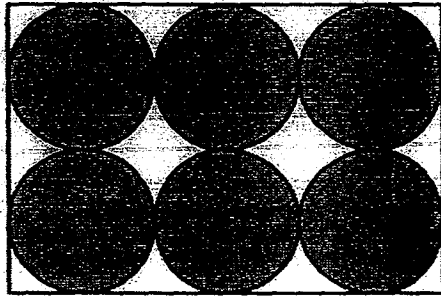
$$\text{Standard Deviation} = \sqrt{\frac{3720}{120} - 5.1^2}$$

$$\begin{aligned}
 &= 2.2338 \\
 &\approx 2.23
 \end{aligned}$$

- 8(d) Seoul has a larger spread of number of hours spent on smartphones in a day among the users.

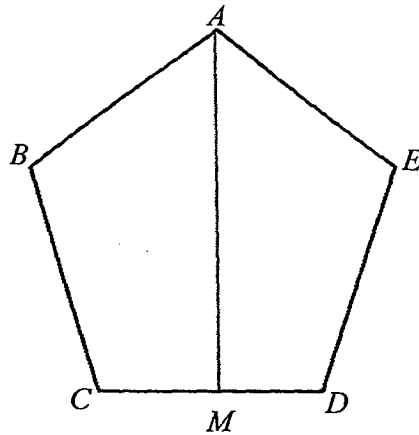
- 9 (a) A box containing 6 spherical chocolates, each of radius 1.2 cm, are packed touching each other and the sides of the box. The diagram below shows the

top view of the box and the chocolates with its lid uncovered.



- (i) Calculate the volume of the box in cm^3 . [2]
- (ii) Calculate the percentage of the total volume of the box **not** filled by the chocolates. [3]
- (iii) All the chocolates are removed from the box and each is uniformly coated with caramel 0.5 cm thick around it. How many complete boxes of spherical chocolates can be coated with 250 ml of caramel? [3]

(b)



A regular pentagon, $ABCDE$, has sides of length 8 cm.

- (i) Find the height of the pentagon, AM , where M is the midpoint of CD . [4]
- (ii) Calculate the area of the pentagon. [2]
- (c) A decagon has eight angles of 134° each. The two remaining angles are $3x^\circ$ and $(x + 60)^\circ$. Find x . Hence, state the largest angle in the decagon. [3]

9(a)(i)

22

Volume of the box

$$= 4.8 \times 7.2 \times 2.4$$

$$= 82.944 \text{ cm}^3$$

(ii)

Volume of the chocolate spheres

$$= 6 \times \frac{4}{3} \times \pi \times 1.2^3$$

$$= 43.4293 \text{ cm}^3$$

Percentage of volume of box not filled by chocolates

$$= \frac{82.944 - 43.4293}{82.944} \times 100\%$$

$$= 47.640$$

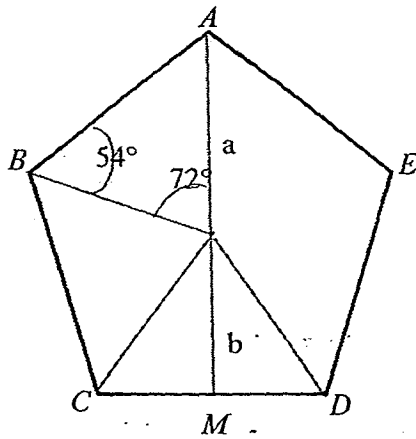
$$\approx 47.6\%$$

(iii)

$$\begin{aligned} \text{Volume of caramel used for one box} &= 6 \times \frac{4}{3} \times \pi \times (1.7^3 - 1.2^3) \\ &= 80.0477 \text{ cm}^3 \end{aligned}$$

$$\begin{aligned} \text{Number of boxes} &= \frac{250}{80.0477} \\ &= 3.123 \\ &\approx 3 \text{ boxes} \end{aligned}$$

9(b)



(i)

$$\frac{\sin 54^\circ}{a} = \frac{\sin 72^\circ}{8}$$

$$a = 6.805206 \text{ cm}$$

$$\sin 54^\circ = \frac{b}{6.805206}$$

$$b = 5.505527$$

$$\begin{aligned} \text{Height of pentagon} &= 6.805206 + 5.505527 \\ &= 12.3107 \\ &\approx 12.3 \text{ cm} \end{aligned}$$

(ii)

$$\begin{aligned} \text{Area of pentagon} &= 5 \times \frac{1}{2} \times 6.805206 \times 6.805206 \times \sin 72^\circ \\ &= 110.110 \\ &\approx 110 \text{ cm}^2 \end{aligned}$$

(c)

$$8 \times 134^\circ + 3x^\circ + x^\circ + 60^\circ = (10 - 2) \times 180^\circ = 1440^\circ$$

$$1072^\circ + 4x + 60^\circ = 1440^\circ$$

$$4x = 308$$

$$x = 77$$

$$\text{Hence largest angle} = 3x^\circ = 3 \times 77^\circ = 231^\circ$$

10

Answer the whole of this question on a sheet of graph paper.

- (a) The volume of a solid is given by the equation $V = x^3 + 3x$. The table below gives some values of x and the corresponding values of V that satisfy the equation.

x	0	1	2	3	4	5
V	0	4	14	p	76	q

- Calculate the value of p and of q . [2]
- (b) Using a scale of 2 cm to represent 1 unit on the horizontal x -axis and a scale of 2 cm to represent 20 units on the vertical V -axis, draw the graph of $V = x^3 + 3x$ for the values of x shown in the table. [3]
- (c) Use your graph to estimate
- (i) the volume of the solid if $x = 3.5$, [1]
- (ii) the value of x if the volume of the solid is 90 cm^3 . [1]
- (d) If this solid is melted down and made into a solid rectangular block of dimensions 3 cm, 5 cm and $(6 - x)$ cm, draw a suitable straight line on the same axes as in part (b) above in order to find the value of x . [3]

1. (a) $p = 36$, $q = 140$

(c) (i) If $x = 3.5$, $V = 52 \text{ cm}^3$

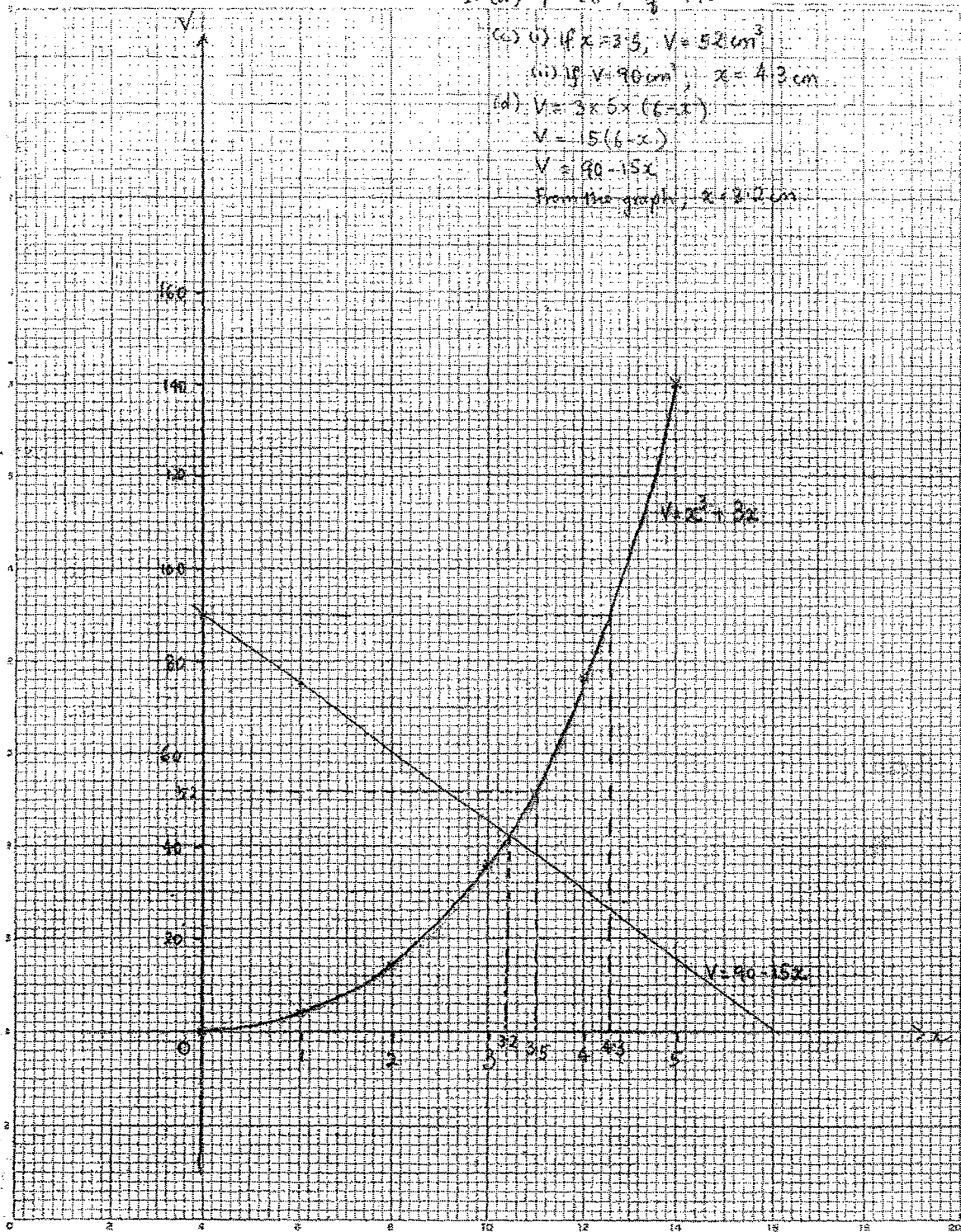
(ii) If $V = 90 \text{ cm}^3$, $x = 4.3 \text{ cm}$

(d) $V = 3 \times 5 \times (6 - x)$

$V = 15(6 - x)$

$V = 90 - 15x$

From the graph, $x = 2.2 \text{ cm}$



Markers

Q1,4, 6 Mr Martin Yong

Q2,3, 8 NgSS

Q9, 10 Ms Jessica Ng

Q5, 7 Mr Mok