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3

- 1 When written as a product of their prime factors,

$$A = 2 \times 3^2 \times 7,$$

$$B = 2^2 \times 3 \times 7^2,$$

Given that the HCF and LCM of A , B and C is 6 and 8820 respectively, find C .

Answer $C = \dots\dots\dots$ [2]

- 2 It is given that y is inversely proportional to x^2 . Find the percentage change in y when x is decreased by 20%.

Answer ... [2]

- 3 Solve the equation $4x^2 = 7x$:
Hence solve $4(y-1)^2 - 7(y-1) = 0$.

Answer $y = \dots\dots\dots$ [2]

- 4 (a) Solve the equation $4^{w^2} = 8^{\frac{4-7w}{3}}$.
- (b) Show that $3^{3m+2} - 9^{\frac{3}{2}m} + \left(\frac{1}{27}\right)^{-m-1}$ is divisible by 7 for all positive integer values of m .

Answer (a) $w = \dots\dots\dots$...[2]

-
- 5 The radius of a hydrogen atom is approximately 5 nanometres.
- (a) Calculate the length, in metres, of 46 of these atoms when they are lined up in a straight line. Give your answer in standard form.
- (b) If the shape of the atom is a sphere, calculate the volume of the atom, in m^3 , giving your answer in standard form correct to 2 decimal places.

Answer (a) $\dots\dots\dots$ m [1]

(b) $\dots\dots\dots$ m^3 [1]

- 6 A cup can hold 250 ml of coffee, correct to the nearest millilitre.
- (a) Find the least possible capacity of coffee in the cup.
- (b) The cup of coffee contains 124.7 mg of caffeine, correct to 1 decimal place.
Find the greatest possible mass of caffeine in 1 ml of coffee.

Answer (a)ml [1]

(b)mg [2]

- 7 During the Great Singapore Sale, a table is sold at a 40% discount. A further reduction of 20% is given on a discounted price if a discount coupon is used. A customer buys a table using a discount coupon. Calculate the original price if she pays only \$172.80 for the table.

Answer \$..... [2]

- 8 (a) When it is 12 00 in London, the time in Singapore is 20 00.
A plane leaves Singapore at 12 30 to fly to London. The flight takes 13 hours 42 minutes.
Calculate the time in London when it arrives.

Answer (a) [1]

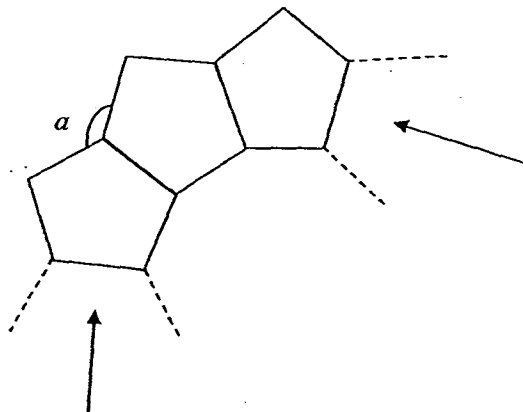
- (b) 8 men working 6 hours per day are to complete a drainage work in 5 days. After working for 2 days, 2 men left the job. How many hours per day must the men work on the remaining 3 days to complete the drainage work on time?

Answer (b)h [2]

- 9 The lengths of three square flower beds P , Q and R on the plan of a garden are in the ratio $2 : 3 : n$ where n is an integer. Given that the actual area of P is 64 m^2 and the total area of P , Q and R is 608 m^2 , find n .

Answer (a) $n = \dots\dots\dots$ [2]

- 10 A number of regular pentagons are placed together as shown in the diagram below to form a ring.



- Find (a) the value of a ,
(b) the number of pentagons in the ring.

Answer (a) $a = \dots\dots\dots$ [1]

(b) $\dots\dots\dots$ [1]

- 11 In the Venn diagram ε is the set of all students in a certain CCA group,

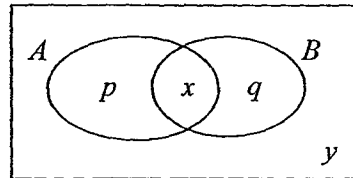
$A = \{\text{student in NCC}\}$

$B = \{\text{student in Brass Band}\}$.

The letters p , q , x and y in the diagram represent the number of student in each subset.

Given that $n(\varepsilon) = 200$, $n(A) = 75$ and $n(B) = 35$, find

- (a) The smallest possible value of y ,
 (b) The largest possible value of x ,
 (c) The value of q if $p = 45$.



Answer (a) $y = \dots\dots\dots$ [1]

(b) $x = \dots\dots\dots$ [1]

(c) $q = \dots\dots\dots$ [1]

- 12 (a) Find all the integers values of x that satisfy $-1 < \frac{2(7+5x)}{7} < 10$.

Answer (a) $x = \dots\dots\dots$ [3]

- 13 Write down an expression, in terms of n , for the n^{th} term of the following sequences.

(a) $-1, 2, 7, 14, 23, 34, \dots$

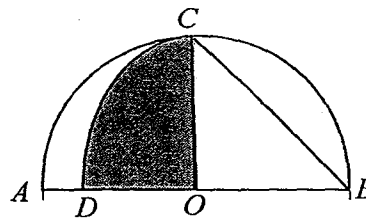
Answer (a) $\dots\dots\dots$ [2]

(b) $\dots\dots\dots$ [1]

- 14 $\{x-y, x-3, x-1, x, x+2, y-1, y, y+4\}$ is arranged in ascending order. The lower quartile and the upper quartile of the dataset are 8 and 14.5 respectively. Find the values of x and y .

Answer $x = \dots\dots\dots$, $y = \dots\dots\dots$ [4]

- 15 The diagram shows a semi-circle ABC with centre O and radius 5 cm such that $\angle BOC = 90^\circ$. Given that CD is an arc of a circle with centre B , calculate
- The length of the arc CD ,
 - The area of the shaded region,



Answer (a) cm [1]

(b) cm² [2]

- 16 (a) The day temperature in Singapore in January ranges from 24°C to 33°C while the day temperature at the Artic ranges from $-h^{\circ}\text{C}$ to $-k^{\circ}\text{C}$ where h and k are positive integers and $h > k$.

Find

- (i) the maximum difference in the day temperature between Singapore and the Artic,
 (ii) the mean day temperature at the Artic.

Answer (i) ... $\dots\dots\dots^{\circ}\text{C}$ [1]

(ii) ... $\dots\dots\dots^{\circ}\text{C}$ [1]

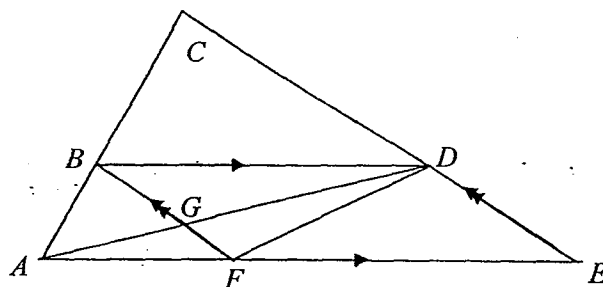
- (b) A car travels at a constant speed of 50 km/h for 60% of a journey, a constant speed of 40 m/h for 30% of the journey and the remaining journey at 20 km/h.

Given that the car travels a total distance of x km, calculate the average speed of the car in m/s.

Answer (b) $\dots\dots\dots\text{m/s}$ [2]

- 17 The points B , F and D on the sides of the $\triangle ACE$ are such that $BD \parallel AE$, $FB \parallel EC$ and BF meets AD at G .

(a) Name one triangle which is congruent to $\triangle FED$. Explain clearly the reason for your choice.



Answer (a)

.....

..... [3]

- (b) If $BD = 6$ cm and $AE = 9$ cm, find the numerical value of

- (i) $\frac{\text{Area of } \triangle BDG}{\text{Area of } \triangle FDG}$
- (ii) $\frac{\text{Area of } \triangle BDG}{\text{Area of } \triangle AED}$

Answer (i) [1]

(ii) [2]

- 18 (a) Express $-x^2 + 12x + 8$ in the form $b - (x - a)^2$, and hence sketch the graph of $-x^2 + 12x + 8$.

Answer (a) ... [3]

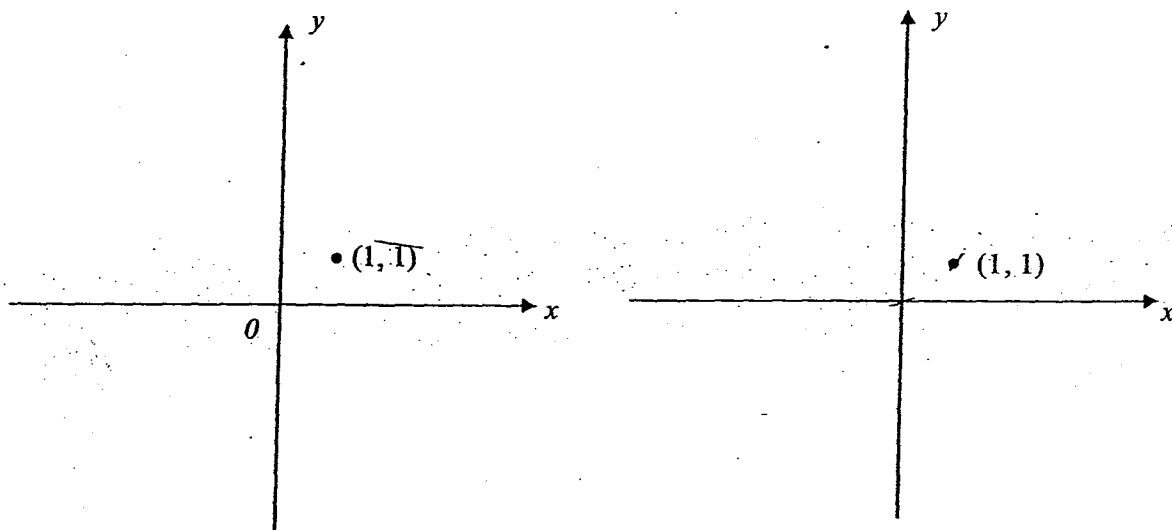
- (b) The point $(1, 1)$ is marked on the diagram below. Sketch the graphs of

(i) $x^2y = 2$ [1]

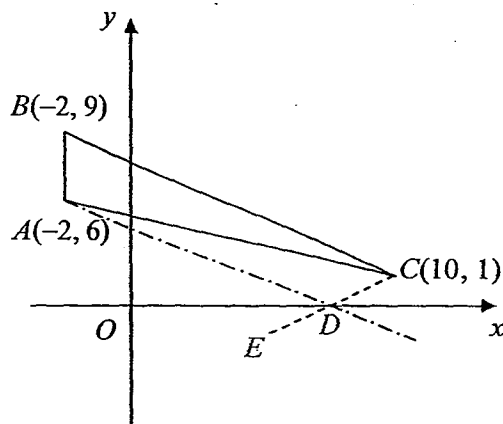
(ii) $y = 2^x - 1$ [1]

(i) $y = \frac{2}{x}$

(ii) $y = 2^x - 1$



19. The points $A(-2, 6)$, $B(-2, 9)$ and $C(10, 1)$ are shown in the diagram.
Find (a) the coordinates of D which lies on the x -axis such that AD is parallel to BC .



- (b) the coordinates of E which lies on CD produced such that $CD = DE$.

- (c) the value of $\cos \angle BAC$.

Answer (a) D (.....) [2]

(b) E (.....) [1]

(c) [2]

- 20 The probability of passing the practical test for a driving license at the first attempt is 0.6. If a candidate fails, he may sit for at most two re-tests and the probability of passing the re-test is 0.8.
- Represent the above information on a probability tree diagram.
 - Find the probability that the candidate passes at the third attempt.
 - Describe a situation where the probability is 0.32.

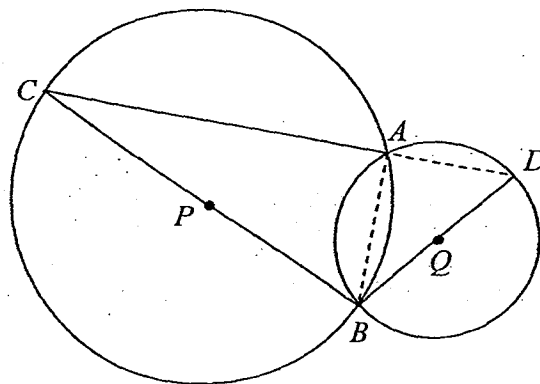
[1]

[1]

Answer (c)

[1]

- 21 In the diagram, a large circle with centre P and a small circle with centre Q intersect at A and B . BC and BD are the diameters of the large circle and the small circle respectively. Given that the diameter of the large circle is twice the diameter of the small circle and $\angle ACB = 20^\circ$.
- Show that CAD is a straight line.
 - Find the size of $\angle CBD$.

Answer $\angle CBD = \dots\dots\dots$ [4]

- 22 A swimming pool can be filled by a large pipe operating alone in 3 hours. If the pool is filled by a small pipe alone, it will take t hours longer. Given that it will take 2 hours for both pipes to fill up the pool, find the value of t .

Answer $t = \dots\dots\dots$ h [2]

- 23 A frustum of height h was cut out from a cone of height $2h$, as shown in Diagram I. It was then attached to a cylinder of height h to form a new container as shown in Diagram II. Water was poured into the empty container in Diagram II at a constant rate and it took 60 minutes to fill to the brim.
- (a) Find the time for water to reach a height of h .
- (b) On the grid in the answer space, sketch the graph of the depth of water (d) against the time (t).

Answer (b) [1]

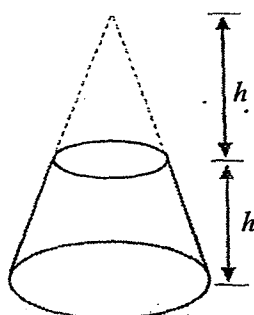


Diagram I

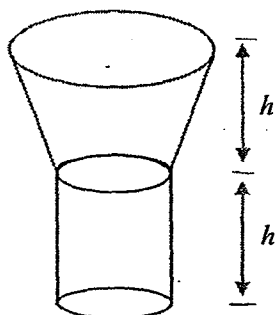
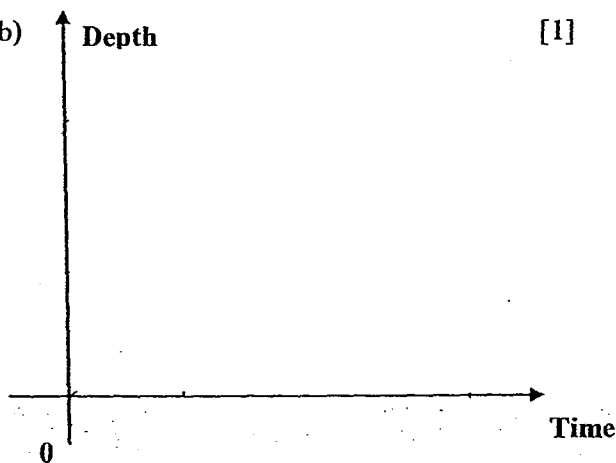


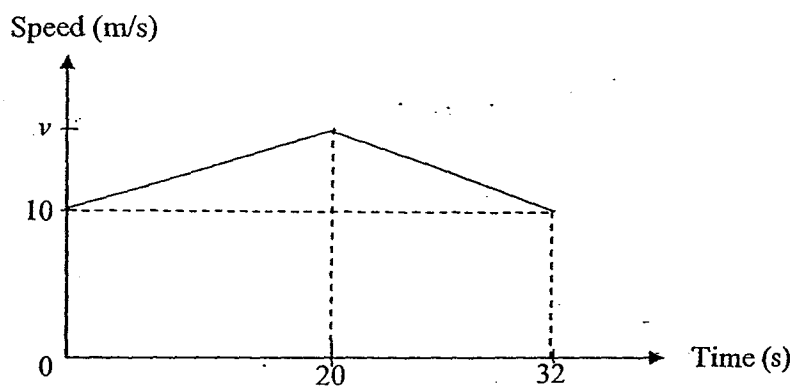
Diagram II



Answer (a) $\dots\dots\dots$ min [2]

- 24 The diagram shows the speed-time graph of a car during a period of 32 seconds. The distance travelled in the first 20 seconds is 250 metres.

- (a) Find the maximum speed v .
 (b) Calculate the retardation during the motion.
 (c) Draw the distance-time graph for the entire journey.



Answer (a) m/s [2]

(b) m/s² [1]

Answer (c)

[1]

- 25 The answer space below shows line AC . Construct $\triangle ABC$ where $AB = 8$ cm and $BC = 9$ cm. [1]

It is given that point X is inside $\triangle ABC$, and X is

- (a) nearer to AC than BC [1]
(b) nearer to A than C . [1]

Shade the region in which point X must lie. [1]

END OF PAPER

3

- 1 When written as a product of their prime factors,

$$A = 2 \times 3^2 \times 7,$$

$$B = 2^2 \times 3 \times 7^2,$$

Given that the HCF and LCM of A , B and C is 6 and 8820 respectively, find C .

$$\text{HCF} = 6 = 2 \times 3$$

M1

$$\text{LCM} = 8820 = 2^2 \times 3^2 \times 5 \times 7^2$$

$$C = 2 \times 3 \times 5 = 30$$

A1

Answer $C = \dots\dots\dots 30 \dots\dots\dots$ [2]

- 2 It is given that y is inversely proportional to x^2 . Find the percentage change in y when x is decreased by 20%.

$$y = \frac{k}{x^2}$$

$$y_1 = \frac{k}{\left(\frac{4}{5}x^2\right)} = \frac{25}{16} \left(\frac{k}{x^2}\right)$$

or

$$y_1 = \frac{k}{(0.8x^2)} = \frac{yx^2}{0.64x^2}$$

M1

$$y_1 = \frac{25}{16}y$$

$$= 1.5625y$$

$$\therefore y \text{ increases by } \frac{9}{16} \times 100\% = 56.25\%$$

A1

Answer y increases by 56.25% [2]

- 3 Solve the equation $4x^2 = 7x$.
Hence solve $4(y-1)^2 - 7(y-1) = 0$.

$$4x^2 - 7x = 0$$

$$x(4x - 7) = 0$$

$$x = 0 \text{ or } x = \frac{7}{4}$$

A1

$$y - 1 = 0 \text{ or } y - 1 = \frac{7}{4}$$

$$\therefore y = 1 \text{ or } y = 2\frac{3}{4}$$

A1

Answer $y = \dots\dots\dots 1 \text{ or } 2\frac{3}{4} \dots\dots\dots$ [2]

- 4 (a) Solve the equation $4^{w^2} = 8^{\frac{4-7w}{3}}$.
- (b) Show that $3^{3m+2} - 9^{\frac{3}{2}m} + \left(\frac{1}{27}\right)^{-m-1}$ is divisible by 7 for all positive integer values of m .

$$\begin{aligned}
 \text{(a)} \quad 4^{w^2} &= 8^{\frac{4-7w}{3}} \\
 2^{2w^2} &= 2^{4-7w} && \text{M1} \\
 2w^2 &= 4-7w \\
 2w^2 + 7w - 4 &= 0 \\
 (2w-1)(w+4) &= 0 \\
 w &= \frac{1}{2} \text{ or } -4 && \text{A1}
 \end{aligned}$$

Answer (a) $w = \dots\dots\dots \frac{1}{2} \text{ or } -4 \dots\dots\dots [2]$

$$\begin{aligned}
 \text{Answer (b)} \quad 3^{3m+2} - 9^{\frac{3}{2}m} + \left(\frac{1}{27}\right)^{-m-1} \\
 &= 3^{3m+2} - 3^{3m} + 3^{3m+3} && \text{M1} \\
 &= 3^2(3^{3m}) - 3^{3m} + 3^3(3^{3m}) \\
 &= 35(3^{3m}) \\
 &= 7 \times 5(3^{3m}) \text{ is divisible by 7} && \text{A1}
 \end{aligned}$$

- 5 The radius of a hydrogen atom is approximately 5 nanometres.
- (a) Calculate the length, in metres, of 46 of these atoms when they are lined up in a straight line. Give your answer in standard form.
- (b) If the shape of the atom is a sphere, calculate the volume of the atom, in m^3 , giving your answer in standard form correct to 2 decimal places.

$$\begin{aligned}
 \text{(a) Length of the atoms} &= 46 \times 5 \times 10^{-9} \times 2 \\
 &= 4.6 \times 10^{-7} \text{ m} && \text{A1}
 \end{aligned}$$

$$\begin{aligned}
 \text{(b) Volume of the atoms} &= \frac{4}{3} \pi (5 \times 10^{-9})^3 \\
 &= 523.60 \times 10^{-27} \\
 &= 5.24 \times 10^{-25} \text{ m}^3 && \text{A1}
 \end{aligned}$$

Answer (a) $\dots\dots\dots 4.6 \times 10^{-7} \dots\dots\dots \text{m} [1]$

(b) $\dots\dots\dots 5.24 \times 10^{-25} \dots\dots\dots \text{m}^3 [1]$

- 6 A cup can hold 250 ml of coffee, correct to the nearest millilitre.
 (a) Find the least possible capacity of coffee in the cup.
 (b) The cup of coffee contains 124.7 mg of caffeine, correct to 1 decimal place.
 Find the greatest possible mass of caffeine in 1 ml of coffee.

$$\begin{aligned} \text{(b) The greatest possible mass} &= \frac{124.75}{249.5} \\ &= 0.5 \text{ mg} \end{aligned}$$

M1

A1

Answer (a) 249.5 A1 ml [1]

(b) 0.5 mg [2]

- 7 During the Great Singapore Sale, a table is sold at a 40% discount. A further reduction of 20% is given on a discounted price if a discount coupon is used. A customer buys a table using a discount coupon. Calculate the original price if she pays only \$172.80 for the table.

$$\begin{aligned} 80\% \times 60\% \text{ CP} &= 172.80 & \text{M1} \\ \text{CP} &= \$360 & \text{A1} \end{aligned}$$

Answer \$ 360 [2]

- 8 (a) When it is 12 00 in London, the time in Singapore is 20 00.
 A plane leaves Singapore at 12 30 to fly to London. The flight takes 13 hours 42 minutes.
 Calculate the time in London when it arrives.

London	Singapore	
12 00	20 00	
04 30	12 30	A1
13 42		
18 12		

Answer (a) 18 12 [1]

- (b) 8 men working 6 hours per day are to complete a drainage work in 5 days. After working for 2 days, 2 men left the job. How many hours per day must the men work on the remaining 3 days to complete the drainage work on time?

8 men working 6 hours per day for 2 days complete $\frac{2}{5}$ of the job.

6 men working x hours per day for 3 days complete $\frac{3}{5}$ of the job.

$$\frac{8 \times 6 \times 2}{6 \times 3x} = \frac{2}{3} \quad \text{M1}$$

$$x = 8 \quad \text{A1}$$

Answer (b) 8 h [2]

- 9 The lengths of three square flower beds P , Q and R on the plan of a garden are in the ratio $2 : 3 : n$ where n is an integer. Given that the actual area of P is 64 m^2 and the total area of P , Q and R is 608 m^2 , find n .

	P	:	Q	:	R
Length	2	:	3	:	n
Area	4	:	9	:	n^2
Actual Area	$64 \text{ m}^2 : 144 \text{ m}^2 : (608 - 64 - 144) \text{ m}^2 = 400 \text{ m}^2$				

M1

$$16n^2 = 400$$

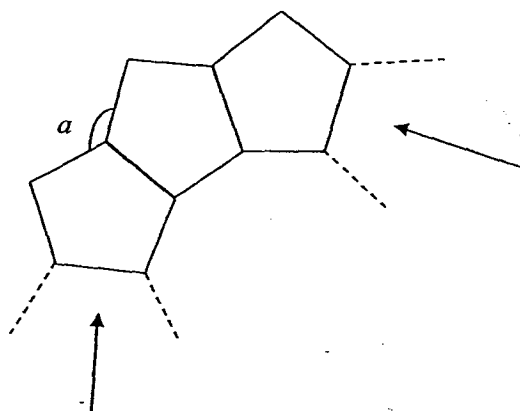
$$n^2 = 25$$

$$n = 5$$

A1

Answer (a) $n = \dots\dots\dots 5 \dots\dots\dots$ [2]

- 10 A number of regular pentagons are placed together as shown in the diagram below to form a ring.



- Find (a) the value of a ,
(b) the number of pentagons in the ring.

(a) Each interior angle of pentagon $= 108^\circ$

$$a = 360^\circ - 2 \times 108^\circ$$

$$= 144^\circ$$

A1

$$\text{No of pentagons} = \frac{360^\circ}{180^\circ - 144^\circ} = \frac{360^\circ}{36^\circ} = 10$$

A1

Answer (a) $a = \dots\dots\dots 144^\circ \dots\dots\dots$ [1]

(b) $\dots\dots\dots 10 \dots\dots\dots$ [1]

- 11 In the Venn diagram ε is the set of all students in a certain CCA group,

$A = \{\text{student in NCC}\}$

$B = \{\text{student in Brass Band}\}$.

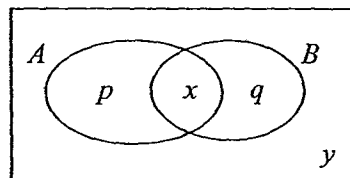
The letters p, q, x and y in the diagram represent the number of student in each subset.

Given that $n(\varepsilon) = 200$, $n(A) = 75$ and $n(B) = 35$, find

(a) The smallest possible value of y ,

(b) The largest possible value of x ,

(c) The value of q if $p = 45$.



(a) When $x = 0$, $y = 200 - 75 - 35 = 90$.

A1

(b) When $B \subset A$, $x = 35$.

A1

(c) When $p = 45$, $x = 30$

$$\therefore q = 35 - 30 = 5$$

A1

Answer (a) $y = \dots\dots\dots 90 \dots\dots\dots$ [1]

(b) $x = \dots\dots\dots 35 \dots\dots\dots$ [1]

(c) $q = \dots\dots\dots 5 \dots\dots\dots$ [1]

- 12 (a) Find all the integers values of x that satisfy $-1 < \frac{2(7+5x)}{7} < 10$.

$$\begin{array}{ll} \text{(a) } -1 < \frac{2(7+5x)}{7} & \text{and} \quad \frac{2(7+5x)}{7} < 10 \\ -7 < 14 + 10x & \text{and} \quad 14 + 10x < 70 \\ -21 < 10x & \text{and} \quad 10x < 56 \\ x > -2.1 & \text{and} \quad x < 5.6 \end{array}$$

M1

$$\therefore -2.1 < x < 5.6$$

A1

A1

Answer (a) $x = \underline{-2, -1, 0, 1, 2, 3, 4 \text{ and } 5}$ [3]

- 13 Write down an expression, in terms of n , for the n^{th} term of the following sequences.

(a) $-1, 2, 7, 14, 23, 34, \dots$

(+2)

$\Rightarrow 1, 4, 9, 16, 25, 36, \dots \quad n^2$

(b) $\frac{2}{3}, \frac{5}{9}, \frac{8}{27}, \frac{11}{81}, \frac{14}{243}, \dots$

Answer (a) $\dots\dots\dots n^2 - 2 \dots\dots\dots$ A2.... [2]

(b) $\dots\dots\dots \frac{3n-1}{3^n} \dots\dots\dots$ A1... [1]

- 14 $\{x-y, x-3, x-1, x, x+2, y-1, y, y+4\}$ is arranged in ascending order. The lower quartile and the upper quartile of the dataset are 8 and 14.5 respectively. Find the values of x and y .

$$\{x-y, x-3, x-1, x, x+2, y-1, y, y+4\}$$

$$\begin{array}{ccc} \downarrow & & \downarrow \\ 8 & \text{Median} & 14.5 \end{array}$$

$$\begin{array}{ll} \text{M1} & \text{M1} \\ \frac{x-3+x-1}{2} = 8 & \text{and} \quad \frac{y-1+y}{2} = 14.5 \\ \text{(b)} \quad 2x-4=16 & 2y-1=29 \\ x=10 & y=15 \end{array}$$

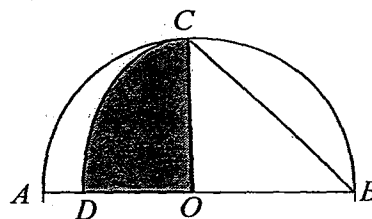
A1

A1

Answer $x = \dots 10 \dots$, $y = \dots 15 \dots$ [4]

- 15 The diagram shows a semi-circle ABC with centre O and radius 5 cm such that $\angle BOC = 90^\circ$. Given that CD is an arc of a circle with centre B , calculate

- (a) The length of the arc CD ,
(b) The area of the shaded region,



$$\begin{aligned} \text{(a)} \quad BC &= \sqrt{5^2 + 5^2} = \sqrt{50} \\ \text{Length of arc } CD &= \frac{45}{360} \times 2\pi\sqrt{50} \\ &= 5.55 \text{ cm} \end{aligned}$$

A1

$$\begin{aligned} \text{(b)} \quad \text{Area of shaded region} &= \frac{45}{360} \times \pi (\sqrt{50})^2 - \frac{1}{2} \times 5 \times 5 \\ &= 7.1349 \text{ cm}^2 \end{aligned}$$

M1

A1

Answer (a) $\dots 5.55 \dots$ cm [1](b) $\dots 7.13 \dots$ cm² [2]

- 16 (a) The day temperature in Singapore in January ranges from 24°C to 33°C while the day temperature at the Arctic ranges from $-h^{\circ}\text{C}$ to $-k^{\circ}\text{C}$ where h and k are positive integers and $h > k$.

Find

- (i) the maximum difference in the day temperature between Singapore and the Arctic,
 (ii) the mean day temperature at the Arctic.

(i) The max difference $= 33^{\circ} - (-h)$

$$= (33 + h)^{\circ}\text{C} \quad \text{A1}$$

(ii) Mean $= \frac{(-h) + (-k)}{2} = -\frac{(h+k)}{2} \quad \text{A1}$

Answer (i) $(33 + h)$ $^{\circ}\text{C}$ [1]

(ii) ... $-\frac{(h+k)}{2}$ $^{\circ}\text{C}$ [1]

- (b) A car travels at a constant speed of 50 km/h for 60% of a journey, a constant speed of 40 km/h for 30% of the journey and the remaining journey at 20 km/h .

Given that the car travels a total distance of $x\text{ km}$, calculate the average speed of the car in m/s .

$$\begin{aligned} \text{Total time taken for the whole journey} &= \frac{0.6x}{50} + \frac{0.3x}{40} + \frac{0.1x}{20} & \text{M1} \\ &= 0.0245x \end{aligned}$$

$$\text{Average speed} = x \div 0.0245x$$

$$= 40.816\text{ km/h}$$

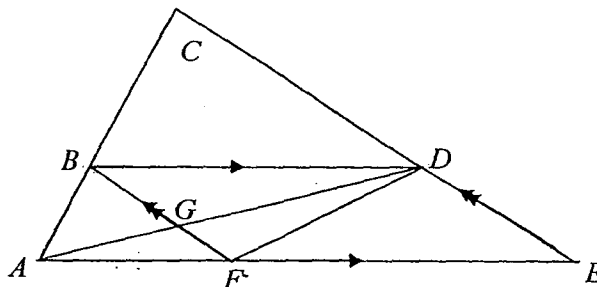
$$= \frac{40816\text{ m}}{3600\text{ s}}$$

$$= 11.3\text{ m/s} \quad \text{A1}$$

Answer (b) 11.3 m/s [2]

17. The points B , F and D on the sides of the $\triangle ACE$ are such that $BD \parallel AE$, $FB \parallel EC$ and BF meets AD at G .

(a) Name one triangle which is congruent to $\triangle FED$. Explain clearly the reason for your choice.



Answer (a) $\triangle FED \equiv \triangle DBF$ A1

In $\triangle FED$ and $\triangle DBF$

$$\angle FDB = \angle DFE \text{ (alt } \angle\text{s, } FE \parallel BD \text{)}$$

$$\angle BFD = \angle EDF \text{ (alt } \angle\text{s, } ED \parallel FB \text{)}$$

DF is a common side

$$\therefore \triangle FED \equiv \triangle DBF \text{ (AAS or ASA)}$$

M1

A1

[3]

- (b) If $BD = 6$ cm and $AE = 9$ cm, find the numerical value of

(i) $\frac{\text{Area of } \triangle BDG}{\text{Area of } \triangle FDG}$

(ii) $\frac{\text{Area of } \triangle BDG}{\text{Area of } \triangle AED}$

- (ii) $\triangle BDG$ is similar to $\triangle EAD$.

$$\frac{\text{Area of } \triangle BDG}{\text{Area of } \triangle AED} = \left(\frac{6}{9}\right)^2 = \frac{4}{9}$$

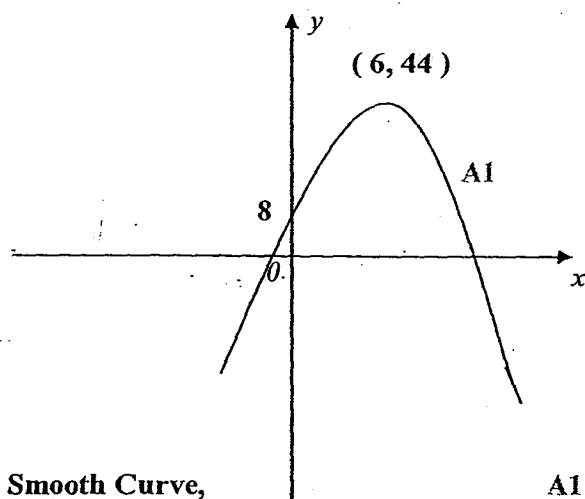
M1 A1

A1

Answer (i)2..... [1]

(ii)4/9 [2]

- 18 (a) Express $-x^2 + 12x + 8$ in the form $b - (x - a)^2$, and hence sketch the graph of $-x^2 + 12x + 8$.



$$\begin{aligned} -x^2 + 12x + 8 &= -(x^2 - 12x + 6^2 - 6^2 - 8) \\ &= -(x - 6)^2 + 44 \quad \text{A1} \end{aligned}$$

Turning Point : (6, 44)
y-intercept : 8

Smooth Curve,
Turning point and y-intercept labelled

A1

Answer (a) $44 - (x - 6)^2$ [3]

- (b) The point (1, 1) is marked on the diagram below. Sketch the graphs of

(i) $x^2y = 2$

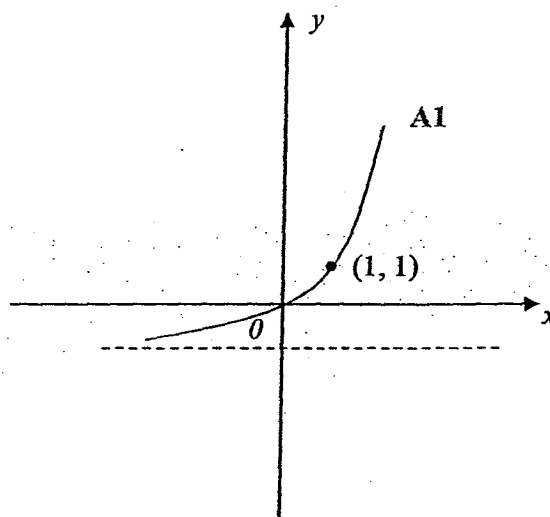
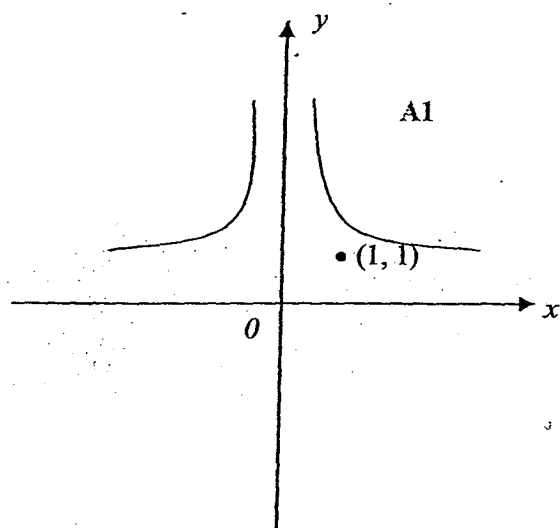
[1]

(ii) $y = 2^x - 1$

[1]

(i) $y = \frac{2}{x}$

(ii) $y = 2^x - 1$



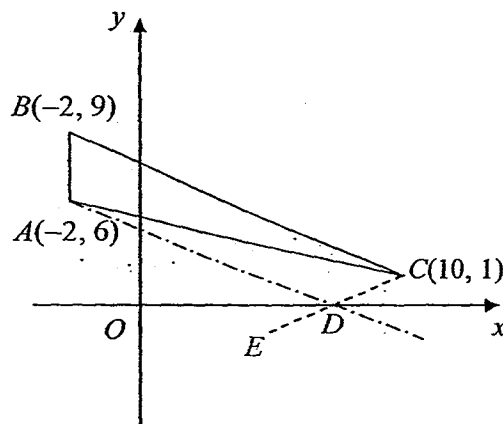
- 19 The points $A(-2, 6)$, $B(-2, 9)$ and $C(10, 1)$ are shown in the diagram.
Find (a) the coordinates of D which lies on the x -axis such that AD is parallel to BC .

(a) Let coordinate of D be $(d, 0)$

$$\frac{9-1}{-2-10} = \frac{0-6}{d-(-2)} \quad \text{M1}$$

$$\frac{8}{-12} = \frac{-6}{d+2} \quad \text{A1}$$

$$d = 7$$



(b) the coordinates of E which lies on CD produced such that $CD = DE$.

Coordinate of $E(4, -1)$ A1

(c) the value of $\cos \angle BAC$.

$$AC = \sqrt{12^2 + 5^2} = 13 \quad \text{M1}$$

$$\therefore \cos \angle BAC = -\frac{5}{13} \quad \text{A1}$$

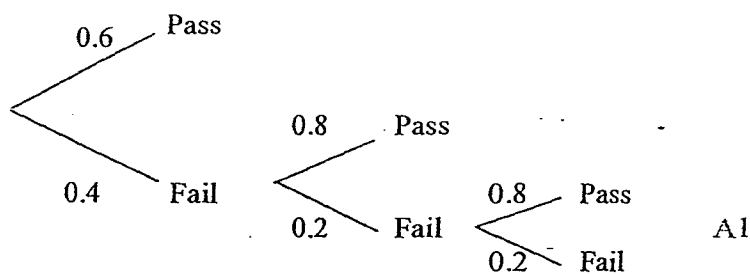
Answer (a) $D(.....7....., ...0.....)$ [2]

(b) $E(.....4....., ...-1.....)$ [1]

(c) $.....-\frac{5}{13}.....$ [2]

- 20 The probability of passing the practical test for a driving license at the first attempt is 0.6. If a candidate fails, he may sit for at most two re-tests and the probability of passing the re-test is 0.8.
- Represent the above information on a probability tree diagram.
 - Find the probability that the candidate passes at the third attempt.
 - Describe a situation where the probability is 0.32.

(a) 1st attempt 2nd attempt 3rd attempt



[1]

(b) $P(\text{passes at the 3rd attempt}) = 0.4 \times 0.2 \times 0.8$
 $= 0.064$

A1

[1]

Answer (c) A candidate passes at the 2nd attempt.

A1

[1]

- 21 In the diagram, a large circle with centre P and a small circle with centre Q intersect at A and B . BC and BD are the diameters of the large circle and the small circle respectively. Given that the diameter of the large circle is twice the diameter of the small circle and $\angle ACB = 20^\circ$.

- Show that CAD is a straight line.
- Find the size of $\angle CBD$.

(a) $\angle BAC = \angle BAD = 90^\circ$ ($\angle$ s in semicircle)

$\therefore CAD$ is a straight line

B1

(b) $\frac{a}{\sin 20^\circ} = \frac{2a}{\sin \angle BDC}$

M1

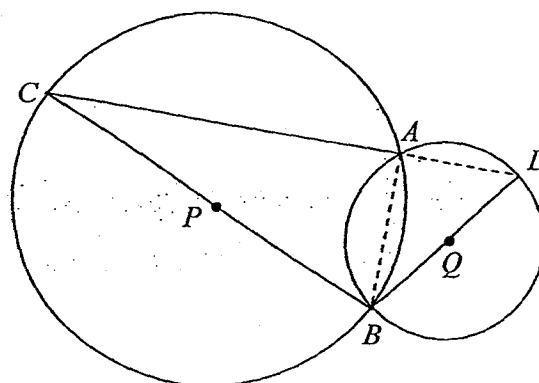
$\angle BDC = 43.16^\circ$

A1

$\therefore \angle CBD = 180^\circ - 20^\circ - 43.16^\circ$

$= 116.8^\circ$

A1



Answer $\angle CBD = \dots\dots\dots 116.8^\circ \dots\dots\dots$ [4]

- 22 A swimming pool can be filled by a large pipe operating alone in 3 hours. If the pool is filled by a small pipe alone, it will take t hours longer. Given that it will take 2 hours for both pipes to fill up the pool, find the value of t .

Large pipe fill up $\frac{1}{3}$ of the pool in 1 hour.

Small pipe fill up $\frac{1}{3+t}$ of the pool in 1 hour.

$$\frac{1}{3} + \frac{1}{3+t} = \frac{1}{2} \quad \text{M1}$$

$$2(3+t) + 6 = 3(3+t)$$

$$6 + 2t + 6 = 9 + t$$

$$t = 3$$

A1

Answer $t = \dots\dots\dots 3 \dots\dots\dots$ h [2]

- 23 A frustum of height h was cut out from a cone of height $2h$, as shown in Diagram I. It was then attached to a cylinder of height h to form a new container as shown in Diagram II. Water was poured into the empty container in Diagram II at a constant rate and it took 60 minutes to fill to the brim.

(a) Find the time for water to reach a height of h .

(b) On the grid in the answer space, sketch the graph of the depth of water (d) against the time (t).

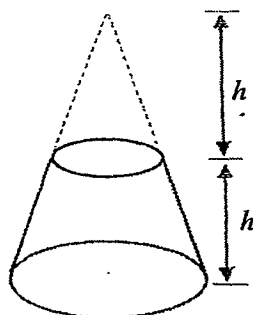


Diagram I

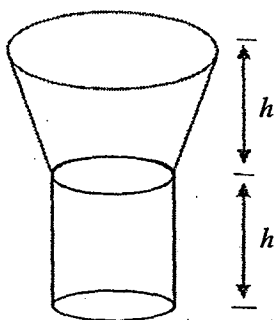
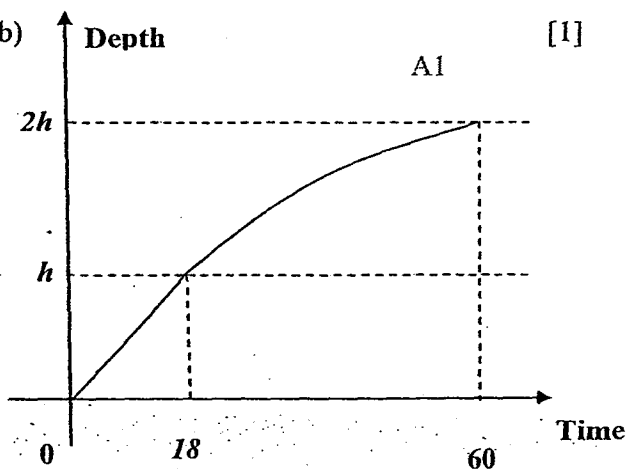


Diagram II

Answer (b)



[1]

A1

(a) $\frac{\text{Volume of Frustum}}{\text{Volume of cylinder}} = \frac{7}{3}$

10 u — 60 min

3 u — 18 min

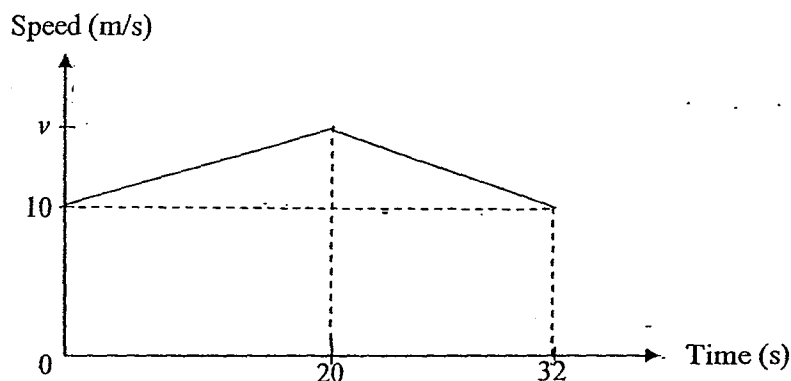
M1

A1

Answer (a) $\dots\dots\dots 18 \dots\dots\dots$ min [2]

24 The diagram shows the speed-time graph of a car during a period of 32 seconds. The distance travelled in the first 20 seconds is 250 metres.

- Find the maximum speed v .
- Calculate the retardation during the motion.
- Draw the distance-time graph for the entire journey.



$$(a) \frac{1}{2}(10 + v) \times 20 = 250 \quad \text{M1}$$

$$10 + v = 25$$

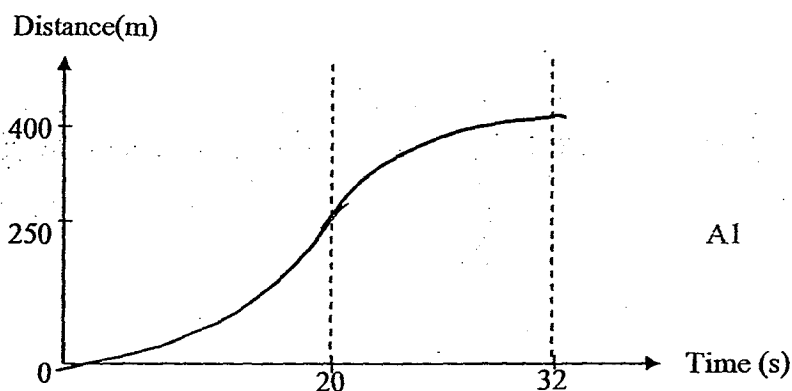
$$v = 15 \text{ m/s} \quad \text{A1}$$

$$(b) \text{Retardation} = \frac{5}{12} \text{ m/s}^2 \quad \text{A1}$$

$$\text{Answer (a)} \dots\dots\dots 15 \dots\dots\dots \text{m/s} \quad [2]$$

$$(b) \dots\dots\dots \frac{5}{12} \dots\dots\dots \text{m/s}^2 \quad [1]$$

$$\text{Answer (c)} \quad [1]$$



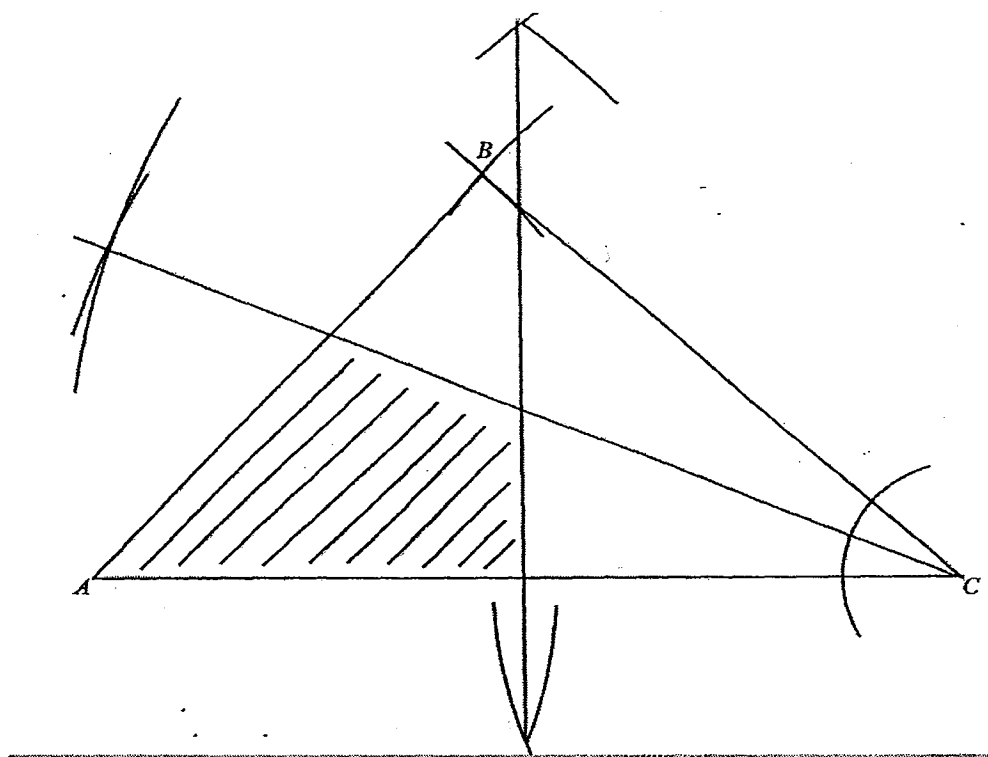
- 25 The answer space below shows line AC . Construct $\triangle ABC$ where $AB = 8$ cm and $BC = 9$ cm. [1]

It is given that point X is inside $\triangle ABC$, and X is

(a) nearer to AC than BC [1]

(b) nearer to A than C . [1]

Shade the region in which point X must lie. [1]



END OF PAPER

Name: Marking Scheme	Register No.:	Class:
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**CRESCENT GIRLS' SCHOOL
SECONDARY FOUR
PRELIMINARY EXAMINATION**

MATHEMATICS

Paper 2

4016/02

20 August 2014

2 hours 30 minutes

Additional Materials: Answer Paper
Graph Paper (1 sheet)
Mark Sheet

READ THESE INSTRUCTIONS FIRST

Write your name, register number and class on all the work you hand in.

Write in dark blue or black pen on both sides of the paper.

You may use a pencil for any diagrams or graphs.

Do not use paper clips, highlighters, glue or correction fluid.

Answer all the questions.

If working is needed for any question it must be shown with the answer.

Omission of essential working will result in loss of marks.

Calculators should be used where appropriate.

If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.

For π , use either your calculator value or 3.142, unless the question requires the answer in terms of π .

At the end of the examination, fasten all your work securely together with the mark sheet at the top.

The number of marks is given in brackets [] at the end of each question or part question.

The total number of marks for this paper is 100.

For Examiner's Use	
	100

Mathematical Formulae

Compound interest

$$\text{Total amount} = P \left(1 + \frac{r}{100} \right)^n$$

Mensuration

$$\text{Curved surface area of a cone} = \pi r l$$

$$\text{Surface area of a sphere} = 4\pi r^2$$

$$\text{Volume of a cone} = \frac{1}{3}\pi r^2 h$$

$$\text{Volume of a sphere} = \frac{4}{3}\pi r^3$$

$$\text{Area of triangle } ABC = \frac{1}{2}ab \sin C$$

$$\text{Arc length} = r\theta, \text{ where } \theta \text{ is in radians}$$

$$\text{Sector area} = \frac{1}{2}r^2\theta, \text{ where } \theta \text{ is in radians}$$

Trigonometry

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

Statistics

$$\text{Mean} = \frac{\sum fx}{\sum f}$$

$$\text{Standard deviation} = \sqrt{\frac{\sum fx^2}{\sum f} - \left(\frac{\sum fx}{\sum f} \right)^2}$$

3

Answer all the questions.

- 1 (a) It is given that $\frac{y}{5} = \frac{3z}{x-4z} - \frac{2}{3(x-4z)}$. Express z in terms of x and y in its lowest terms. [3]
- (b) Factorise completely $p^2 + 5pq + 6q^2 - 3p - 9q$. [2]
- (c) Express $\frac{5}{12x-8y} + \frac{y-2x}{2y^2-xy-3x^2}$ as a single fraction in its simplest form. [2]

Solution:

- 1 (a)
$$\frac{y}{5} = \frac{3z}{x-4z} - \frac{2}{3(x-4z)}$$

$$\frac{y}{5} = \frac{9z-2}{3(x-4z)}$$

$$3xy - 12yz = 45z - 10 \quad [\text{M1}]$$

$$z(45 + 12y) = 3xy + 10 \quad [\text{M1}]$$

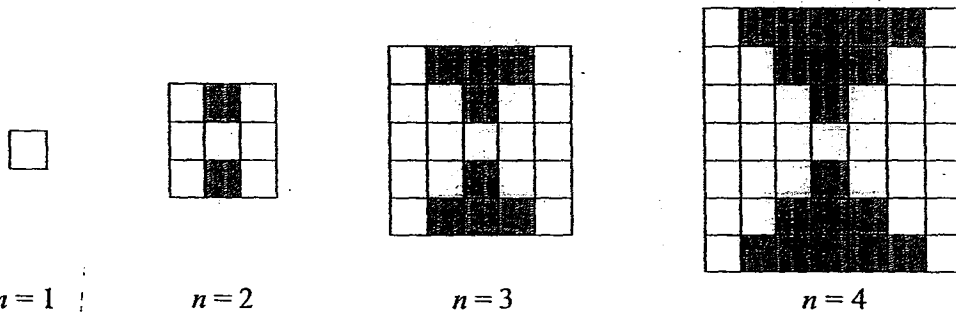
$$z = \frac{3xy + 10}{45 + 12y} \quad [\text{A1}]$$
- (b)
$$p^2 + 5pq + 6q^2 - 3p - 9q = (p+2q)(p+3q) - 3(p+3q) \quad [\text{M1}]$$

$$= (p+3q)(p+2q-3) \quad [\text{A1}]$$
- (c)
$$\frac{5}{12x-8y} + \frac{y-2x}{2y^2-xy-3x^2} = \frac{5}{4(3x-2y)} + \frac{y-2x}{(2y-3x)(y+x)}$$

$$= \frac{5(y+x) - 4(y-2x)}{4(3x-2y)(y+x)} \quad [\text{M1}]$$

$$= \frac{y+13x}{4(3x-2y)(y+x)} \quad [\text{A1}]$$

- 2 The diagram below shows a sequence of shapes. Each shape consists of white squares (W) and shaded squares (S). The letter T represents the total number of squares.



n	No. of shaded squares (S)	No. of white squares (W)	Total squares (T)
1	$0 = 2 \times 0$	1	1
2	$2 = 2 \times 1$	7	9
3	$8 = 2 \times 4$	17	25
4	$18 = 2 \times 9$	31	49
5	x	y	z

- (a) Write down the values of x , y and z . [2]
- (b) Write down, in terms of n , an expression for W . [2]
- (c) If there are 161 white squares, find the number of shaded squares. [2]

Solution:

- 2 (a) $x = 32$, $y = 49$, $z = 81$ [B2 for all correct, B1 for any 2 correct answers]

(b) $W = T - S$

$$= (2n-1)^2 - 2(n-1)^2 \quad [\text{M1}]$$

$$= 4n^2 - 4n + 1 - 2n^2 + 4n - 2$$

$$= 2n^2 - 1 \quad [\text{A1}]$$

(c) $2n^2 - 1 = 161$

$$n^2 = 81$$

$$n = 9 \quad \text{or} \quad n = -9 \quad (\text{NA}) \quad [\text{M1}]$$

$$S = 2(8)^2$$

$$= 128 \quad [\text{A1}]$$

3

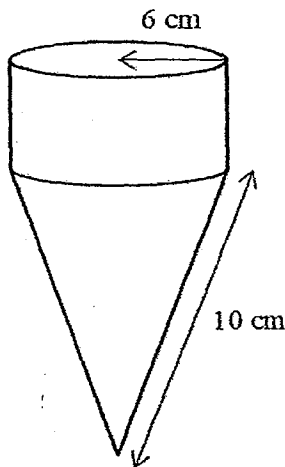


Diagram I

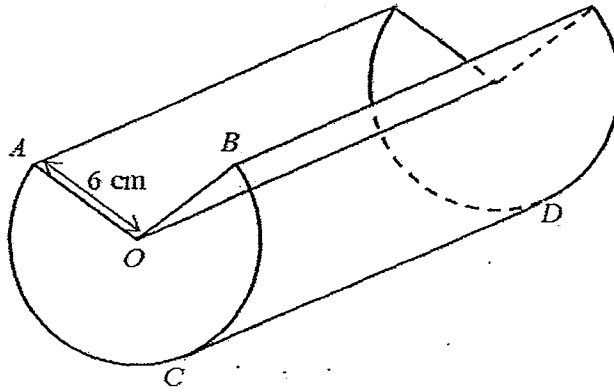


Diagram II

Diagram I shows a solid that is made up of a cone of radius 6 cm and slant height of 10 cm and a cylinder of radius 6 cm.

- (a) Find the volume of the cone, leaving your answer in terms of π . [2]

The volume of the cylinder is twice the volume of the cone. The solid is melted completely and recast to form a solid prism as shown in Diagram II. The cross section of the prism is a sector of a circle with centre O , radius 6 cm and $\angle AOB = \frac{2}{3}\pi$ rad.

- (b) Calculate the length CD of the prism. [2]

- (c) Find the total surface area of the prism. [3]

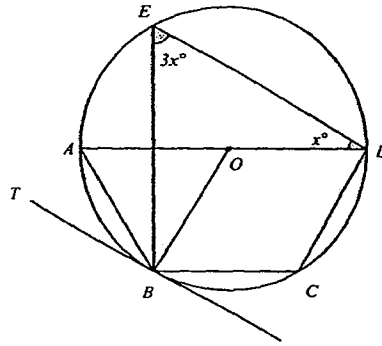
Solution:

$$\begin{aligned} 3 \quad (a) \quad \text{Volume of cone} &= \frac{1}{3} \times \pi \times 6^2 \times \sqrt{10^2 - 6^2} && [\text{M1 - finding height of cone}] \\ &= 192\pi && [\text{A1}] \end{aligned}$$

$$\begin{aligned} (b) \quad \text{Cross sectional area} &= \frac{1}{2} \times 6^2 \times \frac{4}{3}\pi && [\text{M1}] \\ &= 24\pi \\ CD &= \frac{96\pi \times 3}{24\pi} && [\text{A1}] \\ &= 12 \text{ cm} \end{aligned}$$

$$\begin{aligned} (c) \quad \text{Total surface area} &= \overbrace{2(24\pi) + 2(6 \times 12)}^{[\text{M1}]} + \overbrace{\frac{4}{3}\pi \times 6 \times 12}^{[\text{A1}]} \\ &= 48\pi + 144 + 96\pi \\ &= 596 \text{ cm}^2 \text{ (3sf)} \end{aligned}$$

4



In the diagram, O is the centre of the circle $ABCDE$. AOD is a straight line and BT is a tangent to the circle at B . It is given that $\angle EDA = x^\circ$ and $\angle BED = 3\angle EDA$.

- (a) Stating all reasons clearly, find in terms of x , in its simplest form,
- (i) $\angle AOB$, [2]
- (ii) $\angle EBT$. [3]
- (b) Given that $\angle AFD = x^\circ$, does the point F lies on the circumference of the circle, inside the circle or outside the circle? Briefly explain your answer. [1]
- (c) Is it possible for pentagon $ABCDE$ to be a regular pentagon? Briefly explain your answer. [2]

Solution:

- 4 (ai) $\angle BED = 2x^\circ$
 $\angle BED = 4x^\circ$ (angle at centre = 2 angle at circumference) [M1]
 $\angle AOB = 180^\circ - 4x^\circ$ (angles on a straight line) [A1]
- (aii) $\angle ABE = x^\circ$ (angles in the same segment) [M1]
 $\angle OBA = \angle OAB$ (base angles of isosceles triangle)
 $= \angle DEB$ (angles in the same segment)
 $= 3x^\circ$ [M1]
 $\angle OBE = 2x^\circ$
 $\angle EBT = 90^\circ - 2x^\circ$ (tangent perpendicular to radius) [A1]
- (b) Since $x < 90^\circ$, the point F lies outside the circle. [B1]
- (c) interior angle of pentagon $= \frac{180(5-2)}{5}$ [M1]
 $= 108^\circ$
 Since $\angle AED = 90^\circ$ (right angle in semicircle), it is not possible for pentagon $ABCDE$ to be a regular pentagon. [A1]

- 5 (a) The marked price of a limited edition comic book is \$85. When it is sold at a discount of 10%, there is still a profit of 12.5% of the cost price. Find the cost price of the comic book. [2]
- (b) Esther visited her relatives in Japan during the New Year and brought ¥250000 to spend on her food and souvenirs. During her visit in Japan, Esther spent ¥175000 and received ¥ x from her relatives. [2]
- After her visit, she exchanged all her remaining Japanese Yen into Singapore dollars at the exchange rate of ¥1000 = S\$12.21. Assuming that Esther received S\$921.85, find the value of x , leaving your answer correct to the nearest yen.
- (c) Veronica and Betty each decided to buy a new car that is priced at \$180000. After some bargaining, the car dealer agreed to give both of them a 10% discount on the cost price of the car.
- (i) Veronica paid 30% of the selling price as deposit and the rest at monthly instalments over a period of 10 years at a simple interest rate of 3% per annum. Calculate the monthly instalment that she has to pay. [3]
- (ii) Betty decided to borrow money from a financial company to pay for the car in full. She then repaid the loan at the end of 10 years at a rate of 2% per annum compounded half yearly. Calculate the amount that she has to repay the financial company. [2]
-

Solution:

$$\begin{aligned} 5 \quad (a) \quad \text{selling price} &= \frac{90}{100} \times 85 \\ &= \$76.50 \end{aligned}$$

$$\begin{aligned} \text{cost price} &= \frac{100}{112.5} \times 76.50 & [M1] \\ &= \$68 & [A1] \end{aligned}$$

$$\begin{aligned} (b) \quad \text{Amount of Japanese yen left after the trip} &= 250000 - 175000 + x \\ &= 75000 + x \end{aligned}$$

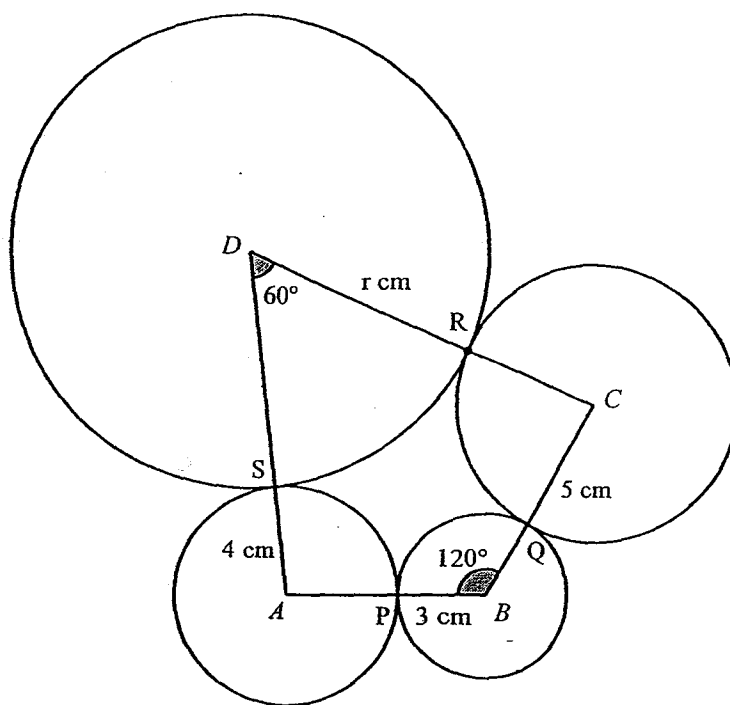
$$\begin{aligned} \frac{12.21}{1000} \times (75000 + x) &= 921.85 & [M1] \\ x &= 500 \text{ (nearest yen)} & [A1] \end{aligned}$$

$$\begin{aligned} (ci) \quad \text{Balance} &= \frac{70}{100} \left(\frac{90}{100} \times 180000 \right) & [M1] \\ &= \$113400 \end{aligned}$$

$$\begin{aligned} \text{Amount to be paid} &= 113400 \left(1 + \frac{3}{100} \times 10 \right) & [M1] \\ &= \$147420 \end{aligned}$$

$$\begin{aligned} \text{Monthly installment} &= \frac{147420}{10 \times 12} \\ &= \$1228.50 & [A1] \end{aligned}$$

$$\begin{aligned} (cii) \quad \text{Amount of money to be repaid} &= \left(\frac{90}{100} \times 180000 \right) \left(1 + \frac{1}{100} \right)^{20} & [M1] \\ &= \$197670.79 & [A1] \end{aligned}$$



In the diagram, the points A , B , C and D are the centres of circles with radius 4 cm, 3 cm, 5 cm and r cm respectively. The circles touch each other at the points P , Q , R and S . It is given that $\angle ADC = 60^\circ$ and $\angle ABC = 120^\circ$.

- (a) Show that $AC = \sqrt{r^2 + 9r + 21}$. [2]
- (b) Form an equation in r and show that it reduces to $r^2 + 9r - 148 = 0$. [2]
- (b) Solve the equation $r^2 + 9r - 148 = 0$. [2]
- (c) Find the area of the quadrilateral $ABCD$. [3]

Solution:

6 (a) By cosine rule,

$$AC = \sqrt{(r+4)^2 + (r+5)^2 - 2(r+4)(r+5)\cos 60^\circ} \quad [\text{M1}]$$

$$= \sqrt{r^2 + 8r + 16 + r^2 + 10r + 25 - r^2 - 9r - 20}$$

$$= \sqrt{r^2 + 9r + 21} \quad (\text{shown}) \quad [\text{A1}]$$

(b) By cosine rule,

$$AC = \sqrt{7^2 + 8^2 - 2(7)(8)\cos 120^\circ} \quad [\text{M1}]$$

$$AC = 13$$

$$\sqrt{r^2 + 9r + 21} = 13$$

$$r^2 + 9r + 21 = 169$$

$$r^2 + 9r - 148 = 0 \quad (\text{shown}) \quad [\text{A1}]$$

(c) $r^2 + 9r - 148 = 0$

$$r = \frac{-9 \pm \sqrt{81 - 4(1)(-148)}}{2(1)} \quad [\text{M1}]$$

$$= \frac{-9 \pm \sqrt{673}}{2}$$

$$= 8.47 \quad \text{or} \quad -17.5 \quad [\text{A1}]$$

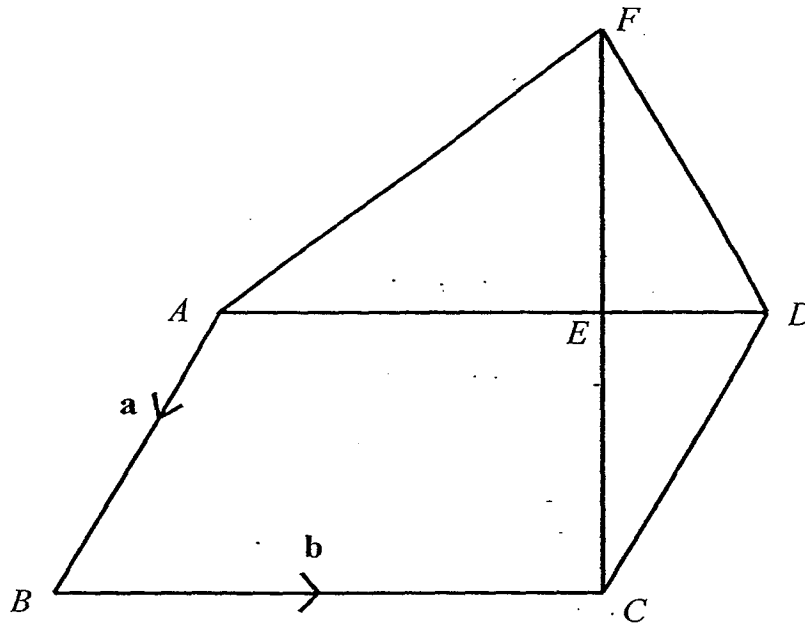
(d) Area of quadrilateral $ABCD$

= Area of triangle ADC + Area of triangle ABC

$$= \left(\frac{1}{2} (4 + 8.4711)(5 + 8.4711) \sin 60^\circ \right) + \left(\frac{1}{2} (7)(8) \sin 120^\circ \right) \quad [\text{M2}]$$

$$= 97.0 \text{ cm}^2 \quad [\text{A1}]$$

- 7 $ABCD$ is a parallelogram. E is a point on AD such that $4AE = 3AD$ and F is a point on CE produced such that $FE = EC$. It is given that $\overrightarrow{AB} = \mathbf{a}$ and $\overrightarrow{BC} = \mathbf{b}$.



- (a) Express, as simply as possible, in terms of $\mathbf{a}$ and/or $\mathbf{b}$,
- (i) $\overrightarrow{CE}$, [2]
 - (ii) $\overrightarrow{AF}$, [2]
 - (iii) $\overrightarrow{BF}$, [2]
 - (iv) $\overrightarrow{BE}$, [1]
 - (v) $\overrightarrow{BG}$. [1]
- (b) Write down two facts about BE and BG . [2]

Solution:

$$7 \quad (\text{ai}) \quad \overline{ED} = \frac{1}{4}\mathbf{b} \quad [\text{M1}]$$

$$\begin{aligned} \overline{CE} &= \overline{CD} + \overline{DE} \\ &= -\mathbf{a} - \frac{1}{4}\mathbf{b} \quad [\text{A1}] \end{aligned}$$

$$\begin{aligned} (\text{aii}) \quad \overline{DF} &= \overline{DE} + \overline{EF} \\ &= -\frac{1}{4}\mathbf{b} - \mathbf{a} - \frac{1}{4}\mathbf{b} \\ &= -\mathbf{a} - \frac{1}{2}\mathbf{b} \quad [\text{M1}] \end{aligned}$$

$$\begin{aligned} \overline{AF} &= \overline{AD} + \overline{DF} \\ &= \mathbf{b} - \mathbf{a} - \frac{1}{2}\mathbf{b} \\ &= -\mathbf{a} + \frac{1}{2}\mathbf{b} \quad [\text{A1}] \end{aligned}$$

$$\begin{aligned} (\text{aiii}) \quad \overline{BF} &= \overline{BC} + \overline{CF} \\ &= \mathbf{b} + 2\left(-\mathbf{a} - \frac{1}{4}\mathbf{b}\right) \quad [\text{M1}] \\ &= -2\mathbf{a} + \frac{1}{2}\mathbf{b} \quad [\text{A1}] \end{aligned}$$

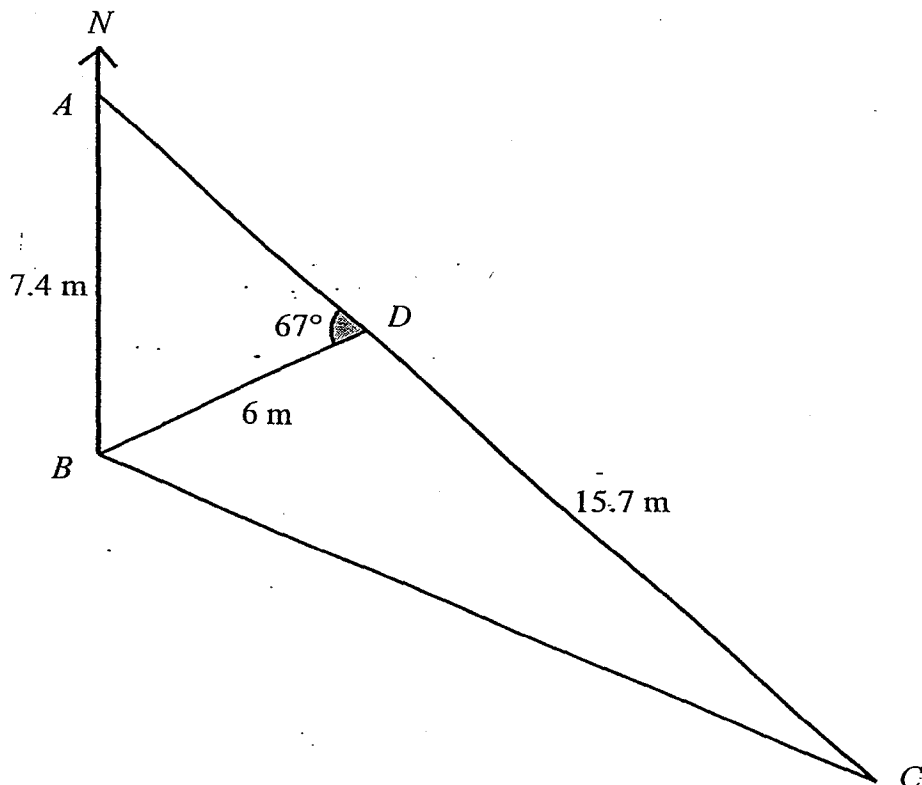
$$(\text{aiv}) \quad \overline{BE} = -\mathbf{a} + \frac{3}{4}\mathbf{b} \quad [\text{B1}]$$

$$(\text{av}) \quad \overline{BG} = \mathbf{b} - \frac{4}{3}\mathbf{a} \quad [\text{B1}]$$

$$(\text{b}) \quad \overline{BE} = \frac{3}{4}\overline{BG} \quad [\text{B1}]$$

B, E and G are collinear and $4BE = 3BG$. [B1]

- 8 The points A , B and C are the vertices of a triangular field where A is due north of B and $AB = 7.4$ m. Daniel is standing at the point D on the triangular field such that $DC = 15.7$ m and $\angle ADB = 67^\circ$.



(a) Calculate

- (i) $\angle BAC$, [2]
- (ii) the bearing of B from D , [1]
- (iii) how far north D is from B . [2]

(b) An apple tree is growing at point B . The angle of elevation of the top of the apple tree from C is 34° .

- (i) Find the height of the apple tree. [2]
- (ii) Find the greatest angle of elevation of the top of the apple tree along the line AC . [3]

Solution:

$$8 \quad (\text{ai}) \quad \frac{\sin \angle BAC}{6} = \frac{\sin 67^\circ}{7.4}$$

$$\sin \angle BAC = \frac{\sin 67^\circ}{7.4} \times 6 \quad [\text{M1}]$$

$$\angle BAC = \sin^{-1} \left(\frac{\sin 67^\circ}{7.4} \times 6 \right)$$

$$= 48.275^\circ$$

$$= 48.3^\circ \text{ (1 dp)} \quad [\text{A1}]$$

$$(\text{aii}) \quad \text{Bearing} = 360^\circ - 67^\circ - 48.275^\circ$$

$$= 244.7^\circ \text{ (1dp)} \quad [\text{B1}]$$

$$(\text{aiii}) \quad \text{Let the foot of perpendicular from } D \text{ to } AB \text{ be } E.$$

$$\angle ABD = 180^\circ - 67^\circ - 48.275^\circ$$

$$= 64.725^\circ$$

$$BE = 6 \cos 64.725^\circ \quad [\text{M1}]$$

$$= 2.5617 \text{ km}$$

$$= 2.56 \text{ km (3sf)} \quad [\text{A1}]$$

$$(\text{bi}) \quad \tan 34^\circ = \frac{h}{18.870} \quad [\text{M1}]$$

$$h = 18.870 \tan 34^\circ$$

$$h = 12.728 \text{ m}$$

$$= 12.7 \text{ m (3sf)} \quad [\text{A1}]$$

$$(\text{bii}) \quad \text{Let the perpendicular distance of } B \text{ to } AC \text{ be } h.$$

$$\sin 67^\circ = \frac{h}{6}$$

$$h = 6 \sin 67^\circ \quad [\text{M1}]$$

Let θ be the greatest angle of elevation.

$$\tan \theta = \frac{12.728}{6 \sin 67^\circ}$$

$$\theta = 66.5^\circ \text{ (1dp)} \quad [\text{A1}]$$

- 9 (a) There are three types of tickets available for sale for the concert by the school orchestra. The number and cost of each type of ticket are summarised in the table below.

	Number of tickets per day	Cost of tickets on Saturday	Cost of tickets on Sunday
Type 1	50	\$18	\$28
Type 2	35	\$48	\$58
Type 3	15	\$68	\$78

- (a) Represent the cost of tickets as a matrix **A** of order 3 by 2. [1]
- (b) Using matrix multiplication, find a matrix **B** of order 1 by 2 whose elements represent the total revenue for each day, assuming that all the tickets were sold. [2]
- (c) Using matrix multiplication, find the total revenue for the weekend, assuming that all the tickets were sold. [2]
- (d) To celebrate National Day, the school orchestra decided to reduce the cost of Type 1 tickets by 30%, Type 2 tickets by 20% and Type 3 tickets by 10%.
- (i) By matrix multiplication, find a matrix **C** of order 3 by 2 whose elements represent the cost of tickets after discount for each type of tickets on Saturday and Sunday respectively. [2]
- (ii) By matrix multiplication, find the total revenue for the weekend, assuming that all the tickets were sold. [3]

Solution:

$$9 \quad (a) \quad A = \begin{pmatrix} 18 & 28 \\ 48 & 58 \\ 68 & 78 \end{pmatrix} \quad [B1]$$

$$(b) \quad B = (50 \quad 35 \quad 15) \begin{pmatrix} 18 & 28 \\ 48 & 58 \\ 68 & 78 \end{pmatrix} \quad [M1]$$

$$= (3600 \quad 4600) \quad [A1]$$

$$(c) \quad (3600 \quad 4600) \begin{pmatrix} 1 \\ 1 \end{pmatrix} = (8200) \quad [M1]$$

Total revenue = \$8200 [A1]

$$(di) \quad C = \begin{pmatrix} 0.7 & 0 & 0 \\ 0 & 0.8 & 0 \\ 0 & 0 & 0.9 \end{pmatrix} \begin{pmatrix} 18 & 25 \\ 48 & 58 \\ 68 & 78 \end{pmatrix} \quad [M1]$$

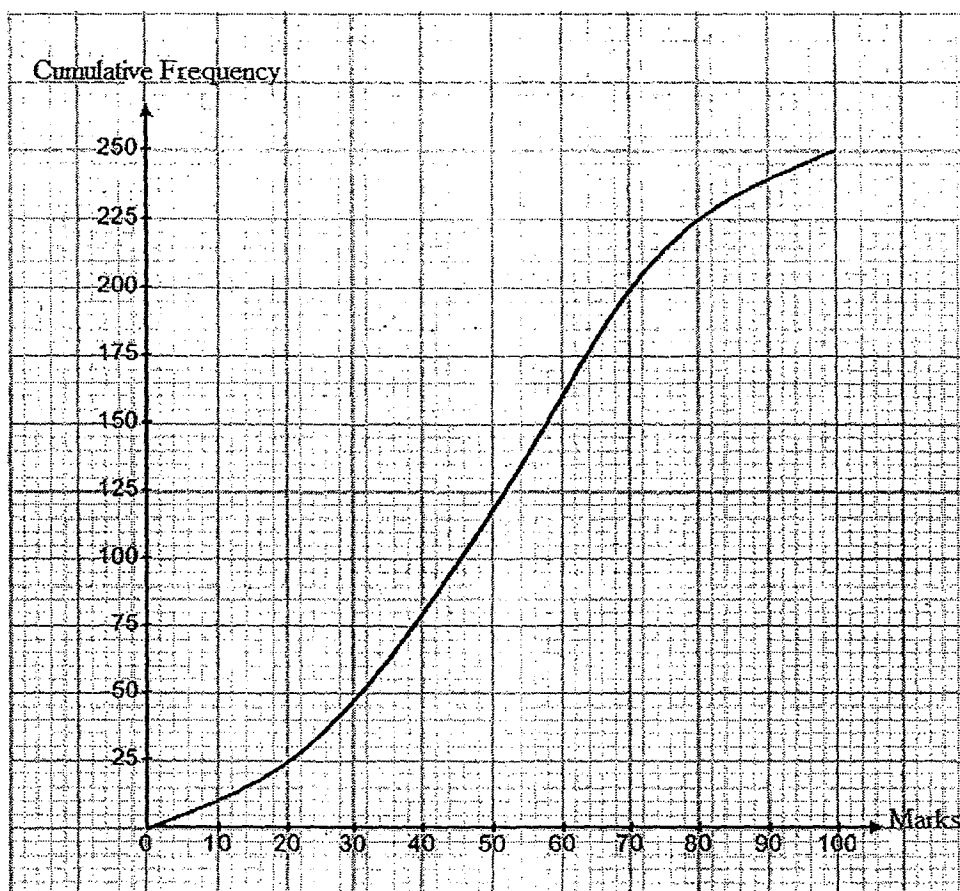
$$= \begin{pmatrix} 12.60 & 19.60 \\ 38.40 & 46.40 \\ 61.20 & 70.20 \end{pmatrix} \quad [A1]$$

$$(dii) \quad (50 \quad 35 \quad 15) \begin{pmatrix} 12.60 & 19.60 \\ 38.40 & 46.40 \\ 61.20 & 70.20 \end{pmatrix} = (2892 \quad 3657) \quad [M1]$$

$$(2892 \quad 3657) \begin{pmatrix} 1 \\ 1 \end{pmatrix} = (6549) \quad [A1]$$

Total revenue = \$6549 [A1]

- 10 The cumulative frequency graph below shows the distribution of marks of 250 students from Sunflower Secondary School in a mathematics examination.



- (a) Use your graph to find the
- (i) median mark, [1]
 - (ii) interquartile range, [2]
 - (iii) the lowest mark a student must obtain in order to get a prize that is only awarded to the top 10% of the cohort. [1]

The grouped frequency table of the marks scored by the 250 students is shown below.

Marks	$0 < x < 20$	$20 < x < 40$	$40 < x < 60$	$60 < x < 80$	$80 < x < 100$
Frequency	25	55	80	65	25

- (b) Using the grouped frequency table, estimate
- (i) the mean mark, [2]
 - (ii) the standard deviation. [2]

- (c) Two students were chosen at random from this distribution. Giving your answers as a fraction in its lowest term, find the probability that

- (i) both students scored more than 40 marks in the test, [2]
- (ii) one student scored more than 80 marks and the other student scored less than 20 marks in the test. [2]

Solution:

10 (ai) Median mark = 52 [B1] (Accept 51 – 53)

(aii) Interquartile range = $67 - 35$ [M1] (Accept 66 – 68 and 34 – 36)
 $= 32$ [A1] (Accept 30 – 34)

(aiii) $\frac{90}{100} \times 250 = 225$
 Lowest mark = 80 [B1] (Accept 79 – 81)

(bi) mean = $\frac{12700}{250}$ [M1]
 $= 50.8$ [A1]

(bii) standard deviation = $\sqrt{\frac{773000}{250} - \left(\frac{12700}{250}\right)^2}$ [M1]
 $= 22.6$ [A1]

(ci) probability = $\frac{170}{250} \times \frac{169}{249}$ [M1]
 $= \frac{2873}{6225}$ [A1]

(cii) probability = $\frac{25}{250} \times \frac{25}{249} \times 2$ [M1]
 $= \frac{5}{249}$ [A1]

11 Answer the whole of this question on a sheet of graph paper.

The variables x and y are connected by the equation

$$y = 10 - 2x - \frac{5}{x^2}.$$

Some corresponding values of x and y , correct to 1 decimal place, are given in the table below.

	1	2	3	4	5	6	7
y	3.0	4.8	3.4	a	-0.2	-2.1	-4.1

- (a) Find the value of a . [1]

- (b) Using a scale of 2 cm to represent 1 unit, draw a horizontal x -axis for $1 \leq x \leq 7$.
Using a scale of 2 cm to represent 1 unit, draw a vertical y -axis for $-5 \leq y \leq 5$.

On your axes, plot the points given in the table and join them with a smooth curve. [3]

- (c) Use your graph to find the values of x in the range $1 \leq x \leq 7$ for which $13 - 4x - \frac{10}{x^2} = 0$. [2]

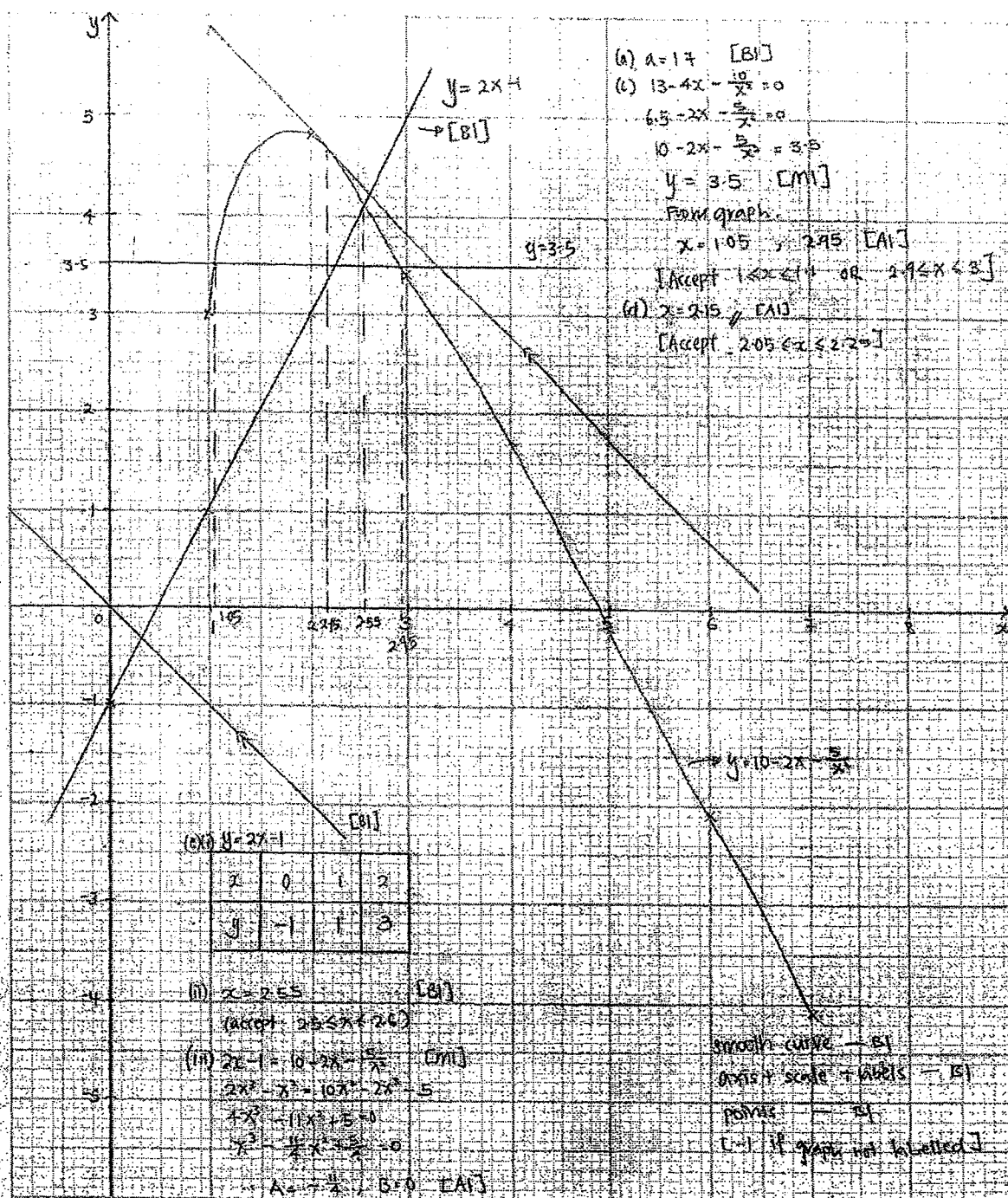
- (d) By drawing a suitable tangent, find the x -coordinate of the point on the curve where the gradient of the curve is -1 . [2]

- (e) (i) On the same axes, draw the line $y = 2x - 1$. [1]

- (ii) Write down the x -coordinate of the point where this line intersects the curve for $1 \leq x \leq 7$. [1]

- (iii) This value of x is one of the solutions of the equation $x^3 + Ax^2 + Bx + \frac{5}{4} = 0$. [2]
Find the value of A and the value of B .

Sec 4 Prelim 2014 Question 11



END OF PAPER