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HOLY INNOCENTS' HIGH SCHOOL

PRELIM EXAMINATION 2014
SECONDARY 4 EXPRESS/ 5 NORMAL (ACADEMIC)

MATHEMATICS **4016 / 01**

Paper 1

Name : _____ **Date** : **14 Aug 2014**

Register No : _____ **Duration** : **2 hours**

Class : _____ **Marks** : **/ 80**

Additional Materials needed:

INSTRUCTIONS TO CANDIDATES

Write your register number and name on all the work you hand in.
Write in dark blue or black pen.
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Answer all questions in the answer space provided.

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If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.
For π, use either your calculator value or 3.142, unless the question requires the answer in terms of π.

The number of marks is given in brackets [ ] at the end of each question or part question.
The total marks for this paper is 80.

Setter: Ms Nicole Goh

This paper consists of 20 printed pages, inclusive of this cover page.

Mathematical Formulae

Compound interest

$$\text{Total amount} = P\left(1+\frac{r}{100}\right)^n$$

Geometry and Measurement

$$\text{Curved surface area of a cone} = \pi rl$$

$$\text{Surface area of a sphere} = 4\pi r^2$$

$$\text{Volume of a cone} = \frac{1}{3}\pi r^2 h$$

$$\text{Volume of a sphere} = \frac{4}{3}\pi r^3$$

$$\text{Area of triangle } ABC = \frac{1}{2}ab\sin C$$

$$\text{Arc length} = r\theta \text{ , where } \theta \text{ is in radians}$$

$$\text{Sector area} = \frac{1}{2}r^2\theta \text{ , where } \theta \text{ is in radians}$$

Trigonometry

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

Statistics

$$\text{Mean} = \frac{\sum fx}{\sum f}$$

$$\text{Standard deviation} = \sqrt{\frac{\sum fx^2}{\sum f} - \left(\frac{\sum fx}{\sum f}\right)^2}$$

Answer **all** the questions.

1. Write as a single fraction in its simplest form $\frac{2}{(x-1)} - \frac{x}{x-x^2}$.

Answer ..... [2]

---

2. In 2014, the world population is estimated to be 7.17 billion.
   (a) 7.17 billion can be written as k trillion.
   Find the value of k.
   (b) The population of Singapore is estimated to be 0.08% of the world's population.
   Find the population of Singapore.
   Express your answer in standard form.

Answer (a) $k =$ ..... [1]

(b) ..... [1]

---

3. The numbers 6048 and 360, written as the product of prime factors are

$6048 = 2^5 \times 3^3 \times 7$ and $360 = 2^3 \times 3^2 \times 5$.

Use these results to find

(a) the smallest positive integer, k, such that $\frac{6048}{k}$ is a perfect square,

(b) the smallest integer, n, such that $360n$ is a multiple of 6048.

Answer (a) $k =$ ..... [1]

(b) $n =$ ..... [1]

---

4. Find the smallest integer value of x which satisfies the inequality

$$\frac{-2x+8}{2} < \frac{7x+3}{7}.$$

Answer $x =$ ..... [2]

---

5. Solve the simultaneous equations

$$\frac{3}{4}x - y = \frac{1}{2},$$
$$-9 + 5y = -2x.$$

Answer $x =$ ....., $y =$ ..... [3]

6. (a) Given $3^{3k} \div 5^{3k} = \frac{27}{125}$, find k.

(b) Simplify $3ab^2 \times \left(9a^4b^{-2}\right)^{\frac{1}{2}}$.

Answer (a) $k =$ ..... [1]

(b) ..... [2]

---

7. The volume of a cylinder of radius r cm and height h cm varies directly as r^2h. If the base radius is increased by 70% and the height is decreased by 50%, find the percentage change in the volume of the cylinder.

Answer .....% [3]

---

8. Two provision shops in Hougang neighbourhood sell the same type of cookies and ice cream.
A box of cookies costs \$2.50 at Shop A and \$$x$ at Shop B.
A tub of ice-cream costs \$$y$ at Shop A and \$4.25 at Shop B.
This information can be represented by the matrix $S = \begin{pmatrix} 2.50 & x \\ y & 4.25 \end{pmatrix}$.

The table below shows the quantities of cookies and ice-cream Adam and Samuel planned to buy.

	Cookies (in boxes)	Ice cream (in tubs)
Adam	7	4
Samuel	6	2

This information can be represented by the matrix $R = \begin{pmatrix} 7 & 4 \\ 6 & 2 \end{pmatrix}$.

Adam needs to pay \$41.50 for his supply regardless of whether he buys from Shop A or Shop B.
Samuel pays \$27 for his supply if he buys from Shop A and \$29.50 if he buys from Shop B.
This information can be represented by the matrix $T = \begin{pmatrix} 41.50 & 41.50 \\ 27 & 29.50 \end{pmatrix}$.

By forming an equation connecting R, S and T, find the values of x and y.

Answer $x =$ ....., $y =$ ..... [3]

9. A floor tile in the shape of a rhombus has sides of length 52 cm and a diagonal of length 48 cm.
Find
(a) the length of the other diagonal,
(b) the area of the floor tile.

Answer (a) ..... cm [2]

(b)..... cm^2 [1]

---

10. $\xi = \{x : x \text{ is a natural number less than } 12\}$
$P = \{x : x \text{ is a multiple of } 3\}$
$Q = \{x : x \text{ is a prime number}\}$

Find
(a) (i) $P \cup Q$,
(ii) $n(P \cap Q')$,
(b) Express, in set notation, the set
$\{x : x \text{ is neither a multiple of 3 nor a prime number}\}$.

Answer (a)(i) ..... [1]

(a)(ii)..... [1]

(b) ..... [1]

---

11. A model of a car is made to a scale of 1 : 30.

(a) If the actual length of the car is 4.47 m, find the length of the model in centimetres.

(b) If the total surface area of the model is 750 cm^2, find the actual surface area of the car in square metres, correct to the nearest square metres.

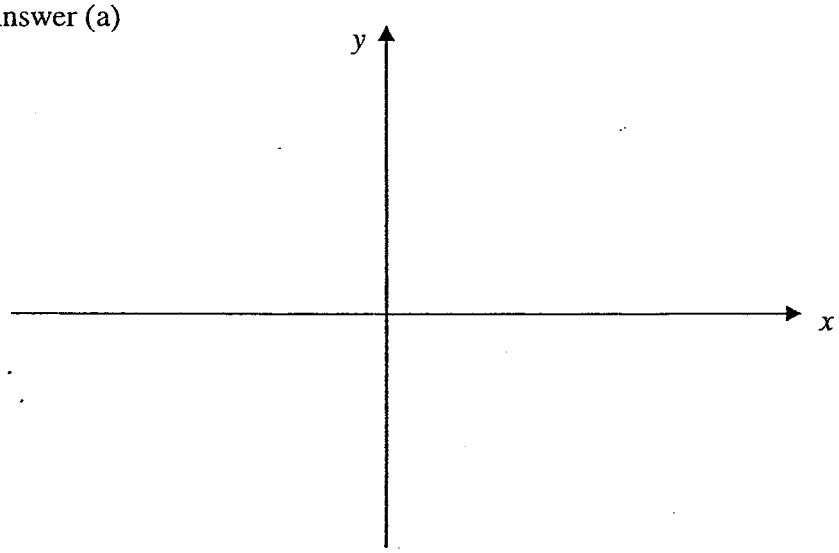
Answer (a) ..... cm [1]

(b)..... m^2 [2]

---

12. (a) Sketch the graph of $y = 2(x+1)(x-3)$. [2]

Answer (a)



(b) (i) Express $x^2 + 4x + 10$ in the form $(x+h)^2 + k$.

(ii) Hence write down the minimum value of $x^2 + 4x + 10$.

Answer (b)(i) ..... [1]

(ii) ..... [1]

---

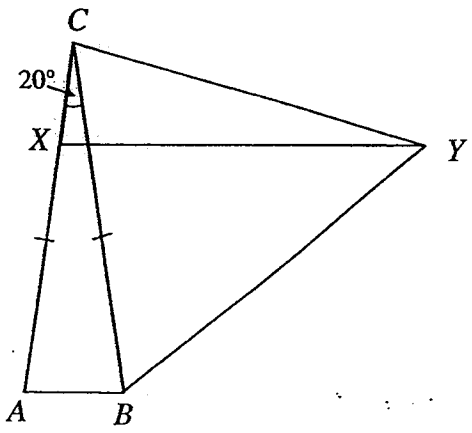
13. (a) Factorise $p^2 - 9q^2 + 6qr - r^2$ completely.

(b) Solve $\frac{1}{6x-1} - \frac{1}{4x+1} = \frac{1}{6}$.

Answer (a) ..... [2]

(b) $x =$ ..... or ..... [2]

14. In the diagram, triangle ABC is isosceles with $AC = BC$ and $\angle ACB = 20°$. BCY is an equilateral triangle and X is a point on AC such that $CX = AB$.



Prove that triangle ABC and triangle CXY are congruent.

Answer .....

.....

.....

.....

.....

.....

..... [4]

15. The line AB passes through the points A (0, –1) and B (2, 1).

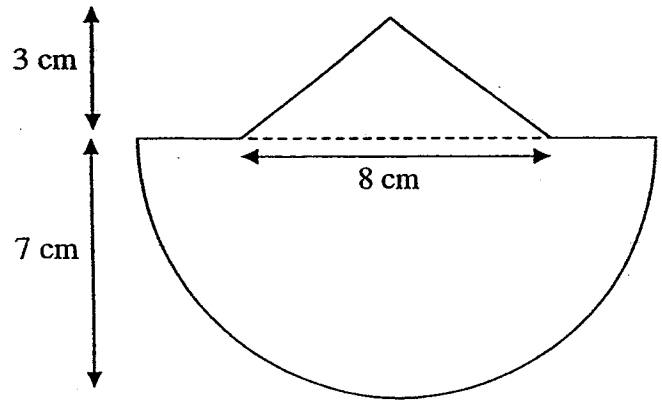
(a) Find the equation of the line AB.

(b) Find the equation of the line which is the reflection of the line AB about the y-axis.

Answer (a)..... [2]

(b) ..... [2]

16. The diagram shows the side view of a solid figure.
It is made up of a hemisphere and a cone.



(a) Calculate the exterior area of the solid figure to be painted, leaving your answer in terms of π.

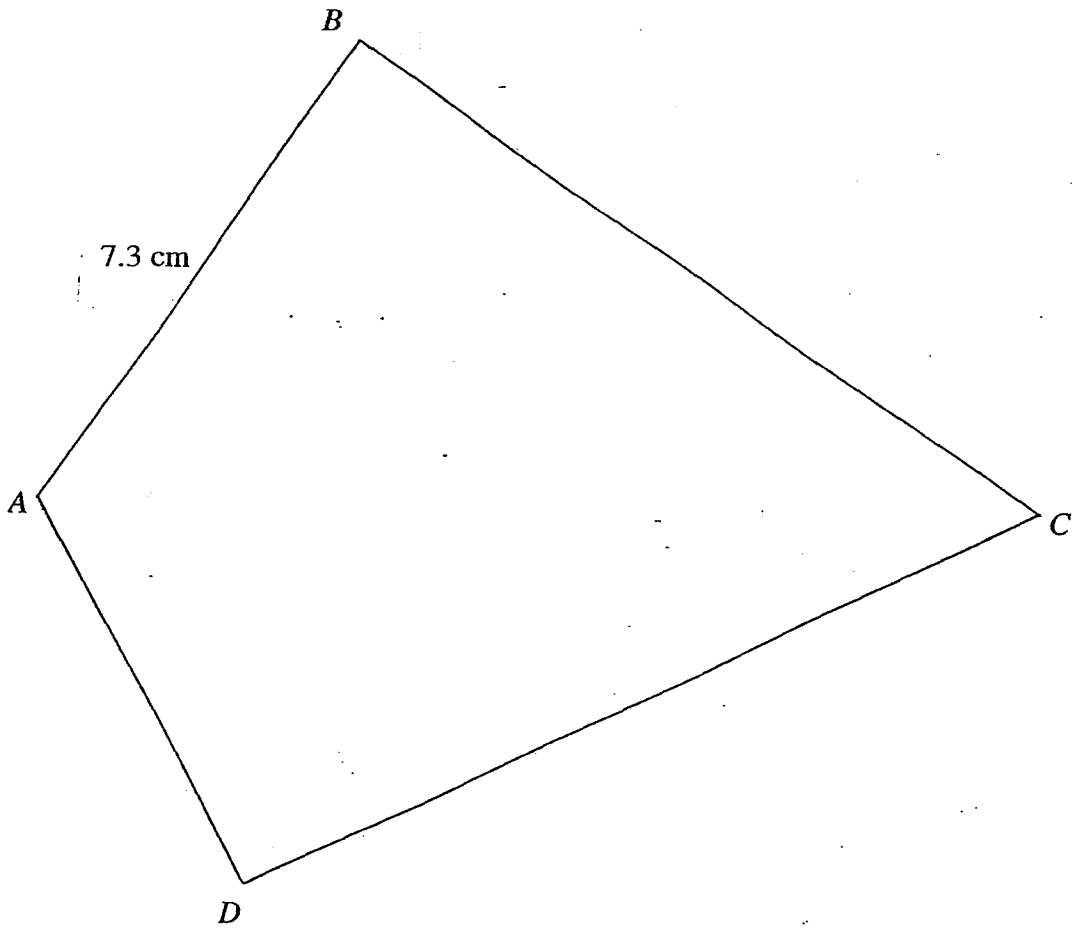
(b) Given that 1 cm^2 of the surface area of the figure requires 2 cm^3 of paint, find the number of identical figures that can be painted with 3 litres of paint.

Answer (a) ..... [3]

(b) ..... [1]

17. The diagram shows a park $ABCD$ in the shape of a quadrilateral.

Answer (b)



(a) If the actual distance of AB is 7.3 km, express the scale in the form of $1 : n$.

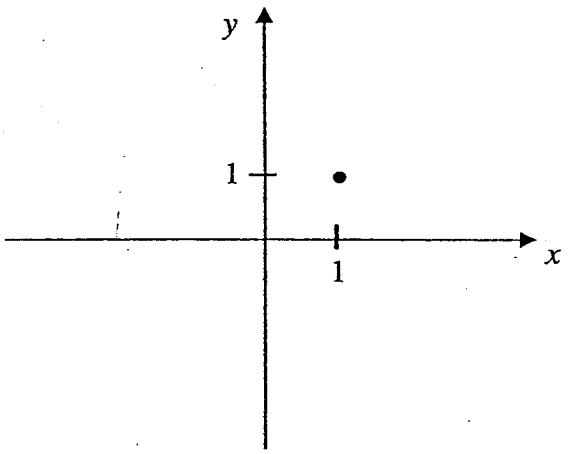
(b) A statue is located at point Q which is inside the park $ABCD$ and it is equidistant from B and C and equidistant from BC and DC.
Mark and label the point Q. [3]

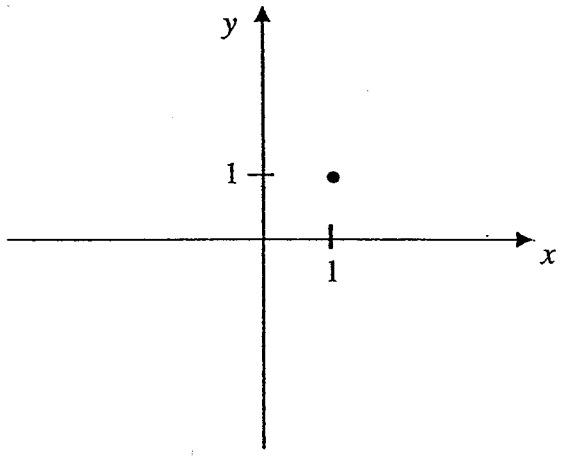
Answer (a) 1 : ..... [1]

18. (a) The point (1, 1) is marked on the diagram in the answer space.
On the axes in the answer space, draw the graphs of

(i) $y = x^3$ (ii) $y = \frac{1}{x^2}$ [2]

Answer (a)(i)(ii)





(b) Explain how you can obtain the number of solutions for $x^3 = \frac{1}{x^2}$ and state the number of solution.

(c) Write down the coordinates for the solution(s) to (b).

Answer (b) .....

.....

The number of solution is ..... [2]

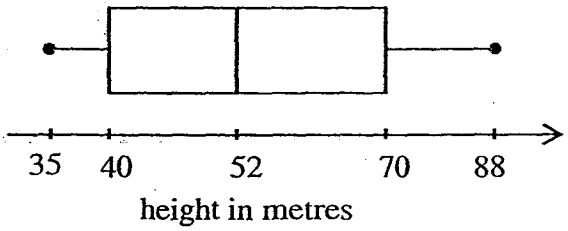
Answer (c) (....., ...)[1]

19. (a) The heights, to the nearest metres, of 12 trees collected from Land A are

35, 38, 40, 42, 45, 57, 58, 58, 63, 67, 70, 75.

(i) Find the interquartile range.

(ii) The box and whisker plot below shows the heights of another group of 12 trees collected from Land B.



Comment briefly on the heights of the trees, in metres, on both lands.

(b) With the addition of one more tree of length x m on Land A, the new median height of the trees on Land A is 57 m.
Write down the range of values of x.

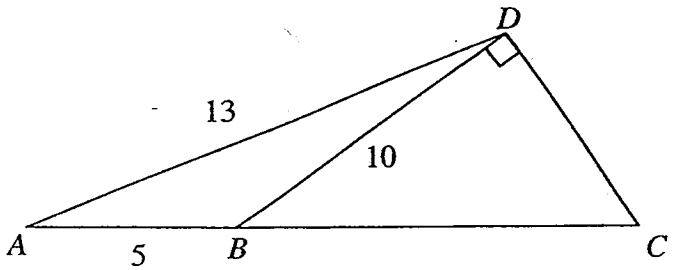
Answer (a)(i) ..... [2]

(ii) .....

..... [2]

(b) ..... [1]

20. In the diagram, ABC is a straight line.
$AB = 5$ m, $BD = 10$ m, $AD = 13$ m and $\angle BDC = 90°$.



(a) Find the value of $\cos \angle ABD$.

(b) (i) Write down the value of $\cos \angle CBD$.

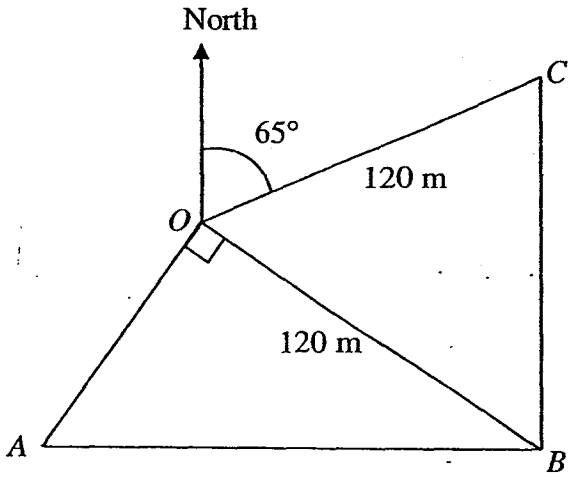
(ii) **Hence** find the length of BC.

Answer (a) ..... [2]

(b)(i) ..... [1]

(ii) .....m [2]

21. O, A, B and C are four points on level ground.
C is on a bearing of 065° from O.
B is due east of A and south of C.
$OB = OC = 120$ m and $\angle AOB = 90°$.



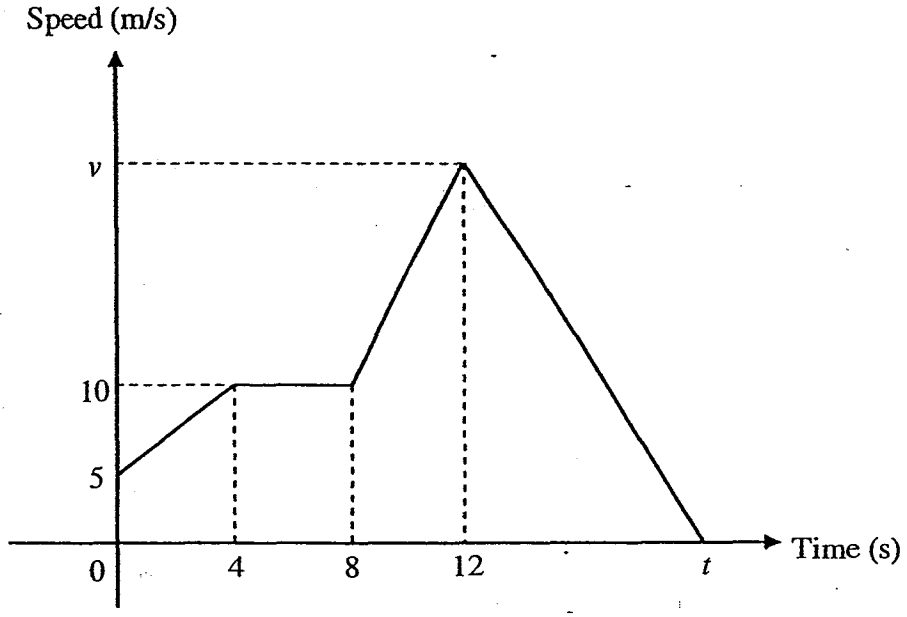
Calculate
(a) the bearing of A from O,
(b) the distance OA,
(c) the area of $OABC$.

Answer (a) ..... ° [2]

(b) ..... m [2]

(c) ..... m^2 [2]

22. The diagram below shows the speed-time graph of a moving body over a period of t seconds.



Calculate

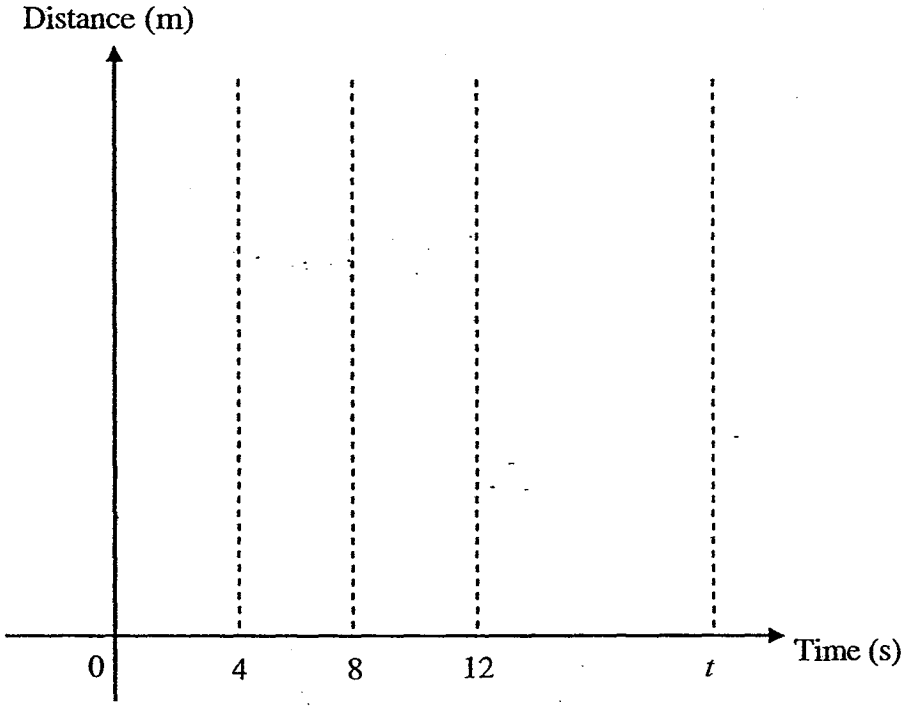
(a) the acceleration during the first 4 seconds,

(b) the value of v if the total distance moved in the first 12 seconds was 146 m,

(c) the value of t, given that the retardation was 4.5 m/s^2.

(d) On the axes in the answer space, sketch the distance-time graph of the object over the period of t seconds.
Mark the distance travelled on the axis clearly. [2]

Answer (d)



Answer (a) .....m/s^2 [1]

(b) ..... m/s [2]

(c) ..... s [1]

End of paper

HIHS Elementary Mathematics Mid-Year Examination 2014
Secondary 4 Express / 5 Normal (Academic)

Answers

1. $\frac{-3}{1-x}$

2(a) $k = 0.00717$ (b) 5.736×10^6

3(a) $k = 2\times3\times7 = 42$ (b) $n = 84$

4. Smallest integer $= 2$

5. x = 2, y = 1

6(a) k = 1 (b) $\frac{b^3}{a}$

7. 44.5%

8. x = \$3.50 y = \$6

9(a) $92.3cm$ (b) $2210cm^2$

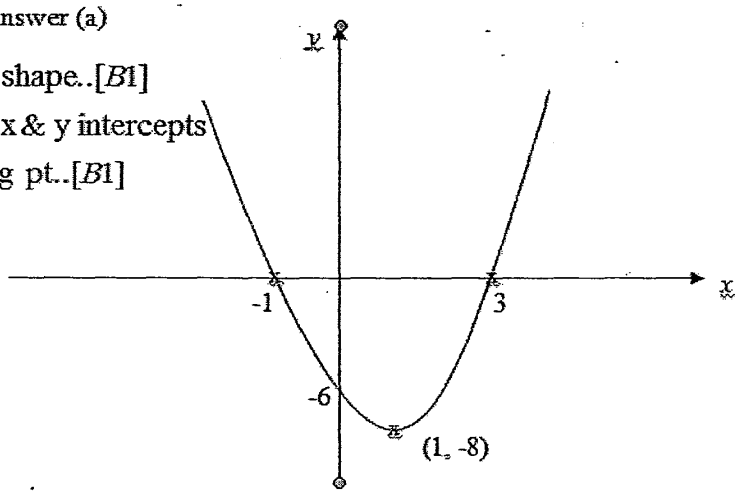
10(a)(i) $P\cup Q = \{2,3,5,6,7,9,11\}$ (ii) $n(P\cap Q') = 2$ (b) $P'\cap Q'$

11(a) $14.9cm$ (b) $68m^2$

12(a)

Answer (a)

(a) Correct shape..[B1]
Correct x & y intercepts & turning pt..[B1]



(b) (i) $(x+2)^2+6$ (ii) 6

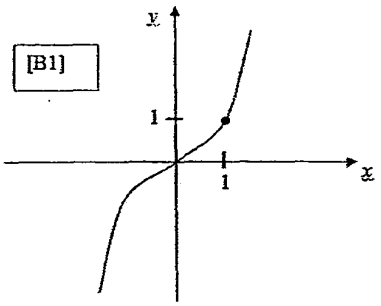
13(a) $(p+3q-r)(p-3q+r)$ (b) $x=\frac{1}{2}$ or $x=-1\frac{1}{12}$

14. proof

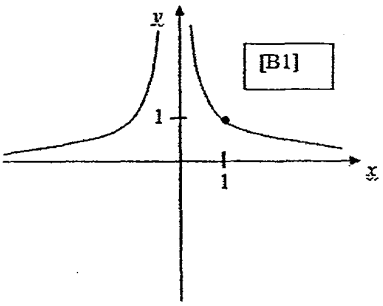
15(a) $y = x-1$ (b) $y = -x-1$

16(a) 151π (b) 3

17(a) construction (b) 100000



[B1]



[B1]

18(a)

18(b)

Draw the two graphs $y = x^3$ and $y = \frac{1}{x^2}$ on the same axes and find the intersection points of the two graphs..[*B*1]

There is only 1 solution..[B1]

(c) (1,1)

19(a)(i) 24 [accept 21] (ii)

$\text{Median}_A < \textit{Median}_B$ The trees on Land A are generally taller than that on Land B..[B1]

$\text{IQR}_A < \textit{IQR}_B$ The heights of trees on Land A are more consistent than that on Land B..[B1]

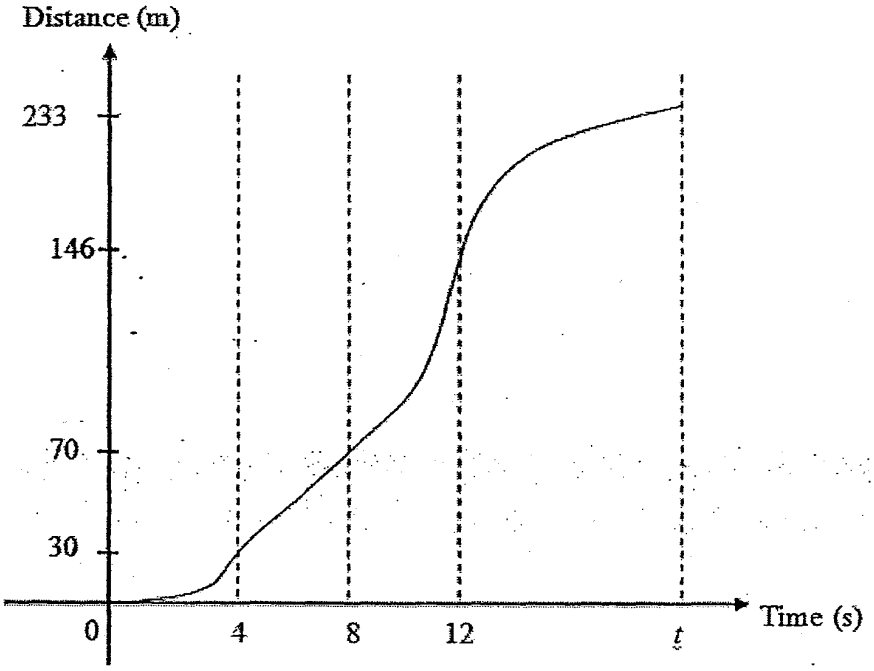
(b) $0 < x \leq 57$

20(a) $\cos \angle ABD = -0.44$ (b)(i) 0.44 (ii) 22.7

21(a) 205° (b) $56.0m$ (c) $8870m^2$

22(a) $1.25m/s^2$ (b) $v = 28m/s$ (c) $t = 18.2s$

(d)



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HOLY INNOCENTS' HIGH SCHOOL

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SECONDARY 4 EXPRESS/ 5 NORMAL (ACADEMIC)

MATHEMATICS **4016 / 01**

Paper 1 [Marking Scheme]

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$$\text{Standard deviation} = \sqrt{\frac{\sum fx^2}{\sum f} - \left(\frac{\sum fx}{\sum f}\right)^2}$$

Answer **all** the questions.

1. Write as a single fraction in its simplest form $\frac{2}{(x-1)}-\frac{x}{x-x^2}$.

$$\frac{2}{(x-1)}-\frac{x}{x-x^2}=\frac{2}{(x-1)}-\frac{x}{x(1-x)}$$

$$=\frac{-2}{(1-x)}-\frac{1}{(1-x)}..[M1]$$

$$=\frac{-3}{1-x}..[A1]$$

Also accept $\frac{3}{x-1}$

Answer ..... [2]

---

2. In 2014, the world population is estimated to be 7.17 billion.
   (a) 7.17 billion can be written as k trillion.
   Find the value of k.
   (b) The population of Singapore is estimated to be 0.08% of the world's population.
   Find the population of Singapore.
   Express your answer in standard form.

(a) $k=0.00717..[B1]$

(b) $\frac{0.08}{100}\times 7.17\times 10^9$

$=5.736\times 10^6..[B1]$

Answer (a) $k=$ ..... [1]

(b) ..... [1]

---

3. The numbers 6048 and 360, written as the product of prime factors are

$6048 = 2^5 \times 3^3 \times 7$ and $360 = 2^3 \times 3^2 \times 5$.

Use these results to find

(a) the smallest positive integer, k, such that $\frac{6048}{k}$ is a perfect square,

(b) the smallest integer, n, such that $360n$ is a multiple of 6048.

(a) $k = 2 \times 3 \times 7 = 42$..[B1]

(b) $LCM = 2^5 \times 3^3 \times 5 \times 7 = 30240$

$360n = 30240$

$n = 84$..[B1]

Answer (a) $k =$ ..... [1]

(b) $n =$ ..... [1]

---

4. Find the smallest integer value of x which satisfies the inequality

$$\frac{-2x+8}{2} < \frac{7x+3}{7}.$$

$7(-2x+8) < 2(7x+3)$..[M1]

$-14x + 56 < 14x + 6$

$-28x < -50$

$x > 1\frac{11}{14}$..also accept 1.79

Smallest integer $= 2$..[A1]

Answer ..... [2]

---

5. Solve the simultaneous equations

$$\frac{3}{4}x - y = \frac{1}{2},$$
$$-9 + 5y = -2x.$$

$(1): y = \frac{3}{4}x - \frac{1}{2}$

Substituition

$-9 + 5\left(\frac{3}{4}x - \frac{1}{2}\right) = -2x$

$-9 + \frac{15}{4}x - \frac{5}{2} = -2x..[M1]$

$\frac{23}{4}x = \frac{23}{2}$

$x = 2..[A1]$

$-9 + 5y = -2(2)$

$5y = 5$

$y = 1..[A1]$

$(1) \times 5: \frac{15}{4}x - 5y = \frac{5}{2}$

Elimination

$\frac{15}{4}x + 2x = \frac{23}{3}..[M1]$

$\frac{23}{4}x = \frac{23}{2}$

$x = 2..[A1]$

$-9 + 5y = -2(2)$

$5y = 5$

$y = 1..[A1]$

Answer $x =$ ....., $y =$ ..... [3]

6. (a) Given $3^{3k} \div 5^{3k} = \frac{27}{125}$, find k.

(b) Simplify $3ab^2 \times \left(9a^4b^{-2}\right)^{\frac{1}{2}}$.

$(a)\left(\frac{3}{5}\right)^{3k} = \left(\frac{3}{5}\right)^3$

$3k = 3$

$k = 1..[B1]$

$(b)3ab^2 \times \frac{1}{3}a^{-2}b..[M1]$

$= 3ab^2 \times \frac{b}{3a^2}$

$= \frac{b^3}{a}..[A1]$

Answer (a) $k =$ ..... [1]

(b) ..... [2]

---

7. The volume of a cylinder of radius r cm and height h cm varies directly as r^2h. If the base radius is increased by 70% and the height is decreased by 50%, find the percentage change in the volume of the cylinder.

$V_{new} = \pi(1.7r)^2(0.5h)..[M1]$

$= 1.445\pi r^2 h$

$\frac{1.445\pi r^2 h - \pi r^2 h}{\pi r^2 h} \times 100\%..[M1]$

$= 0.445 \times 100\%$

$= 44.5\%..[A1]$

Answer .....% [3]

---

8. Two provision shops in Hougang neighbourhood sell the same type of cookies and ice cream.
A box of cookies costs \$2.50 at Shop A and \$$x$ at Shop B.
A tub of ice-cream costs \$$y$ at Shop A and \$4.25 at Shop B.
This information can be represented by the matrix $S = \begin{pmatrix} 2.50 & x \\ y & 4.25 \end{pmatrix}$.
The table below shows the quantities of cookies and ice-cream Adam and Samuel planned to buy.

	Cookies (in boxes)	Ice cream (in tubs)
Adam	7	4
Samuel	6	2

This information can be represented by the matrix $R = \begin{pmatrix} 7 & 4 \\ 6 & 2 \end{pmatrix}$.
Adam needs to pay \$41.50 for his supply regardless of whether he buys from Shop A or Shop B.
Samuel pays \$27 for his supply if he buys from Shop A and \$29.50 if he buys from Shop B.
This information can be represented by the matrix $T = \begin{pmatrix} 41.50 & 41.50 \\ 27 & 29.50 \end{pmatrix}$.
By forming an equation connecting R, S and T, find the values of x and y.

$$\begin{pmatrix} 7 & 4 \\ 6 & 2 \end{pmatrix}\begin{pmatrix} 2.50 & x \\ y & 4.25 \end{pmatrix} = \begin{pmatrix} 41.50 & 41.50 \\ 27 & 29.50 \end{pmatrix} ..[M1]$$

$7(2.50)+4y=41.50$

$4y=24$

$y=\$6 ..[A1]$

$7x+4(4.25)=41.50$

$7x=24.50$

$x=\$3.50 ..[A1]$

9. A floor tile in the shape of a rhombus has sides of length 52 cm and a diagonal of length 48 cm.
Find
(a) the length of the other diagonal,
(b) the area of the floor tile.

$(a)\sqrt{52^2-\left(\frac{48}{2}\right)^2}$ ..[M1 for applying Pythagoras thm]

$=\sqrt{2128}$

Length of diagonal $=2\sqrt{2128}=92.3cm\,(3sf)$..[A1]

$(b)\text{Area}=2\left(\frac{1}{2}(48)\left(\sqrt{2128}\right)\right)$

$=2214.25$

$=2210cm^2$..[B1]

..... cm [2]

..... cm^2 [1]

---

10. $\xi=\{x:x \text{ is a natural number less than } 12\}$
$P=\{x:x \text{ is a multiple of } 3\}$
$Q=\{x:x \text{ is a prime number}\}$

Find
(a) (i) $P\cup Q$,
(ii) $n(P\cap Q')$,
(b) Express, in set notation, the set
$\{x:x \text{ is neither a multiple of 3 nor a prime number}\}$.

$(a)(i)P\cup Q=\{2,3,5,6,7,9,11\}$..[B1]

$(ii)P\cap Q'=\{6,9\}$
$n(P\cap Q')=2$..[B1]

$(b)P'\cap Q'$..[B1] Also accept $(P\cup Q)'$

Answer (a)(i) .....[1]

(a)(ii)..... [1]

(b) ..... [1]

11. A model of a car is made to a scale of 1 : 30.
(a) If the actual length of the car is 4.47 m, find the length of the model in centimetres.
(b) If the total surface area of the model is 750 cm^2, find the actual surface area of the car in square metres, correct to the nearest square metres.

$(a) 4.47 \div 0.3 = 14.9cm$..[B1]

$(b) 1cm^2 : 0.09m^2$..[M1]

750×0.09

$= 67.5$

$\approx 68 m^2$...[A1]

Answer (a) ..... cm [1]

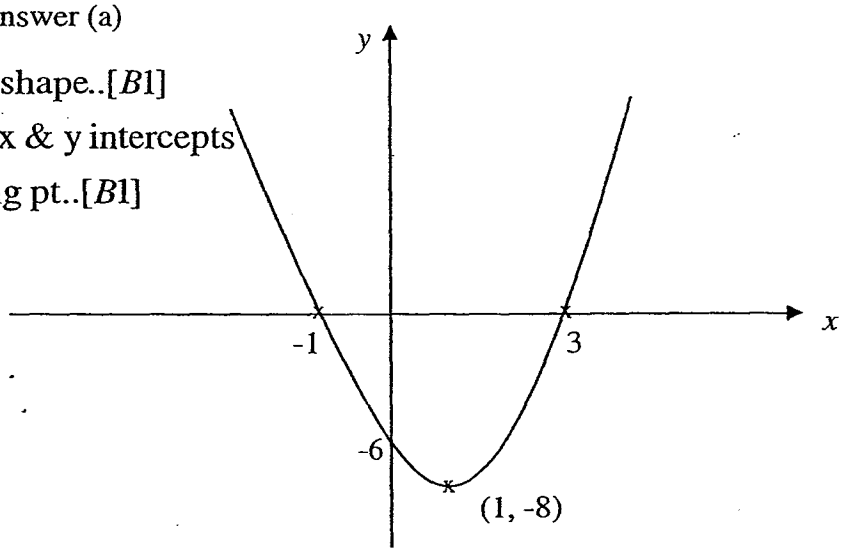
(b)..... m^2 [2]

---

12. (a) Sketch the graph of $y = 2(x+1)(x-3)$. [2]

Answer (a)

(a) Correct shape..[B1]
Correct x & y intercepts & turning pt..[B1]



(b) (i) Express $x^2 + 4x + 10$ in the form $(x+h)^2 + k$.
(ii) Hence write down the minimum value of $x^2 + 4x + 10$.

$(b)(i)(x+2)^2 + 6$..[B1]

(ii) Min value = 6..[B1]

Answer (b)(i) ..... [1]

(ii) ..... [1]

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13. (a) Factorise $p^2-9q^2+6qr-r^2$ completely.

(b) Solve $\frac{1}{6x-1}-\frac{1}{4x+1}=\frac{1}{6}$.

$(a)\, p^2-9q^2+6qr-r^2$

$=p^2-(9q^2-6qr+r^2)$

$=p^2-(3q-r)^2$..[M1 for factorisation]

$=(p+3q-r)(p-3q+r)$..[A1]

$(b)\, 6(4x+1)-6(6x-1)=(6x-1)(4x+1)$

$-12x+12=24x^2+2x$..[M1]

$24x^2+14x-13=0$

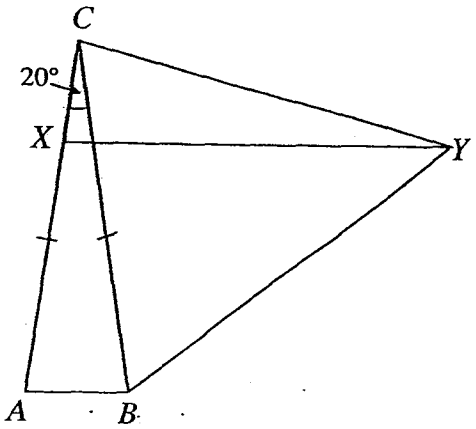
$(2x-1)(12x+13)=0$

$x=\frac{1}{2}$ or $x=-1\frac{1}{12}$..[A1 for both correct answers]

Answer (a) ..... [2]

(b) $x=$ ..... [2]

14. In the diagram, triangle ABC is isosceles with $AC = BC$ and $\angle ACB = 20°$. BCY is an equilateral triangle and X is a point on AC such that $CX = AB$.



Prove that triangle ABC and triangle CXY are congruent.

Since $AC = BC$ (isos triangle),
$AC = CY$ (BC = CY, equilateral triangle)..[M1]

$AB = CX$ (given)..[M1]

$$\angle CAB = \frac{180° - 20°}{2} = 80°$$

Since $\angle YCB = 60°$ (equilateral triangle)
$\angle YCX = 20° + 60° = 80°$
$\angle CAB = \angle YCX$ ..[M1]

$\Delta ABC \equiv \Delta CXY (SAS)$..[A1]

15. The line AB passes through the points A (0, –1) and B (2, 1).

(a) Find the equation of the line AB.

(b) Find the equation of the line which is the reflection of the line AB about the y-axis.

$(a) Gradient$

$$= \frac{1-(-1)}{2-0}$$

$$= 1..[M1]$$

$$y = x - 1..[A1]$$

(b) *Gradient*

$$= -1..[M1]$$

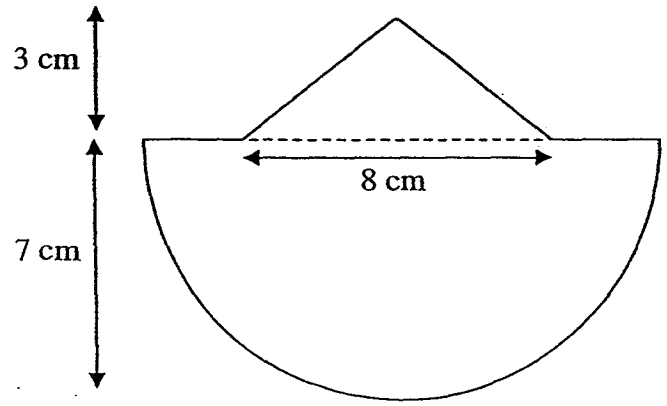
Reflect y - intercept(0,–1) about $y - axis$:

$$y = -x - 1..[A1]$$

Answer (a)..... [2]

(b) ..... [2]

16. The diagram shows the side view of a solid figure.
It is made up of a hemisphere and a cone.



(a) Calculate the exterior area of the solid figure to be painted, leaving your answer in terms of π.

(b) Given that 1 cm^2 of the surface area of the figure requires 2 cm^3 of paint, find the number of identical figures that can be painted with 3 litres of paint.

$$(a)\text{Exterior area} = 2\pi(7)^2 + \pi\left(\frac{8}{2}\right)\left(\sqrt{\left(\frac{8}{2}\right)^2 + 3^2}\right) + \left[\pi(7)^2 - \pi\left(\frac{8}{2}\right)^2\right]$$

$$= 98\pi + 20\pi + 33\pi$$

$$= 151\pi..[A1]$$

[M2 for finding surface area of hemisphere, cone and ring]

$$(b)\ 3000 \div 151\pi(2) = 3.16 \approx 3..[B1]$$

Answer (a) ..... [3]

(b)..... [1]

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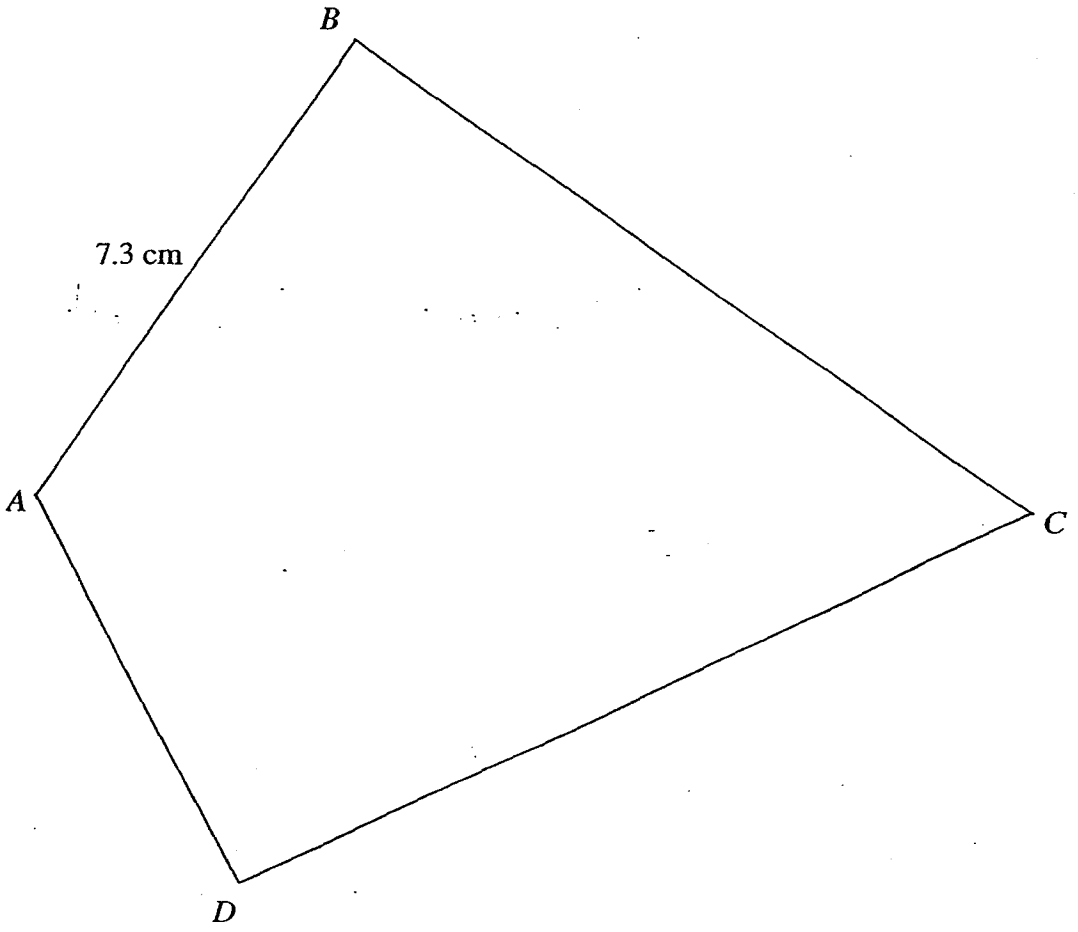
17. The diagram shows a park *ABCD* in the shape of a quadrilateral.

Answer (b)



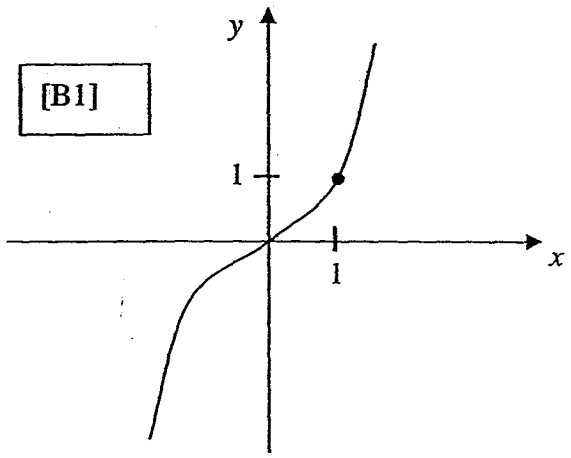
(a) If the actual distance of *AB* is 7.3 km, express the scale in the form of 1 : *n*.

(b) A statue is located at point *Q* which is inside the park *ABCD* and it is equidistant from *B* and *C* and equidistant from *BC* and *DC*.
Mark and label the point *Q*. [3]

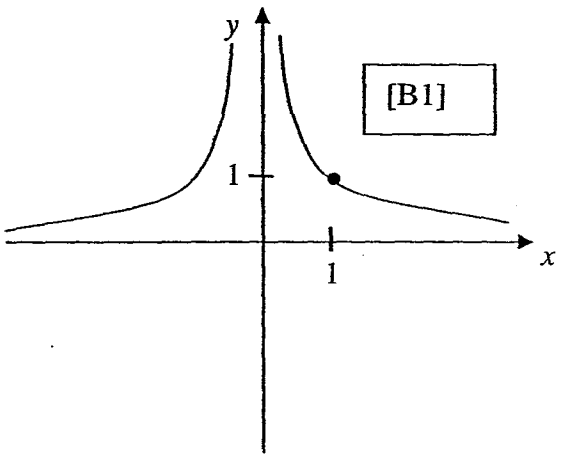
Answer (a) 1 : ..... [1]

18. (a) The point (1, 1) is marked on the diagram in the answer space.
On the axes in the answer space, draw the graphs of

(i) $y = x^3$ (ii) $y = \frac{1}{x^2}$ [2]



[B1]



[B1]

(b) Explain how you can obtain the number of solutions for $x^3 = \frac{1}{x^2}$ and state the number of solution.

(c) Write down the coordinates for the solution(s) to (b).

(*b*) Draw the two graphs $y = x^3$ and $y = \frac{1}{x^2}$ on the same axes and find the intersection points of the two graphs..[*B*1]
There is only 1 solution..[B 1]
[Also accept : Solve the two equations $y = x^3$ and $y = \frac{1}{x^2}$ simultaneously to find thesolution..[B 1]]

(*c*) (1,1)..[*B*1]

Answer (b).....

..... [2]

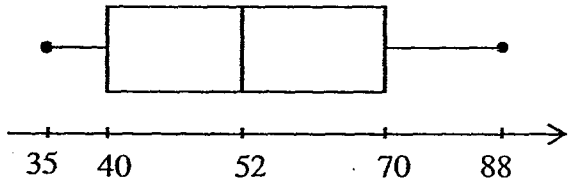
(c) (..... , ...) [1]

19. (a) The heights, to the nearest metres, of 12 trees collected from Land A are

35, 38, 40, 42, 45, 57, 58, 58, 63, 67, 70, 75.

(i) Find the interquartile range.

(ii) The box and whisker plot below shows the heights of another group of 12 trees collected from Land B.



Comment briefly on the heights of the trees, in metres, on both lands.

(b) With the addition of one more tree of length x m on Land A, the new median is 57 m.
Write down the range of values of x.

$(a)(i)$ IQR

$= \left(\frac{63+67}{2}\right) - \left(\frac{40+42}{2}\right)$ ..[M1]

$= 65 - 41$

$= 24$..[A1]

IQR
= 63 – 42 ..[M1] Median is included
= 21 ..[A1] Also accept

(ii) $\text{Median}_A < Median_B$ The trees on Land A are generally taller than that on Land B..[B1]

$\text{IQR}_A < IQR_B$ The heights of trees on Land A are more consistent than that on Land B..[B1]

(b) $0 < x \leq 57$..[B1]

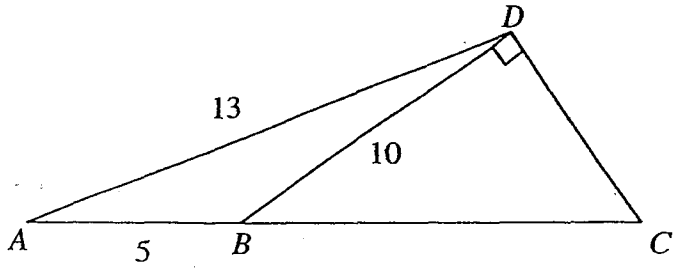
Answer (a)(i) ..... [2]

(ii) ..... [1]

(b) .....

..... [2]

20. In the diagram, ABC is a straight line.
$AB = 5$ m, $BD = 10$ m, $AD = 13$ m and $\angle BDC = 90°$.



(a) Find the value of $\cos \angle ABD$.

(b) (i) Write down the value of $\cos \angle CBD$.

(ii) **Hence** find the length of BC.

$(a)\ 13^2 = 10^2 + 5^2 - 2(10)(5)\cos \angle ABD..[M1]$

$\cos \angle ABD = -0.44..[A1]$

$(b)(i)\ \cos \angle CBD = 0.44..[B1 \quad \text{Also accept } \frac{11}{25}]$

$(ii)\ \cos \angle CBD = \frac{10}{BC}..[M1]$

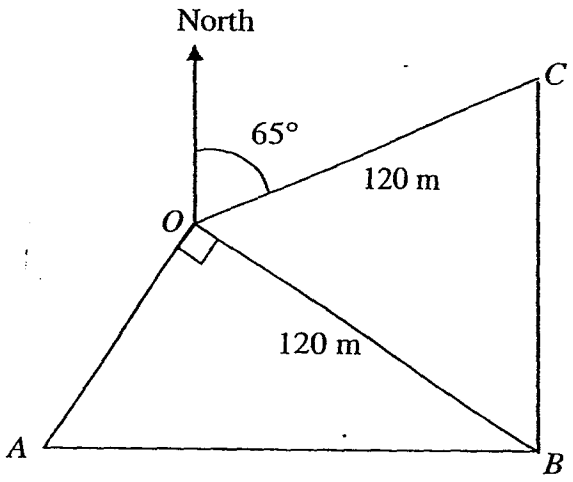
$BC = 22.7m(3sf)..[A1]$

Answer (a) ..... [2]

(b)(i) ..... [1]

(ii) .....m [2]

21. O, A, B and C are four points on level ground.
C is on a bearing of $065°$ from O.
B is due east of A and south of C.
$OB = OC = 120$ m and $\angle AOB = 90°$.



Calculate
(a) the bearing of A from O,
(b) the distance OA,
(c) the area of $OABC$.

(a)Bearing of A from O
$= 65° + 90° + (180° - 65° - 65°)$....[$M1$ for $\angle$COB]
$= 205°$..[$A1$]

$(b)\ \tan 25° = \dfrac{\text{OA}}{120}$...[$M1$]
$OA = 56.0m(3sf)$..[$A1$]

(c)Area
$= \frac{1}{2}(120)(120\tan 25°) + \frac{1}{2}(120)^2 \sin 50°$..[$M1$]
$= 8872.93$
$\approx 8870m^2\,(3sf)$..[$A1$]

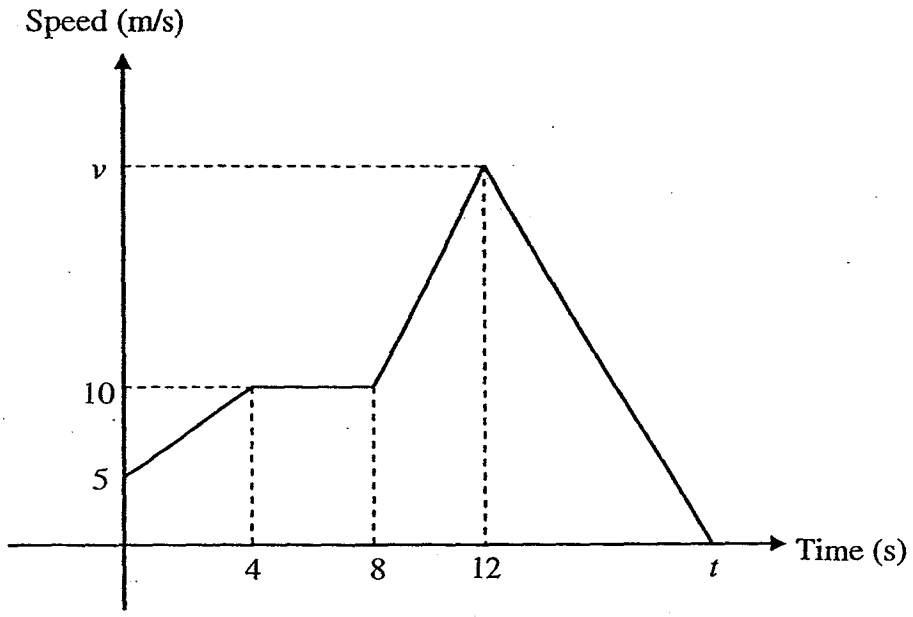
(a) Since $OB = OC$,
$\angle COB = 180° - 2(65°)$
$= 50°$..[$M1$]
Bearing of A from O
$= 65° + 50° + 90°$
$205°$..[$A1$]

Answer (a) ..... ° [2]

(b) ..... m [2]

(c) ..... m [2]

22. The diagram below shows the speed-time graph of a moving body over a period of t seconds.



Calculate

(a) the acceleration during the first 4 seconds,

(b) the value of v if the total distance moved in the first 12 seconds was 146 m,

(c) the value of t, given that the retardation was 4.5 m/s^2.

$$(a)\ \text{Acc.} = \frac{10-5}{4-0} = 1.25 m/s^2 ..[B1]$$

$$(b)\frac{1}{2}(5+10)(4)+4(10)+\frac{1}{2}(10+v)(4)=146..[M1]$$

$$2(10+v)=76$$

$$v=28m/s..[A1]$$

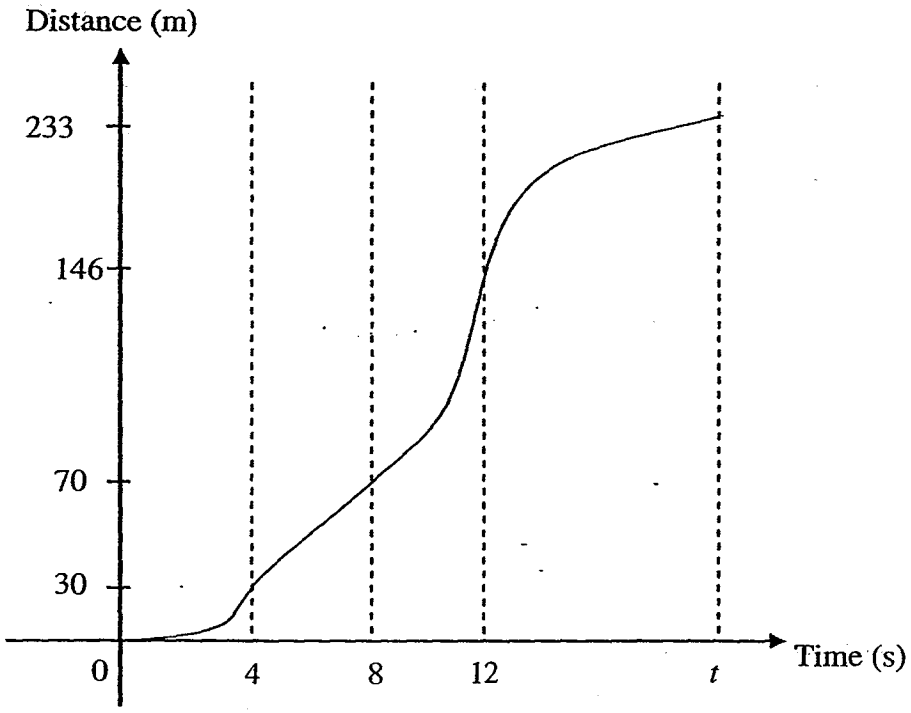
$$(c)-4.5=\frac{28-0}{12-t}$$

$$-54+4.5t=28$$

$$4.5t=82$$

$$t=18.2s..[B1]$$

(d) On the axes in the answer space, sketch the distance-time graph of the object over the period of t seconds.
Mark the distance travelled on the axis clearly. [2]



[B1] for correct shapes for graphs
[B1] correct labelling of values

Answer (a) .....m/s^2 [1]

(b) ..... m/s [2]

(c) ..... m/s [1]

聖嬰中學

HOLY INNOCENTS' HIGH SCHOOL

PRELIMINARY EXAMINATION 2014
SECONDARY 4 EXPRESS / 5 NORMAL (ACADEMIC)

MATHEMATICS **4016/02**

Paper 2

Name : _____ **Date** : **5 August 2014**

Register No : _____ **Duration** : **2 h 30 min**

Class : _____

Additional Materials needed:
6 sheets of writing papers
1 sheet of graph paper

Instructions to Candidates

Write your index number and name on all the work you hand in.
Write in dark blue or black pen.
You may use a pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** questions.

If working is needed for any question, it must be shown with the answer.
Omission of essential working will result in loss of marks.
Calculators should be used where appropriate.

If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.
For π, use either your calculator value or 3.142, unless the question requires the answer in terms of π.

The number of marks is given in brackets [ ] at the end of each question or part question.
The total marks for this paper is 100.

Setter: Mrs Rajammal Nathan

Vetter: Mdm Hayati
Ms Koh Swee Kun

This paper consists of 10 printed pages, inclusive of this cover page.

Mathematical Formulae

Compound interest

$$\text{Total amount} = P\left(1+\frac{r}{100}\right)^n$$

Geometry and Measurement

$$\text{Curved surface area of a cone} = \pi rl$$

$$\text{Surface area of a sphere} = 4\pi r^2$$

$$\text{Volume of a cone} = \frac{1}{3}\pi r^2 h$$

$$\text{Volume of a sphere} = \frac{4}{3}\pi r^3$$

$$\text{Area of triangle } ABC = \frac{1}{2}ab\sin C$$

$$\text{Arc length} = r\theta \text{ , where } \theta \text{ is in radians}$$

$$\text{Sector area} = \frac{1}{2}r^2\theta \text{ , where } \theta \text{ is in radians}$$

Trigonometry

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

Statistics

$$\text{Mean} = \frac{\sum fx}{\sum f}$$

$$\text{Standard deviation} = \sqrt{\frac{\sum fx^2}{\sum f} - \left(\frac{\sum fx}{\sum f}\right)^2}$$

Answer **all** the questions

1 **(a)** **(i)** Factorise completely $n^3 - n$. [2]

(ii) Explain why $n^3 - n$ is divisible by 2 for any integer value of n. [1]

(b) Given that $\frac{3a}{4b} = \frac{1}{2}$, where $b \neq 0$, evaluate $\frac{\sqrt{a^2 + 5b^2}}{b}$. [3]

(c) It is given that $m = \frac{\sqrt[3]{n^2 - l}}{2}$.

(i) Find m when $n = 0.5$ and $l = -3.05$. [1]

(ii) Express l in terms of m and n. [2]

---

2 Numbers are written out in three columns to make the following pattern:

	Column 1	Column 2	Column 3
Row 1	3	8	67
Row 2	12	27	63
Row 3	27	64	59
Row 4	48	125	55
Row 5	p	q	r
Row n	x	y	z

(a) Study the patterns and write down the value of

(i) p, [1]

(ii) q, [1]

(iii) r. [1]

(b) Find expressions, in terms of n, for each of x, y and z. [3]

(c) It is given that Row m, Column 1 contains the first term which is greater than 90 000.
Find the value of m. [1]

(d) It is given that Row s, Column 3 contains the first negative term.
Find the value of s. [1]

3 (a) Mrs Jones saved 12% of her monthly income and spent 25% of the remainder on entertainment.
If Mrs Jones spent \$528 on entertainment, calculate her monthly income. [2]

(b) Mrs Jones bought a new sewing machine for \$1440 at a 20% discount.
Despite giving the 20% discount, the shopkeeper still made a profit of 20% on the cost price.
Calculate

(i) the price Mrs Jones would have to pay for the sewing machine if the shopkeeper had not given the 20% discount, [2]

(ii) the cost price of the sewing machine. [2]

(c) Mrs Jones invested \$500 in an account which pays x % per year compound interest.
She earned an interest of \$220 in two years.
Calculate the value of x. [4]

---

4 The distance between two towns, X and Y, is 350 km.
Andy left Town X on Sunday and reached Town Y at 06 05 on Monday.
He drove at an average speed of x km/h from X to Y.

(a) Write down an expression, in terms of x, for the time, in hours, that he took to complete the journey from X to Y. [1]

(b) On the return journey from Y to X, his average speed was 2 km/h less than his speed from X to Y.
Write down an expression, in terms of x, for the time, in hours, that he took for the journey from Y to X. [1]

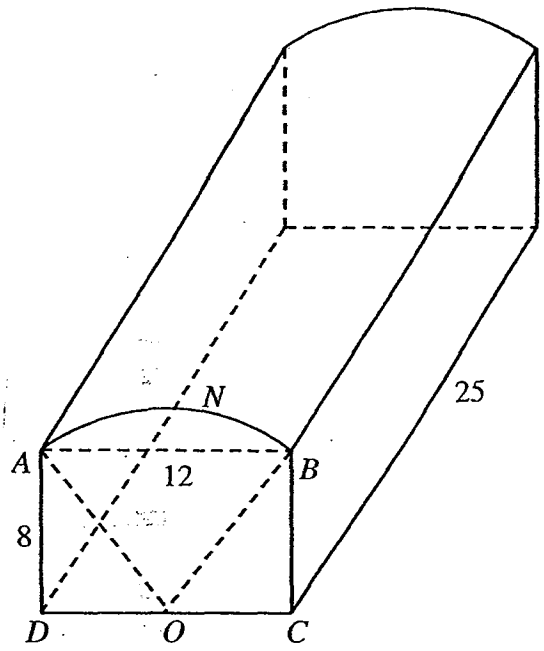
(c) Given also that the return journey took 20 minutes more than the journey from X to Y, form an equation in x, and show that it reduces to $x^2 - 2x - 2100 = 0$. [3]

(d) Solve the equation $x^2 - 2x - 2100 = 0$, giving both answers correct to two decimal places. [4]

(e) Calculate, in hours and minutes, correct to the nearest minute, the total time that Andy spent travelling from X to Y and back to X. [2]

(f) Find the time Andy left Town X on Sunday. [1]

5



The diagram shows a loaf of bread.
It has a uniform cross-section *ANBCD*, in which *ABCD* is a rectangle.
O is the midpoint of *DC* and *ANB* is an arc of a circle, centre *O*.
AB = 12 cm and *AD* = 8 cm.
The length of the loaf of bread is 25 cm.

(a) Show that angle *AOB* = 1.29 radians. [2]

(b) Find

(i) the length of the arc *ANB*, [2]

(ii) the area of the segment *ANB*, [2]

(iii) the total surface area of the loaf of bread. [2]

(c) The mass of the loaf of bread is 600 g.
Another similar loaf of bread is made, with a mass of 400 g.
Find the length of this loaf of bread. [2]

6 Answer the whole of this question on a sheet of graph paper.

When x copies of a book are produced, the cost, \$$y$ of each copy is given by the formula $y = 10 + \frac{2400}{x}$.

(a) The table below gives some values of x and the corresponding values of y.

x	100	200	300	400	600	800	1200
y	34	22	18	16	14	13	p

(i) Find the value of p. [1]

(ii) Using a scale of 2 cm to represent 200 books, draw a horizontal x-axis for $0 \le x \le 1200$.
Using a scale of 2 cm to represent \$5, draw a vertical y-axis for $0 \le y \le 40$.
On your axes, plot the points given in the table and join them with a smooth curve. [3]

(b) Use your graph to estimate the number of books to be printed if the cost of producing each book is \$15. [1]

(c) **(i)** By drawing a tangent, find the gradient of the curve at the point where $x = 300$. [2]

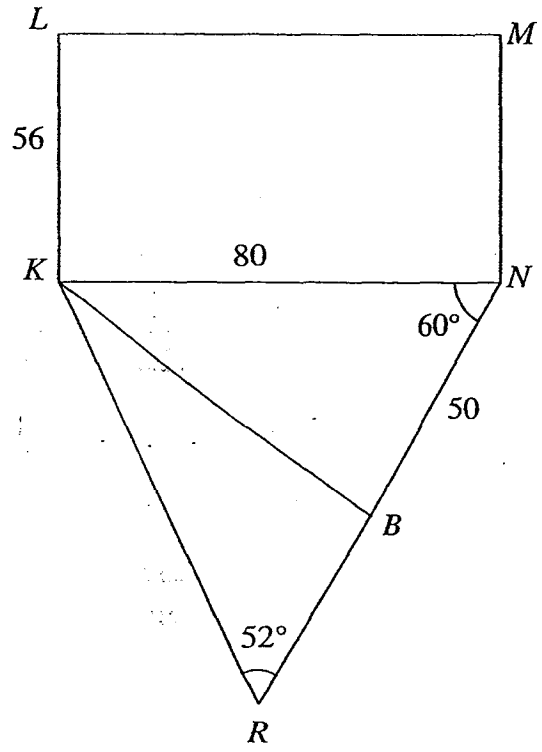
(ii) State briefly what this gradient represents. [1]

(d) In order to sell x books, the selling price of each book must be $\left(25 - \frac{x}{60}\right)$ dollars.

(i) On the axes in part (a), draw the graph of $y = \left(25 - \frac{x}{60}\right)$, for values of x from 0 to 1200. [2]

(ii) Use your graph to find the range of the number of books that should be printed if no loss is to be made, assuming that all books will be sold. [2]

7



In the diagram, the rectangle *KLMN* represents a vertical cliff and *KNBR* represents part of the horizontal surface of the sea.
The sea meets the cliff along the horizontal line *KN*.
A boat is at the point *B*, a rock is at the point *R* and *RBN* is a horizontal straight line.
LK = 56 m, *KN* = 80 m, *NB* = 50 m, angle *KNR* = 60° and angle *KRN* = 52°.

(a) Calculate

(i) *KR*, [2]

(ii) *KB*, [3]

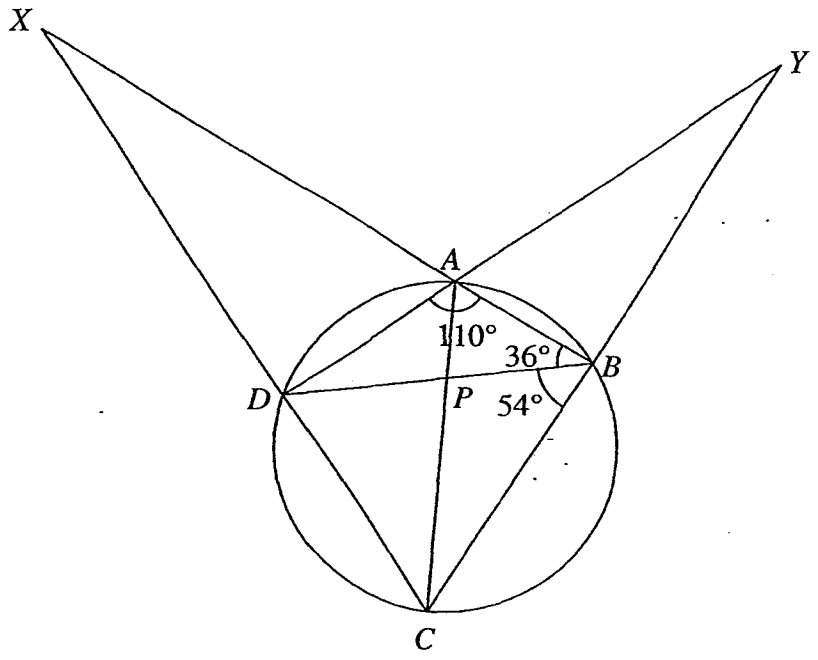
(b) Calculate the angle of depression of the boat when viewed from *L*. [2]

(c) Calculate the shortest distance from the boat to the cliff, *KN*. [2]

(d) Calculate the greatest possible value of the angle of elevation of the top of the cliff when viewed from the boat. [2]

8 **(a)** A polygon has n sides. Two of its interior angles are 140° and 160°, while the remaining interior angles are each equal to 120°.
Find the value of n. [2]

(b)



The diagram shows a circle $ABCD$.
Angle $BAD = 110°$, angle $ABD = 36°$ and angle $DBC = 54°$.
AC cuts BD at P.

(i) Find

(a) angle CAD, [1]

(b) angle BPA. [2]

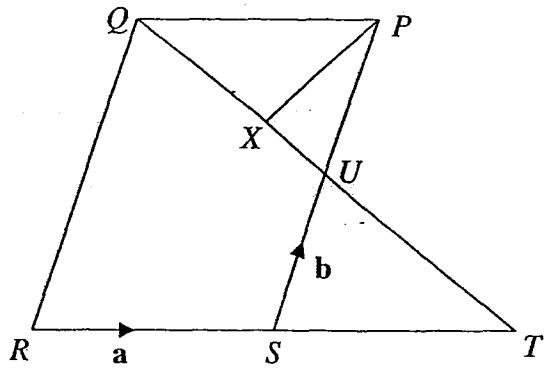
(ii) CDX, BAX and CBY are straight lines.

(a) Calculate angle ADX, showing your reasoning. [2]

(b) Explain why X, D, B and Y lie on a circle. [1]

(c) State where the centre of the circle $XDBY$ lies. [1]

9



In the diagram, $PQRS$ is a parallelogram.
RS and QU produced meet at T such that $RT = 2RS$ and $TU = UQ$.
X is a point on QU such that $XU = \frac{1}{3}QU$.

$\overrightarrow{RS} = \mathbf{a}$ and $\overrightarrow{SU} = \mathbf{b}$.

(a) Express, as simply as possible, in terms of **a** and/or **b**,

(i) $\overrightarrow{SP}$, [1]

(ii) $\overrightarrow{RP}$, [1]

(iii) $\overrightarrow{UQ}$. [1]

(b) Show that R, X and P lie on a straight line. [2]

(c) Find

(i) $\dfrac{\text{Area of } \Delta TSU}{\text{Area of } \Delta TRQ}$, [1]

(ii) $\dfrac{\text{Area of } \Delta PXQ}{\text{Area of } \Delta PQU}$, [1]

(iii) $\dfrac{\text{Area of } \Delta PQX}{\text{Area of } PQRS}$. [2]

10 (a) A survey is carried out to find the waiting times at two clinics.
The results are shown in the tables below.

Clinic *A*

Time (x minutes)	$6 < x \leq 10$	$10 < x \leq 14$	$14 < x \leq 18$	$18 < x \leq 22$	$22 < x \leq 26$
Number of patients	1	6	4	5	4

Clinic *B*

Mean = 17 minutes
Standard deviation = 3.28 minutes

(i) For Clinic *A*, calculate

(a) the mean, [1]

(b) the standard deviation. [2]

(ii) With regards to waiting time, which clinic is better in its service?
Justify your answer. [1]

(b)

	X				*Q*	*Y*		*P*

The diagram shows a grid of squares.
A button is placed on one of the squares.
A fair die is thrown.
If 1, 2, 3 or 4 appears, the button is moved one square to the left.
If 5 or 6 appears, the button is moved one square to the right.

(i) The button is placed on square *X*.
The die is thrown once.
Find, in its simplest form, the probability that the button is moved to the right. [1]

(ii) On another occasion the button is placed on square *Y*.
The die is thrown once and the button is moved.
The die is thrown a second time and the button is moved again.
Find, in its simplest form, the probability that the button finishes at

(a) *P*, [1]

(b) *Q*, [1]

(c) *Y*, [2]

(d) *P* or *Q* or *Y*. [1]

End of Paper

Answers

1 a (i) $n(n+1)(n-1)$

(ii) $n^3 - n$ is a product of 3 consecutive integers with one or two of the integers being even. Therefore it is divisible by two. B1

[OR If n is odd, then n + 1 is even. ∴ $n(n+1)(n-1)$ is divisible by 2. If n is even, then $n(n+1)(n-1)$ is divisible by 2.]

b $2\frac{1}{3}$

(c) (i) 0.744 (ii) $l = n^2 - 8m^3$

2 (a) (i) $p = 75$ (ii) $q = 216$ (iii) $r = 51$

(b) $x = 3n^2$, $y = (n+1)^3$, $z = 67 - 4(n-1)$ **OR** $\mathbf{74 - 4n}$

(c) m = 174 (d) n = 18

3 (a) \$2400 (b) (i) \$1800 (ii) \$1200

(c) $x = 20$

4 (a) $\frac{350}{x}$ h (b) $\frac{350}{x-2}$ h

(c) $\frac{350}{x-2} - \frac{350}{x} = \frac{20}{60}$

$\frac{350x - 350(x-2)}{x(x-2)} = \frac{1}{3}$

$\frac{700}{x^2 - 2x} = \frac{1}{3}$

$x^2 - 2x - 2100 = 0$ (shown)

(d) $x \approx 46.84$ or -44.84 (e) 15 h 17 min (f) 22 37

5 (a) Let M be the mid-point of AB.

$\tan \angle AOM = \frac{6}{8}$

$\angle AOB = 2 \times \tan^{-1} \frac{6}{8}$

$= 1.287 \approx 1.29$ radians (shown)

OR [tan $\angle AOD = \frac{8}{6}$

$\angle AOB = \pi - 2\tan^{-1}\frac{8}{6}$

≈ 1.29 radians]

[OR using cosine rule:

$12^2 = 10^2 + 10^2 - 2\,(10)(10)\cos\angle AOB$

$\angle AOB \approx 1.29$ radians]

(b) (i) 12.9 cm

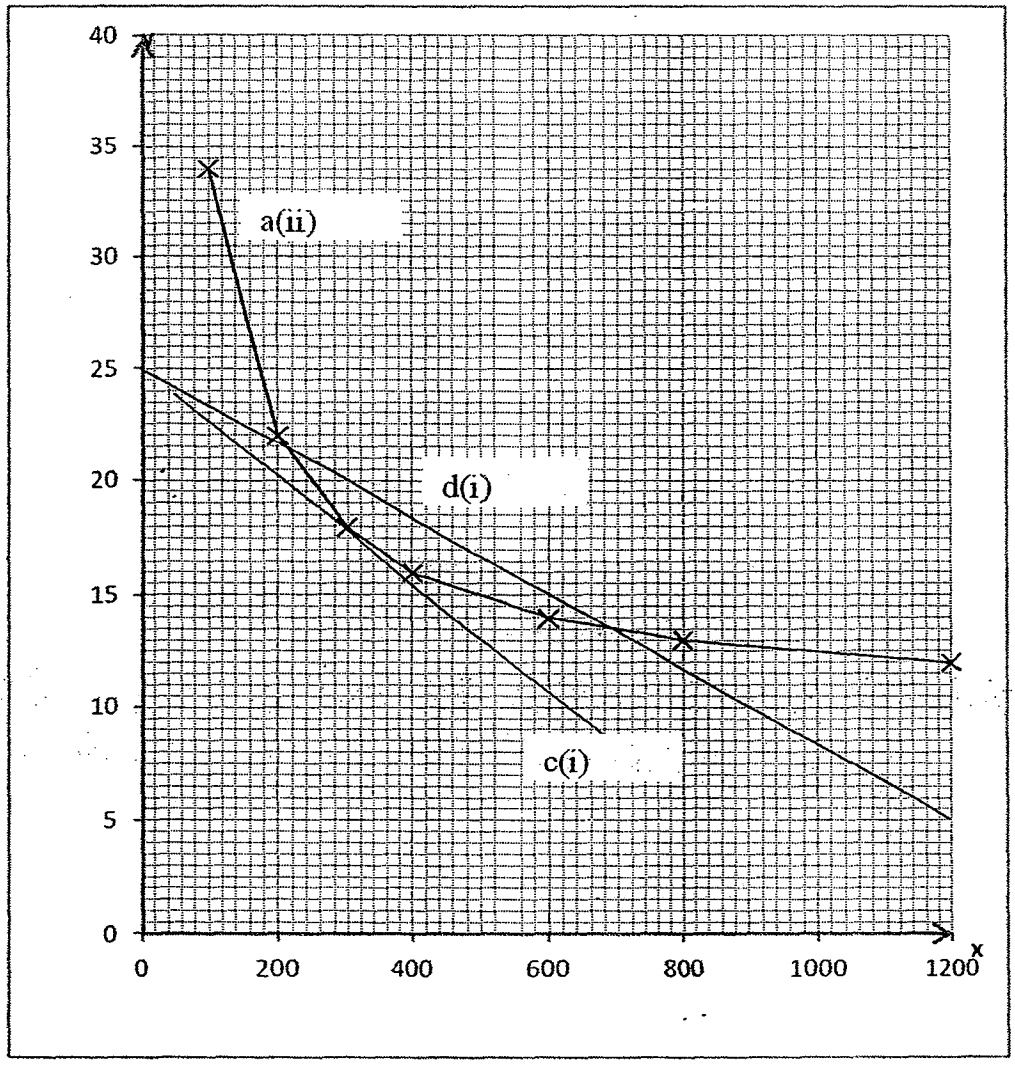
(ii) If $\angle AOB \approx 1.287$ radians is used, area of segment $ANB \approx 16.4$ cm^2

[If $\angle AOB \approx 1.29$ radians, Area ≈ 16.5 cm^2]

(iii) 1250 cm^2

(c) 21.8 cm

6 (a) (i) $p = 12$



b 480 ±10

c (i) Drawing of tangent line at $x = 300$

gradient = -0.027 ± 0.005

(ii) Possible answers:
The cost decreases at a rate of \$0.027 per book
The rate at which the cost is decreasing
The unit cost is decreasing at \$0.027 per book
The rate of change in the cost

(d) (i) The straight line must pass through (0, 25) and (1200, 5).
(ii) $210 \leq x \leq 700 (\pm 10)$

7 (a) (i) 87.9 m (ii) $KB = 70$ m
(b) 38.7° (c) 43.3 m (d) 52.3°

8 (a) $n = 7$

(b) (i) (a) 54° (b) 88°

(ii) (a) 90°

(b) $\angle ABY = 180° - (54° + 36°)$ ($\angle$s on a st. line)
$= 90°$
$= \angle ADX$
Since it satisfies the property of angles in the same segment, X, D, B and Y lie on a circle.

(c) mid-point of XY.

9 (a) (i) $2\mathbf{b}$ (ii) $\mathbf{a} + 2\mathbf{b}$ (iii) $-\mathbf{a} + \mathbf{b}$

(b) $\overrightarrow{RX} = \overrightarrow{RS} + \overrightarrow{SU} + \overrightarrow{UX}$

$= \mathbf{a} + \mathbf{b} + \frac{1}{3}\overrightarrow{UQ}$

$= \mathbf{a} + \mathbf{b} + \frac{1}{3}(\mathbf{b} - \mathbf{a})$

$= \frac{2}{3}\mathbf{a} + \frac{4}{3}\mathbf{b}$

$= \frac{2}{3}(\mathbf{a} + 2\mathbf{b})$

$= \frac{2}{3}\overrightarrow{RP}$

Therefore R, X and P lie on a straight line.

[OR $\overrightarrow{XP} = \overrightarrow{XU} + \overrightarrow{UP}$

$= \frac{1}{3}(\mathbf{a} - \mathbf{b}) + \mathbf{b}$

$= \frac{1}{3}\mathbf{a} + \frac{2}{3}\mathbf{b}$

$= \frac{1}{3}(\mathbf{a} + 2\mathbf{b})$

$= \frac{1}{3}\overrightarrow{RP}$

Therefore R, X and P lie on a straight line.]

(c) (i) $\frac{1}{4}$

(ii) $\frac{2}{3}$

(iii) $\frac{1}{6}$

10 (a) (i) (a) mean = 17 minutes
(b) standard deviation = 4.88 minutes

(ii) Clinic B is better in its service as it has a smaller standard deviation than that of clinic A. The spread of its waiting time is lesser and hence more consistent.

(b) (i) $\frac{1}{3}$

(ii) (a) $\frac{1}{9}$

(b) 0

(c) $\frac{4}{9}$

(d) $\frac{5}{9}$

Marking Scheme
Preliminary Examination 2014
Paper 2
Sec 4 Express

1 a (i) $n^3 - n$

$= n(n^2 - 1)$

$= n(n+1)(n-1)$ B2

(B1 for partial factorisation)

(ii) $n^3 - n$ is a product of 3 consecutive integers with one or two of the integers being even. Therefore it is divisible by two. B1

[OR If n is odd, then n + 1 is even. ∴ $n(n+1)(n-1)$ is divisible by 2. If n is even, then $n(n+1)(n-1)$ is divisible by 2.]

b $\frac{3a}{4b} = \frac{1}{2}$

$a = \frac{2}{3}b$ M1

Subst $a = \frac{2}{3}b$ into $\frac{\sqrt{a^2+5b^2}}{b}$

$\frac{\sqrt{a^2+5b^2}}{b} = \frac{\sqrt{\left(\frac{2}{3}b\right)^2+5b^2}}{b}$ M1

$= \frac{\sqrt{\frac{49}{9}b^2}}{b} = \frac{\frac{7}{3}b}{b}$

$= 2\frac{1}{3}$ A1

[OR $b = \frac{3}{2}a$ **M1**

$\frac{\sqrt{a^2+5b^2}}{b} = \frac{\sqrt{a^2+5\left(\frac{3}{2}a\right)^2}}{\frac{3}{2}a}$ **M1**

$= \frac{\sqrt{\frac{49}{4}a^2}}{\frac{3}{2}a} = \frac{\frac{7}{2}a}{\frac{3}{2}a}$

$= 2\frac{1}{3}$ **A1]**

(c) (i) 0.744 B1

(ii) $8m^3 = n^2 - l$ M1

$l = n^2 - 8m^3$ A1

2 (a) (i) $p = 75$ B1

(ii) $q = 216$ B1

(iii) $r = 51$ B1

(b) $x = 3n^2$ B1

$y = (n + 1)^3$ B1

$z = 67 - 4(n - 1)$ B1

OR 71 – 4*n*

(c) $3m^2 > 90000$

$m^2 > 30000$

$m > 173.2$

$m = 174$ B1

(Accept guess and check method)

(d) $71 - 4s < 0$

$s > 17\frac{3}{4}$

$s = 18$ B1

(Accept guess and check method)

3 (a) $\frac{25}{100} \times 88\% = 22\%$

$\frac{\$528}{22} \times 100$ M1

$= \$2400$ A1

[OR 25% of remainder = \$528

Remainder = \$528 × 4 = \$2112 **M1**

\$2112 rep 88% of monthly income

∴ monthly income = $\frac{2112}{88} \times 100 = \$2400$ **A1]**

(b) (i) $\frac{\$1440}{80} \times 100$ M1

$= \$1800$ A1

(ii) $\frac{\$1440}{120} \times 100$ M1

$= \$1200$ A1

(c) $220 = 500\left(1+\frac{x}{100}\right)^2 - 500$ M2

(M1 for $500\left(1+\frac{x}{100}\right)^2$)

$\left(1+\frac{x}{100}\right)^2 = \frac{720}{500}$ M1

$\left(1+\frac{x}{100}\right) = \sqrt{\frac{720}{500}}$

$\left(\frac{x}{100}\right) = \sqrt{\frac{720}{500}} - 1$

$x = 20$ A1

4 (a) $\frac{350}{x}$ h B1

(b) $\frac{350}{x-2}$ h B1

(c) $\frac{350}{x-2} - \frac{350}{x} = \frac{20}{60}$ M1

$\frac{350x - 350(x-2)}{x(x-2)} = \frac{1}{3}$ M1

$\frac{700}{x^2-2x} = \frac{1}{3}$

$x^2 - 2x - 2100 = 0$ A1

(d) $x = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(1)(-2100)}}{2(1)}$ M2

$x = \frac{2 \pm \sqrt{8404}}{2}$

$= 46.836$ or -44.836

≈ 46.84 or -44.84 A2

(e) $\frac{350}{46.836} + \frac{350}{46.836 - 2}$ M1

= 15 h 17 min A1

(f) Time taken from X to $Y = \frac{350}{46.836} = 7.472$ h

= 7 h 28 min

He left Town X at 22 37 B1

5 (a) Let M be the mid-point of AB.

$\tan \angle ACM = \frac{6}{8}$ M1

$\angle ACB = 2 \times \tan^{-1} \frac{6}{8}$

$= 1.287 \approx 1.29$ radians (shown) A1

OR [tan $\angle AOD = \frac{8}{6}$ **M1**

$\angle AOB = \pi - 2\tan^{-1} \frac{8}{6}$

≈ 1.29 **radians. A1]**

[OR using cosine rule:

$\mathbf{12^2 = 10^2 + 10^2 - 2\,(10)(10)\cos \angle AOB}$ **M1**

$\angle AOB \approx 1.29$ radians **A1]**

(b) (i) radius = OA

∴ by Pythagoras Theorem,

$OA = \sqrt{8^2 + 6^2} = 10$ M1

Length of arc $AB = 10 \times 1.287 = 12.87$

≈ 12.9 cm (shown) A1

(ii) Area of segment $ANB = \frac{1}{2}(10)^2(1.287) - \frac{1}{2}(10)^2 \sin 1.287$ M1

$= 16.35 \text{ cm}^2 \approx 16.4 \text{ cm}^2$ A1

[For working, accept $\angle AOB = 1.29$**, Area = 16. 45** $\approx$ **16.5 cm^2]**

(iii) Total surface area $= 12.87 \times 25 + 2(8 \times 12 + 16.35) + 2(8 \times 25) + 25 \times 12$ M1

$= 1246.45$

$\approx 1250 \text{ cm}^2$ A1

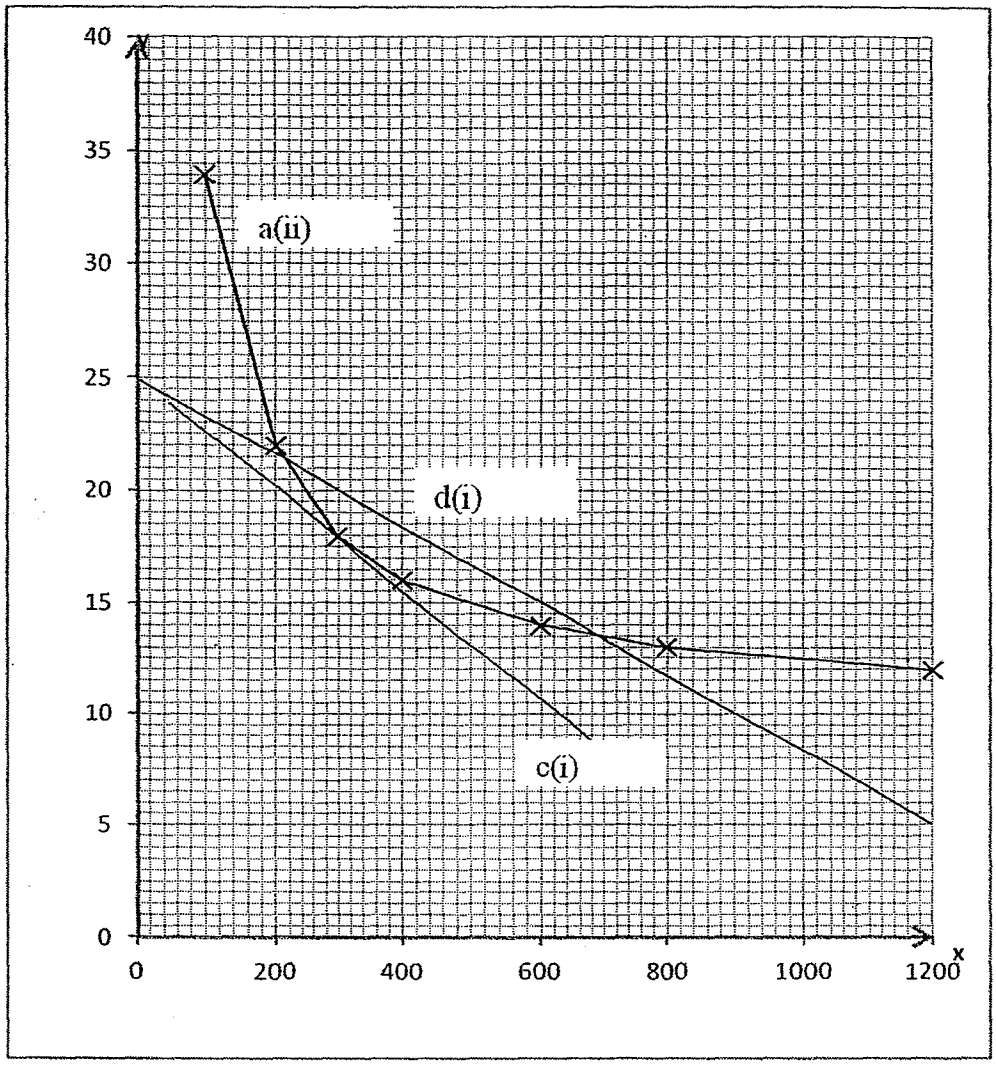
[OR $2(8 \times 12 + 16.35) + [(12.87 + 12 + 8 + 8) \times 25]$

[For working, accept $\angle AOB = 1.29$**, Total surface area = 1247.4** $\approx$ **1250 cm^2]**

(c) length of loaf of bread = $\sqrt[3]{\frac{400}{600}} \times 25$ M1

≈ 21.8 cm A1

6 (a) (i) $p = 12$ B1



(ii) Draw the axes with correct scale and plot all given points. P2
(Allow P1 of not more than 3 errors)
Draw a smooth curve through all plots. C1

b 480 ±10 B1

c (i) Drawing of tangent line at $x = 300$ B1
gradient = -0.027 ± 0.005 B1

(ii) Possible answers: B1
The cost decreases at a rate of \$0.027 per book
The rate at which the cost is decreasing
The unit cost is decreasing at \$0.027 per book
The rate of change in the cost

(d) (i) The straight line must pass through (0, 25) and (1200, 5). B2
(Inaccurate drawing B1)

(ii) $210 \leq x \leq 700\ (\pm 10)$ (B1 for 210 and B1 for 700) B2

7 (a) (i) By sine rule,

$$\frac{KR}{\sin 60°} = \frac{80}{\sin 52°}$$

$$KR = \frac{80}{\sin 52°} \times \sin 60° \quad \text{M1}$$

$$= 87.92 \approx 87.9 \text{ m} \quad \text{A1}$$

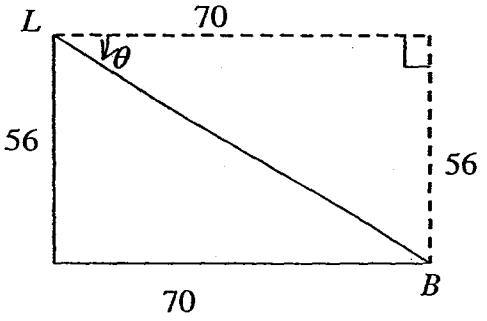
(ii) By cosine rule,

$KB^2 = 80^2 + 50^2 - 2(80)(50)\cos 60°$ (correct substitution) M2

M1 **M1**

$KB = 70$ m A1

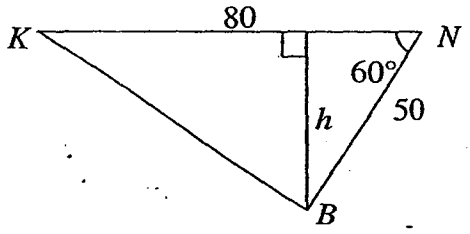
(b) Let the angle of depression be θ.



$$\tan\theta = \frac{56}{70} \quad \text{M1}$$

$$\theta = 38.65° \approx 38.7° \quad \text{A1}$$

(c) Let h be the shortest distance from the boat to the cliff.



h

$$\sin 60° = \frac{h}{50}$$

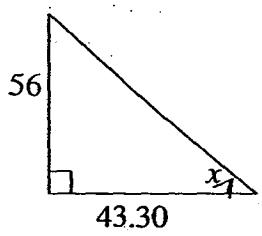
$$= 50 \sin 60° \quad \text{M1}$$

$$= 43.30 \text{ m} \approx 43.3 \text{ m} \quad \text{A1}$$

(OR $\frac{1}{2} \times 80 \times h = \frac{1}{2} \times 80 \times 50 \times \sin 60°$ M1

$h = 43.30$ m $\approx$ 43.3 m A1)

(d) Let x be the greatest angle of elevation



$$\tan x = \frac{56}{43.30} \quad \text{M1}$$

$$x = 52.28° \approx 52.3° \quad \text{A1}$$

8 (a) sum of ext angles = 360°

$(n-2)\,60° + 40° + 20° = 360°$ M1

$n - 2 = \dfrac{360-60}{60}$

$n = 7$ A1

[OR using sum of interior angles = $(n-2)180°$

$(n-2)120° + 140° + 160° = (n-2)180°$ M1

$(n-2)180° - (n-2)120° = 300°$

$60(n-2) = 300$

$n = 7$ A1]

(b) (i) (a) $\angle CAD = \angle CBD = 54°$ ($\angle$s in the same segment) B1

(b) $\angle BPA = 180° - (110° - 54°) - 36°$ (ext $\angle$ of Δ) M1

$= 88°$ A1

(accept other correct methods)

(ii) (a) $\angle ADX = \angle ABC$ (exterior $\angle$ of cyclic quadrilateral) B1 (for reason)

$= 36° + 54° = 90°$ B1

[OR $\angle ADC = 180° - \angle ABC$

$= 180° - 90° = 90°$

(opposite $\angle$s of cyclic quad) B1

Or $\angle$s in opposite segments)

$\angle ADX = 180° - 90°$ ($\angle$s on a st. line)

$= 90°$ A1]

[OR $\angle ADX = \angle ACD + \angle CAD$ (ext $\angle$ of Δ) B1

$= \angle ABD + \angle CBD$ ($\angle$s in the same segment)

$= 36° + 54° = 90°$ B1]

(b) $\angle ABY = 180° - (54° + 36°)$ ($\angle$s on a st. line)

$= 90°$

$= \angle ADX$

Since it satisfies the property of angles in the same segment, X, D, B and Y lie on a circle. B1

(c) mid-point of XY. B1

9 (a) (i) $\overrightarrow{SP} = \overrightarrow{RQ}$ (ΔTQR and ΔTUS are similar Δs)

$= 2\mathbf{b}$ B1

(ii) $\overrightarrow{RP} = \overrightarrow{RS} + \overrightarrow{SP}$

$= \mathbf{a} + 2\mathbf{b}$ B1

(iii) $\overrightarrow{UQ} = \overrightarrow{TU}$

$= \overrightarrow{TS} + \overrightarrow{SU}$

$= -\mathbf{a} + \mathbf{b}$ B1

[OR $\overrightarrow{UQ} = \frac{1}{2}\overrightarrow{TQ}$

$= \frac{1}{2}(-2\mathbf{a} + 2\mathbf{b})$

$= -\mathbf{a} + \mathbf{b}$ **B1**

OR $\overrightarrow{UQ} = \overrightarrow{US} + \overrightarrow{SR} + \overrightarrow{RQ}$

$= -\mathbf{b} - \mathbf{a} + 2\mathbf{b}$

$= \mathbf{b} - \mathbf{a}$ **B1]**

(b) $\overrightarrow{RX} = \overrightarrow{RS} + \overrightarrow{SU} + \overrightarrow{UX}$

$= \mathbf{a} + \mathbf{b} + \frac{1}{3}\overrightarrow{UQ}$

$= \mathbf{a} + \mathbf{b} + \frac{1}{3}(\mathbf{b} - \mathbf{a})$

$= \frac{2}{3}\mathbf{a} + \frac{4}{3}\mathbf{b}$ M1

$= \frac{2}{3}(\mathbf{a} + 2\mathbf{b})$

$= \frac{2}{3}\overrightarrow{RP}$ A1

Therefore R, X and P lie on a straight line.

[OR $\overrightarrow{XP} = \overrightarrow{XU} + \overrightarrow{UP}$

$= \frac{1}{3}(\mathbf{a} - \mathbf{b}) + \mathbf{b}$

$= \frac{1}{3}\mathbf{a} + \frac{2}{3}\mathbf{b}$ **M1**

$= \frac{1}{3}(\mathbf{a} + 2\mathbf{b})$

$= \frac{1}{3}\overrightarrow{RP}$ **A1**

Therefore R, X and P lie on a straight line.]

(c) (i) $\dfrac{\text{Area of } \Delta TSU}{\text{Area of } \Delta TRQ} = \left(\dfrac{TS}{TR}\right)^2$ (ΔTQR and ΔTUS are similar Δs)

$= \left(\dfrac{1}{2}\right)^2 = \dfrac{1}{4}$ B1

(ii) $\dfrac{\text{Area of } \Delta PXQ}{\text{Area of } \Delta PQU} = \dfrac{\frac{1}{2} \times QX \times h}{\frac{1}{2} \times QU \times h}$ (The triangles have the same height, h)

$= \dfrac{QX}{QU} = \dfrac{2}{3}$ B1

(iii) $\dfrac{\text{Area of } \Delta PQX}{\text{Area of } PQRS} = \dfrac{\frac{2}{3} \text{ Area of } \Delta PQU}{\text{Area of } PQRS}$ (from (ii))

$= \dfrac{\frac{2}{3} \text{ Area of } \Delta TSU}{\text{Area of } \Delta TQR}$ (ΔPQX and ΔTSU are congruent Δs)

$= \dfrac{2}{3} \times \dfrac{1}{4}$ (from (i)) M1

$= \dfrac{1}{6}$ A1

[OR Area of parallelogram = 4 × Area of ΔPQU

= 12 units M1

$\dfrac{\textbf{Area of } \Delta PQX}{\textbf{Area of } PQRS} = \dfrac{2}{12} = \dfrac{1}{6}$ **A1]**

10 (a) (i) (a) mean = 17 minutes B1

(b) standard deviation = 4.88 minutes B2

(ii) Clinic B is better in its service as it has a smaller standard deviation than that of clinic *A*. The spread of its waiting time is lesser and hence more consistent. B1

(b) (i) $\frac{2}{6}=\frac{1}{3}$ B1

(ii) (a) $P(P)=\frac{2}{6}\times\frac{2}{6}=\frac{1}{9}$ B1

(b) $P(Q)=0$ B1

(c) $P(Y)=\frac{4}{6}\times\frac{2}{6}+\frac{2}{6}\times\frac{4}{6}$ M1

$=\frac{4}{9}$ A1

(d) $\frac{1}{9}+0+\frac{4}{9}=\frac{5}{9}$ B1