Calculator Model:

Name: Mark Scheme	Class	Class Register Number/ Centre No./Index No.

中正中学

CHUNG CHENG HIGH SCHOOL (MAIN)

Parent's Signature

PRELIMINARY EXAMINATION 2015
SECONDARY 4

Mathematics 4016/01

Paper 1

Friday 21 August 2015

2 hours

READ THESE INSTRUCTIONS FIRST

Do not open this booklet until you are told to do so.

Write your name, class and index number clearly in the spaces provided at the top of this page.
Write in dark blue or black pen on both sides of the paper.
You may use a pencil for any diagrams, graphs or rough working.
Do not use highlighters or correction fluid.

Answer all questions.
If working is needed for any question it must be shown with the answer.
Omission of essential working will result in loss of marks.
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For π, use either your calculator value or 3.142, unless the question requires the answer in terms of π.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 80.

/ 80

This document consists of 18 printed pages

$\frac{7}{0}$

115

Mathematical Formulae

Compound interest

$$\text{Total amount} = P\left(1+\frac{r}{100}\right)^n$$

Mensuration

$$\text{Curved surface area of a cone} = \pi r l$$

$$\text{Surface area of a sphere} = 4\pi r^2$$

$$\text{Volume of a cone} = \frac{1}{3}\pi r^2 h$$

$$\text{Volume of a sphere} = \frac{4}{3}\pi r^3$$

$$\text{Area of triangle } ABC = \frac{1}{2}ab\sin C$$

$$\text{Arc length} = r\theta \text{, where } \theta \text{ is in radians}$$

$$\text{Sector area} = \frac{1}{2}r^2\theta \text{, where } \theta \text{ is in radians}$$

Trigonometry

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

Statistics

$$\text{Mean} = \frac{\Sigma f x}{\Sigma f}$$

$$\text{Standard deviation} = \sqrt{\frac{\Sigma f x^2}{\Sigma f} - \left(\frac{\Sigma f x}{\Sigma f}\right)^2}$$

Answer all the questions.

1 From the set of numbers $\left\{-\frac{2}{7}, -5, 0, 2.1, \sqrt{0.4}, \sqrt{4}, \sqrt{40}, \sqrt[3]{0.064}\right\}$, write down

(a) all the integers,

Answer (a) $-5, 0, \sqrt{4}$ [1]

(b) all the rational numbers.

Answer (b) $-\frac{2}{7}, -5, 0, 2.1, \sqrt{4}, \sqrt[3]{0.064}$ [1]

2 Factorise completely $x^2 + a^2 - 4b^2 - 2ax$.

$$x^2 + a^2 - 4b^2 - 2ax = (x-a)^2 - 4b^2$$
$$= (x-a+2b)(x-a-2b)$$

or

$$x^2 + a^2 - 4b^2 - 2ax = (a-x)^2 - 4b^2$$
$$= (a-x+2b)(a-x-2b)$$

Answer .... [2]

3 During the Great Singapore Sale, a refrigerator is sold at a 25% discount. A further reduction of 10% is applied on a discounted price if a discount coupon is used.
A customer pays \$2430 for a refrigerator using a discount coupon.
Calculate the original price.

90% × 75% × original price = \$2430
Original price = \$2430 ÷ (0.9 × 0.75)
= \$ 3600

Answer \$.................................. [2]

116

4 (a) Simplify $\left(\dfrac{2x^{-\frac{1}{2}}y^{3}}{y^{-2}}\right)^{-2}$.

$$\left(\frac{2x^{-\frac{1}{2}}y^{3}}{y^{-2}}\right)^{-2}=\left(2x^{-\frac{1}{2}}y^{5}\right)^{-2}$$
$$=2^{-2}xy^{-10}$$
$$=\frac{x}{4y^{10}}$$

Answer *(a)*.............................. [2]

(b) Given that $\dfrac{(343)^{x+1}}{7}=49^{2x}$, find the value of x.

$$\frac{(343)^{x+1}}{7}=49^{2x}$$
$$\left(7^{3}\right)^{x+1}=7\left(7^{2}\right)^{2x}$$
$$7^{3x+3}=7^{4x+1}$$
Comparingpowers, $3x+3=4x+1$
$$x=2$$

Answer *(b)* x = ... [2]

---

5 When written as a product of their prime factors,
$A=2^{2}\times 3^{2}\times 5$,
$B=2\times 3\times 5^{2}$.
Given that the HCF and LCM of A, B and C is 6 and 6300 respectively, find the smallest possible value of C.

HCF $=6=2\times3$
LCM $=6300=2^{2}\times 3^{2}\times 5^{2}\times7$

$C=2\times 3\times7=42$

Answer C =.............................. [2]

6 (a) Solve the equation $1+\frac{3}{4x-13}=\frac{2}{5}$.

$$1+\frac{3}{4x-13}=\frac{2}{5}$$
$$\frac{3}{4x-13}=-\frac{3}{5}$$
$$12x-39=-15$$
$$12x=24$$
$$x=2$$

Answer (a) $x=$ .... [2]

(b) The relationship between p and q can be expressed as $\frac{2p+3q}{p+q}=5$.

Find the value of $\frac{2p}{q}$.

$$\frac{2p+3q}{p+q}=5$$
$$5p+5q=2p+3q$$
$$3p=-2q$$
$$\frac{p}{q}=-\frac{2}{3}$$
$$\therefore\frac{2p}{q}=2\left(-\frac{2}{3}\right)$$
$$=-1\frac{1}{3}$$

Answer (b) $\frac{2p}{q}=$ .... [2]

---

7 Find all the integer values of x that satisfy the inequality $-1<\frac{2(4-5x)}{3}<8$.

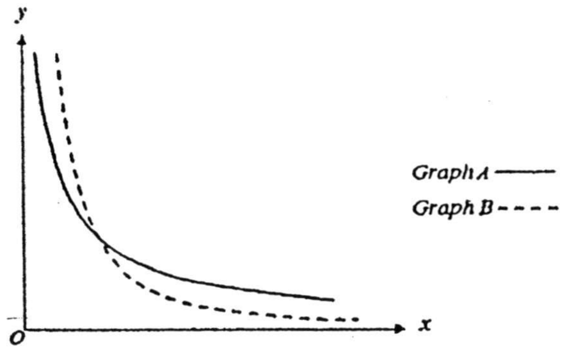
$$-1<\frac{2(4-5x)}{3}<8$$
$$-1<\frac{2(4-5x)}{3} \quad\text{and}\quad \frac{2(4-5x)}{3}<8$$
$$-3<8-10x \quad\text{and}\quad 8-10x<24$$
$$10x<11 \quad\text{and}\quad -10x<16$$
$$x<1.1 \quad\text{and}\quad x>-1.6$$
$$\therefore -1.6<x<1.1$$

Hence the integer values of x are -1, 0 and 1.

50

Answer $x=$ .... [3]

8 (a) The graphs of $y=\frac{3}{x}$ and $y=\frac{3}{x^2}$ are shown on the axes below.



(i) Identify whether Graph A or B is the graph for $y=\frac{3}{x^2}$.

Answer (a)(i) Graph...B............... [1]

(ii) Write down the coordinates of the intersection point of the two graphs.

Answer (a)(ii) (1 , 3) [1]

(b) (i) Express $y=-x^2+5x-2$ in the form $y=-(x-a)^2+b$.

$y=-x^2+5x-2$

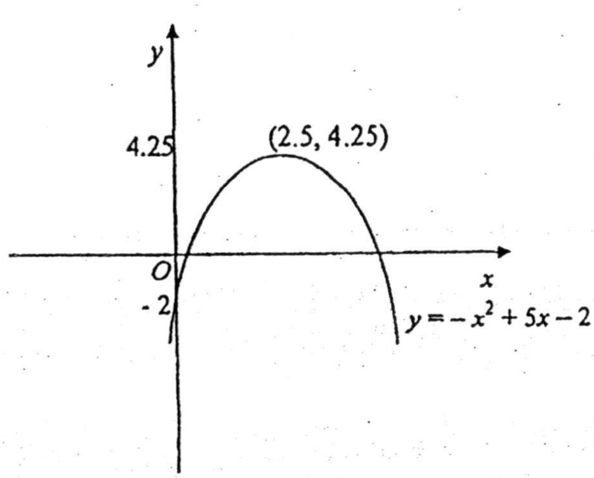
$y=-(x^2-5x+2)$

$y=-\left[x^2-5x+\left(\frac{-5}{2}\right)^2+2-\left(\frac{-5}{2}\right)^2\right]$

$y=-\left(x-2\frac{1}{2}\right)^2+4\frac{1}{4}$

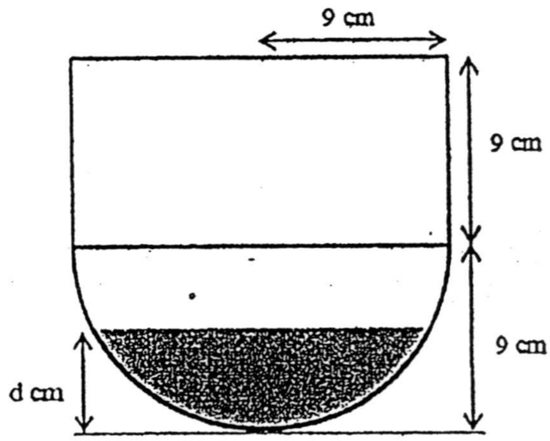
Answer (b)(i) $y=$...................... [2]

(ii) Hence, sketch the graph of $y=-x^2+5x-2$.

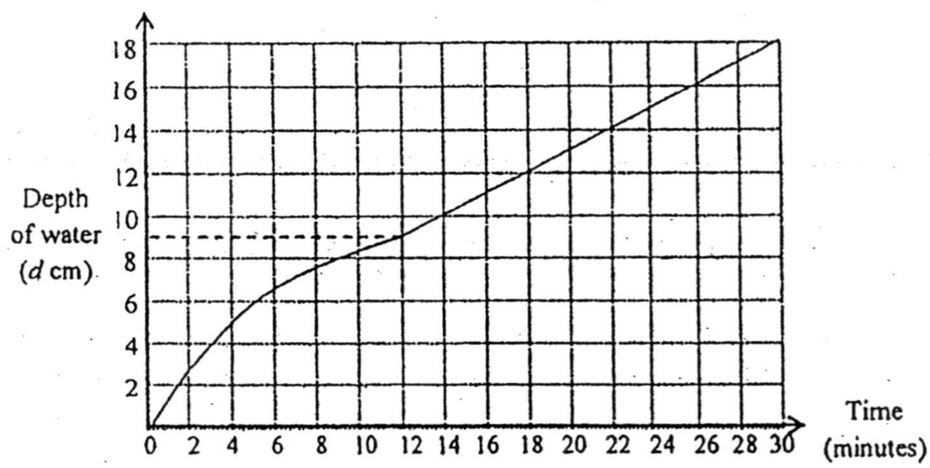


[2]

9 The diagram shows the cross-section of a container, made up of a cylinder and a hemisphere. The cylinder has height 9 cm and a circular base with radius 9 cm. Water is poured into the empty container at a constant rate and fills it in 30 minutes. On the given axes, sketch the graph showing the relationship between the depth of the water, d cm, and the time, t minutes, as the container is being filled up during the 30 minutes.



Answer



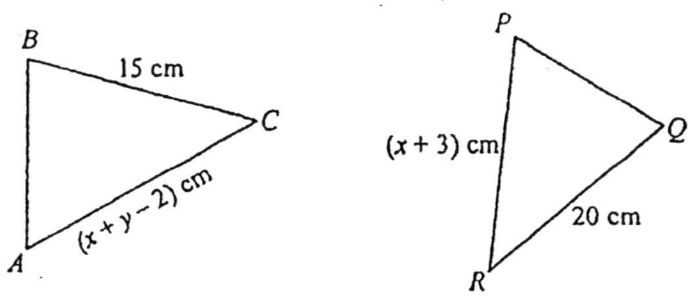
$$\frac{\text{Vol of hemisphere}}{\text{Vol of cylinder}} = \frac{\frac{2}{3}\pi(9)^3}{\pi(9)^2(9)}$$

$$= \frac{2}{3}$$

Time taken to fill up hemisphere $= \frac{2}{5} \times 30\text{mins}$

$= 12$ mins

118



[1]

he diagram above, triangle ABC is similar to triangle PQR.

Form an equation in x and y.

Since triangle ABC is similar to triangle PQR,

$$\frac{15}{20}=\frac{x+y-2}{x+3}$$

$$20x+20y-40=15x+45$$

$$5x+20y=85$$

$$x+4y=17$$

[2]

Answer *(a)* .... [2]

) Find the values of x and y if $2x+3y=20$.

$2x+3y=20$ ———————— (1)

$x+4y=17$ ———————— (2)

$(2)\times 2: 2x+8y=34$ ———————— (3)

$(3)-(1)$: $5y=14$

[1]

$$y=2.8 \text{ or } 2\frac{4}{5}$$

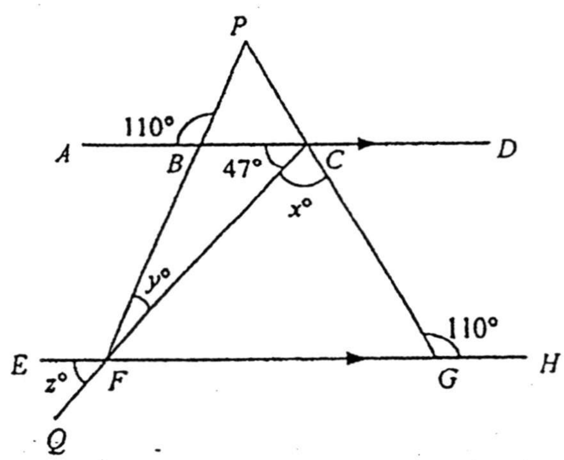
When $y=2.8$ or $2\frac{4}{5}$, $x=5.8$ or $5\frac{4}{5}$

[1]

Answer *(b)* $x=$........., $y=$ .... [2]

[1]

13 In the diagram, $ABCD$ is parallel to $EFGH$. QFC, PBF and PCG are straight lines.



(a) Find the value of

(i) x,

$x° + 47° = 110°$ (alt. $\angle$s, $AD//EH$)

$x = 63$

Answer (a)(i) $x =$ .... [1]

(ii) y,

$\angle FBC = 110°$ (vert. opp. $\angle$s)

$y° = 180° - 110° - 47°$ ($\angle$s sum of Δ)

$y = 23$

Answer (a)(ii) $y =$ .... [1]

(iii) z.

$z° = 47°$ (corr. $\angle$s, $AD//EH$)

$z = 47$

Answer (a)(iii) $z =$ .... [1]

(b) Explain, stating the reasons, whether PB is or is not equal to PC.

Answer (b) PB .... equal to PC because

.... [1]

PB is equal to PC because $\angle PBC = \angle PCB = 70°$, thus they are base angles of an isosceles triangle PBC.

119

14 The lengths of three square flower beds P, Q and R are in the ratio $2:3:n$, where n is an integer. Given that the actual area of P is $64\ \text{m}^2$ and the total area of P, Q and R is $784\ \text{m}^2$, find the value of n.

	P : Q : R
Length	$2 : 3 : n$
Area	$4 : 9 : n^2$
Actual Area	$64\ \text{m}^2 : 144\ \text{m}^2 : (784-64-144)\ \text{m}^2 = 576\ \text{m}^2$

$$16n^2 = 576$$
$$n^2 = 36$$
$$n = 6$$

Answer $n=$ .... [3]

---

15 80 students took a Physical Fitness test and the number of push-ups that they did in one minute are shown in the frequency table below.

No. of push-ups (x)	Frequency
$0 < x \le 10$	8
$10 < x \le 20$	17
$20 < x \le 30$	21
$30 < x \le 40$	22
$40 < x \le 50$	10
$50 < x \le 60$	2

Find the mean and the standard deviation of the number of push-ups that the students did in one minute.

No. of push-ups (x)	Mid-class value, x_m	Frequency
$0 < x \le 10$	5	8
$10 < x \le 20$	15	17
$20 < x \le 30$	25	21
$30 < x \le 40$	35	22
$40 < x \le 50$	45	10
$50 < x \le 60$	55	2

$$\text{Mean} = \frac{5(8)+(15)(17)+(25)(21)+(35)(22)+(45)(10)+(55)(2)}{80}$$
$$= 26\frac{7}{8}$$

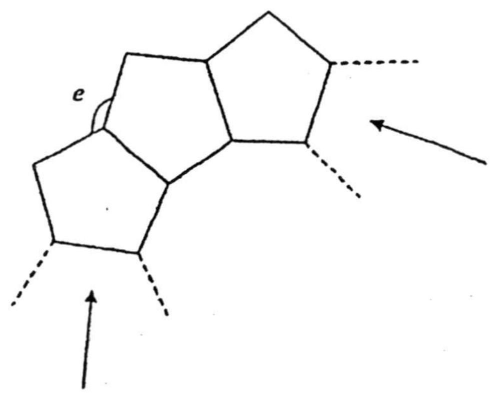
Using calculator, Standard deviation = 12.6 (3 sig.fig.)

$$\underline{\text{Or}}\ \text{Standard deviation} = \sqrt{\frac{\sum fx^2}{\sum f} - \left(\frac{\sum fx}{\sum f}\right)^2}$$
$$= \sqrt{\frac{70400}{80} - (26\frac{7}{8})^2}$$
$$= 12.6$$

Answer Mean = ... [2]

Standard deviation = ... [2]

17 The diagram shows part of a ring formed by a number of regular pentagons placed together.



Find

(a) the value of e,

Each interior angle of pentagon $= 108°$

$$e = 360° - 2 \times 108°$$
$$= 144°$$

Answer *(a) e =* .... [2]

(b) the number of pentagons in the ring.

$$\text{No of pentagons} = \frac{360°}{180° - 144°} = \frac{360°}{36°} = 10$$

Answer *(b)* .... [1]

---

18 One solution of the equation $3x^2 + x + k = 0$, where k is a constant, is $x = -2$. Find

(a) the value of k,

$$3x^2 + x + k = 0$$
$$\text{When } x = -2, 3(-2)^2 + (-2) + k = 0$$
$$12 - 2 + k = 0$$
$$\therefore k = -10$$

Answer (a) $k =$ .... [1]

(b) the second solution of the equation.

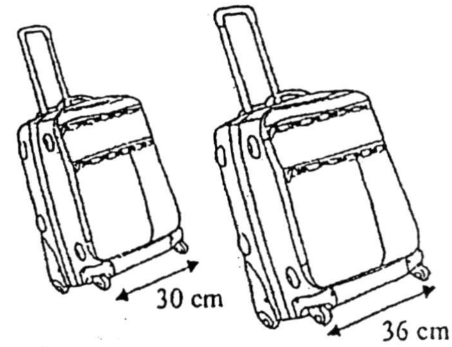
$$3x^2 + x - 10 = 0$$
$$(3x - 5)(x + 2) = 0$$
$$x = 1\frac{2}{3} \text{ or } x = -2$$

The second solution of the equation is $x = 1\frac{2}{3}$.

Answer (b) $x =$ .... [1]

120

19 The diagram shows two luggage bags which are geometrically similar. The cost of the luggage bag is proportional to its volume. The larger luggage bag has a length of 36 cm and the smaller one has a length of 30 cm. If the smaller luggage bag costs \$175, find the cost of the larger luggage bag.



$$\frac{C_1}{C_2}=\frac{V_1}{V_2}=\left(\frac{36}{30}\right)^3$$

$$\frac{C_1}{175}=\left(\frac{6}{5}\right)^3$$

$$C_1=\left(\frac{6}{5}\right)^3\times 175$$

$$=302.40$$

The larger luggage bag costs \$302.40.

Answer \$.............................. [2]

---

20 (a) p is inversely proportional to q^2. It is known that $p = 36$ for a particular value of q. Find the value of p when this value of q is trebled.

$p=\frac{k}{q^2}$, k is a non-zero constant

$k = pq^2$

$=36q^2$ for a particular value of q

$p=\frac{36q^2}{(3q)^2}$ when q is trebled

$=4$

Answer (a) p =.............................. [2]

(b) If Alex and Benji work together, they can complete a job in 12 days. Benji alone takes 5 days to complete $\frac{1}{6}$ of the job. How many days will Alex take to complete the job alone?

Benji takes $6\times5=30$ days to complete the job alone.

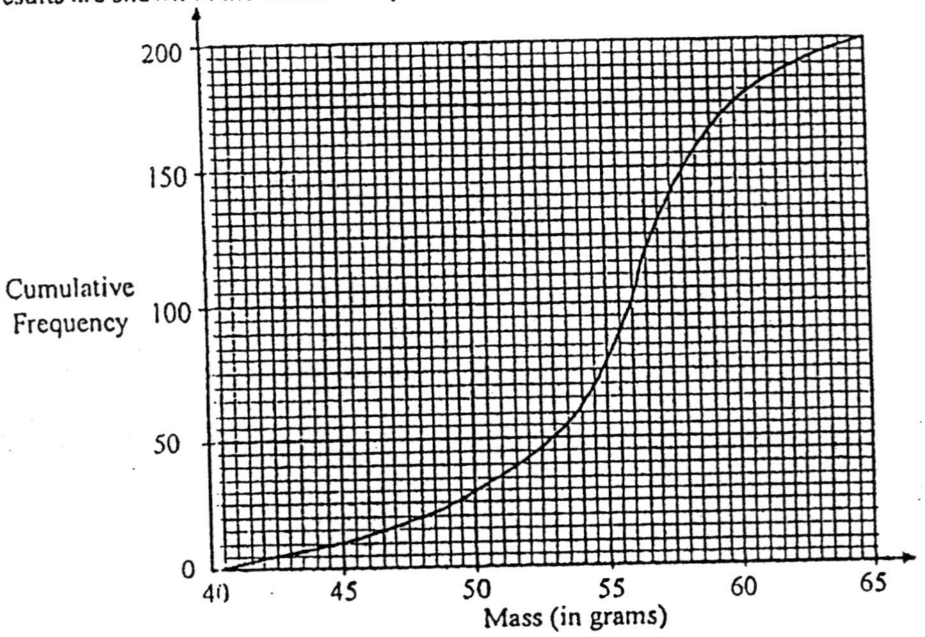
In 12 days, Benji completes $\frac{12}{30}=\frac{2}{5}$ of the job

and Alex completes $\frac{3}{5}$ of the job.

Therefore Alex takes $\frac{5}{3}\times12=20$ days to complete the job alone.

Answer (b) ....days [3]

23 The masses of a sample of 200 eggs from Rainbow Farm were measured and the results are shown in the cumulative curve below.



The heaviest 25% of the eggs are classified as "Grade 1".
The lightest 20% of the eggs are classified as "Grade 3".
The remaining eggs are classified as "Grade 2".

Using the graph,

(a) find an estimate for the least possible difference between the mass of an egg classified as "Grade 3" and the mass of an egg classified as "Grade 1".

Least possible difference $= 58 - 51.5$ g
$= 6.5$ g

Answer *(a)* ....grams [1]

Two eggs are chosen from the sample group.

(b) Find the probability that both eggs are classified as "Grade 2".

$$P(\text{both eggs are Grade 2}) = \frac{110}{200} \times \frac{109}{199}$$
$$= \frac{1199}{3980}$$

53

Answer *(b)* .... [2]

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Name	Class	Class Register Number

中正中学

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PRELIMINARY EXAMINATION 2015
SECONDARY 4

4016/02

MATHEMATICS
Paper 2

Friday 28 August 2015
2 hours 30 minutes

Additional Materials: Answer Paper
Graph Paper

READ THESE INSTRUCTIONS FIRST

Do not open this booklet until you are told to do so.

Write your name, class and register number on all the work you hand in.
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The total number of marks for this paper is 100.

	100

This Question Paper consists of 7 printed pages and 1 blank page.

[Turn Over

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$$\text{Sector area} = \frac{1}{2}r^2\theta\text{, where } \theta \text{ is in radians}$$

Trigonometry

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

Statistics

$$\text{Mean} = \frac{\Sigma fx}{\Sigma f}$$

$$\text{Standard deviation} = \sqrt{\frac{\Sigma fx^2}{\Sigma f} - \left(\frac{\Sigma fx}{\Sigma f}\right)^2}$$

3 **Answer the whole of this question on a sheet of graph paper.**

The variables x and y are connected by the equation

$$y = \frac{1}{2}(8x - x^2 - \frac{12}{x})$$

Some of the corresponding values of x and y, correct to 2 decimal places, are given in the following table.

x	1	2	3	4	5	6	7	8
y	–2.50	p	5.50	6.50	6.30	5.00	2.64	–0.75

(a) Find the value of p. [1]

(b) Using a scale of 2 cm to 1 unit, draw a horizontal axis for $0 \le x \le 8$. [3]
Using a scale of 2 cm to 1 unit, draw a vertical axis for $-3 \le y \le 7$.
On your axes, plot the points given in the table and join them with a smooth curve.

(c) By drawing a tangent, find the gradient of the curve at $x = 3.5$. [2]

(d) (i) On the same axes, draw the line $y = \frac{1}{3}x$ for $0 \le x \le 8$. [1]

(ii) Write down the x-coordinate of the points where this line intersects the curve. [1]

(iii) This value of x is a solution of the equation $Ax - 2Bx^2 - \frac{12}{x} = 0$.

Find the values of A and B. [2]

---

4 The vertices of a triangle ABC are $A(2,6)$, $B(16,6)$ and $C(8,12)$.

Find

(a) (i) $\overrightarrow{BC}$, [2]

(ii) $|\overrightarrow{BC}|$. [2]

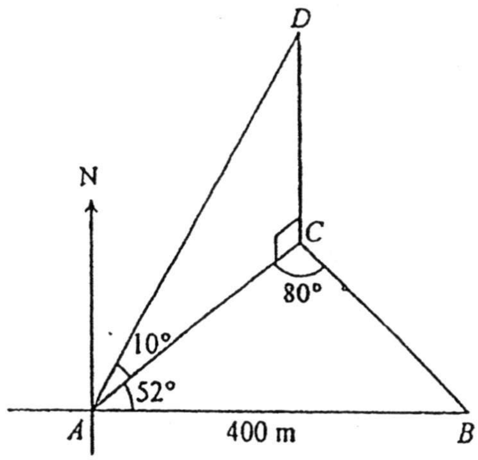
(b) (i) Find the coordinates of point M on BC where $BC : BM = 4 : 1$. [4]

(ii) State the position vector of point M. [1]

(iii) Express $\overrightarrow{AM}$ as a column vector. [2]

,23

7



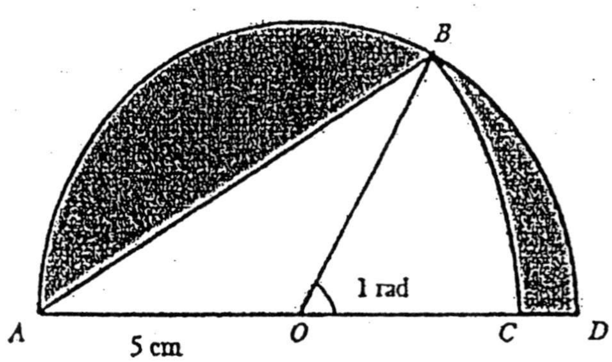
In the diagram, points A, B and C are on level ground and B is due east of A. CD is a vertical building at point C, angle $ACB = 80°$, angle $BAC = 52°$, $AB = 400$ m and the angle of elevation of the top of the building from A is $10°$. Find

(a) the distance AC, [3]
(b) the height of the building CD, [2]
(c) the maximum angle of elevation from AB to the top of the building. [3]

The bearing of point E from point A is $112°$. If AE is 500 m, find
(d) (i) angle CAE [2]
(ii) the distance from C to E. [2]

---

8 $ABDCO$ is a semi-circle with centre at O, radius 5 cm and diameter AD. Arc BC is part of a circle with centre at A. Given angle $BOC = 1$ rad, find



(a) the length of arc BD, [1]
(b) the length of arc BC, [3]
(c) perimeter of the shaded region, [3]
(d) area of the shaded region. [3]

	Answer Key
	$\dfrac{2}{(x-3)(x-4)}$
(i)	$x=\pm\sqrt{\dfrac{2py^2}{k}+5}$
(ii)	1) $\dfrac{2py^2}{k}+5\geq 0$ 2) $k\neq 0$
(i)	$(4\times 5)-8=12$
(ii)	$m=11$ $p=22$ $t=110$
(iii)	$r=n^2-n$ or $n(n-1)$
	$n=14.5$ or -13.5 $\Rightarrow n\notin \mathbb{Z}^+$ Therefore it's not one of the results of an equation of the sequence.
	$p=3$
	See attached
	Gradient = 1.01
(i)	See attached
(ii)	1.45 and 7.10
(iii)	$A=7\frac{1}{3}, B=\frac{1}{2}$
(i)	$\overline{BC}=\begin{pmatrix}-8\\6\end{pmatrix}$
(ii)	10 units
(i)	$M(14,7\frac{1}{2})$
(ii)	$\overline{OM}=\begin{pmatrix}14\\7\frac{1}{2}\end{pmatrix}$
(iii)	$\overline{AM}=\begin{pmatrix}12\\1\frac{1}{2}\end{pmatrix}$
	(i) $\dfrac{800}{x}$ km/l (ii) $\dfrac{800}{x-10}$ km/l
	$x=68.44l$ or $-58.44l$(2d.p)
	937 km
	$\mathrm{A}=\begin{pmatrix}8\\30\\20\end{pmatrix}$
	$\mathrm{B}=\begin{pmatrix}2 & 1 & 1\end{pmatrix}$ $\mathrm{BA}=(66)$

Calculator Model :

Name	Class	Class Register Number

中正中学

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PRELIMINARY EXAMINATION 2015
SECONDARY 4

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MATHEMATICS
Paper 2

SOLUTIONS

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2 hours 30 minutes

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/100

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This Question Paper consists of 7 printed pages and 1 blank page.

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Turn Over

Mathematical Formulae

Compound interest

$$\text{Total amount} = P\left(1+\frac{r}{100}\right)^n$$

Mensuration

$$\text{Curved surface area of a cone} = \pi r l$$

$$\text{Surface area of a sphere} = 4\pi r^2$$

$$\text{Volume of a cone} = \frac{1}{3}\pi r^2 h$$

$$\text{Volume of a sphere} = \frac{4}{3}\pi r^3$$

$$\text{Area of triangle } ABC = \frac{1}{2}ab\sin C$$

$$\text{Arc length} = r\theta \text{, where } \theta \text{ is in radians}$$

$$\text{Sector area} = \frac{1}{2}r^2\theta \text{, where } \theta \text{ is in radians}$$

Trigonometry

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

Statistics

$$\text{Mean} = \frac{\sum fx}{\sum f}$$

$$\text{Standard deviation} = \sqrt{\frac{\sum fx^2}{\sum f} - \left(\frac{\sum fx}{\sum f}\right)^2}$$

Answer all the questions.

1 (a) Simplify $\dfrac{4}{(x-2)(x-4)}-\dfrac{2}{(x-2)(x-3)}$. [3]

(b) (i) Make x the subject for the following equation. [3]

$$y=\sqrt{\frac{(x^2-5)k}{2p}}$$

(ii) State the 2 conditions such that x is a real value. [2]

Soln:

(a)

$$\frac{4}{(x-2)(x-4)}-\frac{2}{(x-2)(x-3)}$$

$$=\frac{4(x-3)-2(x-4)}{(x-2)(x-3)(x-4)}$$

$$=\frac{4x-12-2x+8}{(x-2)(x-3)(x-4)}$$

$$=\frac{2x-4}{(x-2)(x-3)(x-4)}$$

$$=\frac{2(x-2)}{(x-2)(x-3)(x-4)}$$

$$=\frac{2}{(x-3)(x-4)}$$

(b) (i)

$$y=\sqrt{\frac{(x^2-5)k}{2p}}$$

$$y^2=\frac{(x^2-5)k}{2p}$$

$$\frac{2py^2}{k}=x^2-5$$

$$x^2=\frac{2py^2}{k}+5$$

$$x=\pm\sqrt{\frac{2py^2}{k}+5}$$

(ii) 1) $\dfrac{2py^2}{k}+5\geq 0$

2) $k\neq 0$

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2 (a) In the following sequence of equations,

$$\begin{aligned}(1\times2)-2 &= 0\\(2\times3)-4 &= 2\\(3\times4)-6 &= 6\\ \vdots \\ (m\times12)-p &= t\\ \vdots \\ (n\times y)-z &= r\end{aligned}$$

(i) State the 4^{th} equation [1]

(ii) Find the values of m, p and t. [3]

(iii) Express r in terms of n. [2]

(b) Determine if $r=196$ can be the result of an equation in the sequence. [2]

Soln:

(a) (i) 4^{th} equation: $(4\times5)-8=12$

(ii) $m=11$

$p=22$

$t=(11\times12)-22$

$=110$

(iii) $(n\times y)-z=r$

$(n\times(n+1))-2n=r$

$n^2+n-2n=r$

$n^2-n=r$

$r=n^2-n$ or $n(n-1)$

(b) $r=n^2-n$

$196=n^2-n$

$n^2-n-196=0$

$n=14.5$ or -13.5

$\Rightarrow n\notin\mathbb{Z}^+$

Therefore it's not one of the results of an equation of the sequence.

3 **Answer the whole of this question on a sheet of graph paper.**

The variables x and y are connected by the equation

$$y = \frac{1}{2}(8x - x^2 - \frac{12}{x})$$

Some of the corresponding values of x and y, correct to 2 decimal places, are given in the following table.

x	1	2	3	4	5	6	7	8
y	–2.50	p	5.50	6.50	6.30	5.00	2.64	–0.75

(a) Find the value of p. [1]

(b) Using a scale of 2 cm to 1 unit, draw a horizontal axis for $0 \le x \le 8$. [3]
Using a scale of 2 cm to 1 unit, draw a vertical axis for $-3 \le y \le 7$.
On your axes, plot the points given in the table and join them with a smooth curve.

(c) By drawing a tangent, find the gradient of the curve at $x = 3.5$. [2]

(d) (i) On the same axes, draw the line $y = \frac{1}{3}x$ for $0 \le x \le 8$. [1]

(ii) Write down the x-coordinate of the points where this line intersects the curve. [1]

(iii) This value of x is a solution of the equation $Ax - 2Bx^2 - \frac{12}{x} = 0$.

Find the values of A and B. [2]

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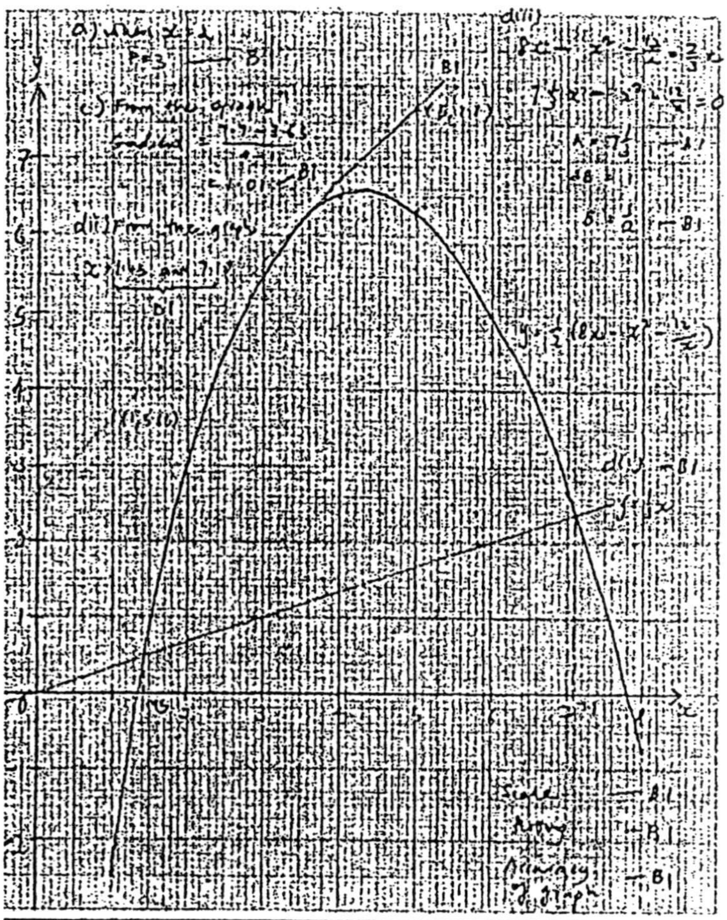

ces of a triangle ABC are $A(2,6)$, $B(16,6)$ and $C(8,12)$.

$\overrightarrow{BC}$, [2]

$\left|\overrightarrow{BC}\right|$. [2]

Find the coordinates of point M on BC where $BC : BM = 4:1$. [4]

State the position vector of point M. [1]

Express $\overrightarrow{AM}$ as a column vector. [2]

Maximum distance

$$=\frac{800}{68.4428-10}\times 68.4428$$

$$=936.9$$

$$=937\text{km (3s.f)}$$

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6 The table below shows the price of a ticket for each category for a musical, Swan Lake.

Child (below 12 years old)	\$8
Adult	\$30
Senior Citizen(above 55 years old)	\$20

(a) Write down a column matrix A to represent the above information [1]

(b) Mrs Lim bought four tickets for her 70 year old father, her 9 year old and 11 year old daughters, and herself.
Write down a matrix B such that the product BA gives the total amount of money Mrs Lim paid for the tickets.
Hence, find this product. [3]

(c) The table below shows information about the musical.

Number of tickets sold

	Child	Adult	Senior Citizen
Saturday	37	u	25
Sunday	44	85	v

(i) Form a matrix multiplication if the ticket sales collected on Saturday and Sunday are \$2686 and \$3522 respectively. [1]

(ii) Find the values of u and v. [2]

(d) (i) Evaluate the matrix $P=\begin{pmatrix}1.2 & 2\end{pmatrix}T$ where T is the matrix representing the revenue collected on Saturday and Sunday. [1]

(ii) Explain what the matrix $\begin{pmatrix}1.2 & 2\end{pmatrix}$ means and what the answer in d(i) represents. [2]

Soln

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(a)

$$A = \begin{pmatrix} 8 \\ 30 \\ 20 \end{pmatrix}$$

(b)

$$B = \begin{pmatrix} 2 & 1 & 1 \end{pmatrix}$$

$$BA = \begin{pmatrix} 2 & 1 & 1 \end{pmatrix}\begin{pmatrix} 8 \\ 30 \\ 20 \end{pmatrix}$$

$$= (16 + 30 + 20)$$

$$= (66)$$

(c) (i)

$$\begin{pmatrix} 37 & u & 25 \\ 44 & 85 & v \end{pmatrix}\begin{pmatrix} 8 \\ 30 \\ 20 \end{pmatrix} = \begin{pmatrix} 2686 \\ 3522 \end{pmatrix}$$

(ii)

$$(37 \times 8) + (u \times 30) + (25 \times 20) = 2686$$

$$u = 63$$

$$(44 \times 8) + (85 \times 30) + (v \times 20) = 3522$$

$$v = 31$$

(d) (i)

$$\begin{pmatrix} 1.2 & 2 \end{pmatrix}\begin{pmatrix} 2686 \\ 3522 \end{pmatrix}$$

$$= (10267.2)$$

(ii) Increase in price by 20% and 100% on Saturday and Sunday respectively.
Total sales of tickets for the Saturday and Sunday

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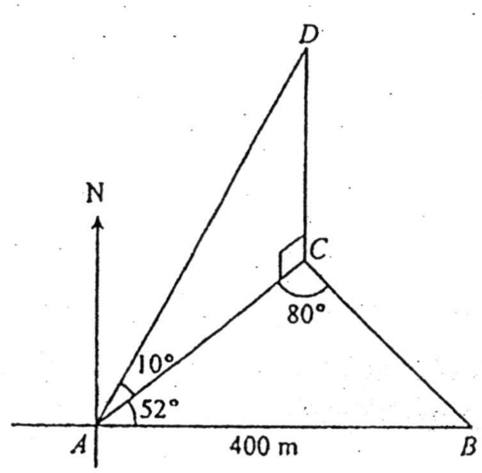

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In the diagram, points A, B and C are on level ground and B is due east of A. CD is a vertical building at point C, angle $ACB = 80°$, angle $BAC = 52°$, $AB = 400$ m and the angle of elevation of the top of the building from A is $10°$. Find

(a) the distance AC, [3]
(b) the height of the building, [2]
(c) the maximum angle of elevation from AB to the top of the building. [3]

The bearing of point E from point A is $112°$. If AE is 500 m, find
(d) (i) angle CAE [2]
(ii) the distance from C to E. [2]

Soln:

(a)

$$\angle ABC = 180° - 52° - 80° \text{ (sum of angles of triangle)}$$
$$= 48°$$

Using Sine Rule:

$$\frac{AC}{\sin 48°} = \frac{AB}{\sin 80°}$$
$$AC = \frac{400}{\sin 80°} \times \sin 48°$$
$$= 301.8436$$
$$= 302\text{m(3 sig. fig)}$$

(b)

$$\tan \angle DAC = \frac{DC}{AC}$$
$$\tan 10° = \frac{DC}{301.8436}$$
$$DC = 53.2231$$
$$= 53.2\text{m(3 sig. fig)}$$

(c)

Let perpendicular distance from C to AB be h

$$\sin 52° = \frac{h}{AC}$$
$$\sin 52° = \frac{h}{301.8436}$$
$$h = 237.856$$

Maximum angle of elevation

$$= \tan^{-1}\left(\frac{53.2231}{237.856}\right)$$
$$= 12.613°$$
$$= 12.6°\text{(1 dec. place)}$$

$$\sin 1 = \frac{BM}{5}$$
$$BM = 4.20735 \text{ cm}$$

$$\sin 0.5 = \frac{BM}{AB}$$
$$= \frac{4.20735}{AB}$$
$$AB = 8.7758\text{cm}$$

$$\text{Arc } BC = r\theta$$
$$= 8.7758 \times 0.5$$
$$= 4.3879$$
$$= 4.39\text{cm (3 sig. fig.)}$$

(c) Arc $AB = 5(\pi - 1)$
$$= 10.70796$$

Perimeter
$= Arc\ AB +$ Line $AB + BD + BC + CD$
$= Arc\ AB +$ Line $AB + BD + BC + (OD - OC)$
$= Arc\ AB +$ Line $AB + BD + BC + (OD - ($Line $AB - 5)$
$= 10.70796 + 8.7758 + 5 + 4.3879 + (5 - (8.7758 - 5))$
$= 30.09586$
$= 30.1$cm (3sig.fig)

(d) Shaded area
$$= \frac{\pi 5^2}{2} - \frac{1}{2}(8.7758^2)(0.5)$$
$$= 20.0162$$
$$= 20.0\text{cm}^2 \text{(3sig.fig)}$$

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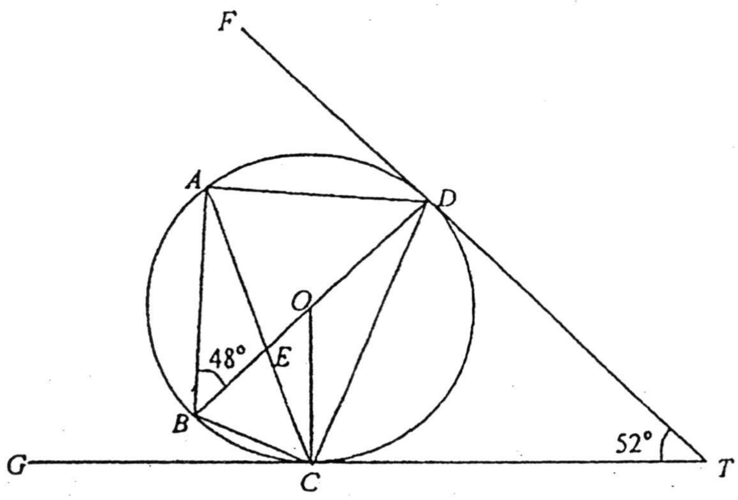

The diagram, shows a circle with centre O and points A, B, C and D lie on its circumference. TDF and TCG are tangents to the circle at D and C respectively. $BEOD$ is a straight line. Given that angle $CTD = 52°$ and angle $ABD = 48°$, calculate

(i) angle COD, [1]
(ii) angle ACG, [2]
(iii) angle DEA. [2]

Given that the radius of the circle is 6 cm, find
(i) CT, [2]
(ii) the area of triangle DCT. [2]

If given two points S and R on the circumference of the circle, Mary commented that tangent lines drawn from these points will always meet. Do you agree? Explain. [2]

Soln:

(i) $\angle OCT = \angle ODT = 90°$(tan $\perp$ radius)

$\angle COD = 360° - 90° - 90° - 52°$(angle sum of quadrilateral)

$= 128°$

(ii)

$\angle ACD = \angle ABD$

$= 48°$(angles in the same segment)

$\angle DCT = \dfrac{180° - 522°}{2}$(base angles in isos. triangle, $DT = CT$)

$= 64°$

$\angle ACG = 180° - 48° - 64°$(sum of angles in triangle)

$= 68°$

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