| Index Number | Class | Marks: / 80 |
|---|---|---|

Anglo-Chinese School
(Barker Road)

# PRELIMINARY EXAMINATION 2015

## SECONDARY FOUR EXPRESS/FIVE NORMAL ACADEMIC

## MATHEMATICS 4016
## PAPER ONE

### 2 HOURS

**READ THESE INSTRUCTIONS FIRST**

**Do not open this booklet until you are told to do so.**

Write your class and candidate number on the cover sheet.
Write in dark blue or black pen.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** questions.

If working is needed for any question it must be shown with the answer.
Omission of essential working will result in loss of marks.
Calculators should be used where appropriate.
If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.
For $\pi$, use either the calculator value or 3.142, unless the question requires the answer in terms of $\pi$.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total marks for Paper One is 80.

| | | | |
|---|---|---|---|
| 3 s.f. | | Simplify fraction | |
| 1 d.p. | | Truncation error | |

## Compound Interest

$$\text{Total amount} = P\left(1+\frac{r}{100}\right)^n$$

## Mensuration

$$\text{Curved Surface area of cone} = \pi rl$$

$$\text{Surface area of a sphere} = 4\pi r^2$$

$$\text{Volume of a cone} = \frac{1}{3}\pi r^2 h$$

$$\text{Volume of a sphere} = \frac{4}{3}\pi r^3$$

$$\text{Area of a triangle} = \frac{1}{2}ab\sin C$$

$$\text{Arc length} = r\theta, \text{ where } \theta \text{ is in radians}$$

$$\text{Sector area} = \frac{1}{2}r^2\theta, \text{ where } \theta \text{ is in radians}$$

## Trigonometry

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

## Statistics

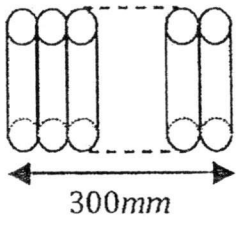
$$\text{Mean} = \frac{\sum fx}{\sum f}$$

$$\text{Standard deviation} = \sqrt{\frac{\sum fx^2}{\sum f} - \left(\frac{\sum fx}{\sum f}\right)^2}$$

Answer **all** the questions

For Examiner's Use

**1** The diameter of polyester fiber is 9.8 micrometres each.
They are woven into sheets of width 300 millimetres.
If each sheet is made by arranging polyester fibre strands side-by-side without overlapping, find the number of fiber strands in each sheet, expressing the answer in standard form.




*Answer* _____ [2]

**2** Charlie invested $25000 into a business which gave a guaranteed return at 4% per annum compounded every quarterly.
Find the total return earned after 3 years, giving your answers to the nearest cents.

*Answer* _____ [2]

For Examiner's Use

3 At 12 noon, the temperature of the Sahara Desert is $45.4°C$ while the temperature at recorded at 12 midnight is 49°C. Assuming that the temperature decreases uniformly as the time passes, find the time when the temperature first reaches $26.5°C$.

*Answer* _____ a.m./p.m. [2]

For Examiner's Use

4 A factory can assemble 4000 watches in 8 hours by 50 workers.
In their latest project, they are to assemble 13500 watches.
Assuming that all workers work at the same rate, calculate how many more workers are required if they are to complete the task in 18 hours.

*Answer* _____ workers [2]

For Examiner's Use

5 When the Goods and Services Tax (GST) increases by 2%, James decreases his expenditure by 2%. James claimed that his expenditure neither increases nor decreases. Explain if he is right or wrong.

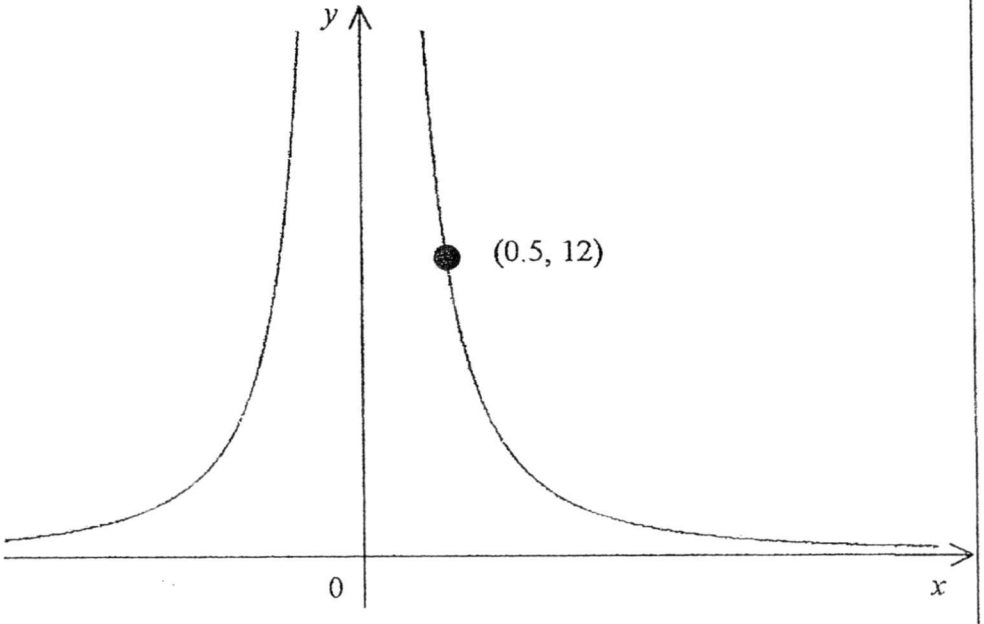
*Answer:* .....

.....

..... [2]

For Examiner's Use

6 The diagram shows the graph of $y = kx^{-2}$.




Given that it passes through (0.5, 12),

**(i)** find the value of $k$.

**(ii)** State the equation of the line of symmetry for this graph.

*Answer* (i) $k =$ ..... [1]

(ii) ..... [1]

For Examiner's Use

7 An area of 1250 km$^2$ is represented on a map by an area of 50 cm$^2$.

Find

**(a)** the scale of the map in the form of $1:n$,

*Answer* _____ [1]

**(b)** the length of a road on the map with an actual distance of 27 km.

*Answer* _____ cm [1]

For Examiner's Use

8 Simplify $\dfrac{\left(3a^{-\frac{1}{2}}\right)^2 \times 4a^2b}{12b^{-2}}$, leaving your answers as positive indices.

*Answer* _____ [2]

4

For Examiner's Use

9 The probability that an amateur dart archer actually hits the bull's eye is $\frac{1}{6}$.

If three consecutive attempts are made, find the probability that the bull's eye will be hit

**(a)** three times,

*Answer* (a) _____ [1]

**(b)** not at all,

*Answer* (b) _____ [1]

**(c)** at least twice.

*Answer* (c) _____ [1]

For Examiner's Use

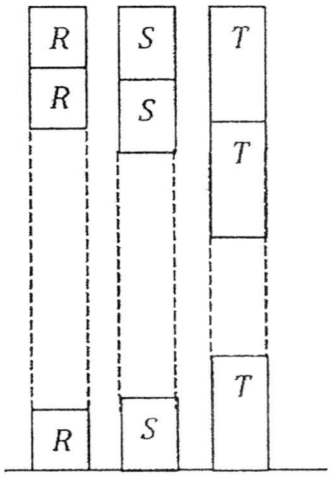
**10** **(a)** 495 and a number $N$ have a LCM of 4950 and a HCF of 15.

Given that $4950 = 2 \times 3^2 \times 5^2 \times 11$, find the number $N$.

*Answer* (a) $N =$ _____ [1]

**(b)** Three types of cylindrical cans, $R$, $S$ and $T$, of the same radii, have heights 50 cm, 0.6 m and 0.72 m. They are stacked to the **same** height for a game to be played.

Find the minimum number of can $T$.




*Answer* (b) _____ cans [2]

**11** The volume, $V$, of a cylinder varies directly to the square of its base radius $r$ cm ~~and height $h$ cm~~. If the base radius decreases by 50% and the height increases by 40%, find the percentage change in the volume of the cylinder.

*Answer* _____ % [3]

5

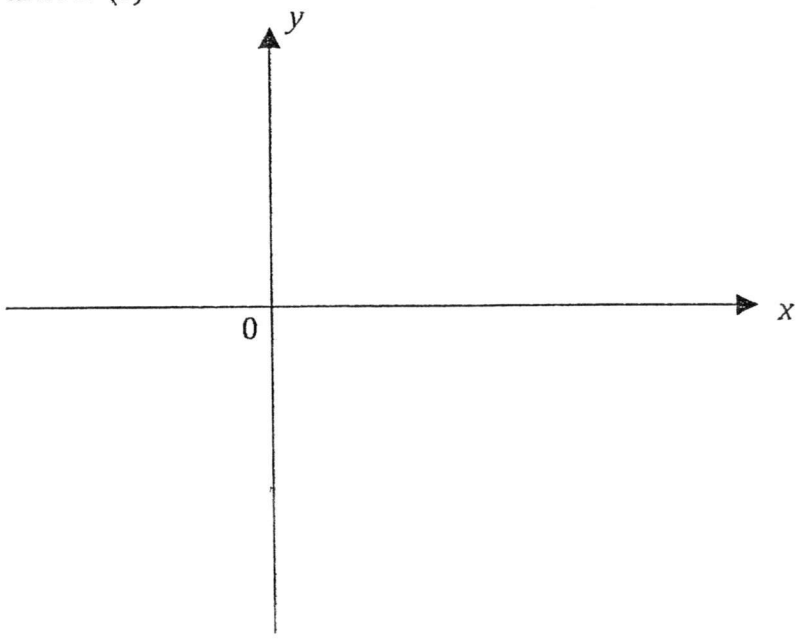
For Examiner's Use

**12** **(a)** Express $y = -x^2 + 4x - 9$ in the form $y = -(x-a)^2 + b$.

*Answer* (a) _____ [1]

**(b)** Hence, sketch the graph of $y = -x^2 + 4x - 9$ and write down the coordinates of the turning point and $y$-intercept.

*Answer* (b) [2]




**13** The first four terms of a number sequence are 5, 12, 19 and 26.

**(a)** If the $n$th term of the number sequence can be expressed in the form of $pn + q$, find the values of $p$ and $q$.

*Answer* (a) $p =$ _____ [1]

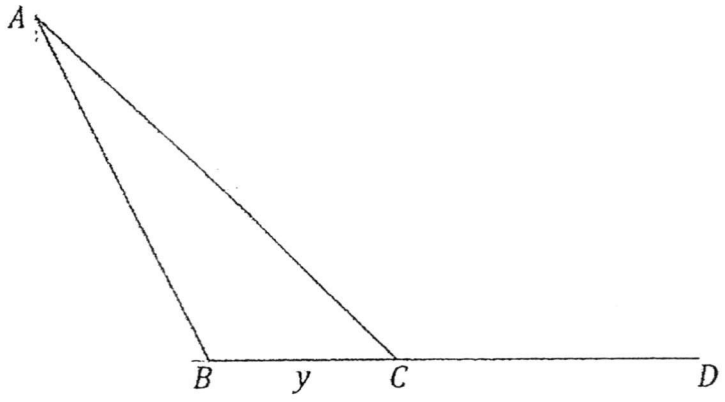
$q =$ _____ [1]

**(b)** Also, deduce the $n$th term of a sequence that has –3, 4, 11 and 18 as its first four terms.

*Answer* (b) _____ [1]

For Examiner's Use

**14** In the diagram, $BCD$ is a straight line.




Given that $BC = y$ cm, $AC = 2BC$ and the area of $\Delta ABC$ is $0.8y^2$ cm$^2$, find the exact value of

**(a)** $\sin \angle ACD$,

*Answer* (a) _____ [2]

**(b)** $\cos \angle ACD$.

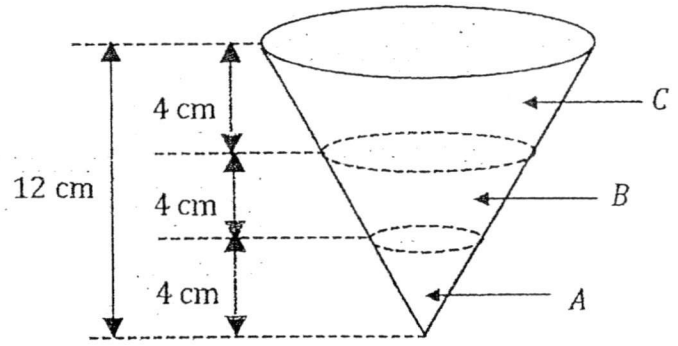
*Answer* (b) _____ [1]

For Examiner's Use

6

For Examiner's Use

15 A solid cone is cut into three equal parts, $A$, $B$ and $C$, by planes parallel to the base, as shown below.
The height of the cone is 12 cm.




Find the ratio of the

**(a)** diameters of the bases $C$ to $B$,

*Answer* (a) _____ [1]

**(b)** the volumes of $A$ to $C$.

*Answer* (b) _____ [2]

For Examiner's Use

**16** **(a)** Given that $x + y = 6$ and $xy = 2$, find the value of $\frac{2}{x^2} + \frac{2}{y^2}$.

For Examiner's Use

*Answer* (a) _____ [2]

**(b)** Given that $\frac{3x - 2y}{y - 4x} = \frac{3}{2}$, find the value of $\frac{3x}{7y}$.

*Answer* (b) _____ [2]

7

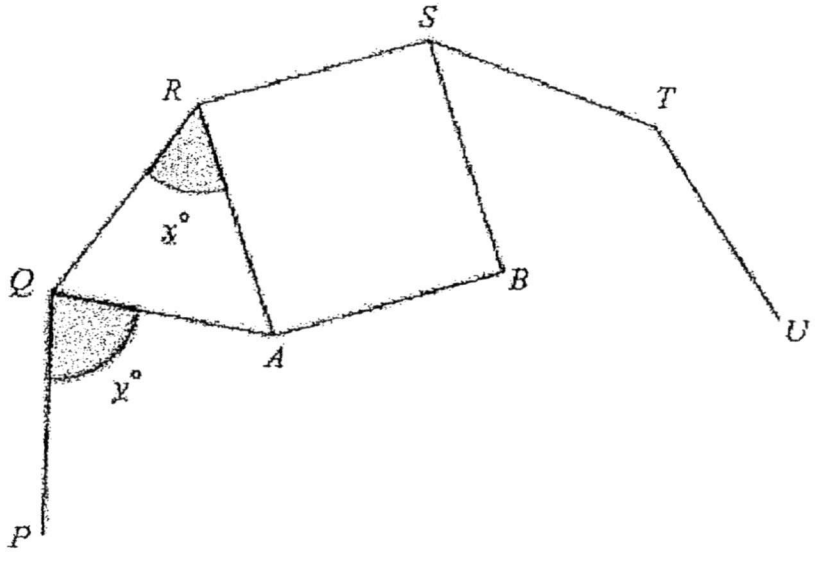
For Examiner's Use

17 **(a)** Find the size of each exterior angle of a regular decagon.

For Examiner's Use

*Answer* (a) _____ ° [1]

**(b)** *PQRSTU*... is part of a regular polygon which has interior angles of 144°. *RABS* is a square.




Find

**(i)** the value of $x$,

*Answer* (b) (i) _____ ° [1]

**(ii)** the value of $y$.

*Answer* (b) (ii) _____ ° [2]

For Examiner's Use

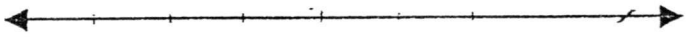
**18** **(a)** Solve the inequalities $\frac{x}{2}+2<\frac{2x+7}{3}\leq 9-2x$.

For Examiner's Use

*Answer* (a) _____ [2]

**(b)** Represent your solutions in **part (a)** on the number line in the answer space below.

*Answer* (b) [1]



**(c)** With reference to your solutions in **part (a)**, write down the smallest value of $x^2$.

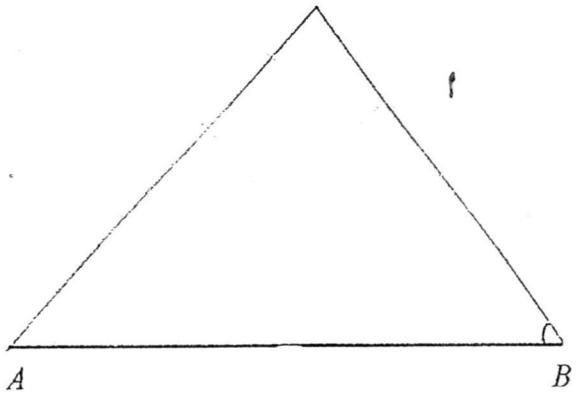
*Answer* (c) _____ [1]

8

For Examiner's Use

19 (a) In the spaces below, construct a triangle $ABC$ such that $AB = 7.4$ cm, $BC = 5.5$ cm and $\angle ABC = 54°$.

*Answer* (a)




[2]

(b)(c) A point $S$ is on the diagram.

It is equidistant from the lines $AC$ and $AB$ and points $A$ and $B$.

With suitable construction lines shown, mark point $S$. [3]

For Examiner's Use

**20** $A$ is a point $(1, 2)$, $\overrightarrow{AB} = \begin{pmatrix} 4 \\ 0 \end{pmatrix}$ and $\overrightarrow{BC} = \begin{pmatrix} 2 \\ 3 \end{pmatrix}$.

**(a)** Find $\left|\overrightarrow{BC}\right|$.

*Answer* (a) _____ units [1]

**(b)** Express $\overrightarrow{AC}$ as a column vector.

*Answer* (b) _____ [2]

**(c)** Given that $\overrightarrow{CD} = \begin{pmatrix} -2 \\ s \end{pmatrix}$, find the two possible values of $s$ such that $ABCD$ is a trapezium.

*Answer* (c) $s$ = _____ or _____ [3]

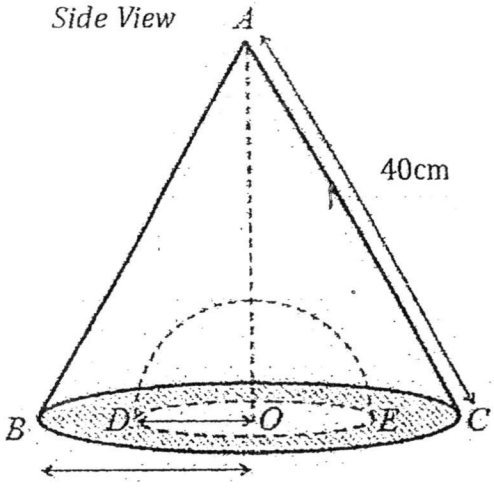
For Examiner's Use

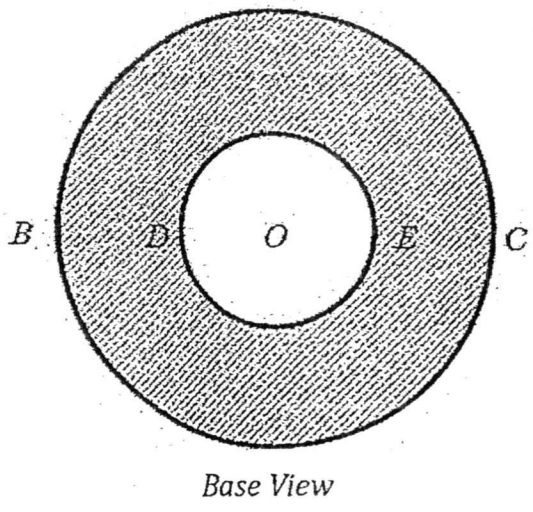
9

For Examiner's Use

21 The diagram shows the side and base views of a solid party hat.

It is made up of a cone and a hemisphere cut out from the bottom of the cone.




Base View

Given that the slant height, $AC$, and radius, $OB$, of the cone are 40 cm and 15 cm respectively and the shaded area of the base is $176\pi$ cm$^2$, find

**(a)** the radius, $OD$, of the hemisphere,

*Answer* (a) _____ cm [2]

**(b)** the volume of the party hat,

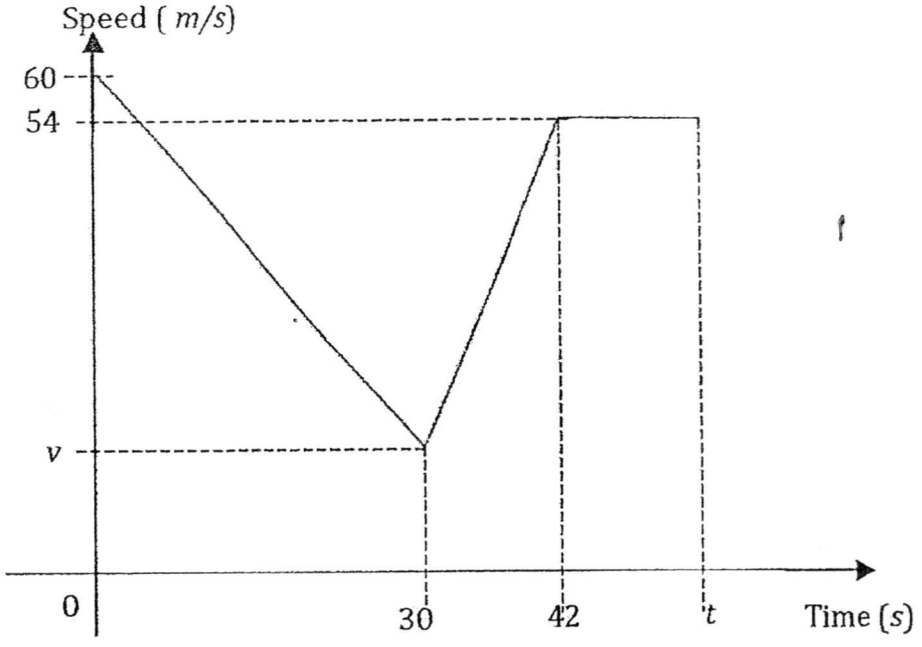
*Answer* (b) _____ cm$^3$ [2]

**(c)** the total surface area of the party hat.

*Answer* (c) _____ cm$^2$ [2]

For Examiner's Use

22 The diagram shows the speed-time graph for the first $t$ seconds of the motion of a particle.




**(a)** Given that the particle undergoes a retardation of $1.2\text{m/s}^2$ at 20s, in the spaces below, show that the value of $v$ is 24.

*Answer* (a)

[2]

**(b)** The particle travelled a distance of 702 metres between the 42 and $t$ second, find the value of $t$.

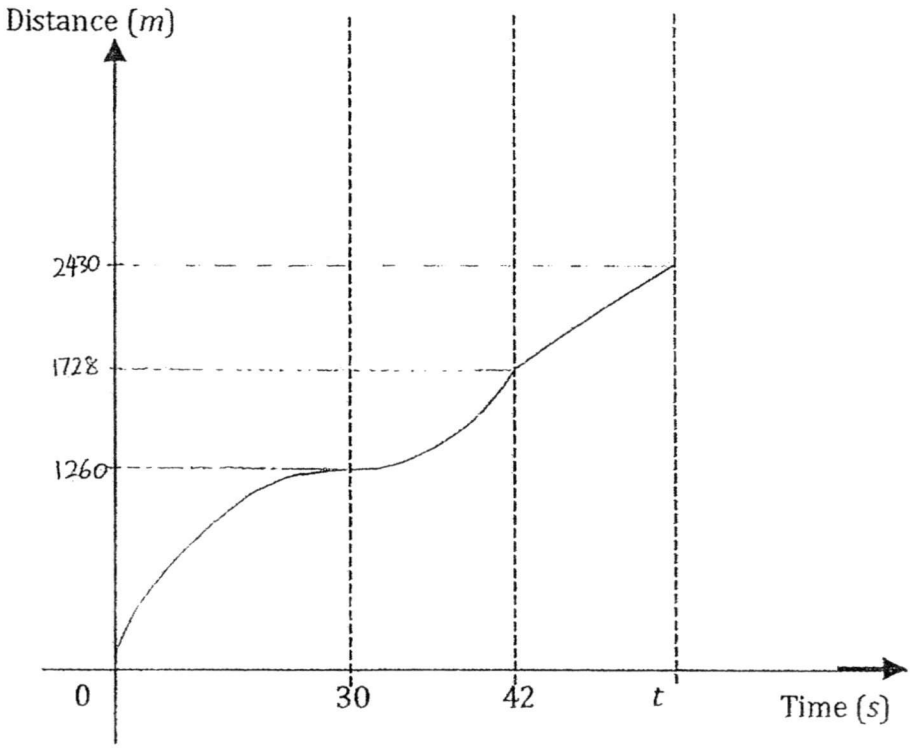
*Answer* (b) $t =$ _____ [1]

10

For Examiner's Use

**(c)** In the axes below, sketch the distance-time graph of the particle for the entire duration of $t$ seconds.

*Answer* (c)




[3]

For Examiner's Use

For Examiner's Use

23 The equation of the line $l$ is $3x + 4y = 24$.

**(a)** Given that line $l$ crosses the $x$-axis and $y$-axis at points $A$ and $B$ respectively, find the coordinates of the points $A$ and $B$.

*Answer* (a) $A$ ( ) [1]
$B$ ( ) [1]

**(b)** Find the shortest distance from the origin to $AB$.

*Answer* (b) units [3]

**(c)** Line $l$ intersects another line $h$ at a point $C$.

**(i)** Given that the gradient of line $h$ is $\frac{3}{4}$ and it passes through the origin, write down the equation of line $h$.

*Answer* (c) (i) [1]

**(ii)** Hence, find the coordinates of $C$.

*Answer* (c) (ii) $C$ ( ) [2]

11

***[END OF PAPER 1]***

1. $3.06x10^4$
2. $3170.63
3. 5.36pm
4. 25 workers
5. Let the expenditure be $x & original GSt of 5%
   Total expense is $1.05x.
   With the increment of GST by 2%, with 2% drop in expenditure,ie.$0.98x, the GST is now 7% x 0.98x=$0.0686x.
   New total expenses is 0.98x+ 0.0686x=$1.0486x
   So there is actually a decrease in total expenses.
   Or
   If figures is rounded to 2dp, James is correct to say that there is no change.

6.i K=3

6ii. x=0

7a.1:500000

7b.5.4cm

8.$3ab^3$

9a.$\frac{1}{216}$

9b. $\frac{125}{216}$

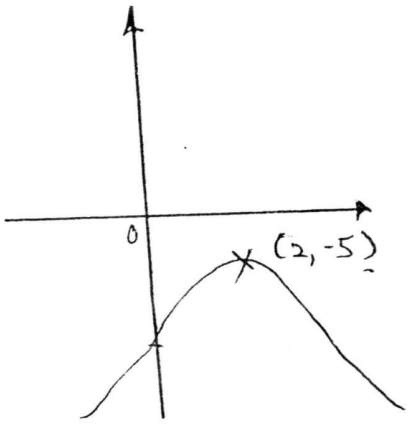
9c .$\frac{2}{27}$

10a.N=150

10b.25 cans

11.65%

12.a-$(x-2)^2$-5




13a.P=7 q=-2

13b.7n-10

14a.$\frac{4}{5}$

14b.$-\frac{3}{5}$

15a.3:2

15b.1:19

16a.16

16b.$\frac{1}{6}$

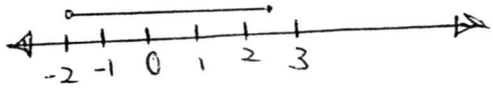
17a.$36^0$

17bi.$54^0$

17bii.$81^0$

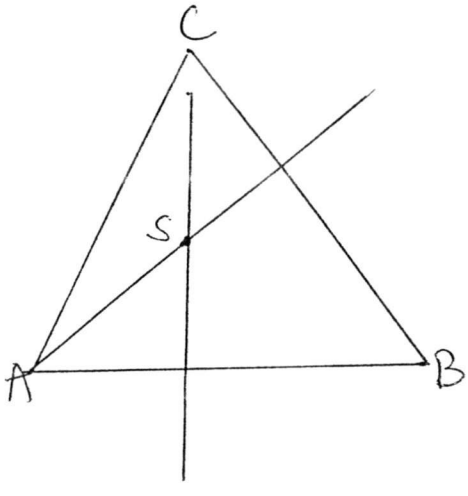
18a.$-2<x\leq\frac{5}{2}$

18b.




18c.0

19.



20a.3.61units

20b. $\binom{6}{3}$

20c. s=0 or 3

21a. 7 cm

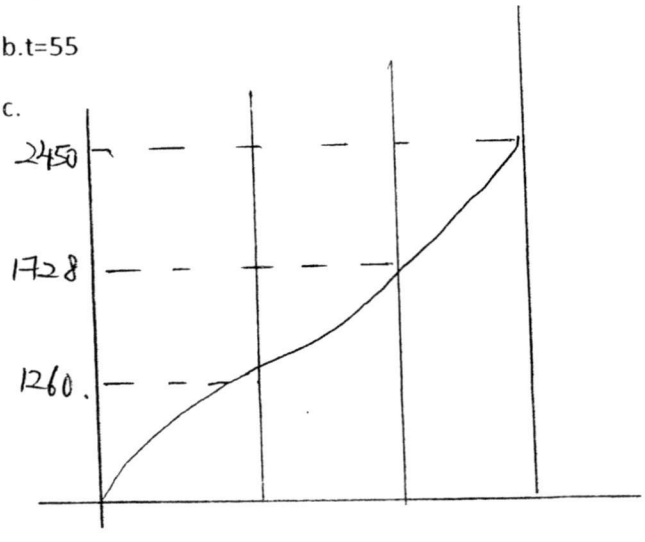
21b. $\frac{1}{3}\pi(15)^2(\sqrt{40^2-15^2})-\frac{1}{2}$

21c.2750cm$^2$

22.a.24

22b.t=55

22c.




23a.A(8,0) B(0,6)

23b. → Let the shortest distance from the origin to AB be d cm

23c.y=$\frac{3}{4}$x

23.ii.c(4,3)

Area of $\Delta = \frac{1}{2} \times 8 \times 6$

$\frac{1}{2} \times AB \times d = 24$

$\sqrt{8^2+6^2} \times d.$

Anglo-Chinese School
(Barker Road)

# PRELIMINARY EXAMINATION 2015

## SECONDARY FOUR EXPRESS/FIVE NORMAL ACADEMIC

## MATHEMATICS 4016
## PAPER TWO

**2 HOURS 30 MINUTES**

Additional Materials: Answer Paper
Graph Paper (1 sheet)

**READ THESE INSTRUCTIONS FIRST**

**Do not open this booklet until you are told to do so.**

Write your class and candidate number on the cover sheet.
Write in dark blue or black pen on both sides of the paper.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** questions.
If working is needed for any question it must be shown with the answer.
Omission of essential working will result in loss of marks.
Calculators should be used where appropriate.
If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.
For $\pi$, use either the calculator value or 3.142, unless the question requires the answer in terms of $\pi$.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 100.

---

*This paper consists of **10** printed pages inclusive of this page.* [Turn over

14

# *Mathematical Formulae*

*Compound interest*

$$\text{Total amount} = P\left(1+\frac{r}{100}\right)^n$$

*Mensuration*

$$\text{Curved surface area of a cone} = \pi rl$$

$$\text{Surface area of a sphere} = 4\pi r^2$$

$$\text{Volume of a cone} = \frac{1}{3}\pi r^2 h$$

$$\text{Volume of a sphere} = \frac{4}{3}\pi r^3$$

$$\text{Area of triangle ABC} = \frac{1}{2}ab\sin C$$

$$\text{Arc length} = r\theta, \text{ where } \theta \text{ is in radians}$$

$$\text{Sector area} = \frac{1}{2}r^2\theta, \text{ where } \theta \text{ is in radians}$$

*Trigonometry*

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

*Statistics*

$$\text{Mean} = \frac{\sum fx}{\sum f}$$

$$\text{Standard deviation} = \sqrt{\frac{\sum fx^2}{\sum f} - \left(\frac{\sum fx}{\sum f}\right)^2}$$

Answer **all** the questions
[100 marks]

**1.** Given that a universal set $\xi = \{x : x \text{ is a positive integer and } x \leq 14\}$,
$A = \{x : x \text{ is an odd integer and } 3x + 5 > 13\}$ and $B = \{x : x \text{ is a factor of } 28\}$.

**(a)** Find the following

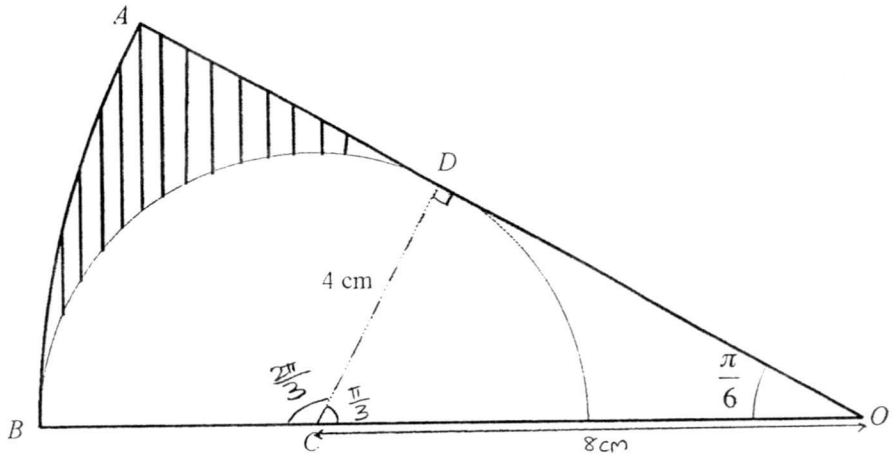
**(i)** $A \cup B$ [1]

**(ii)** $n(A \cap B')$ [2]

**(b)** With the help of a Venn diagram, list out the elements illustrating the sets $\xi$, $A$ and $B$. [2]

---

**2.** The diagram shows the sector $OAB$ of a circle with centre $O$ and the radius $OA$. A semi-circle with centre $C$ and radius 4 cm is drawn inside the sector such that $OA$ is the tangent to the semi-circle at $D$.




Given that $\angle AOB = \frac{\pi}{6}$ radians, calculate

**(a)** the perimeter of the shaded region, [3]

**(b)** the area of the shaded region, [3]

---

15

**3.** A shop sells three varieties of ice-creams, Vanilla, Chocolate and Strawberry. Each variety is sold in either cone or bowl. The sales of the ice-creams in two consecutive days are shown in the table below.

| | DAY ONE | | DAY TWO | |
|---|---|---|---|---|
| | Cone | Bowl | Cone | Bowl |
| Vanilla ice-cream | 20 | 11 | 27 | 16 |
| Chocolate ice-cream | 35 | 19 | 33 | 21 |
| Strawberry ice-cream | 28 | 20 | 19 | 22 |

The information for the Day One's sales can be represented by

the matrix $\boldsymbol{R} = \begin{pmatrix} 20 & 11 \\ 35 & 19 \\ 28 & 20 \end{pmatrix}$.

The information for Day Two is represented by a matrix **S**.

**(a)** Write down the matrix $\boldsymbol{S}$. [1]

**(b)** Calculate $\boldsymbol{R} + \boldsymbol{S}$. [1]

**(c)** Describe what is represented by the elements $\boldsymbol{R} + \boldsymbol{S}$. [1]

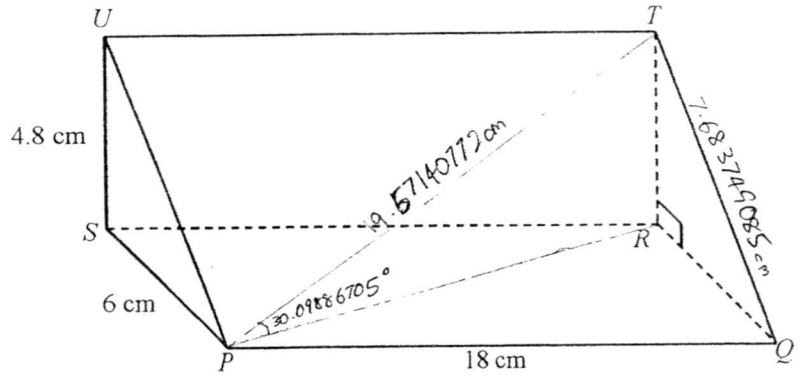
Suppose the price of Vanilla ice-cream is \$1, Chocolate ice-cream is \$2 and Strawberry ice-cream is \$2.

**(d)** Represent these prices by a $1 \times 3$ matrix $\boldsymbol{T}$. [1]

**(e)** Calculate $\boldsymbol{TS}$. [1]

**(f)** Describe what is represented by the elements of $\boldsymbol{TS}$. [1]

**4.**




The diagram shows a floatation device shaped in the form of a triangular prism in which $\angle QRT = 90°$, $US = 4.8$ cm, $SP = 6$ cm and $PQ = 18$ cm.

**(a)** Calculate

**(i)** the length of $PT$, [2]

**(ii)** $\angle RPT$. [2]

**(b)** Prove that triangles $PRU$ and $QST$ are congruent. [2]

**(c)** Styrofoam is needed to make such a floatation device.
Calculate the volume of styrofoam needed to make 10 such floatation devices. [2]

---

**5.** **(a)** Given that $w = \sqrt{\dfrac{u-v}{1+u}}$, express $u$ in terms of $w$ and $v$.

State the value(s) of $w$ that does not exist in the equation. [3]

**(b)** Solve the equation $\dfrac{x+1}{2x-3} - \dfrac{14-3x}{4x^2-9} = 0$. [3]

**(c)** Solve the equation $4^{x^2+3} = 8^{\frac{5x}{3}+3}$. [2]

16

---

6. A car travels from town $A$ to town $B$ at an average speed of $k$ km/h and returns from town $B$ to town $A$ at an average speed of $k + 15$ km/h. Given that the distance between town $A$ and town $B$ is 360 km and that the average speed of the entire journey is 60 km/h,

**(a)** Find the total time taken for the entire journey. [2]

**(b)** **(i)** Find an expression in terms of $k$ for the time taken from town $A$ to town $B$. [1]

**(ii)** Find an expression in terms of $k$ for the time taken from town $B$ to town $A$. [1]

**(c)** Show that $k$ can be expressed in an equation, and that it simplifies to $k^2 - 45k - 450 = 0$. [3]

**(d)** Solve $k^2 - 45k - 450 = 0$. [3]

---

7. **Answer the whole of this question on a single sheet of graph paper.**

The variables $x$ and $y$ are connected by the equation $y = \frac{1}{2}x^2(3 - x)$.

Some corresponding values of $y$, corrected to 1 decimal place where necessary, are given in the following table.

| $x$ | -1.5 | - 1.3 | -1 | -0.5 | 0 | 0.5 | 1 | 1.5 | 2 | 2.5 | 3 | 3.3 | 3.5 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $y$ | 5.1 | 3.6 | 2 | $p$ | 0 | 0.3 | 1 | 1.7 | 2 | 1.6 | 0 | -1.6 | -3.1 |

**(a)** Find the value of $p$. [1]

**(b)** Taking 2 cm to represent 1 unit on each axis, draw the graph of $y = \frac{1}{2}x^2(3 - x)$, for values $-1.5 \le x \le 3.5$. [3]

**(c)** By drawing a tangent, find the gradient of the curve at (2.5, 1.6). [2]

**(d)** **(i)** By adding a suitable line on the graph, use your graph to find the solution(s) of the equation $\frac{1}{4}x^2(3 - x) = x + 2$. [2]

**(ii)** This value of $x$ is a solution of the equation $x^3 + Ax^2 + Bx + C = 0$.
Find the value of $A$, of $B$ and of $C$. [1]

---

**8.** **(a)** 40 students from class **Y** were weighed and the results, recorded to the nearest kilogram were as follows.

| **Weight ($x$ kilograms)** | **Frequency** |
|---|---|
| $54 < x \leq 58$ | 3 |
| $58 < x \leq 62$ | $p$ |
| $62 < x \leq 66$ | 9 |
| $66 < x \leq 70$ | $q$ |
| $70 < x \leq 74$ | 14 |
| $74 < x \leq 78$ | 5 |

**(i)** Show that the values of $p$ and $q$ are 3 and 6 respectively given that the mean is 68 kg. [3]

**(ii)** Find the standard deviation of the weights of the students in the class. [2]

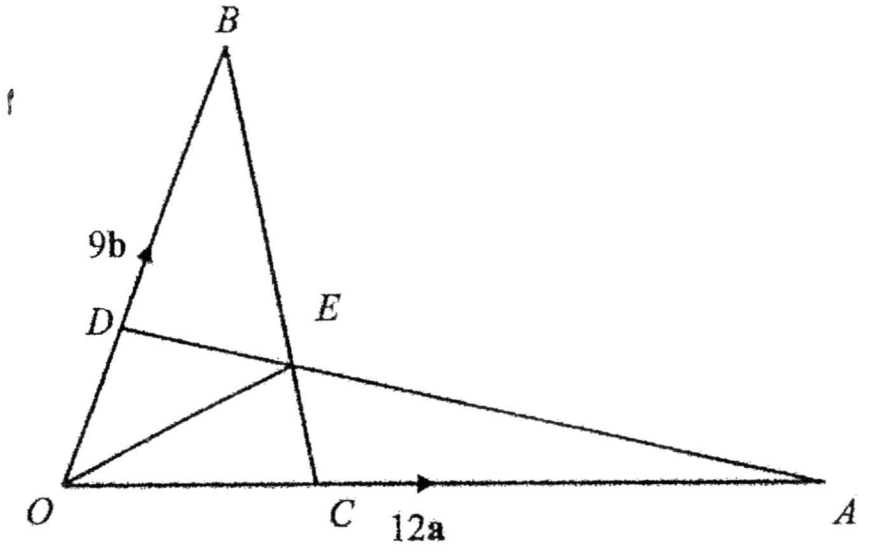
**(iii)** Given that the mean and standard deviation of another class **Z** is 60 kg and 7 kg respectively, comment on the distribution of the weights of the students in both classes. [2]

**(b)** To achieve the ideal weight, a student must be 70 kg and below.

**(i)** Calculate the probability that if two students in class **Y** are chosen at random, one achieved an ideal weight but the other did not. [2]

**(ii)** Calculate the probability that if three students in class **Y** are chosen at random, at least one student achieved the ideal weight. [2]

**9.** In the diagram, $\overrightarrow{OA} = 12\mathbf{a}$ and $\overrightarrow{OB} = 9\mathbf{b}$. It is given that $OD = \frac{1}{2}DB$ and $OC = \frac{1}{3}OA$.




**(a)** Express, as simply as possible, in terms of **a** and **b**,

**(i)** $\overrightarrow{BC}$, [1]

**(ii)** $\overrightarrow{DA}$. [1]

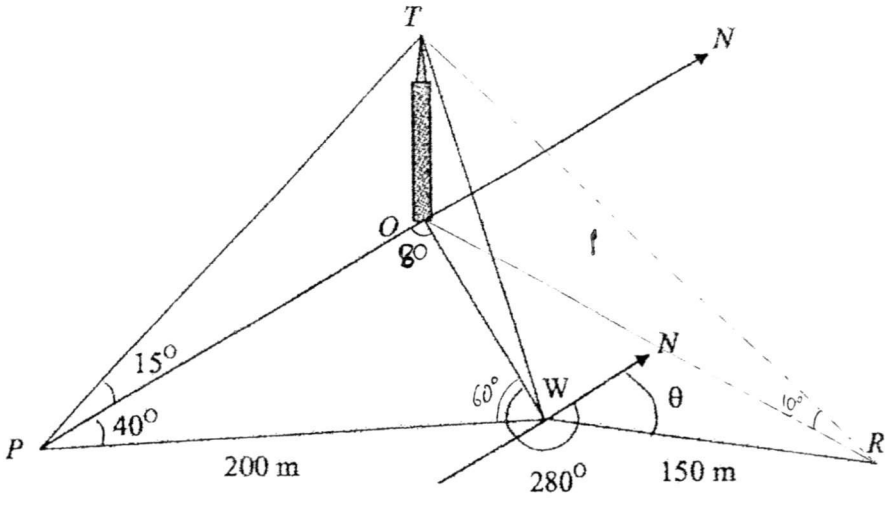
**(b)** Given that $\frac{\text{area of } \Delta ODE}{\text{area of } \Delta ODA} = \frac{1}{4}$, find the position vector of $E$ in terms of **a** and **b.** [2]

**(c)** Calculate the numerical value of $\frac{BE}{EC}$. [2]

**(d)** Show that the areas of $\Delta OCE$ and $\Delta ODE$ are equal. [3]

**(e)** Find $\frac{\text{area of } \Delta BDE}{\text{area of } \Delta BOC}$. [2]

**10.**




$O$, $P$, $W$ and $R$ are points on a horizontal plane. A vertical memorial tower, $OT$, is due north of $P$. The angle of elevation of a man at $P$ to the top of the tower is $15^{\circ}$. He walks a distance of 200 metres to point $W$. Given that the bearing of $W$ from $P$ is $040^{\circ}$ and the bearing of $O$ from $W$ *is* $280^{\circ}$,

calculate

**(a)** $\angle PWO$, [2]

**(b)** $OP$, [2]

**(c)** the height of the tower $OT$, [2]

**(d)** using the answer from part (c), the angle of depression of $W$ from $T$. [3]

From $W$, the man walks 150 metres to a point $R$ on a bearing of $\theta^{\circ}$ on the horizontal plane. At $R$, the angle of elevation of the man to the top of the tower is $10^{\circ}$.

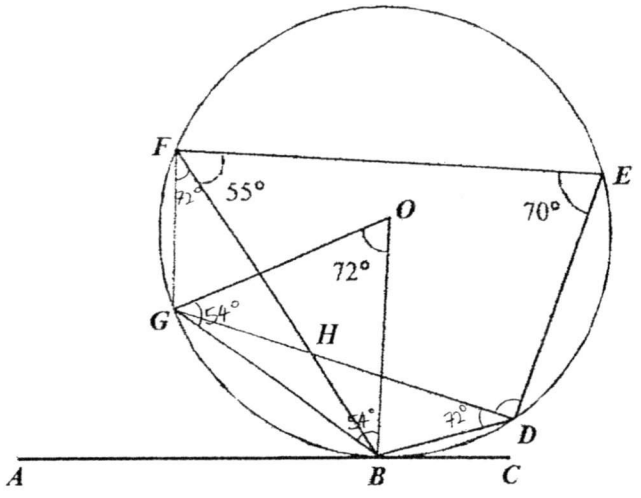
Calculate

**(e)** angle $OWR$, [3]

**(f)** the bearing of $\theta$ on the horizontal plane. [1]

18

**11.**




**(a)** In the diagram, $ABC$ is a tangent to the circle centre $O$. Given that $\angle BFE = 55^\circ$, $\angle FED = 70^\circ$ and $\angle GOB = 72^\circ$, calculate

**(i)** $\angle OBG$, [1]

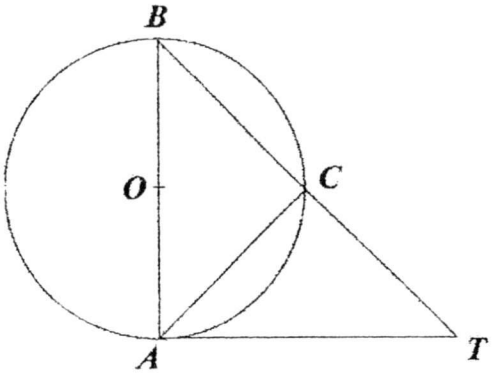
**(ii)** $\angle GDB$, [1]

**(iii)** $\angle EDG$, [1]

**(iv)** $\angle GHB$, [2]

**(v)** the sum of interior angles of the pentagon $BDEFG$. [2]

**(b)** In the diagram, $AB$ is the diameter of the circle with centre $O$. $AT$ is the tangent to the circle and $TB$ cuts the circle at $C$.




**(i)** Prove that $\Delta ABT$ is similar to $\Delta CAT$. [3]

**(ii)** Given that $BC = 18$ cm and $AT = 12$ cm, calculate the length of $CT$. [3]

---

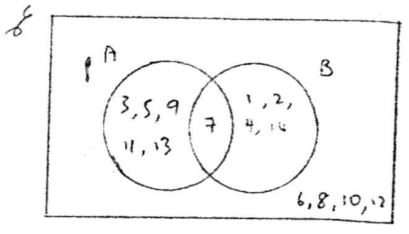
**END OF PAPER 2**

Answer Prelims 2015 Sec4/5 Paper 2

**1.** (a)(i) $A \cup B = \{1.2,3,4,5,7,9,11,13.14\}$ [B1]

(ii) $n(A \cap B') = 5$ [B2]

(b) [B2]




**2.** **(a)**

$$\sin\frac{\pi}{6} = \frac{4}{OC}$$

$$OC = 8cm$$

$$OD = \sqrt{8^2 - 4^2} = \sqrt{48}$$

$$OA = OB = 12cm$$

$$AD = 12 - \sqrt{48} = 5.0718$$ [M1]

$$\text{Arc } AB = r\theta = 12 \times \frac{\pi}{6} = 2\pi$$

$$\text{Angle } BCD = \frac{\pi}{2} + \frac{\pi}{6} = \frac{2\pi}{3}$$

$$\text{Arc } BD = 4 \times \frac{2\pi}{3} = \frac{8\pi}{3}$$ [M1]

$$\text{Per of the shaded region} = = \frac{8\pi}{3} + 2\pi + 5.0718 = 19.7cm$$ [A1]

**(b)** $$\text{Area of sector } OAB = \frac{1}{2}(8+4)^2\left(\frac{\pi}{6}\right) = 12\pi$$ [M1]

$$\text{Area of Triangle } OCD = \frac{1}{2}\left(\sqrt{48}\right)(4) = 2\sqrt{48}$$

$$\angle BCD = \frac{\pi}{2} + \frac{\pi}{6} = \frac{2\pi}{3}$$

$$\text{Area of sector } BCD = \frac{1}{2}(4)^2\left(\frac{2\pi}{3}\right) = \frac{16}{3}\pi$$ [M1]

$$\text{Area of shaded region} = 12\pi - 2\sqrt{48} - \frac{16}{3}\pi \approx 7.09 \text{ cm}^2$$ [A1]

١٩

3. **(a)** $\begin{pmatrix} 27 & 16 \\ 33 & 21 \\ 19 & 22 \end{pmatrix}$ [B1]

**(b)** $\begin{pmatrix} 47 & 27 \\ 68 & 40 \\ 47 & 42 \end{pmatrix}$ [B1]

**(c)** Total sales of each flavour in both bowls n cones respectively. [B1]

**(d)** $\begin{pmatrix} 1 & 2 & 2 \end{pmatrix}$ [B1]

**(e)** $\begin{pmatrix} 1 & 2 & 2 \end{pmatrix}\begin{pmatrix} 27 & 16 \\ 33 & 21 \\ 19 & 22 \end{pmatrix} = \begin{pmatrix} 131 & 102 \end{pmatrix}$ [B1]

**(f)** Total sales for Day 2 sold in cones and bowls respectively [B1]

4 **(a)** **(i)** $RP = \sqrt{6^2 + 18^2} = 18.973665$ [M1]
$\approx 19.0cm$

$PU = \sqrt{4.8^2 + 6^2} \approx 7.6837$

$PT = \sqrt{18^2 + 7.6837^2} \approx 19.6cm$ [A1]

OR

$PT = \sqrt{18.97366^2 + 4.8^2} \approx 19.6cm$

**(ii)** $\tan \angle RPT = \dfrac{4.8}{18.9737}$ [M1]

$\angle \text{RPT} \approx 14.2^{\circ}$ [A1]

**(b)** $PU = QT$ (Prop of Rect), $UR = TS$ (Diagonals of Rect), [B2,1]
$PR = QS$ (Diagonals of Rect)
Therefore by (SSS), triangles $PRU$ and $QST$ are congruent. (proven)

**(c)** Vol of device $= \dfrac{1}{2} \times 4.8 \times 6 \times 18 = 259.2cm^3$ [M1]

Vol of 10 devices $= 259.2 \times 10 = 2592cm^3$ [A1]

**5.** **(a)**

$$w = \sqrt{\frac{u-v}{1+u}}$$

$$w^2(1+u) = u - v \qquad \text{[M1]}$$

$$u(w^2 - 1) = -w^2 - v$$

$$u = \frac{w^2 + v}{1 - w^2} \text{ or } \frac{-w^2 - v}{w^2 - 1} \qquad \text{[A1]}$$

$$w \neq 1 \text{ or } w \neq -1 \qquad \text{[B1]}$$

**(b)**

$$\frac{x+1}{2x-3} - \frac{14-3x}{(2x-3)(2x+3)} = 0$$

$$\frac{(x+1)(2x+3) - (14-3x)}{(2x-3)(2x+3)} = 0 \qquad \text{[M1]}$$

$$2x^2 + 3x + 2x + 3 - 14 + 3x = 0$$

$$2x^2 + 8x - 11 = 0$$

$$x = \frac{-8 \pm \sqrt{64 - 4(2)(-11)}}{2(2)} \qquad \text{[B1]}$$

$$= 1.08 \text{ or } -5.08 \qquad \text{[A1]}$$

**(c)**

$$2^{2x^2+6} = 2^{5x+9} \qquad \text{[M1]}$$

$$\therefore 2x^2 + 6 = 5x + 9$$

$$(2x+1)(x-3) = 0 \qquad \text{[A1]}$$

$$\therefore x = -0.5 \text{ or } 3$$

**6.** **(a)**

$$\text{Av Speed} = \frac{720}{T} \qquad \text{[M1]}$$

$$60 = \frac{720}{T}$$

$$T = 12 \text{ hrs} \qquad \text{[A1]}$$

**(b)** $\text{Time}_{\text{A to B}} = \frac{360}{k}$ [B1]

$\text{Time}_{B \text{ to A}} = \frac{360}{k+15}$ [B1]

20

**(c)**

$$\frac{360}{k}+\frac{360}{k+15}=12$$

$$360(k+15)+360k=12k(k+15)$$ [M1]

$$360k+5400+360k=12k^2+180k$$ [M1]

$$0=12k^2-540k-5400$$

$$k^2-45k-450=0 \text{ (shown)}$$ [M1]

**(d)**

$$k^2-45k-450=0$$

$$=\frac{-(-45)\pm\sqrt{2025-4(1)(-450)}}{2(1)}$$ [M1]

$$=-8.42 \text{ or } 53.4$$ [A2]

8 **(a)(i)**

$$31+p+q=40$$

$$p+q=9----------(1)$$ [M1]

$$2132+60p+68q=40\times 68$$

$$15p+17q=147----------(2)$$

$$(1)\times 17:\ 17p+17q=153-------(3)$$

$$(3)-(2)\ 2p=6$$

$$p=3$$

$$q=6 \text{ (shown)}$$ [M1] [A1]

**(ii)** $\overline{X}=68$ [M1]

$$\text{std dev}=\sqrt{\frac{186272}{40}-(68)^2}$$

$$\approx 5.73kg$$ [A1]

**(iii)**

Mean of Class Y > Mean of Class Z

$\therefore$ Wt of Class Y > Wt of Class Z

Std Dev of Class Y < Std Dev of Class Z

$\therefore$ Spread of Class Y < Spread of Class Z [B2]

**(b)(i)** $$\text{P(ideal weight)}=\left(\frac{21}{40}\right)\left(\frac{19}{39}\right)+\left(\frac{19}{40}\right)\left(\frac{21}{39}\right)$$ [M1]

$$\text{P(ideal weight)}=\frac{133}{260}$$ [A1]

**(ii)** $$\text{P(at least 1 student ach ideal wt)}=1-\left(\frac{19}{40}\right)\left(\frac{18}{39}\right)\left(\frac{17}{38}\right)$$ [M1]

$$=\frac{469}{520}$$ [A1]

9.

**(a)** (i) $\overrightarrow{BC} = \overrightarrow{OC} - \overrightarrow{OB} = 4\mathbf{a} - 9\mathbf{b}$ [B1]

(ii) $\overrightarrow{DA} = \overrightarrow{OA} - \overrightarrow{OD} = 12\mathbf{a} - 3\mathbf{b}$ [B1]

**(b)** since

$$\frac{\text{Area of } \Delta ODE}{\text{Area of } \Delta ODA} = \frac{1}{4}$$

$$\frac{DE}{DA} = \frac{1}{4}$$ [M1]

$$\overrightarrow{DE} = \frac{1}{4}(12\mathbf{a} - 3\mathbf{b}) = 3\mathbf{a} - \frac{3}{4}\mathbf{b}$$

$$\overrightarrow{OE} = 3\mathbf{a} - \frac{3}{4}\mathbf{b} + 3\mathbf{b} = 3\mathbf{a} + \frac{9}{4}\mathbf{b}$$ [A1]

**(c)**

$$\overrightarrow{BE} = 3\mathbf{a} + \frac{9}{4}\mathbf{b} - 9\mathbf{b} = 3\left(\mathbf{a} - \frac{9}{4}\mathbf{b}\right)$$

$$\overrightarrow{EC} = \overrightarrow{OC} - \overrightarrow{OE} = 4\mathbf{a} - 3\mathbf{a} - \frac{9}{4}\mathbf{b} = \mathbf{a} - \frac{9}{4}\mathbf{b}$$ [M1]

$$\therefore \frac{BE}{EC} = \frac{3}{1}$$ [A1]

**(d)**

$$\frac{\text{Area of } \Delta OEC}{\text{Area of } \Delta OEB} = \frac{\frac{1}{2} \times EC \times h}{\frac{1}{2} \times BE \times h} = \frac{1}{3}$$ [M1]

$$\frac{\text{Area of } \Delta ODE}{\text{Area of } \Delta OEB} = \frac{\frac{1}{2} \times OD \times h}{\frac{1}{2} \times OB \times h} = \frac{1}{3}$$ [M1]

$$\therefore \frac{\text{Area of } \Delta OEC}{\text{Area of } \Delta ODE} = \frac{1}{1}$$ Hence, they are equal. [A1]

**(e)**

$$\therefore \frac{\text{Area of } \Delta BDE}{\text{Area of } \Delta BOC} = \frac{\text{Area of } \Delta BDE}{\text{Area of } \Delta BDE + \Delta ODE + \Delta OCE}$$ [M1]

$$= \frac{2}{2+1+1} = \frac{1}{4}$$ [A1]

21

**10** **(a)** $\angle PWO = 280^\circ - 180^\circ - 40^\circ \text{ (alt } \angle s)$ [M1]

$= 60^\circ$ [A1]

**(b)** $\dfrac{OP}{\sin 60} = \dfrac{200}{\sin 80}$ [M1]

$OP = 175.877$

$\approx 176 \text{ cm}$ [A1]

**(c)** $\tan 15 = \dfrac{OT}{175.877}$ [M1]

$\therefore\ OT = 47.1261 \approx 47.1m$ [A1]

**(d)** $OW^2 = 175.877^2 + 200^2 - 2(175.877)(200)\cos 40$ [M1]

$= 130.5407$

$\tan \angle OWT = \dfrac{47.1261}{130.5407}$ [M1]

$\angle$ of depression $= 19.8494$

$\approx 19.8^\circ$ [A1]

**(e)** $\tan \angle OWT = \dfrac{47.1261}{OR}$ [M1]

$\approx 2.67.265$

$\cos \angle OWR = \dfrac{130.5407^2 + 150^2 - 267.265^2}{2(130.5407)(150)}$ [M1]

$\angle OWR = 144.518 \approx 144.5^\circ$ [A1]

**(f)**

$\angle OWN = 180 - 100$

$\approx 80$

$\theta = 144.518 - 80$ [B1]

$\approx 064.5^\circ$

**11.** **(a)** **(i)** $\angle OBG = \dfrac{180 - 72}{2} = 54^\circ$ ($\angle$ sum of isoc $\Delta$) [B1]

**(ii)** $\angle GDB = \dfrac{72}{2} = 36^\circ$ ($\angle$ at ctr $= 2\angle$s at cir.) [B1]

**(iii)** $\angle GFB = 36^\circ$ ($\angle$ in same seg)

$\angle EDG = 180 - 36 - 55 = 89^\circ$ (opp$\angle$ cyclic quad) [B1]

**(iv)** $\angle FHD = 360 - 70 - 55 - 89 = 146^\circ$ ($\angle$sum of Quad) [M1]

$\angle GHB = \angle FHD = 146^\circ$ (vert opp $\angle$s) [A1]

**(v)** sum of int $\angle$s of $BDEFG = (5 - 2)180$ [M1]

$= 540^\circ$ [A1]

**(b)** **(i)** $\angle BAT = 90^{\circ}(\tan \perp rad)$ [M1]

$\angle BCA = 90^{\circ}$ ($\angle$ *in semicircle*) [M1]

$\angle ACT = 90^{\circ}$ (*adj*$\angle s$ on a st line)

$\angle BAT = \angle ACT = 90^{\circ}$

$\angle ATB = \angle CTA$(*common*$\angle$) [M1]

$\therefore \Delta ABT$ is similar to $\Delta CAT$

**(ii)** $\dfrac{CT}{AT} = \dfrac{AT}{BT}$ [M1]

$\dfrac{CT}{12} = \dfrac{12}{18+CT}$

$CT^2 + 18CT - 144 = 0$ [M1]

$(CT + 24)(CT - 6) = 0$

$CT = -24$ (NA) or 6 cm [A1]

22