Answer **all** the questions.

1 (a) Given $16u^2v^2 - 56uv + 54 = 5$, express v in terms of u. [3]

(b) Express as a single fraction in its simplest form $\frac{a^2+5ab+4b^2}{a^2-16b^2} - \frac{b}{4b-a}$. [4]

(c) Make y the subject of the formula $\frac{m(nx-y^2)}{p} = 3n$. [3]

2 (a) Fred, Glen and Spencer invested \$20 000, \$30 000 and \$46 000 respectively in an eatery business. After two years, they earned a profit of \$40 800. The profit was shared in the ratio of their invested amounts.

(i) Calculate the profit that Fred received. [1]

(ii) Glen thought that the effort put in by each person should be taken into consideration for profit sharing. Hence, he suggested sharing the profit in the ratio of 14 : 22 : 15 (Fred : Glen : Spencer). If Glen's suggestion is accepted, what is the percentage change in the profit that Fred receives? [2]

(b) With the profit that Fred received in (a)(ii), he decides to deposit them into a Gold\$aver account with BSD bank. The account pays compound interest at 1.88% p.a. compounded quarterly. Find the amount of money that Fred will get when he withdraws the money one and a half years later. [2]

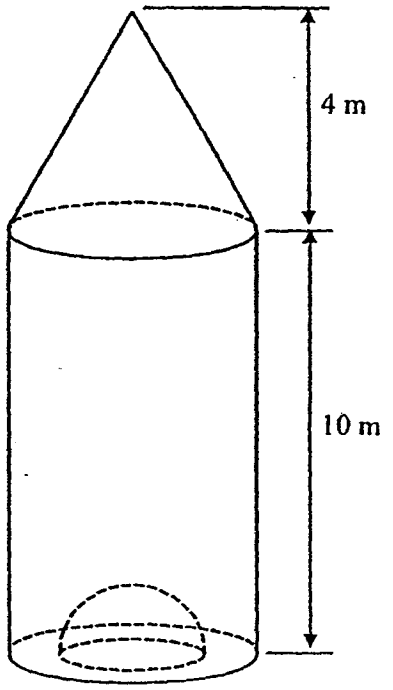
(c) Spencer suggests going to Spain for a holiday.

On a certain day, the exchange rate between the euros and United States dollars (USD) was 0.7332 euros = 1 USD.

On the same day, the exchange rate between United States dollars (USD) and Singapore dollars (SGD) was 1 USD = 1.2479 SGD.

If Spencer has to exchange 4500 USD that Glen passed to him and an additional 1200 SGD for the Spain trip, calculate how many euros he will receive. [3]

3 A wooden sculpture is made by joining a cylinder and a cone together as shown in the diagram. A hemisphere of radius 1.5 m is dug out from the base of the cylinder. The height of the cone is 4 m and its base area is 38.5 m^2. The height of the cylinder is 10 m.



(a) Calculate the radius of the cone. [1]

(b) Calculate the volume of the wooden sculpture. [2]

(c) Given that the wooden sculpture is made from walnut of density 0.59 g/cm^3, calculate its mass in kilograms. [2]

(d) Find the surface area of the wooden sculpture. [3]

(e) A model of height 875 cm is geometrically similar to the wooden sculpture. Calculate the volume of the model in cubic centimetres. [3]

4 A train usually departs at 07 00 from Beijing South Station to Danyang North Station, 1120 km apart.

If the average speed of the train is x km/h,

(a) find in terms of x, the time taken by the train to travel from Beijing South Station to Danyang North Station. [1]

On a particular Sunday, the train increased its average speed by 10 km/h. It arrived at Danyang North Station 50 minutes earlier than usual.

(b) Find, in terms of x, the time taken for this journey. [1]

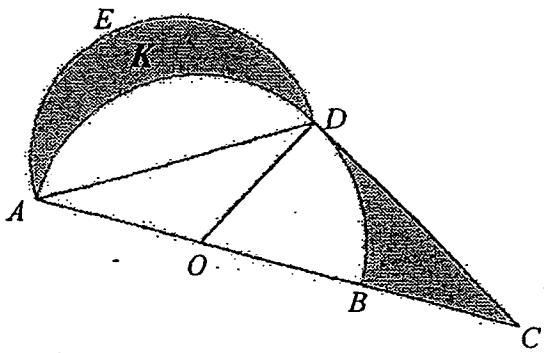
(c) Form an equation in x and show that it reduces to

$$x^2+10x-13440=0.$$ [2]

(d) Solve the equation $x^2+10x-13440=0$, giving your answers correct to 2 decimal places. [3]

(e) Hence, find the time taken by the train to travel from Beijing South Station to Danyang North Station on that Sunday. Give your answer in hours and minutes, correct to the nearest minute. [2]

5 In the diagram, ADB is a semicircle of the centre O, radius 10 cm. The tangent to the circle at point D meets AB produced at C. AED is a semicircle with AD as the diameter.



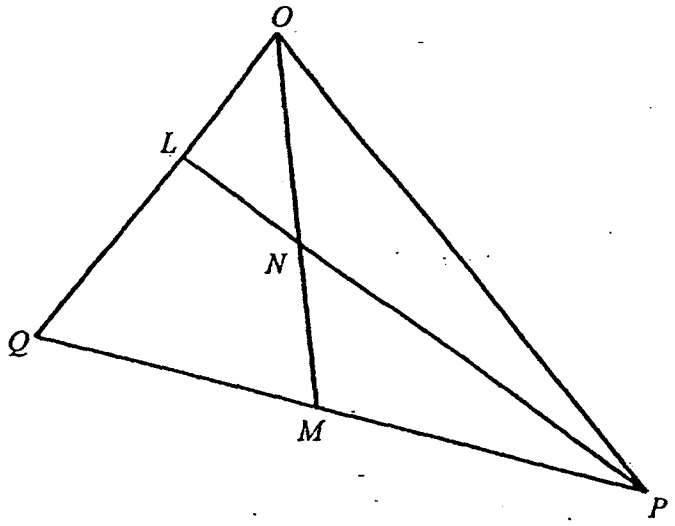
Given that $BC=10$ cm,

(a) **(i)** show that $\angle BOD=\frac{\pi}{3}$ radians, [1]

(ii) find the area of the shaded region BCD. [2]

(b) Calculate the perimeter of the shaded region K. [3]

6 The position vectors of point P and Q relative to an origin O are 5**a** and 5**b** respectively. M is the midpoint of PQ. L is the point on OQ such that $OL:OQ=2:5$ and N is the point on OM such that $\overrightarrow{ON}=\frac{4}{7}\overrightarrow{OM}$.



(a) Express, as simply as possible, in terms of **a** and/or **b**,

(i) $\overrightarrow{OM}$, [1]

(ii) $\overrightarrow{PN}$ [1]

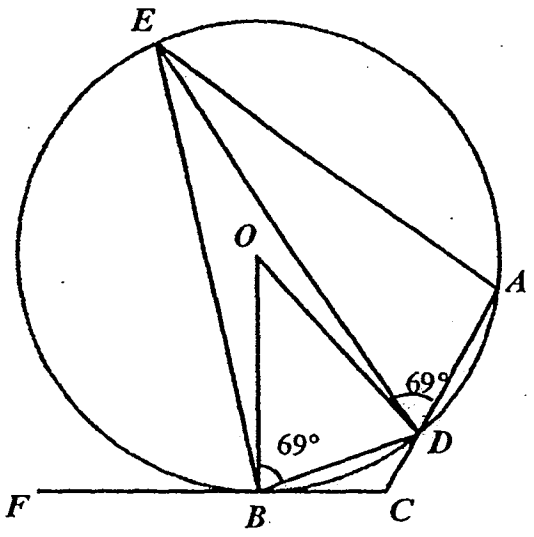
(iii) $\overrightarrow{NL}$. [1]

(b) Hence show that N lies on PL and state the ratio of PN: NL. [2]

(c) Find $\frac{\text{area of } \Delta PNM}{\text{area of } \Delta PON}$. [1]

(d) Find $\frac{\text{area of } \Delta PON}{\text{area of quadrilateral } LNMQ}$. [2]

7 In the diagram, O is the centre of the circle which passes through points B, D, A and E. FC is a tangent to the circle at B, ADC is a straight line and $\angle ODA = \angle OBD = 69°$.



(a) Find,

(i) $\angle AED$, [3]

(ii) $\angle ABO$, [2]

(iii) $\angle BCD$. [2]

(b) Show that the line BD bisects $\angle ABC$. [1]

8 Joyce put toothpicks into jelly-tots to make a series of shapes. The first three of these shapes are shown below.

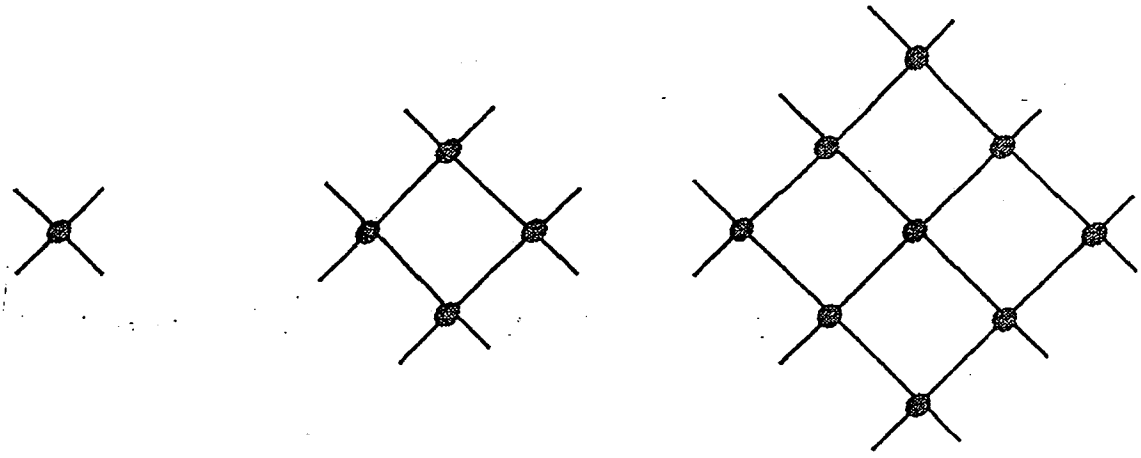


Figure 1 Figure 2 Figure 3

The total number of jelly-tots and the number of toothpicks needed in each figure are shown in the following table.

Figure	Number of jelly-tots	Number of toothpicks
1	1	4
2	4	12
3	9	24
4	a	b
. . .	. . .	. . .
n	p	q

(a) Write down the values of a and b. [1]

(b) Find the total number of jelly-tots needed to form figure 10. [1]

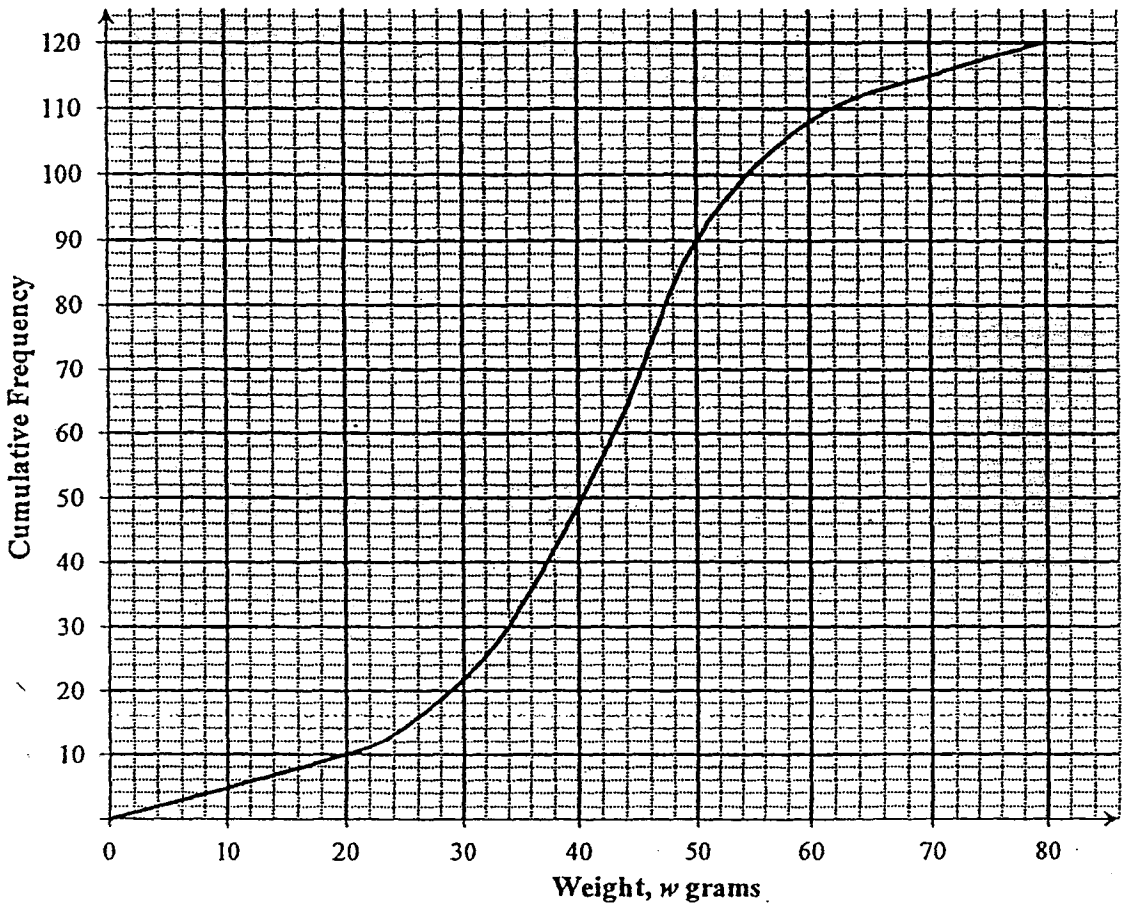
(c) Find the number of toothpicks needed in figure 5. [1]

(d) **(i)** Express p in terms of n. [1]

(ii) Express q in terms of n. [2]

(e) Which figure has 364 toothpicks? [1]

9 Danny took a sample of 120 pebbles from Bondi Beach, Sydney.
He weighed each pebble and recorded its weight. The cumulative frequency curve below shows the weight of these pebbles, in grams.



(a) Use the curve to find

(i) the median weight of these pebbles, [1]

(ii) the interquartile range of the distribution, [2]

(iii) the number of pebbles with a weight more than 60 grams. [1]

(b) Danny also took a sample of 100 pebbles from Pirates Bay, Tasmania.
The table below shows the distribution of the weights of the pebbles in the sample from Pirates Bay, Tasmania.

Weight (w grams)	Cumulative Frequency
$0 < w \leq 20$	1
$0 < w \leq 30$	15
$0 < w \leq 40$	36
$0 < w \leq 50$	65
$0 < w \leq 60$	84
$0 < w \leq 70$	94
$0 < w \leq 80$	100

(i) Copy and complete the following frequency table. [2]

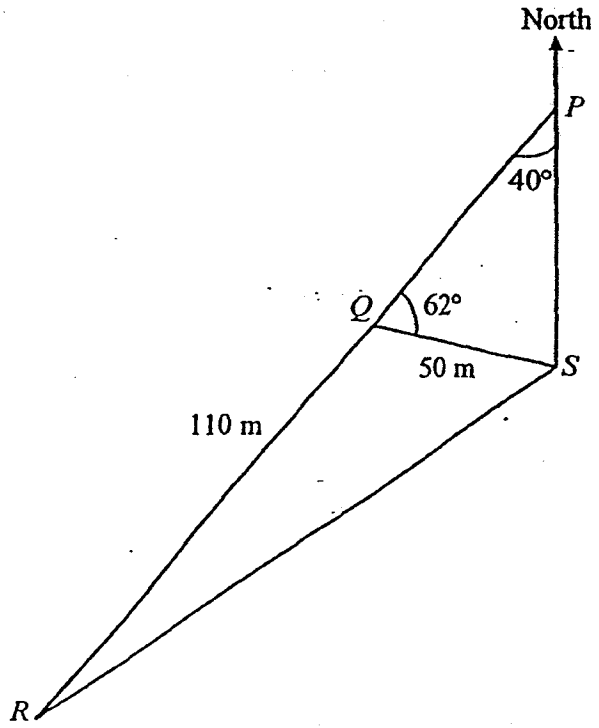
Weight (w grams)	Frequency
$0 < w \leq 20$	
$20 < w \leq 30$	
$30 < w \leq 40$	
$40 < w \leq 50$	
$50 < w \leq 60$	
$60 < w \leq 70$	
$70 < w \leq 80$	

(ii) Estimate the mean and standard deviation of the pebbles from Pirates Bay, Tasmania. [3]

(c) Danny takes one pebble, at random, from his sample from Bondi Beach, Sydney and one pebble, at random, from his sample from Pirates Bay, Tasmania. Find the probability that the weight of the pebble from Bondi Beach, Sydney is more than 60 grams and the weight of the pebble Pirates Bay, Tasmania is less than 40 grams [1]

10 The diagram shows two horizontal fields PQS and SRQ which are surrounded by hedges, with P north of S.

PQR is a straight line, $RQ = 110$ m, $QS = 50$ m, $\angle QPS = 40°$ and $\angle PQS = 62°$.



Calculate

(a) (i) the length of the hedge PS, [2]

(ii) the area of the field QRS, [2]

(iii) the length of the hedge RS. [2]

(b) Given that X is the point on QR such that $QX = 50$ m, calculate the bearing of S from X. [1]

(c) A vertical tree of height 16.2 m is growing at S.

(i) Find the greatest angle of elevation of the top of the tree from a point on PR. [3]

(ii) A boy has climbed exactly half way up the tree. Calculate the angle of depression of R when viewed by the boy. [2]

11 Answer the whole of this question on a sheet of graph paper.

In any month, Vogue Garment Company estimated it can make a net profit of y thousand dollars by producing x thousand jeans where $y=-6x^2+27x-2$. The monthly cost of running the business consists of rental fee and production cost.

The table below shows some values of x and the corresponding values of y, correct to the nearest whole number.

x **(in thousands)**	0	1	2	3	4	5
y **(in thousand dollars)**	−2	19	28	p	10	−17

(a) Calculate the value of p. [1]

(b) Using a scale of 2 cm to represent 1 unit, draw a horizontal x-axis for $0\le x\le 5$.
Using a scale of 1 cm to represent 2 units, draw a vertical y-axis for $-18\le y\le 30$.
On your axes, plot the points given in the table and join them with a smooth curve. [3]

(c) Use your graph to find

(i) the monthly rental fee of the company, [1]

(ii) the number of jeans the company must produce to maximize profit, [1]

(iii) the minimum number of jeans the company must produce to just break even, [1]

(iv) the range for the number of jeans the company must produce to earn at least \$12 000. [1]

(d) **(i)** By drawing a tangent, find the gradient of the curve at the point where $x=3.5$. [2]

(ii) What does this gradient represent? [1]

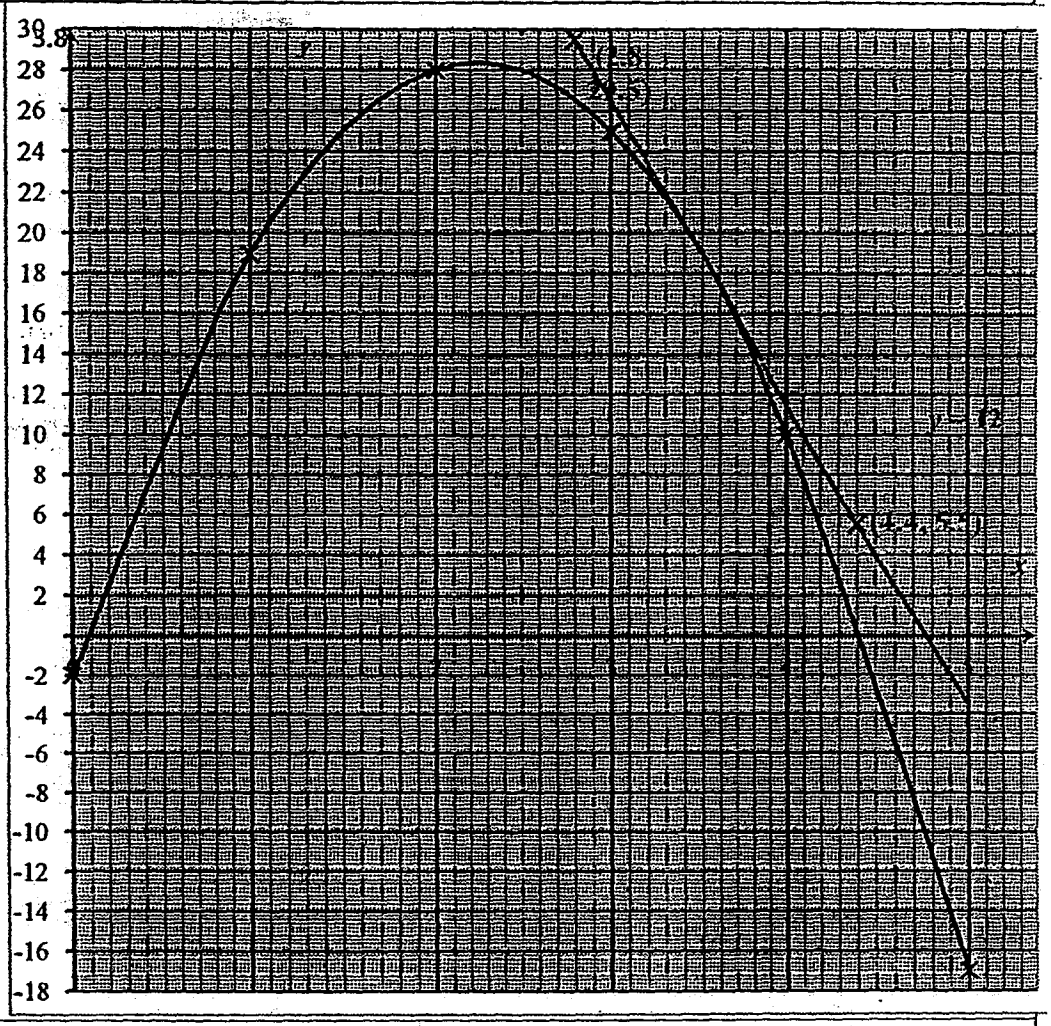
~ End of Paper ~

Answer Key

1(a)	$v=\frac{7}{4u}$
1(b)	$\frac{a+2b}{a-4b}$
1(c)	$y=\pm\sqrt{\frac{n(mx-3p)}{m}}$ or $\pm\sqrt{\frac{mnx-3np}{m}}$ or $\pm\sqrt{nx-\frac{3np}{m}}$
2 (a)(i)	\$8500
2 (a)(ii)	31.8%
2(b)	\$11519.57 (2 d.p)
2(c)	4004.46 euros (2 d.p.)
3(a)	3.50 m (3 s.f.)
3(b)	429 m^3 (3s.f)
3(c)	253000 kg (3 s.f.)
3(d)	324 m^2 (3 s.f.)
3 (e)	105000000 cm^3 / 1.05×10^8 cm^3
4(a)	$\frac{1120}{x}$ h
4(b)	$\frac{1120}{x+10}$ h
4(c)	$\frac{1120}{x}-\frac{1120}{x+10}=\frac{50}{60}$ $\frac{1120x+11200-1120x}{x(x+10)}=\frac{5}{6}$ $11200(6)=5x(x+10)$ $5x^2+50x-67200=0$ $x^2+10x-13440=0$
4(d)	$x=111.04$ (2 d.p.) $=-121.04$ (2 d.p.)
4(e)	9 hr 16 min (rounded up so as to ensure train reach Danyang North Station)
5(a)(ii)	34.2 cm2 (3 s.f)

5(b)	48.2 cm (3 s.f.)
6(a)(i)	$\frac{5}{2}(\mathbf{a}+\mathbf{b})$ or $\frac{5}{2}\mathbf{a}+\frac{5}{2}\mathbf{b}$
6(a)(ii)	$\frac{5}{7}(2\mathbf{b}-5\mathbf{a})$ or $\frac{10}{7}\mathbf{b}-\frac{25}{7}\mathbf{a}$
6(a)(iii)	$\frac{2}{7}(2\mathbf{b}-5\mathbf{a})$ or $\frac{4}{7}\mathbf{b}-\frac{10}{7}\mathbf{a}$
6(b)	From the results in (a)(ii) and (a)(iii), $\overrightarrow{PN}=\frac{5}{2}\overrightarrow{NL}$ PN is parallel to NL and since they have a common point N, P, N and L are collinear. $\therefore$ N lies on PL. $PN:NL=5:2$
6(c)	$\frac{3}{4}$
6(d)	$\frac{20}{27}$
7(a)(i)	$\angle AED=21°$
7(a)(ii)	$\angle ABO=48°$
7(a)(iii)	$\angle BCD=117°$
7 (b)	$\angle ABD=\angle AED=21°$ ($\angle$s in same segment) $\angle DBC=90°-69°=21°$ (tangent $\perp$ radius) Since $\angle ABD=\angle DBC$ $\therefore$ the line BD bisects $\angle ABC$.
8(a)	$a=16$, $b=40$
8(b)	100
8(c)	60
8(d)(i)	$p=n^2$
8(d)(ii)	$q=2n^2+2n$ or $q=2n(n+1)$
8(e)	Figure 13
9(a)(i)	Median weight of pebbles from Bondi Beach, Sydney= 43 g
9(a)(ii)	Interquartile range of the distribution = 50 – 34 = 16 g
9(a)(iii)	Number of pebbles with a weight more than 60 grams = 12

9(b)(i)	<table><tr><th>Weight (w grams)</th><th>Frequency</th></tr><tr><td>$0 < w \leq 20$</td><td>1</td></tr><tr><td>$20 < w \leq 30$</td><td>14</td></tr><tr><td>$30 < w \leq 40$</td><td>21</td></tr><tr><td>$40 < w \leq 50$</td><td>29</td></tr><tr><td>$50 < w \leq 60$</td><td>19</td></tr><tr><td>$60 < w \leq 70$</td><td>10</td></tr><tr><td>$70 < w \leq 80$</td><td>6</td></tr></table>
9(b)(ii)	Mean = 45.5 g (3.s.f.), Standard deviation = 14.2 g (3.s.f.))
9(c)	0.036 or $\frac{9}{250}$
10(a)(i)	68.7 m (3 s.f.)
10(a)(ii)	2430 m^2(3s.f.)
10(a)(iii)	141 m (3 s.f.)
10(b)	071°
10(c)(i)	20.2°(1d.p.)
10(c)(ii)	3.3° (1 d.p.)

11(a)	$p = 25$
10(b)	
10(c)(i)	\$2000
11(c)(ii)	2250
11(c)iii)	100
11(c)(iv)	$600 \le x \le 3900$
11(d)(i)	-15
11(d)(ii)	It represents the rate at which the profit is changing at $x = 3.5$.

Name		( )	Class	

南 华 中 学

NAN HUA HIGH SCHOOL

PRELIMINARY EXAMINATION 2014

Subject	:	**Mathematics**
Paper	:	**4016/01**
Level	:	**Secondary Four Express**
Date	:	**01 September 2014**
Duration	:	**2 hours**

READ THESE INSTRUCTIONS FIRST

Write your name, class and index number in the space provided.
Write in dark blue or black pen.
You may use a pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** questions.
If working is needed for any question it must be shown with the answer.
Omission of essential working will result in loss of marks.
The use of an approved scientific calculator is expected, where appropriate.
If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.
For π, use either your calculator value or 3.142, unless the question requires the answer in terms of π.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total of the marks for this paper is 80.

For Examiner's Use

Mathematical Formulae

Compound interest

$$\text{Total amount} = P\left(1+\frac{r}{100}\right)^n$$

Mensuration

$$\text{Curved surface area of a cone} = \pi rl$$

$$\text{Surface area of a sphere} = 4\pi r^2$$

$$\text{Volume of a cone} = \frac{1}{3}\pi r^2 h$$

$$\text{Volume of a sphere} = \frac{4}{3}\pi r^3$$

$$\text{Area of triangle } ABC = \frac{1}{2}ab\sin C$$

$$\text{Arc length} = r\theta, \text{ where } \theta \text{ is in radians}$$

$$\text{Sector area} = \frac{1}{2}r^2\theta, \text{ where } \theta \text{ is in radians}$$

Trigonometry

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

Statistics

$$\text{Mean} = \frac{\Sigma fx}{\Sigma f}$$

$$\text{Standard deviation} = \sqrt{\frac{\Sigma fx^2}{\Sigma f} - \left(\frac{\Sigma fx}{\Sigma f}\right)^2}$$

For Examiner's Use

Answer **all** questions

1. (a) Without using calculator, estimate the value of $\frac{\sqrt{3620}\times 31.2}{6.9^2}$ to 1 significant figure, showing your working clearly.

Answer (a) ..... [1]

(b) Simplify $\frac{(-2mn)^3}{\sqrt{m^5}} \times \frac{3}{4n^{-2}}$, giving your answer with positive indices.

Answer (b) ..... [2]

2. The estimated atomic mass of 10 billion nitrogen atoms is 2.325×10^{-13} g.

(a) 2.325×10^{-13} can be written as x pico. Find the value of x.

(b) The atomic mass of a helium atom is 6.684×10^{-24} g. How much heavier is a nitrogen atom when compared to a helium atom? Give your answer in standard form.

Answer (a) $x =$ ..... [1]

(b) ..... g [2]

For Examiner's Use

3. A watch-shop owner bought a watch for \$560 and was intending to sell it for \$$x$. He eventually sold the watch at a 20% discount of \$$x$, and made a profit of 15%. Find x.

Answer $x =$ ..... [2]

For Examiner's Use

4. A contractor was awarded a tender to construct 4 houses within 180 days. He hired 24 men for the job. After 60 days, he found that they had completed only 1 house. How many more men should he employ to complete the job within 180 days?

Answer ..... men [3]

For Examiner's Use

5. In a factory, the temperature of a packet of frozen meat in a freezer was $-20.4°C$. The temperature of the frozen meat became 3.6°C 12 minutes after it was removed from the freezer. Given that the temperature of the frozen meat increases at a constant rate, calculate

(a) the change in temperature of the frozen meat,
(b) the time taken for the frozen meat to thaw to $-4°C$.

Answer (a) ..... °C [1]

(b) ..... minutes [1]

For Examiner's Use

6. **(a)** Solve the inequalities $\frac{3+2x}{3} < \frac{3x+1}{7} < 5 + x$.

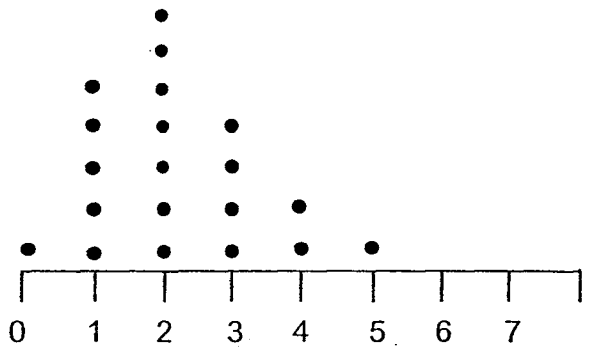
(b) Hence write down the greatest integer value of x which satisfies $\frac{3+2x}{3} < \frac{3x+1}{7} < 5 + x$.

Answer (a) ..... [2]

(b) $x =$ ..... [1]

For Examiner's Use

7. A group of 20 students were surveyed on the number of hours they spent on online gaming in a day. The results of the survey were represented in the following dot diagram.



Find

(a) the mean length of time.

(b) the median gaming hours.

Answer (a) ..... hours [1]

(b) ..... hours [1]

For Examiner's Use

8. Given that $\boldsymbol{A} = \begin{pmatrix} 3 \\ 5 \end{pmatrix}$, $\boldsymbol{B} = \begin{pmatrix} -2 & 4 \\ 8 & -3 \end{pmatrix}$ and $\boldsymbol{C} = \begin{pmatrix} -3 & 6 \\ 12 & x \end{pmatrix}$.

(a) Find $\boldsymbol{BA}$.

(b) Find x if $\boldsymbol{B} = \frac{1}{k}\boldsymbol{C}$, where k is a constant.

Answer (a) ..... [1]

(b) x = ... [2]

For Examiner's Use

9. The graph of $\frac{x}{2}+\frac{y}{5}=\frac{3}{2}$ cuts the x-axis at A and the y-axis at B. Find

(a) the coordinates of A and B,

(b) the length AB,

(c) the equation of the line passing through point A, parallel to $2y + 3x = -7$.

For Examiner's Use

Answer (a) A (..... , ...) [1]

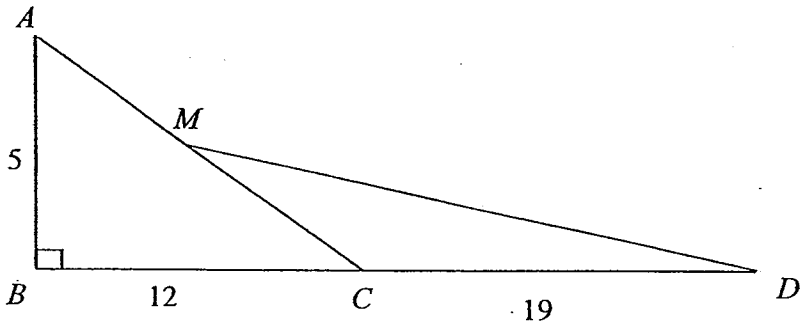
B (..... , ...) [1]

(b) .....units [1]

(c) ..... [3]

For Examiner's Use

10. In the diagram below, $AB = 5$ cm, $BC = 12$ cm and $CD = 19$ cm.
M is the midpoint of AC, and the points B, C and D lie on a straight line.



(a) Express $\cos \angle ACD$ as a fraction in its lowest term.

(b) Hence calculate MD.

Answer (a) ..... [1]

(b) ..... cm [2]

For Examiner's Use

For Examiner's Use

11. If $27m^{3x} = 1$ and $m > 0$, find $m^{2x} - m^{-x}$.

Answer ..... [2]

12. A cylindrical container has a height of 18 cm and a radius of 3 cm, both correct to the nearest centimetre.

(a) Find the least possible height of the container.

(b) Find the greatest possible volume of this container, giving your answer correct to 2 decimal places.

Answer (a) ..... cm [1]

(b) ..... cm^3 [1]

For Examiner's Use

13. **(a)** Given that $\varepsilon = \{x \text{ is an integer} : -3 \leq x \leq 8\}$, $A = \{x \text{ is a prime number}\}$ and $B = \{x \text{ is an odd number}\}$.

(i) List the element in A.

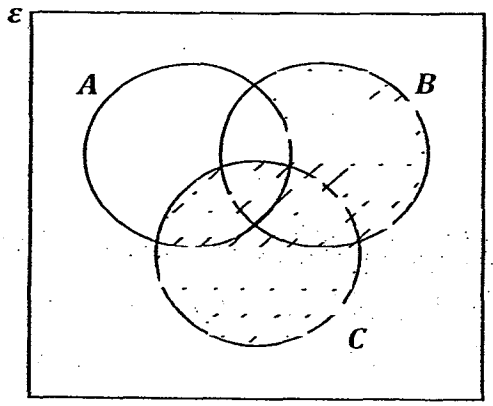
(ii) Find $n(A \cup B)'$.

Answer (a)(i) ..... [1]

(ii) ..... [1]

(b) On the Venn diagram provided below, shade the set $C \cup (A' \cap B)$.

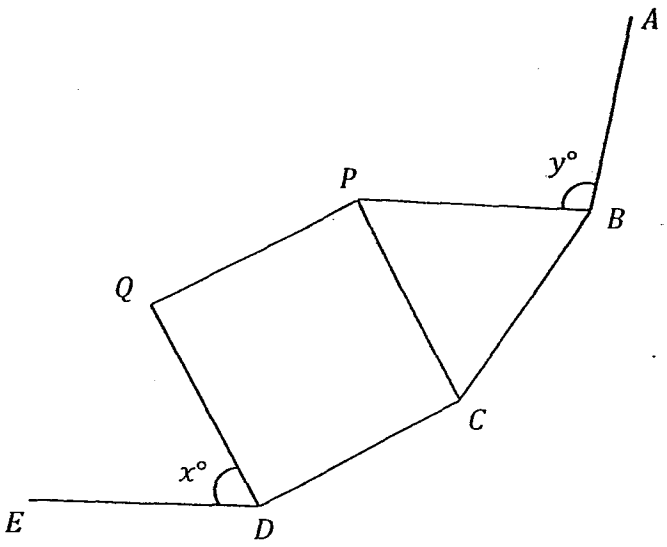
Answer (b) [1]



For Examiner's Use

For Examiner's Use

14. $ABCDE$ . . . . is part of an n-sided regular polygon which has interior angles of 165°. $CDQP$ is a square.



Find the values of

(a) n,

(b) x,

(c) y.

Answer (a) $n =$ ..... [1]

(b) $x =$ ..... [1]

(c) $y =$ ..... [1]

For Examiner's Use

For Examiner's Use

15. Written as the product of its prime factors, $360 = 2^3 \times 3^2 \times 5$.

(a) Given that $360k$ is a perfect square, write down the smallest integer value of k.

(b) (i) Express 132 as the product of its prime factors.

(ii) Hence, write down the smallest integer value of n such that $132n$ is a multiple of 360.

Answer (a) $k =$ ..... [1]

(b) (i) ..... [1]

(ii) $n =$ ..... [1]

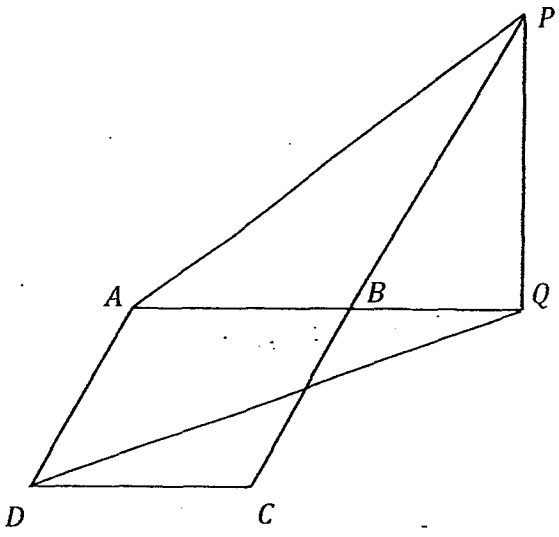
For Examiner's Use

For Examiner's Use

16. In the figure $ABCD$ is a rhombus, ABQ and PBC are straight lines, and $AQ = BP$.

(a) State the triangle that is congruent to triangle APB and

(b) prove that they are congruent.



For Examiner's Use

Answer (a) ..... [1]

(b)

.....

.....

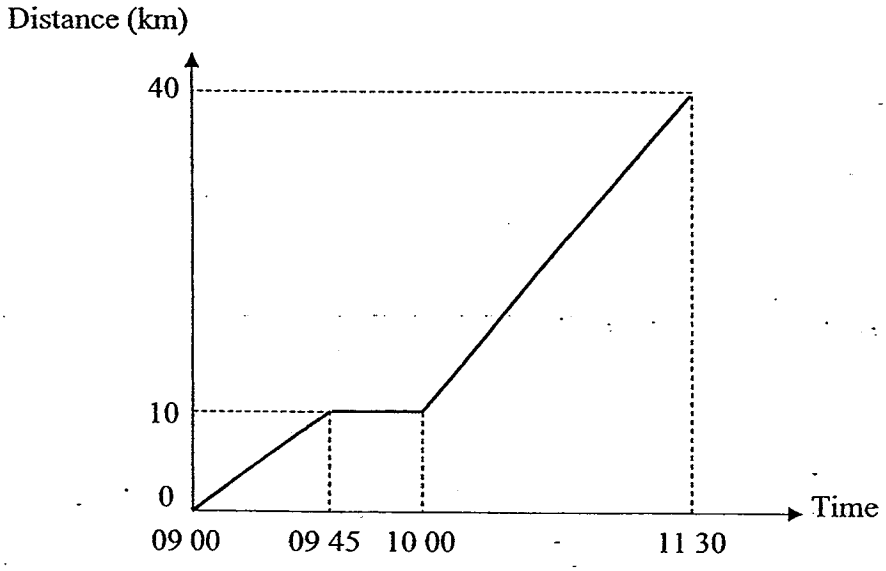
.....

.....

..... [2]

For Examiner's Use

17. The diagram shows the distance-time graph of a cyclist who begins his journey of 40 km at 09 00 and completes it at 11 30.



Calculate

(a) the speed of the cyclist for the first 30 minutes in km/h,

(b) his acceleration at 10 30 in m/s^2,

(c) his average speed from 09 00 to 11 00, in km/h.

Answer (a) ..... km/h [1]

(b) ..... m/s^2 [1]

(c) ..... km/h [2]

For Examiner's Use

For Examiner's Use

18. (a) Construct a rhombus $PQRS$ in which $PR = 8$ cm and $QS = 6$ cm. [2]

Answer (a), (bi), (bii) and (c)

(b) From **(a)**, construct

(i) the perpendicular bisector of line PQ, [1]

(ii) the angle bisector of $\angle PQS$, [1]

(c) Mark clearly a possible point, X, inside the rhombus such that it is closer to point P than point Q, and also nearer to QS than PQ. [1]

For Examiner's Use

For Examiner's Use

19. y is inversely proportional to the square of x.

(a) If x is increased by 50%, find the percentage change in y.

(b) Given that $y = 25$ when $x = \frac{1}{2}$, find

(i) the equation connecting y and x,

(ii) the values of x when $y = 400$.

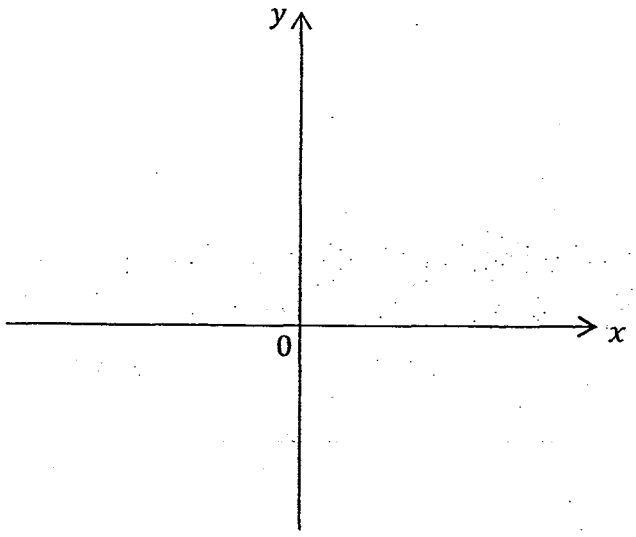
(c) Sketch the graph which represents the relation between y and x.

Answer (a) ..... % [2]

(b)(i) ..... [2]

(ii) $x =$ ..... , $x =$ ..... [2]

Answer (c) [1]



For Examiner's Use

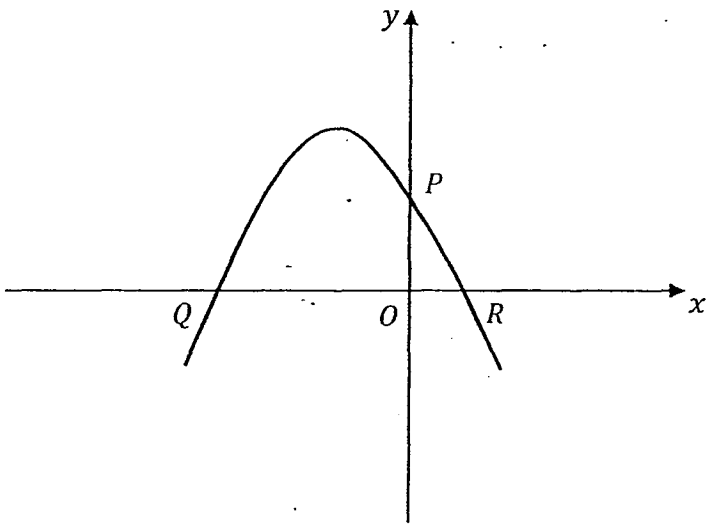
For Examiner's Use

20. The diagram shows the graph of $y = -2x^2 - 8x + 10$.

(a) The graph cuts the y-axis at $P(0, p)$.
Write down the value of p.

(b) The graph cuts the x-axis at Q and R.
Find the coordinates of Q and R.

(c) Hence, find the coordinates of the maximum point.



Answer (a) $p =$ ..... [1]

(b) $Q =$ (..... , ...) [1]

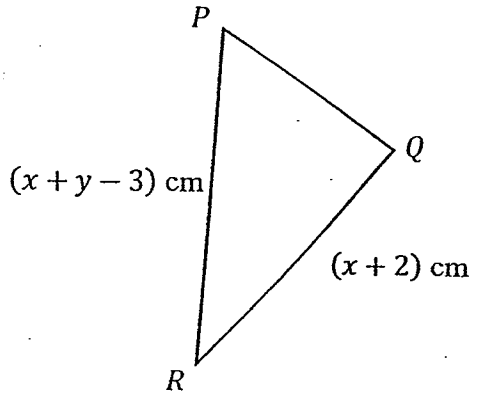
$R =$ (..... , ...) [1]

(c) (..... , ...) [2]

For Examiner's Use

similar to ΔPQR,

For Examiner's Use



Answer (a) ..... [2]

(b) $x =$ ..... , $y =$ ..... [2]

h $PR = 8$ cm and $QS = 6$ cm. [2]

For Examiner's Use

[1]

[1]

h that it is closer

[1]

For Examiner's Use

22. Given that P is the point (1 , 2), $\overrightarrow{PQ} = \begin{pmatrix} 4 \\ -5 \end{pmatrix}$ and $\overrightarrow{PR} = \begin{pmatrix} 6 \\ 3 \end{pmatrix}$, find

(a) $\left|\overrightarrow{PQ}\right|$,

(b) $\overrightarrow{QR}$,

(c) the coordinates of the point S such that $PQRS$ is a parallelogram.

Answer (a) $\left|\overrightarrow{PQ}\right|$ = ... units [1]

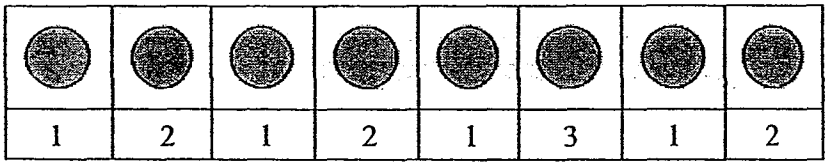
(b) $\overrightarrow{QR}$ = ... [1]

(c) S = (..... , ...) [3]

For Examiner's Use

For Examiner's Use

23. A ball is dropped at random into one of the eight holes, numbered as shown in the diagram. The number under each hole gives the score obtained when the ball drops into that hole. Each hole can only fit one ball.



(a) What is the probability of scoring 2?.

(b) If a second ball is dropped into the holes, without removing the first ball, find the probability of scoring

(i) a total of 6,

(ii) a total of 4.

Answer (a) ..... [1]

(b)(i) ..... [1]

(ii) ..... [2]

For Examiner's Use

End of Paper

Name		( )	Class	

南 华 中 学

NAN HUA HIGH SCHOOL

PRELIMINARY EXAMINATION 2014

Subject : Mathematics
Paper : 4016/01
Level : Secondary Four Express
Date : 01 September 2014
Duration : 2 hours

READ THESE INSTRUCTIONS FIRST

Write your name, class and index number in the space provided.
Write in dark blue or black pen.
You may use a pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** questions.
If working is needed for any question it must be shown with the answer.
Omission of essential working will result in loss of marks.
The use of an approved scientific calculator is expected, where appropriate.
If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.
For π, use either your calculator value or 3.142, unless the question requires the answer in terms of π.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total of the marks for this paper is 80.

For Examiner's Use

Mathematical Formulae

Compound interest

$$\text{Total amount} = P\left(1+\frac{r}{100}\right)^n$$

Mensuration

$$\text{Curved surface area of a cone} = \pi rl$$

$$\text{Surface area of a sphere} = 4\pi r^2$$

$$\text{Volume of a cone} = \frac{1}{3}\pi r^2 h$$

$$\text{Volume of a sphere} = \frac{4}{3}\pi r^3$$

$$\text{Area of triangle } ABC = \frac{1}{2}ab\sin C$$

$$\text{Arc length} = r\theta, \text{ where } \theta \text{ is in radians}$$

$$\text{Sector area} = \frac{1}{2}r^2\theta, \text{ where } \theta \text{ is in radians}$$

Trigonometry

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

Statistics

$$\text{Mean} = \frac{\Sigma fx}{\Sigma f}$$

$$\text{Standard deviation} = \sqrt{\frac{\Sigma fx^2}{\Sigma f} - \left(\frac{\Sigma fx}{\Sigma f}\right)^2}$$

For Examiner's Use

Answer **all** questions

1. (a) Without using calculator, estimate the value of $\frac{\sqrt{3620}\times 31.2}{6.9^2}$ to 1 significant figure, showing your working clearly.

$$\frac{60\times 30}{50}=36$$

Answer (a) 40 [1]

(b) Simplify $\frac{(-2mn)^3}{\sqrt{m^5}}\times\frac{3}{4n^{-2}}$, giving your answer with positive indices.

$$\frac{-8m^3n^3}{m^{\frac{5}{2}}}\times\frac{3n^2}{4}=-6m^{\frac{1}{2}}n^5$$

Answer (b) $-6m^{\frac{1}{2}}n^5$ [2]

For Examiner's Use

2. The estimated atomic mass of 10 billion nitrogen atoms is 2.325×10^{-13} g.

(a) 2.325×10^{-13} can be written as x pico. Find the value of x.

(b) The atomic mass of a helium atom is 6.684×10^{-24} g. How much heavier is a nitrogen atom when compared to a helium atom? Give your answer in standard form.

(a) $2.325\times 10^{-1}\times 10^{-12}=0.2325\times 10^{-12}$

(b) $2.325\times 10^{-13}\times 10^{-10}-6.684\times 10^{-24}$
$=2.325\times 10^{-23}-0.6684\times 10^{-23}$
$=1.6566\times 10^{-23}$

Answer (a) $x =$ 0.2325 [1]

(b) 1.6566×10^{-23} g [2]

For Examiner's Use

3. A watch-shop owner bought a watch for \$560 and was intending to sell it for \$$x$. He eventually sold the watch at a 20% discount of \$$x$, and made a profit of 15%. Find x.

$$0.8x = 1.15 \times 560$$
$$= 644$$
$$x = 805$$

Answer $x =$ 805 [2]

For Examiner's Use

4. A contractor was awarded a tender to construct 4 houses within 180 days. He hired 24 men for the job. After 60 days, he found that they had completed only 1 house. How many more men should he employ to complete the job within 180 days?

1 house was completed in 60 days with 24 men doing the job.
With 1 man, number of days needed to build 3 houses $= 60 \times 24 \times 3$
$= 4320$

Number of men needed to complete in 120 days $= 4320 \div 120$
$= 36$

Therefore number of extra men needed to complete the job $= 36 - 24$
$= 12$

Answer 12 men [3]

For Examiner's Use

5. In a factory, the temperature of a packet of frozen meat in a freezer was $-20.4°C$. The temperature of the frozen meat became $3.6°C$ 12 minutes after it was removed from the freezer. Given that the temperature of the frozen meat increases at a constant rate, calculate

(a) the change in temperature of the frozen meat,
(b) the time taken for the frozen meat to thaw to $-4°C$.

(b) Change of 24°C requires 12 minutes.
Change of 16.4°C $(-4-(-20.4))$ will require $= \frac{16.4}{24} \times 12$
$= 8.2$

Answer (a) ..... 24 ..... °C [1]

(b) ..... 8.2 ..... minutes [1]

For Examiner's Use

6. **(a)** Solve the inequalities $\frac{3+2x}{3} < \frac{3x+1}{7} < 5+x$.

(b) Hence write down the greatest integer value of x which satisfies $\frac{3+2x}{3} < \frac{3x+1}{7} < 5+x$.

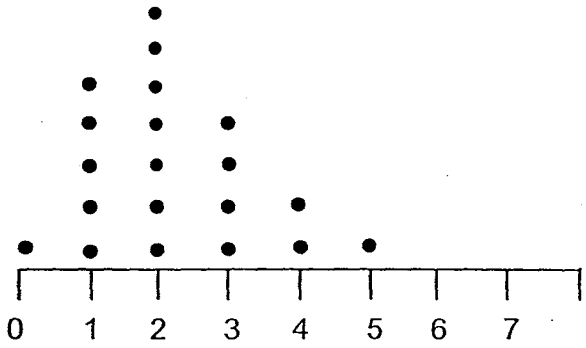
$7(3+2x) < 3(3x+1)$ and $3x+1 < 7(5+x)$
$21+14x < 9x+3$ $3x+1 < 35+7x$
$5x < -18$ $-34 < 4x$
$x < -\frac{18}{5}$ $x > -\frac{34}{4}$
$x < -3.6$ $x > -8.5$

Answer (a) ..... $-8.5 < x < -3.6$ ..... [2]

(b) $x =$ ..... -4 ..... [1]

For Examiner's Use

7. A group of 20 students were surveyed on the number of hours they spent on online gaming in a day. The results of the survey were represented in the following dot diagram.



Find

(a) the mean length of time.

(b) the median gaming hours.

(a) $\text{Mean} = \frac{5+14+12+8+5}{20}$

$= 2.2$

Answer (a) ..... 2.2 ..... hours [1]

(b) ..... 2 ..... hours [1]

For Examiner's Use

8. Given that $\boldsymbol{A} = \begin{pmatrix} 3 \\ 5 \end{pmatrix}$, $\boldsymbol{B} = \begin{pmatrix} -2 & 4 \\ 8 & -3 \end{pmatrix}$ and $\boldsymbol{C} = \begin{pmatrix} -3 & 6 \\ 12 & x \end{pmatrix}$.

(a) Find $\boldsymbol{BA}$.

(b) Find x if $\boldsymbol{B} = \frac{1}{k}\boldsymbol{C}$, where k is a constant.

(a) $\boldsymbol{BA} = \begin{pmatrix} -2 & 4 \\ 8 & -3 \end{pmatrix}\begin{pmatrix} 3 \\ 5 \end{pmatrix} = \begin{pmatrix} 14 \\ 9 \end{pmatrix}$

(b) $k\begin{pmatrix} -2 & 4 \\ 8 & -3 \end{pmatrix} = \begin{pmatrix} -3 & 6 \\ 12 & x \end{pmatrix}$

$\Rightarrow k = \frac{3}{2}$

$x = -4\frac{1}{2}$

Answer (a) ..... $\begin{pmatrix} 14 \\ 9 \end{pmatrix}$ ..... [1]

(b) $x =$ ..... $-4\frac{1}{2}$ ..... [2]

For Examiner's Use

9. The graph of $\frac{x}{2}+\frac{y}{5}=\frac{3}{2}$ cuts the x-axis at A and the y-axis at B. Find

(a) the coordinates of A and B,

(b) the length AB,

(c) the equation of the line passing through point A, parallel to $2y + 3x = -7$.

(b) $AB = \sqrt{3^2 + 7.5^2} = 8.077$

(c) $y = -\frac{3}{2}x + C$

At (3 , 0), $0 = -\frac{3}{2}(3) + c$

$c = 4\frac{1}{2}$

Equation of line : $y = -\frac{3}{2}x + \frac{9}{2}$

Or $2y = -3x + 9$

Answer (a) A (3 , 0) [1]

B (0 , 7.5) [1]

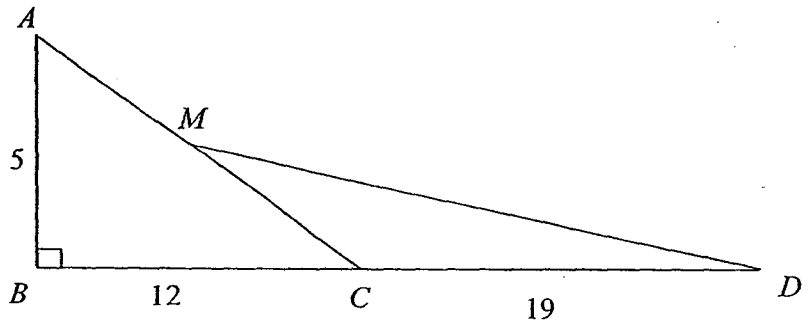
(b) 8.08 units [1]

(c) $2y = -3x + 9$ [3]

For Examiner's Use

For Examiner's Use

10. In the diagram below, $AB = 5$ cm, $BC = 12$ cm and $CD = 19$ cm.
M is the midpoint of AC, and the points B, C and D lie on a straight line.



(a) Express $\cos \angle ACD$ as a fraction in its lowest term.

(b) Hence calculate MD.

(a) $AC = \sqrt{5^2 + 12^2} = 13$

$\cos \angle ACD = -\frac{12}{13}$

(b) $MD^2 = 6.5^2 + 19^2 - 2(6.5)(19)\left(-\frac{12}{13}\right)$
$= 631.25$
$\therefore MD = 25.12$

Answer (a) $-\frac{12}{13}$ [1]

(b) 25.1 cm [2]

For Examiner's Use

For Examiner's Use

11. If $27m^{3x} = 1$ and $m > 0$, find $m^{2x} - m^{-x}$.

$$m^{3x} = \frac{1}{27}$$

$$m^{x} = \frac{1}{3}$$

$$m^{2x} - \frac{1}{m^{x}} = \left(\frac{1}{3}\right)^{2} - 3 = -2\frac{8}{9}$$

Answer $-2\frac{8}{9}$ [2]

For Examiner's Use

12. A cylindrical container has a height of 18 cm and a radius of 3 cm, both correct to the nearest centimetre.

(a) Find the least possible height of the container.

(b) Find the greatest possible volume of this container, giving your answer correct to 2 decimal places.

(b) Volume $= (3.5)^2(18.5)\pi$
$= 711.96 \text{ cm}^3$ (2 d.p.)

Answer (a) 17.5 cm [1]

(b) 711.96 cm^3 [1]

For Examiner's Use

13. **(a)** Given that $\varepsilon = \{x \text{ is an integer} : -3 \leq x \leq 8\}$, $A = \{x \text{ is a prime number}\}$ and $B = \{x \text{ is an odd number}\}$.

(i) List the element in A.

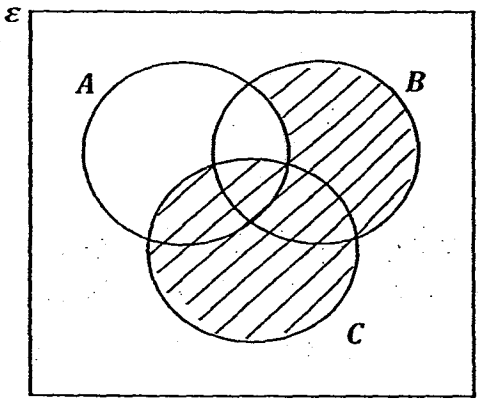
(ii) Find $n(A \cup B)'$.

Answer (a)(i) $\{2, 3, 5, 7\}$ [1]

(ii) 5 [1]

(b) On the Venn diagram provided below, shade the set $C \cup (A' \cap B)$.

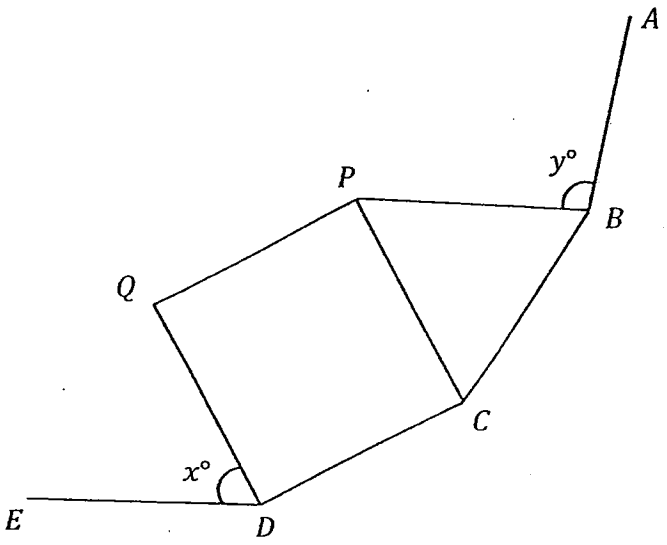
Answer (b) [1]



For Examiner's Use

For Examiner's Use

14. $ABCDE$ . . . . is part of an n-sided regular polygon which has interior angles of 165°. $CDQP$ is a square.



Find the values of

(a) n,

(b) x,

(c) y.

(a) $n = \frac{360}{15} = 24$

(b) $x = 165 - 90 = 75$

(c) $\angle CBP = \frac{180° - 75°}{2} = 52.5°$

$y = 165 - 52.5$

$= 112.5$

Answer (a) $n =$ ..24.. [1]

(b) $x =$ ..75.. [1]

(c) $y =$ ..112.5.. [1]

For Examiner's Use

15. Written as the product of its prime factors, $360 = 2^3 \times 3^2 \times 5$.

(a) Given that $360k$ is a perfect square, write down the smallest integer value of k.

(b) (i) Express 132 as the product of its prime factors.

(ii) Hence, write down the smallest integer value of n such that $132n$ is a multiple of 360.

(b) (ii) LCM of 360 and $132 = 2^3 \times 3^2 \times 5 \times 11$
$= 3960$

$132n = 3960$
$n = 30$

Answer (a) $k =$ 10 [1]

(b) (i) $2^2 \times 3 \times 11$ [1]

(ii) $n =$ 30 [1]

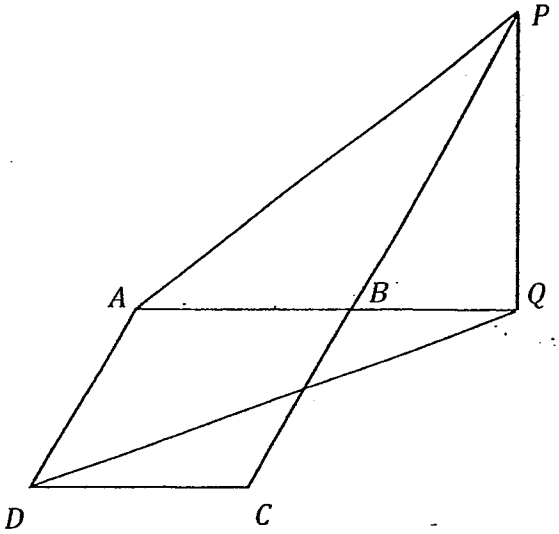
For Examiner's Use

For Examiner's Use

16. In the figure $ABCD$ is a rhombus, ABQ and PBC are straight lines, and $AQ = BP$.

(a) State the triangle that is congruent to triangle APB and

(b) prove that they are congruent.



For Examiner's Use

Answer (a) ..... [1]

(b) $\angle DAQ = \angle ABP$ (alternate angles, $AD//BC$ and BP)

.....

$AD = AB$ (given $ABCD$ is a rhombus)

.....

$AQ = BP$ (given)

.....

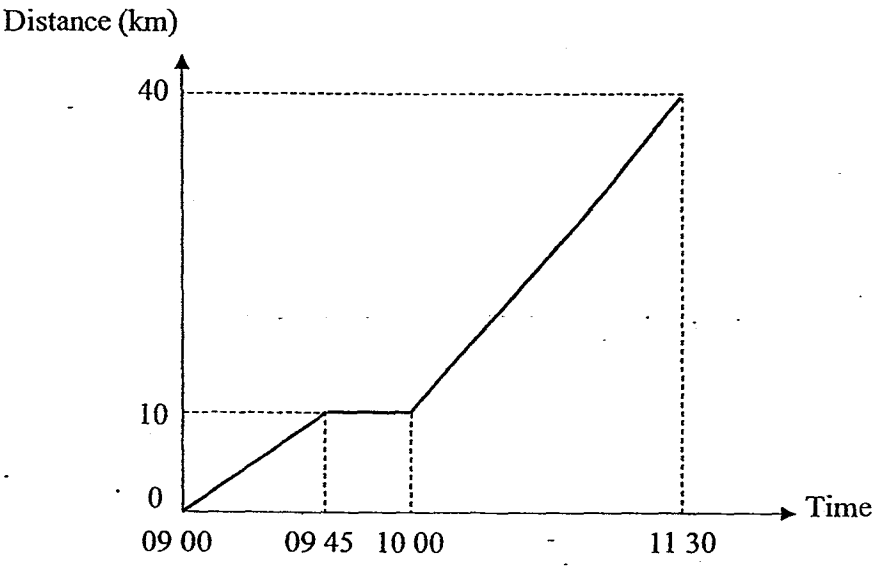
$\therefore \Delta DQA$ is congruent to ΔAPB (SAS)

.....

..... [2]

For Examiner's Use

17. The diagram shows the distance-time graph of a cyclist who begins his journey of 40 km at 09 00 and completes it at 11 30.



Calculate

(a) the speed of the cyclist for the first 30 minutes in km/h,

(b) his acceleration at 10 30 in m/s^2,

(c) his average speed from 09 00 to 11 00, in km/h.

(a) Speed $= 10 \div \frac{3}{4}$

$= 13\frac{1}{3}$

(c) Let x be the distance travelled from 1000 to 1100

$\frac{30}{1.5} = \frac{x}{1} \Rightarrow x = 20$

Average speed $= (10 + 20) \div 2$

$= 15$ km/h

Answer (a) $13\frac{1}{3}$ or 13.3 km/h [1]

(b) 0 m/s^2 [1]

(c) 15 km/h [2]

For Examiner's Use

18. **(a)** Construct a rhombus $PQRS$ in which $PR = 8$ cm and $QS = 6$ cm. [2]

Answer (a), (bi), (bii) and (c)

(b) From **(a)**, construct

(i) the perpendicular bisector of line PQ, [1]

(ii) the angle bisector of $\angle PQS$, [1]

(c) Mark clearly a possible point, X, inside the rhombus such that it is closer to point P than point Q, and also nearer to QS than PQ. [1]

For Examiner's Use

For Examiner's Use

19. y is inversely proportional to the square of x.

(a) If x is increased by 50%, find the percentage change in y.

(b) Given that $y = 25$ when $x = \frac{1}{2}$, find

(i) the equation connecting y and x,

(ii) the values of x when $y = 400$.

(c) Sketch the graph which represents the relation between y and x.

(a) $y = \frac{k}{x^2}$

$y_1 = \frac{k}{(1.5x)^2}$

$= \frac{4}{9}y$

Percentage change $= \frac{\frac{4}{9}y - y}{y} \times 100\%$

$= -55.56\%$

(b)(i) $k = \frac{25}{4}$

$y = \frac{25}{4x^2}$

(ii) $400 = \frac{25}{4x^2}$

$x^2 = \frac{25}{1600}$

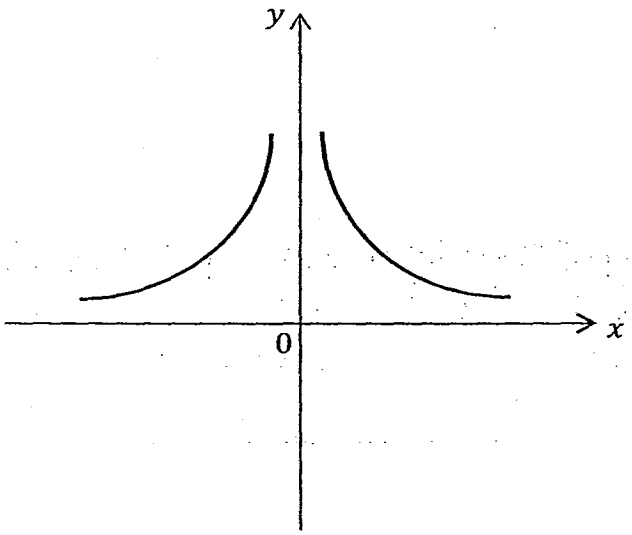
$x = \pm\frac{1}{8}$

Answer (a) ..... -55.6 ..... % [2]

(b)(i) ..... $y = \frac{25}{4x^2}$ ..... [2]

(ii) $x =$ ..... $\frac{1}{8}$ ..... , $x =$ ..... $-\frac{1}{8}$ ..... [2]

Answer (c) [1]



For Examiner's Use

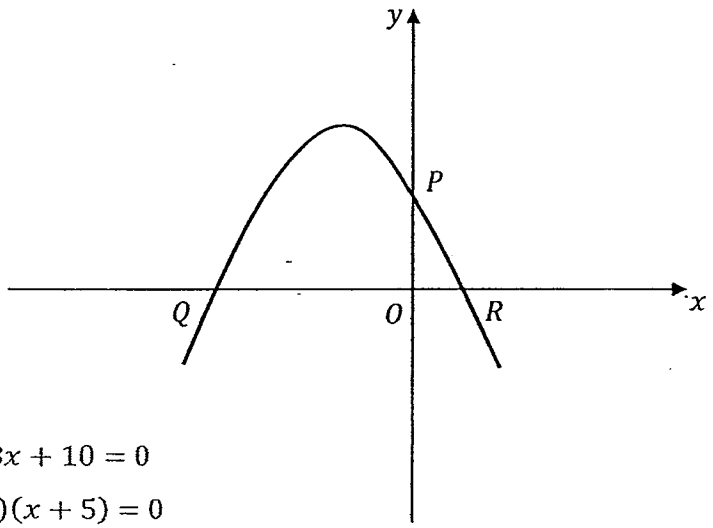
For Examiner's Use

20. The diagram shows the graph of $y = -2x^2 - 8x + 10$.

(a) The graph cuts the y-axis at $P(0, p)$.
Write down the value of p.

(b) The graph cuts the x-axis at Q and R.
Find the coordinates of Q and R.

(c) Hence, find the coordinates of the maximum point.



(b) $-2x^2 - 8x + 10 = 0$

$(-2x + 2)(x + 5) = 0$

$x = 1$ or $x = -5$

(c) mid-point of $QR = \left(\frac{-5+1}{2}, 0\right) = (-2, 0)$

When $= -2$,

$y = -2(-2)^2 - 8(-2) + 10 = 18$

Answer (a) $p =$ ..10.. [1]

(b) $Q =$ (..−5.. , ..0..) [1]

$R =$ (..1.. , ..0..) [1]

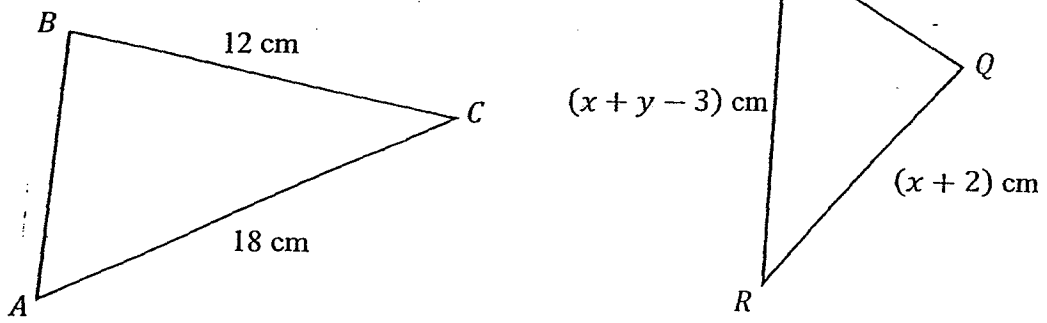
(c) (..−2.. , ..18..) [2]

For Examiner's Use

21. Given that ΔABC is similar to ΔPQR,

(a) form an equation in x and y.

(b) find the values of x and y if $3x - y = 9$.



(a) $\frac{12}{18} = \frac{x+2}{x+y-3}$

$12x + 12y - 36 = 18x + 36$

$6x - 12y = -72$

$x - 2y = -12$ ------- (1)

(b) $3x - y = 9$ ------- (2)

(1) × 2 : $6x - 2y = 18$ ------- (3)

(3) − (1) : $5x = 30$

$\Rightarrow \quad x = 6$

When $x = 6$, $y = 9$

$x - 2y = -12$ or
$2y = x + 12$

Answer (a) ..... [2]

(b) $x =$ ..... 6 ..... , $y =$ ..... 9 ..... [2]

For Examiner's Use

For Examiner's Use

22. Given that P is the point $(1, 2)$, $\overrightarrow{PQ} = \begin{pmatrix} 4 \\ -5 \end{pmatrix}$ and $\overrightarrow{PR} = \begin{pmatrix} 6 \\ 3 \end{pmatrix}$, find

(a) $|\overrightarrow{PQ}|$,

(b) $\overrightarrow{QR}$,

(c) the coordinates of the point S such that $PQRS$ is a parallelogram.

(a) $|\overrightarrow{PQ}| = \sqrt{4^2 + (-5)^2} = \sqrt{41}$

(b) $\overrightarrow{QR} = \overrightarrow{QP} + \overrightarrow{PR}$

$= \begin{pmatrix} -4 \\ 5 \end{pmatrix} + \begin{pmatrix} 6 \\ 3 \end{pmatrix}$

$= \begin{pmatrix} 2 \\ 8 \end{pmatrix}$

(c) Let the position vector of S be $\overrightarrow{OS}$.

$\overrightarrow{PQ} = \overrightarrow{SR}$

$\begin{pmatrix} -4 \\ 5 \end{pmatrix} = \overrightarrow{SO} + \overrightarrow{OR}$

Given $\overrightarrow{PR} = \begin{pmatrix} 6 \\ 3 \end{pmatrix} = \overrightarrow{PO} + \overrightarrow{OR}$

$\begin{pmatrix} 6 \\ 3 \end{pmatrix} = \begin{pmatrix} -1 \\ -2 \end{pmatrix} + \overrightarrow{OR}$

$\overrightarrow{OR} = \begin{pmatrix} 7 \\ 5 \end{pmatrix}$

$\overrightarrow{SO} = \begin{pmatrix} 4 \\ -5 \end{pmatrix} - \begin{pmatrix} 7 \\ 5 \end{pmatrix} = \begin{pmatrix} -3 \\ -10 \end{pmatrix}$

$\overrightarrow{OS} = \begin{pmatrix} 3 \\ 10 \end{pmatrix}$

Answer (a) $|\overrightarrow{PQ}| =$ ..... 6.40 ..... units [1]

(b) $\overrightarrow{QR} =$ ..... $\begin{pmatrix} 2 \\ 8 \end{pmatrix}$ ..... [1]

(c) $S =$ (..... 3 ..... , ... 10 .....) [3]

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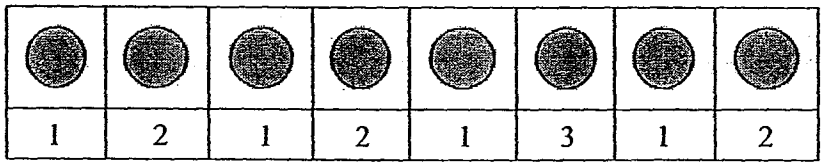
23. A ball is dropped at random into one of the eight holes, numbered as shown in the diagram. The number under each hole gives the score obtained when the ball drops into that hole. Each hole can only fit one ball.



1	2	1	2	1	3	1	2

(a) What is the probability of scoring 2?

(b) If a second ball is dropped into the holes, without removing the first ball, find the probability of scoring

(i) a total of 6,

(ii) a total of 4.

(a) P(scoring 2) $= \frac{3}{8}$

(b) (i) P(scoring a total of 6) = 0

(ii) P(scoring a total of 4) $= P(1, 3) + P(3, 1) + P(2, 2)$

$$= \left(\frac{4}{8} \times \frac{1}{7}\right) + \left(\frac{1}{8} \times \frac{4}{7}\right) + \left(\frac{3}{8} \times \frac{2}{7}\right)$$

$$= \frac{1}{14} + \frac{1}{14} + \frac{3}{28}$$

$$= \frac{1}{4}$$

Answer (a) $\frac{3}{8}$ ..... [1]

(b)(i) 0 ..... [1]

(ii) $\frac{1}{4}$ ..... [2]

End of Paper

Qn	Solutions
1(a)	$16u^2v^2 - 56uv + 54 = 5$ $16(uv)^2 - 56uv + 49 = 0$ $(4uv-7)^2 = 0$ $4uv = 7$ $uv = \frac{7}{4}$ $v = \frac{7}{4u}$
1(b)	$\frac{a^2+5ab+4b^2}{a^2-16b^2} - \frac{b}{4b-a}$ $= \frac{a^2+5ab+4b^2}{(a+4b)(a-4b)} - \frac{b}{4b-a}$ $= \frac{a^2+5ab+4b^2}{(a+4b)(a-4b)} + \frac{b}{a-4b}$ $= \frac{a^2+5ab+4b^2+ab+4b^2}{(a+4b)(a-4b)}$ $= \frac{a^2+6ab+8b^2}{(a+4b)(a-4b)}$ $= \frac{(a+4b)(a+2b)}{(a+4b)(a-4b)}$ $= \frac{a+2b}{a-4b}$
1(c)	$\frac{m(nx-y^2)}{p} = 3n$ $mnx - my^2 = 3np$ $my^2 = mnx - 3np$ $y^2 = \frac{nmx-3np}{m}$ $y^2 = \frac{n(mx-3p)}{m}$ $y = \pm\sqrt{\frac{n(mx-3p)}{m}}$ or $\pm\sqrt{\frac{mnx-3np}{m}}$ or $\pm\sqrt{nx-\frac{3np}{m}}$
2 (a)(i)	Profit that Fred will get $= \frac{20000}{20000+30000+46000} \times 40800 = \$8500$

2 (a)(ii)	Fred's new amount of profit $= \frac{14}{14+22+15} \times 40800 = \$11200$ Percentage change $= \frac{11200-8500}{8500} \times 100\%$ $= 31\frac{13}{17}\%$ $= 31.7647\%$ $= 31.8\%$ (3 s.f.)
2(b)	$A = 11200(1+\frac{1.88}{4}\%)^6$ $= 11519.57446$ $= \$11519.57$ (2 d.p)
2(c)	0.7332 euros = 1 USD 4500 USD $= 4500 \times 0.7332 = 3299.40$ euros 1 USD = 1.2479 SGD 1200 SGD $= \frac{1200}{1.2479} = 961.6155$ USD 961.6155 USD $= 961.6155 \times 0.7332 = 705.0564$ euros Total amount received $= 3299.40 + 705.0564$ $= 4004.4564$ $= 4004.46$ euros (2 d.p.)
3(a)	Radius of cone $= \sqrt{\frac{38.5}{\pi}} = 3.50$ m (3 s.f.)
3(b)	Volume of wooden sculpture $= (38.5)(10) + \frac{1}{3}(38.5)(4) - \left(\frac{1}{2}\right)\left(\frac{4}{3}\right)\pi(1.5)^3$ $= 429.2647$ $= 429\ \text{m}^3$ (3s.f)
3(c)	$0.59 = \frac{m}{429.2647 \times 100^3}$ $m = 253266173$ g $= 253266.173$ kg $= 253000$ kg (3 s.f.)

3(d)	Curved surface area of cone $=\pi(3.5007)(\sqrt{4^2+3.5007^2})$ $=58.459037\ \text{m}^2$ Surface area of cylinder $=2\pi(3.5007)(10)+38.5-\pi(1.5)^2+\left(\frac{1}{2}\right)4\pi(1.5)^2$ $=265.5240\,\text{m}^2$ Surface area of wooden sculpture $=58.459037+265.5240\,\text{m}^2$ $=323.983089\,\text{m}^2$ $=324\,\text{m}^2$ (3 s.f.)
3 (e)	$\frac{V_{model}}{V_{solid}}=\left(\frac{Height_{model}}{Height_{solid}}\right)^3=\left(\frac{875}{1400}\right)^3=\frac{125}{512}$ $V_{model}=\frac{125}{512}\times 429.2647$ $=104.8010\ \text{m}^3$ $=105\ \text{m}^3$ (3 s.f.) $=105000000\ \text{cm}^3 / 1.05\times10^8\ \text{cm}^3$
4(a)	Time taken by the train to travel from Beijing South Station to Danyang North Station $=\frac{1120}{x}$ h
4(b)	The time taken in hours for this journey $=\frac{1120}{x+10}$ h
4(c)	$\frac{1120}{x}-\frac{1120}{x+10}=\frac{50}{60}$ $\frac{1120x+11200-1120x}{x(x+10)}=\frac{5}{6}$ $11200(6)=5x(x+10)$ $5x^2+50x-67200=0$ $x^2+10x-13440=0$
4(d)	$x=\frac{-10\pm\sqrt{(10)^2-4(1)(-13440)}}{2(1)}$ $=\frac{-10\pm\sqrt{53860}}{2}$

	$x = 111.0388$ or $x = -121.0388$ $= 111.04$ (2 d.p.) $= -121.04$ (2 d.p.)
4(e)	$x = -121.04$ (rej. Θ $x > 0$) Time taken $= \dfrac{1120}{111.0388 + 10}$ $= 9.2532$ hr $= 9$ hr 15.192 min $= 9$ hr 16 min (rounded up so as to ensure train reach Danyang North Station) The time taken by the train to reach Danyang North Station on that Sunday is 9 hr 16 min.
5(a)(i)	In ΔCOD, $\angle ODC = \dfrac{\pi}{2}$ (tangent $\perp$ radius) $\cos \angle BOD = \dfrac{10}{20}$ $\angle BOD = \dfrac{\pi}{3}$ (Shown)
5(a)(ii)	Area of $\Delta COD = \dfrac{1}{2} \times 10 \times 20 \times \sin\left(\dfrac{\pi}{3}\right)$ $= 86.6025$ cm^2 Area of sector $BOD = \dfrac{1}{2} \times 10^2 \times \left(\dfrac{\pi}{3}\right)$ $= 52.3599$ cm^2 Area of the shaded region $BCD = 86.6025 - 52.3599$ $= 34.2426$ cm^2 $= 34.2$ cm^2 (3 s.f)
5(b)	By Cosine Rule, $AD^2 = 10^2 + 10^2 - 2(10)(10)\cos\left(\dfrac{2\pi}{3}\right)$ $= 300$ $AD = \sqrt{300}$ cm

	Arc length $AD = 10\left(\frac{2\pi}{3}\right) = \frac{20\pi}{3}$ cm Length of semicircle $AED = \pi\left(\frac{\sqrt{300}}{2}\right)$ cm Perimeter of shaded region, $K = \frac{20\pi}{3} + \pi\left(\frac{\sqrt{300}}{2}\right)$ $= 48.1509$ $= 48.2$ cm (3 s.f.)
6(a)(i)	$\overrightarrow{OM} = \overrightarrow{OP} + \frac{1}{2}\overrightarrow{PQ}$ $= 5\mathbf{a} + \frac{1}{2}(5\mathbf{b} - 5\mathbf{a})$ $= \frac{5}{2}(\mathbf{a}+\mathbf{b})$ or $\frac{5}{2}\mathbf{a} + \frac{5}{2}\mathbf{b}$
6(a)(ii)	$\overrightarrow{PN} = \overrightarrow{ON} - \overrightarrow{OP}$ $= \frac{4}{7}\overrightarrow{OM} - \overrightarrow{OP}$ $= \frac{4}{7}\left(\frac{5}{2}\right)(\mathbf{a}+\mathbf{b}) - 5\mathbf{a}$ $= \frac{10}{7}(\mathbf{a}+\mathbf{b}) - 5\mathbf{a}$ $= \frac{5}{7}(2\mathbf{b} - 5\mathbf{a})$ or $\frac{10}{7}\mathbf{b} - \frac{25}{7}\mathbf{a}$
6(a)(iii)	$\overrightarrow{NL} = \overrightarrow{OL} - \overrightarrow{ON}$ $= \frac{2}{5}(5\mathbf{b}) - \frac{10}{7}(\mathbf{a}+\mathbf{b})$ $= \frac{2}{7}(2\mathbf{b} - 5\mathbf{a})$ or $\frac{4}{7}\mathbf{b} - \frac{10}{7}\mathbf{a}$
6(b)	From the results in (a)(ii) and (a)(iii), $\overrightarrow{PN} = \frac{5}{7}(2\mathbf{b} - 5\mathbf{a})$ $= \frac{5}{2}[\frac{2}{7}(2\mathbf{b} - 5\mathbf{a})]$ $= \frac{5}{2}\overrightarrow{NL}$ PN is parallel to NL and since they have a common point N, P, N and L are collinear. $\therefore$ N lies on PL. $PN : NL = 5 : 2$

6(c)	$\frac{\text{area of } \Delta PNM}{\text{area of } \Delta PON} = \frac{\frac{1}{2} \times h \times NM}{\frac{1}{2} \times h \times ON}$ $= \frac{3}{4}$
6(d)	$\Delta PON : \Delta PNM$ $= 4:3$ $= 20:15$ $\Delta PON : \Delta OLN$ $= 5:2$ $= 20:8$ $\Delta OQM : \Delta OMP$ $= 35:35$ $= \Delta OLN + \text{Quadrilateral } LNQM : \Delta PON + \Delta PNM$ $= 8+27:20+15$ $\therefore \frac{\text{area of } \Delta PON}{\text{area of quadrilateral } LNMQ} = \frac{20}{27}$
7(a)(i)	$\angle ODB = 69°$ (base $\angle s$, isos Δ) $\angle BEA = 180° - 69° - 69°$ ($\angle s$ in opposite segments) $= 42°$ $\angle BOD = 42°$ $\angle BED = 42° \div 2$ ($\angle$ at centre $= 2\ \angle$ at circumference) $= 21°$ $\angle AED = 42° - 21°$ $= 21°$
7(a)(ii)	$\angle ABD = \angle AED = 21°$ ($\angle$s in same segment) $\angle ABO = 69° - 21°$ $= 48°$
7(a)(iii)	$\angle ADB = 69° + 69° = 138°$ $\angle OBC = 90°$ (tangent $\perp$ radius) $\angle DBC = 90° - 69°$ $= 21°$ $\angle DBC + \angle BCD° = 138°$ (ext. $\angle$ of Δ) $\angle BCD = 138° - 21°$ $= 117°$
7 (b)	$\angle ABD = \angle AED = 21°$ ($\angle$s in same segment) $\angle DBC = 90° - 69° = 21°$ (tangent $\perp$ radius) Since $\angle ABD = \angle DBC$ $\therefore$ the line BD bisects $\angle ABC$.

8(a)	$a=16,\ b=40$
8(b)	100
8(c)	60
8(d)(i)	$p=n^2$
8(d)(ii)	Let $q=an^2+bn+c$ $n=1:\ a+b+c=4$.....(1) $n=2:\ 4a+2b+c=12$.....(2) $n=3:\ 9a+3b+c=24$.....(3) (2) − (1) $3a+b=8$.....(4) (3) − (2) $5a+b=12$.....(5) (5) − (4) $2a=4$, $a=2$ Subst. $a=2$ into (4): $b=2$ Subst. $a=2$ and $b=2$ into (1): $c=0$ $\therefore q=2n^2+2n$ or $q=2n(n+1)$
8(e)	$364=2n^2+2n$ $n^2+n-182=0$ $(n+14)(n-13)=0$ $n=13$ or $n=-14$ (rejected) $\therefore$ figure 13 has 364 toothpicks.
9(a)(i)	Median weight of pebbles from Bondi Beach, Sydney = 43 g
9(a)(ii)	Interquartile range of the distribution = 50 − 34 = 16 g
9(a)(iii)	Number of pebbles with a weight more than 60 grams. = 120 − 108 = 12
9(b)(i)	

Weight (w grams)	Frequency
$0<w\leq 20$	1
$20<w\leq 30$	14
$30<w\leq 40$	21
$40<w\leq 50$	29
$50<w\leq 60$	19
$60<w\leq 70$	10
$70<w\leq 80$	6

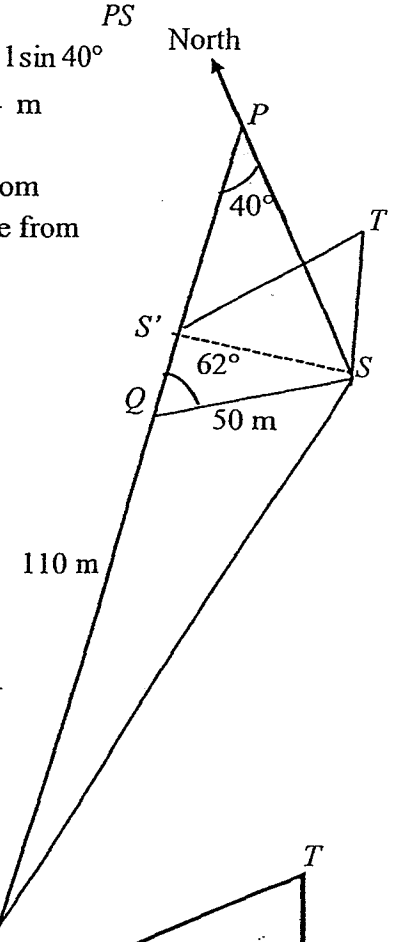
9(b)(ii)	$= \frac{10(1)+25(14)+35(21)+45(29)+55(19)+65(10)+75(6)}{100}$ Mean $= 45.45\,\text{g}$ $= 45.5\,\text{g}$ (3.s.f.)
	$\frac{\sum fx^2}{\sum f} = \frac{10^2(1)+25^2(14)+35^2(21)+45^2(29)+55^2(19)+65^2(10)+75^2(6)}{100}$ $= 2267.75$ Standard Deviation $= \sqrt{2267.75-45.45^2} = \sqrt{\frac{812191}{400}} = 14.2\,\text{g}$ (3.s.f.)
9(c)	P (the weight of the pebble from Bondi Beach, Sydney is more than 60 grams and the weight of the pebble Pirates Bay, Tasmania is less than 40 grams) $= \left(\frac{12}{120}\right)\left(\frac{36}{100}\right)$ $= 0.036$ or $\frac{9}{250}$
10(a)(i)	By Sine Rule, $\frac{PS}{\sin 62°} = \frac{50}{\sin 40°}$ $PS = \frac{50\sin 62°}{\sin 40°}$ $PS = 68.6811$ $= 68.7\,\text{m}$ (3 s.f.)
10(a)(ii)	$\angle RQS = 180° - 62°$ (adj. $\angle$s on a str.line) $= 118°$ Area of the field $QRS = \frac{1}{2}\times 110\times 50\times \sin 118°$ $= 2428.10588$ $= 2430\ \text{m}^2$ (3s.f.)
10(a)(iii)	By Cosine Rule, $RS^2 = 110^2 + 50^2 - 2(110)(50)\cos 118°$ $= 19764.187$ $RS = 140.5852$ $= 141\,\text{m}$ (3 s.f.)
10(b)	Bearing of S from $X = (40° + 31°) = 071°$

10(c)(i)

$$\sin 40° = \frac{\text{shortest distance from S to } QP(SS')}{PS}$$

shortest distance from S to $QP(SS') = 68.68711 \sin 40°$

$= 44.1474$ m



The angle of elevation of the top of the tree from a point on PR is the greatest when the distance from S to PR is the shortest.

$\therefore$ greatest angle of elevation = $\angle TS'S$

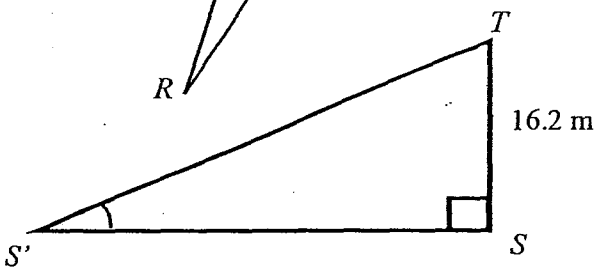
In $\Delta TSS'$,

$$\tan \angle TSS' = \frac{TS}{SS'}$$

$$\tan \angle TSS' = \frac{16.2}{44.1474}$$

$$\angle TSS' = 20.1507°$$

$$= 20.2° \text{ (1 d.p.)}$$



$\therefore$ greatest angle of elevation = $\angle TS'S = 20.2°$.

10(c)(ii)

Let T' be the point where the boy is half way up the tree.

$\therefore$ angle of depression = $\angle T'RS$ (alt. $\angle s$, // lines)

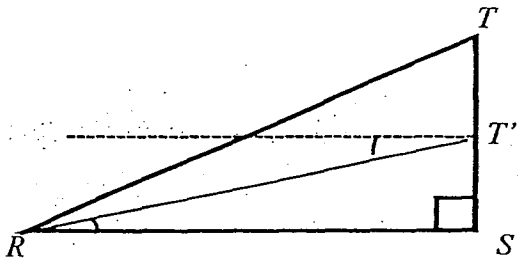
$$T'S = \frac{16.2}{2} = 8.1 \text{ m}$$

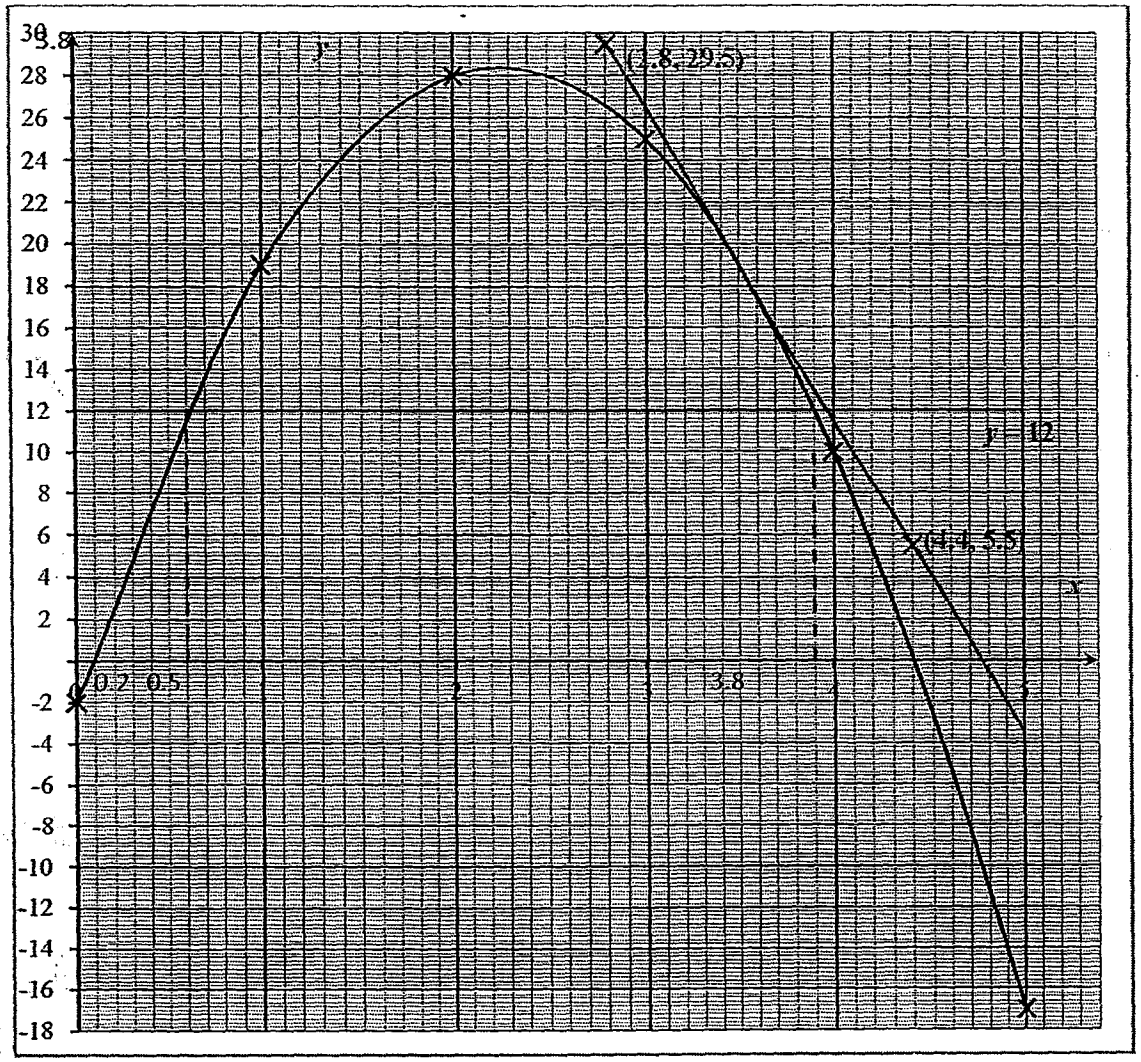
$$\tan \angle T'RS = \frac{T'S}{RS}$$

$$\tan \angle T'RS = \frac{8.1}{140.5852}$$

$$\angle T'RS = 3.2975°$$

$$= 3.3° \text{ (1 d.p.)}$$



11(a)	$p = 25$
11(b)	
11(c)(i)	From the graph, when $x = 0$, $y = -2$. $\therefore$ monthly rental fee = \$2000.
11(c)(ii)	From the graph, maximum point of the graph is (2.25, 28.4) $\therefore$ number of jeans to produce to maximise profit $= 2.25 \times 1000$ $= 2250$
11(c)iii)	To just break even, profit = 0. From the graph, when $y = 0$, $x = 0.1$ (accept 0.05 to 0.15) or $x = 4.4$ $\therefore$ minimum number of jeans to produce to break even $= 0.1 \times 1000$ $= 100$

11(c)(iv)	From the graph, when $y=12$, $x=0.6$ or $x=3.9$ $\therefore$ for profit of at least \$12 000, range of number of jeans to be produced is $600 \le x \le 3900$.
11(d)(i)	Using points (2.8, 29.5) and (4.4, 5.5) Gradient $= \dfrac{5.5-29.5}{4.4-2.8}$ $= \dfrac{-24}{1.6}$ $= -15$
11(d)(ii)	It represents the rate at which the profit is changing at $x=3.5$.