Name:		Index Number:		Class:	

CATHOLIC HIGH SCHOOL
2015 Preliminary Examination 3
Secondary 4

MATHEMATICS

4016/01

Paper 1

1 September 2015
2 hours

Candidates answer in the space provided on the Question Paper.

READ THESE INSTRUCTIONS FIRST

Write your name, register number and class on all the work you hand in.
Write in dark blue or black pen.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.
DO **NOT** WRITE ON THE MARGINS.

Answer **all** questions.
If working is needed for any question, it must be shown with the answer.
Omission of essential working will result in loss of marks.
The use of an approved scientific calculator is expected, where appropriate.
If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to **three significant figures**. Give answers in **degrees to one decimal place**.
For π, use either your calculator value or 3.142, unless the question requires the answer in terms of π.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total of the marks for this paper is **80**.

For Examiner's Use only
80

This document consists of **20** printed pages. 80

Mathematical Formulae

Compound interest

$$\text{Total amount} = P\left(1+\frac{r}{100}\right)^n$$

Mensuration

$$\text{Curved surface area of a cone} = \pi r l$$

$$\text{Surface area of a sphere} = 4\pi r^2$$

$$\text{Volume of a cone} = \frac{1}{3}\pi r^2 h$$

$$\text{Volume of a sphere} = \frac{4}{3}\pi r^3$$

$$\text{Area of triangle } ABC = \frac{1}{2}ab\sin C$$

$$\text{Arc length} = r\theta\text{, where } \theta \text{ is in radians}$$

$$\text{Sector area} = \frac{1}{2}r^2\theta\text{, where } \theta \text{ is in radians}$$

Trigonometry

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

Statistics

$$\text{Mean} = \frac{\sum fx}{\sum f}$$

$$\text{Standard deviation} = \sqrt{\frac{\sum fx^2}{\sum f} - \left(\frac{\sum fx}{\sum f}\right)^2}$$

Answer **all** the questions.

1 **(a)** Express 864 as the product of its prime factors.

Answer (*a*)................................... [1]

(b) Written as a product of its prime factors, $720 = 2^4 \times 3^2 \times 5$.

Find the smallest possible integer value of n such $720n$ is a multiple of 864.

Answer (*b*) $n =$ .... [2]

---

2 The proton, which is the nucleus of a hydrogen atom, can be pictured as a sphere whose radius is 160 nanometres.

(a) Express 160 nanometres in metres, giving your answer in standard form.

Answer (*a*)...................................m [1]

(b) How many protons must be placed in a straight line to make a length of 64 cm? Give your answer in standard form.

Answer (*b*)................................... [1]

3 Factorise the following expressions completely.

(a) $2a^2 + 5ab - 3b^2$

Answer (*a*)................................... [1]

(b) $x^2 y - 4y - 12 + 3x^2$

Answer (*b*).. [3]

4 (a) Solve the inequality $x + 5 \le 3 + 2x < 24 - x$.

Answer (*a*)..................................[2]

(b) Hence, write down all the prime numbers that satisfy the inequality in **(a)**.

Answer (*b*)................................... [1]

5 An open field has an area of 112.5 km^2.
It is represented by an area of 18 cm^2 on map X.

(a) Find the scale of the map in the form $1 : n$.

Answer (*a*) 1 : .... [2]

(b) Map Y has a scale of $1 : 400\ 000$.
A road is measured 2.4 cm on Map X.

Find, in centimetres, the length representing this road on Map Y.

Answer (*b*)................................ cm [2]

---

6 Simplify $\left(\dfrac{x^6}{1000}\right)^{\frac{1}{3}} \div \left(256x^4\right)^{-\frac{1}{4}}$, leaving your answer in positive index.

Answer .... [2]

---

7 Solve the equation $\dfrac{2x}{2x-3}+1=\dfrac{1}{2-3x}$.

Answer $x=$ ....[3]

8 It is given that $\xi=\{x:0<x<15,\ x \text{ is an integer}\}$.
$A=\{x:3<x\le 9\}$ and $B=\{x:4\le x<12,\ x \text{ is a prime number}\}$.

(a) Draw a Venn diagram in the space below to illustrate this information.

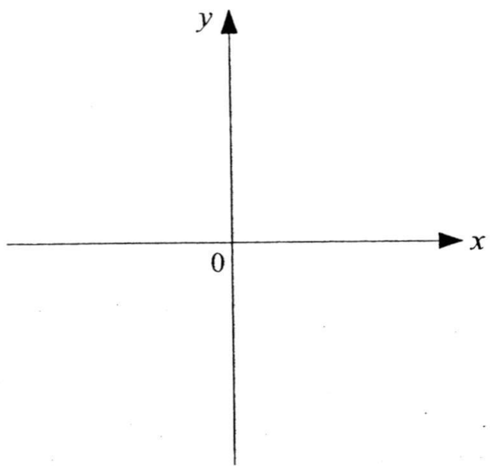
Answer (a) [2]

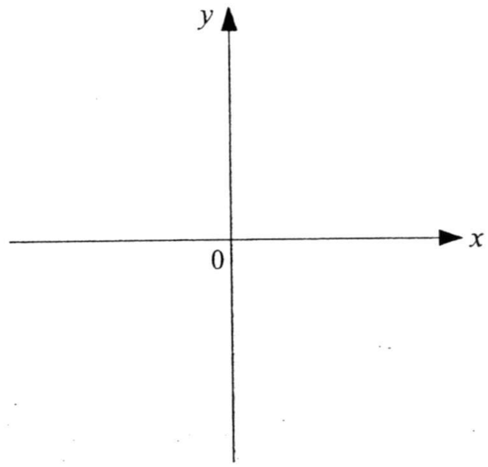
(b) Write down $n(A'\cap B)$.

Answer (b)..................................[1]

9 **(a)** On the axes given, sketch the following graphs, indicating the x and y intercepts where relevant.

(i) $y=-x^2$ **(ii)** $y=2^x$ [2]





(b) Hence explain why the equation $2^x+x^2=0$ has no solution.

Answer (b)..

....

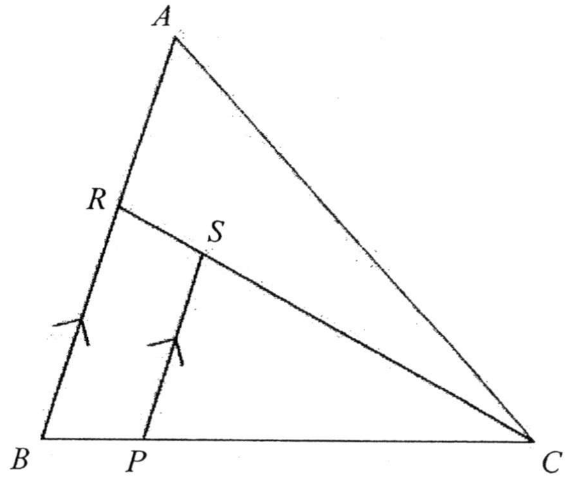
.... [1]

---

10 Simplify $\dfrac{3pq^3}{2r^2}\times\dfrac{qr^3}{5p^2}\div\dfrac{q^2}{10pr}$.

Answer .... [2]

11 In the figure, *AB* and *SP* are parallel lines. *P* lies on the line *BC* such that $BP : PC = 1 : 4$ and *R* lies on the line *AB* such that $BR : RA = 5 : 3$.



(a) Explain why triangles *BCR* and *PCS* are similar.

Answer (a) In triangles *BCR* and *PCS*, ...

....

....

....

.... [1]

(b) If the area of ΔBCR is 50 cm^2, calculate the area of ΔPCS.

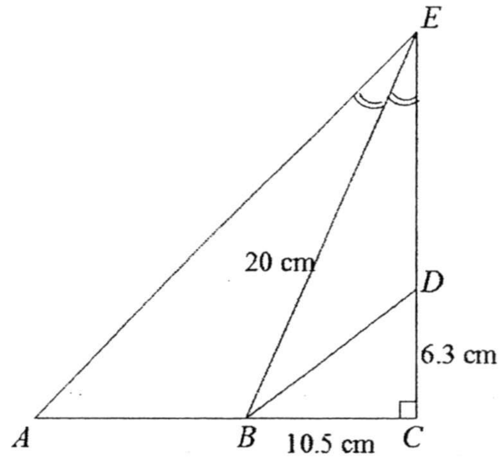
Answer (b).................................cm^2 [2]

(c) Find $\dfrac{\text{area of } \Delta PCS}{\text{area of } \Delta ABC}$, giving your answer as a fraction in the simplest form.

Answer (c).................................... [1]

12 In the figure, triangle ACE is a right-angle triangle.
B and D are points on AC and CE respectively such that $BC = 10.5$ cm, $BE = 20$ cm and $CD = 6.3$ cm.
The line BE bisects the angle AEC.

Find



(a) angle BEC,

Answer (a)..................................° [1]

(b) the length of AE,

Answer (b).............................. cm [2]

(c) $\tan \angle BDE$, giving your answer as a fraction in the simplest form.

Answer (c) .... [1]

13 Two stores advertise the same LED television set during the Great Singapore Sale.

STORE *A*

$1500 + 7% GST*

STORE *B*

- Deposit of $220

plus 12 monthly instalments of $120

- Price includes 7% GST*

**GST* : *Goods and Services Tax*

(a) Which store sells the television set at a lower price? Justify your answer.

Answer (*a*) Store .... offers a lower price because ....

....

....

....

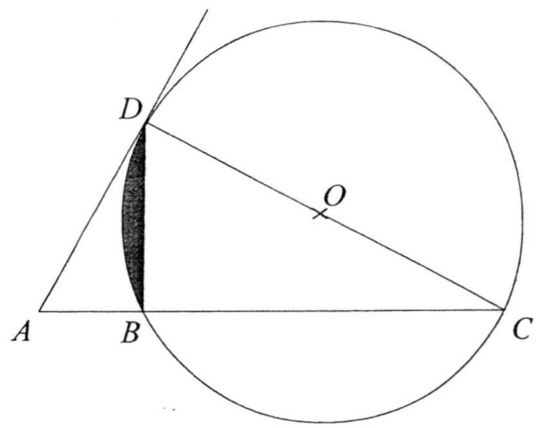
....

.... [2]

(b) Calculate the amount of GST charged on the television set in Store *B*.

Answer (*b*) \$.................................. [1]

14 In the diagram, O is the centre of circle BCD with radius 20 cm and CD is a diameter of the circle.
A is a point on BC produced such that AD is a tangent to the circle at D.



(a) Given that $\dfrac{AB}{AD} = \dfrac{1}{2}$, explain why angle $BCD = \dfrac{\pi}{6}$ radian.

Answer (*a*) [2]

(b) Calculate the area of the shaded region.

Answer (*b*)............................ cm^2 [3]

85

15 The illumination, I units, of a bulb varies inversely as the square of the distance, d metres. Given that the illumination is 9 units when the distance is 2 m,

(a) express I in terms of d,

Answer (a) $I =$ .... [1]

(b) find the percentage change in the illumination of the bulb which is required to reduce the distance of the bulb to $\frac{1}{5}$ of its original value.

Answer (b).............................. % [2]

---

16 Express $\dfrac{2}{x+2} - \dfrac{9x+20}{(x+3)(x+2)}$ as a fraction in its simplest form.

Answer .... [2]

---

17 Han Brothers Travel has tour packages to Australia, South Korea and Japan.

The table below shows the number of people who have signed up for the respective packages from January to June and from July to December.

	Period 1	Period 2
	From January to June	From July to December
Australia	15	29
South Korea	25	24
Japan	20	31

The information for the number of people who signed up for packages to Australia, South Korea and Japan can be represented by the matrix $\mathbf{P} = \begin{pmatrix} 15 & 29 \\ 25 & 24 \\ 20 & 31 \end{pmatrix}$.

(a) Write down a 1×3 matrix **Q** such that the matrix multiplication $\mathbf{C} = \mathbf{QP}$ gives the total number of people who signed up for the tour packages in period 1 and period 2 respectively.
Hence find **C**.

Answer (a) **Q** = ... [1]

C = ... [1]

(b) Given that $\mathbf{R} = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$, calculate $\mathbf{D} = \mathbf{CR}$.

Answer (b) **D** = ... [1]

(c) Describe what is represented by the element(s) of **D**.

Answer (c) ....

....

.... [1]

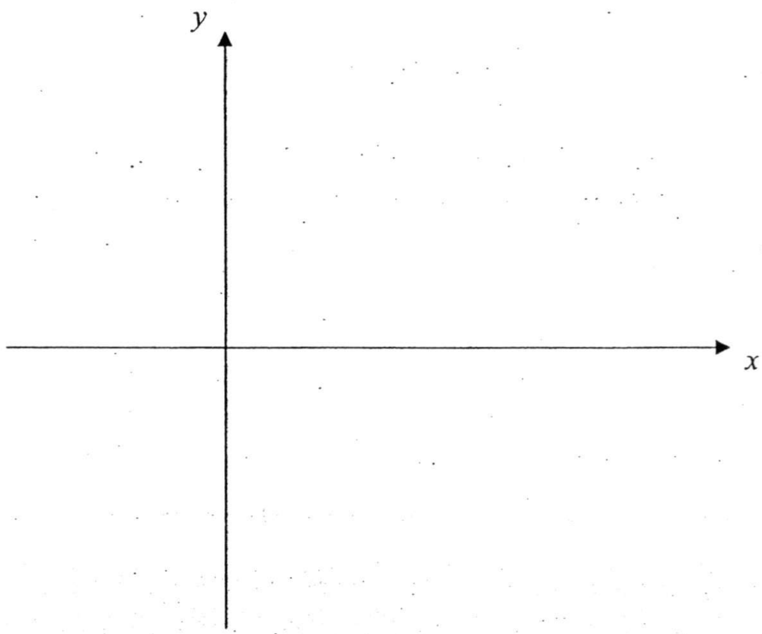
---

18 **(a)** Express $y = 4x - x^2$ in the form $y = a - (x+b)^2$, where a and b are constants.

Answer (*a*) $y =$ .... [2]

(b) Sketch the graph of $y = 4x - x^2$, showing the axes intercepts and coordinates of the turning point clearly.

Answer (*b*) [2]



19 Point A and B have coordinates (1, 3) and (1, 15) respectively.
Point P lies on the line $y = 6 + x$ and is equidistant from A and B.

(a) Explain, with working clearly shown, why P is **not** the mid-point of AB.

Answer (a) [2]

(b) Find the coordinates of P.

Answer (b) P (.............. , ...) [2]

---

20 The average daily temperature, in degrees Celsius (°C), of City A measured in 11 days are shown below.

29.5 32.1 33.2 30.1 33.9 32.7 29.9 33.0 30.7 33.6 29.9

(a) Find

(i) the median temperature,

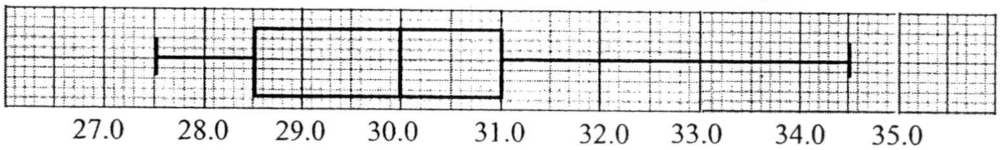
Answer (a)(i) .... °C [1]

87

(ii) the interquartile range.

Answer (*a*)(ii) ....°C [2]

(b) The box-and-whisker diagram shows the average temperature of City B in the same period.



A report commented that the average temperature of City B during these 11 days is generally higher than that of City A.

Do you agree? Give a reason to support your answer.

Answer (*b*) ....

....

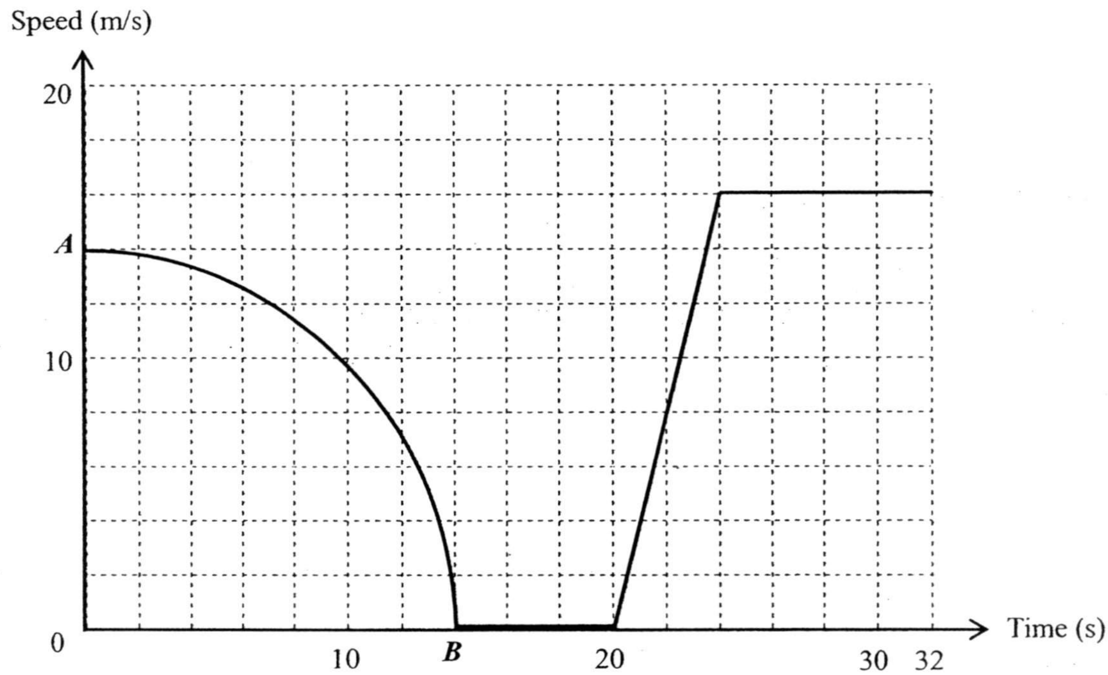
.... [1]

[Turn over

21 The diagram below shows the speed-time graph of a car for a journey.

The car decelerates from an initial speed of 14 m/s for the first 14 seconds and remained at rest for the next 6 seconds. It then accelerates uniformly for the next 4 seconds and remained at a constant speed of 16 m/s thereafter.

AB is an arc of a quadrant.



(a) Find the acceleration at $t = 23$ s.

Answer (a) .... m/s^2 [1]

(b) Taking π to be $\frac{22}{7}$, find the total distance travelled for the entire journey.

Answer (b).................................... m [2]

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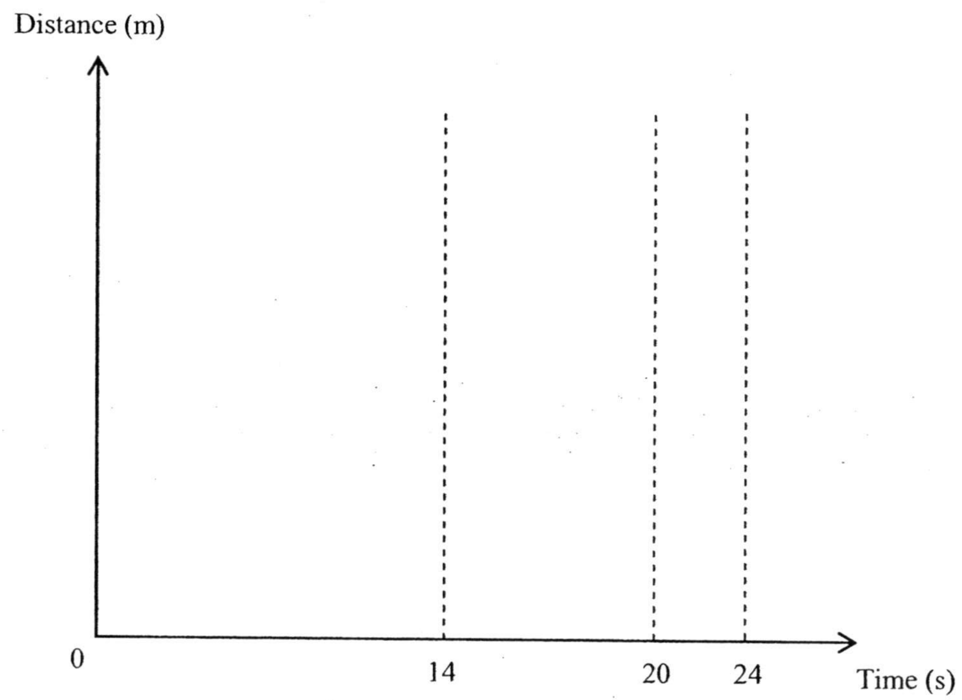
(c) On the axes provided below, sketch the distance-time graph of the vehicle for the first 24 seconds of the journey, indicating the distances travelled after $t = 14$ s, $t = 20$ s and $t = 24$ s on the vertical axis clearly.

Answer (*c*) [2]



22 Magician William randomly draws 2 cards from a stack of 5 cards labelled 5 to 9, one after another without replacement.

The sum of the numbers on the two cards is obtained.

(a) Complete the possibility diagram in the answer space below.

Answer (*a*) [2]

+	5	6	7	8	9
5		11			
6	11				
7					
8					
9					

(b) Calculate the probability that the sum obtained is a multiple of 5.

Answer (*b*) .... [1]

(c) A third card is chosen at random from the stack without replacement.
Find the probability that the sum of the numbers on the three cards is 24.

Answer (*c*) .... [2]

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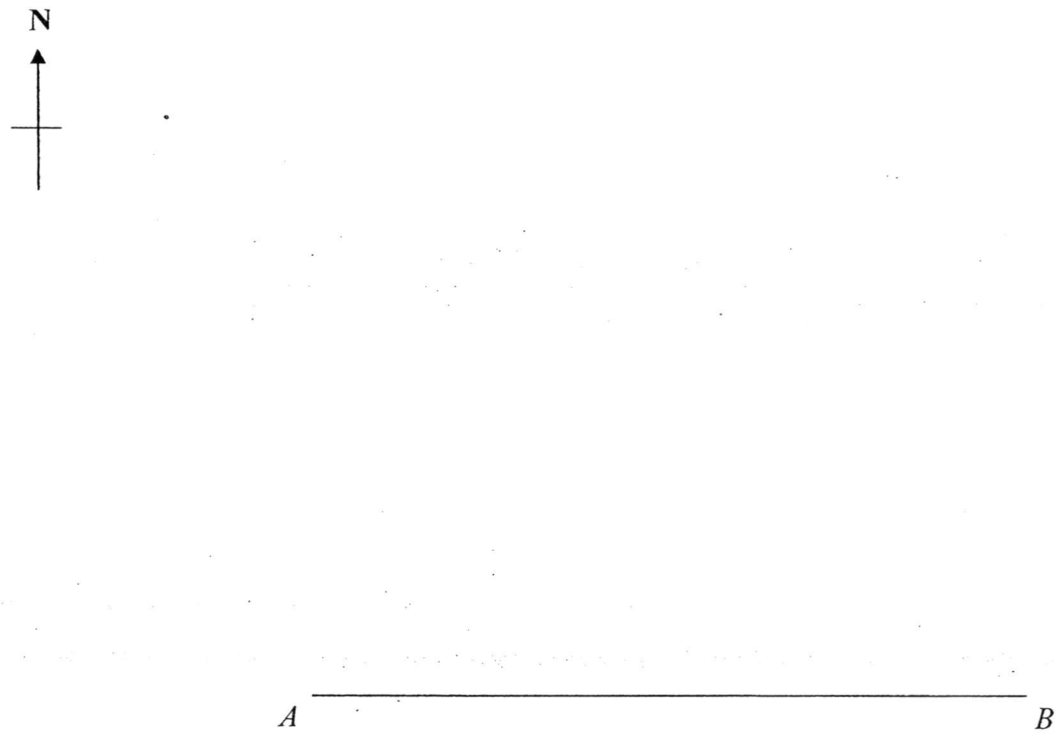
23 In a triangular field ABC, B is due east of A, C is at a bearing of $025°$ from A and $AC = 460$ m.

AB has already been drawn.

(a) Using a scale of 1 cm to represent 50 m, construct the triangular field ABC.

Answer (a) [1]



(b) A statue is located at point Q which is equidistant from the points B and C and equidistant from the lines AC and AB.

By constructing the appropriate perpendicular and angle bisectors, mark the point Q and estimate the actual distance CQ.

Answer (b) $CQ =$ ....m [3]

END OF PAPER

Answer **all** the questions.

1 **(a)** Express 864 as the product of its prime factors.

Answer (a) $2^5 \times 3^3$ [1]

(b) Written as a product of its prime factors, $720 = 2^4 \times 3^2 \times 5$.

Find the smallest possible integer value of n such $720n$ is a multiple of 864.

LCM of 720 and 864 $= 2^5 \times 3^3 \times 5 = 4320$

$$\therefore \; n = \frac{4320}{720} = 6$$

Answer (b) $n =$ 6 [2]

---

2 The proton, which is the nucleus of a hydrogen atom, can be pictured as a sphere whose radius is 160 nanometres.

(a) Express 160 nanometres in metres, giving your answer in standard form.

Answer (a) 1.6×10^{-7} m [1]

(b) How many protons must be placed in a straight line to make a length of 64 cm? Give your answer in standard form.

Answer (b) 2×10^6 [1]

---

3 Factorise the following expressions completely.

(a) $2a^2 + 5ab - 3b^2$

Answer (a) $(2a-b)(a+3b)$ [1]

(b) $x^2y - 4y - 12 + 3x^2$

$$x^2y - 4y - 12 + 3x^2 = y(x^2-4) + 3(x^2-4)$$

$$= (x^2-4)(y+3)$$

$$= (x+2)(x-2)(y+3)$$

Answer (b) $(x+2)(x-2)(y+3)$ [3]

---

4 (a) Solve the inequality $x + 5 \le 3 + 2x < 24 - x$.

$x + 5 \le 3 + 2x < 24 - x$

$x + 5 \le 3 + 2x$ and $3 + 2x < 24 - x$

$x \ge 2$ $\quad$ $3x < 21$

$x < 7$

Hence, $2 \le x < 7$

Answer (a) $2 \le x < 7$ [2]

(b) Hence, write down all the prime numbers that satisfy the inequality in **(a)**.

Answer (b) 2 , 3 , 5 [1]

5 An open field has an area of 112.5 km^2.

It is represented by an area of 18 cm^2 on map X.

(a) Find the scale of the map in the form 1 : n.

$18cm^2 : 112.5km^2$

$1cm^2 : 6.25km^2$

$1cm : 2.5km$

$1cm : 250000cm$

Answer (*a*) 1 : 250 000 [2]

(b) Map Y has a scale of $1 : 400\ 000$.
A road is measured 2.4 cm on Map X.

Find, in centimetres, the length representing this road on Map Y.

$MapX : 1cm \rightarrow 2.5km$

$\therefore 2.4cm \rightarrow 6km$

$MapY : 1cm \rightarrow 4km$

$\therefore 6km \rightarrow 1.5cm$

Answer (*b*) 1.5 cm [2]

---

6 Simplify $\left(\frac{x^6}{1000}\right)^{\frac{1}{3}} \div \left(256x^4\right)^{-\frac{1}{4}}$, leaving your answer in positive index.

$$\left(\frac{x^6}{1000}\right)^{\frac{1}{3}} \div \left(256x^4\right)^{-\frac{1}{4}} = \frac{x^2}{10} \div \frac{1}{4x}$$

$$= \frac{2x^3}{5}$$

Answer $\frac{2x^3}{5}$ [2]

---

7 Solve the equation $\dfrac{2x}{2x-3}+1=\dfrac{1}{2-3x}$.

$$\frac{2x}{2x-3}+1=\frac{1}{2-3x}$$

$$\frac{2x+(2x-3)}{2x-3}=\frac{1}{2-3x}$$

$$\frac{4x-3}{2x-3}=\frac{1}{2-3x}$$

$$(4x-3)(2-3x)=2x-3$$

$$-12x^2+17x-6=2x-3$$

$$12x^2-15x+3=0$$

$$4x^2-5x+1=0$$

$$(4x-1)(x-1)=0$$

$$x=\frac{1}{4} \quad \text{or} \quad x=1$$

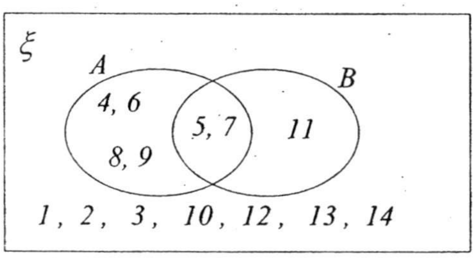
Answer $x=\dfrac{1}{4}$ or 1 [3]

---

8 It is given that $\xi=\{x:0<x<15,\ x \text{ is an integer}\}$.
$A=\{x:3<x\le 9\}$ and $B=\{x:4\le x<12,\ x \text{ is a prime number}\}$.

(a) Draw a Venn diagram in the space below to illustrate this information.

Answer (*a*) [2]



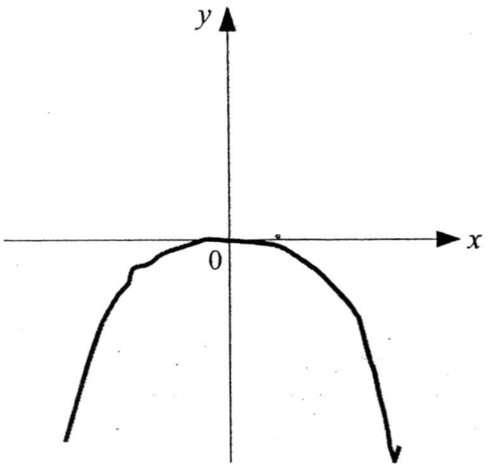
(b) Write down $n(A' \cap B)$.

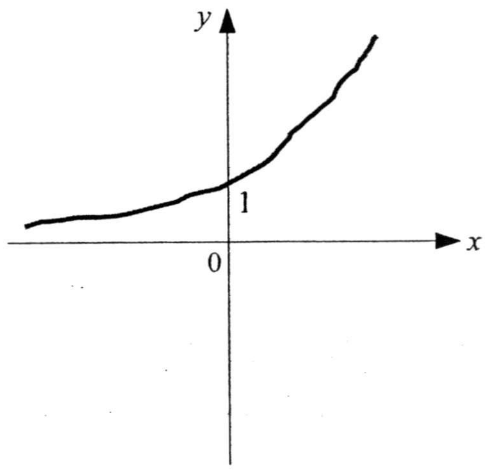
Answer (*b*) 1 [1]

---

9 **(a)** On the axes given, sketch the following graphs, indicating the x and y intercepts where relevant.

(i) $y=-x^2$ **(ii)** $y=2^x$ [2]





(b) Hence explain why the equation $2^x+x^2=0$ has no solution.

Answer (*b*) $2^x+x^2=0$ has no solution since the graphs of $y=2^x$ and $y=-x^2$ do not intersect. [1]

---

10 Simplify $\dfrac{3pq^3}{2r^2}\times\dfrac{qr^3}{5p^2}\div\dfrac{q^2}{10pr}$.

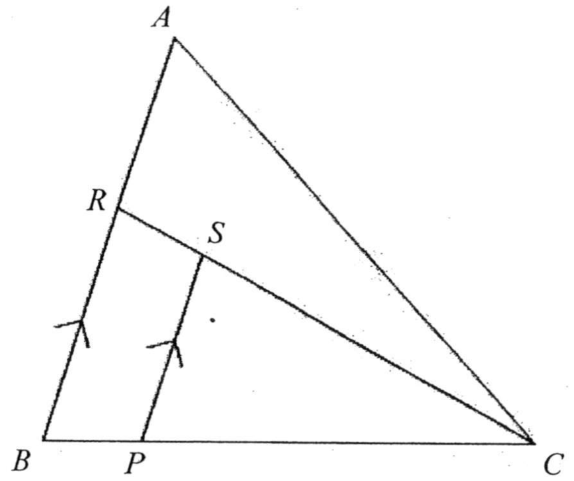
$$\frac{3pq^3}{2r^2}\times\frac{qr^3}{5p^2}\div\frac{2q^2}{10pr}=\frac{3pq^3}{2r^2}\times\frac{qr^3}{5p^2}\times\frac{10pr}{q^2}$$

$$=3q^2r^2$$

Answer $3q^2r^2$ [2]

92

11 In the figure, AB and SP are parallel lines. P lies on the line BC such that $BP : PC = 1 : 4$ and R lies on the line AB such that $BR : RA = 5 : 3$.



(a) Explain why triangles BCR and PCS are similar.

In triangles BCR and PCS,

$\angle BCR$ is shared

$\angle CBR = \angle CPS$ (corresponding angles)

$\angle CRB = \angle CSP$ (angle sum of triangle)

(b) If the area of ΔBCR is 50 cm^2, calculate the area of ΔPCS.

$$\frac{\text{Area of }\Delta PCS}{\text{Area of }\Delta BCR} = \left(\frac{4}{5}\right)^2 = \frac{16}{25}$$

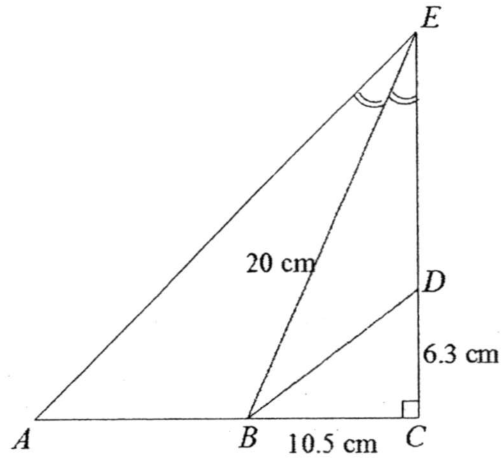
$$\text{Area of }\Delta PCS = \frac{16}{25} \times 50 = 32 \text{ cm}^2$$

Answer (*b*) 32 cm^2 [2]

(c) Find $\dfrac{\text{area of }\Delta PCS}{\text{area of }\Delta ABC}$, giving your answer as a fraction in the simplest form.

Answer (*c*) $\dfrac{2}{5}$ [1]

12 In the figure, triangle ACE is a right-angle triangle.
B and D are points on AC and CE respectively such that $BC = 10.5$ cm, $BE = 20$ cm and $CD = 6.3$ cm.
The line BE bisects the angle AEC.



Find

(a) angle BEC,

$$\sin\angle BEC = \frac{10.5}{20}$$

$$\angle BEC = 31.66° \approx 31.7°$$

Answer (*a*) 31.7 ° [1]

(b) the length of AE,

$$CE = \sqrt{20^2 - 10.5^2} = \sqrt{289.75}$$

$$\angle AEC = 2(31.66) = 63.32°$$

$$\cos 63.32 = \frac{\sqrt{289.75}}{AE}$$

$$AE = 37.9$$

Answer (*b*) 37.9 cm [2]

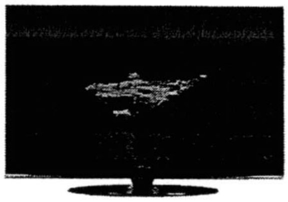
(c) $\tan\angle BDE$, giving your answer as a fraction in the simplest form.

Answer (*c*) $-\frac{5}{3}$ [1]

13 Two stores advertise the same LED television set during the Great Singapore Sale.

STORE *A*

$1500 + 7% GST*



STORE *B*

- Deposit of $220

  plus 12 monthly instalments of $120

- Price includes 7% GST*

**GST : Goods and Services Tax*

(a) Which store sells the television set at a lower price? Justify your answer.

Answer (*a*) Store **A** offers a lower price because

$$\text{Price of LCD TV (Store A)} = \frac{107}{100} \times \$1500$$
$$= \$1605$$
$$\text{Price of LCD TV (Store B)} = \$220 + 12 \times \$120$$
$$= \$1660$$

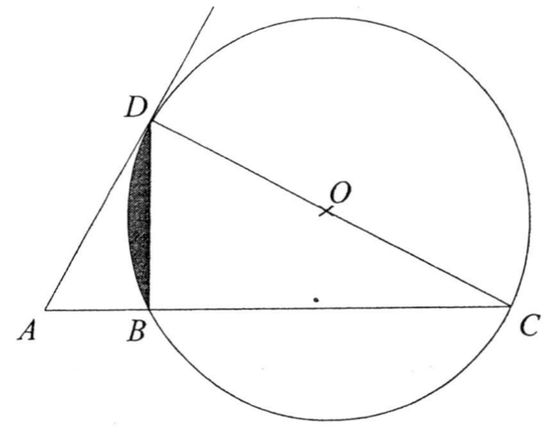
Store A offers better deal since the cost of the TV set is lower at Store A. [2]

(b) Calculate the amount of GST charged on the television set in Store *B*.

$$\text{Amount of GST of the TV set in Store B} = \frac{7}{107} \times \$1650$$
$$= \$107.94$$

Answer (*b*) $107.94 $ [1]

14 In the diagram, O is the centre of circle BCD with radius 20 cm and CD is a diameter of the circle.
A is a point on BC produced such that AD is a tangent to the circle at D.



(a) Given that $\frac{AB}{AD}=\frac{1}{2}$, explain why angle $BCD=\frac{\pi}{6}$ radian.

Answer (*a*) [2]

$$\angle ADB=\sin^{-1}\frac{1}{2}=\frac{\pi}{6}$$

$$\angle BDC=\frac{\pi}{2}-\frac{\pi}{6} \text{ (tangent perpendicular radius)}$$

$$=\frac{\pi}{3}$$

$$BCD=\pi-\frac{\pi}{2}-\frac{\pi}{3}=\frac{\pi}{6}$$

(b) Calculate the area of the shaded region.

$$\angle BOD=2\left(\frac{\pi}{6}\right)=\frac{\pi}{3}$$

$$\text{shaded area}=\frac{1}{2}\left(20^2\right)\left(\frac{\pi}{3}\right)-\frac{1}{2}\left(20^2\right)\sin\frac{\pi}{3}$$

$$=36.2$$

Answer (*b*) 36.2 cm^2 [3]

15 The illumination, I units, of a bulb varies inversely as the square of the distance, d metres. Given that the illumination is 9 units when the distance is 2 m,

(a) express I in terms of d,

$$I = \frac{k}{d^2}$$

$$k = (9)(2^2)$$

$$= 36$$

$$\therefore I = \frac{36}{d^2}$$

Answer (a) $I = \frac{36}{d^2}$ [1]

(b) find the percentage change in the illumination of the bulb which is required to reduce the distance of the bulb to $\frac{1}{5}$ of its original value.

$$I_O = \frac{k}{d^2}$$

$$I_N = \frac{k}{(0.2d)^2} = \frac{25k}{d^2}$$

$$\% \text{ change} = \frac{\frac{25k}{d^2} - \frac{k}{d^2}}{\frac{k}{d^2}} \times 100 = 2400$$

Answer (b) 2400% [2]

---

16 Express $\frac{2}{x+2} - \frac{9x+20}{(x+3)(x+2)}$ as a fraction in its simplest form.

$$\frac{2}{x+2} - \frac{9x+20}{(x+3)(x+2)} = \frac{2(x+3)-(9x+20)}{(x+3)(x+2)}$$

$$= \frac{2x+6-9x-20}{(x+3)(x+2)}$$

$$= \frac{-7x-14}{(x+2)(x+3)}$$

$$= \frac{-7(x+2)}{(x+2)(x+3)} = \frac{-7}{x+3}$$

Answer $\frac{-7}{x+3}$ [2]

17 Han Brothers Travel has tour packages to Australia, South Korea and Japan.

The table below shows the number of people who have signed up for the respective packages from January to June and from July to December.

	Period 1	Period 2
	From January to June	From July to December
Australia	15	29
South Korea	25	24
Japan	20	31

The information for the number of people who signed up for packages to Australia, South Korea and Japan can be represented by the matrix $\mathbf{P} = \begin{pmatrix} 15 & 29 \\ 25 & 24 \\ 20 & 31 \end{pmatrix}$.

(a) Write down a 1×3 matrix **Q** such that the matrix multiplication $\mathbf{C} = \mathbf{QP}$ gives the total number of people who signed up for the tour packages in period 1 and period 2 respectively.
Hence find **C**.

Answer (a) $\mathbf{Q} = \begin{pmatrix} 1 & 1 & 1 \end{pmatrix}$ [1]

$\mathbf{C} = \begin{pmatrix} 60 & 84 \end{pmatrix}$ [1]

(b) Given that $\mathbf{R} = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$, calculate $\mathbf{D} = \mathbf{CR}$.

$$D = \begin{pmatrix} 60 & 84 \end{pmatrix} \begin{pmatrix} 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 144 \end{pmatrix}$$

Answer (b) $\mathbf{D} = \begin{pmatrix} 144 \end{pmatrix}$ [1]

(c) Describe what is represented by the element(s) of **D**.

Answer (*c*) Total number of people who signed up for the tour packages in the year. [1]

---

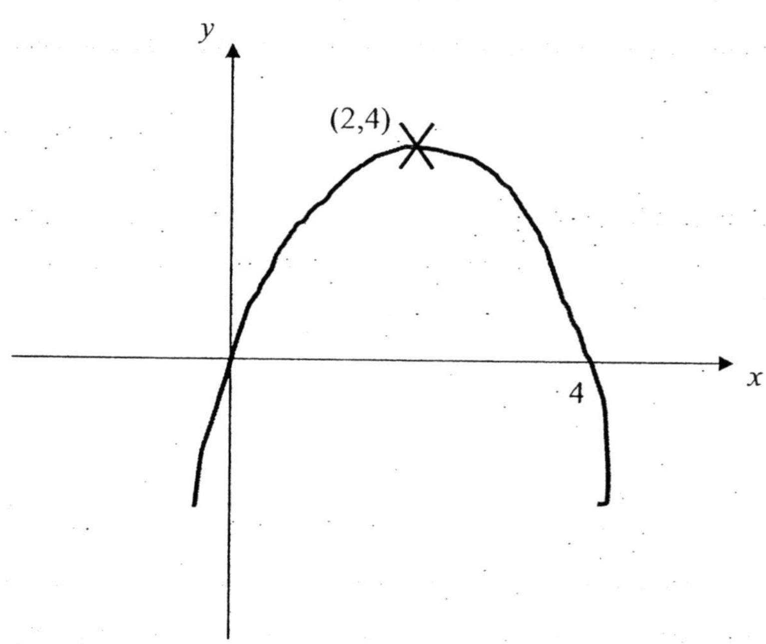
18 **(a)** Express $y = 4x - x^2$ in the form $y = a - (x+b)^2$, where a and b are constants.

$$\begin{aligned} y &= -(x^2 - 4x) \\ &= -[(x-2)^2 - 4] \\ &= -(x-2)^2 + 4 \end{aligned}$$

Answer (*a*) $y = 4 - (x-2)^2$ [2]

(b) Sketch the graph of $y = 4x - x^2$, showing the axes intercepts and coordinates of the turning point clearly.

Answer (*b*) [2]



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19 Point A and B have coordinates $(1, 3)$ and $(1, 15)$ respectively.
Point P lies on the line $y = 6 + x$ and is equidistant from A and B.

(a) Explain, with working clearly shown, why P is **not** the mid-point of AB.

Answer (*a*) [2]

Mid-point of $AB = (1, 9)$

Substituting $x = 1$ into $y = 6 + x$, $y = 7 \neq 9$

Thus, the mid-point of AB does not lie on the line $y = 6 + x$ and P cannot be the mid-point.

(b) Find the coordinates of P.

Point P lies along the line $y = 9$ since it is at equidistant from A and from B.

Since P lies on the line $y = 6 + x$, substituting $y = 9$, $x = 3$

Answer (*b*) $P(3, 9)$ [2]

---

20 The average daily temperature, in degrees Celsius (°C), of City A measured in 11 days are shown below.

29.5 32.1 33.2 30.1 33.9 32.7 29.9 33.0 30.7 33.6 29.9

(a) Find

(i) the median temperature,

Answer (*a*)(i) 32.1°C [1]

96

(ii) the interquartile range.

$Q_1 = 29.9$

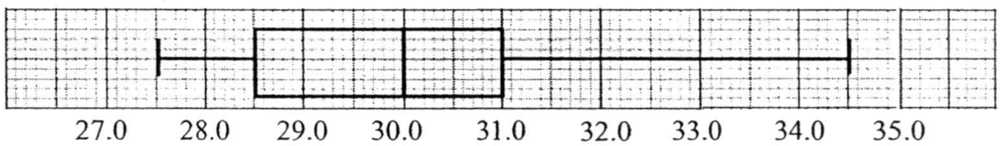
$Q_3 = 33.2$

Interquartile range $= 33.2 - 29.9$

$= 3.3^\circ C$

Answer (*a*)(ii) 3.3°C [2]

(b) The box-and-whisker diagram shows the average temperature of City B in the same period.



A report commented that the average temperature of City B during these 11 days is generally higher than that of City A.

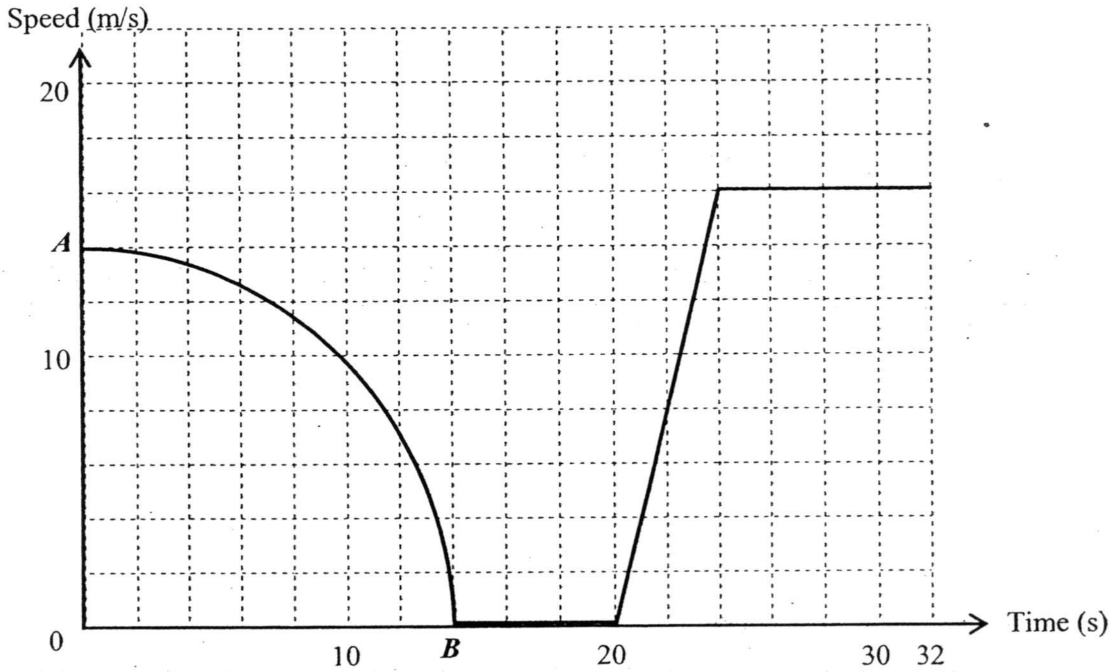
Do you agree? Give a reason to support your answer.

Answer (*b*) No. Median of City A is 32.1°C, higher than that of B, 30.0°C. [1]

21 The diagram below shows the speed-time graph of a car for a journey.

The car decelerates from an initial speed of 14 m/s for the first 14 seconds and remained at rest for the next 6 seconds. It then accelerates uniformly for the next 4 seconds and remained at a constant speed of 16 m/s thereafter.

AB is an arc of a quadrant.



(a) Find the acceleration at $t = 23$ s.

$$\frac{16-0}{24-20} = 4 \text{ m/s}^2$$

Answer (a) 4 m/s^2 [1]

(b) Taking π to be $\frac{22}{7}$, find the total distance travelled for the entire journey.

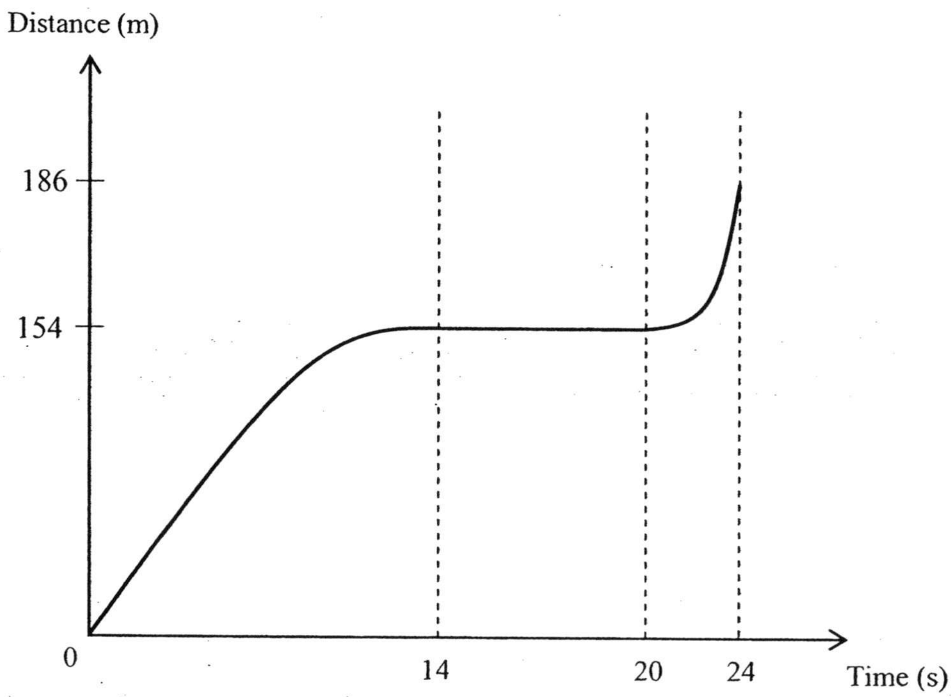
$$\text{Total distance} = \frac{1}{4}\left(\frac{22}{7}\right)(14^2) + \frac{1}{2}(16)(8+12)$$
$$= 154 + 160$$
$$= 314$$

Answer (b) 314 m [2]

97

(c) On the axes provided below, sketch the distance-time graph of the vehicle for the first 24 seconds of the journey, indicating the distances travelled at $t = 14$ s and $t = 24$ s on the vertical axis clearly.

Answer (*c*) [2]



22 Magician William randomly draws 2 cards from a stack of 5 cards labelled 5 to 9, one after another without replacement.

The sum of the numbers on the two cards is obtained.

(a) Complete the possibility diagram in the answer space below.

Answer (a) [2]

+	5	6	7	8	9
5	X	11	12	13	14
6	11	X	13	14	15
7	12	13	X	15	16
8	13	14	15	X	17
9	14	15	16	17	X

(b) Calculate the probability that the sum obtained is a multiple of 5.

Answer (b) $\frac{1}{5}$ [1]

(c) A third card is chosen at random from the stack without replacement. Find the probability that the sum of the numbers on the three cards is 24.

$$P(\text{sum is } 24)=6\left(\frac{1}{5}\times\frac{1}{4}\times\frac{1}{3}\right)=\frac{1}{10}$$

OR $$P(\text{sum is } 24)=\frac{6}{20}\times\frac{1}{3}=\frac{1}{10}$$

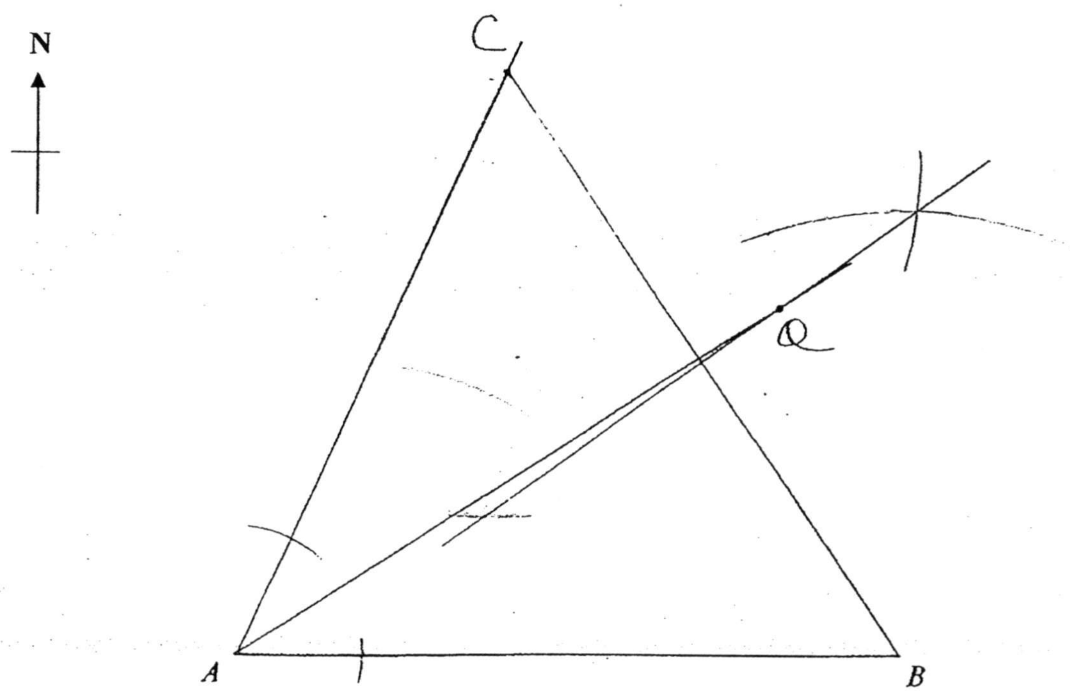
Answer (c) $\frac{1}{10}$ [2]

23 In a triangular field ABC, B is due east of A, C is at a bearing of $025°$ from A and $AC = 460$ m .

AB has already been drawn.

(a) Using a scale of 1 cm to represent 50 m, construct the triangular field ABC.

Answer (*a*) [1]



(b) A statue is located at point Q which is equidistant from the points B and C and equidistant from the lines AC and AB.

By constructing the appropriate perpendicular and angle bisectors, mark the point Q and estimate the actual distance CQ.

Answer (*b*) 250 - 300 m [3]

END OF PAPER

Name:		Index Number:		Class:	

CATHOLIC HIGH SCHOOL
2015 Preliminary Examination 3
Secondary 4

MATHEMATICS **4016/02**

Paper 2

2 September 2015
2 hours 30 minutes

Additional materials: Answer booklets **A, B** and **C**

READ THESE INSTRUCTIONS FIRST

Write your name, register number and class on all the work you hand in.
Write in dark blue or black pen.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.
DO **NOT** WRITE ON THE MARGINS.

Answer **all** questions.
Attempt **questions 1 to 4** on **Answer booklet A**,
questions 5 to 8 on **Answer booklet B**,
questions 9 to 11 on **Answer booklet C**.

If working is needed for any question, it must be shown with the answer.
Omission of essential working will result in loss of marks.
The use of an approved scientific calculator is expected, where appropriate.
If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to **three significant figures**. Give answers in **degrees to one decimal place**.
For π, use either your calculator value or 3.142, unless the question requires the answer in terms of π.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total of the marks for this paper is **100**.

This document consists of **12** printed pages. 99

Mathematical Formulae

Compound interest

$$\text{Total amount} = P\left(1+\frac{r}{100}\right)^n$$

Mensuration

$$\text{Curved surface area of a cone} = \pi r l$$

$$\text{Surface area of a sphere} = 4\pi r^2$$

$$\text{Volume of a cone} = \frac{1}{3}\pi r^2 h$$

$$\text{Volume of a sphere} = \frac{4}{3}\pi r^3$$

$$\text{Area of triangle } ABC = \frac{1}{2}ab\sin C$$

$$\text{Arc length} = r\theta \text{, where } \theta \text{ is in radians}$$

$$\text{Sector area} = \frac{1}{2}r^2\theta \text{, where } \theta \text{ is in radians}$$

Trigonometry

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

Statistics

$$\text{Mean} = \frac{\Sigma f x}{\Sigma f}$$

$$\text{Standard deviation} = \sqrt{\frac{\Sigma f x^2}{\Sigma f} - \left(\frac{\Sigma f x}{\Sigma f}\right)^2}$$

Answer **all** the questions.

Attempt **questions 1 to 4** on **Answer Booklet A.**

1 **(a)** Given that $x^2 + y^2 = a$ and $xy = b$, find $(2x - 2y)^2$ in terms of a and b. [2]

(b) Given that $\dfrac{3a - bc}{2ac - 5b} = \dfrac{1}{2}$,

(i) find the exact value of c when $a = -3$ and $b = 2$, [1]

(ii) express c in terms of a and b, [2]

(c) Given that $\dfrac{y}{x} = 2015$ and $\dfrac{z}{y} = 2015$, where $x \neq 0$ and $y \neq 0$.

Find the value of $\dfrac{y+z}{x+y}$. [2]

---

2 Teddy, Colin and Azmat each decided to buy a new motorbike that was priced at \$8 500.

(a) Teddy was given a discount and he paid \$8202.50 for the motorbike in cash.

(i) Calculate the percentage discount he received. [1]

(ii) The value of the new motorbike depreciates by 12% during the first year. In the second year, its value depreciates by 20% of its value at the beginning of the year.

If Teddy sold off his motorbike at the end of the second year, how much money would he lose? [3]

(b) Colin paid a down payment of 20% of the price of the motorbike and the balance to be paid by instalments at a simple interest rate of 5% per annum for a period of 2.5 years.

Calculate

(i) the amount of down payment paid, [1]

(ii) the amount of instalment payable in a month. [2]

(c) Azmat paid a down payment of \$2612 and the balance to be paid at the end of 2 years with compound interest rate of 3.75% per annum.

How much would the motorbike cost him altogether? [2]

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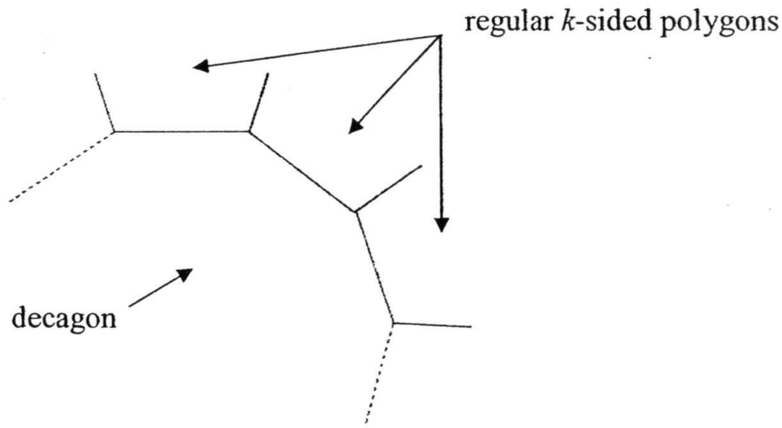
3 A number of regular k-sided polygons are placed together in a ring to form a regular decagon as shown in the diagram below.



(a) Find

(i) the interior angle of the decagon, [2]

(ii) the value of k. [2]

(b) If n-sided regular polygons that are placed together in a ring to form a N-sided polygon,

(i) show that $N = \dfrac{2n}{n-4}$. [3]

(ii) Hence or otherwise, explain why a regular octagon cannot be formed by placing a number of regular polygons in a ring. [1]

---

4 **(a)** The position vectors of a point A, B and C are $\begin{pmatrix}-3\\5\end{pmatrix}$, $\begin{pmatrix}5\\20\end{pmatrix}$ and $\begin{pmatrix}10\\0\end{pmatrix}$ respectively.

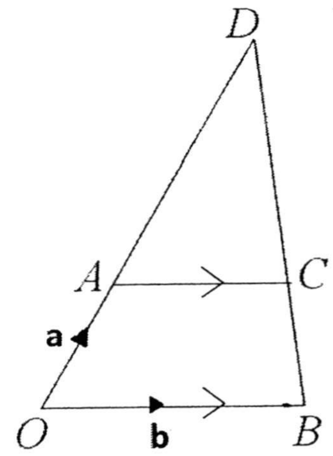
Find

(i) the column vector $\overrightarrow{AB}$, [1]

(ii) the coordinates of M such that M is the result of the translation of point A by $\begin{pmatrix}-6\\-8\end{pmatrix}$, [2]

(iii) the equation of the line AM. [2]

(b) In the diagram, $OACB$ is a trapezium where AC is parallel to OB.
The line OA is produced to the point D such that $\frac{OA}{AD} = \frac{1}{2}$.



(i) Given that $\overrightarrow{OA} = \mathbf{a}$ and $\overrightarrow{OB} = \mathbf{b}$, express, as simply as possible, in terms of $\mathbf{a}$ and/or $\mathbf{b}$,

(a) $\overrightarrow{BD}$, [1]

(b) $\overrightarrow{OC}$. [2]

(ii) Given that $\overrightarrow{OE} = 3\mathbf{a} + 2\mathbf{b}$,

(a) state the name of the quadrilateral $ODEB$, [1]

(b) explain why O, C and E lie in a straight line. [1]

(iii) Find, giving your answers as fractions in the simplest form,

(a) $\dfrac{\text{area of } \Delta ADC}{\text{area of } \Delta ODB}$, [1]

(b) $\dfrac{\text{area of } \Delta ADC}{\text{area of quadrilateral } ODEB}$. [1]

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Attempt **questions 5 to 8** on **Answer Booklet B**.

5 The first four terms in a sequence of numbers, T_1, T_2, T_3 and T_4, ... are given below.

$$T_1 = 4 - 3 = 1$$
$$T_2 = 9 - 6 = 3$$
$$T_3 = 16 - 9 = 7$$
$$T_4 = 25 - 12 = 13$$

(i) Study the pattern and write down the line for T_5. [1]

(ii) T_n can be expressed in the form $an^2 + bn + c$, where a, b and c are constants.

Find the values of a, b and c. [3]

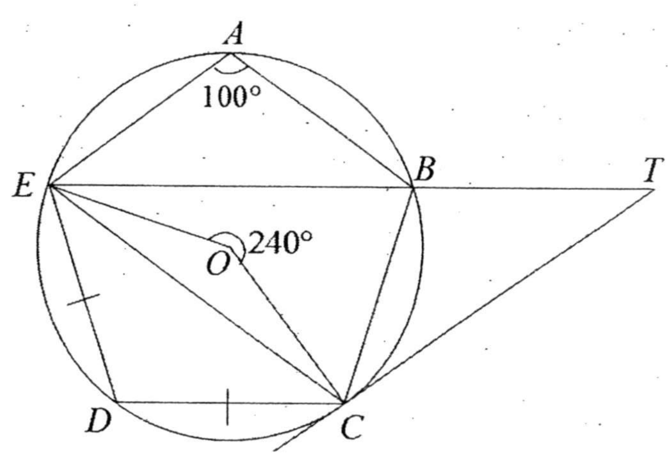
(iii) Find k such that $T_k = 73$. [2]

The first four terms of another sequence are 3, 7, 13, 21.

(iv) By using part **(i)** and **(ii)** or otherwise, write down an expression, in terms of n, for the nth term T_n of this sequence. [1]

---

6 In the diagram, O is the centre of circle $ABCDE$ where $DE = DC$.
Reflex angle $EOC = 240°$ and angle $EAB = 100°$.
(*You must not assume CT is a tangent to the circle at C.*)



(a) Find, giving reasons for each answer,

(i) angle DCE, [2]

(ii) angle CBE, [1]

(iii) angle CEB. [2]

(b) Given that $CTE = 20°$ and EBT is a straight line, show that CT is a tangent to the circle at C. [2]

7 The organizing committee of the national day parade is expecting 30 000 spectators at the floating platform.

The committee plans to have two entrances, the East Entrance and the West Entrance. The East Entrance will allow x number of spectators to enter in a minute while the West Entrance will allow y number of spectators to enter in a minute.

(i) Write down an expression, in terms of x, for the time taken in minutes for 30 000 spectators to enter the floating platform via the East Entrance only. [1]

Opening the East Entrance only will take 30 more minutes for all spectators to enter than opening the West Entrance only.

(ii) Show that $y = \dfrac{1000x}{1000 - x}$. [2]

(iii) If both entrances are opened at the same time, 30 000 spectators will take exactly 2.5 hours to enter the venue.

Form an equation in x and show that it reduces to $x^2 - 2200x + 200\ 000 = 0$. [2]

(iv) Solve the equation $x^2 - 2200x + 200\ 000 = 0$, giving your answers correct to the nearest whole number. [2]

(v) Explain why one of the answers in **(iv)** has to be rejected. [1]

ɪoɔ

8 **Answer the whole of this question on a sheet of graph paper.**

The variables x and y are connected by the equation $y = x + \dfrac{5}{x} - 3,\ x > 0$.

Some corresponding values of x and y are given in the table below.

x	0.5	1	1.5	2	2.5	3	4	5
y	7.5	3	1.83	1.5	1.5	1.67	2.25	3

(a) Using a scale of 2 cm to represent 1 unit, draw a horizontal x-axis for $0 \le x \le 5$.
Using a scale of 2 cm to represent 1 unit, draw a vertical y-axis for $0 \le y \le 8$.

On your axes, plot the points given in the table and join them with a smooth curve. [3]

(b) Use the graph to find the x-coordinate of the minimum point. [1]

(c) By drawing a tangent, find the gradient of the curve at the point where $x = 1.5$. [2]

(d) **(i)** On the same axes in **(a)**, draw the line $y = \dfrac{1}{3}x + 1$. [1]

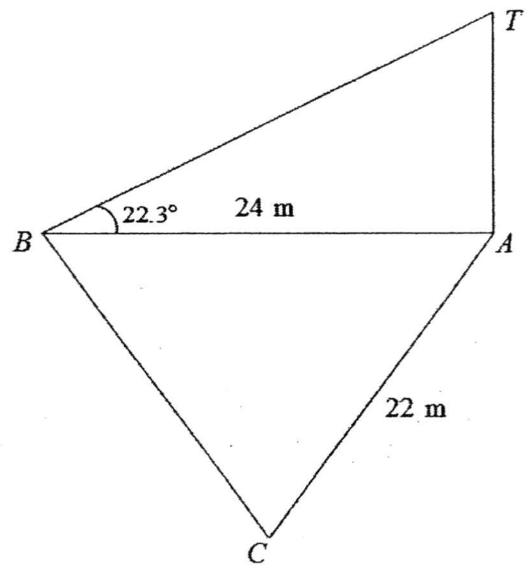
(ii) Write down the x-coordinates of the intersection points between this line and the curve for $0 \le x \le 5$. [1]

(iii) The values of x in **(d)(ii)** are solutions of the equation $2x^2 + Ax + B = 0$, where A and B are integer constants.

Find the values of A and B. [3]

Attempt **questions 9 to 11** on **Answer Booklet C**.

9 In the diagram, A is the foot of a cliff and B and C are yachts in the sea.
A is due east of B and the bearing of C from A is $214°$.
$AB = 24$ m and $AC = 22$ m .



(a) The angle of elevation of the top of the cliff, T, from B is $22.3°$.

Find the height of the cliff, TA. [2]

(b) Find the distance BC and hence, determine the angle BCA. [4]

(c) Calculate

(i) the bearing of B from C, [2]

(ii) the area of triangle ABC. [2]

(d) Determine the shortest distance from A to BC. [2]

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10 **Diagram I** shows a water bottle.
The cover of the water bottle is a hemisphere of radius 6 cm.
The portion of the water bottle that contains water is a cylinder of height 16 cm.

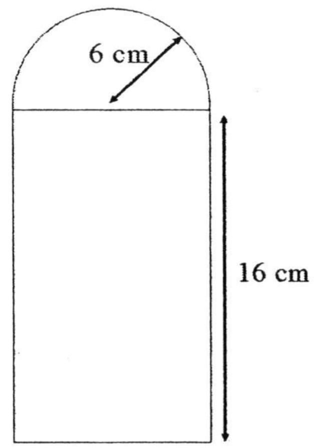


Diagram I

(a) Calculate the total surface area, including the base, of the outside of the water bottle. [2]

Diagram II shows a container which is a prism and whose cross-section is a trapezium.
The lengths of the parallel sides of the trapezium are 40 cm and 60 cm.
The depth of the container is 30 cm.

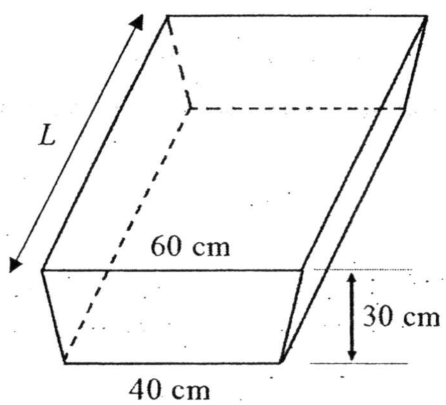


Diagram II

(b) 20 water bottles in **Diagram I** are filled with water to the brim of the cylinder and then poured into an empty trapezoidal container as shown in **Diagram II**.

Given that the trapezoidal container is completely filled with water, calculate the length of the container, L, giving your answer correct to 2 decimal places. [3]

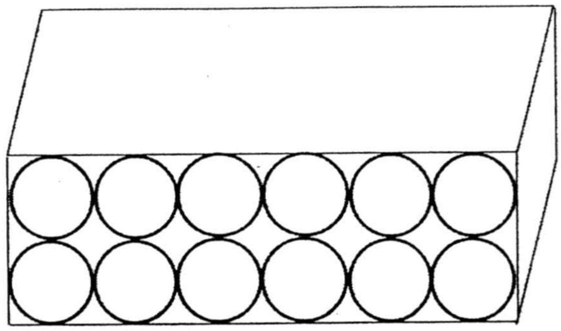


Diagram III

Diagram III shows twelve of these water bottles all facing in the same direction, which just fit into a box.

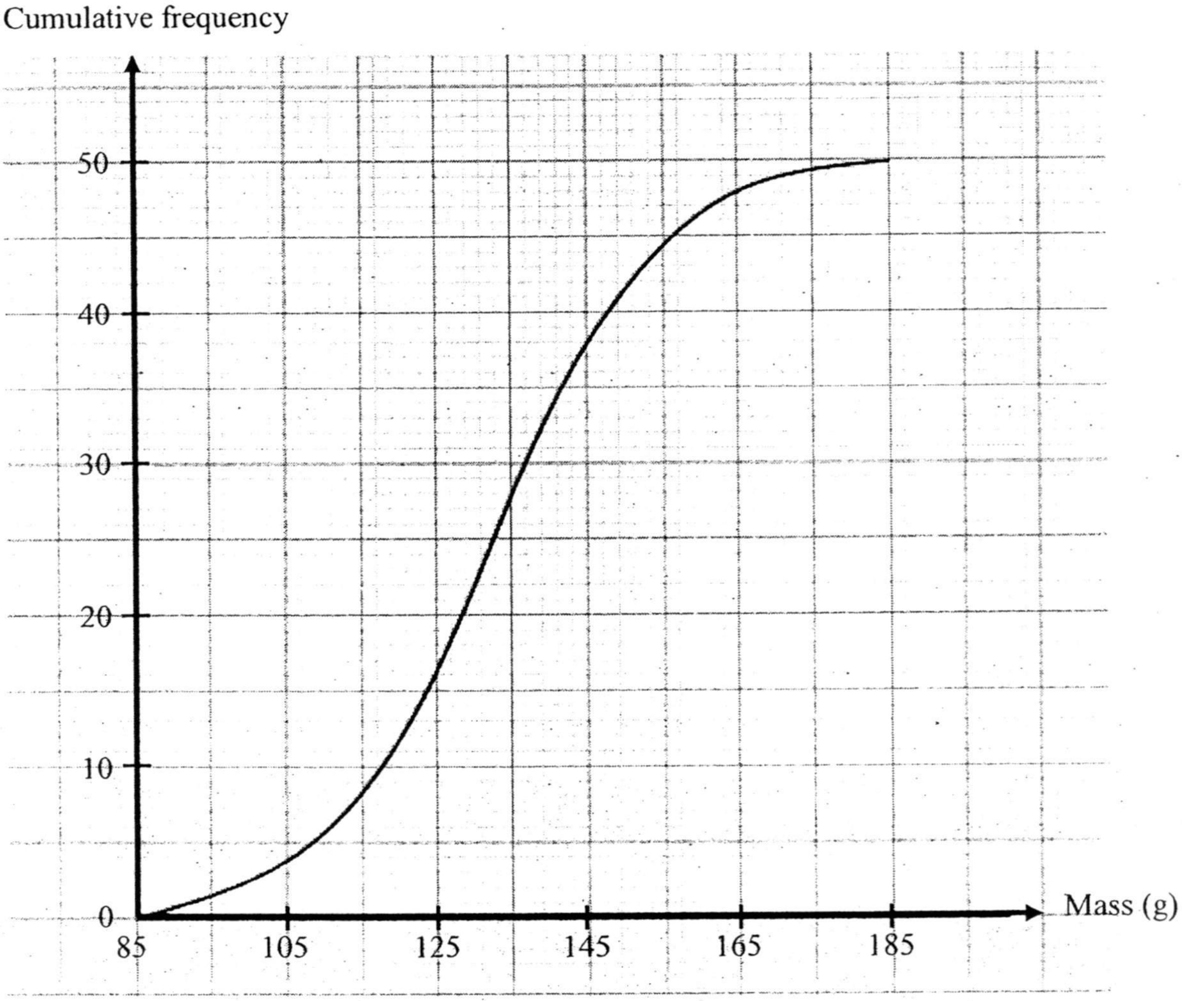
(c) Calculate

(i) the volume of the inside of the box. [2]

(ii) the percentage of volume of the box that is **not** occupied by the water bottles. [2]

---

11 The graph below shows a cumulative frequency curve depicting the masses of 50 oranges.



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(a) From the cumulative frequency curve, find the values of p and q in the grouped frequency table shown below. [2]

Mass, x (g)	$x \le 85$	$85 < x \le 105$	$105 < x \le 125$	$125 < x \le 145$	$145 < x \le 165$	$165 < x \le 185$
No. of oranges	0	4	p	q	10	2

(b) Hence, calculate an estimate of the

(i) mean mass, [2]

(ii) standard deviation of the 50 oranges. [2]

(c) Two oranges are chosen at random from the crate of 50 oranges, one after another without replacement.

Find, as a fraction in its simplest form, the probability that

(i) both oranges weigh at most 145 g, [1]

(ii) one orange weighs more than 165 g but the other weighs at most 125 g. [2]

(d) Another crate of 50 oranges have the same median but a smaller interquartile range. Describe how this cumulative frequency curve will differ from the given curve. [1]

END OF PAPER

Answer **all** the questions.

1 **(a)** Given that $x^2 + y^2 = a$ and $xy = b$, find $(2x-2y)^2$ in terms of a and b. [2]

(b) Given that $\dfrac{3a-bc}{2ac-5b} = \dfrac{1}{2}$,

(i) find the exact value of c when $a=-3$ and $b=2$, [1]

(ii) express c in terms of a and b, [2]

(c) Given that $\dfrac{y}{x} = 2015$ and $\dfrac{z}{y} = 2015$, where $x \neq 0$ and $y \neq 0$.

Find the value of $\dfrac{y+z}{x+y}$. [2]

Solution

(a)

$$(2x-2y)^2 = 4(x-y)^2$$

$$= 4(x^2+y^2-2xy)$$

$$= 4(a-2b) \quad (\text{or } 4a-8b)$$

(b)(i)

$$c = \frac{6(-3)+5(2)}{2(-3+2)} = \frac{-8}{-2} = 4$$

(b)(ii)

$$\frac{3a-bc}{2ac-5b} = \frac{1}{2}$$

$$6a-2bc = 2ac-5b$$

$$2ac+2bc = 6a+5b$$

$$2c(a+b) = 6a+5b$$

$$c = \frac{6a+5b}{2(a+b)}$$

(c)

$$\frac{y}{x}=2015 \Rightarrow y=2015x$$

$$\frac{z}{y}=2015 \Rightarrow z=2015y$$

$$y+z=2015(x+y)$$

$$\frac{y+z}{x+y}=2015$$

Alternative

$$\frac{y}{x}=2015 \Rightarrow y=2015x$$

$$\frac{z}{y}=2015 \Rightarrow z=2015y=\left(2015^2\right)x$$

$$\frac{y+z}{x+y}=\frac{2015x+\left(2015^2\right)x}{x+2015x}$$

$$=\frac{4062240x}{2016x}=2015$$

---

2 Teddy, Colin and Azmat each decided to buy a new motorbike that was priced at \$8 500.

(a) Teddy was given a discount and he paid \$8202.50 for the motorbike in cash.

(i) Calculate the percentage discount he received. [1]

(ii) The value of the new motorbike depreciates by 12% during the first year.
In the second year, its value depreciates by 20% of its value at the beginning of the year.

If Teddy sold off his motorbike at the end of the second year, how much money would he lose? [3]

(b) Colin paid a down payment of 20% of the price of the motorbike and the balance to be paid by instalments at a simple interest rate of 5% per annum for a period of 2.5 years.

Calculate

(i) the amount of down payment paid, [1]

(ii) the amount of instalment payable in a month. [2]

(c) Azmat paid a down payment of \$2612 and the balance to be paid at the end of 2 years with compound interest rate of 3.75% per annum.

How much would the motorbike cost him altogether? [2]

Solution

(a)(i)

$$\% \text{ discount}=\frac{8500-8202.5}{8500}\times 100=3.5$$

(a)(ii)

$$\text{Value of car at end of second year} = 0.8 \times 0.88 \times 8500$$
$$= \$5984$$

$$\text{Total amount Cody will lose} = 8202.50 - 5984$$
$$= \$2218.50$$

(b)(i)

$$\text{Down payment} = \frac{20}{100} \times \$8500 = \$1700$$

(b)(ii)

$$\text{Balance} = \$8500 - \$1700 = \$6800$$

$$\text{Interest} = \frac{6800 \times 2.5 \times 5}{100} = \$850$$

$$\text{Monthly instalment} = \frac{6800 + 850}{30} = \$255$$

(c)

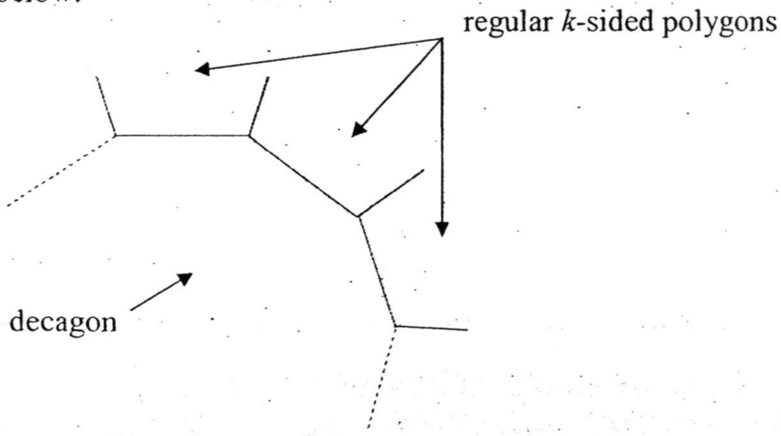
$$\text{Balance} = \$8500 - \$2612 = \$5888$$

$$\text{Total amount} = \$5888\left(1 + \frac{3.75}{100}\right)^2 = \$6337.88$$

$$\text{Total cost} = \$6337.88 + \$2612 = \$8949.88$$

---

3 A number of regular k-sided polygons are placed together in a ring to form a regular decagon as shown in the diagram below.



(a) Find

(i) the interior angle of the decagon, [2]

(ii) the value of k. [2]

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(b) If n-sided regular polygons that are placed together in a ring to form a N-sided polygon,

(i) show that $N = \dfrac{2n}{n-4}$. [3]

(ii) Hence or otherwise, explain why a regular octagon cannot be formed by placing a number of regular polygons in a ring. [1]

Solution

(a)(i)

$$\text{Size of each interior angle } = \frac{180(10-2)}{10}$$

$$= 144°$$

(a)(ii)

$$\text{interior angle of } k\text{-sided polygon} = \frac{360-144}{2}$$
$$= 108°$$

$108k = 180(k-2)$

$72k = 360$

$k = 5$

(b)(i)

$$\frac{180(N-2)}{N} + 2\left(\frac{180(n-2)}{n}\right) = 360$$

$$\frac{(N-2)}{N} + \frac{2(n-2)}{n} = 2$$

$$\frac{nN - 2n + 2Nn - 4N}{Nn} = 2$$

$3Nn - 2n - 4N = 2Nn$

$Nn - 4N = 2n$

$$N = \frac{2n}{n-4}$$

(b)(ii)

When $N = 8$,

$$8 = \frac{2n}{n-4}$$

$8n - 32 = 2n$

$6n = 32$

$n = 5.33$ (N.A. since n must be integer)

4 **(a)** The position vectors of a point A, B and C are $\begin{pmatrix}-3\\5\end{pmatrix}$, $\begin{pmatrix}5\\20\end{pmatrix}$ and $\begin{pmatrix}10\\0\end{pmatrix}$ respectively.

Find

(i) the column vector $\overrightarrow{AB}$, [1]

(ii) the coordinates of M such that M is the result of the translation of point A by $\begin{pmatrix}-6\\-8\end{pmatrix}$, [2]

(iii) the equation of the line AM. [2]

<u>Solutions</u>

(a)(i)

$$\overrightarrow{AB}=\begin{pmatrix}8\\15\end{pmatrix}$$

(a)(ii)

$$\overrightarrow{OM}=\begin{pmatrix}-3\\5\end{pmatrix}+\begin{pmatrix}-6\\-8\end{pmatrix}$$

$$=\begin{pmatrix}-9\\-3\end{pmatrix}$$

M is $(-9\ ,\ -3\ )$

(b)

$$m_{AM}=\frac{-8}{-6}=\frac{4}{3}$$

Equation of AM is $y=\frac{4}{3}x+c$.

At $A(-3\ ,\ 5)$,

$$5=\frac{4}{3}(-3)+c$$

$$c=9$$

$$y=\frac{4}{3}x+9\ (\text{ or } 3y=4x+27)$$

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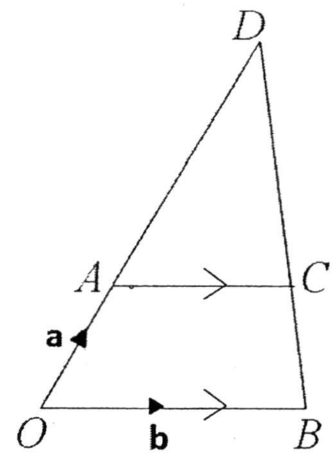
(b) In the diagram, $OACB$ is a trapezium where AC is parallel to OB.
The line OA is produced to the point D such that $\frac{OA}{AD} = \frac{1}{2}$.



(i) Given that $\overrightarrow{OA} = \mathbf{a}$ and $\overrightarrow{OB} = \mathbf{b}$, express, as simply as possible, in terms of $\mathbf{a}$ and/or $\mathbf{b}$,

(a) $\overrightarrow{BD}$, [1]

(b) $\overrightarrow{OC}$. [2]

(ii) Given that $\overrightarrow{OE} = 3\mathbf{a} + 2\mathbf{b}$,

(a) state the name of the quadrilateral $ODEB$, [1]

(b) explain why O, C and E lie in a straight line. [1]

(iii) Find, giving your answers as fractions in the simplest form,

(a) $\dfrac{\text{area of } \Delta ADC}{\text{area of } \Delta ODB}$, [1]

(b) $\dfrac{\text{area of } \Delta ADC}{\text{area of quadrilateral } ODEB}$. [1]

<u>**Solutions**</u>
(i)(a)

$\overrightarrow{BD} = 3\mathbf{a} - \mathbf{b}$

(i)(b)

$$\overrightarrow{OC} = \overrightarrow{OD} + \overrightarrow{DC}$$
$$= 3\mathbf{a} + \frac{2}{3}(\mathbf{b} - 3\mathbf{a})$$
$$= \mathbf{a} + \frac{2}{3}\mathbf{b}$$

(ii)(a)

Trapezium

(ii)(b)

$$\overrightarrow{OE} = 3\mathbf{a} + 2\mathbf{b}$$
$$= 3\left(\mathbf{a} + \frac{2}{3}\mathbf{b}\right)$$
$$= 3\overrightarrow{OC}$$

So, O, C and E are collinear.

(iii)(a)

$$\frac{\text{area of } \Delta ADC}{\text{area of } \Delta ODB} = \frac{4}{9}$$

(iii)(b)

$$\frac{\text{area of } \Delta ADC}{\text{area of quadrilateral } ODEB} = \frac{4}{9} \times \frac{1}{3}$$
$$= \frac{4}{27}$$

---

5 The first four terms in a sequence of numbers, T_1, T_2, T_3 and T_4, ... are given below.

$$T_1 = 4 - 3 = 1$$
$$T_2 = 9 - 6 = 3$$
$$T_3 = 16 - 9 = 7$$
$$T_4 = 25 - 12 = 13$$

(i) Study the pattern and write down the line for T_5. [1]

(ii) T_n can be expressed in the form $an^2 + bn + c$, where a, b and c are constants.

Find the values of a, b and c. [3]

(iii) Find k such that $T_k = 73$. [2]

The first four terms of another sequence are 3, 7, 13, 21.

(iv) By using part **(i)** and **(ii)** or otherwise, write down an expression, in terms of n, for the nth term T_n of this sequence. [1]

Solution

(i)

$$T_5 = 36 - 15 = 21$$

(ii)

$$T_n = (n+1)^2 - 3(n)$$
$$= n^2 - n + 1$$

$a = 1$ $\quad$ $b = -1$ $\quad$ $c = 1$

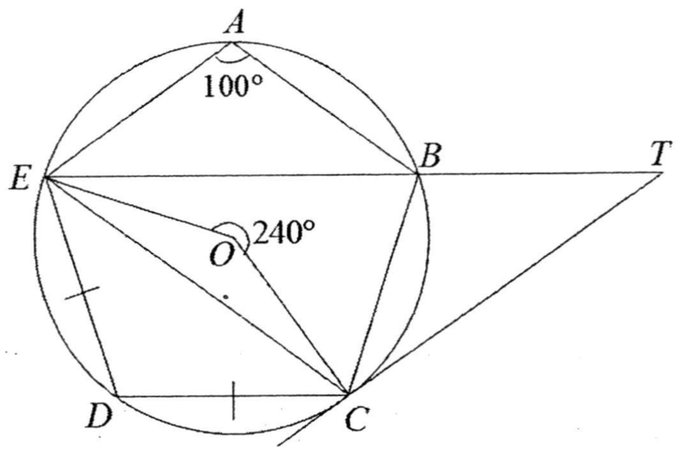
(iii)

$$n^2 - n + 1 = 73$$
$$n^2 - n - 72 = 0$$
$$(n+8)(n-9) = 0$$
$$n = -8 \text{(N.A.)} \quad \text{or} \quad n = 9$$

(iv)

$$T_n = (n+1)^2 - (n+1) + 1$$
$$= n^2 + n + 1$$

6 In the diagram, O is the centre of circle $ABCDE$ where $DE = DC$.
Reflex angle $EOC = 240°$ and angle $EAB = 100°$.
(*You must not assume CT is a tangent to the circle at C.*)



(a) Find, giving reasons for each answer,

(i) angle DCE, [2]

(ii) angle CBE, [1]

(iii) angle CEB. [2]

(b) Given that $CTE = 20°$ and EBT is a straight line, show that CT is a tangent to the circle at C. [2]

<u>Solution</u>
(a)(i)

$\angle CDE = 120°$ (angle at center = twice angle at circumference)

$\angle DCE = \dfrac{180° - 120°}{2}$ (isosceles triangle)

$= 30°$

(a)(ii)

$\angle CBE = 180° - 120°$ (opposite angles of cyclic quadrilateral)

$= 60°$

(a)(iii)

$\angle BCE = 180° - 100°$ (opposite angles of cyclic quadrilateral)

$= 80°$

$\angle CEB = 180° - 80° - 60°$ (angle sum of triangle)

$= 40°$

(b)

$\angle BCT = 180 - 20 - (180 - 60)$ (angle sum of triangle)

$= 40°$

$\angle BCO = 80 - \frac{1}{2}(180 - 120)$ (isosceles triangle)

$= 50°$

$\angle OCT = 40 + 50$

$= 90°$

Hence by tangent perpendicular radius,
CT is a tangent to the circle at C.

---

7 The organizing committee of the national day parade is expecting 30 000 spectators at the floating platform.

The committee plans to have two entrances, the East Entrance and the West Entrance. The East Entrance will allow x number of spectators to enter in a minute while the West Entrance will allow y number of spectators to enter in a minute.

(i) Write down an expression, in terms of x, for the time taken in minutes for 30 000 spectators to enter the floating platform via the East Entrance only. [1]

Opening the East Entrance only will take 30 more minutes for all spectators to enter than opening the West Entrance only.

(ii) Show that $y = \dfrac{1000x}{1000 - x}$. [2]

(iii) If both entrances are opened at the same time, 30 000 spectators will take exactly 2.5 hours to enter the venue.

Form an equation in x and show that it reduces to $x^2 - 2200x + 200\ 000 = 0$. [2]

(iv) Solve the equation $x^2 - 2200x + 200\ 000 = 0$, giving your answers correct to the nearest whole number. [2]

(v) Explain why one of the answers in **(iv)** has to be rejected. [1]

Solution

(i)

$\dfrac{30\ 000}{x}$

(ii)

$$\frac{30\,000}{x}-\frac{30\,000}{y}=30$$

$$\frac{1000}{x}-\frac{1000}{y}=1$$

$$\frac{1000y-1000x}{xy}=1$$

$$1000y-1000x=xy$$

$$1000y-xy=1000x$$

$$y=\frac{1000x}{1000-x}$$

(iii)

$$x+y=\frac{30000}{150}\ \left(\text{ or } 150x+150y=30000\right)$$

$$x+\frac{1000x}{1000-x}=200$$

$$x\left(1000-x\right)+1000x=200\left(1000-x\right)$$

$$-x^2+2000x=200000-200x$$

$$x^2-2200x+200000=0$$

(iv)

$$x=\frac{2200\pm\sqrt{(-2200)^2-4(1)(200000)}}{2}$$

$$=\frac{2200\pm\sqrt{4040000}}{2}$$

$$=2104.99 \quad \text{or} \quad x=95.01$$

$$\approx 2105 \qquad\qquad \approx 95$$

(v)

$x=2105$ has to be rejected since $y=200-2105<0$.

8 **Answer the whole of this question on a sheet of graph paper.**

The variables x and y are connected by the equation $y = x + \frac{5}{x} - 3$, $x > 0$.

Some corresponding values of x and y are given in the table below.

x	0.5	1	1.5	2	2.5	3	4	5
y	7.5	3	1.83	1.5	1.5	1.67	2.25	3

(a) Using a scale of 2 cm to represent 1 unit, draw a horizontal x-axis for $0 \le x \le 5$.
Using a scale of 2 cm to represent 1 unit, draw a vertical y-axis for $0 \le y \le 8$.

On your axes, plot the points given in the table and join them with a smooth curve. [3]

(b) Use the graph to find the x-coordinate of the minimum point. [1]

(c) By drawing a tangent, find the gradient of the curve at the point where $x = 1.5$. [2]

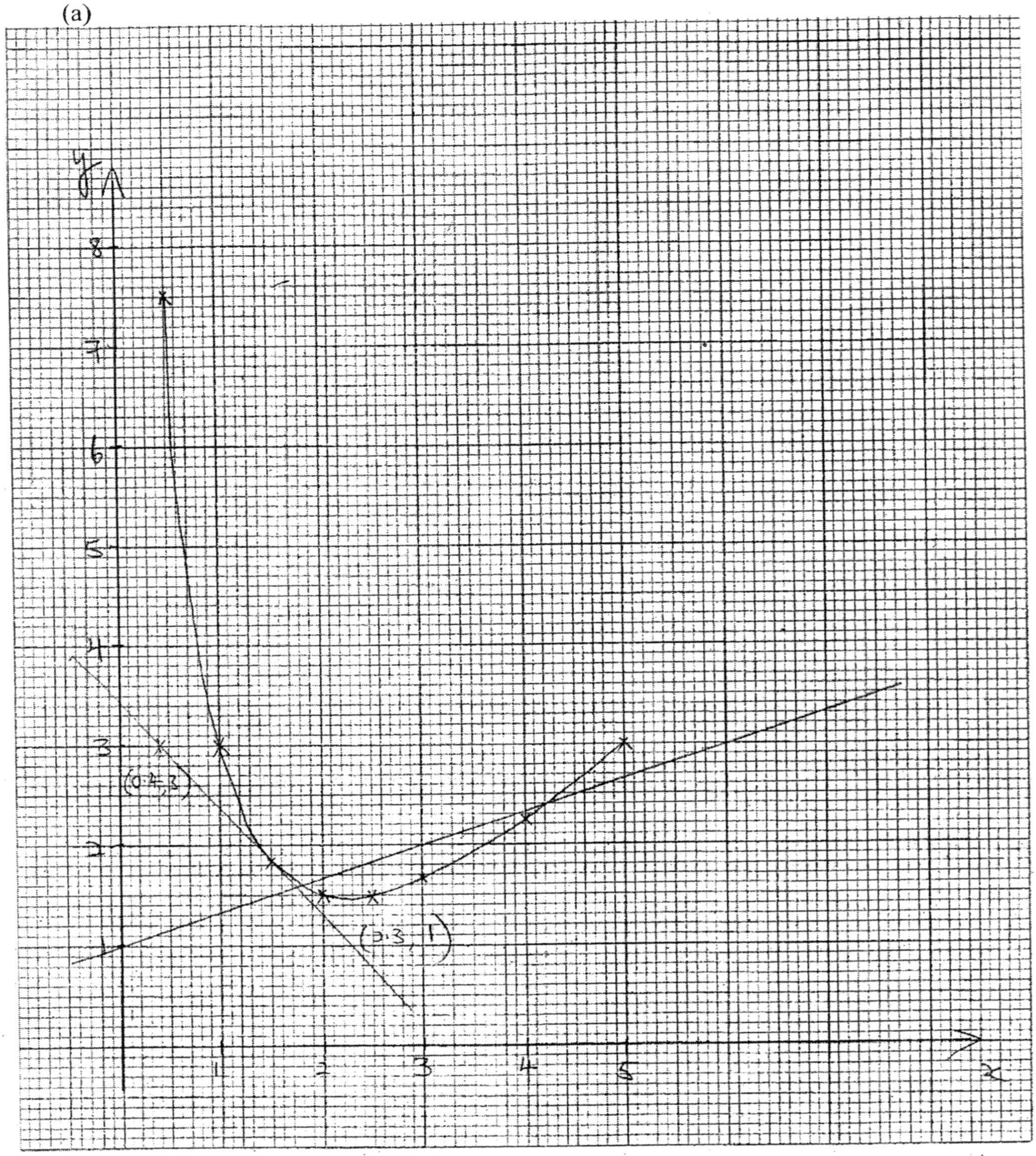
(d) **(i)** On the same axes in **(a)**, draw the line $y = \frac{1}{3}x + 1$. [1]

(ii) Write down the x-coordinates of the intersection points between this line and the curve for $0 \le x \le 5$. [1]

(iii) The values of x in **(d)(ii)** are solutions of the equation $2x^2 + Ax + B = 0$, where A and B are integer constants.

Find the values of A and B. [3]

(a)



(a)

On graph paper.

(b)

$x = 2.3$ (accept coordinate ± 0.1)

(c)

Gradient $= -1.05$ (accept gradient ± 0.2)

(d)(i)

On graph

(d)(ii)

$x = 1.8$ (accept 1.7 to 1.9), 4.2 (accept 4.1 to 4.3)

(d)(iii)

$$\frac{1}{3}x + 1 = x + \frac{5}{x} - 3$$

$$\frac{2}{3}x + \frac{5}{x} - 4 = 0$$

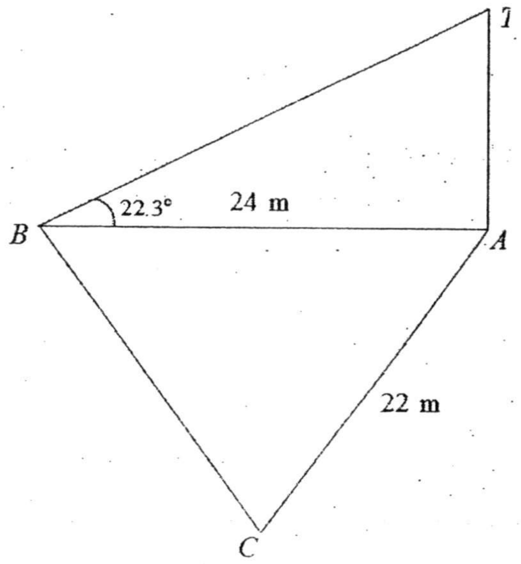
$$2x^2 - 12x + 15 = 0$$

$$A = -12$$

$$B = 15$$

---

9 In the diagram, A is the foot of a cliff and B and C are yachts in the sea.
A is due east of B and the bearing of C from A is $214°$.
$AB = 24$ m and $AC = 22$ m .



(a) Given that the angle of elevation of the top of the cliff, T, from B is $22.3°$.

Find the height of the cliff, TA. [2]

(b) Find the distance BC and hence, determine the angle BCA. [4]

(c) Calculate

(i) the bearing of B from C, [2]

(ii) the area of triangle ABC. [2]

(d) Determine the shortest distance from A to BC. [2]

Solution

(a)

$$\tan 22.3^\circ = \frac{TA}{24}$$

$$TA = 9.843 \approx 9.84 \text{ m}$$

(b)

$$\angle BAC = 56^\circ$$

$$BC = \sqrt{24^2 + 22^2 - 2(24)(22)\cos 56^\circ}$$

$$= 21.667 \approx 21.7 \text{ m}$$

$$\frac{\sin \angle BCA}{24} = \frac{\sin 56^\circ}{21.66}$$

$$\angle BCA = 66.674^\circ \approx 66.7^\circ$$

(c)(i)

$$\angle BCN_C = 66.67 - 34$$
$$= 32.67^\circ$$

$$\text{bearing} = 360 - 32.67$$
$$= 327.325^\circ$$
$$\approx 327.3^\circ$$

(c)(ii)

$$\text{Area of triangle} = \frac{1}{2}(24)(22)\sin 56^\circ$$
$$= 218.86$$
$$\approx 219 \text{ m}^2$$

(d)

$$\sin 66.67^\circ = \frac{AP}{22}$$

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$$AP = 20.201 \approx 20.2 \text{ m}$$

10 **Diagram I** shows a water bottle.
The cover of the water bottle is a hemisphere of radius 6 cm.
The portion of the water bottle that contains water is a cylinder of height 16 cm.

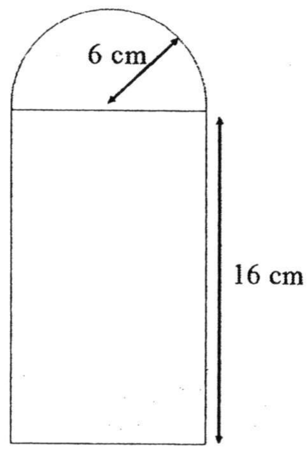


Diagram I

(a) Calculate the total surface area, including the base, of the outside of the water bottle. [2]

Diagram II shows a container which is a prism and whose cross-section is a trapezium.
The lengths of the parallel sides of the trapezium are 40 cm and 60 cm.
The depth of the container is 30 cm.

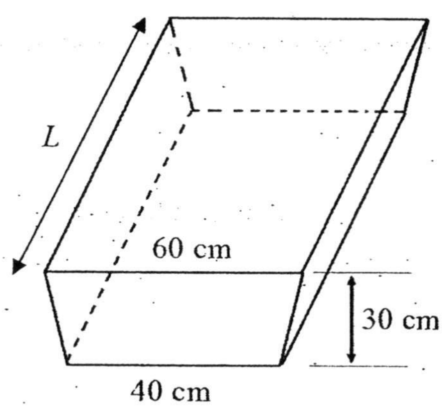


Diagram II

(b) 20 water bottles in **Diagram I** are filled with water to the brim of the cylinder and then poured into an empty trapezoidal container as shown in **Diagram II**.

Given that the trapezoidal container is completely filled with water, calculate the length of the container, L, giving your answer correct to 2 decimal places. [3]

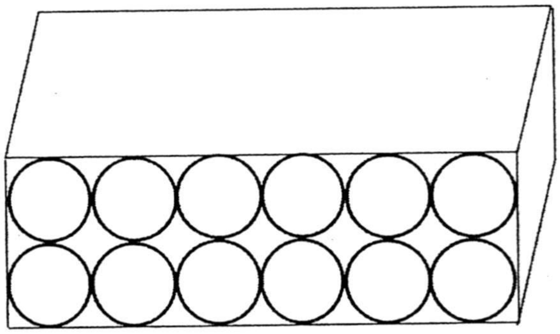


Diagram III

Diagram III shows twelve of these water bottles all facing in the same direction, which just fit into a box.

(c) Calculate

(i) the volume of the inside of the box. [2]

(ii) the percentage of volume of the box that is **not** occupied by the water bottles. [2]

Solution

(a)

$$\text{total surface area} = \frac{1}{2}(4\pi)(6)^2 + (2\pi)(6)(16) + (\pi)(6)^2$$

$$= 300\pi \text{ cm}^2$$

$$= 942.47$$

$$\approx 942 \text{ cm}^2$$

(b)

$$\text{volume of water} = (\pi)(6)^2(16)\times 20$$

$$= 11520\pi = 36190 \text{ cm}^3$$

$$\frac{1}{2}(30)(60+40)L = 36190$$

$$L \approx 24.13$$

(c)(i)

$$\text{volume of box} = 6(2\times 6)\times 2(2\times 6)\times 22$$

$$= 38016 \text{ cm}^3$$

(c)(ii)

Volume of 1 water bottle, including the cap $= \frac{1}{2}(\frac{4}{3}\pi)(6)^3 + 576\pi$

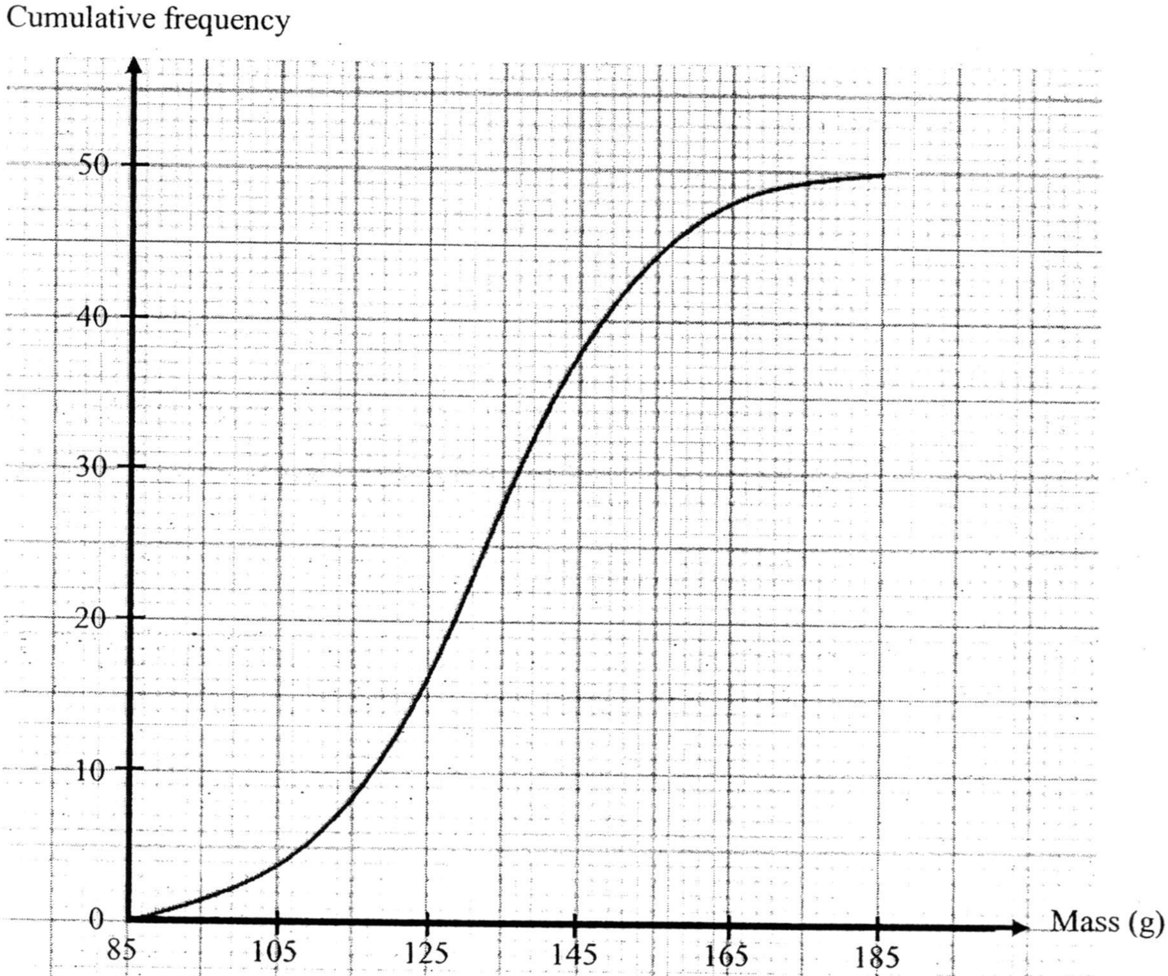
$= 720\pi = 2261 \text{ cm}^3$

% not occupied $= \dfrac{38016 - 2261 \times 12}{38016} \times 100$

$=28.6\%$

---

11 The graph below shows a cumulative frequency curve depicting the masses of 50 oranges.



(a) From the cumulative frequency curve, find the values of p and q in the grouped frequency table shown below. [2]

Mass, x (g)	$x \leq 85$	$85 < x \leq 105$	$105 < x \leq 125$	$125 < x \leq 145$	$145 < x \leq 165$	$165 < x \leq 185$
No. of oranges	0	4	p	q	10	2

(b) Hence, calculate an estimate of the

(i) mean mass, [2]

(ii) standard deviation of the 50 oranges. [2]

(c) Two oranges are chosen at random from the crate of 50 oranges, one after another without replacement.

Find, as a fraction in its simplest form, the probability that

(i) both oranges weigh at most 145 g, [1]

(ii) one orange weighs more than 165 g but the other weighs at most 125 g. [2]

(d) Another crate of 50 oranges have the same median but a smaller interquartile range. Describe how this cumulative frequency curve will differ from the given curve. [1]

Solution

(a)

$p = 12$

$q = 22$

(b)(i)

$$\text{Mean mass} = \frac{95\times4+115\times12+135\times22+155\times10+175\times2}{50}$$

$$= \frac{6630}{50}$$

$$= 132.6 \text{ g}$$

(b)(ii)

$$\text{Standard deviation} = \sqrt{\frac{4(95)^2+12(115)^2+22(135)^2+10(155)^2+2(175)^2}{50}-(132.6)^2}$$

$$= 19.03$$

$$\approx 19.0 \text{ g}$$

(c)(i)

$$\frac{38}{50}\times\frac{37}{49}=\frac{703}{1225}$$

(c)(ii)

$$\frac{2}{50}\times\frac{16}{49}\times2=\frac{32}{1225}$$

(d)

The cumulative frequency curve for this crate of 50 oranges will be steeper.