NAME: _____ ( ) CLASS: _____

FAIRFIELD METHODIST SCHOOL (SECONDARY)

PRELIMINARY EXAMINATION 2015
SECONDARY 4 EXPRESS / 5 NORMAL (ACADEMIC)

MATHEMATICS 4016/01

Paper 1

Date: 20 August 2015 **Duration: 2 hours**

Candidates answer on the Question Paper.

READ THESE INSTRUCTIONS FIRST

Write your name, index number and class on all the work you hand in.
Write in dark blue or black pen.
You may use a pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** questions.
If working is needed for any question it must be shown with the answer.
Omission of essential working will result in loss of marks.
You are expected to use a scientific calculator to evaluate explicit numerical expressions.
If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.
For π, use either your calculator value or 3.142, unless the question requires the answer in terms of π.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 80.

For Examiner's Use	
Paper 1	/ 80
Paper 2	/ 100
Total	%

Setters: Mr Alester Tan

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This question paper consists of 19 printed pages including the cover page.

Name: _____ ( ) Class: _____

Mathematical Formulae

Compound interest

$$\text{Total amount} = P\left(1+\frac{r}{100}\right)^n$$

Mensuration

Curved surface area of a cone = πrl

Surface area of a sphere = $4\pi r^2$

Volume of a cone = $\frac{1}{3}\pi r^2 h$

Volume of a sphere = $\frac{4}{3}\pi r^3$

Area of a triangle $ABC = \frac{1}{2}ab\sin C$

Arc length = $r\theta$, where θ is in radians

Sector area = $\frac{1}{2}r^2\theta$, where θ is in radians

Trigonometry

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

Statistics

$$\text{Mean} = \frac{\sum fx}{\sum f}$$

$$\text{Standard deviation} = \sqrt{\frac{\sum fx^2}{\sum f} - \left(\frac{\sum fx}{\sum f}\right)^2}$$

Name: _____ ( ) Class: _____

Answer **all** the questions.

1 Calculate $\sqrt{\dfrac{28\cdot047}{3\cdot743\times10^{-4}}}$.

(a) Give your answer as displayed in the calculator.

(b) Write down your answer in part (a) correct to 2 significant figures.

Answer (a) ..... [1]

(b) ..... [1]

---

2 Alan bought a smart watch for \$240. When he sold it, he made a profit of 115% of his cost. Find the selling price.

Answer \$..... [2]

---

3 **(a)** Given that $a^5 \times a^2 \div \sqrt[3]{a} = a^k$, find the value of k.

(b) Simplify $\left(\dfrac{2b}{3}\right)^{-3}$.

Answer (a) k = ... [1]

(b) ..... [1]

Name: _____ ( ) Class: _____

4 Trevor bought p apples at q cents per half a dozen. He sold them for r cents each.
Write down, in its simplest form, an expression in terms of p, q and r, for the profit, in dollars, that he made.

Answer $\$$..... [2]

---

5 The scale of the floor plan of a house can be expressed as $\frac{1}{2}$ cm represents 2 m.

(a) Express this scale in the form 1 : n.

(b) Calculate, in cm^2, the area of a room on the map which has an actual area of 32 m^2.

Answer (a) 1 : ..... [1]

(b) ..... cm^2 [2]

Name: _____ ( ) Class: _____

6 The number of books that students borrowed is shown in the table below.

No. of books	0	1	2	3	4	5	6
No. of students	5	6	x	11	5	2	1

Find

(a) the largest possible value of x if the median is 3,

(b) the value of x if the mean number of books borrowed is $2\frac{1}{3}$.

Answer (a) ..... [1]

(b) ..... [2]

---

7 Express as a single fraction in its simplest form.

$$\frac{16x^2-1}{(4x-1)^2}-\frac{2x}{4x-1}$$

Answer ..... [3]

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Name: _____ ( ) Class: _____

8 The thickness of a sheet of paper is 0.05 millimetre.

(a) Express 0.05 millimetre in metre in standard form.

(b) Find the total number of sheets of paper from a pile of paper, which has a height of 6×10^{-2} m.

Answer (a) ..... [1]

(b) ..... sheets [2]

---

9 Given a sequence

$$1, \frac{2}{3}, \frac{4}{7}, \frac{8}{15}, \frac{16}{31}, \ldots$$

(a) Write down the next term in the sequence.

(b) One of the terms in the sequence is $\frac{x}{y}$. Find in **terms of** x and y, the term which comes immediately after $\frac{x}{y}$.

(c) Write down an expression in terms of n, for the nth term of the sequence.

Answer (a) ..... [1]

(b) ..... [1]

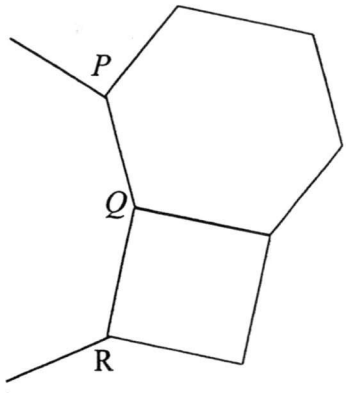
(c) ..... [1]

Name: _____ ( ) Class: _____

10 The figure is made up of a square and a regular hexagon. PQ and QR are two sides of another regular polygon. Find

(a) the reflex $\angle PQR$,

(b) the number of sides of this polygon.



Answer (a) ..... ° [1]

(b) ..... [2]

---

11 **(i)** Express 14520 as a product of prime factors.

(ii) Given that $14520k$ is a perfect square, write down the smallest positive integer k.

(iii) Find the largest integer which is a factor of 14520 and 4400.

Answer (i) ..... [1]

(ii) ..... [1]

(iii) ..... [1]

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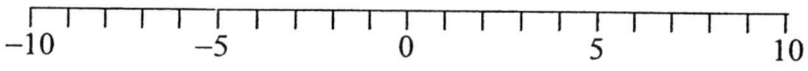
Name: _____ ( ) Class: _____

12 **(a)** Solve the inequality $-2-3x \leq 8+2x \leq x+12$.

Show your solution on the number line in the answer space.

Answer (a)



[2]

(b) Write down all the prime numbers which satisfy $-2-3x \leq 8+2x \leq x+12$.

Answer (b) ..... [1]

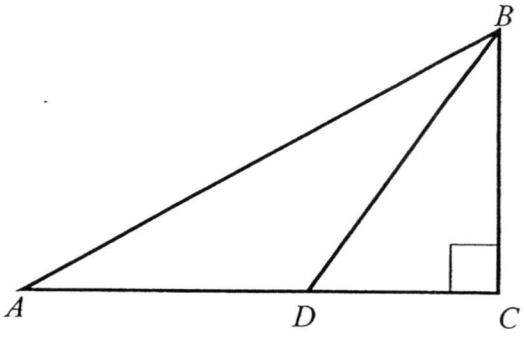
---

13 ABC is a right-angled triangle where $AB = 17$ cm, $BC = 8$ cm and $\angle ACB = 90°$.

Given that D is a point on AC such that $\frac{CD}{AD} = \frac{2}{3}$, find, giving your answer as a fraction in its simplest form, the value of

(a) tan $\angle CBD$,

(b) cos $\angle ADB$.



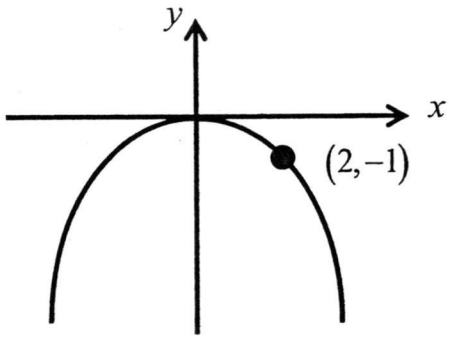
Answer (a) tan $\angle CBD$ =..... [2]

(b) cos $\angle ADB$ =..... [2]

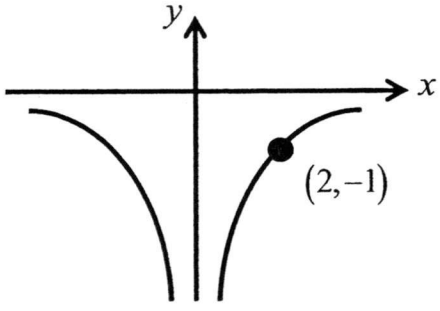
Name: _____ ( ) Class: _____

14 The equation of the following curves is of the form $y = ax^n$, where n is an integer. Each of the curves passes through the point $(2,-1)$. State a possible value for a and n for each curve.

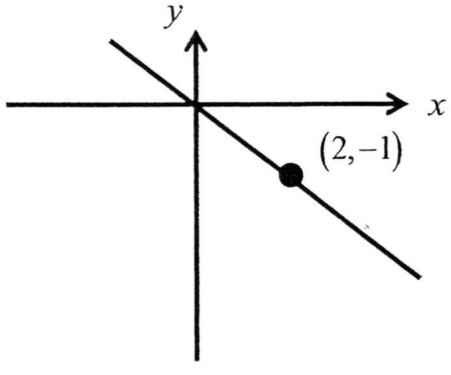
(a)



(b)



(c)



Answer (a) a = ..., n = ... [1]

(b) a = ..., n = ... [1]

(c) a = ..., n = ... [1]

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Name: _____ ( ) Class: _____

15 The table below shows the number of goals scored by a soccer team in 16 matches.

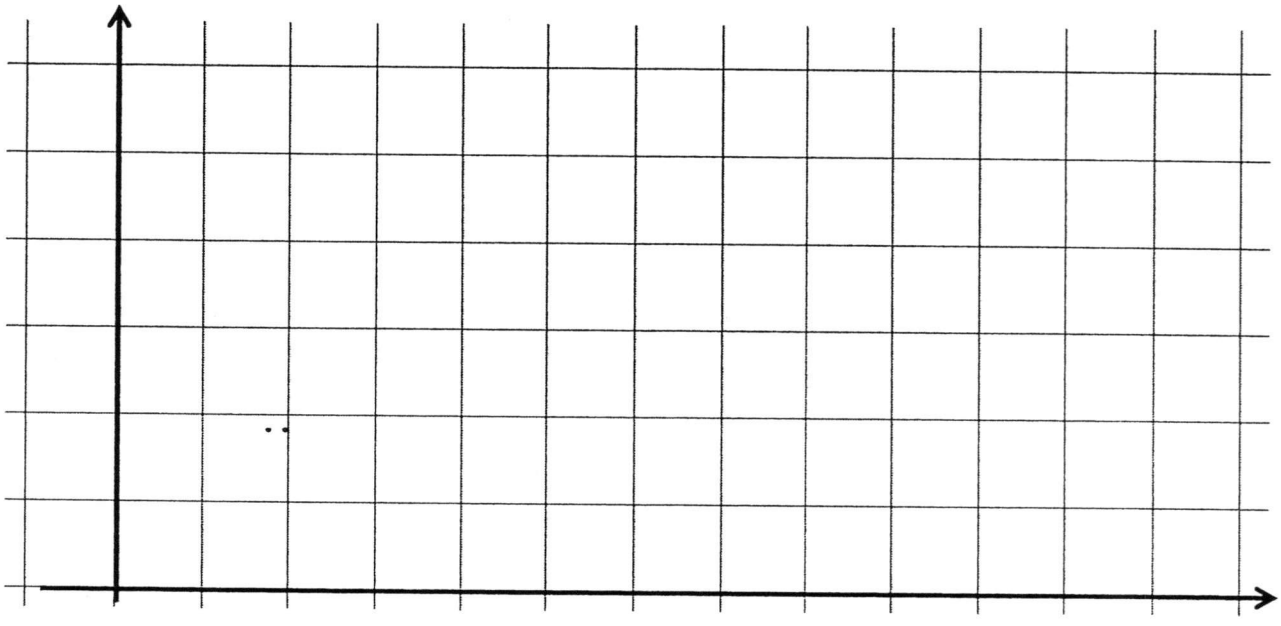
No. of goals	0	1	2	3	4	5
No. of matches	3	5	1	4	2	1

(a) Complete, with labels, the bar chart below using the data from the table above.

(b) Find the median of the distribution.

Answer (a)

[2]



Answer (b) .....goal(s) [1]

Name: _____ ( ) Class: _____

16 **(a)** Express $x^2 - 6x - 16$ in the form $(x-a)^2 + b$.

(b) Hence solve the equation $\frac{x^2 - 16}{2} = 3x$.

Answer (a) ..... [1]

(b) x = ... or x = ... [2]

---

17 **(a)** 6 men can paint a building in 15 days.

Find the number of days taken for 5 men to paint the same building.

(b) y is directly proportional to x^2. It is known that $y = 14$ for a particular value of x.

Find the value of y when this value of x is halved.

Answer (a) ..... days [1]

(b) y = ... [2]

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Name: _____ ( ) Class: _____

18 Solve the following simultaneous equations

$$3a+2b=3$$
$$4a-3b=-13$$

Answer a = ...

b = ... [3]

Name: _____ ( ) Class: _____

19 Two coins are placed in a bag. One of the coins is fair while the other is biased such that when tossed, the probability of obtaining a tail is $\frac{2}{3}$. A coin is chosen at random from the bag and is tossed twice.

Calculate the probability that

(i) two tails are obtained,

(ii) a head is obtained at second toss.

Answer (i) ..... [2]

(ii) .....[2]

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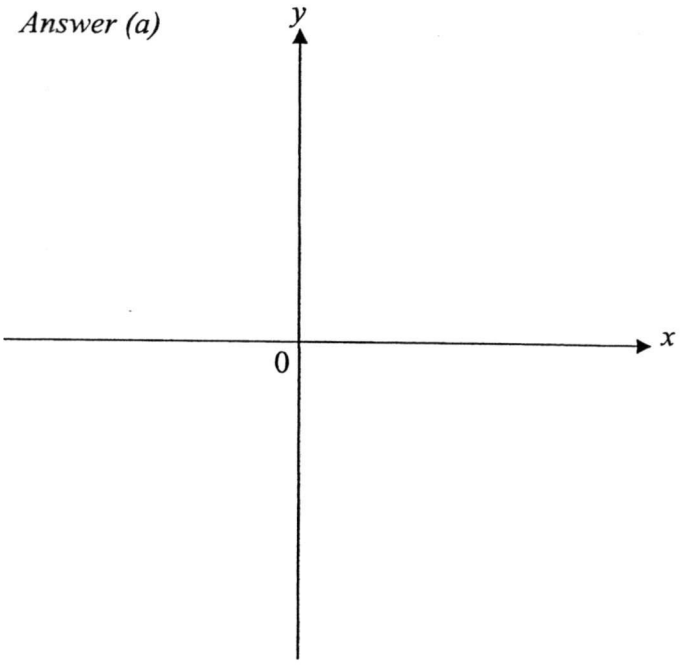
Name: _____ ( ) Class: _____

20 **(a)** Sketch the graph of $y = (x-2)(x+3)$.

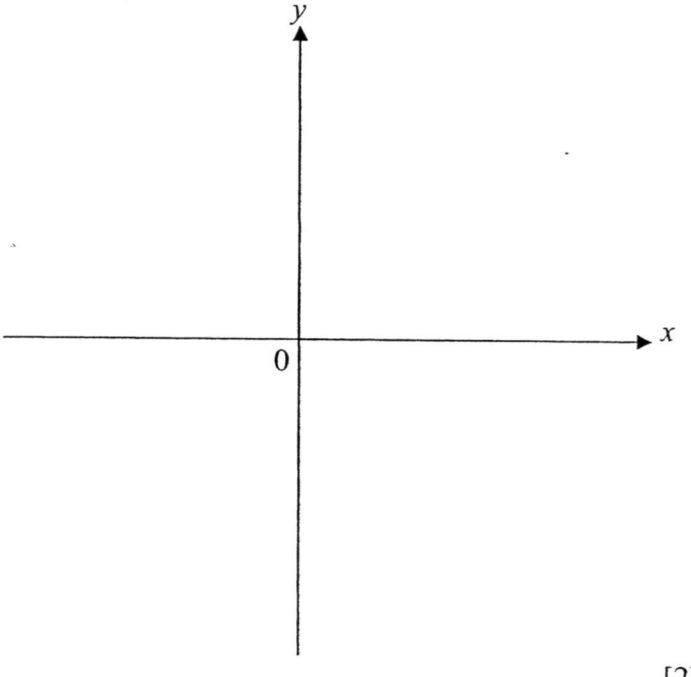
Answer (a)



[2]

(b) Sketch the graph of $y = 5-(x+1)^2$.

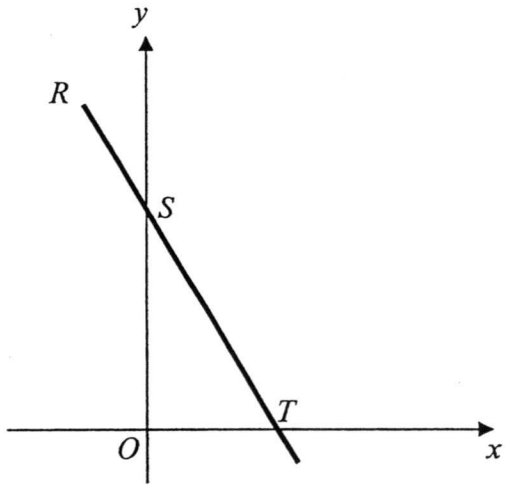
Answer *(b)*



[2]

Name: _____ ( ) Class: _____

21 In the diagram, which is not drawn to scale, the line RST has the equation $y+2x=5$.



(a) Write down the coordinates of the points S and T.

(b) The y-coordinate of the point R is 9. Find

(i) the x-coordinate of R,

(ii) the area of triangle RSO.

Answer (a) S (,) [1]

T (,) [1]

(b)(i) $x =$ ..... [1]

(ii) ..... units^2 [1]

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Name: _____ ( ) Class: _____

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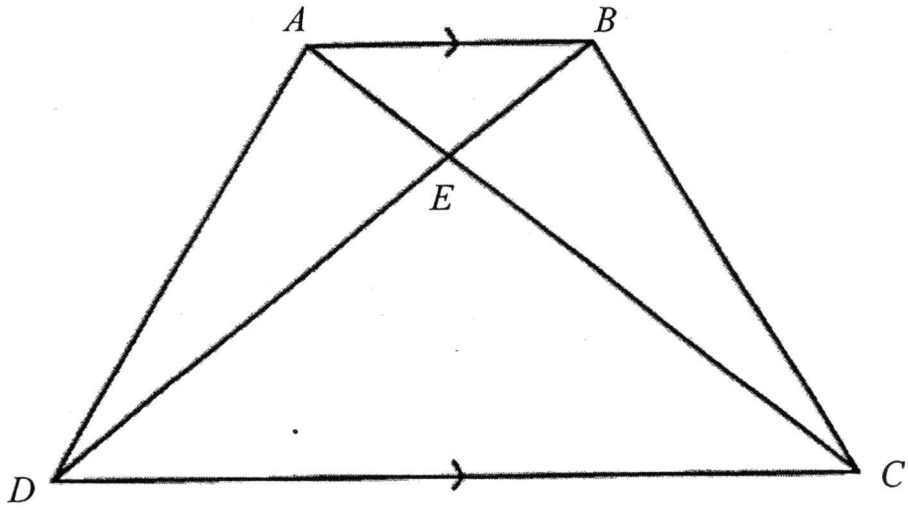


The diagram shows a trapezium $ABCD$ where $AB = 3$ cm and $DC = 9$ cm.
The diagonals AC and BD meet at E.

(a) Show that triangle AEB is similar to triangle CED.

.....

.....

.....

.....[2]

(b) Given that the area of triangle AEB is 7 cm^2, find the area of triangle CED.

Answer (b) .....cm^2 [1]

(c) Find $\dfrac{\text{area of triangle } DEC}{\text{area of trapezium } ABCD}$.

Answer (c) ..... [2]

Name: _____ ( ) Class: _____

23 Three shops A, B and C are located such that B is 500 m due South of A and C is 1000 m from B on a bearing of 220°.

A map is being drawn to a scale of 1 cm to 100 m. The positions of A, B and C are shown below.

Another shop D is 1200 m from C on a bearing of 330°.

(a) Complete the map to show the position of D. [2]

(b) A new shop is to be built [3]

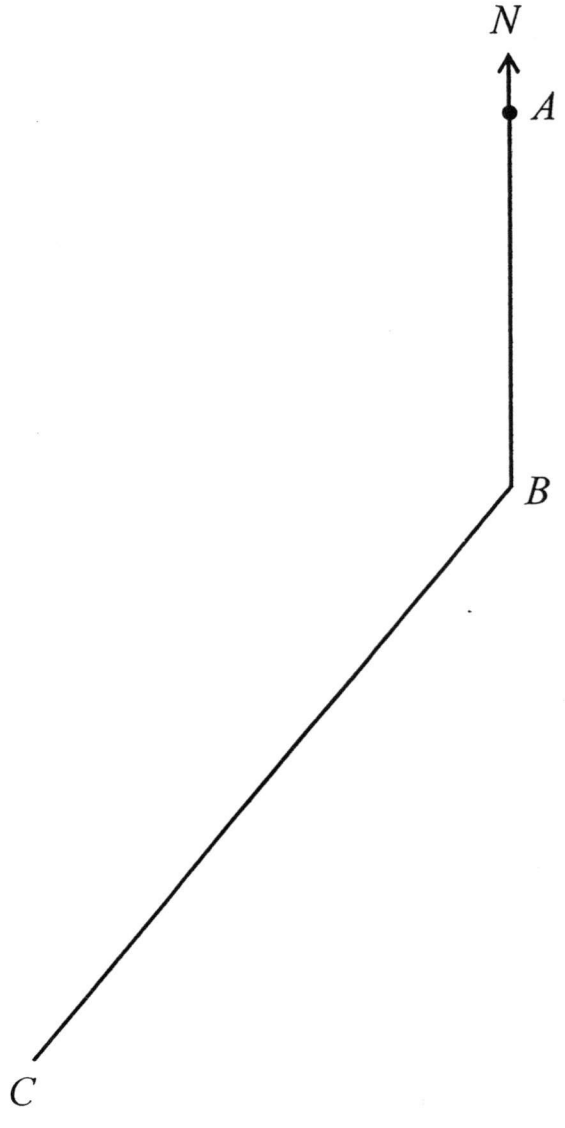
I equidistant from B and C,

II equidistant from BC and DC.

Using the ruler and compasses only, find and label the position of the new shop X.

(c) Find the bearing of shop X from D.

Answer (a) and *(b)*



Answer (c) ..... ° [1]

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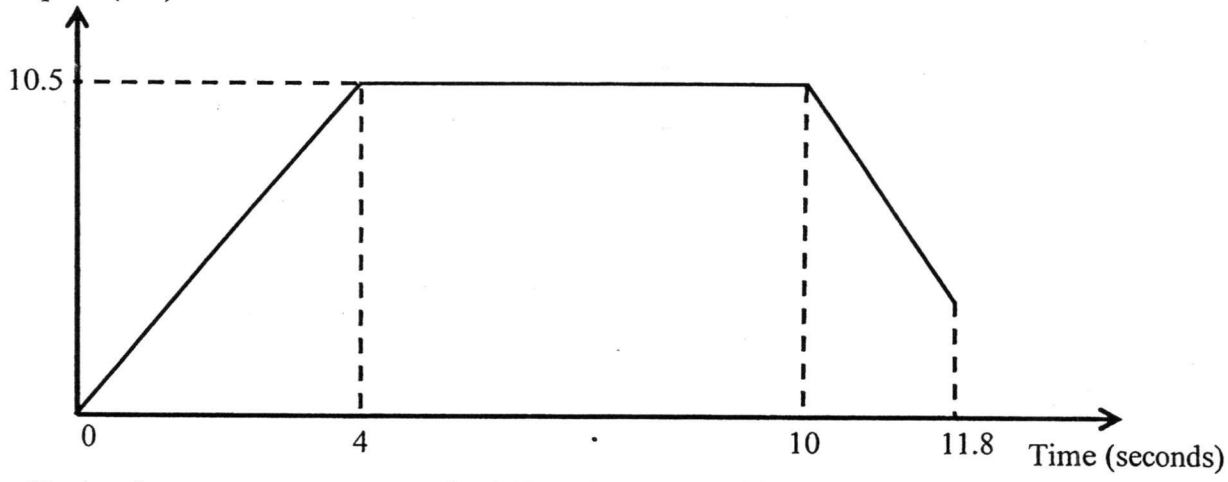
Name: _____ ( ) Class: _____

24 Speed (m/s)



During the recent sports meet at the S.E.A. Games, an athlete accelerated to a speed of 10.5 m/s in 4 seconds. She ran at the same speed for the next 6 seconds and slowed down over the last 1.8 seconds as shown in the speed-time graph. She crossed the finish line after 11.8 seconds.

(a) Find the acceleration of the athlete in the first 3 seconds.

(b) If the total distance she ran was 100 m, calculate her speed when she crossed the finishing line.

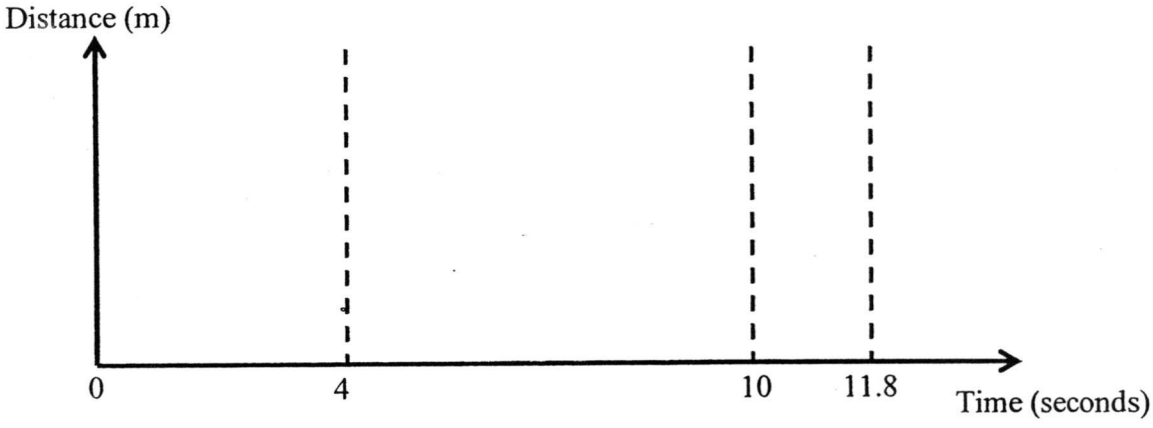
Answer (a) .....m/s^2 [1]

(b) ..... m/s [2]

Name: _____ ( ) Class: _____

24 **(c)** Use the grid below to sketch the distance-time graph for the first 11.8 seconds.

Answer (c) [3]



End of paper

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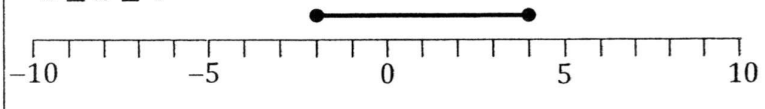
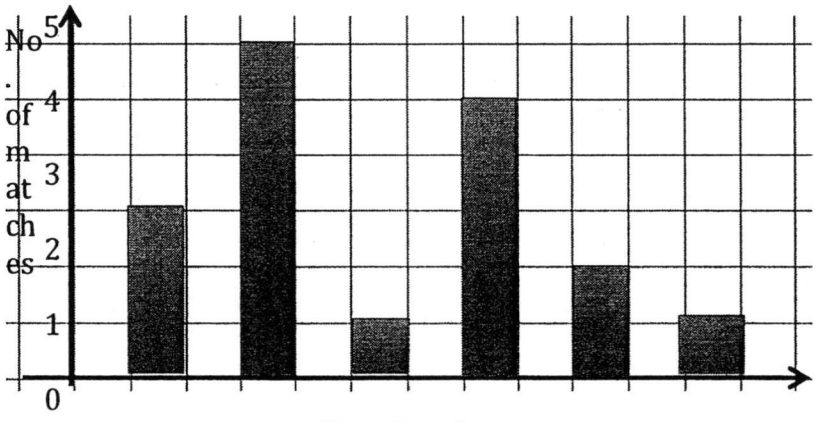
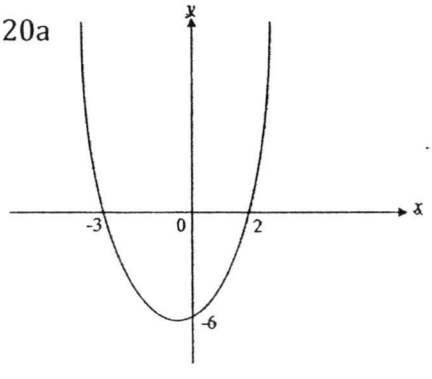
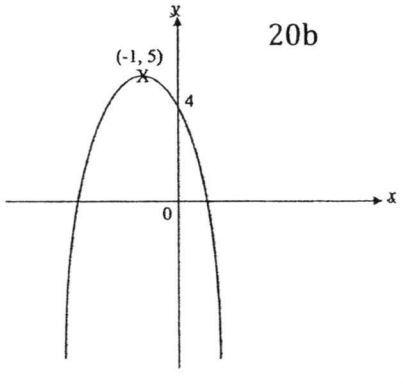
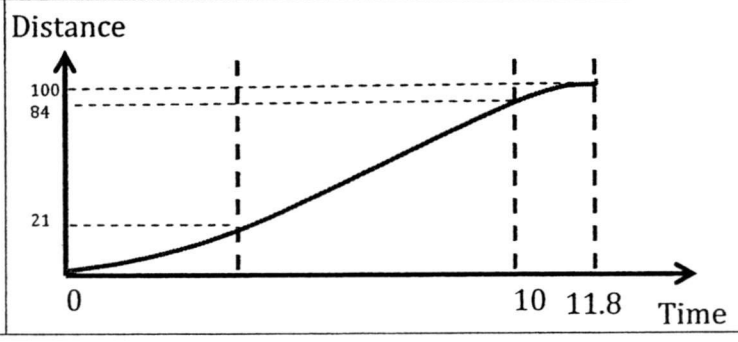
Answer Key for FMS(S) Sec 4/5 Prelim 2015 Maths P1

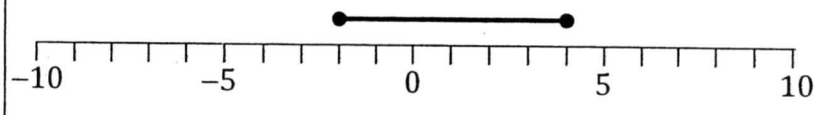
1a	273.7368679
1b	270
2	\$516
3a	$k = 6\frac{2}{3}$
3b	$\frac{27}{8b}$
4	$\$\frac{1}{100}(pr - \frac{pq}{6})$
5a	1 : 400
5b	2 cm^2
6a	7
6b	$x = 15$
7	$\frac{2x+1}{4x-1}$
8a	5×10^{-5}
8b	1200
9a	$\frac{32}{63}$
9b	$\frac{2x}{2x+y}$ or $\frac{2x}{2y+1}$
9c	$\frac{2^{n-1}}{2^n - 1}$
10a	210°
10b	n = 12
11i	$2^3 \times 3 \times 5 \times 11^2$
11ii	30
11iii	440
12a	$-2 \leq x \leq 4$ 
12b	2, 3
13a	$\frac{3}{4}$
13b	$-\frac{3}{5}$

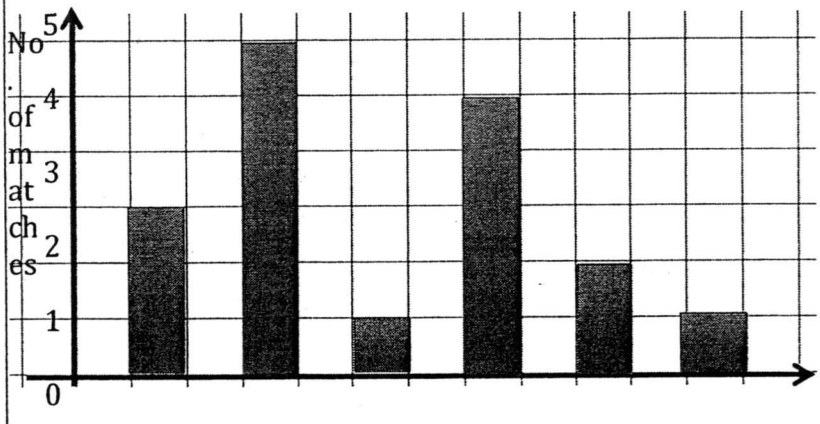
14a	$a=-\frac{1}{4}, n=2$
14b	$a=-4, n=-2$
14c	$a=-\frac{1}{2}, n=1$
15a	 No. of goals
15b	1.5
16a	$(x-3)^2-25$
	$x=8 \text{ or } -2$
17a	18
17b	3.5
18	$a=-1$ $b=3$
19i	$\frac{25}{36}$
19ii	$\frac{5}{6}$
20	20a  20b 
21a	$S(0,5)$ $T(2.5,0)$

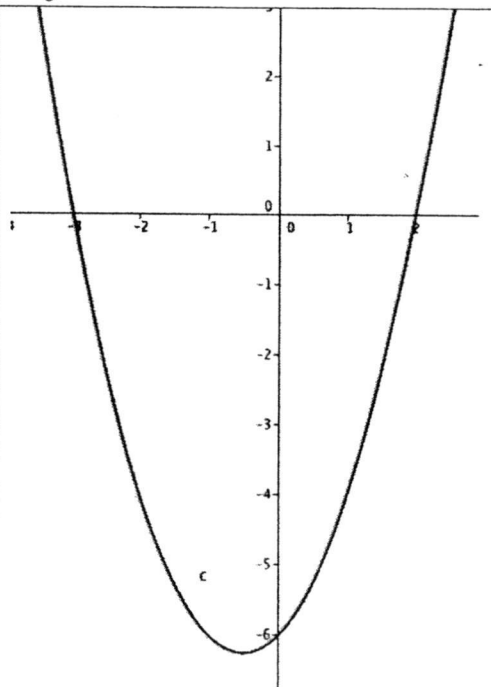
21bi	$x = -2$
21bii	5 units^2
22a	$\angle AEB = \angle CEB$ (*vert opp* $\angle s$) $\angle ABE = \angle CDE$ (*alt* $\angle s$) $\angle BAE = \angle DCE$ (*alt* $\angle s$) ΔAEB *is similar to* ΔCED (*AAA*)
22b	63
22c	$\frac{9}{16}$
23c	123.5 ± 1
24a	2.625
24b	7.28 (3*s.f.*)
24c	

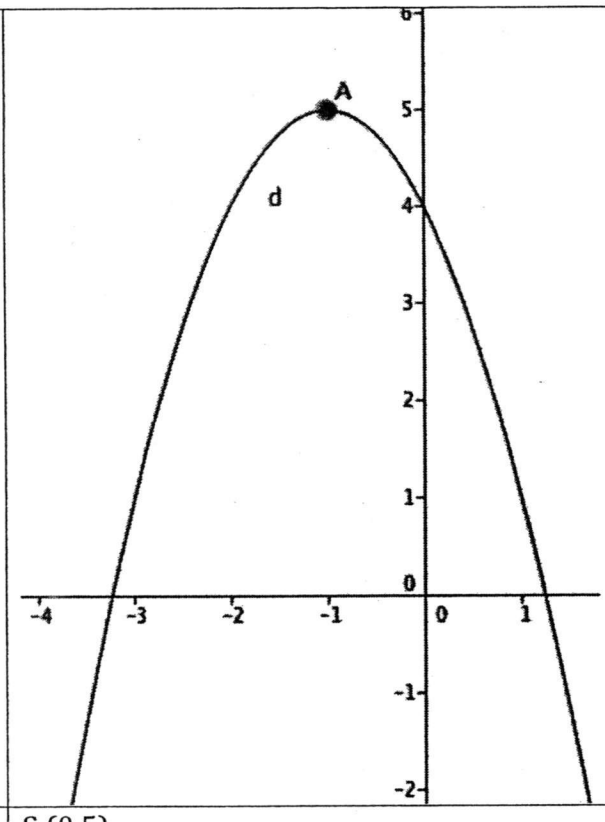
Marking scheme for FMS(S) Sec 4/5 Prelim 2015 Maths P1

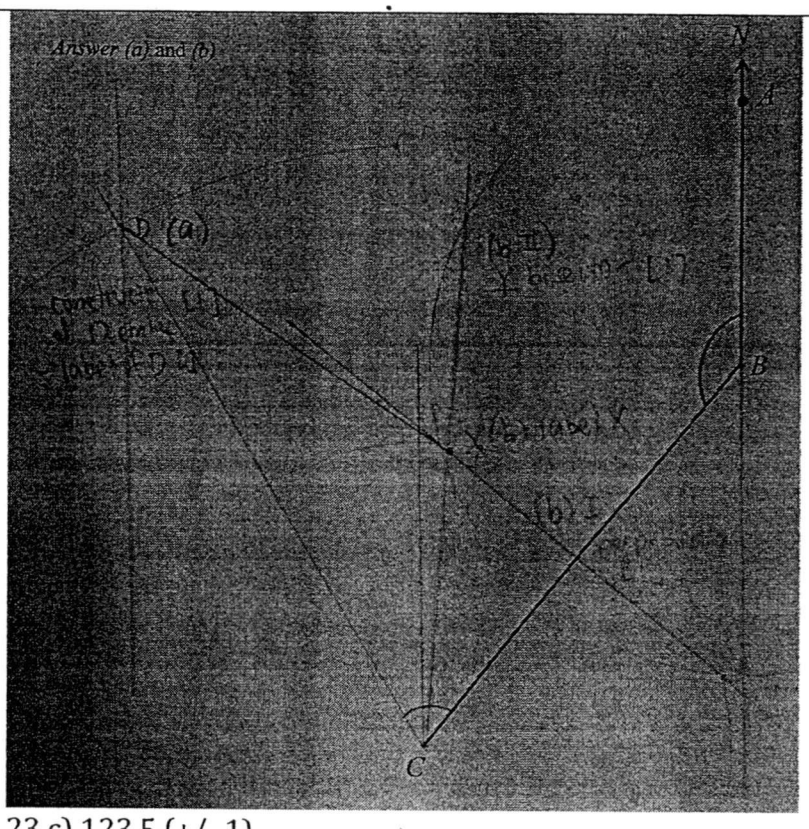
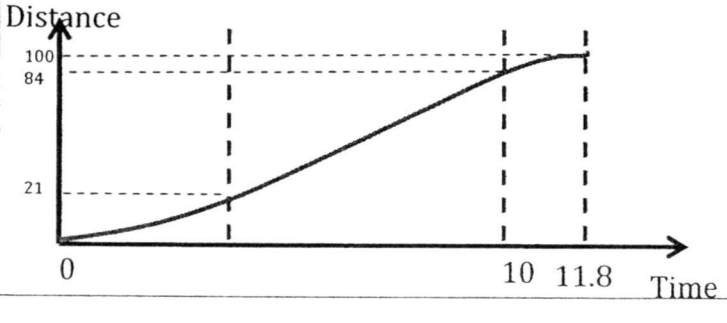
1a	273.7368679	B1
1b	270	B1
2	240 x 2.15 = $516	M1 A1
3a	k = $6\frac{2}{3}$	B1
3b	$\frac{27}{8b}$	B1
4	Cost of apples = $\frac{pq}{6}$ cents Selling price = pr cents Profit in dollars = $\$\frac{1}{100}(pr - \frac{pq}{6})$	B1 B1
5a	1 : 400	B1
5b	1 cm^2 : 16 m^2 Area = 32 / 16 = 2 cm^2	M1 – finding area ratio A1 or B2
6a	7	B1
6b	Mean = $\frac{2x+75}{x+30} = \frac{7}{3}$ $x = 15$	B1 – form an equation B1
7	$\frac{(4x-1)(4x+1)}{(4x-1)^2} - \frac{2x}{4x-1}$ $= \frac{4x+1-2x}{4x-1}$ $= \frac{2x+1}{4x-1}$	M1 – factorise $16x^2-1$ M1 – Common denominator A1
8a	5×10^{-5}	B1
8b	$\frac{6 \times 10^{-2}}{5 \times 10^{-5}}$ =1200	M1 A1
9a	$\frac{32}{63}$	B1
9b	$\frac{2x}{2x+y}$ or $\frac{2x}{2y+1}$	B1
9c	$\frac{2^{n-1}}{2^n - 1}$	B1
10a	210^o	B1
10b	150n = (n-2) 180 n = 12	M1 – form the equation A1
11i	$2^3 \times 3 \times 5 \times 11^2$	B1
11ii	$2 \times 3 \times 5 = 30$	B1
11iii	$2^3 \times 5 \times 11 = 440$	B1

12a	$-2-3x \le 8+2x \qquad 8+2x \le x+12$ $-10 \le 5x \qquad\qquad x \le 4$ $-2 \le x \le 4$  	B1 – correct inequality B1 for correct number line drawn
12b	2, 3	B1
13a	$AC=\sqrt{17^2-8^2}=15$ $DC=\frac{15\times 2}{5}=6$ $\tan\angle CBD=\frac{3}{4}$	 M1 A1
13b	$BD=\sqrt{8^2-6^2}=10$ $\cos\angle ABD=-\cos\angle BDC$ $=-\frac{3}{5}$	M1 A1
14a	$a=-\frac{1}{4}, n=2$ ..	B1 (n can be any even positive number; a is the corresponding value to n)
14b	$a=-4, n=-2$	B1 (n can be any even negative number; a is the corresponding value to n)
14c	$a=-\frac{1}{2}, n=1$	B1

15a		[B2] - 1 per mistake from bar chart - 1 for no label
15b	1.5	B1
16a	$(x-3)^2-25$	B1
	$\frac{x^2-16}{2}=3x$ $x^2-16=6x$ $x-6x-16=0$ $(x-3)^2-25=0$ $(x-3)^2=25$ $x-3=\pm\sqrt{25}$ $x=3+\sqrt{25}$ *or* $3-\sqrt{25}$ $x=8$ *or* -2	M1 A1
17a	D = 18	B1
17b	$k=\frac{14}{x^2}$ $y=\frac{14}{x^2}(\frac{1}{2}x)^2$ =3.5	M1 A1

18	$3a+2b=3 \quad -(1)$ $4a-3b=-13 \quad -(2)$ $(1)\times 3,$ $9a+6b=9 \quad -(3)$ $(2)\times 2,$ $8a-6b=-26 \quad -(4)$ $(3)+(4),$ $17a=-17$ $a=-1$ $Sub\ a=-1\ into\ o\ (1)$ $3a+2b=3$ $3(-1)+2b=3$ $b=3$	M1- Using elimination or substitution method A1 A1
19i	$P(2\ tails)=\frac{1}{2}\times\frac{1}{2}+\frac{2}{3}\times\frac{2}{3}$ $=\frac{25}{36}$	M1 A1 or B2
19ii	$P(H\ at\ 2nd\ toss)=\left[\left(\frac{1}{2}\times\frac{1}{2}\right)\times 2\right]+\left(\frac{1}{3}\times\frac{1}{3}\right)+\left(\frac{2}{3}\times\frac{1}{3}\right)$ $=\frac{5}{6}$	M1 A1
20a		B1 – "U" curve B1- mark and label correct x-intercepts (x=2 and x=-3) and y-intercept (y=-6)

20b		B1 – "∩" curve B1- mark & label correct maximum point (-1,5) and y-intercept (y=4)
21a	S (0,5) T (2.5, 0)	B1 B1
21bi	X = -2	B1
21bii	Area = 0.5 x 5 x 2 =5 units^2	B1
22a	$\angle AEB = \angle CEB$ (*vert opp* $\angle s$) $\angle ABE = \angle CDE$ (*alt* $\angle s$) $\angle BAE = \angle DCE$ (*alt* $\angle s$) ΔAEB *is similar to* ΔCED (*AAA*)	} } B1 – Show } 2 angles with valid reasons B1- Final statement with valid test of similarity
22b	$\frac{A_1}{A_2} = \left(\frac{1}{3}\right)^2$ $\frac{1}{9} = \frac{7}{A_2}$ $A_2 = 63$	B1

22c	$\frac{\textit{Area of triangle DEC}}{\textit{Area of trapezium ABCD}} = \frac{\text{Area of DEC}}{\text{Area of DBC}} \times \frac{\text{Area of DBC}}{\text{Area of Trapezium}}$ $= \frac{3}{4} \times \frac{\frac{1}{2} \times DC \times h}{\frac{1}{2} \times (DC + AB) \times h}$ $= \frac{3}{4} \times \frac{3}{4}$ $= \frac{9}{16}$	M1 A1
23	 23 c) 123.5 (+/- 1)	
24a	2.625	B1
24b	$\frac{1}{2}(4 \times 10.5) + (6 \times 10.5) + \textit{Area of trapezium} = 100$ $\textit{Area of trapezium} = 100 - 84 = 16$ $\textit{Speed} = \left[16 \div (\frac{1}{2} \times 1.8)\right] - 10.5 = 7.28\ (3s.f.)$	M1 A1
24c		Acurate sketch of graph with labelling of distance for time: 0-4 [B1], 4-10 [B1], 10-11.8 [B1] -1 if no label of distance/the graph is not smooth.

NAME: _____ ( ) CLASS: _____

FAIRFIELD METHODIST SCHOOL (SECONDARY)

PRELIMINARY EXAMINATION 2015
SECONDARY 4 EXPRESS / 5 NORMAL (ACADEMIC)

MATHEMATICS 4016/02

Paper 2

Date: 24 August 2015 **Duration: 2 hours 30 minutes**

Additional Materials: Answer Paper
Graph paper (1 sheet)

READ THESE INSTRUCTIONS FIRST

Write your name, index number and class on all the work you hand in.
Write in dark blue or black pen.
You may use a pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** questions.
If working is needed for any question it must be shown with the answer.
Omission of essential working will result in loss of marks.
Calculators should be used where appropriate.
If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.
For π, use either your calculator value or 3.142, unless the question requires the answer in terms of π.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 100.

At the end of the examination, fasten all your work securely together.

For Examiner's Use	
Total	/ 100

Setters: Mdm Haliza and Mr Joel Li

This question paper consists of 13 printed pages including the cover page.

[9]

Name: _____ ( ) Class: _____

Mathematical Formulae

Compound interest

$$\text{Total amount} = P\left(1+\frac{r}{100}\right)^n$$

Mensuration

Curved surface area of a cone = πrl

Surface area of a sphere = $4\pi r^2$

Volume of a cone = $\frac{1}{3}\pi r^2 h$

Volume of a sphere = $\frac{4}{3}\pi r^3$

Area of a triangle $ABC = \frac{1}{2}ab\sin C$

Arc length = $r\theta$, where θ is in radians

Sector area = $\frac{1}{2}r^2\theta$, where θ is in radians

Trigonometry

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

Statistics

$$\text{Mean} = \frac{\sum fx}{\sum f}$$

$$\text{Standard deviation} = \sqrt{\frac{\sum fx^2}{\sum f} - \left(\frac{\sum fx}{\sum f}\right)^2}$$

Name: _____ ( ) Class: _____

Answer **all** the questions.

1 A bag contains 10 identical balls of which 4 are red and 6 are blue. Peter and Glen take turns to draw a ball from the bag **without** replacement. The first student to take a blue ball wins the game and the game ends.

(a) Given that Peter takes the first draw, Glen takes the second draw, Peter then takes the third draw and so on, state, as a fraction, the probability that

(i) Peter wins at the first draw, [1]

(ii) Glen wins at the second draw, [1]

(iii) no fourth draw is required. [2]

(b) If the game ends at the nth draw, state the largest value of n. [1]

---

2 **(a)** Simplify $\dfrac{2ab-a^2}{a^2-4b^2}$. [2]

(b) Given that $b=\dfrac{2a-3}{4+a}$, express a in terms of b. [2]

(c) It is given that

$$\xi=\{x: x \text{ is an integer, } 1<x<16\},$$

$$D=\{x: x \text{ is a perfect square}\},$$

$$E=\{x: x \text{ is a multiple of } 3\},$$

$$F=\{x: x \text{ is a prime number}\}.$$

(i) List the elements of D. [1]

(ii) Find $n(D \cap E \cap F)$. [1]

(iii) Draw a Venn diagram to represent the given sets. [1]

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Name: _____ ( ) Class: _____

3 During weekdays, a restaurant serves set lunches and set dinners. 2 variations of lunches and dinners are served. The number of meals sold from Monday to Wednesday, for each type of meal is as shown in the table.

	Set Lunch *A*	**Set Lunch *B***	**Set Dinner *C***	**Set Dinner *D***
Monday	56	54	60	68
Tuesday	63	69	67	74
Wednesday	70	66	78	68

The prices of the set lunches and set dinners are as follows:

Set Lunch *A*	$ 12.50
Set Lunch *B*	$ 13.20
Set Dinner *C*	$ 19.90
Set Dinner *D*	$ 22.50

(a) Write down a 3 × 4 matrix, **M**, that represents the number of set meals sold from Monday to Wednesday. [1]

(b) Write down a 4 × 1 matrix, **N**, to represent the prices of the set meals. [1]

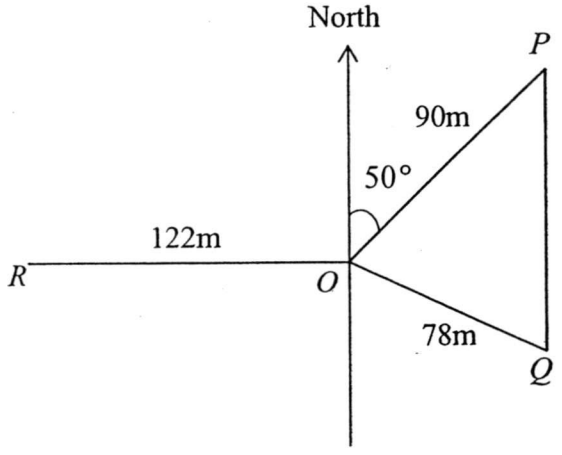
(c) Using matrix multiplication, evaluate matrix **MN**. [2]

(d) Explain what the matrix **MN** represents. [1]

(e) Using matrix multiplication, calculate the total earnings for the set lunches sold for the 3 days. [1]

Name: _____ ( ) Class: _____

4



(a) A surveyor is carrying out a survey on horizontal ground. From a point O, she observes a point P which is 90 m from O on a bearing of 050° and a point Q which is 78 m from O and due south of the point P.

Calculate

(i) the bearing of O from P, [1]

(ii) the angle OQP. [2]

(b) The point R is 122 m due west of O.

The surveyor walks directly from R to P. Calculate the distance she walked. [3]

(c) The surveyor then walks from P towards Q until she reaches a point T, where RT is a minimum. Calculate RT. [2]

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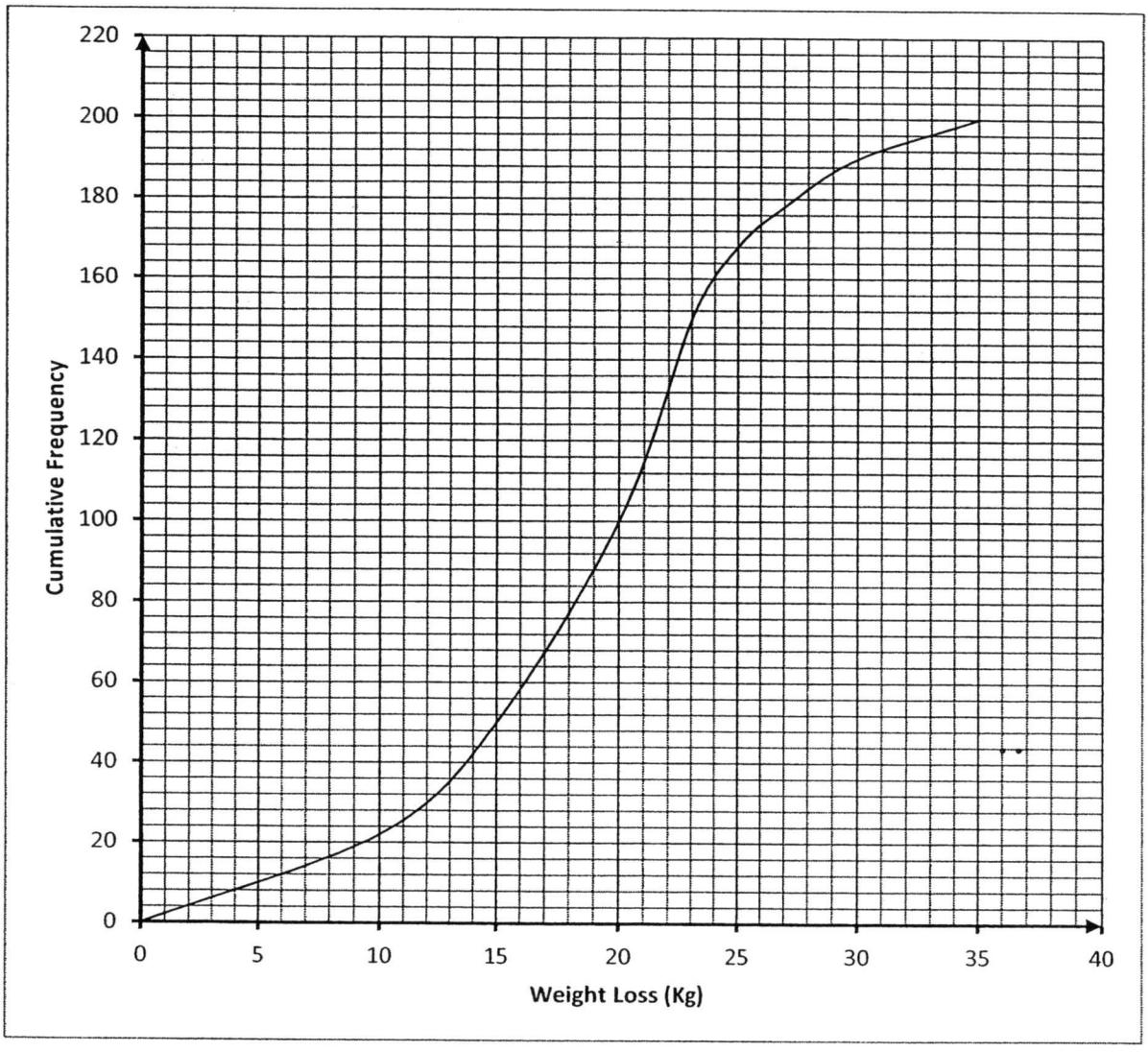
Name: _____ ( ) Class: _____

5 The cumulative frequency curve below shows the amount of weight loss, measured to the nearest kg, of 200 men in Glow Fitness Centre.



The corresponding cumulative frequency table for this distribution is as shown below.

Weight loss (x kg)	≤ 10	≤ 15	≤ 20	≤ 25	≤ 30	≤ 35
Cumulative Frequency	22	a	100	168	190	200

(a) Determine the value of a. [1]

(b) Use your graph to find

(i) the median weight loss, [1]

(ii) the interquartile range, [2]

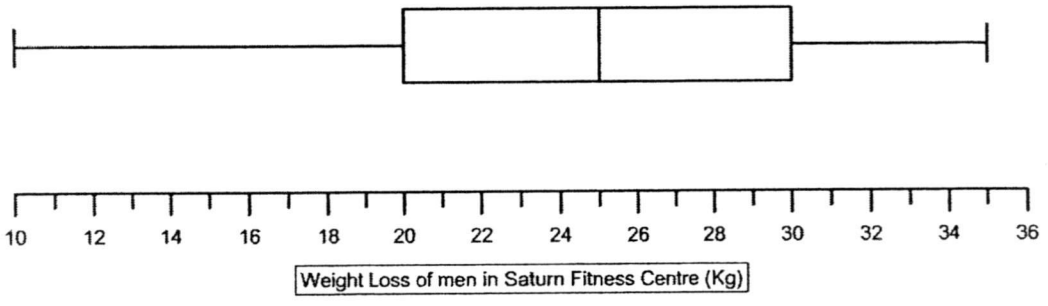
(iii) the 44^{th} percentile weight loss. [1]

Name: _____ ( ) Class: _____

5 (c) A second fitness centre, Saturn Fitness Centre also measured the amount of weight loss by 200 of their men. The box-and-whisker diagram below illustrates their weight loss achieved.

(i) Find the interquartile range. [1]

(ii) A physical instructor claimed that the weight loss program at Glow Fitness Centre is more effective than Saturn Fitness Centre. Explain if his statement is true. [2]



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6 (a) It is given that $\mathbf{u} = \begin{pmatrix} 7 \\ -3 \end{pmatrix}$ and $\mathbf{v} = \begin{pmatrix} -5 \\ 6 \end{pmatrix}$, find

(i) $|\mathbf{u}|$, [1]

(ii) $2\mathbf{u} - \mathbf{v}$. [1]

(b) $ABCD$ is a quadrilateral in which $\overrightarrow{AB} = 3\mathbf{p}$, $\overrightarrow{DC} = 4.5\mathbf{p}$ and $\overrightarrow{DA} = 2\mathbf{t}$.

(i) Sketch the quadrilateral $ABCD$. [1]

(ii) Write down two important facts about the pair of sides AB and DC. [2]

(iii) Express $\overrightarrow{BC}$, as simply as possible, in terms of $\mathbf{p}$ and/or $\mathbf{t}$. [1]

(iv) The sides DA and CB, when produced, meet at X.

Express $\overrightarrow{XA}$, as simply as possible, in terms of $\mathbf{p}$ and/or $\mathbf{t}$. [1]

(v) Write down the value of

(a) $\dfrac{\text{Area of } \Delta ABD}{\text{Area of } \Delta BDC}$, [1]

(b) $\dfrac{\text{Area of } \Delta ABX}{\text{Area of } ABCD}$. [2]

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Name: _____ ( ) Class: _____

7 **Answer the whole of this question on a graph paper.**

The variables x and y are connected by the equation $y = \frac{1}{2}x(x^2 - 4)$.

The table below shows some values of x and the corresponding values of y correct to 1 decimal place.

x	-0.5	0	0.5	1	1.5	2	2.5
y	0.9	0	-0.9	-1.5	-1.3	0	2.8

(a) Taking 4 cm to represent 1 unit on each axis, draw a horizontal x-axis for $-1 \le x \le 3$ and a vertical y-axis for $-2 \le y \le 3$. Draw the graph of $y = \frac{1}{2}x(x^2 - 4)$ for $-0.5 \le x \le 2.5$. [3]

(b) Use your graph to solve the following equation in the range $-0.5 \le x \le 2.5$.

$$x(x^2 - 4) = 2 \text{ ..}$$ [1]

(c) By drawing a suitable tangent, find the gradient of the curve at (0.5, -0.9). [2]

(d) **(i)** On the same axes, draw the graph of $y = \frac{x}{2} + 1$. [1]

(ii) Write down the x coordinates of the points at which the two graphs intersect. [1]

(iii) Find the equation, in the form $\frac{1}{2}x^3 + bx^2 + cx + d = 0$, which is satisfied by the values of x found in part **(d)(ii)**. [1]

8 **(a)** Mr Ang went to Australia for a short holiday in 2013.

(i) He exchanged some Singapore dollars (S$) for 3000 Australian dollars (A$) from a money changer at an exchange rate of S$1.00 = A$0.92. The money changer also charges a commission of 0.6% on the amount exchanged.

Calculate the amount of Singapore dollars he has to pay. [2]

(ii) He booked a hotel at a cost of A$232 ++ per night. This means that he would have to pay a basic charge of A$232 per night plus a service charge of 10%. On top of these, he would have to pay an additional Goods and Service Tax (GST) of 10% on the sum of the basic charge and service charge.

If he stayed a total of 6 nights at the hotel, calculate the amount of A$ he has to pay for the hotel stay. [2]

(b) **(i)** Peter deposits $7680 in a fund that pays compound interest of 3.35% per annum, compounded half-yearly. Calculate the total amount of money that Peter will have in the fund at the end of five years. [2]

(ii) At the end of five years, Peter withdrew $6500 from the fund and invested this amount in a bank which offered simple interest rate of 4.8% per annum. Find the minimum number of full years he had to leave the money in the bank in order for it to be more than $12000. [3]

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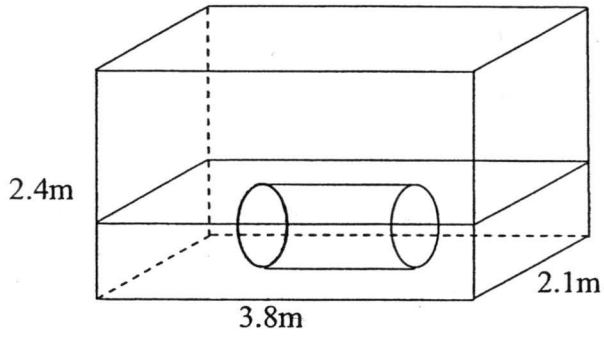
9



An open rectangular tank of depth 2.4 m has a horizontal base of length 3.8 m and breadth 2.1 m.

A solid metal cylinder of volume 0.865 m^3 rests with its curved surface on the base of the tank as shown in the figure above.

6 400 litres of water is poured into the tank at a rate of 900 cm^3/s.

(a) Calculate how many hours and minutes it takes for all the water to be poured in, giving your answer to the nearest minute. [2]

(b) Given that the water just covers the cylinder as shown in the figure, calculate, in metres,

(i) the depth of the water, [3]

(ii) the radius of the cylinder, [1]

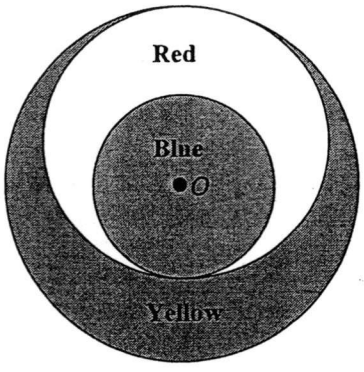
(iii) the length of the cylinder. [2]

(c) The cylinder is now removed from the tank.

Calculate by how much the water level falls. [2]

Name: _____ ( ) Class: _____

10 The diagram below shows the design of a company logo. It consists of three circles.



The largest and the smallest circles have the same centre O.

The largest circle has radius $2x$ cm and the smallest circle has radius $2y$ cm.

The third circle touches both the other two circles.

The three regions formed are coloured red, blue and yellow.

(a) Explain fully why the radius of the third circle is $(x + y)$ centimetres. [2]

(b) Write down and simplify, in terms of π, x and y, expressions for the area of the region that is coloured

(i) red, [1]

(ii) yellow. [1]

(c) The area of the yellow region is twice the area of the red region.

Use this information to write down an equation involving x and y, and show that it simplifies to $x^2 - 6xy + 5y^2 = 0$. [3]

(d) Solve the equation $x^2 - 6xy + 5y^2 = 0$, expressing x in terms of y. [3]

(e) Calculate the fraction of the logo that is coloured yellow. [2]

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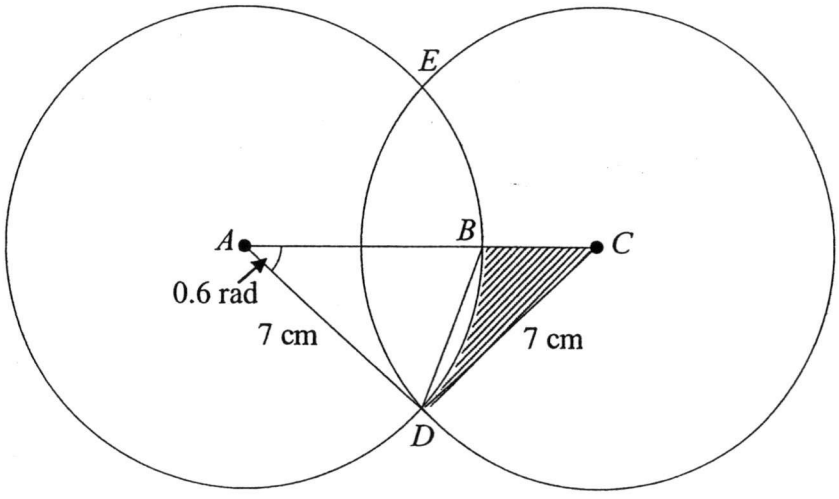
Name: _____ ( ) Class: _____

11 (a) The diagram shows two circles, centres A and C, with equal radii of 7 cm. The circles meet at D and E. AC meets the circle with centre A at B. Angle $DAB = 0.6$ radians.



Find

(i) the area of sector ABD, [2]

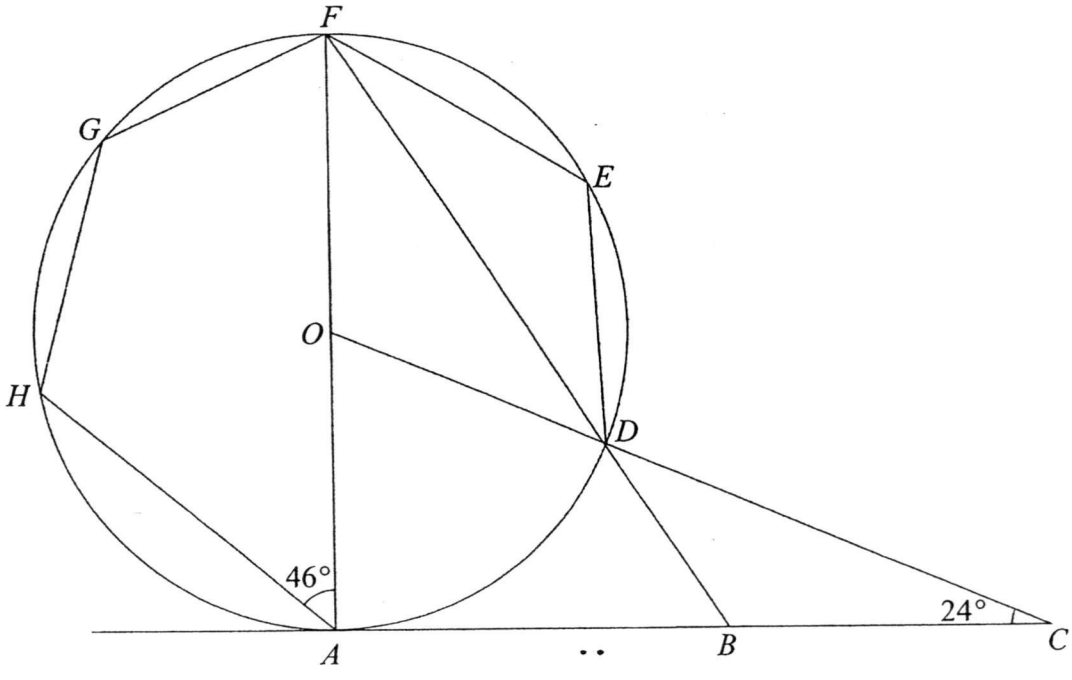
(ii) the area of shaded region. [3]

(iii) the length of arc BD, [2]

(iv) the perimeter of the shaded region, [3]

Name: _____ ( ) Class: _____

11 (b) The diagram shows a circle with centre O. AC is a tangent to the circle. AOF, FDB and ODC are straight lines. Given that $\angle ACO = 24°$ and $\angle HAF = 46°$.



(i) Write down why $\angle FAB = 90°$. [1]

(ii) Calculate

(a) $\angle HEF$, [1]

(b) $\angle AFB$, [1]

(c) $\angle HGF$, [1]

(d) $\angle FED$. [2]

-- End of Paper –

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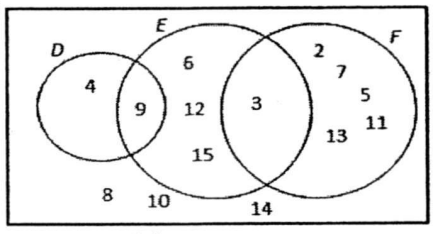
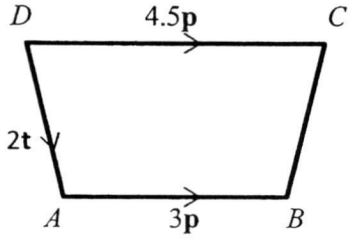
SEC 4 MATHEMATICS PRELIMINARY EXAM PAPER 2 2015 ANSWER KEY

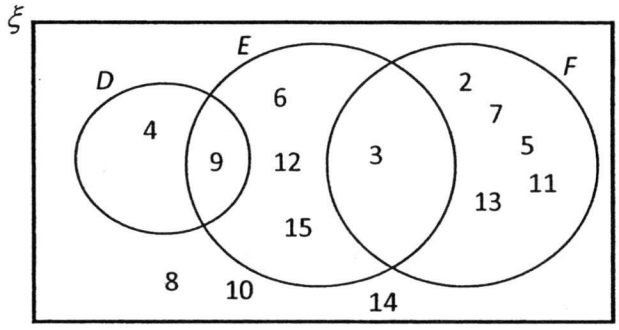
1	(a)(i)	$\frac{3}{5}$
	(ii)	$\frac{4}{15}$
	(iii)	$\frac{29}{30}$
	(iv)	n = 5
2	(a)	$\frac{-a}{a+2b}$
	(b)	$a = \frac{3+4b}{2-b}$
	(c)(i)	D = { 4, 9}
	(c)(ii)	$n(D \cap E \cap F) = 0$
	(c)(iii)	
3	(i)	$M = \begin{pmatrix} 56 & 54 & 60 & 68 \\ 63 & 69 & 67 & 74 \\ 70 & 66 & 78 & 68 \end{pmatrix}$
	(ii)	$N = \begin{pmatrix} 12.50 \\ 13.20 \\ 19.90 \\ 22.50 \end{pmatrix}$
	(iii)	$\begin{pmatrix} 4136.80 \\ 4696.60 \\ 4828.40 \end{pmatrix}$
	(iv)	Matrix MN represents the total amount of money (revenue) collected for all set meals for each day respectively.
	(v)	(13661.80)
4	(a)(i)	230^{o}
	(ii)	$62.1°$
	(b)	200 m
	(c)	191 m
5	(a)	a = 50
	(b)(i)	20kg
	(b)(ii)	Interquartile Range = 23 – 15 = 8 kg

	(b)(iii)	44th Percentile = 19 kg
	(c)(i)	Interquartile Range = 30 – 20 = 10 kg
	(c)(ii)	The median weight loss of Saturn Fitness Centre is higher than Glow Fitness Centre (25kg > 20 kg) The Interquartile Range of Glow Fitness Centre is slightly lower than Saturn Fitness Centre (8kg < 10 kg) Since the median weight loss of Saturn Fitness Centre is higher and the interquartile range difference between 2 centres is comparable. It is untrue that Glow Fitness Centre is more effective at weight loss.
6(a)	(i)	7.62 units
	(a)(ii)	$\begin{pmatrix} 19 \\ -12 \end{pmatrix}$
	(b)(i)	
	(b)(ii)	AB is parallel to DC. DC is 1.5 times of AB or $\lvert\overrightarrow{DC}\rvert = \frac{3}{2}\lvert\overrightarrow{AB}\rvert$.
	(b)(iii)	1.5**p** – 2**t**
	(b)(iv)	– 4**t**
	(b)(v)(a)	$\frac{2}{3}$
	(b)(v)(b)	$\frac{4}{5}$
7	(a)	Draw axes and plot all given points Draw a smooth curve
	(b)	Draw y = 1, x = -0.55 or 2.25 (±0.1)
	(c)	Gradient = $\frac{-1.95}{1.25} = -1.56$ (Accept – 1.425 to – 1.825)
	(d)(i)	Plot and draw $y = \frac{x}{2} + 1$
	(d)(ii)	x = 2.45 or x = -0.425 (±0.1)
	(d)(iii)	$\frac{1}{2}x^3 - \frac{5}{2}x - 1 = 0$
8	(a)(i)	S$3280.43 (2dp)
	(a)(ii)	A$1684.32
	(b)(i)	$9067.82
	(b)(ii)	T = 17.628 years Minimum number of full years = 18 years

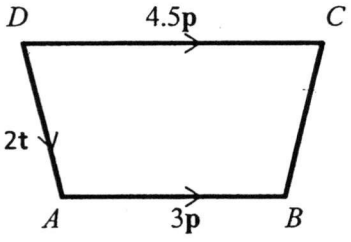
9	(a)(i)	1 h 59 min
	(b)(i)	0.910 m
	(b)(ii)	0.455 m
	(b)(iii)	1.33 m
	(c)	0.108 m
10	(a)	Diameter of 3rd circle is $2x + 2y$ cm. Therefore its radius is $\frac{2x+2y}{2} = (x+y)$ cm (Shown)
	(b)(i)	$\pi(x^2 + 2xy - 3y^2)\,\text{cm}^2$
	(b)(ii)	$\pi(3x^2 - 2xy - y^2)\,\text{cm}^2$
	(c)	$\pi(3x^2 - 2xy - y^2) = 2\pi(x^2 + 2xy - 3y^2)$
	(d)	$x = 5y$ or x = y
	(e)	$\frac{16}{25}$
11	(a)(i)	14.7 cm^2
	(a)(ii)	8.13 cm^2
	(a)(iii)	4.2 cm
	(a)(iv)	15.8 cm
	(b)(i)	AC is a tangent to circle Therefore $\angle FAB = 90°$
	(ii)(a)	$\angle HEF = 46°$ (Angles in same segment)
	(ii)(b)	$\angle AFB = 66° \div 2$ $= 33°$ (Angle at centre = 2 x angle at circumference)
	(ii)(c)	$\angle HCF = 180° - 46°$ $= 134°$ (opp angles of cyclic quad)
	(ii)(d)	$\angle FED = 246° \div 2$ (Angle at centre = 2 x angle at circumference) $= 123°$

SEC 4 MATHEMATICS PELIMINARY EXAM MARKING SCHEME 2015 Maths P2

1.	(a)(i)	$\frac{3}{5}$	B1
	(ii)	P(Student B wins at 2nd draw) = $\frac{4}{10}\times\frac{6}{9}$ $= \frac{4}{15}$	 B1
	(iii)	P(No fourth draw is required) = P(student A wins 1st draw) + P(student B wins 2nd draw) + P(student A wins 3rd draw) $= \frac{3}{5}+\frac{4}{15}+\left(\frac{4}{10}\times\frac{3}{9}\times\frac{6}{8}\right)$ $= \frac{29}{30}$	 M1 A1
	(iv)	n = 5	B1
2	(a)	$\frac{2ab-a^2}{a^2-4b^2} = \frac{a(2b-a)}{(a+2b)(a-2b)}$ $= \frac{-a(a-2b)}{(a+2b)(a-2b)}$ $= \frac{-a}{a+2b}$	M1 – factorize both numerator and denominator A1
	(b)	$b = \frac{2a-3}{4+a}$ $4b+ab = 2a-3$ $ab-2a = -3-4b$ $2a-ab = 3+4b$ $a(2-b) = 3+4b$ $a = \frac{3+4b}{2-b}$	 M1 – factorize A1
	(c)(i)	D = { 4, 9}	B1
	(ii)	$n(D \cap E \cap F) = 0$	B1

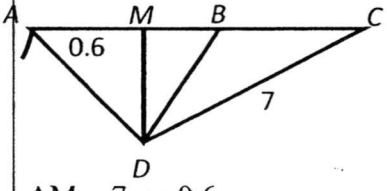
	(iii)		B1 (Must label clearly D,E, F) Zero marks if all 3 sets intersect each other
3	(i)	$M = \begin{pmatrix} 56 & 54 & 60 & 68 \\ 63 & 69 & 67 & 74 \\ 70 & 66 & 78 & 68 \end{pmatrix}$	B1
	(ii)	$N = \begin{pmatrix} 12.50 \\ 13.20 \\ 19.90 \\ 22.50 \end{pmatrix}$	
	(iii)	$MN = \begin{pmatrix} 56 & 54 & 60 & 68 \\ 63 & 69 & 67 & 74 \\ 70 & 66 & 78 & 68 \end{pmatrix}\begin{pmatrix} 12.50 \\ 13.20 \\ 19.90 \\ 22.50 \end{pmatrix}$ $= \begin{pmatrix} 700+712.8+1194+1530 \\ 787.5+910.8+1333.3+1665 \\ 875+871.2+1552.2+1530 \end{pmatrix}$ $= \begin{pmatrix} 4136.80 \\ 4696.60 \\ 4828.40 \end{pmatrix}$	M1 A1
	(iv)	Matrix MN represents the total amount of money (revenue) collected for all set meals for each day respectively.	B1
	(v)	$\begin{pmatrix} 1 & 1 & 1 \end{pmatrix}\begin{pmatrix} 4136.80 \\ 4696.60 \\ 4828.40 \end{pmatrix} = \begin{pmatrix} 13661.80 \end{pmatrix}$	B1

4	(a)(i)	Bearing of O from $P = 230^{\circ}$	B1
	(ii)	$\sin \angle OQP = \dfrac{\sin 50^{\circ}}{78} \times 90$ $\angle OQP = 62.116$ $= 62.1^{\circ}$ (to 1 d.p.)	M1 A1
	(b)	$RP^2 = 90^2 + 122^2 - 2(90)(122)\cos 140^{\circ}$ $RP = \sqrt{39806.33597}$ $= 199.52$ $= 200$ m	M1 M1 A1
	(c)	$OT = 90 \sin 50^{\circ}$ $= 68.944$ $\therefore RT = 122 + 68.944$ $= 190.944$ $= 191$ m	M1 A1
5	(a)	a = 50	B1
	(b)(i)	20kg	B1
	(b)(ii)	Lower Quartile = 15 kg Upper Quartile = 23 kg Interquartile Range = 23 – 15 = 8 kg	M1 A1
	(b)(iii)	44^{th} Percentile = 19 kg	B1
	(c)(i)	Interquartile Range = 30 – 20 = 10 kg	B1
	(c)(ii)	The median weight loss of Saturn Fitness Centre is higher than Glow Fitness Centre (25kg > 20 kg) The Interquartile Range of Glow Fitness Centre is slightly lower than Saturn Fitness Centre (8kg < 10 kg) Since the median weight loss of Saturn Fitness Centre is higher and the interquartile range difference between 2 centres is comparable. It is untrue that Glow Fitness Centre is more effective at weight loss.	M1 for both median and IQR A1 for conclusion
6(a)	(i)	$\lvert\mathbf{u}\rvert = \sqrt{7^2 + (-3)^2} = 7.62$ units	B1

6	(a)(ii)	$2\mathbf{u}-\mathbf{v}=2\begin{pmatrix}7\\-3\end{pmatrix}-\begin{pmatrix}-5\\6\end{pmatrix}$ $=\begin{pmatrix}19\\-12\end{pmatrix}$	B1
	(b)(i)		B1 (2 // sides, correct labelling of vertices)
	(b)(ii)	AB is parallel to DC. DC is 1.5 times of AB. $\lvert\overrightarrow{DC}\rvert=\frac{3}{2}\lvert\overrightarrow{AB}\rvert$.	Any 2 facts. [B1,1]
	(b)(iii)	$\overrightarrow{BC}=-3\mathbf{p}-2\mathbf{t}+4.5\mathbf{p}$ $=1.5\mathbf{p}-2\mathbf{t}$	B1
	(b)(iv)	$\overrightarrow{XA}=-4\mathbf{t}$	B1
	(b)(v)(a)	$\frac{\text{Area of } \Delta ABD}{\text{Area of } \Delta BDC}=\frac{2}{3}$	B1
	(b)(v)(b)	$\frac{\text{Area of } \Delta ABX}{\text{Area of } \Delta DCX}=\left(\frac{2}{3}\right)^2=\frac{4}{9}$ $\therefore \frac{\text{Area of } \Delta ABX}{\text{Area of } ABCD}=\frac{4}{5}$	M1 A1
7	(a)	Draw axes and plot all given points Draw a smooth curve	P2 S1
	(b)	Draw y = 1, x = -0.55 or 2.25 (± 0.1)	B1
	(c)	Gradient = $\frac{-1.95}{1.25}=-1.56$ (Accept – 1.425 to – 1.825)	B1 – Draw tangent B1 – Gradient
	(d)(i)	Plot and draw $y=\frac{x}{2}+1$	B1
	(d)(ii)	x = 2.45 or x = -0.425 (± 0.1)	B1

	(d)(iii)	$\frac{1}{2}x(x^2-4)=\frac{1}{2}x+1$ $\frac{1}{2}x^3-2x-\frac{1}{2}x-1=0$ $\frac{1}{2}x^3-\frac{5}{2}x-1=0$	B1
8	(a)(i)	A\$3000 $=\frac{3000}{0.92}$ = S\$ 3260.8696 Commission $=\frac{0.6}{100}\times 3260.8696$ = \$19.5652 Amount paid = \$3260.8696 + \$19.5652 = S\$3280.43 (2dp)	M1 A1
	(a)(ii)	Cost of Service Charge $=\frac{10}{100}\times A\$232$ = A\$23.20 Cost of room and Service Charge = A\$232 + A\$23.20 = A\$255.20 Cost of GST = 0.1 x A\$255.20 = A\$25.52 Total cost for 1 night = A\$255.20 + A\$25.32 = A\$280.72 Total cost for 6 nights = A\$280.72 x 6 = A\$1684.32	M1 A1
	(b)(i)	Total amount = $7680\left(1+\frac{1.675}{100}\right)^{5\times 2}$ = \$9067.8229 = \$9067.82	M1 A1
	(b)(ii)	Amount of interest to earn for it to be \$12 000 = 12 000 – 6500 = \$5500 $5500=\frac{(6500)(4.8)(T)}{100}$ T = 17.628 years Minimum number of full years = 18 years	M1 M1 A1
9	(a)(i)	$\frac{6400000}{900}$ = 7111.11 s = 1 h 59 min	M1 A1

9	(b)(i)	Depth of water $= \dfrac{6.4+0.865}{3.8\times 2.1}$ $= 0.91040$ $= 0.910\,\text{m}$	M1 (Total vol.) + M1 (Base area) + units must be the same A1
	(b)(ii)	Radius of cylinder $= \dfrac{0.910040}{2}$ $= 0.4552$ $= 0.455\,\text{m}$	B1 ft from (b)(i)
	(b)(iii)	Length of cylinder $= \dfrac{0.865}{\pi(0.4552)^2}$ $= 1.3288$ $= 1.33\,\text{m}$	M1 A1
	(c)	Ht. of water level after cylinder is removed $= \dfrac{6.4}{3.8\times 2.1}$ $= 0.80201$ Water level falls by $0.91040 - 0.80201 = 0.10839$ $= 0.108\,\text{m}$ OR $= \dfrac{0.865}{(3.8\times 2.1)}$ $= 0.108\,\text{m}$	M1 A1 any unit
10	(a)	Diameter of 3rd circle is $2x + 2y$ cm. Therefore its radius is $\dfrac{2x+2y}{2} = (x+y)$ cm (Shown)	M1 A1
	(b)(i)	Area of red region $= \pi(x+y)^2 - \pi(2y)^2$ $= \pi(x^2+2xy-3y^2)\,\text{cm}^2$	B1
	(b)(ii)	Area of yellow region $= \pi(2x)^2 - \pi(x+y)^2$ $= \pi(3x^2-2xy-y^2)\,\text{cm}^2$	B1

	(c)	$\pi(3x^2-2xy-y^2)=2\pi(x^2+2xy-3y^2)$ $3x^2-2xy-y^2-2x^2-4xy+6y^2=0$ $x^2-6xy+5y^2=0$ (Shown)	M1 M1 AG1
	(d)	$x^2-6xy+5y^2=0$ $(x-5y)(x-y)=0$ $x=5y$ or x = y	 M1 A1, A1
	(e)	Yellow $=\frac{\pi[3(5y)^2-2(5y)y-y^2]}{\pi[2(5y)]^2}$ $=\frac{75-10-1}{100}$ $=\frac{64}{100}$ $=\frac{16}{25}$	 M1 A1
11	(a)(i)	Area of sector ABD = $\frac{1}{2}(7)^2(0.6)$ $= 14.7\ cm^2$	M1 A1
	(a)(ii)	Area of shaded region = Area of Triangle ADC – Area of Sector Area of Triangle ADC = $\frac{1}{2}(7)(2AM)\sin 0.6$ $= 22.83476\ cm^2$ Shaded Area = 22.83476 – 14.7 = 8.13476 = 8.13 cm^2	 M1 M1 A1
	(a)(iii)	Arc Length BD = $r\theta$ = (7)(0.6) = 4.2 cm	 M1 A1
	(a)(iv)	 AM = 7cos 0.6 = 5.7773 cm BM = 7 – 5.7773 = 1.2227 cm BC = 7 – 2BM = 4.5546	 M1 M1

		Perimeter of shaded region $= 7 + 4.2 + 4.5546$ $= 15.7546$ $= 15.8$ cm	 A1
11	(b)(i)	AC is a tangent to circle Therefore $\angle FAB = 90°$	B1
	(ii)(a)	$\angle HEF = 46°$ (Angles in same segment)	B1
	(ii)(b)	$\angle AOC = 90° - 24°$ $= 66°$ (Sum of Angles in Triangle) $\angle AFB = 66° \div 2$ $= 33°$ (Angle at centre = 2 x angle at circumference)	 B1
	(ii)(c)	$\angle HCF = 180° - 46°$ $= 134°$ (opp angles of cyclic quad)	 B1
	(ii)(d)	$\angle FOD = 180° - 66°$ (Angles on straight line) $= 114°$ Reflex $\angle FOD = 246°$ (Angles at a pt) $\angle FED = 246° \div 2$ (Angle at centre = 2 x angle at circumference) $= 123°$	 M1 A1