ANDERSON SECONDARY SCHOOL
Preliminary Examination 2015
Secondary Four Express & Five Normal

CANDIDATE NAME: _____

CLASS: _____ / _____ INDEX NUMBER: _____

MATHEMATICS **4016/01**

Paper 1

24 August 2015

2 hours

0800 – 1000h

Additional Materials: Writing paper

READ THESE INSTRUCTIONS FIRST

Write your name, centre number and index number in the spaces at the top of this page and on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use a pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer all the questions.
If working is needed for any question it must be **neatly and clearly** shown in the space below the question.
Omission of essential working will result in loss of marks.
The use of an approved scientific calculator is expected, where appropriate.
If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.
For π, use either your calculator value or 3.142, unless the question requires the answer in terms of π.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total of the marks for this paper is 80.

F

This document consists of **17** printed pages. 23

Setter: Mdm Wong Lai Fong

Mathematical Formulae

Compound Interest

$$\text{Total amount} = P\left(1 + \frac{r}{100}\right)^n$$

Mensuration

Curved surface area of a cone = πrl

Surface area of a sphere = $4\pi r^2$

$$\text{Volume of a cone} = \frac{1}{3}\pi r^2 h$$

$$\text{Volume of a sphere} = \frac{4}{3}\pi r^3$$

$$\text{Area of triangle } ABC = \frac{1}{2}ab\sin C$$

Arc length = $r\theta$, where θ is in radians

$$\text{Sector area} = \frac{1}{2}r^2\theta, \text{ where } \theta \text{ is in radians}$$

Trigonometry

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

Statistics

$$\text{Mean} = \frac{\Sigma fx}{\Sigma f}$$

$$\text{Standard deviation} = \sqrt{\frac{\Sigma fx^2}{\Sigma f} - \left(\frac{\Sigma fx}{\Sigma f}\right)^2}$$

For Examiner's Use

Answer **ALL** questions.

For Examiner's Use

1 Consider the following numbers:

$$27^{\frac{1}{3}},\ \pi,\ \frac{22}{7},\ 3.1\dot{},\ 3.33,\ \sqrt{10}$$

(a) Write the above numbers in order of size, smaller first.

(b) State which of the above numbers are irrational.

Answer (a) _____ [1]

(b) _____ [1]

2 (a) Evaluate $\sqrt{\frac{1-\sin 12.3}{e^5}}$, giving your answer correct to 3 significant figures.

(b) At room temperature, an oxygen atom in the air travels 500 metres per second. Find the time taken, in minutes, for an oxygen atom to travel 750 megametres in room temperature. Give your answer in standard form.

Answer (a) _____ [1]

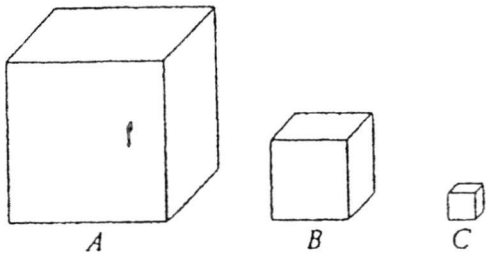
(b) _____ min [1]

For Examiner's Use

3 The weekly sales of three brands of bread A, B and C are shown in the following table:

Bread	A	B	C
Sales (in number of loaves)	600	300	100

Brady draws three *geometrically similar* cubes, labelled A, B and C (as shown below) to represent the above information on the weekly sales of the three brands A, B and C respectively. Explain why this representation is misleading.



Answer _____

_____

_____ [1]

4 Given a sequence

$$2, \frac{3}{2}, \frac{5}{4}, \frac{9}{8}, \frac{17}{16}, \ldots$$

(a) Write down the next term in the sequence.

(b) One of the terms in the sequence is $\frac{x}{y}$.

Write down, in terms of y only, the term that is immediately before $\frac{x}{y}$.

(c) Without finding further terms, explain if $\frac{201}{200}$ is a term in the sequence.

Answer (a) _____ [1]

(b) _____ [1]

(c) _____ [1]

For Examiner's Use

5 (a) Solve $(x-2)^6 = \pi$, leaving your answer(s) correct to 4 significant figures.

(b) Given that $3^{\sqrt{m}} = 4 - e^0$, find the value(s) of m.

Answer (a) $x =$ _____ [2]

(b) $m =$ _____ [2]

6 (a) A piece of string can be cut into exact number of 15-cm pieces.
Find the least number of 12-cm pieces that the original piece of string could be cut into.

(b) Solve the inequalities $\frac{x}{2} + 1 \le 2x - 5 < 5(x+1) - 22$.

25

Answer (a) _____ [1]

(b) _____ [3]

For Examiner's Use

7 (a) A polygon has n sides. Two of its exterior angles are 70° and 80°, and the remaining exterior angles are each 14°. Find the value of n.

(b) A contractor is looking at two different type of tiles, P and H.
Each tile of type P is in the shape of a regular pentagon while that of type H is a regular hexagon.
If the contractor needs to fit the same type of tiles together on a floor without any gaps, explain which type of tiles he should use.

Answer (a) $n =$ _____ [2]

(b) _____

_____

_____

_____

_____ [2]

8 An area of 12.5 m^2 is represented by 8 cm^2 on a map.

(a) Calculate, in m^2, the actual **exact** area of a field which is represented by a circle of area 16π cm^2.

(b) Find the scale of the map in the form $1 : n$

Answer (a) _____ m^2 [1]

(b) _____ [2]

For Examiner's Use

9 (a) Given that $t = 1 \pm \sqrt{\frac{x}{y^3}}$, make x the subject.

(b) Express $\frac{2}{4m^2 - 1} - \frac{3}{1+2m}$ as a single fraction in its simplest form.

Answer (a) _____ [2]

(b) _____ [2]

r's

10 A survey was carried out with a group of drivers to find out the average number of cups of coffee they consume daily. The survey findings are as shown in the table.

For Examiner's Use

Number of cups	0	1	2	3	4
Number of drivers	2	8	7	3	1

(a) Find
(i) the mode,
(ii) the median.

(b) A driver who consumes x cups of coffee daily is later excluded from the survey. If the mode and the median remain unchanged, find the possible value(s) of x.

Answer (a) (i) _____ [1]

(ii) _____ [1]

(b) _____ [2]

11 (a) If Nathan sells his branded watch for \$7500, he will lose 30%.
Find the amount he must sell his watch in order for him to make a profit of 40%.

(b) Nathan borrowed \$12 000 from a money-lender who charged a compound interest of R% per month.
After $2\frac{1}{3}$ years, Nathan found that he owed a total of \$15 000.
Calculate the value of R.

Answer (a) \$ _____ [2]

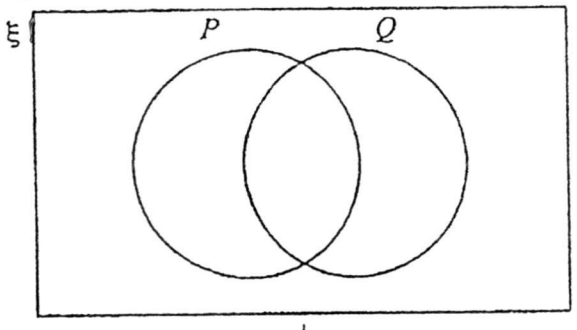
(b) $R =$ _____ [2]

For Examiner's Use

12 (a) Given that ξ = {all points on a plane},
L = {all points on a given straight line},
and C = {all points on the circumference of a given circle}.
Write down the possible value(s) of $n(L \cap C)$.

(b) Given that $A = \{(x, y): y = \sin x\}$ and $B = \{(x, y): y = \cos(k - x)\}$.
Write down a possible value of k, other than $k = 0$, such that $A = B$.

(c) The Venn diagram shows a universal set ξ and the two sets P and Q.



In this Venn diagram, add a set T which is such that
$P \cap T = \varnothing$ and $Q \cap T = T$.

Answer (a) _____ [1]

(b) _____ [1]

(c) *In the diagram above.* [2]

13 Solve the equations

(a) $5x = 10x^2$

(b) $(y^2 + 1)(y^2 - 1) = 0$

Answer (a) $x =$ _____ [2]

(b) $y =$ _____ [2]

For Examiner's Use

14 Given that $\mathbf{P} = \begin{pmatrix} 3 & 1 & -2 \end{pmatrix}$ and $\mathbf{L} = \begin{pmatrix} 2 \\ 4 \\ 3 \end{pmatrix}$.

(a) Calculate **LP**.

(b) M is a $a \times b$ matrix which has elements that are either 1 or 0 such that

$$\mathbf{ML} = \begin{pmatrix} 3 \\ 4 \\ 2 \end{pmatrix}.$$

(i) State the value of a and of b.

(ii) Find M.

Answer (a) _____ [1]

(b) (i) $a =$ _____ ; $b =$ _____ [2]

(ii) _____ [1]

For Examiner's Use

15 The point (0, 1) is marked on each axes in the answer space.
On these axes, sketch the graphs of

(a) $2(x + y) = 1$

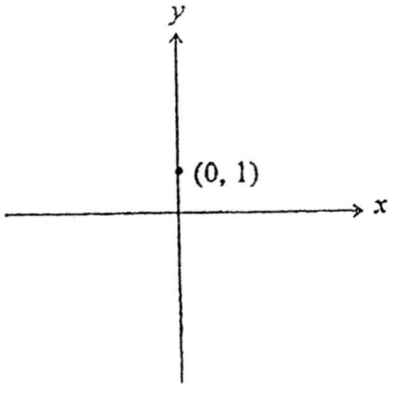
(b) $y = \frac{1}{x} + 1$

(c) $y = 2^{-x}$

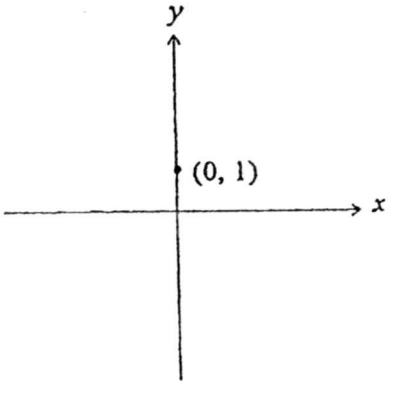
[3]

Answer

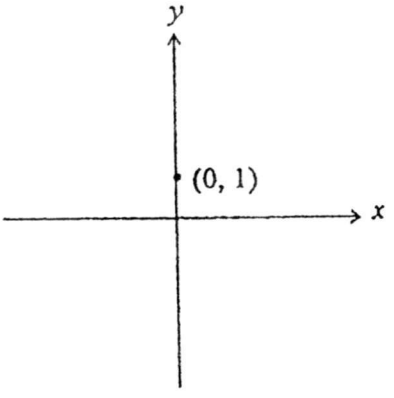
(a)



(b)

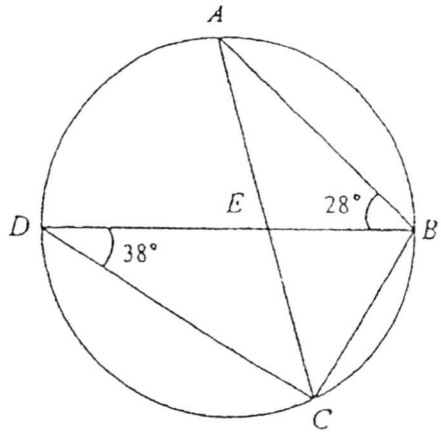


(c)



For Examiner's Use

16 Given A, B, C and D are points on the circumference of the circle such that $\angle BDC = 38°$, $\angle ABD = 28°$ and $\angle BCD = 90°$.
Chords AC and BD intersect at E.
Using appropriate geometrical reasons,
(a) find $\angle ACD$,
(b) explain where the centre of the circle is,
(c) determine, without further calculation, if EC is longer than ED.



Answer (a) $\angle ACD =$ _____ (Reason: _____) [1]

(b) _____
_____
_____ [1]

(c) _____
_____
_____ [1]

17 A mark, M on a cliff is p m below the sea level at high tide.
At low tide, M is above sea level and the sea level is q m lower than at high tide.
(a) Given that p and q are positive integers, find, in terms of p and q, the distance of M above sea level at low tide.
(b) If $p = 1.3$ and $q = 2.8$, calculate how far M is above or below sea level when the sea level is exactly halfway between high tide and low tide.

Answer (a) _____ m [1]

(b) _____ [1]

For Examiner's Use

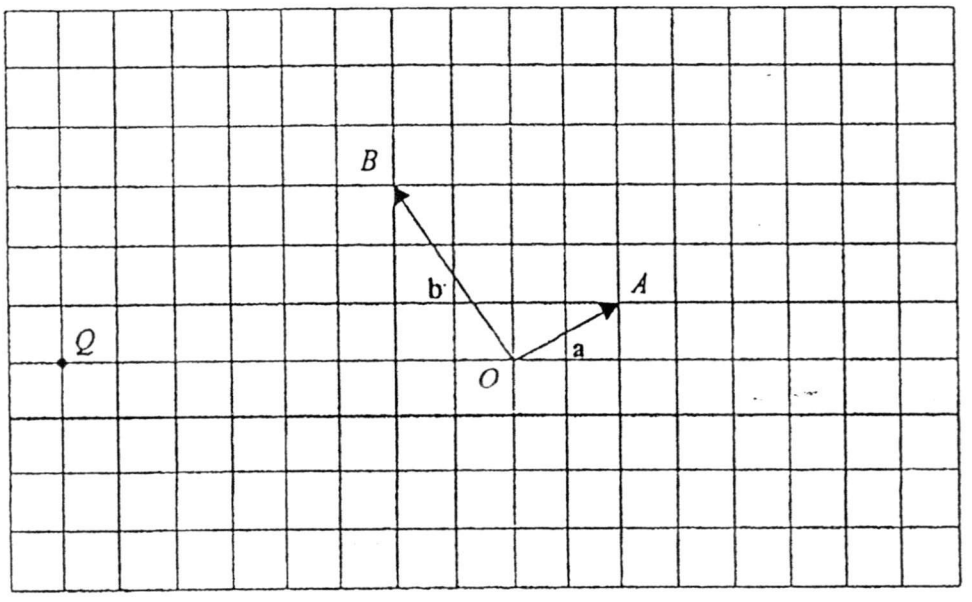
18 In the diagram, $\overrightarrow{OA} = \mathbf{a}$ and $\overrightarrow{OB} = \mathbf{b}$.

(a) Mark and label clearly in the diagram, the point P such that $\overrightarrow{OP} = 2\mathbf{a} - \mathbf{b}$.

(b) Determine the value of λ such that $\overrightarrow{OQ} = \lambda\mathbf{a} + \mathbf{b}$.

(c) If $\mathbf{a} = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$, find $\left|\overrightarrow{AQ}\right|$.

Answer (a)



Answer (a) *In the diagram above.* [1]

(b) $\lambda =$ _____ [1]

(c) _____ [1]

For Examiner's Use

19 (a) A man is deciding which of the following options will give him more water flowing into his flowerbed:

Option A	Option B
A hose with a diameter of 8 cm	Two hoses with diameter of 5 cm each.

The man chooses option B and his reasoning is that the two hoses have a bigger combined diameter of $5+5=10>8$. Is he right? Explain.

(b) (i) Sketch the graph of $y=9-(2-x)^2$ on the axes provided, indicating clearly its turning point and the intercepts on the axes.

(ii) The graph of $y=9-(2-x)^2$ is reflected in the y-axis.
Write down the coordinates of the turning point of the new graph.

For Examiner's Use

Answer (a) _____

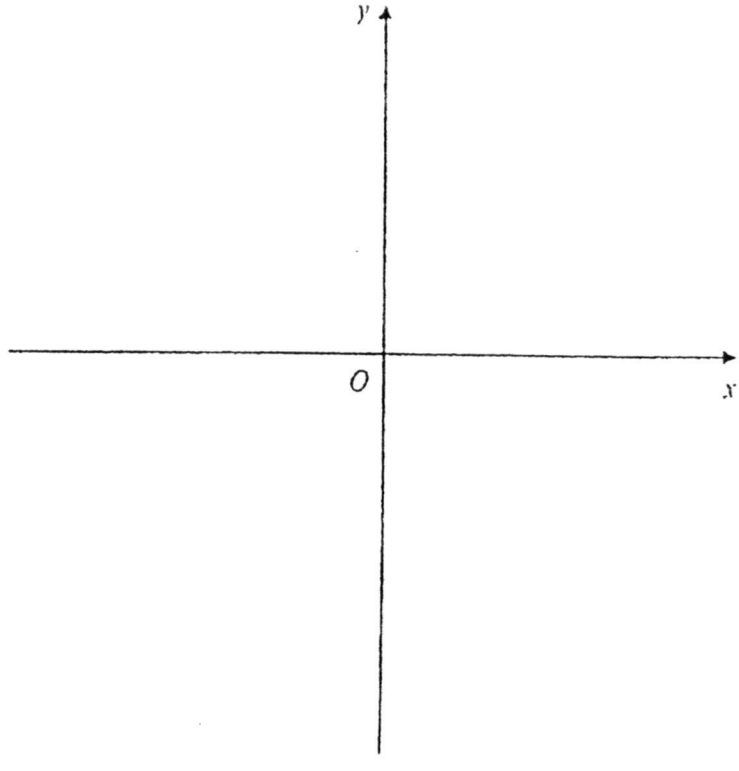
_____

_____

_____

_____ [2]

(b) (i)



[2]

(ii) _____ [1]

For Examiner's Use

20 (a) A researcher claims that the weight loss, W, of a mice in an experiment is directly proportional to the square of the dosage, D, of a herb administered to this mice. If the weight loss is to be multiplied by 10, find the factor the dosage should be multiplied.

(b) It is thought that x is inversely proportional to y^n.
The following table shows some values of x and the corresponding values of y.

x	0.5	2
y	1	0.5

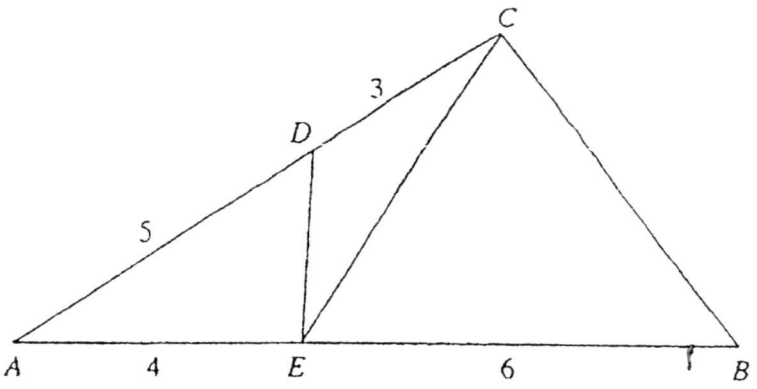
Form an equation relating x and y.

Answer (a) _____ [2]

(b) _____ [2]

For Examiner's Use

21 In triangle ABC, $AE = 4$ cm, $BE = 6$ cm, $AD = 5$ cm and $CD = 3$ cm.



(a) Name a pair of similar triangles and show that they are similar.

(b) Given that area of ΔCDE is x cm^2, find the area of ΔBCE in terms of x.

(c) A point is selected at random from inside ΔABC.
Find the probability that the point lies inside ΔADE.

Answer (a) _____

_____

_____ [3]

(b) _____ cm^2 [3]

(c) _____ [1]

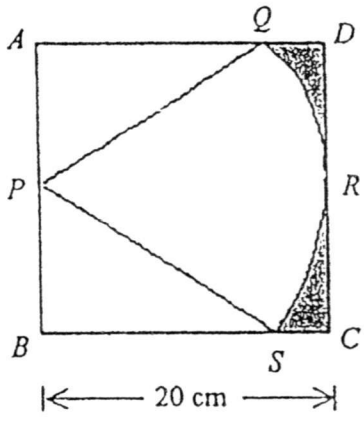
For Examiner's Use

22 In the figure, P is a mid-point of AB. Sector $PQRS$, with centre P, is inscribed in the square $ABCD$ of side 20 cm.

(a) State the exact value of $\cos \angle QPB$.

(b) Find $\angle QPS$ in radians.

(c) Express the shaded region as a percentage of the area of square $ABCD$.



For Examiner's Use

Answer (a) _____ [1]

(b) _____ [2]

(c) _____ [3]

ANDERSON SECONDARY SCHOOL
2015 Preliminary Examination
Secondary Four Express / Five Normal
MATHEMATICS PAPER 1 (4016/01)

Answer Key

1 (a) $27^{\frac{1}{3}}$, 3.1, π, $\frac{22}{7}$, $\sqrt{10}$, 3.33

(b) π, $\sqrt{10}$

2 (a) 0.0923

(b) 2.5×10^{4}

3 Sales represented by the heights of the similar cubes or by the volumes

4 (a) $\frac{33}{32}$

(b) $\frac{y+2}{y}$

(c) denominator is not 2^n

5 (a) 3.210 or 0.7898

(b) 1

6 (a) 5

(b) $x>4$

7 (a) 17

(b) Interior $\angle$ of tile $P = 108°$
Interior $\angle$ of tile $H = 120°$
Since 360 is divisible by 120 but not by 108, use type H that fit together without gaps.

8 (a) 25π

(b) 1 : 125

9 (a) $x = y^3(t-1)^2$

(b) $\frac{5-6m}{(2m+1)(2m-1)}$

10 (a) (i) 1

(ii) 2

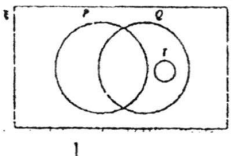
(b) 0

11 (a) 15 000

(b) 0.800

12 (a) 0, 1 or 2

(b) Any odd multiple of $\frac{\pi}{2}$

(c) 

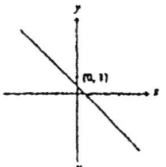
13 (a) 0 or $\frac{1}{2}$

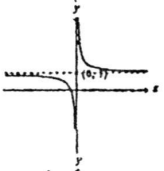
(b) ±1

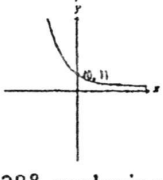
14 (a) $\begin{pmatrix} 6 & 2 & -4 \\ 12 & 4 & -8 \\ 9 & 3 & -6 \end{pmatrix}$

(b) (i) $a=3$, $b=3$

(ii) $\begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix}$

15 (a) 

(b) 

(c) 

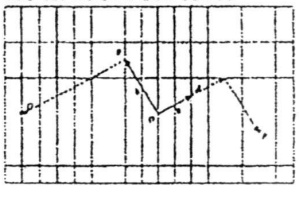
16 (a) 28°, angles in same segment

(b) Since $\angle BCD = 90°$ (rt. $\angle$ in semicircle), DB is a diameter. Centre is midpoint of DB.

(c) Since $\angle EDC > \angle ECD = 28°$, EC is longer than ED.

17 (a) $q-p$

(b) 0.1 m above sea level

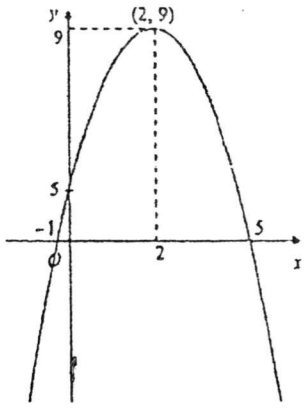
18 (a) 

(b) −3

(c) 10.0

19 (a) Wrong. Cross-sectional area of A > cross-sectional of B

19 (b) (i)



(ii) (−2, 9)

20 (a) 3.16

(b) $x = \frac{1}{2y^2}$

21 (a) $\frac{AD}{AB} = \frac{AE}{AC} = \frac{1}{2}$ (given)

$\angle DAE = \angle BAC$ (common)

ΔABC is similar to ΔADE (SAS)

(b) $4x$

(c) $\frac{1}{4}$

22 (a) $-\frac{1}{2}$

(b) $\frac{\pi}{3}$

(c) 4.34%

ANDERSON SECONDARY SCHOOL
Preliminary Examination 2015
Secondary Four Express & Five Normal

CANDIDATE NAME: _____

CLASS: _____ / _____ INDEX NUMBER: _____

MATHEMATICS 4016/01

Paper 2

25 August 2015

2 hours 30 minutes

0800 – 1030h

Additional Materials: Writing paper (10 sheets)
Graph paper (2 sheets)

READ THESE INSTRUCTIONS FIRST

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For π, use either your calculator value or 3.142, unless the question requires the answer in terms of π.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total of the marks for this paper is 100.

Mathematical Formulae

Compound Interest

$$\text{Total amount} = P\left(1+\frac{r}{100}\right)^{n}$$

Mensuration

Curved surface area of a cone $= \pi rl$

Surface area of a sphere $= 4\pi r^2$

Volume of a cone $= \frac{1}{3}\pi r^2 h$

Volume of a sphere $= \frac{4}{3}\pi r^3$

Area of triangle $ABC = \frac{1}{2}ab\sin C$

Arc length $= r\theta$, where θ is in radians

Sector area $= \frac{1}{2}r^2\theta$, where θ is in radians

Trigonometry

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

Statistics

$$\text{Mean} = \frac{\Sigma fx}{\Sigma f}$$

$$\text{Standard deviation} = \sqrt{\frac{\Sigma fx^2}{\Sigma f} - \left(\frac{\Sigma fx}{\Sigma f}\right)^2}$$

Answer all the questions.

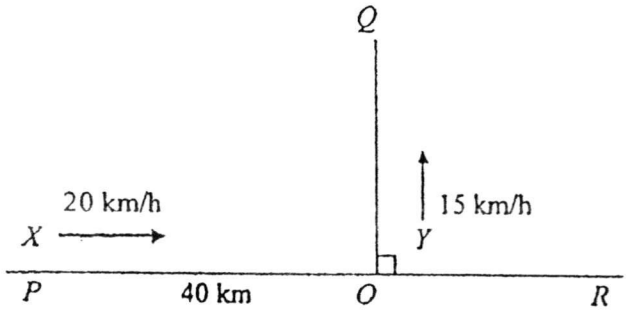
1 (a) A straight line passes through the points $A(8, 4)$ and $B(14, 12)$.

(i) Another line, parallel to the y-axis and passing through the point $(9, 2)$, meets the line segment AB at the point D.
Calculate the coordinates of D. [2]

(ii) The line segment AB undergoes a clockwise rotation R about the point A such that B', the image of B, lies on the negative y-axis.
Find the coordinates of B'. [2]

(iii) Hence, find the equation of the image of the line AB under rotation R. [1]

(b) The diagram shows a road junction at O with PR perpendicular to OQ and $OP = 40$ km. A cyclist X starting from P travels towards R at a constant speed of 20 km/h. At the same time, another cyclist Y starting from O travels towards Q at a constant speed of 15 km/h.



(i) Find the distance between the cyclists 1 hour after the start. [1]

(ii) Given that after t hours, the cyclists are 60 km apart, show that $25t^2 - 64t - 80 = 0$. [3]

(iii) Solve $25t^2 - 64t - 80 = 0$ and hence find the distance cyclist X is from O when the cyclists are 60 km apart. [3]

2 (a) Michelle wants to buy a car which costs \$130 000.
She decides to make a 20% down payment and take a loan from Ah Long Finance Company to pay for the remaining cost of the car.

The table below shows the monthly repayment plan (per \$1 000 borrowed) based on a simple interest rate of 3.3% per annum offered by Ah Long Finance Company.

Term of loan (in years)	3	5	7	10
Monthly repayment (for every \$1000 borrowed)	\$30.53	\$19.42	\$$m$	\$11.08

(i) Find the value of m. [2]

(ii) Using the table above, calculate the amount of interest Michelle will have to pay if she chooses the term of loan of 10 years. [2]

(b) Math Powerhouse supplies electricity to households and each household has to pay a fixed charge of \$12 and a further charge of 12 cents for each unit of electricity used in a month.

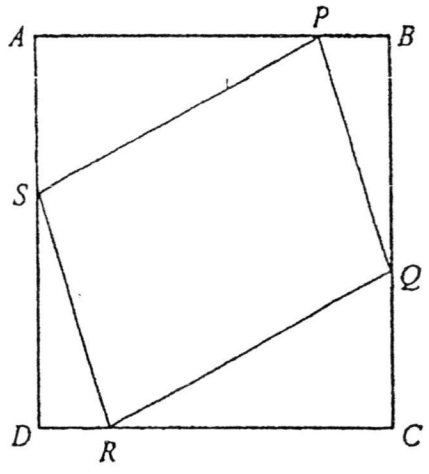
(i) Write down a formula connecting the total cost, C dollars, and the number of units, n, of electricity used. [1]

(ii) Calculate the number of units used when the total cost was \$31.92. [1]

(iii) Math Powerhouse offers another method of payment: 17 cents for each unit of electricity used in a month but no fixed charge.
Each household can choose to pay either using the method with fixed charge or the one without.
The Wong household uses N units of electricity each month.

Suggest to the Wong household which method of payment is a better choice. [3]

3



In the diagram, $ABCD$ is a rectangle. Points P, Q, R and S lie on AB, BC, CD and DA such that $AP = BQ = CR = DS$.

(a) Showing all your reasons clearly, prove that

(i) $AS = CQ$, [2]

(ii) triangle APS is congruent to triangle CRQ. [3]

(b) Write down two reasons to prove that $PQRS$ is a parallelogram. [2]

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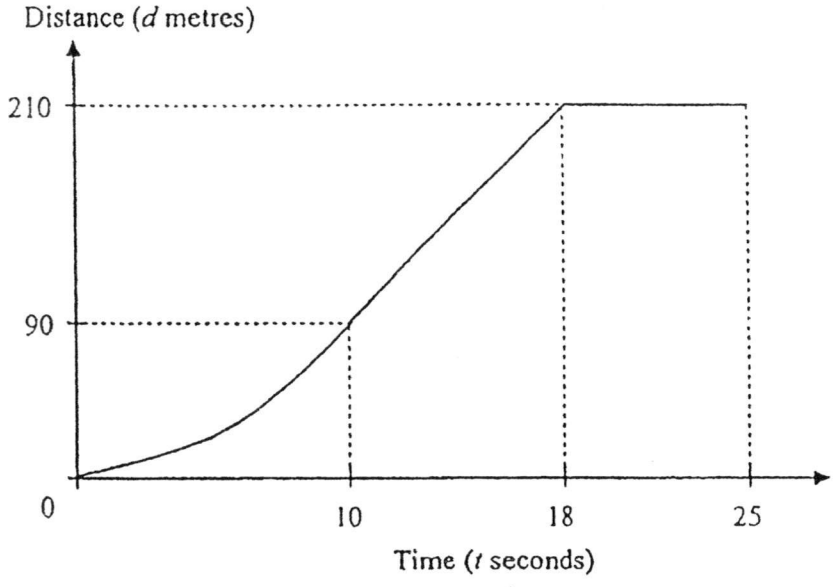
4 The diagram below shows the distance-time graph of an object A for the first 25 seconds of its motion along a straight path. The object increases its speed uniformly from 3 m/s for the first 10 seconds of the journey.



(a) Find the average speed of object A for the first 25 seconds of its journey. [1]

(b) Find the speed of object A when $t = 15$. [1]

(c) Find the acceleration of object A when $10 < t < 18$. [1]

(d) On a sheet of graph paper, draw the speed-time graph of object A for the first 25 seconds, using a scale of 2 cm to represent 5 seconds on the horizontal axis and a scale of 2 cm to represent 5 m/s for the vertical axis. [3]

Another object B also starts its motion together with object A along the same straight path. The speed of object B, v m/s, is given by $v = t$.

(e) On the same axes drawn in (d), draw the speed-time graph of object B for the first 25 seconds. [1]

(f) Write down the significance of the point where the two graphs intersect. [1]

(g) Without any calculation, use your graphs to explain which object travels a greater distance during the first 15 seconds. [1]

5 **Answer the whole of this question on a sheet of graph paper.**

Anderson Printing Company makes of a profit of y thousand dollars from the printing of x thousand T-shirts where $y = 8 - x - \frac{10}{x+1}$.

The table below shows some corresponding values of x and y for this equation.

x	0	0.5	1	2	3	4	5	6	7
y	−2	0.83	2	2.67	2.5	2	1.33	0.57	−0.25

(a) Explain what does the value of −2 for y signifies. [1]

(b) Using a scale of 2 cm to represent 1 unit on the x-axis and 2 cm to represent 0.5 unit on the y-axis, draw the graph of $y = 8 - x - \frac{10}{x+1}$ for $0 \le x \le 7$. [3]

(c) Using your graph, find

(i) the solution(s) of the equation $8 - x - \frac{10}{x+1} = 0$, [1]

(ii) the number of T-shirts the company should print in order to obtain the maximum profit. [1]

(d) Explain the significance of the solution(s) in (c)(i). [1]

(e) By drawing a suitable straight line, find the number of T-shirts the company would be printing when the profit per shirt is $1. [2]

(f) (i) Determine graphically the x-coordinate of point P on the curve such that the gradient of the curve at P is −0.5. [2]

(ii) State what the value of −0.5 in part (i) represents. [1]

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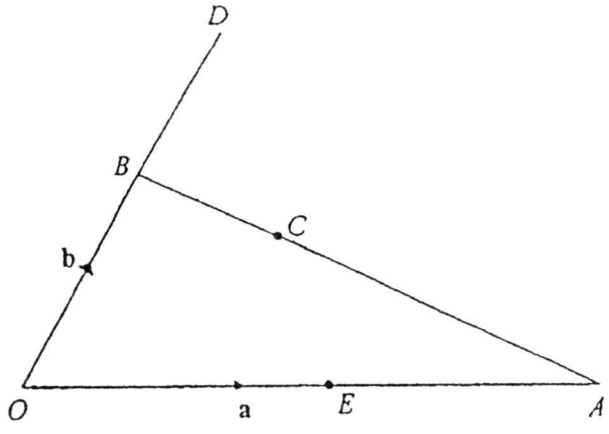
6 In the diagram, OAB is a triangle. C is a point on AB such that $AC : CB = 2 : 1$. The side OB is produced to the point D such that $OB : BD = 3 : 2$.



It is given that $\overrightarrow{OA} = \mathbf{a}$ and $\overrightarrow{OB} = \mathbf{b}$.

(a) Express, as simply as possible, in terms of a and/or b,

(i) $\overrightarrow{AB}$, [1]

(ii) $\overrightarrow{AC}$, [1]

(iii) $\overrightarrow{OC}$, [1]

(iv) $\overrightarrow{OD}$. [1]

(b) Show that $\overrightarrow{CD} = \mathbf{b} - \frac{1}{3}\mathbf{a}$. [2]

(c) It is given that E is a point on OA such that $\overrightarrow{OE} = \frac{5}{9}\mathbf{a}$.

Express, as simply as possible, in terms of a and b, the vector $\overrightarrow{ED}$. [1]

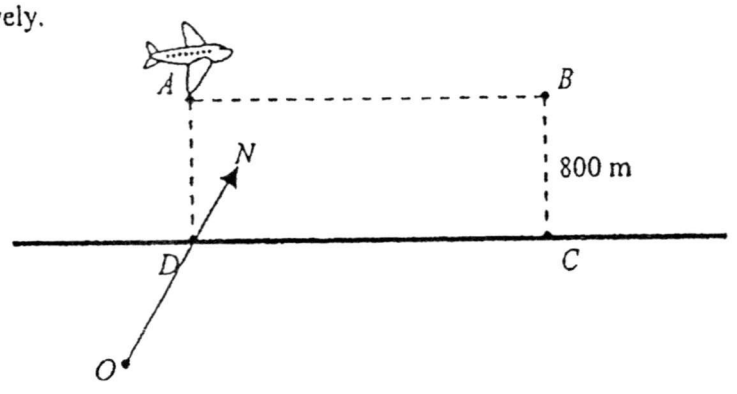
(d) With reasons clearly provided, show that D, C and E are collinear. [2]

(e) Given that $|\mathbf{a}| = 8$, $|\mathbf{b}| = 6$ and $\angle BOA = 60°$, calculate

(i) $\left|\overrightarrow{AB}\right|$, [2]

(ii) the area of triangle ABE. [2]

7 Three points O, C and D lie on the ground such that D is due north of O and C is due east of D. An aeroplane flies eastwards from A, a point 800 m directly above D. After 30 seconds, it reaches a point B, a point 800 m directly above C.
The angles of elevation of the plane from O at A and at B are 45° and 30° respectively.



Calculate

(a) the distance OD and OC, [2]

(b) the bearing of C from O, [2]

(c) the distance AB travelled by the plane, giving your answer to the nearest metre, [2]

(d) the speed of the plane, giving your answer to the nearest km/h. [2]

37

8 Mr Eu is planning to make a pet feeding bowl for his terrier.
He first obtains a solid wooden right circular cone with base diameter 50 cm and vertical height 50 cm as shown in Diagram I.

To make the feeding bowl, a smaller right circular cone of radius r cm is first removed from the original cone to form a frustum of height h cm as shown in Diagram II.

A hemispherical depression of r cm is then removed from the frustum to make a solid wooden feeding bowl as shown in Diagram III.

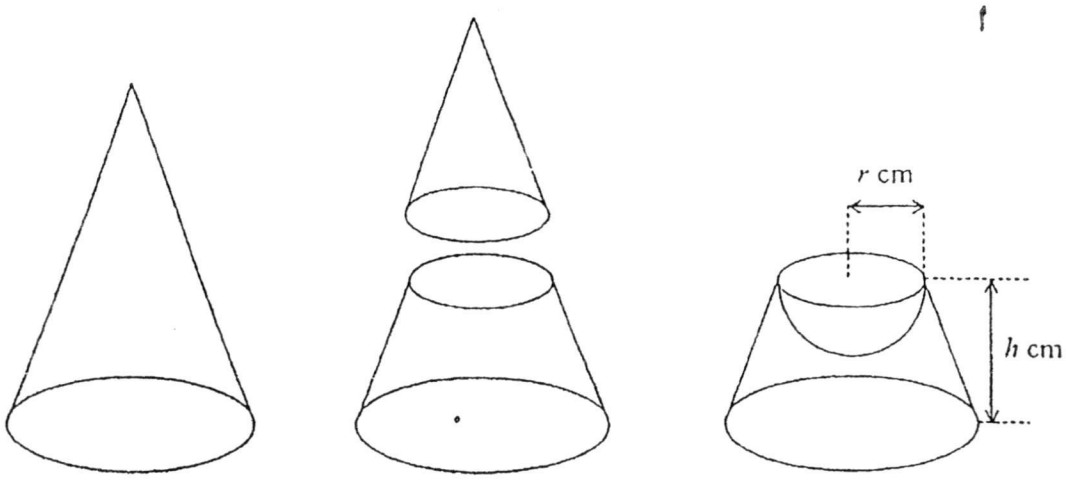


Diagram I Diagram II Diagram III

(a) Find the volume of the original wooden cone.
Leave your answer in terms of π. [1]

(b) Given that the volume of the cone removed is 21.6% of the original cone, show that the height of the frustum is 20 cm. [2]

(c) Find the volume of the hemispherical portion removed. [2]

(d) Mr Eu wants to paint the entire (both inner and outer) feeding bowl red.
Calculate the total surface area to be painted. [3]

(e) After using the feeding bowl for a week, Mr Eu thinks that the hemispherical volume created to fill the dog feeds is insufficient for his greedy terrier.
He says, "I should have created a hemispherical depression of radius 18 cm."
Explain to Mr Eu, with mathematical evidence, why his hypothetical feeding bowl cannot be created. [2]

9 Two bus companies use 3 different types of buses – small, medium and big. The table below shows the number of bus trips run on 3 consecutive days by the two bus companies.

			Bus Companies	
			SAS	SNRT
No. of bus trips	Day 1	Small	8	15
		Medium	20	30
		Big	6	5
	Day 2	Small	10	15
		Medium	20	35
		Big	12	6
	Day 3	Small	6	8
		Medium	13	14
		Big	9	5

The information for the number of trips run on Days 1, 2 and 3 can be represented by the matrices $P = \begin{pmatrix} 8 & 15 \\ 20 & 30 \\ 6 & 5 \end{pmatrix}$, $Q = \begin{pmatrix} 10 & 15 \\ 20 & 35 \\ 12 & 6 \end{pmatrix}$ and $R = \begin{pmatrix} 6 & 8 \\ 13 & 14 \\ 9 & 5 \end{pmatrix}$ respectively.

(a) Evaluate $A = \frac{1}{3}(P+Q+R)$ and describe what is represented by the elements of A. [2]

(b) Let $B = \begin{pmatrix} 1 & 1 & 1 \end{pmatrix}$.

(i) Explain which matrix, 3AB or 3BA, exists. [1]

(ii) Describe what the elements of the matrix that exists in (b)(i) represent. [1]

(c) The number of passengers per trip on a small, medium and big bus is 12, 25 and 50 respectively.

Write down the product of P and two other matrices such that the elements in this product represent the number of passengers the two bus companies carry on Day 1. **You need not** evaluate this product. [2]

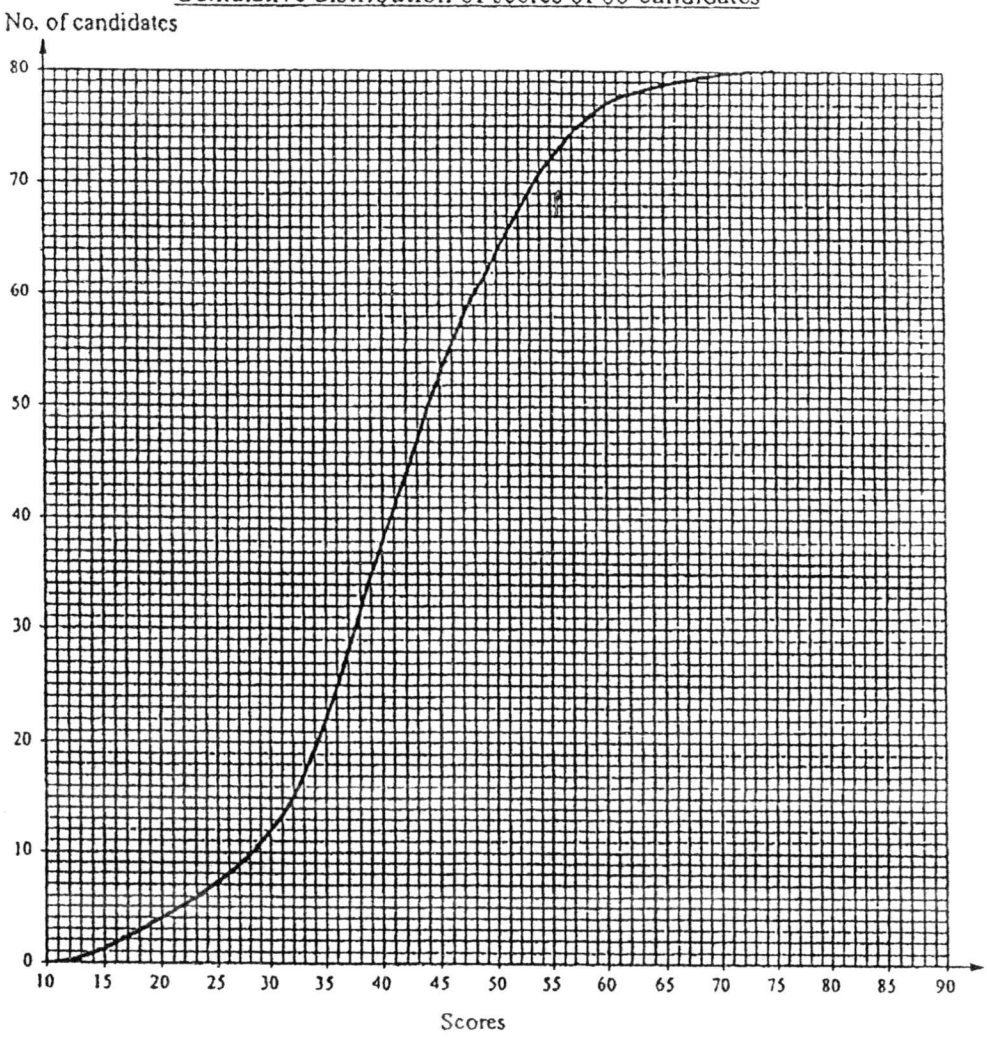
(d) Company SAS charges \$22 per passenger while company SNRT charges \$25 per passenger.

(i) Write down, but **do not** evaluate, the product of 3 matrices that gives the total amount each bus company collects on Day 1. [2]

(ii) Let the matrix in (d)(i) be M.

Explain what the matrix $M\begin{pmatrix} -1 \\ 1 \end{pmatrix}$ represents. [1]

10 The cumulative distribution of scores gained by a group of 80 candidates in the first round of aptitude test for admission to ANDE School is shown below.



(a) Using the cumulative frequency curve, find

(i) the median score, [1]

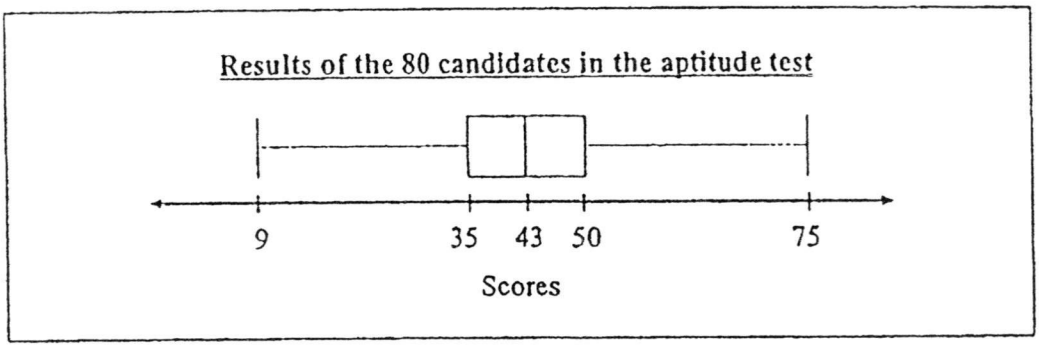
(ii) the interquartile range. [1]

(b) If 20% of the candidates qualified for the direct admission, use your graph to estimate the qualifying score for direct admission to ANDE School. [1]

(c) Candidates who scored at least 35 proceeded to the second round of test. Find the probability that a candidate, selected at random, failed to proceed to the second round of test by a score difference of 2. [2]

(d) Several days later, it was noticed that there was an error in the marking, and that all candidates should get 3 more marks in the first round of test. State what adjustment, if any, should be made to the above cumulative frequency graph to reflect the correct information. [1]

(e) Another group of 80 candidates also took the same aptitude test in the first round. The results of this second group are illustrated in the box-and-whisker plot below.



(i) Write down the median and the range of this distribution. [2]

(ii) If the probability that a candidate chosen random scored between a and b is 0.5, suggest a possible pair of values of a and b. [1]

(iii) Mr Loh compared the scores of the second group of candidates with the corrected scores of the first group and remarked that the performance of the two groups was comparable.
Do you agree with him? Justify your answer. [2]

ANDERSON SECONDARY SCHOOL
2015 Preliminary Examination
Secondary Four Express / Five Normal
MATHEMATICS PAPER 2 (4016/02)

Answer Key

1 (a) (i) $\left(9, 5\frac{1}{3}\right)$

(ii) $(0, -2)$

(iii) $y = \frac{3}{4}x - 2$

(b) (i) 25 km

(iii) $t = 3.48$; 29.6 km

2 (a) (i) $m = 14.65$

(ii) \$34278.40

(b) (i) $C = 12 + 0.12n$

(ii) $n = 166$

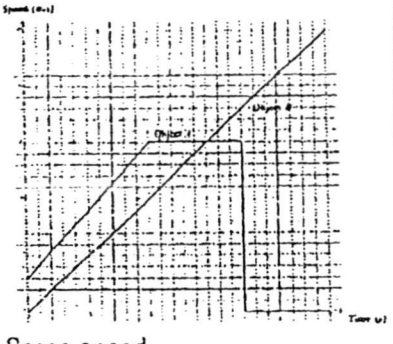
(iii) If uses 240 units each month, either payment; if uses more than 240 units, use fixed charge payment, else use no fixed charge payment.

4 (a) $8\frac{2}{5}$ m/s

(b) 15 m/s

(c) 0 m/s^2

(d), (e)

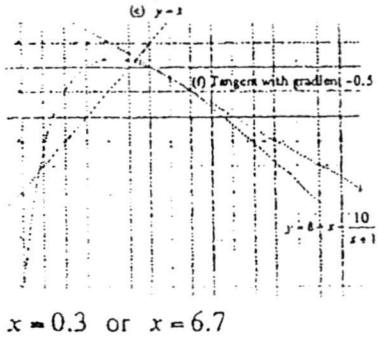


(f) Same speed

(g) Object A

5 (a) Loss of \$2000 incurred when no T-shirt is printed.

(b)



(c) (i) $x = 0.3$ or $x = 6.7$

(ii) 2150 T-shirts

5 (d) The company makes a profit when it prints more than 300 but less than 6700 T-shirts.

(e) 400 or 2600 T-shirts

(f) (i) $x = 3.5$

(ii) The rate of decrease of profit per T-shirt printed.

6 (a) (i) $b - a$

(ii) $\frac{2}{3}(b - a)$

(iii) $\frac{1}{3}(a + 2b)$

(iv) $\frac{5}{3}b$

(c) $\frac{5}{3}(b - \frac{1}{3}a)$

(e) (i) $\sqrt{52}$

(ii) 9.24 units^2

7 (a) $OD = 800$ m; $OC = 800\sqrt{3}$ m

(b) 054.7°

(c) 1131 m

(d) 136 km/h

8 (a) $\frac{31250\pi}{3}$ cm^3

(c) 7070 cm^3

(d) 6190 cm^2

(e) No. If the hemispherical depression is of radius 18 cm, the height of the cone to be removed would be 36 cm. Then the height of the frustum would be only 14 cm, which is insufficient to accommodate a hemispherical depression of 18 cm.

40

9 (a) $\begin{pmatrix} 8 & 12\frac{2}{3} \\ 17\frac{2}{3} & 26\frac{1}{3} \\ 9 & 5\frac{1}{3} \end{pmatrix}$

The average number of trips per day made by each type of buses run by each of the two bus companies.

(b) 3BA exists since the dimension of matrix B is 1×3 and the that of A is 3×2.

(c) $\begin{pmatrix} 12 & 25 & 50 \end{pmatrix}\begin{pmatrix} 8 & 15 \\ 20 & 30 \\ 6 & 5 \end{pmatrix}\begin{pmatrix} 1 \\ 1 \end{pmatrix}$

(d) (i) $\begin{pmatrix} 12 & 25 & 50 \end{pmatrix}\begin{pmatrix} 8 & 15 \\ 20 & 30 \\ 6 & 5 \end{pmatrix}\begin{pmatrix} 22 & 0 \\ 0 & 25 \end{pmatrix}$

(ii) The amount bus company SNRT collects more than SAS does on Day 1.

10 (a) (i) 40.5

(ii) 14

(b) 50

(c) $\frac{1}{16}$

(d) Move the graph 3 marks to the right.

(e) (i) Median = 43; Range = 66

(ii) Possible pairs are:
(1) $a = 9$ and $b = 43$
(2) $a = 35$ and $b = 50$
(3) $a = 43$ and $b = 75$

(iii) I agree with Mr Loh that the performance of the two groups was comparable as their median scores of 43;5 and 43 were close. Furthermore, their scores for interquartile range of 14 and 15 were also approximately the same.