Class Register Number

Name

4016/01 **14/S4PR1/EM/1**

MATHEMATICS **PAPER 1**

Tuesday **6 May 2014** **2 hours**

VICTORIA SCHOOL

PRELIMINARY EXAMINATION ONE
SECONDARY FOUR

Candidates answer on the Question Paper.

READ THESE INSTRUCTIONS FIRST

Write your name, class and register number on all the work you hand in.
Write in dark blue or black pen.
You may use a pencil for any diagrams or graphs.
Do not use paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.
If working is needed for any question it must be shown with the answer.
Omission of essential working will result in loss of marks.
You are expected to use a scientific calculator to evaluate explicit numerical expressions.
If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.
For π, use either your calculator value or 3.142, unless the question requires the answer in terms of π.

The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 80.

This paper consists of 17 printed pages, including the cover page.

[Turn over

Mathematical Formulae

Compound interest

$$\text{Total amount} = P\left(1+\frac{r}{100}\right)^n$$

Mensuration

$$\text{Curved surface area of a cone} = \pi r l$$

$$\text{Surface area of a sphere} = 4\pi r^2$$

$$\text{Volume of a cone} = \frac{1}{3}\pi r^2 h$$

$$\text{Volume of a sphere} = \frac{4}{3}\pi r^3$$

$$\text{Area of triangle } ABC = \frac{1}{2}ab\sin C$$

$$\text{Arc length} = r\theta\text{, where } \theta \text{ is in radians}$$

$$\text{Sector area} = \frac{1}{2}r^2\theta\text{, where } \theta \text{ is in radians}$$

Trigonometry

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

Statistics

$$\text{Mean} = \frac{\sum fx}{\sum f}$$

$$\text{Standard deviation} = \sqrt{\frac{\sum fx^2}{\sum f} - \left(\frac{\sum fx}{\sum f}\right)^2}$$

1 A polygon has n sides. Three of its exterior angles are $30°$, $12°$ and $63°$ respectively while the remaining $(n-3)$ exterior angles are $51°$ each. Find n.

Answer $n =$ .... [2]

---

2 It is given that y is inversely proportional to x^2. It is known that $y = 256$ for a particular value of x. Find the value of y when this value of x is decreased by 60%.

Answer $y =$ .... [2]

---

3 **(a)** The profits of a company were \$880 million in 2012 and \$1.1 billion in 2013. Find the percentage increase in profits from 2012 to 2013.

Answer (a) ....% [1]

(b) The length of a cord is 0.062 metres, correct to the nearest millimetre. Write down, in millimetres, the lower bound of this length.

Answer (b) .... mm [1]

4 Trains following route A leave station D every five minutes.
Trains following route B leave station D every eight minutes.
Trains following route C leave station D every twelve minutes.
Three trains, following routes A, B and C respectively, leave station D together at 14 15.
At what time will the three trains next leave station D together?

Answer .... [2]

---

5 J, K and L are the points $(1, 2)$, $(4, 6)$ and $(-5, 3)$. Find the

(a) equation of the line which passes through L and is parallel to JK,

Answer (a) .... [2]

(b) coordinates of two possible points N, such that J, K, L and N are the four vertices of a parallelogram.

Answer (b) (.............. , ...)

(.............. , ...) [1]

---

6 **(a)** Factorise $9x^3+36x^2-x-4$ completely.

Answer (a) .... [2]

(b) Hence, solve $9x^3+36x^2-x-4=0$.

Answer (b) $x=$ .... [1]

7 Some students were asked how many books they have read in the previous week. The table shows the results.

Number of books	0	1	2	3
Number of students	7	3	1	y

(a) If the mean number of books is 2, find the value of y.

Answer (a) $y =$ .... [1]

(b) If the median number of books is 1, find the greatest possible value of y.

Answer (b) $y =$ .... [1]

(c) If the modal number of books is 0, find the range of possible values of y.

Answer (c) .... [1]

---

8 The temperature at 0400 is -5 °C.
The temperature at 1200 is 19 °C.

(a) Find the difference between the two temperatures.

Answer (a) ....°C [1]

(b) Assuming that the temperature rises at a steady rate, find

(i) the temperature at 0930,

Answer (b)(i) ....°C [1]

(ii) the time when the temperature is 8 °C.

Answer (b)(ii) .... [1]

9 A cone D has a volume of 135 cm^3.

(a) Calculate the volume of a second cone, E, which has a radius half that of cone D and height ten times that of cone D.

Answer (*a*) .... cm^3 [2]

(b) A third cone F is similar to cone D and has a volume of 5 cm^3. Find the ratio of the surface area of cone D to the surface area of cone F.

Answer (*b*) .... : .... [1]

10 The cash price of a smart LED television set is \$3600. Ruth paid a 15% down payment. The outstanding balance plus interest is paid over 24 months. Simple interest on the outstanding balance was charged at 9.5% per annum.

(a) Find the cost of the smart LED television set if it is bought on hire purchase.

Answer (*a*) \$ .... [2]

(b) Find the amount Ruth paid for each monthly instalment.

Answer (*b*) \$ .... [1]

11 Given that $\overrightarrow{PQ}=\begin{pmatrix}6\\1\end{pmatrix}$, $A(-3,\ k)$ and $B(-2,\ 5)$.

(a) Express $\overrightarrow{AB}$ as a column vector, in terms of k.

Answer (*a*) .... [1]

(b) If $\left|\overrightarrow{AB}\right|=\left|\overrightarrow{PQ}\right|$, find the possible values of k.

Answer (*b*) $k =$ .... or .... [2]

---

12 A field has an area of $112.5\ \text{km}^2$. It is represented by an area of $18\ \text{cm}^2$ on Map P.

(a) Find the scale of the map in the form 1 : n.

Answer (*a*) 1 : .... [1]

Map Q has a scale of 1 : 400 000.

(b) A road is measured 2.4 cm on Map P. Find, in centimetres, the length representing this road on Map Q.

Answer (*b*) ....cm [2]

13 (a) It is given that $\frac{1}{25^{m-3}} \times 625 = 5^m$. Find the value of m.

Answer (*a*) $m =$ .... [2]

(b) Evaluate $\frac{9\times2^{102}-3\times2^{100}}{3\times2^{98}}$.

Answer (*b*) .... [2]

---

14 The waiting times for the arrival of a particular bus service, in minutes, of 40 students are given as follows.

Time (minutes)	1–5	6–10	11–15	16–20	21–25
Number of students	5	9	14	8	4

Calculate an estimate for the

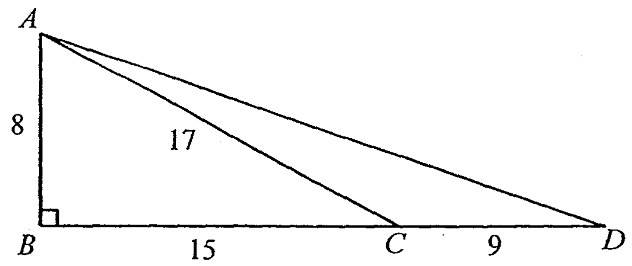
(a) mean waiting time,

Answer (*a*) .... minutes [2]

(b) standard deviation of the waiting times.

Answer (*b*) .... minutes [2]

15



In $\Delta\ ABC$, angle $ABC = 90°$, $AB = 8$ cm, $BC = 15$ cm and $AC = 17$ cm.
BC is produced to D and $CD = 9$ cm.

(a) Find the area of triangle ACD.

Answer (*a*) .... cm^2 [1]

(b) Write down $\cos D\hat{C}A$.

Answer (*b*) .... [1]

(c) Calculate the perpendicular distance from C to AD.

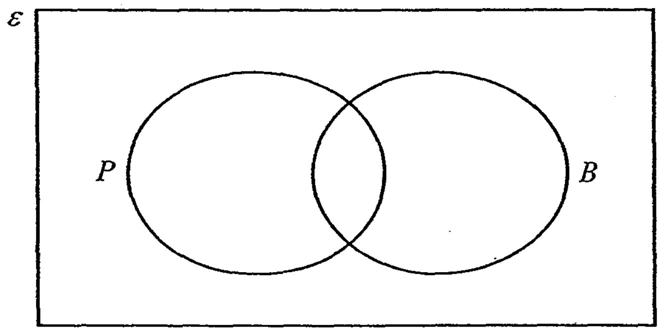
Answer (*c*) .... cm [2]

634

16 In a class of 100 students, 75 study Physics (P) and 40 study Biology (B).
x students study Physics and Biology.
y students study neither Physics nor Biology.

(a) Complete the Venn diagram below to illustrate the information. Show clearly the elements in each region.

Answer (a)



[1]

(b) Expressed in set notation, the value of x is $n(P \cap B)$.
Express the value of y in set notation.

Answer (b) $y =$ ....[1]

(c) Find the

(i) least possible value of x,

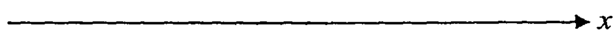
Answer (c)(i) $x =$ .... [1]

(ii) greatest possible value of y.

Answer (c)(ii) $y =$ .... [1]

17 **(a)** Solve the inequality $\frac{8x-17}{2} < 5x-8 \leq \frac{11}{4}x+10$. Show your solution on the number line below.

Answer (*a*)

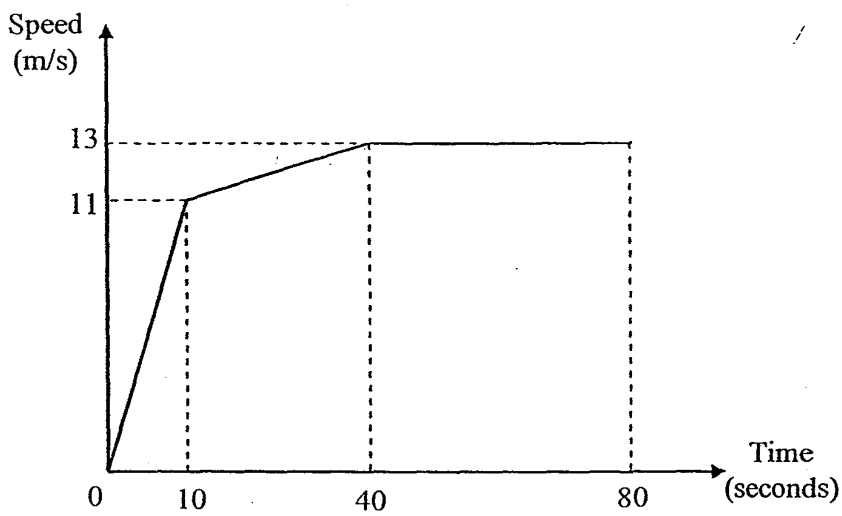


[3]

(b) Hence, write down the smallest integer value of x which satisfies $\frac{8x-17}{2} < 5x-8 \leq \frac{11}{4}x+10$.

Answer (*b*) .... [1]

18



The diagram is the speed-time graph of a cyclist during the first 80 seconds of a race.

(a) Calculate the acceleration during the first 5 seconds.

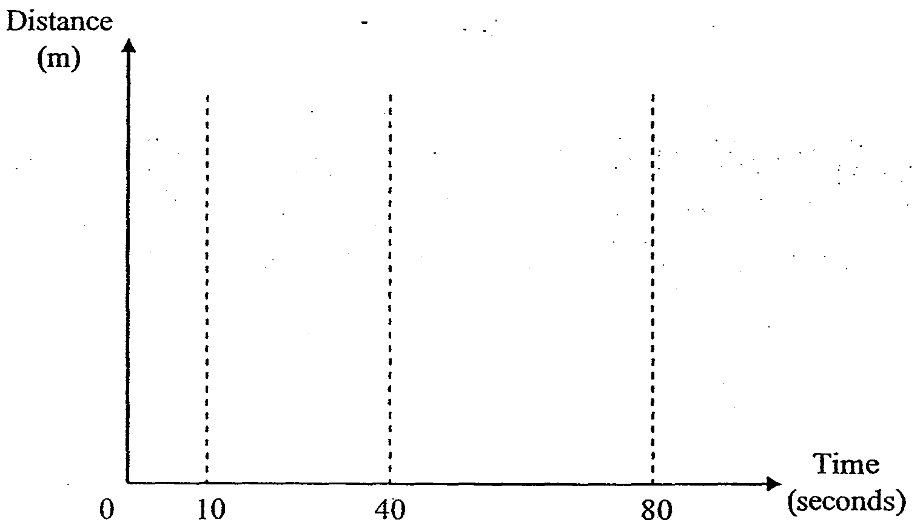
Answer (*a*) .... m/s^2 [1]

(b) Calculate the distance, in metres, travelled in the first 80 seconds.

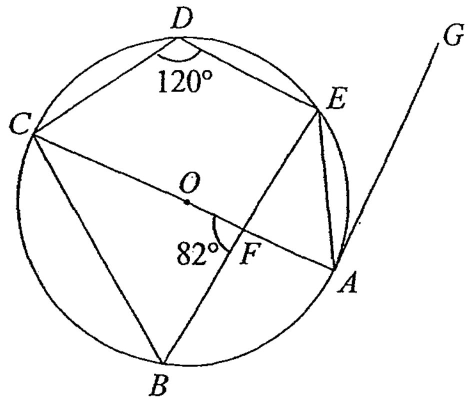
Answer (*b*) .... m [1]

(c) On the axes in the answer space below, sketch the distance-time graph of the cyclist in the first 80 seconds of the race.

Answer (*c*)



[2]



The points A, B, C, D and E lie on a circle, centre O.
AC is a diameter of the circle.
AC and BE meet at F.
GA is a tangent to the circle at A.
Angle $BFC = 82°$ and angle $CDE = 120°$.
Find

(a) $C\hat{B}E$,

Answer (*a*) $C\hat{B}E =$ .... [1]

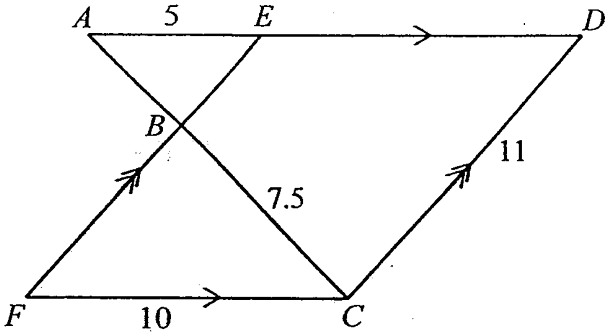
(b) $A\hat{E}B$,

Answer (*b*) $A\hat{E}B =$ .... [1]

(c) $E\hat{A}G$.

Answer (*c*) $E\hat{A}G =$ .... [2]

20



In the diagram, *CDEF* is a parallelogram.
B is a point on *AC*.
ABC and *AED* are straight lines.
$AE = 5$ cm, $BC = 7.5$ cm, $FC = 10$ cm and $CD = 11$ cm.

(a) Show that triangles *ADC* and *CFB* are similar.

Answer (*a*) ....

....

.... [2]

(b) Calculate

(i) *AC*,

Answer (*b*)(*i*) .... cm [1]

(ii) *EB*.

Answer (*b*)(*ii*) .... cm [1]

21 **(a)** Look at the number triangle below. Complete the next two rows.

Answer (*a*)

1

3 5

7 9 11

13 15 17 19

____

____

[1]

(b) Using the number triangle in **(a)**, complete the following table.

Answer (*b*)

Row number, R	Number of terms, n	Average of the first and the last term, m	The first term, a
1	1	1	1
2	2	4	3
3	3	9	7
4			
5			

[1]

(c) Express m in terms of n.

Answer (*c*) $m =$ .... [1]

(d) Form an equation connection n, m and a.

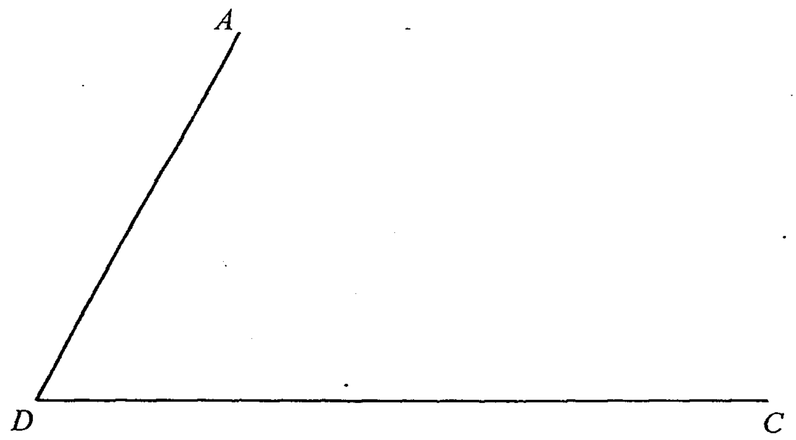
Answer (*d*) .... [1]

(e) Hence, determine the first term of the 31st row.

Answer (*e*) $a =$ .... [1]

22 Three points A, C and D are shown below.

Answer (a), (c) and (d)



(a) Construct the parallelogram $ABCD$. [1]

(b) Measure the size of angle ABC.

Answer (b) $A\hat{B}C$ = ... [1]

(c) Construct the perpendicular bisector of DC. [1]

(d) Construct the angle bisector of angle ADC. [1]

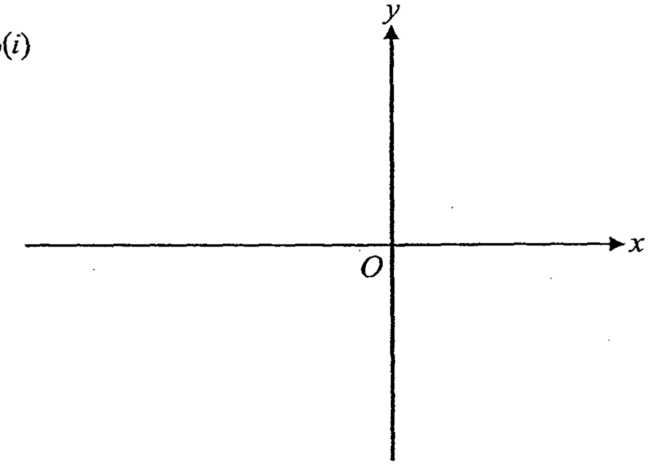
(e) These two bisectors meet at X.
Complete the statement below.

Answer (e) The point X is equidistant from the lines .... and ....

and equidistant from the points .... and .... [1]

23 (a) (i) Sketch the graph of $y=(x+5)(1-x)$.

Answer (*a*)(*i*)



[1]

(ii) Write down the equation of the line of symmetry of $y=(x+5)(1-x)$.

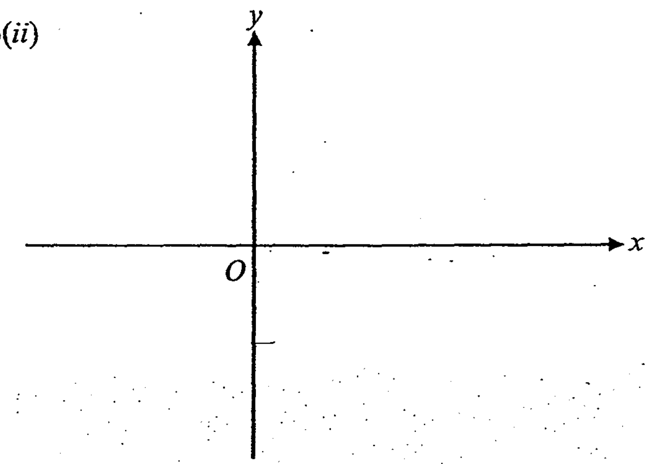
Answer (*a*)(*ii*) .... [1]

(b) (i) Express x^2-8x+3 in the form $(x-a)^2+b$, where a and b are integers.

Answer (*b*)(*i*) .... [1]

(ii) Hence, sketch the graph of $y=x^2-8x+3$.

Answer (*b*)(*ii*)



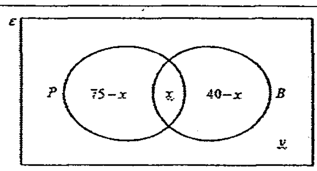
[2]

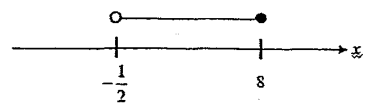
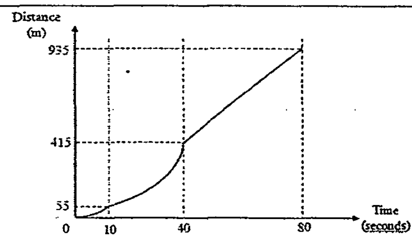
(iii) Write down the coordinates of the minimum point of the curve.

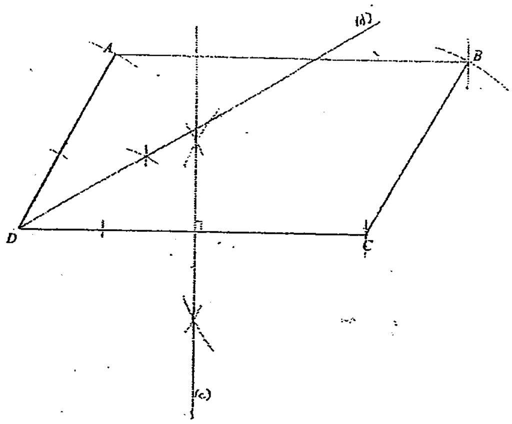
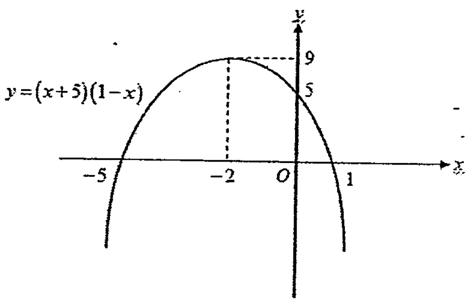
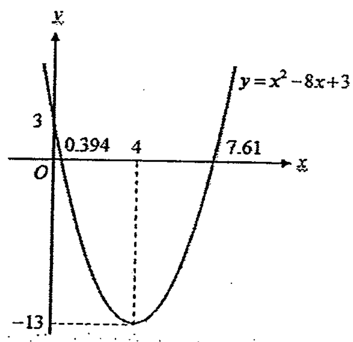
Answer (*b*)(*iii*) (............. , ...) [1]

End of Paper

642

1	8
2	1600
3a	25
3b	61.5
4	16 15
5a	$y=\frac{4}{3}x+9\frac{2}{3}$
5b	$N(-2,\ 7)$ or $N(10,\ 5)$ or $N(-8,\ -1)$
6a	$(3x+1)(3x-1)(x+4)$
6b	$-\frac{1}{3}$ or $\frac{1}{3}$ or -4
7a	17
7b	8
7c	$0\le y<7$ or $0\le y\le 6$
8a	24
8bi	11.5
8bii	08 20
9a	337.5
9b	9 : 1
10a	4181.40
10b	151.73
11a	$\begin{pmatrix}1\\5-k\end{pmatrix}$
11b	11 or -1
12a	250 000
12b	1.5
13a	$3\frac{1}{3}$
13b	44
14a	$12\frac{5}{8}$
14b	5.74
15a	36
15b	$-\frac{15}{17}$
15c	2.85
16a	

16b	$n(P\cup B)'$ or $n(P'\cap B')$
16ci	15
16cii	25
17a	$-\frac{1}{2}<x\le 8$ 
17b	0
18a	1.1
18b	935
18c	
19a	60°
19b	38°
19c	30°
20a	$\angle ADC=\angle CFB$ (opp. $\angle$s of a parallelogram) $\angle ACD=\angle CBF$ (alt. $\angle$s, $FB//CD$) $\angle DAC=\angle FCB$ (alt. $\angle$s, $AD//FC$) Hence, ΔADC and ΔCFB are similar.
20bi	11.25
20bii	$3\frac{2}{3}$
21a	21 23 25 27 29 31 33 35 37 39 41
21b	4 | 16 | 13 5 | 25 | 21
21c	n^2
21d	$n+a=m+1$
21e	931

22a **22c** **22d**	
22b	62°
22e	AD, DC, D, C
23ai	
23aii	$x=-2$
23bi	$x^2-8x+3=(x-4)^2-13$
23bii	
23biii	$(4,\ -13)$

Class Register Number 55

Name MARKING SCHEME

4016/01 **14/S4PR1/EM/1**

MATHEMATICS **PAPER 1**

Tuesday **6 May 2014** **2 hours**

NIL SINE LABORE

VICTORIA SCHOOL

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SECONDARY FOUR

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$$\text{Mean} = \frac{\sum fx}{\sum f}$$

$$\text{Standard deviation} = \sqrt{\frac{\sum fx^2}{\sum f} - \left(\frac{\sum fx}{\sum f}\right)^2}$$

1 A polygon has n sides. Three of its exterior angles are $30°$, $12°$ and $63°$ while the remaining $(n-3)$ exterior angles are each equal to $51°$. Find n.

$$30+12+63+51(n-3)=360 \quad \leftarrow \text{[M1]}$$
$$105+51n-153=360$$
$$51n=408$$
$$n=8 \quad \leftarrow \text{[A1]}$$

Answer $n =$ ....8.......... [2]

---

2 It is given that y is inversely proportional to x^2. It is known that $y = 256$ for a particular value of x. Find the value of y when this value of x is decreased by 60%.

$$y=\frac{k}{x^2},\ k \text{ is a constant}$$
$$\text{new } y=\frac{k}{(0.4x)^2} \quad \leftarrow \text{[M1]}$$
$$=\frac{25}{4}\left(\frac{k}{x^2}\right)$$
$$=\frac{25}{4}y$$

$$\text{New value of } y=\frac{25}{4}(256)$$
$$=1600 \quad \leftarrow \text{[A1]}$$

Answer $y =$ ....1600.......... [2]

---

3 **(a)** The profits of a company were \$880 million in 2012 and \$1.1 billion in 2013. Find the percentage increase in profits from 2012 to 2013.

$$\text{Percentage increase}=\frac{1.1\times10^9-880\times10^6}{880\times10^6}\times 100\%$$
$$=25\ \% \quad \leftarrow \text{[A1]}$$

Answer (a) ....25..........%[1]

(b) The length of a cord is 0.062 metres, correct to the nearest millimetre. Write down, in millimetres, the lower bound of this length.

0.062 m = 62 mm

62 ± 0.05

Lower bound is 61.5 mm. ← [B1]

Answer (b) ....61.5.......... mm [1]

4 Trains following route A leave the station every five minutes.
Trains following route B leave the station every eight minutes.
Trains following route C leave the station every twelve minutes.
Three trains, following routes A, B and C, leave the station together at 14 15.
At what time will the three trains next leave the station together?

$$\begin{array}{r|lll} 4 & 5, & 8, & 12 \\ \hline & 5, & 2, & 3 \end{array}$$

LCM of 5, 8 and 12 $= 4 \times 5 \times 2 \times 3 \leftarrow$ [M1]
$= 120$

120 min = 2 hr

Time $= 1415 + 0200$
$= 1615 \leftarrow$ [A1]

Answer 16 15 [2]

---

5 J, K and L are the points $(1, 2)$, $(4, 6)$ and $(-5, 3)$. Find

(a) the equation of the line which passes through L and is parallel to JK,

Gradient of $JK = \dfrac{6-2}{4-1} \leftarrow$ [M1]
$= \dfrac{4}{3}$

At $(-5, 3)$, $3 = \dfrac{4}{3}(-5) + c$

$c = 9\dfrac{2}{3}$

Equation of line is $y = \dfrac{4}{3}x + 9\dfrac{2}{3}$. $\leftarrow$ [A1]

Answer (c) $y = \dfrac{4}{3}x + 9\dfrac{2}{3}$ [2]

(b) the coordinates of two possible points N, such that J, K, L and N are the four vertices of a parallelogram.

Answer (d) (............ , ...)

$N(-2, 7)$ or $N(10, 5)$ or $N(-8, -1)$ $\leftarrow$ [B1]

(............ , ...) [1]

---

6 **(a)** Factorise $9x^3 + 36x^2 - x - 4$ completely.

$$9x^3 + 36x^2 - x - 4 = 9x^2(x+4) - (x+4)$$
$$= (9x^2 - 1)(x+4) \leftarrow \text{[A1]}$$
$$= (3x+1)(3x-1)(x+4) \leftarrow \text{[A1]}$$

Answer (a) $(3x+1)(3x-1)(x+4)$ [2]

(b) Hence, solve $9x^3 + 36x^2 - x - 4 = 0$.

$9x^3 + 36x^2 - x - 4 = 0$

$(3x+1)(3x-1)(x+4) = 0$

$3x+1=0$ or $3x-1=0$ or $x+4=0$

$x = -\dfrac{1}{3}$ $\quad x = \dfrac{1}{3}$ $\quad x = -4 \leftarrow$ [A1]

Answer (b) $x =$ $-\dfrac{1}{3}$ or $\dfrac{1}{3}$ or -4 [1]

7 Some students were asked how many books they have read in the previous day. The table shows the results.

Number of books	0	1	2	3.
Number of students	7	3	1	y

(a) If the mean number of books is 2, find the value of y.

$$\frac{0+3+2+3y}{7+3+1+y}=2$$

$$\frac{3y+5}{y+11}=2$$

$$3y+5=2y+22$$

$$y=17 \quad \leftarrow \quad [A1]$$

Answer (a) $y=$ 17 [1]

(b) If the median number of books is 1, find the greatest possible value of y.

$$7+2=1+y$$

$$y=8 \quad \leftarrow \quad [B1]$$

Answer (b) $y=$ 8 [1]

(c) If the modal number of books is 0, find the range of possible values of y.

$0 \le y < 7$ or [B1]

Answer (c) $y=$ $0 \le y \le 6$ [1]

---

8 The temperature at 0400 is -5 °C.
The temperature at 1200 is 19 °C.

(a) Find the difference between the two temperatures.

Difference $=19+5$

$=24$ °C $\leftarrow$ [B1]

Answer (a) 24 °C [1]

(b) Assuming that the temperature rises at a steady rate, find

(i) the temperature at 0930,

$$\text{Temperature}=\left(\frac{24}{8}\times 5\frac{1}{2}\right)+(-5)$$

$$=16.5-5$$

$$=11.5\text{ °C} \quad \leftarrow \quad [A1]$$

Answer (b)(i) 11.5 °C [1]

(ii) the time when the temperature is 8 °C.

$$\text{Duration}=\frac{8}{24}\times(8+5)$$

$$=\frac{8}{24}\times 13$$

$$=4\frac{1}{3}\text{ h}$$

$$=4\text{h }20\text{min}$$

Time $=$ 04 00 $+$ 04 20

$=$ 08 20 $\leftarrow$ [A1]

Answer (b)(ii) 08 20 [1]

9 A cone D has a volume of 135 cm^3.

(a) Calculate the volume of a second cone, E, which has a radius half that of cone D and height ten times that of cone D.

$$\text{Vol. of cone } D = 135$$
$$\frac{1}{3}\pi r^2 h = 135$$
$$\pi r^2 h = 405$$
$$\text{Vol. of cone } E = \frac{1}{3}\pi\left(\frac{1}{2}r\right)^2(10h) \quad \leftarrow \text{[M1]}$$

$$\text{Vol. of cone } E = \frac{5}{6}\pi r^2 h$$
$$= \frac{5}{6}(405)$$
$$= 337.5 \text{ cm}^3 \quad \leftarrow \text{[A1]}$$

Answer (a) 337.5 cm^3 [2]

(b) A third cone F is similar to cone D and has a volume of 5 cm^3. Find the ratio of the surface area of cone D to the surface area of cone F.

$$\frac{\text{Vol. of cone } D}{\text{Vol. of cone } F} = \frac{135}{5}$$
$$= 27$$

$$\frac{\text{Surface area of cone } D}{\text{Surface area of cone } F} = \left(\sqrt[3]{\frac{27}{1}}\right)^2$$
$$= 9$$

Ratio is 9 : 1. ← [A1]

Answer (b) 9 : 1 [1]

---

10 The cash price of a smart LED television set is \$3600. Ruth paid a 15% down payment. The outstanding balance plus interest is paid over 24 months. Simple interest on the outstanding balance was charged at 9.5% per annum.

(a) Find the cost of the smart LED television set if it bought on hire purchase.

$$\text{Outstanding balance} = \frac{85}{100} \times 3600$$
$$= \$\ 3060$$
$$\text{Hire purchase cost} = 3600 + \frac{3060 \times 9.5 \times 2}{100} \quad \leftarrow \text{[M1]}$$
$$= 3600 + 581.40$$
$$= \$\ 4181.40 \quad \leftarrow \text{[A1]}$$

Answer (a) \$ 4181.40 [2]

(b) Find the amount Ruth paid for each monthly instalment.

$$\text{Monthly instalments} = \frac{3060 + 581.40}{24}$$
$$= \frac{3641.40}{24}$$
$$= \$\ 151.73 \text{ (2 d.p.)} \quad \leftarrow \text{[A1]}$$

Answer (b) \$ 151.73 [1]

11 Given that $\overrightarrow{PQ}=\begin{pmatrix}6\\1\end{pmatrix}$, $A(-3,\ k)$ and $B(-2,\ 5)$.

(a) Express $\overrightarrow{AB}$ as a column vector, in terms of k.

$$\overrightarrow{AB}=\overrightarrow{OB}-\overrightarrow{OA}$$
$$=\begin{pmatrix}-2\\5\end{pmatrix}-\begin{pmatrix}-3\\k\end{pmatrix}$$
$$=\begin{pmatrix}1\\5-k\end{pmatrix} \leftarrow \text{[A1]}$$

Answer (*a*) $\begin{pmatrix}1\\5-k\end{pmatrix}$ [1]

(b) If $\left|\overrightarrow{AB}\right|=\left|\overrightarrow{PQ}\right|$, find the possible values of k.

$$\left|\overrightarrow{AB}\right|=\left|\overrightarrow{PQ}\right|$$
$$\sqrt{1^2+(5-k)^2}=\sqrt{6^2+1^2} \leftarrow \text{[M1]}$$
$$1+25-10k+k^2=37$$
$$k^2-10k-11=0$$
$$(k-11)(k+1)=0$$
$$k-11=0 \text{ or } k+1=0$$
$$k=11 \qquad k=-1 \leftarrow \text{[A1]}$$

Answer (*b*) $k=$ 11 or −1 [2]

---

12 A field has an area of 112.5 km^2. It is represented by an area of 18 cm^2 on Map P.

(a) Find the scale of the map in the form 1 : n.

$$\text{area scale}=18\text{ cm}^2:112.5\text{ km}^2$$
$$=1\text{ cm}^2:6.25\text{ km}^2$$
$$\text{linear scale}=1\text{ cm}:2.5\text{ km}$$
$$=1:250\ 000 \leftarrow \text{[A1]}$$

Answer (*a*) 1 : 250 000 [1]

Map Q has a scale of 1 : 400 000.

(b) A road measured 2.4 cm on Map P. Find, in centimetres, the length representing this road on Map Q.

$$\text{actual length of road}=2.4\times 250\ 000$$
$$=600\ 000\text{ cm}$$
$$\text{length of road on Map } Q=\frac{600\ 000}{400\ 000} \leftarrow \text{[M1]}$$
$$=1.5 \leftarrow \text{[A1]}$$

Answer (*b*) 1.5 cm [2]

13 (a) It is given that $\frac{1}{25^{m-3}} \times 625 = 5^m$. Write down the value of m.

$$\frac{1}{25^{m-3}} \times 625 = 5^m$$
$$\frac{1}{5^{2m-6}} \times 5^4 = 5^m$$
$$5^{4-2m+6} = 5^m$$
$$10 - 2m = m \quad \leftarrow \text{[M1]}$$
$$3m = 10$$
$$m = 3\frac{1}{3} \quad \leftarrow \text{[A1]}$$

Answer (*a*) $m =$ $3\frac{1}{3}$ [2]

(b) Evaluate $\frac{9\times2^{102} - 3\times2^{100}}{3\times2^{98}}$.

$$\frac{9\times2^{102} - 3\times2^{100}}{3\times2^{98}} = \frac{3\times2^{100}\left(3\times2^2 - 1\right)}{3\times2^{98}} \quad \leftarrow \text{[M1]}$$
$$= 2^2(11)$$
$$= 44 \quad \leftarrow \text{[A1]}$$

Answer (*b*) 44 [2]

---

14 The waiting times for the arrival of a particular bus service, in minutes, of 40 students are given as follows.

Time (minutes)	1–5	6–10	11–15	16–20	21–25
Number of students	5	9	14	8	4

Calculate an estimate for

(a) the mean waiting time,

$$\text{Mean} = \frac{505}{40} \quad \leftarrow \text{[B1]}$$
$$= 12\frac{5}{8}$$
$$= 12.625 \text{ min} \quad \leftarrow \text{[A1]}$$

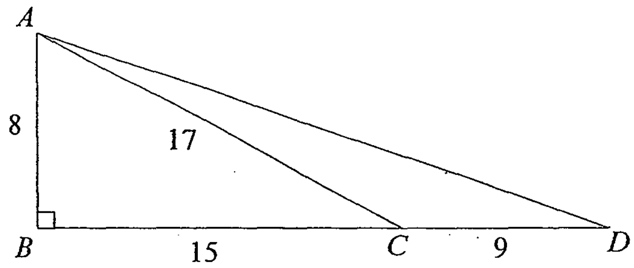
Answer (*a*) $12\frac{5}{8}$ or 12.625 minutes [2]

(b) the standard deviation of the waiting times.

$$\text{Standard deviation} = \sqrt{\frac{7695}{40} - \left(\frac{505}{40}\right)^2} \quad \leftarrow \text{[M1]}$$
$$= 5.74 \text{ min (3 s.f.)} \quad \leftarrow \text{[A1]}$$

Answer (*b*) 5.74 minutes [2]

15



In $\Delta\ ABC$, angle $ABC = 90°$, $AB = 8$ cm, $BC = 15$ cm and $AC = 17$ cm.
BC is produced to D and $CD = 9$ cm.

(a) Find the area of triangle ACD.

$$\text{Area of } \Delta\ ACD = \frac{1}{2} \times 9 \times 8$$
$$= 36 \text{ cm}^2 \quad \leftarrow \quad [\text{A1}]$$

Answer (*a*) ......36...... cm^2 [1]

(b) Write down $\cos D\hat{C}A$.

$$\cos D\hat{C}A = -\cos A\hat{C}B$$
$$= -\frac{15}{17} \quad \leftarrow \quad [\text{B1}]$$

Answer (*b*) ......$-\frac{15}{17}$...... [1]

(c) Calculate the perpendicular distance from C to AD.

Let the perpendicular distance be h cm.

$$AD = \sqrt{8^2 + 24^2}$$
$$= \sqrt{640} \text{ cm}$$

$$\frac{1}{2} \times \sqrt{640} \times h = 36 \quad \leftarrow \quad [\text{M1}]$$
$$h = \frac{36 \times 2}{\sqrt{640}}$$
$$h = 2.85 \text{ (3 s.f.)} \quad \leftarrow \quad [\text{A1}]$$

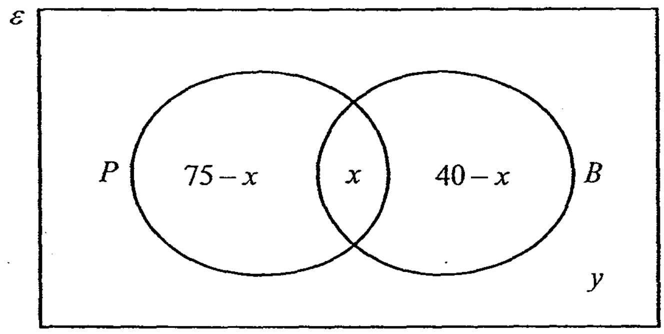
Answer (*c*) ......2.85...... units [2]

664

16 In a class of 100 students, 75 study Physics and 40 study Biology.
x students study Physics and Biology.
y students study neither Physics nor Biology.

(a) Complete the Venn diagram below to illustrate the information. Show clearly the elements in each region.

Answer (*a*)



[1]

(b) Expressed in set notation, the value of x is $\mathrm{n}(P \cap B)$.
Express the value of y in set notation.

$\mathrm{n}(P \cup B)'$
or
Answer (*b*) $y =$ $\mathrm{n}(P' \cap B')$ [1]

(c) Find

(i) the least possible value of x,

when $y = 0$, $75 - x + x + 40 - x = 100$

$115 - x = 100$

$x = 15 \leftarrow$ [B1]

Answer (*c*)(*i*) 15 [1]

(ii) the greatest possible value of y.

$y = 100 - 75$

$= 25 \leftarrow$ [B1]

Answer (*c*)(*ii*) 25 [1]

17 **(a)** Solve the inequality $\frac{8x-17}{2} < 5x-8 \leq \frac{11}{4}x+10$. Show your solution on the number line below.

Answer (*a*)

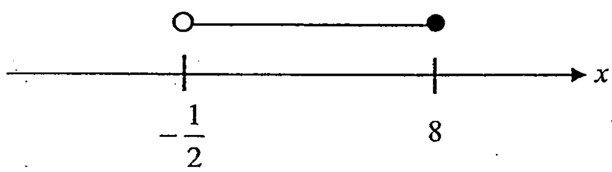
$\frac{8x-17}{2} < 5x-8$ and $5x-8 \leq \frac{11}{4}x+10$ ← [B1]

$8x-17 < 10x-16$ $\qquad$ $\frac{9}{4}x \leq 18$

$2x > -1$ $\qquad$ $x \leq 8$

$x > \frac{-1}{2}$

$-\frac{1}{2} < x \leq 8$ ← [A1]



← [B1]

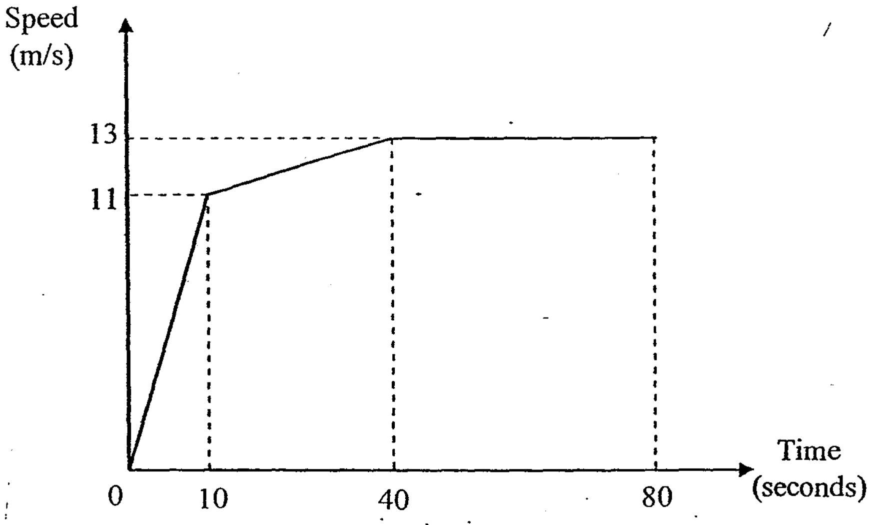
[3]

(b) Hence, write down the smallest integer value of x which satisfies $\frac{8x-17}{2} < 5x-8 \leq \frac{11}{4}x+10$.

Smallest integer $= 0$ ← [B1]

Answer (*b*) ....0.......... [1]

666 **18**



The diagram is the speed-time graph of a cyclist during the first 80 seconds of a race.

(a) Calculate the acceleration during the first 5 seconds.

$$\text{Acceleration} = \frac{11}{10}$$
$$= 1.\dot{1}\ \text{m/s}^2 \quad \leftarrow \quad [\text{B1}]$$

Answer (a) ....1.1.......... m/s^2 [1]

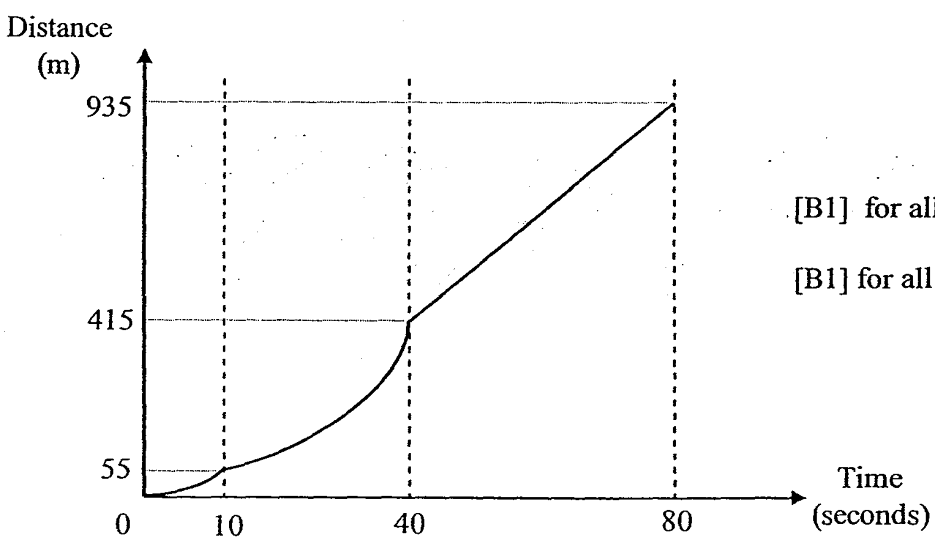
(b) Calculate the distance, in metres, travelled in the first 80 seconds.

$$\text{Distance} = \left(\frac{1}{2}\times10\times11\right)+\left[\frac{1}{2}\times(11+13)\times30\right]+(40\times13)$$
$$= 55+360+520$$
$$= 935\ \text{m} \quad \leftarrow \quad [\text{A1}]$$

Answer (b) ....935.......... m [1]

(c) On the axes in the answer space, sketch the distance-time graph of the cyclist in the first 80 seconds of the race.

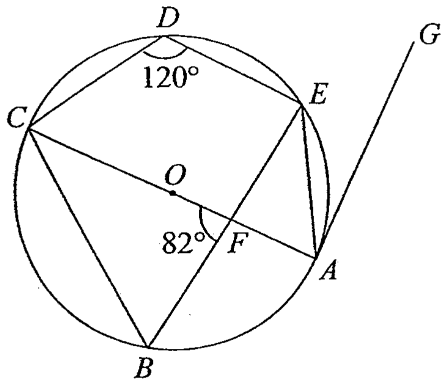
Answer (c)



[B1] for all 3 shapes

[B1] for all 3 distances

[2]



The points A, B, C, D and E lie on a circle, centre O.
AC is a diameter of the circle.
AC and BE meet at F.
GA is a tangent to the circle at A.
Angle $BFA = 82°$ and angle $CDE = 120°$.
Find

(a) $C\hat{B}E$,

$C\hat{B}E = 180° - 120°$ (opp. $\angle$s of cyclicquad.)
$= 60°$ $\leftarrow$ [A1]

Answer (*a*) $C\hat{B}E =$ 60° [1]

(b) $A\hat{E}B$,

$B\hat{C}F = 180° - 60° - 82°$ ($\angle$ sum of Δ)
$= 38°$
$A\hat{E}B = B\hat{C}F$ ($\angle$s in the same segment)
$= 38°$ $\leftarrow$ [A1]

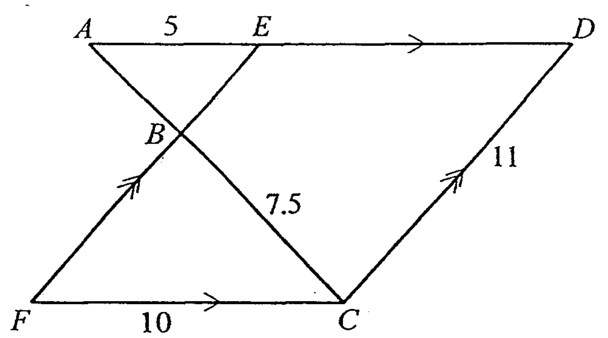
Answer (*b*) $A\hat{E}B =$ 38° [1]

(c) $E\hat{A}G$.

$O\hat{A}G = 90°$ (tan $\perp$ rad) $\leftarrow$ [B1]
$E\hat{A}C = C\hat{B}E$ ($\angle$s in the same segment)
$= 60°$
$E\hat{A}G = 90° - 60°$ (complementary $\angle$s)
$= 30°$ $\leftarrow$ [A1]

Answer (*c*) $E\hat{A}G =$ 30° [2]

20



In the diagram, $CDEF$ is a parallelogram.
B is a point on AC.
ABC and AED are straight lines.
$AE = 5$ cm, $BC = 7.5$ cm, $FC = 10$ cm and $CD = 11$ cm.

(a) Show that triangles ADC and CFB are similar.

Answer (a) $\angle ADC = \angle CFB$ (opp. $\angle$s of a parallelogram)

$\angle ACD = \angle CBF$ (alt. $\angle$s, $FB // CD$)

$\angle DAC = \angle FCB$ (alt. $\angle$s, $AD // FC$) [A2]

Hence, ΔADC and ΔCFB are similar. [2]

(b) Calculate

(i) AC,

$$\frac{AD}{CF} = \frac{AC}{CB}$$

$$\frac{15}{10} = \frac{AC}{7.5}$$

$AC = 11.25$ cm $\leftarrow$ [A1]

Answer (b)(i) 11.25 cm [1]

(ii) EB.

ΔAEB and ΔCFB are similar

$$\frac{EB}{FB} = \frac{AE}{CF}$$

$$\frac{EB}{11 - EB} = \frac{5}{10}$$

$2EB = 11 - EB$

$3EB = 11$

$$EB = \frac{11}{3}$$

$EB = 3\frac{2}{3}$ cm $\leftarrow$ [A1]

Answer (b)(ii) $3\frac{2}{3}$ cm [1]

21 **(a)** Look at the number triangle. Complete the next two rows.

Answer (a)

					1					
				3		5				
			7		9		11			
		13		15		17		19		
	21		23		25		27		29	
31		33		35		37		39		41

← [B1]

[1]

(b) Using the number triangle in **(a)**, complete the following table.

Answer (b)

Row number, R	Number of terms, n	Average of the first and the last term, m	The first term, a
1	1	1	1
2	2	4	3
3	3	9	7
4	4	**16**	**13**
5	5	**25**	**21**

[1]

(c) Express m in terms of n.

Answer (c) $m =$ n^2 [B1] [1]

(d) Form an equation connection n, m and a.

Answer (b) $n + a = m + 1$ [B1] [1]

(e) Hence, determine the first term of the 31st row.

When $R = 31$, $31 + a = 31^2 + 1$

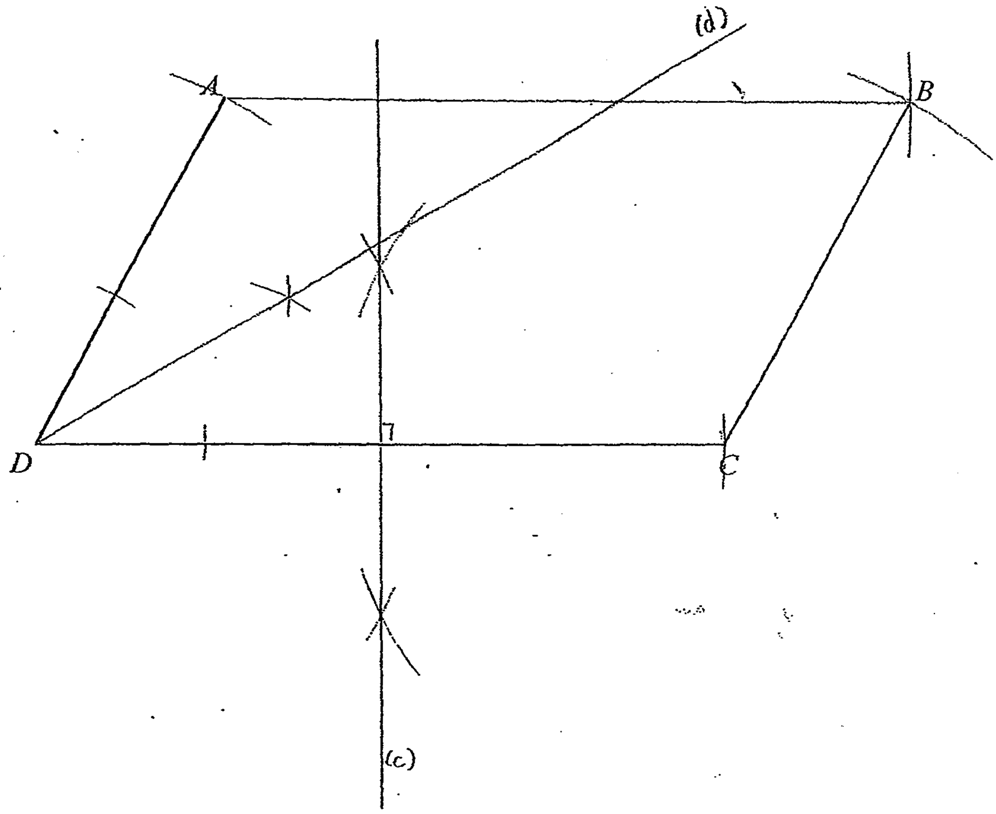
$a = 931$ ← [A1]

Answer (c) $a =$ 931 [1]

670

22 Three points A, C and D are shown below.

Answer (*a*), (*c*) and (*d*)



(a) Construct the parallelogram $ABCD$. [1]

(b) Measure the size of angle ABC.

Answer (*b*) $A\hat{B}C$ = 62° [1]

(c) Construct the perpendicular bisector of DC. [1]

(d) Construct the angle bisector of angle ADC. [1]

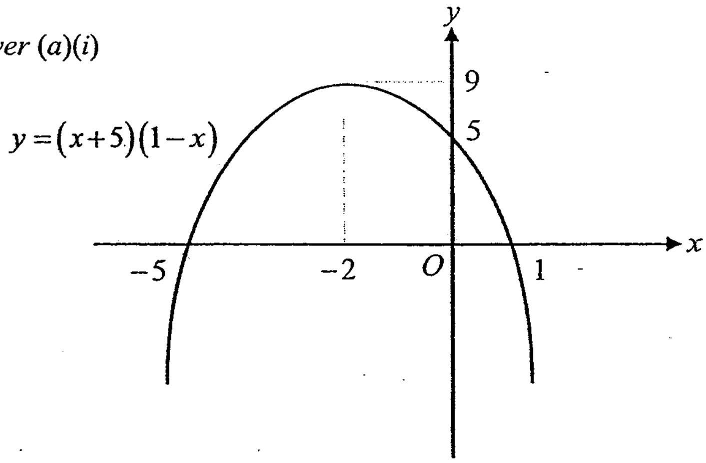
(e) These two bisectors meet at X.
Complete the statement below.

Answer (*e*) The point X is equidistant from the lines *AD* and *DC*

and equidistant from the points *D* and *C* . [1]

23 **(a)** **(i)** Sketch the graph of $y=(x+5)(1-x)$.

Answer (a)(i)



[1]

(ii) Write down the equation of the line of symmetry of $y=(x+5)(1-x)$.

Answer (a)(ii) $x=-2$ [1]

(b) **(i)** Express x^2-8x+3 in the form $(x-a)^2+b$, where a and b are integers.

$$x^2-8x+3=(x-4)^2+3-16$$
$$=(x-4)^2-13 \quad \leftarrow \text{[A1]}$$

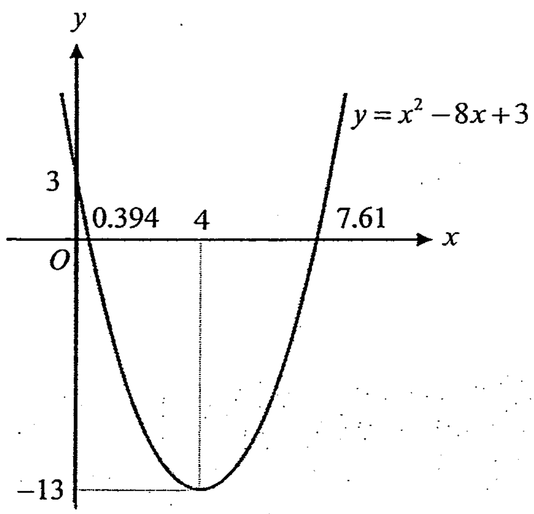
Answer (b)(i) $x^2-8x+3=(x-4)^2-13$ [1]

(ii) Hence, sketch the graph of $y=x^2-8x+3$.

Answer (b)(ii)

[B1] for shape

[B1] for all values



[2]

(iii) Write down the coordinates of the minimum point of the curve.

Answer (b)(iii) (4 , −13) [1]

End of Paper

	Class	Register Number
Name		

4016/02 **14/S4PR1/EM/2**

MATHEMATICS **PAPER 2**

Wednesday **7 May 2014** **2 hours 30 minutes**

NIL SINE LABORE

VICTORIA SCHOOL

PRELIMINARY EXAMINATION ONE
SECONDARY FOUR

Additional Materials: Answer Paper
Graph Paper

READ THESE INSTRUCTIONS FIRST

Write your name, class and register number on all the work you hand in.
Write in dark blue or black pen.
You may use a pencil for any diagrams or graphs.
Do not use paper clips, highlighters, glue or correction fluid.

Answer **all** questions.
If working is needed for any question it must be shown with the answer.
Omission of essential working will result in loss of marks.
You are expected to use a scientific calculator to evaluate explicit numerical expressions.
If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.
For π, use either your calculator value or 3.142, unless the question requires the answer in terms of π.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 100.

Compound interest

$$\text{Total amount} = P\left(1+\frac{r}{100}\right)^n$$

Mensuration

$$\text{Curved surface area of a cone} = \pi r l$$

$$\text{Surface area of a sphere} = 4\pi r^2$$

$$\text{Volume of a cone} = \frac{1}{3}\pi r^2 h$$

$$\text{Volume of a sphere} = \frac{4}{3}\pi r^3$$

$$\text{Area of triangle } ABC = \frac{1}{2}ab\sin C$$

$$\text{Arc length} = r\theta\text{, where } \theta \text{ is in radians}$$

$$\text{Sector area} = \frac{1}{2}r^2\theta\text{, where } \theta \text{ is in radians}$$

Trigonometry

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

Statistics

$$\text{Mean} = \frac{\sum fx}{\sum f}$$

$$\text{Standard deviation} = \sqrt{\frac{\sum fx^2}{\sum f} - \left(\frac{\sum fx}{\sum f}\right)^2}$$

Answer **all** the questions.

1 (a) Factorise $18x^2+21xy-60y^2$ completely. [2]

(b) Express $\dfrac{2}{(3-4y)^2}+\dfrac{1}{60y^2-21y-18}$ as a single fraction in its simplest form. [3]

---

2 The table below shows the number of tickets sold at two ticketing counters for a locally produced musical on a Friday, Saturday and Sunday.

		Standard Ticket	Premium Ticket
Counter 1	Friday	220	140
	Saturday	350	200
	Sunday	200	170
Counter 2	Friday	300	140
	Saturday	250	180
	Sunday	180	150

The information for Counter 1 can be represented by the matrix

$$\mathbf{A}=\begin{pmatrix}220 & 140\\ 350 & 200\\ 200 & 170\end{pmatrix}$$

The information for Counter 2 can be represented by matrix **B**.

(a) Write down matrix **B**. [1]

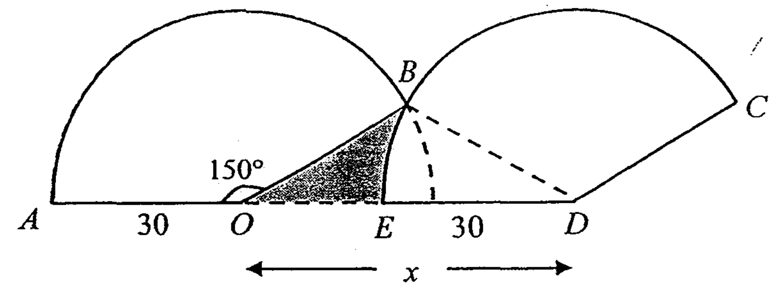
(b) Evaluate $\mathbf{C}=\mathbf{A}+\mathbf{B}$. [1]

A standard ticket costs \$100 while a premium ticket costs \$140. This can be represented by matrix **D**, where $\mathbf{D}=\begin{pmatrix}100\\ 140\end{pmatrix}$.

(c) Evaluate $\mathbf{E}=\mathbf{CD}$. [1]

(d) State what the elements of **E** represent. [1]

(e) Using matrix multiplication of matrix **E** and a further matrix, find the total amount of money collected from both counters. [2]

3 

Two identical screen wipers OA and DE, 30 cm long, are pivoted at point O and point D respectively and sweeps through $150°$ from left to right. Points O and D are placed x cm apart.

Calculate the value of x. Hence, or otherwise, find the area and perimeter of the shaded region OBE. [6]

---

4 **(a)** Given that $p = \dfrac{mv}{\sqrt{1-\dfrac{v^2}{c^2}}}$, express v in terms of c, m and p. [4]

(b) Simplify $\left(\dfrac{2a^3}{b^{-4}}\right)^{-2} \div \dfrac{b^{-2}}{\left(4a^{-1}\right)^3 b^7}$, leaving your answer in positive indices. [3]

---

5 **(a)** A man deposited \$8000 with a bank at a certain simple interest rate per annum. After 3 years, the deposit with interest was \$8180.

(i) Calculate the simple interest rate that the bank pays per annum. [2]

(ii) The same amount of money is deposited with a bank that offers a compound interest rate of 0.6% per annum, compounded half yearly. Find the total amount of money in the bank after 3 years. [2]

(b) The income tax rates for Singaporeans are given in the table below:

Chargeable Income	Rate (%)	Gross Tax Payable (\$)
First \$20 000	0	0
Next \$10 000	2	200
First \$30 000	-	200
Next \$10 000	3.50	350
First \$40 000	-	550
Next \$40 000	7	2800

(i) A man's chargeable income for the year was \$63 000, calculate his income tax. [2]

(ii) His mother paid a total of \$515 in income tax, how much was her chargeable income? [2]

6 There are 9 red, 6 blue and 3 yellow cards in a box. 2 cards are drawn at random, without replacement, from the box.

(a) Draw a probability tree to show the possible outcomes. [2]

(b) Find the probability that both cards drawn are of

(i) the same colour, [1]

(ii) different colours. [1]

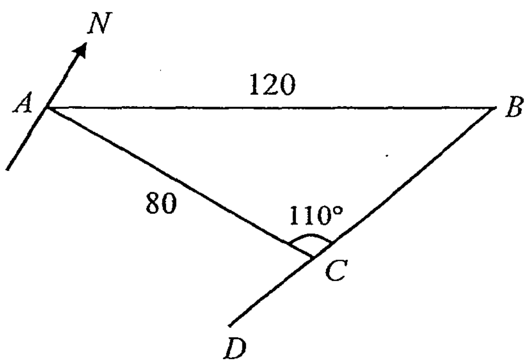
(c) All the cards are placed back in the box and are drawn at random, with replacement.

(i) Find the probability that exactly 2 blue cards are being drawn out of 3 draws. [1]

(ii) The process is repeated 8 times. Find the probability that a red card is being drawn each time. [1]

(d) How many yellow cards must be added to the box such that the probability of drawing a yellow card becomes $\frac{1}{4}$. [2]

7



A, B and C are points on level ground as shown in the diagram. $AB = 120$ m, $AC = 80$ m, angle $ACB = 110°$ and the bearing of C from B is $215°$.

(a) Find the

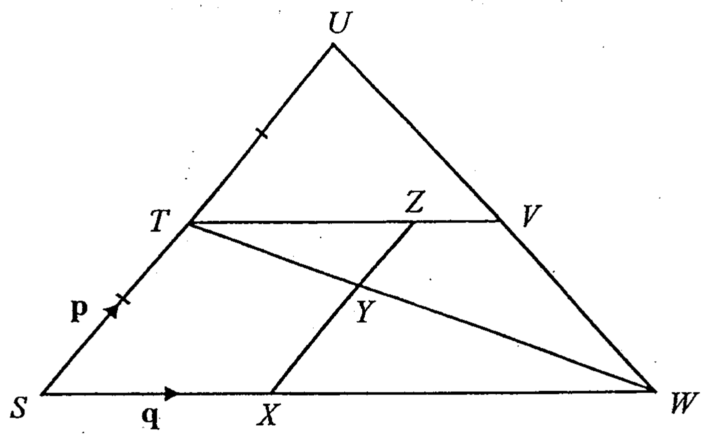
(i) bearing of B from A, [3]

(ii) bearing of A from C, [3]

(iii) area of triangle ABC. [2]

(b) A vertical tower AT stands at A. The angle of depression of B from T is $20°$, find the height of the tower. [1]

(c) D is a point on BC produced such that the angle of elevation of T from D is the greatest. Calculate the angle of elevation of T from D. [3]



$STZX$ is a parallelogram. T is the midpoint of SU, Z is a point on VT such that $VT = 4VZ$ and X is a point on SW such that $3\overrightarrow{SW} = 8\overrightarrow{SX}$. TW intersects XZ at Y. It is given that $\overrightarrow{ST} = \mathbf{p}$ and $\overrightarrow{SX} = \mathbf{q}$.

(a) Express, as simply as possible, in terms of $\mathbf{p}$ and $\mathbf{q}$,

(i) $\overrightarrow{WT}$, [1]

(ii) $\overrightarrow{WV}$, [1]

(iii) $\overrightarrow{UW}$. [1]

(b) Find the value of

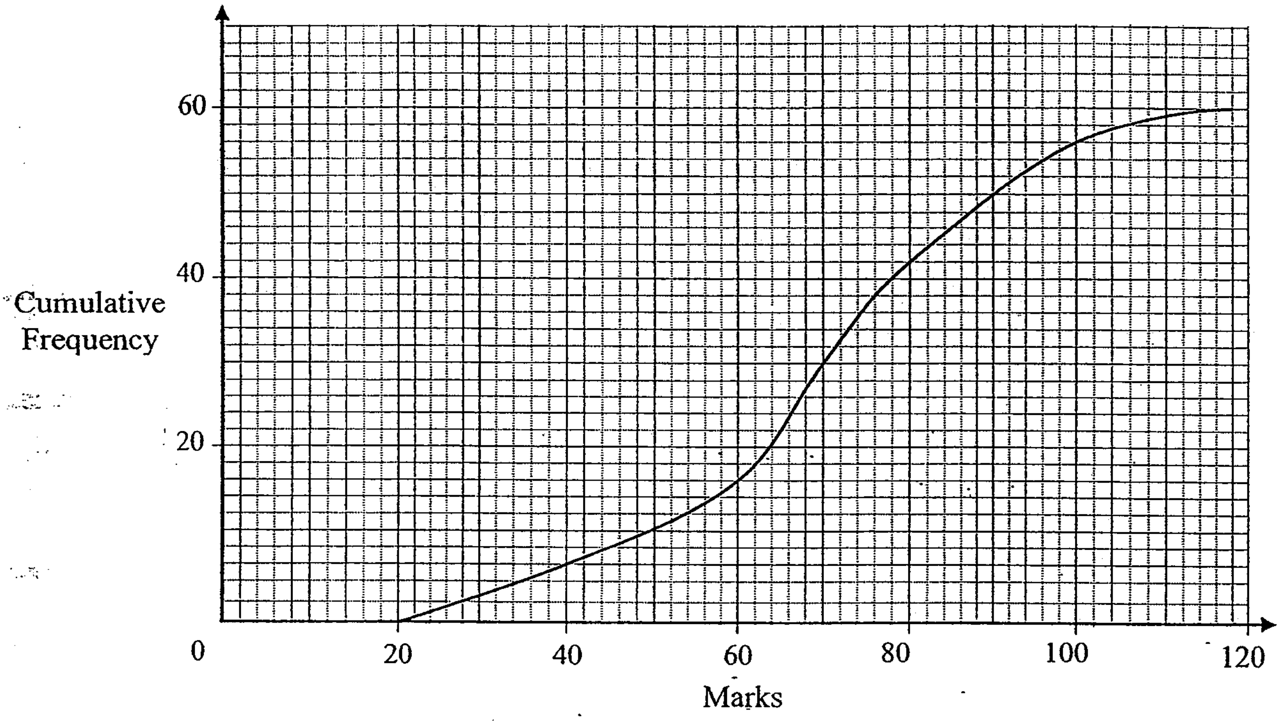
(i) $\dfrac{WY}{WT}$, [1]

(ii) $\dfrac{\text{area of triangle } WYX}{\text{area of triangle } WTS}$, [1]

(iii) $\dfrac{\text{area of triangle } WYX}{\text{area of triangle } WTV}$. [2]

(c) Given that the point U is $(2, 11)$ and $\overrightarrow{UZ} = \begin{pmatrix} 2 \\ -8 \end{pmatrix}$. Find $\left|\overrightarrow{OZ}\right|$, where O is the origin. [2]

9 The cumulative frequency graph below shows the distribution of the marks, out of a full score of 120, obtained by 60 students in Group A during a quiz.



(a) Use the graph to find the

(i) median mark, [1]

(ii) interquartile range of the distribution, [2]

(iii) number of students who obtained a score more than 80% of the marks. [2]

(b) Copy and complete the grouped frequency table of the marks obtained by students in Group A.

Marks	Mid-value (x)	Frequency (f)
$20 < x \leq 40$	30	6
$40 < x \leq 60$		
$60 < x \leq 80$		
$80 < x \leq 100$		
$100 < x \leq 120$		

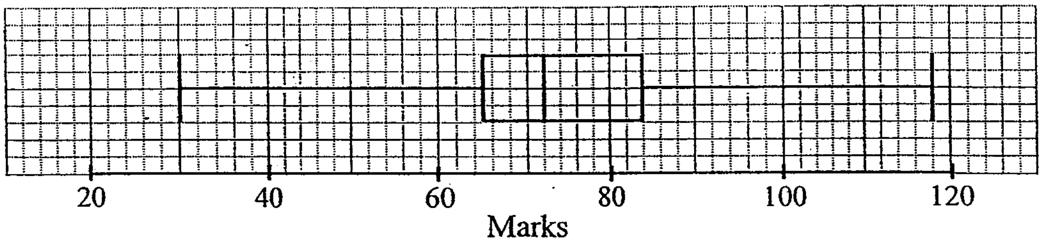
[2]

(c) Using your grouped frequency table, calculate an estimate of the

(i) mean marks obtained by students in Group A, [1]

(ii) standard deviation of the marks. [2]

(d) The marks obtained by 50 students in Group B who took the same quiz is illustrated by the box-and-whisker plot below.



Using your results from **(a)**, compare and comment, in 2 different ways, on the marks obtained by the two groups of students. [2]

---

10 **Answer the whole of this question on a sheet of graph paper.**

The variables x and y are connected by the equation $y = x^3 - 2x^2 + 1$. Some corresponding values of x and y are given in the following table.

x	-1	-0.5	0	0.5	1	1.5	2
y	-2	0.375	1	0.625	0	p	1

(a) Find the value of p. [1]

(b) Using a scale of 2 cm to represent 0.5 units on each axis, draw a horizontal x-axis for $-1 \le x \le 2$ and a vertical y-axis for $-3 \le y \le 2$. On your axes, plot the points given in the table and join them with a smooth curve. [3]

(c) Use your graph to find the solutions of $x^3 - 2x^2 + 1 = 0$. [2]

(d) By drawing a suitable tangent to your curve, find the gradient at the point (1, 0). [2]

(e) **(i)** On the same axes, draw the line $y = 4x + \frac{1}{2}$. [1]

(ii) Write down the x-coordinate of the point where this line intersects the curve. [1]

(iii) This value of x is a solution of the equation $2x^3 - 4x^2 + Ax + B = 0$. Find the value of A and of B. [2]

11 A conical frustum can be created by removing the top of a cone as shown in **Diagram I**. The ratio of the volume of the cone removed to the volume of the original cone is 8 : 27. The original cone has a height of 12 cm and a slant height of 13 cm.

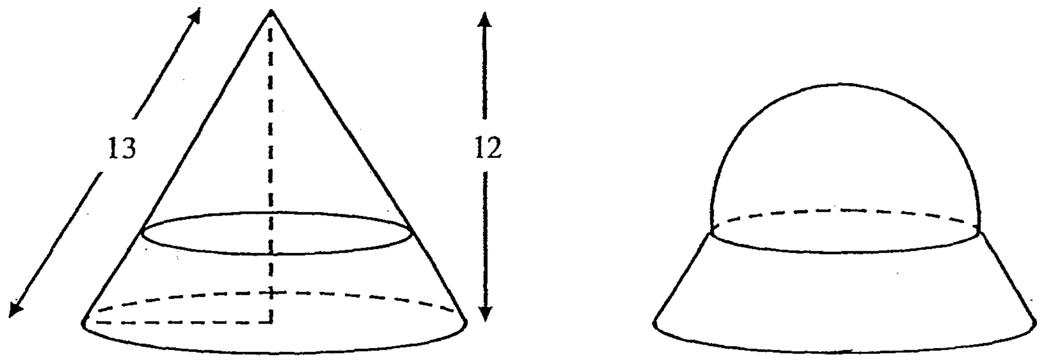


Diagram I **Diagram II**

(a) Without finding the volume of the original cone, show that the radius of the cone removed is $3\frac{1}{3}$ cm. [3]

(b) **Diagram II** shows the model of a UFO, obtained by attaching a hemisphere to the top of the conical frustum in **Diagram I**. The model is to be coated with a layer of paint, except the flat base. Find the total surface area of the model to be coated. [4]

Two machines are used to coat the models. Machine A takes x minutes to coat a model and Machine B takes 30 seconds more than Machine A for each model.

(c) Write down, in terms of x, the rate of production per minute of

(i) Machine A, [1]

(ii) Machine B. [1]

(d) A total of 5 models can be coated by the two machines together in 4 minutes. Write down an equation in x to represent the information and show that it reduces to $10x^2 - 11x - 4 = 0$. [2]

(e) Solve the equation $10x^2 - 11x - 4 = 0$, giving your answers correct to two decimal places. [3]

(f) Find, correct to the nearest whole number, the number of models coated by Machine B in 2 minutes. [1]

End of Paper

Name SOLUTION

Class	Register Number

4016/02 **14/S4PR1/EM/2**

MATHEMATICS **PAPER 2**

Wednesday **7 May 2014** **2 hours 30 minutes**

VICTORIA SCHOOL

PRELIMINARY EXAMINATION ONE
SECONDARY FOUR

Additional Materials: Answer Paper
Graph Paper

READ THESE INSTRUCTIONS FIRST

Write your name, class and register number on all the work you hand in.
Write in dark blue or black pen.
You may use a pencil for any diagrams or graphs.
Do not use paper clips, highlighters, glue or correction fluid.

Answer **all** questions.
If working is needed for any question it must be shown with the answer.
Omission of essential working will result in loss of marks.
You are expected to use a scientific calculator to evaluate explicit numerical expressions.
If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.
For π, use either your calculator value or 3.142, unless the question requires the answer in terms of π.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 100.

This paper consists of 9 printed pages, including the cover page.

[Turn over

674 **Question 1**

[2] **(a)** $18x^2+21xy-60y^2=3\left(6x^2+7xy-20y^2\right)$

$=3(3x-4y)(2x+5y)$ [A2]

[3] **(b)**

$$\frac{2}{(3-4y)^2}+\frac{1}{60y^2-21y-18}$$

$$=\frac{2}{(3-4y)^2}+\frac{1}{3(4y-3)(5y+2)}$$

$$=\frac{2}{(3-4y)^2}-\frac{1}{3(3-4y)(5y+2)} \quad \text{[M1]}$$

$$=\frac{2(3)(5y+2)-(3-4y)}{3(5y+2)(3-4y)^2} \quad \text{[M1]}$$

$$=\frac{30y+12-3+4y}{3(5y+2)(3-4y)^2}$$

$$=\frac{34y+9}{3(5y+2)(3-4y)^2} \quad \text{[A1]}$$

Alternative Solution:

$$\frac{2}{(3-4y)^2}+\frac{1}{60y^2-21y-18}$$

$$=\frac{2}{(3-4y)^2}+\frac{1}{3(4y-3)(5y+2)}$$

$$=\frac{2}{(4y-3)^2}+\frac{1}{3(4y-3)(5y+2)} \quad \text{[M1]}$$

$$=\frac{2(3)(5y+2)+(4y-3)}{3(5y+2)(4y-3)^2} \quad \text{[M1]}$$

$$=\frac{30y+12+4y-3}{3(5y+2)(4y-3)^2}$$

$$=\frac{34y+9}{3(5y+2)(4y-3)^2} \quad \text{[A1]}$$

Question 2

[1] **(a)** $\mathbf{B}=\begin{pmatrix}300 & 140\\ 250 & 180\\ 180 & 150\end{pmatrix}$ [B1]

[1] **(b)** $\mathbf{C}=\begin{pmatrix}220 & 140\\ 350 & 200\\ 200 & 170\end{pmatrix}+\begin{pmatrix}300 & 140\\ 250 & 180\\ 180 & 150\end{pmatrix}$

$=\begin{pmatrix}520 & 280\\ 600 & 380\\ 380 & 320\end{pmatrix}$ [B1]

[1] **(c)** $\mathbf{E}=\begin{pmatrix}520 & 280\\ 600 & 380\\ 380 & 320\end{pmatrix}\begin{pmatrix}100\\ 140\end{pmatrix}$

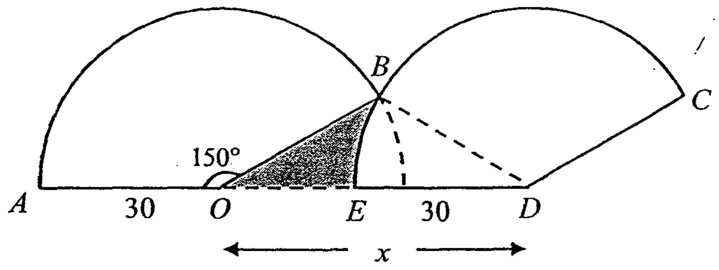
$=\begin{pmatrix}91\,200\\ 113\,200\\ 82\,800\end{pmatrix}$ [A1]

[1] **(d)** The elements represent the total amount of money collect by the two counters from the tickets sales for Friday, Saturday and Sunday respectively. [B1]

[2] **(e)** $\begin{pmatrix}1 & 1 & 1\end{pmatrix}\begin{pmatrix}91\,200\\ 113\,200\\ 82\,800\end{pmatrix}=\begin{pmatrix}287\,200\end{pmatrix}$ [M1]

Total amount of money collected = \$287 200 [A1]

Question 3
[6]



$\angle BOE = 180° - 150°$ (adj. $\angle$s on a straight line)
$= 30°$

$\cos 30° = \dfrac{\frac{1}{2}x}{30}$
$x = 60\cos 30°$
$= 51.96$
$= 52.0$ (3 sf) [A1]

Alternative Solution:
$\angle BDO = \angle BOD$ (base $\angle$s of isosceles Δ)
$= 30°$
$\angle OBD = 180° - 30° - 30°$ ($\angle$ sum of Δ)
$= 120°$
$x^2 = 30^2 + 30^2 - 2(30)(30)\cos 120°$
$x^2 = 2700$
$x = 52.0$

Area of region OBE
= Area of ΔOBD − Area of sector BDE
$= \frac{1}{2}(30)(60\cos 30°)(\sin 30°) - \frac{30°}{360°} \times \pi (30)^2$ [M2]
$= 450\cos 30° - 75\pi$
$= 154\text{ cm}^2$ (3 sf) [A1]

Area of $\Delta OBD = \frac{1}{2}(30)(30)\sin 120°$

Perimeter $= \frac{30°}{360°} \times 2\pi(30) + 30 + (51.96 - 30)$ [M1]
$= 67.7$ cm [A1]

Question 4

[4] (a)

$$p = \frac{mv}{\sqrt{1-\frac{v^2}{c^2}}}$$

$$p^2 = \frac{m^2v^2}{1-\frac{v^2}{c^2}} \quad \text{[M1]}$$

$$p^2 - \frac{p^2v^2}{c^2} = m^2v^2$$

$$c^2p^2 - p^2v^2 = c^2m^2v^2$$

$$c^2m^2v^2 + p^2v^2 = c^2p^2 \quad \text{[M1]}$$

$$v^2\left(c^2m^2 + p^2\right) = c^2p^2 \quad \text{[M1]}$$

$$v^2 = \frac{c^2p^2}{c^2m^2 + p^2}$$

$$v = \pm\frac{cp}{\sqrt{c^2m^2 + p^2}} \quad \text{[A1]}$$

Alternative Solution:

$$p = \frac{mv}{\sqrt{1-\frac{v^2}{c^2}}}$$

$$p^2 = \frac{m^2v^2}{1-\frac{v^2}{c^2}} \quad \text{[M1]}$$

$$p^2 = m^2v^2 \div \frac{c^2 - v^2}{c^2}$$

$$p^2 = m^2v^2 \times \frac{c^2}{c^2 - v^2}$$

$$c^2p^2 - p^2v^2 = c^2m^2v^2$$

$$c^2m^2v^2 + p^2v^2 = c^2p^2 \quad \text{[M1]}$$

$$v^2\left(c^2m^2 + p^2\right) = c^2p^2 \quad \text{[M1]}$$

$$v^2 = \frac{c^2p^2}{c^2m^2 + p^2}$$

$$v = \pm\frac{cp}{\sqrt{c^2m^2 + p^2}} \quad \text{[A1]}$$

[3] (b)

$$\left(\frac{2a^3}{b^{-4}}\right)^{-2} \div \frac{b^{-2}}{\left(4a^{-1}\right)^3 b^7} = \left(\frac{b^{-4}}{2a^3}\right)^2 \times \frac{\left(4a^{-1}\right)^3 b^7}{b^{-2}}$$

$$= \frac{b^{-8}}{4a^6} \times \frac{64a^{-3}b^7}{b^{-2}}$$

$$= \frac{16b}{a^9} \quad \text{[A3]}$$

678

Question 5

(a)

[2] **(i)** $8180-8000=\frac{8000(R)(3)}{100}$ [M1]

$24000R=18000$

$R=0.75$

$\therefore$ Simple interest rate $=0.75\%$ or $\frac{3}{4}\%$ [A1]

[2] **(ii)** Total amount $=8000\left(1+\frac{\frac{1}{2}(0.6)}{100}\right)^{6}$ [M1]

$=\$8145.08$ (2 dp) [A1]

(b)

[2] **(i)** Income tax $=550+\frac{7}{100}(63\,000-40\,000)$ [M1]

$=550+1610$

$=\$2160$ [A1]

[2] **(ii)** Chargeable income $=30\,000+\frac{100}{3.5}(515-200)$ [M1]

$=30\,000+9000$

$=\$39\,000$ [A1]

<u>Alternative Solution:</u>

Let her chargeable income be $\$x$

$200+(x-30000)\times\frac{3.5}{100}=515$

$x-30000=315\times\frac{100}{3.5}$

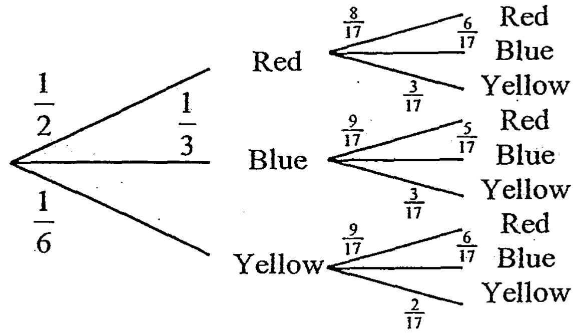
$x-30000=9000$

$x=39000$

Question 6

[2] **(a)**

[B1] [B1]



(b)

[1] **(i)** P(cards drawn are of the same colour) $=\left(\frac{1}{2}\times\frac{8}{17}\right)+\left(\frac{1}{3}\times\frac{5}{17}\right)+\left(\frac{1}{6}\times\frac{2}{17}\right)$

$=\frac{6}{17}$ [A1]

[1] **(ii)** P(cards drawn are of different colours) $=1-\frac{6}{17}$

$=\frac{11}{17}$ [A1]

(c)

[1] **(i)** P(exactly 2 blue cards are drawn) $=3\left(\frac{1}{3}\times\frac{1}{3}\times\frac{2}{3}\right)$

$=\frac{2}{9}$ [A1]

[1] **(ii)** P(red card is drawn each time) $=\left(\frac{1}{2}\right)^8$

$=\frac{1}{256}$ [A1]

[2] **(d)**

Let x be the number of cards to be added.

$\frac{3+x}{18+x}=\frac{1}{4}$ [M1]

$12+4x=18+x$

$x=2$ [A1]

Alternative Solution:

New number of yellow cards

$=\frac{9+6}{3}$

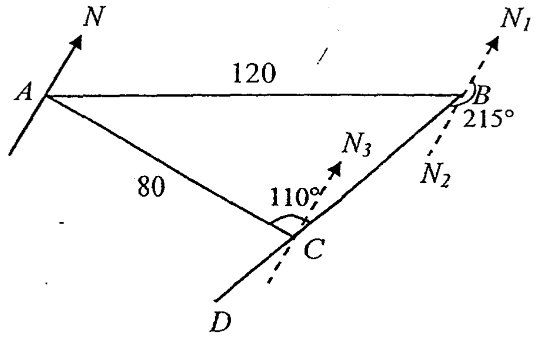
$=5$

Number of yellow cards to be added

$=5-3$

$=2$

Question 7
(a)



[3] **(i)** $\frac{\sin A\hat{B}C}{80} = \frac{\sin 110°}{120}$ [M1]

$A\hat{B}C = \sin^{-1}\left(\frac{80\sin 110°}{120}\right)$

$= 38.79°$

$A\hat{B}N_1 = 360° - 215° - 38.79°$ ($\angle$s at a point) [M1]

$= 106.21°$

Bearing of B from $A = 180° - 106.21°$ (int $\angle$s, $AN // BN_1$)

$= 073.79°$

$= 073.8°$ (1 dp) [A1]

<u>Alternative Solution:</u>

$\frac{\sin A\hat{B}C}{80} = \frac{\sin 110°}{120}$ [M1]

$A\hat{B}C = \sin^{-1}\left(\frac{80\sin 110°}{120}\right)$

$= 38.79°$

$N_2\hat{B}C = 215° - 180°$

$= 35°$

$A\hat{B}N_2 = 35° + 38.79°$ [M1]

$= 73.79°$

Bearing of B from $A = A\hat{B}N_2$ (alt. $\angle$s, $AN // BN_2$)

$= 073.79°$

$= 073.8°$ (1 dp) [A1]

[3] **(ii)** $N_2\hat{B}C = 215° - 180°$

$= 35°$

$N_3\hat{C}B = N_2\hat{B}C$ (alt. $\angle$s, $N_3C // BN_2$) [M1]

$= 35°$

Bearing of A from $C = 360° - 110° + 35°$ ($\angle$s at a point) [M1]

$= 285°$ [M1]

[2] **(iii)** $B\hat{A}C = 180° - 110° - 38.79°$ ($\angle$ sum of Δ) [M1]

$= 31.21°$

Area of $\Delta ABC = \frac{1}{2}(80)(120)\sin 31.21°$

$= 2487.2$

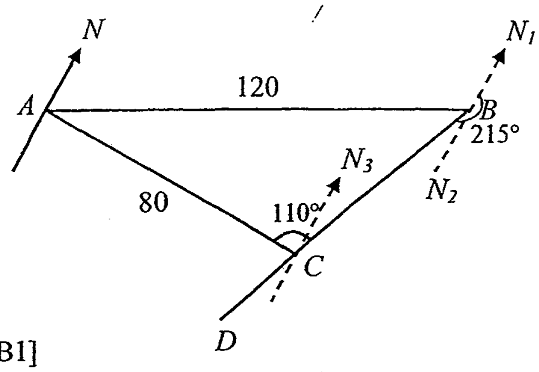
$= 2490 \text{ m}^2$ (3 sf) [A1]

[1] **(b)** $\tan 20° = \dfrac{AT}{120}$

Height, $AT = 120 \tan 20°$

$= 43.68$

$= 43.7$ m [A1]



[3] **(c)** $A\hat{C}D = 180° - 110°$ ($\angle$ sum of Δ)

$= 70°$

$\sin 70° = \dfrac{AD}{80}$

$AD = 80 \sin 70°$

$AD = 75.18$ cm [B1]

$\tan\theta = \dfrac{120 \tan 20°}{80 \sin 70°}$ [M1]

$\theta = 30.16°$ [A1]

Greatest angle of elevation $= 30.2°$

<u>Alternative Solution:</u>

$BC^2 = 80^2 + 120^2 - 2(80)(120)\cos 31.21°$

$BC^2 = 20800 - 19200 \cos 31.21°$

$BC^2 = 4378.74$

$BC = 66.17$ m

$\frac{1}{2}(66.17)(AD) = 2487.2$

$AD = 75.18$ cm [M1]

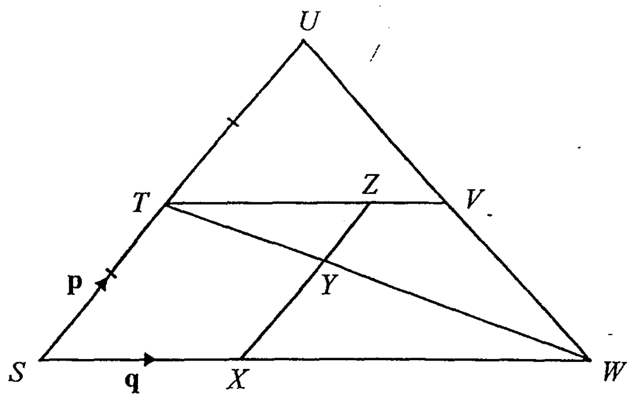
Question 8

(a)

[1] **(i)** $\overrightarrow{WT}=\overrightarrow{WS}+\overrightarrow{ST}$

$=-\frac{8}{3}\mathbf{q}+\mathbf{p}$

$=\frac{1}{3}(3\mathbf{p}-8\mathbf{q})$ [A1]



[1] **(ii)** $\overrightarrow{WV}=\overrightarrow{WT}+\overrightarrow{TV}$

$=-\frac{8}{3}\mathbf{q}+\mathbf{p}+\frac{4}{3}\mathbf{q}$

$=-\frac{4}{3}\mathbf{q}+\mathbf{p}$

$=\frac{1}{3}(3\mathbf{p}-4\mathbf{q})$ [A1]

[1] **(iii)** $\overrightarrow{UW}=\overrightarrow{US}+\overrightarrow{SW}$

$=-2\mathbf{p}+\frac{8}{3}\mathbf{q}$

$=\frac{2}{3}(4\mathbf{q}-3\mathbf{p})$ [A1]

(b)

[1] **(i)** $\frac{WY}{WT}=\frac{WS}{WX}=\frac{5}{8}$ [B1]

[1] **(ii)** ΔWYX and ΔWTS are similar,

$\frac{\text{Area of triangle } WYX}{\text{Area of triangle } WTS}=\left(\frac{5}{8}\right)^2=\frac{25}{64}$ [A1]

(b)

[2] **(iii)** $\frac{\text{Area of } \Delta WTS}{\text{Area of } \Delta WTV}=\frac{\frac{1}{2}\times 8\times h}{\frac{1}{2}\times 4\times h}$, where h is common height [M1]

$=\frac{64}{32}$

Area of ΔWYX : Area of ΔWTS : Area of ΔWTV

$=$ 25 : 64 : 32

$\therefore \frac{\text{Area of } \Delta WYX}{\text{Area of } \Delta WTV}=\frac{25}{32}$ [A1]

<u>Alternative Solution:</u>

$\frac{\text{Area of } \Delta WTS}{\text{Area of } \Delta WTV}=\frac{\frac{1}{2}\times 8\times h}{\frac{1}{2}\times 4\times h}$, where h is common height

$=2$

$\frac{\text{Area of } \Delta WYX}{\text{Area of } \Delta WTV}=\frac{\text{Area of } \Delta WYX}{\text{Area of } \Delta WTS}\times\frac{\text{Area of } \Delta WTS}{\text{Area of } \Delta WTV}$

$=\frac{25}{64}\times 2$

$=\frac{25}{32}$

[2] **(c)** $\overrightarrow{UZ}=\overrightarrow{OZ}-\overrightarrow{OU}$

$\begin{pmatrix}2\\-8\end{pmatrix}=\overrightarrow{OZ}-\begin{pmatrix}2\\11\end{pmatrix}$ [M1]

$\overrightarrow{OZ}=\begin{pmatrix}4\\3\end{pmatrix}$

$|\overrightarrow{OZ}|=\sqrt{4^2+3^2}$

$=5$ units [A1]

Question 9
(a)

[1] **(i)** Median mark = 70 marks [B1]

[2] **(ii)** Interquartile range = 84 – 59 [M1] or = 83 – 59
= 25 marks [A1] = 24 marks

[2] **(iii)** 80% of 120 marks = 96 marks [M1]
Number of students = 60 – 54
= 6 [A1]

[2] **(b)**

Marks	Mid-value (x) [B1]	Frequency (f) [B1]
$20 < x \leq 40$	30	6
$40 < x \leq 60$	**50**	**10**
$60 < x \leq 80$	**70**	**26**
$80 < x \leq 100$	**90**	**14**
$100 < x \leq 120$	**110**	**4**

(c)

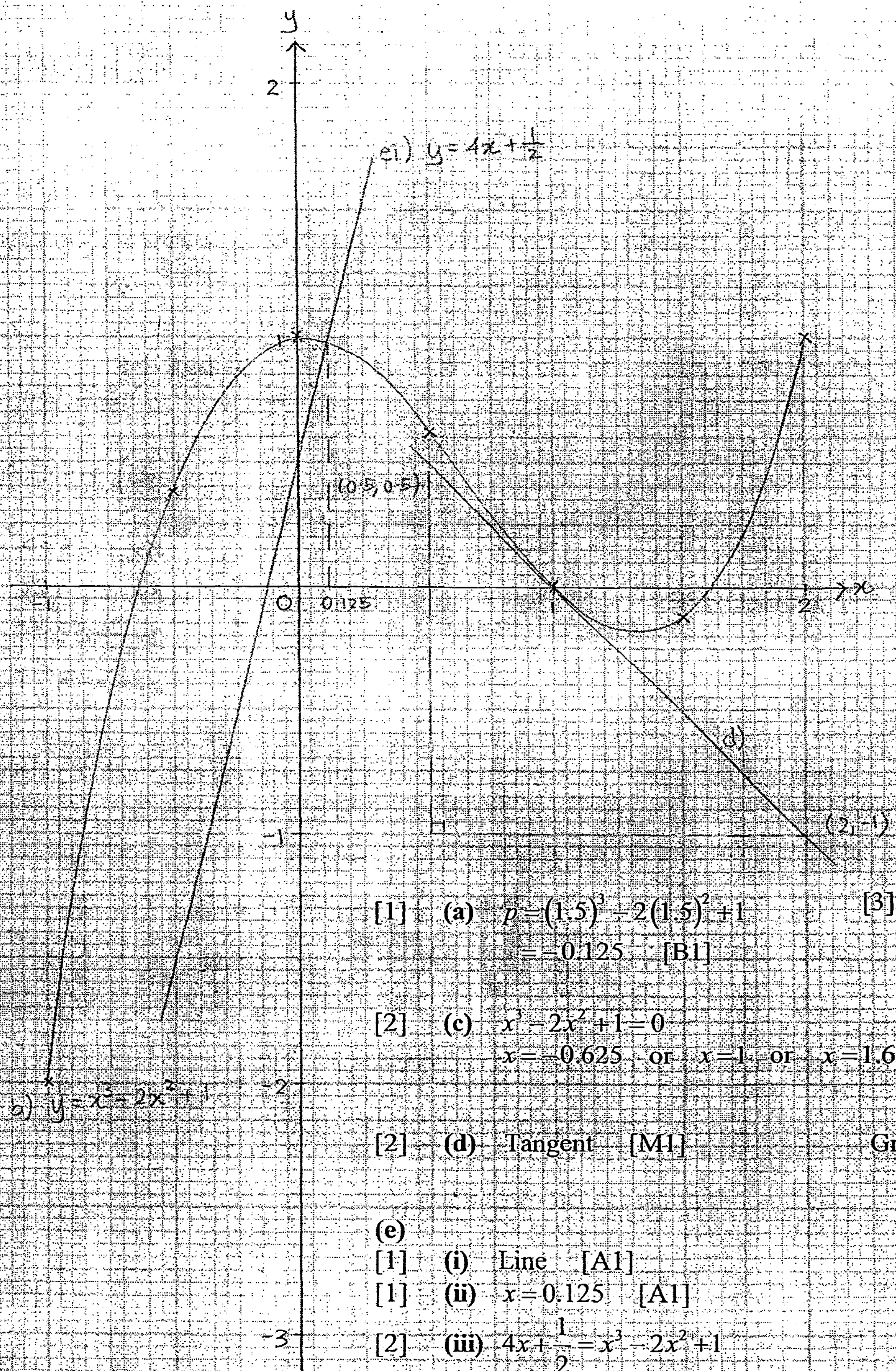
[1] **(i)** Mean $= \dfrac{4200}{60}$
$= 70$ marks [A1]

[2] **(ii)** Standard deviation $= \sqrt{\dfrac{319600}{60} - \left(\dfrac{4200}{60}\right)^2}$ [M1]
$= 20.7$ marks (3 sf) [A1]

[2] **(d)** Median of Group B = 72 marks
Interquartile range of Group B = 84 – 65 = 19 marks

Group B has a higher median than Group A, students in Group B performed better. [A1]
Group B has a smaller interquartile range than Group A, the spread of marks in Group B is smaller. [A1]

Question 10



[1] **(a)** $p = (1.5)^3 - 2(1.5)^2 + 1$
$= -0.125$ [B1]

[3] **(b)** Points [M1]
Curve [M1]
Label [M1]

[2] **(c)** $x^3 - 2x^2 + 1 = 0$
$x = -0.625$ or $x = 1$ or $x = 1.625$ [A2]

[2] **(d)** Tangent [M1]

Gradient $= \dfrac{0.5 - (-1)}{0.5 - 2}$ [M1]
$= -1$

(e)

[1] **(i)** Line [A1]

[1] **(ii)** $x = 0.125$ [A1]

[2] **(iii)** $4x + \dfrac{1}{2} = x^3 - 2x^2 + 1$

$8x + 1 = 2x^3 - 4x^2 + 2$

$2x^3 - 4x^2 - 8x + 1 = 0$

$A = -8$ or $B = 1$ [A2]

Question 11

[3] **(a)** $\dfrac{\text{Radius of smaller cone}}{\text{Radius of original cone}}=\sqrt[3]{\dfrac{8}{27}}$ [B1]

$=\dfrac{2}{3}$

Radius of original cone $=\sqrt{13^2-12^2}$ [M1]

$=5$ cm

Radius of cone removed $=\dfrac{2}{3}\times 5$

$=3\dfrac{1}{3}$ cm (shown) [A1]

[4] **(b)** Slant height of smaller cone $=\dfrac{2}{3}\times 13$ [M1]

$=8\dfrac{2}{3}$ cm

Total surface area $=\dfrac{1}{2}\times 4\pi\left(3\dfrac{1}{3}\right)^2+\pi(5)(13)-\pi\left(3\dfrac{1}{3}\right)\left(8\dfrac{2}{3}\right)$ [M2]

$=\dfrac{200}{9}\pi+65\pi-\dfrac{260}{9}\pi$

$=\dfrac{175}{3}\pi$

$=183\text{ cm}^2$ (3 sf) [A1]

(c)

[1] **(i)** Rate of Machine $A=\dfrac{1}{x}$ [B1]

[1] **(ii)** Rate of Machine $B=\dfrac{2}{2x+1}$ [B1]

or $\dfrac{1}{x+0.5}$ or $\dfrac{1}{x+\frac{1}{2}}$

[2] **(d)** $4\left(\dfrac{1}{x}+\dfrac{2}{2x+1}\right)=5$ [M1]

$\dfrac{2x+1+2x}{2x^2+x}=\dfrac{5}{4}$

$8x+4+8x=10x^2+5x$

$10x^2-11x-4=0$ [A1]

[3] **(e)** $10x^2-11x-4=0$

$x=\dfrac{-(-11)\pm\sqrt{(-11)^2-4(10)(-4)}}{2(10)}$ [B1]

$=\dfrac{11\pm\sqrt{281}}{20}$

$x=1.388$ or $x=-0.288$

$=1.39$ (2 dp) $=-0.29$ (2 dp)

[A1] [A1]

[1] **(f)** Number of models $=2\left(\dfrac{2}{2(1.388)+1}\right)$

$=1.059$

$=1$ (nearest whole number) [A1]

End of Paper

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1a	$3(3x-4y)(2x+5y)$
b	$\dfrac{34y+9}{3(5y+2)(3-4y)^2}$ or $\dfrac{34y+9}{3(5y+2)(4y-3)^2}$
2a	$\begin{pmatrix} 300 & 140 \\ 250 & 180 \\ 180 & 150 \end{pmatrix}$
b	$\begin{pmatrix} 520 & 280 \\ 600 & 380 \\ 380 & 320 \end{pmatrix}$
c	$\begin{pmatrix} 91\,200 \\ 113\,200 \\ 82\,800 \end{pmatrix}$
d	The elements represent the total amount of money collect by the two counters from the tickets sales for Friday, Saturday and Sunday respectively.
e	$\begin{pmatrix} 1 & 1 & 1 \end{pmatrix}\begin{pmatrix} 91\,200 \\ 113\,200 \\ 82\,800 \end{pmatrix}$, \$287 200
3	$x=52.0$, 154 cm^2 , 67.7 cm
4a	$v=\pm\dfrac{cp}{\sqrt{c^2m^2+p^2}}$
b	$\dfrac{16b}{a^9}$
5ai	0.75%
aii	\$8145.08
bi	\$2160
bii	\$39 000
6bi	$\dfrac{6}{17}$; **bii** $\dfrac{11}{17}$
ci	$\dfrac{2}{9}$; **cii** $\dfrac{1}{256}$
d	2
7ai	073.8°
aii	285°
aiii	2490 m^2

b	43.7 m
c	30.2°
8ai	$\dfrac{1}{3}(3\mathbf{p}-8\mathbf{q})$
aii	$\dfrac{1}{3}(3\mathbf{p}-4\mathbf{q})$
aiii	$\dfrac{1}{3}(8\mathbf{q}-6\mathbf{p})$
bi	$\dfrac{5}{8}$
bii	$\dfrac{25}{64}$
biii	$\dfrac{25}{32}$
c	$\begin{pmatrix} 4 \\ 3 \end{pmatrix}$, 5 units
9ai	70 marks
aii	25 marks
aiii	6
B	

50	10
70	26
90	14
110	4

ci	70 marks
ii	20.7 marks
d	B: Higher median, perform better B: Smaller interquartile range, smaller spread
10a	-0.125
c	$x=-0.625$ or 1 or 1.625
d	-1
eii	$x=0.125$
iii	$A=-8$ or $B=1$
11a	$\sqrt[3]{\dfrac{8}{27}}\times\sqrt{13^2-12^2}=3\dfrac{1}{3}$
b	183 cm^2
ci	$\dfrac{1}{x}$
cii	$\dfrac{2}{2x+1}$
d	$4\left(\dfrac{1}{x}+\dfrac{2}{2x+1}\right)=5$
e	$x=1.39$ or $x=-0.29$
f	1