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**SWISS COTTAGE SECONDARY SCHOOL**  
**SECONDARY FOUR EXPRESS**  
**PRELIMINARY EXAMINATIONS**

Name: \_\_\_\_\_ (        )        Class: Sec \_\_\_\_\_

**MATHEMATICS**

Paper 1

**4016/01**

**Friday 14 August 2015**

**2 hours**

Candidates answer on the Question Paper.

**READ THESE INSTRUCTIONS FIRST**

Write your name, class and index number on all the work you hand in.

Write in dark blue or black pen on both sides of the paper.

You may use a soft pencil for any diagrams or graphs.

Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.

If working is needed for any question it must be shown with the answer.

Omission of essential working will result in loss of marks.

The use of an approved scientific calculator is expected, where appropriate.

If the degree of accuracy is not specified in the question, and if the answer is not exact, give your answer to three significant figures. Give answers in degrees to one decimal place.

For  $\pi$ , use either your calculator value or 3.142, unless the question requires the answer in terms of  $\pi$ .

At the end of the examination, fasten all your work securely together.

The number of marks is given in brackets [    ] at the end of each question or part question.

The total number of marks for this paper is 80.

| For Examiner's Use |
|--------------------|
| <b>80</b>          |

This question paper consists of **18** printed pages.

**Setter:** Ms Yeo Koon Koon

**Vetter :** Mr Ang Hanping

[Turn over

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267

**Mathematical Formulae***Compound Interest*

$$\text{Total amount} = P \left( 1 + \frac{r}{100} \right)^n$$

*Measurement*

$$\text{Curved surface area of a cone} = \pi r l$$

$$\text{Surface area of a sphere} = 4\pi r^2$$

$$\text{Volume of a cone} = \frac{1}{3} \pi r^2 h$$

$$\text{Volume of a sphere} = \frac{4}{3} \pi r^3$$

$$\text{Area of triangle } ABC = \frac{1}{2} ab \sin C$$

$$\text{Arc length} = r\theta, \text{ where } \theta \text{ is in radians}$$

$$\text{Sector area} = \frac{1}{2} r^2 \theta, \text{ where } \theta \text{ is in radians}$$

*Trigonometry*

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

*Statistic*

$$\text{Mean} = \frac{\sum fx}{\sum f}$$

$$\text{Standard deviation} = \sqrt{\frac{\sum fx^2}{\sum f} - \left( \frac{\sum fx}{\sum f} \right)^2}$$

Answer **all** the questions.

1 (a) Calculate  $\frac{3 + \sqrt[3]{27 - 8 \times \sqrt{16} \times (-3)}}{4^{\frac{5}{2}}}$ ,

showing the first five digits on your calculator display.

Answer ..... [1]

(b) Give your answer to part (a) correct to 3 significant figures.

- Answer ..... [1]

2 These are the first five terms in a sequence.

1      3      7      15      31

(a) Write down the sixth and seventh term in the sequence.

Answer ..... , ..... [1]

(b) Write down an expression, in terms of  $n$ , for the  $n$ th term in the sequence.

Answer ..... [1]

3 (a) Given that  $7^{\frac{1}{2n}} \div 49^{-2} = 7^{\frac{3}{5}}$ , find the value of  $n$ .

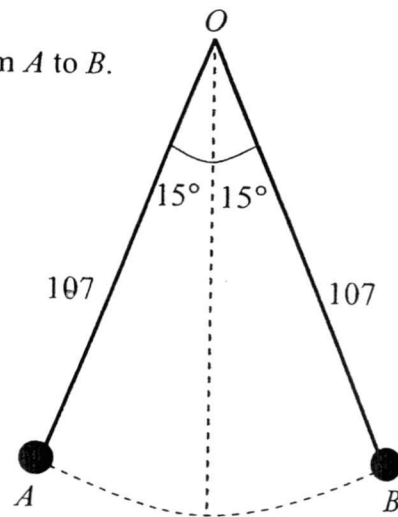
Answer  $n =$  ..... [2]

268

- (b) Given that  $\frac{1}{500} \pi = k \times 10^{-3}$ , find the value of  $k$ , rounding your answer to 1 significant figure.

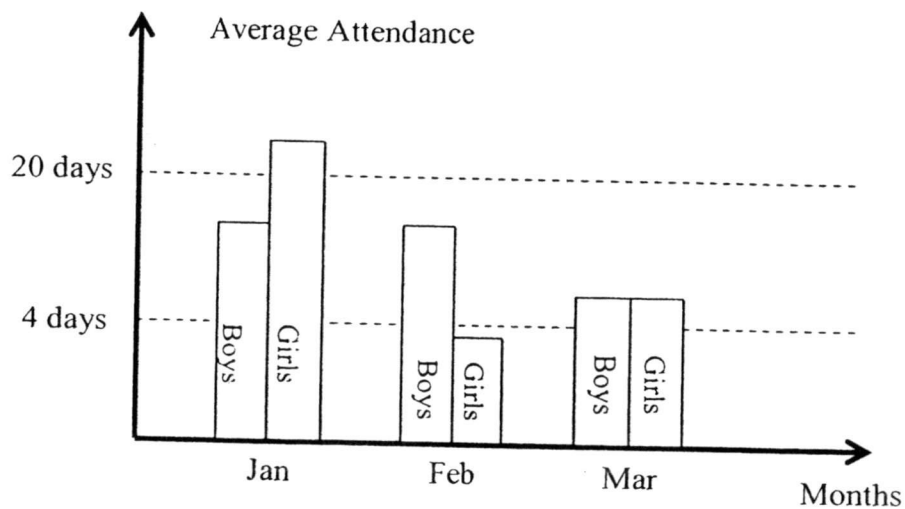
Answer  $k = \dots\dots\dots$  [1]

- 4 A pendulum of length 107 cm is suspended from  $O$ . Its end swings  $15^\circ$  on either side of the vertical from  $A$  to  $B$ . Calculate the length of the arc  $AB$ .



Answer  $\dots\dots\dots$  cm [2]

- 5 The graph shows the average attendance of boys and girls respectively for a school over three months.



Explain one way in which the graph is misleading.

Answer .....

.....

.....

..... [1]

6 Simplify  $(x-4)^2 - 16(x-4) + 64$ .

Answer ..... [2]

7  $V$  is inversely proportional to the cube of  $T$ . Given that  $V$  is increased by 237.5%, calculate the percentage change in  $T$ .

Answer ..... % [2]

269

- 8  $\varepsilon = \{ \text{integers } x : 0 < x \leq 16 \}$   
 $A = \{ \text{multiples of 3} \}$   
 $B = \{ \text{perfect squares} \}$   
 $C = \{ \text{integers } x : x > \sqrt{90} \}$

List the elements in

- (a)  $A'$ ,

Answer ..... [1]

- (b)  $A \cap B$ ,

Answer ..... [1]

- (c)  $(A \cup B \cup C)'$ .

Answer ..... [1]

- 9 A polygon has  $n$  sides. When the number of sides is doubled, the interior angle is increased by  $30^\circ$ .

Find the value of  $n$ .

Answer  $n =$  ..... [3]

10 The volume of a cone of radius  $r$  cm and height  $h$  cm is  $400 \text{ cm}^3$ .

- (a) Calculate the volume of a similar cone whose height is thrice that of the given cone.

Answer .....  $\text{cm}^3$  [1]

- (b) Find the volume of a cone whose radius is  $\frac{1}{2}r$  cm and whose height is  $\frac{3}{5}h$  cm.

Answer .....  $\text{cm}^3$  [2]

---

11 (a) Factorise completely  $9x^2 + 12x + 4 - 25y^2$ .

Answer ..... [2]

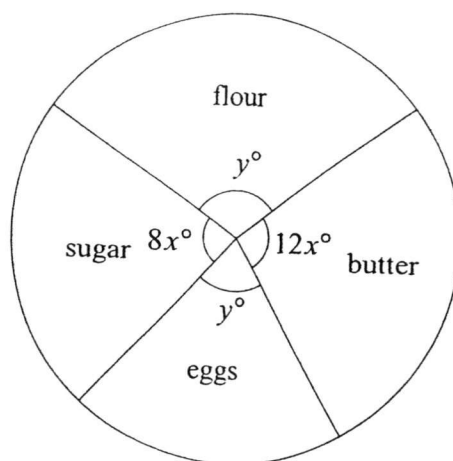
- (b) Simplify  $\frac{x^{2n} - y^{2n}}{x^{n+2} - x^2 y^n}$ .

Answer ..... [2]

---

- 12 The mass of ingredients used for making a cake are represented on the pie chart. Given that the ratio of the mass of butter to eggs is 5 : 2, find the value of  $x$ .

If 360 grams of butter is used, calculate the mass of eggs used.



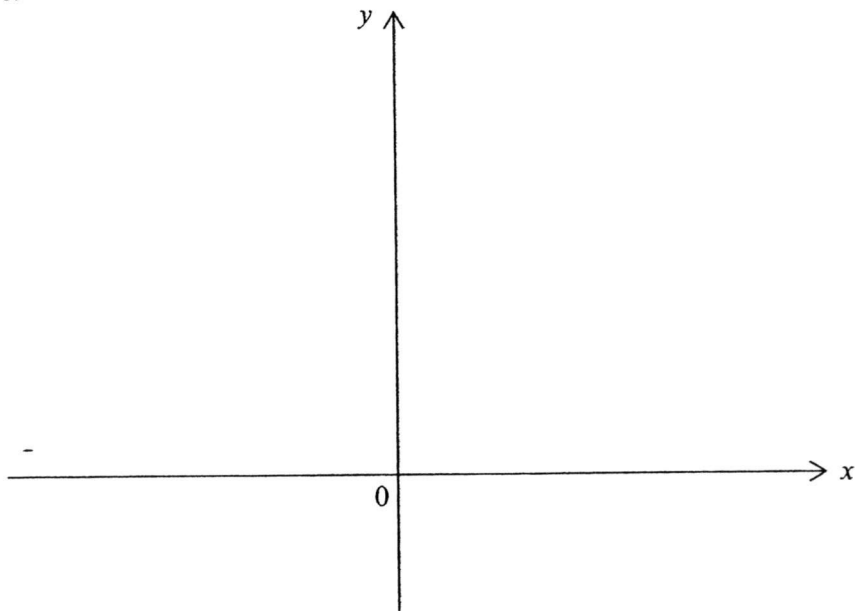
Answer  $x = \dots\dots\dots$  [2]

$\dots\dots\dots$  g [2]

---

- 13 (a) Sketch the graph of  $y = -(x+1)(x-3)$ .

Answer



[2]

- (b) Write down the equation of the line of symmetry of the graph of  $y = -(x+1)(x-3)$ .

Answer ..... [1]

- 14 (a) Solve  $\frac{3}{5}x(x-4)(3x+5) = 0$ .

Answer  $x =$  ..... [2]

- (b) Solve the equation  $2x^2 - 6x - 9 = 0$ , giving your answers correct to two decimal places.

Answer  $x =$  ..... , ..... [2]

- 15 (a) Alan bought an Ipad mini for \$329. He sold it online and made a profit of 135% on the price he paid for it.

Find the price at which he sold the Ipad Mini.

Answer \$ ..... [2]

- (b) The proposed area of a pond of  $0.00078 \text{ km}^2$  is represented by  $2 \text{ cm}^2$  on the floor plan of a garden.

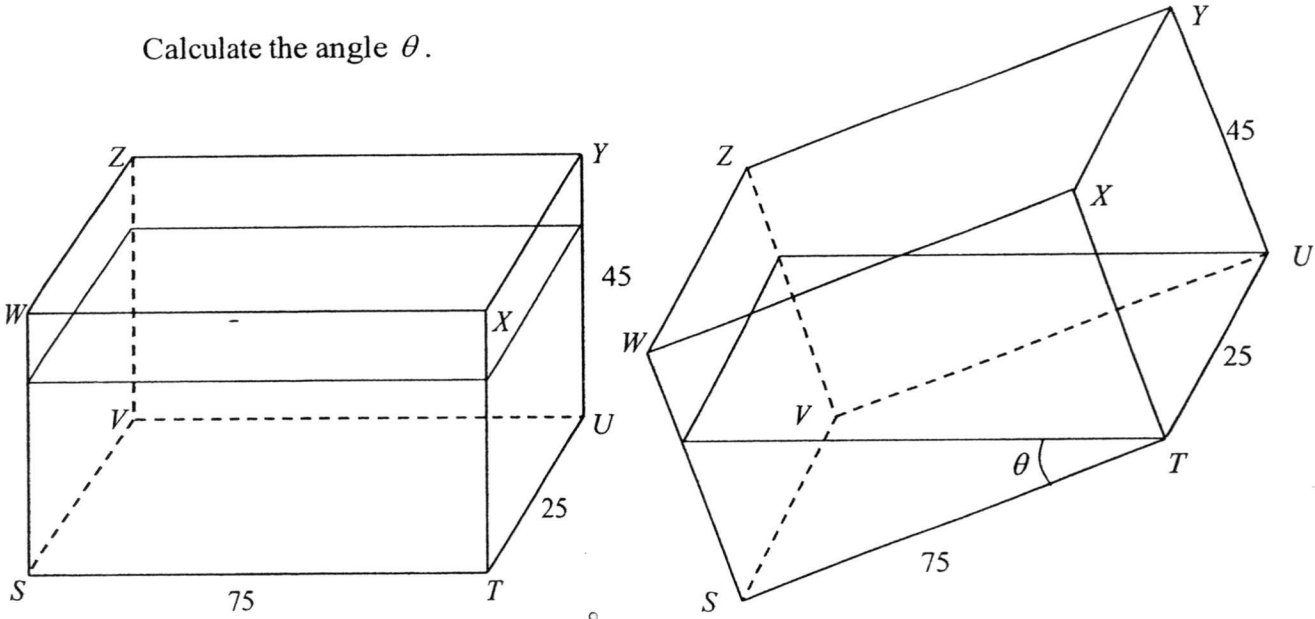
Given that the floor plan is drawn to a scale of  $1 : n$ , find the value of  $n$ .

Answer  $n = \dots\dots\dots$  [2]

---

- 16 Figure A shows an open rectangular container  $STUVWXYZ$  which is  $\frac{4}{5}$  filled with water. The internal dimensions of the container are 75 cm long, 25 cm wide and 45 cm high. The base,  $STUV$ , of the container is resting on a flat surface. In figure B, the container is tilted about the edge  $SV$  and 40% of the amount of water has been poured out.

Calculate the angle  $\theta$ .

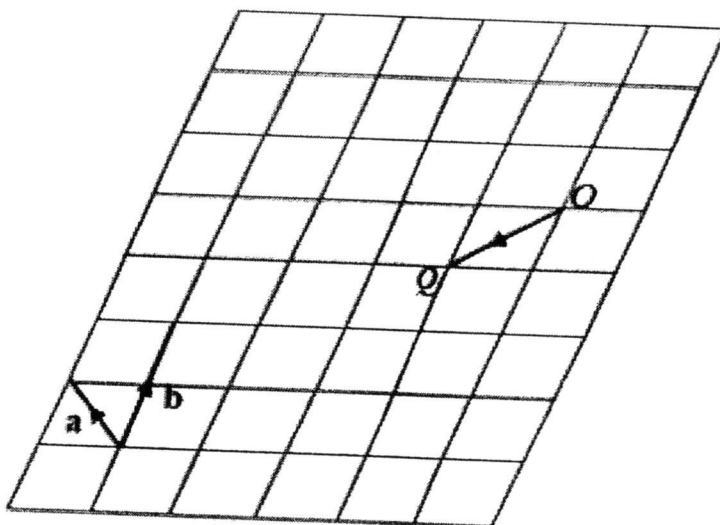


Answer  $\theta = \dots\dots\dots$  [4]

272

- 17 The grid in the answer space below shows the vectors  $\mathbf{a}$  and  $\mathbf{b}$ .  $O$  and  $Q$  are two points on the given grid.

Answer



- (i) Write down an expression for  $\overrightarrow{OQ}$  in terms of  $\mathbf{a}$  and  $\mathbf{b}$ .

Answer  $\overrightarrow{OQ} = \dots\dots\dots$  [1]

- (ii) Mark and label clearly on the grid the point  $P$  such that  $\overrightarrow{OP} = 3\mathbf{a} - 2\mathbf{b}$ .

Answer On the grid [1]

- (iii) A point  $R$  is such that  $\overrightarrow{OR} = h \overrightarrow{OQ}$  and  $\overrightarrow{PR} = k \mathbf{b}$ , where  $h$  and  $k$  are scalars.

- (a) Mark and label clearly on the grid the point  $R$ .

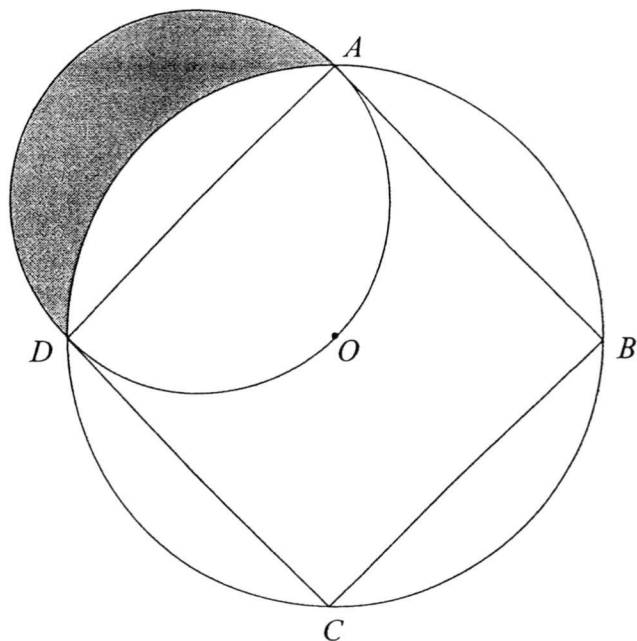
Answer On the grid [1]

- (b) Find the value of  $h$  and of  $k$ .

Answer  $h = \dots\dots\dots$  ,  $k = \dots\dots\dots$  [1]

- 18 In the diagram,  $ABCD$  is a square whose diagonals are each 2 cm long. Given that  $O$  is the centre of the bigger circle and  $BD$  and  $AD$  are the diameters of the bigger and smaller circle respectively.

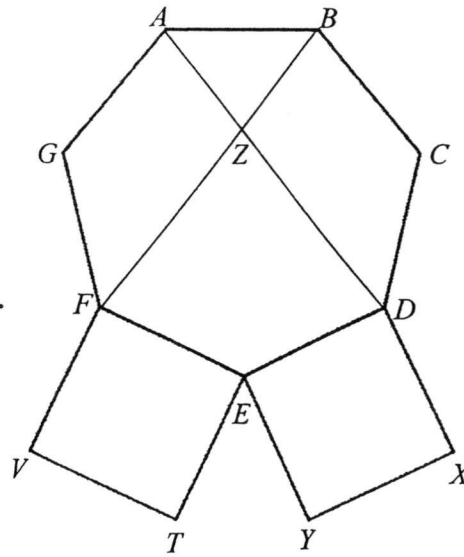
Find the area of the shaded region.



Answer .....  $\text{cm}^2$  [4]

- 19 (a) Given that figure  $ABCDEFG$  is a regular polygon.

Name the figure  $ABCDEFG$ .



Answer ..... [1]

- (b) It is given that figure  $FETV$  and figure  $EDXY$  are squares, find

(i) angle  $YET$ ,

Answer  $\angle YET =$  ..... [2]

(ii) angle  $AZF$ .

Answer  $\angle AZF =$  ..... [2]

---

20 (a) Solve the equation  $\frac{3x}{4} + \frac{2x-9}{7} = 5$ .

Answer  $x = \dots\dots\dots$  [2]

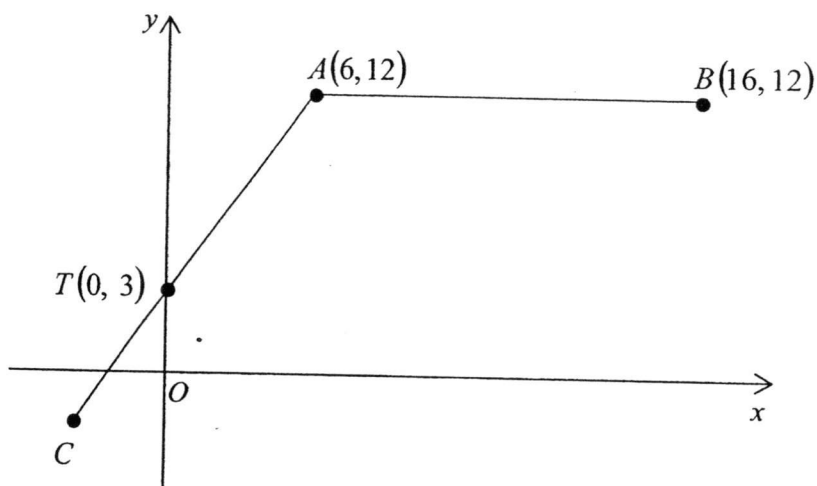
(b)(i) Express  $x^2 + 6x - 7$  in the form of  $(x + p)^2 + q$ .

Answer  $\dots\dots\dots$  [2]

(ii) Hence solve  $x^2 + 6x - 7 = 0$ .

Answer  $x = \dots\dots\dots$  or  $\dots\dots\dots$  [1]

- 21 The diagram shows a sketch of two straight lines  $ATC$  and  $AB$ . It is given that  $AC = 3TC$  and  $A(6, 12)$ ,  $B(16, 12)$  and  $T(0, 3)$ .



- (a) Find the coordinates of  $C$ .

Answer  $C$  (....., .....) [1]

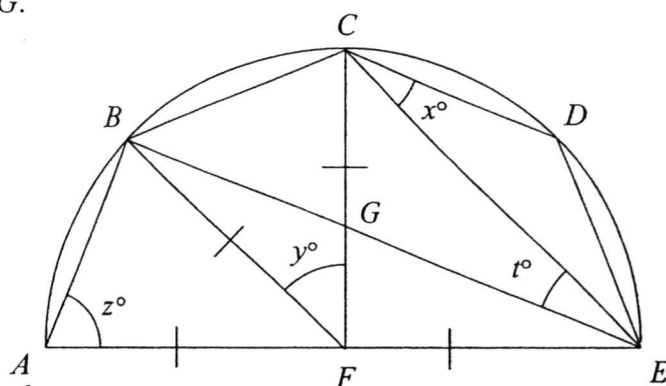
- (b) Find the equation of the line  $ATC$ .

Answer ..... [2]

- (c) Calculate the area of triangle  $ATB$ .

Answer .....  $\text{units}^2$  [2]

- 22 In the diagram,  $AB$ ,  $BC$ ,  $CD$  and  $DE$  are four adjacent sides of a regular octagon.  
 $FA = FB = FC = FE = 5$  cm.  
 $CF$  meets  $BE$  at  $G$ .



(i) Calculate

(a)  $x$ ,

Answer  $x = \dots\dots\dots$  [1]

(b)  $y$ ,

Answer  $y = \dots\dots\dots$  [1]

(c)  $z$ ,

Answer  $z = \dots\dots\dots$  [1]

(d)  $t$ .

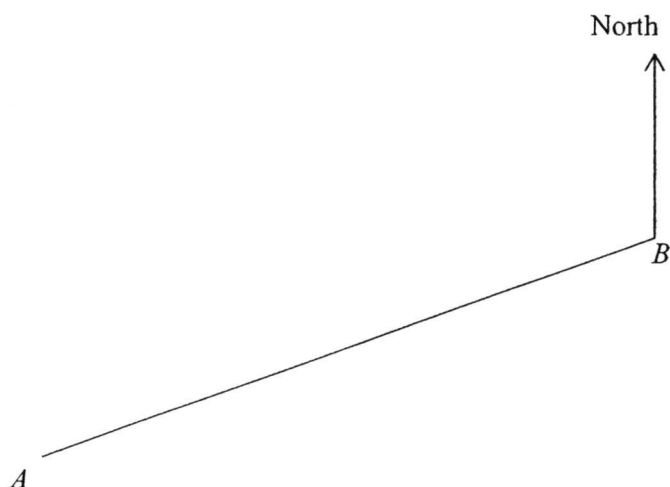
Answer  $t = \dots\dots\dots$  [1]

- (ii) If semi-circle  $ABCDE$  is a dartboard, and a dart is thrown towards the board at random, calculate the probability that the dart lands on the octagonal region.  
 (Assume that the dart will always land on the dartboard.)

Answer  $\dots\dots\dots$  [2]

275

- 23 A map is drawn to a scale of 1 cm to 3 km.  
The diagram below shows the positions of two villages  $A$  and  $B$  on the map.



- (a) A third village,  $C$  lies north of the line  $AB$ .  
It is 21 km from  $A$  and 18 km from  $B$ .  
Using ruler and compasses only, construct triangle  $ABC$ . [2]
- (b) Construct the perpendicular bisector of  $AB$ . [1]
- (c) Draw a circle of radius 9 km taking village  $C$  as the centre of the circle. [1]
- (d) A petrol station is to be built so that it is equidistant from  $A$  and  $B$  and 9 km from village  $C$ .  
Mark, with letters  $F$  and  $G$ , the two possible positions of the petrol station. [1]

## Answers to 2015 S4E EM PRELIMINARY EXAMINATION Paper 1

|     |                                                                                                                                                                                                                                           |
|-----|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| 1a  | 0.2491                                                                                                                                                                                                                                    |
| 1b  | 0.249                                                                                                                                                                                                                                     |
| 2a  | 63, 127                                                                                                                                                                                                                                   |
| 2b  | $n$ th term : $2^n - 1$                                                                                                                                                                                                                   |
| 3a  | $n = -\frac{5}{34}$                                                                                                                                                                                                                       |
| 3b  | $k = 6$ (1 sf)                                                                                                                                                                                                                            |
| 4   | 56.0 cm                                                                                                                                                                                                                                   |
| 5   | The vertical interval range is not proportionate thus giving people wrong impression. <u>or</u><br>The vertical axis should be measured in terms of percentage as it gives a clearer and consistent projection of the average attendance. |
| 6   | $x^2 - 24x + 144$                                                                                                                                                                                                                         |
| 7   | -70.4%                                                                                                                                                                                                                                    |
| 8a  | $A' = \{1, 2, 4, 5, 7, 8, 10, 11, 13, 14, 16\}$                                                                                                                                                                                           |
| 8b  | $A \cap B = \{9\}$                                                                                                                                                                                                                        |
| 8c  | $(A \cup B \cup C)' = \{2, 5, 7, 8\}$                                                                                                                                                                                                     |
| 9   | $n = 6$                                                                                                                                                                                                                                   |
| 10a | $10800 \text{ cm}^3$                                                                                                                                                                                                                      |
| 10b | $60 \text{ cm}^3$                                                                                                                                                                                                                         |
| 11a | $(3x+2+5y)(3x+2-5y)$                                                                                                                                                                                                                      |
| 11b | $\frac{(x^n + y^n)}{x^2}$                                                                                                                                                                                                                 |
| 12  | $y + 10x = 180$ ----- (1)<br>$y = \frac{24x}{5}$ ----- (2)<br>$x = 12\frac{6}{37}$ $y = 58\frac{14}{37}$<br>144 grams                                                                                                                     |
| 13a |                                                                                                                                                                                                                                           |
| 13b | $x = 1$                                                                                                                                                                                                                                   |

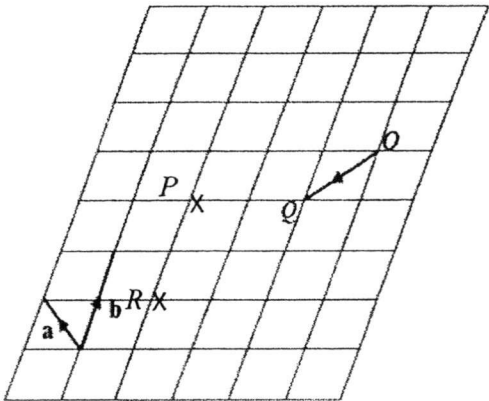
|        |                                                             |
|--------|-------------------------------------------------------------|
| 14a    | $x = 0, 4, -\frac{5}{3}$                                    |
| 14b    | $x = 4.10$ or $-1.10$                                       |
| 15a    | \$773.15                                                    |
| 15b    | 1 cm : 1970 cm                                              |
| 16     | $\theta = 29.9^\circ$                                       |
| 17i    | $\mathbf{a} - \mathbf{b}$                                   |
| 17iiib | $h = 3, k = -1$                                             |
|        |                                                             |
| 18     | $0.5 \text{ cm}^2$                                          |
| 19a    | Heptagon                                                    |
| 19bi   | $51.4^\circ$                                                |
| 19bii  | $102.9^\circ$                                               |
| 20a    | $x = 6\frac{2}{29}$ or $6.07$                               |
| 20bi   | $(x+3)^2 - 16$                                              |
| 20bii  | $x = -7$ or $1$                                             |
| 21a    | $(-3, -1.5)$                                                |
| 21b    | Equation of $ATC$ : $y = \frac{3}{2}x + 3$ or $2y = 3x + 6$ |
| 21c    | Area of triangle $ATB = 45 \text{ units}^2$                 |
| 22ia   | $22.5^\circ$                                                |
| 22ib   | $45^\circ$                                                  |
| 22ic   | $67.5^\circ$                                                |
| 22id   | $22.5^\circ$                                                |
| 22ii   | 0.900                                                       |

Answers to 2015 S4E EM PRELIMINARY EXAMINATION Paper 1

|    |                                                                                                                                                                                                                                                      |          |
|----|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|----------|
| 1a | $3 + \frac{\sqrt[3]{27 - 8 \times \sqrt{16} \times (-3)}}{4^{\frac{5}{2}}}$ $= 0.249162182$ $= 0.2491 \text{ (5 digits)}$                                                                                                                            | B1       |
| 1b | 0.249 (3 sf)                                                                                                                                                                                                                                         | B1       |
| 2a | 63, 127                                                                                                                                                                                                                                              | B1       |
| 2b | $n$ th term : $2^n - 1$                                                                                                                                                                                                                              | B1       |
| 3a | $7^{\frac{1}{2n}} \div 49^{-2} = 7^{\frac{3}{5}}$ $\frac{1}{2n} + 4 = \frac{3}{5}$ $n = -\frac{5}{34}$                                                                                                                                               | M1<br>A1 |
| 3b | $\frac{1}{500} \pi = k \times 10^{-3}$ $k \times 10^{-3} = 6.283185307 \times 10^{-3}$ $k = 6 \text{ (1 sf)}$                                                                                                                                        | B1       |
| 4  | <p>Arc length <math>AB</math></p> $= \frac{30}{360} \times 2\pi(107)$ $= 56.02507 \text{ cm}$ $= 56.0 \text{ cm}$                                                                                                                                    | M1<br>A1 |
| 5  | <p>The vertical interval range is not proportionate thus giving people wrong impression. <u>or</u></p> <p>The vertical axis should be measured in terms of percentage as it gives a clearer and consistent projection of the average attendance.</p> | B1       |
| 6  | $(x-4)^2 - 16(x-4) + 64$ $= (x-12)^2 \text{ or } x^2 - 8x + 16 - 16x + 64 + 64$ $= x^2 - 24x + 144$                                                                                                                                                  | M1<br>A1 |

|     |                                                                                                                                                                                                                                   |                |
|-----|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|----------------|
| 7   | $V = \frac{k}{t^3} \Rightarrow k = Vt^3$ $3.375 V = \frac{k}{T^3}$ $T^3 = \frac{Vt^3}{3.375 V}$ $T^3 = \frac{8}{27} t^3$ $\text{Percentage change in } T = \left(1 - \frac{8}{27}\right) \times 100 = -70.4\%$                    | M1<br>A1       |
| 8a  | $A' = \{1, 2, 4, 5, 7, 8, 10, 11, 13, 14, 16\}$                                                                                                                                                                                   | B1             |
| 8b  | $A \cap B = \{9\}$                                                                                                                                                                                                                | B1             |
| 8c  | $(A \cup B \cup C)' = \{2, 5, 7, 8\}$                                                                                                                                                                                             | B1             |
| 9   | $\frac{(2n-2) \times 180}{2n} - \frac{(n-2) \times 180}{n} = 30$ $\frac{360n - 360 - 360n + 720}{2n} = 30$ $60n = 360$ $n = 6$                                                                                                    | M1<br>M1<br>A1 |
| 10a | $\text{Volume of new cone} = \left(\frac{3}{1}\right)^3 \times 400$ $= 10800 \text{ cm}^3$                                                                                                                                        | B1             |
| 10b | $\text{Volume of the new cone}$ $= \frac{\frac{1}{3} \times \pi \times \left(\frac{1}{2}r\right)^2 \times \frac{3}{5}h}{\frac{1}{3} \times \pi \times (r)^2 \times h} \times 400$ $= \frac{3}{20} \times 400$ $= 60 \text{ cm}^3$ | M1<br>A1       |
| 11a | $9x^2 + 12x + 4 - 25y^2$ $= (3x+2)^2 - 25y^2$ $= (3x+2)^2 - (5y)^2$ $= (3x+2+5y)(3x+2-5y)$                                                                                                                                        | M1<br>A1       |

|     |                                                                                                                                                                                                                                                                                                                                        |                                                                                                                  |
|-----|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|------------------------------------------------------------------------------------------------------------------|
| 11b | $\frac{x^{2n} - y^{2n}}{x^{n+2} - x^2 y^n}$ $= \frac{(x^n)^2 - (y^n)^2}{x^2 (x^n - y^n)}$ $= \frac{(x^n + y^n)(x^n - y^n)}{x^2 (x^n - y^n)}$ $= \frac{(x^n + y^n)}{x^2}$                                                                                                                                                               | M1<br><br>A1                                                                                                     |
| 12  | $2y + 20x = 360$<br>$y + 10x = 180 \text{ ----- (1)}$<br><br>$\frac{12x}{y} = \frac{5}{2}$<br>$y = \frac{24x}{5} \text{ ----- (2)}$<br><br>Sub (2) into (1), $\frac{24x}{5} + 10x = 180$<br>$x = 12\frac{6}{37}$<br>$y = 58\frac{14}{37}$<br><br>$12x = 145\frac{35}{37} : 360 \text{ grams}$<br>$58\frac{14}{37} : 144 \text{ grams}$ | M1<br><br><br><br>M1<br><br>M1<br><br>A1                                                                         |
| 13a |                                                                                                                                                                                                                                                                                                                                        | M1 –<br>x & y-intercept<br><br>A1 –<br>shape & critical pts<br><br><br><div style="text-align: right;">578</div> |

|                |                                                                                                                                                                                                                                                                                                                         |                                           |
|----------------|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-------------------------------------------|
| 13b            | $x = 1$                                                                                                                                                                                                                                                                                                                 | A1                                        |
| 14a            | $x = 0, 4, -\frac{5}{3}$                                                                                                                                                                                                                                                                                                | M1 –<br>Any 2 answers<br>A1 – All answers |
| 14b            | $x = \frac{6 \pm \sqrt{108}}{4}$<br>$x = 4.10$ or $-1.10$                                                                                                                                                                                                                                                               | M1<br>A1                                  |
| 15a            | Price that he sold = $\frac{235}{100} \times 329$<br>= \$773.15                                                                                                                                                                                                                                                         | M1 – ratio<br>A1                          |
| 15b            | $2 \text{ cm}^2 : 0.00078 \text{ km}^2$<br>$1 \text{ cm} : 0.019748417 \text{ km}$<br>$1 \text{ cm} : 1970 \text{ cm}$                                                                                                                                                                                                  | M1<br>A1                                  |
| 16             | Amount of water in container = $0.6 \times 0.8 \times (75 \times 25 \times 45)$<br>= $40\,500 \text{ cm}^3$<br><br>Volume of water in container = $40\,500 \text{ cm}^3$<br>$\frac{1}{2} \times 75 \times 25 \times h = 40500$<br>$h = 43.2 \text{ cm}$<br><br>$\tan \theta = \frac{43.2}{75}$<br>$\theta = 29.9^\circ$ | M1<br><br><br><br>M1<br><br>M1<br>A1      |
| 17i            | $\mathbf{a} - \mathbf{b}$                                                                                                                                                                                                                                                                                               | B1                                        |
| 17iiib         | $h = 3, k = -1$                                                                                                                                                                                                                                                                                                         | B1                                        |
| 17ii<br>17iiia |                                                                                                                                                                                                                                      |                                           |

|  |        |
|--|--------|
|  | B1, B1 |
|--|--------|

|       |                                                                                                                                                                                                                                                                                                                                                                                                                                                            |                                                                 |
|-------|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-----------------------------------------------------------------|
| 18    | <p>By pythagoras' thm</p> $AD = \sqrt{2} \Rightarrow \text{Radius of small circle} = \frac{\sqrt{2}}{2}$ $\text{Area of semi-circle} = \frac{1}{2} \pi \left( \frac{\sqrt{2}}{2} \right)^2 = \frac{\pi}{4} \approx 0.785398163 \text{ cm}^2$ $\text{Area of segment } AD = \frac{1}{2} (1)^2 \left[ \frac{\pi}{2} - \sin \frac{\pi}{2} \right] = 0.285398163 \text{ cm}^2$ $\text{Area of shaded region} = \frac{\pi}{4} - 0.285398163 = 0.5 \text{ cm}^2$ | <p>M1</p> <p>M1</p> <p>M1</p> <p>A1</p>                         |
| 19a   | Heptagon                                                                                                                                                                                                                                                                                                                                                                                                                                                   | B1                                                              |
| 19bi  | <p>One interior angle = <math>\frac{(7-2) \times 180}{7} = 128.5714286^\circ</math></p> <p>Angle <math>YET = 360^\circ - 128.57^\circ - 90^\circ - 90^\circ</math><br/> <math>= 51.4^\circ</math></p>                                                                                                                                                                                                                                                      | <p>M1</p> <p>A1</p>                                             |
| 19bii | <p>Angle <math>ABF = \frac{360^\circ - 128.57^\circ - 128.57^\circ}{2}</math><br/> <math>= 51.43^\circ</math></p> <p>Angle <math>AZF = 51.43^\circ \times 2</math><br/> <math>= 102.9^\circ</math></p>                                                                                                                                                                                                                                                     | <p>M1</p> <p>A1</p>                                             |
| 20a   | $\frac{3x}{4} + \frac{2x-9}{7} = 5$ $\frac{21x+8x-36}{28} = 5$ $29x = 176$ $x = 6\frac{2}{29} \text{ or } 6.07$                                                                                                                                                                                                                                                                                                                                            | <p>M1</p> <p>A1</p>                                             |
| 20bi  | $x^2 + 6x - 7$ $= x^2 + 6x - 7$ $= (x+3)^2 - (3)^2 - 7$ $= (x+3)^2 - 9 - 7$ $= (x+3)^2 - 16$                                                                                                                                                                                                                                                                                                                                                               | <p>M1</p> <p>A1 <span style="margin-left: 20px;">279</span></p> |

|       |                                                                |    |
|-------|----------------------------------------------------------------|----|
| 20bii | $(x+3)^2 - 16 = 0$<br>$(x+3)^2 = 16$<br>$x = -7 \text{ or } 1$ | A1 |
|-------|----------------------------------------------------------------|----|

|      |                                                                                                                                                                                                                                                                                         |              |
|------|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|--------------|
| 21a  | Coordinates $C$ $(0-3, 3-4.5)$<br>$(-3, -1.5)$                                                                                                                                                                                                                                          | B1           |
| 21b  | Gradient $= \frac{3}{2}$<br><br>Equation of $ATC$ : $y = \frac{3}{2}x + 3$ or $2y = 3x + 6$                                                                                                                                                                                             | M1<br>A1     |
| 21c  | Area of triangle $ATB = \frac{1}{2} \times 10 \times 9$<br>$= 45 \text{ units}^2$                                                                                                                                                                                                       | M1<br>A1     |
| 22ia | One interior angle $= 135^\circ$<br>Angle $x = \frac{180-135}{2}$<br>$= 22.5^\circ$                                                                                                                                                                                                     | B1           |
| 22ib | $y = \frac{360}{8}$<br>$= 45^\circ$                                                                                                                                                                                                                                                     | B1           |
| 22ic | $z = \frac{180-45}{2}$<br>$= 67.5^\circ$                                                                                                                                                                                                                                                | B1           |
| 22id | $t = \frac{1}{2}y = 22.5^\circ$                                                                                                                                                                                                                                                         | B1           |
| 22ii | Area of dartboard $= 4 \times \frac{1}{2} \times 5 \times 5 \times \sin 45^\circ = 35.35533906 \text{ cm}^2$<br><br>Area of semi-circle $= \frac{1}{2} \pi (5)^2 = \frac{25}{2} \pi \text{ cm}^2$<br><br>$P(\text{dart land on } ABCDE) = \frac{35.35533906}{\frac{25}{2} \pi} = 0.900$ | M1<br><br>A1 |



**SWISS COTTAGE SECONDARY SCHOOL**  
**SECONDARY FOUR EXPRESS**  
**PRELIMINARY EXAMINATIONS**

Name: \_\_\_\_\_ (                      )

Class: Sec \_\_\_\_\_

**MATHEMATICS**

Paper 2

**4016/02**

**Monday 17 August 2015**

**2 hours 30 minutes**

Additional materials: Answer paper (8 sheets)

Graph paper (1 sheet)

**READ THESE INSTRUCTIONS FIRST**

Write your name, class and index number on all the work you hand in.

Write in dark blue or black pen on both sides of the paper.

You may use a pencil for any diagrams or graphs.

Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.

If working is needed for any question it must be shown with the answer.

Omission of essential working will result in loss of marks.

Calculators should be used where appropriate.

If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.

For  $\pi$ , use either your calculator value or 3.142, unless the question requires the answer in terms of  $\pi$ .

At the end of the examination, fasten all your work securely together.

**SUBMIT SECTION A AND B SEPERATELY**

The number of marks is given in brackets [    ] at the end of each question or part question.

The total number of marks for this paper is 100.

This question paper consists of **10** printed pages.

**Setter:** Mr Ang Hanping

**Vetter:** Ms Zoe Pow

[Turn over

380

*We Nurture Students to Think Care and Lead with P.R.I.D.E.*

*Mathematical Formulae*

## Compound Interest

$$\text{Total amount} = P \left( 1 + \frac{r}{100} \right)^n$$

## Mensuration

$$\text{Curved surface area of a cone} = \pi r l$$

$$\text{Surface area of a sphere} = 4\pi r^2$$

$$\text{Volume of a cone} = \frac{1}{3} \pi r^2 h$$

$$\text{Volume of a sphere} = \frac{4}{3} \pi r^3$$

$$\text{Area of triangle } ABC = \frac{1}{2} ab \sin C$$

$$\text{Arc length} = r\theta, \text{ where } \theta \text{ is in radians}$$

$$\text{Sector area} = \frac{1}{2} r^2 \theta, \text{ where } \theta \text{ is in radians}$$

## Trigonometry

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

## Statistics

$$\text{Mean} = \frac{\sum fx}{\sum f}$$

$$\text{Standard Deviation} = \sqrt{\frac{\sum fx^2}{\sum f} - \left( \frac{\sum fx}{\sum f} \right)^2}$$

Answer **all** the questions.

**Section A (61 marks)**

- 1 (a) Factorise completely  $a^3b + 2ab^3 - 2a^2 - 4b^2$ . [2]
- (b) Express as a single fraction in its simplest form  $\frac{2}{3x^2 + 11x - 20} - \frac{3}{3x - 4}$ . [3]
- (c) Given that  $2xy + 3x = \sqrt{3yz - x^2}$ , express  $x$  in terms of  $y$  and  $z$ . [3]
- (d) (i) Simplify  $(2x + y)(2x - y) - 4x^2$ . [1]
- (ii) Hence evaluate  $23 \times 17 - 400$ . [1]
- 
- 2 (a) (i) Express 3780 as the product of its prime factors. [1]
- (ii) Find the smallest integer  $p$  such that  $3780p$  is a perfect cube. [1]
- (iii) Find the smallest integer  $q$  such that  $3780q$  is a multiple of 350. [1]
- (iv) The lowest common multiple of 3780 and integer  $k$  is 7560.  
The highest common factor of 3780 and integer  $k$  is 60.  
Find the smallest possible value of integer  $k$ . [1]
- (b) The first four terms in a sequence of numbers are
- 3, 6, 11, 18, ...
- (i) Write down an expression, in terms of  $n$ , for the  $n$ th term of the sequence. [1]
- (ii) Find and simplify an expression for the difference between two consecutive terms in the sequence. [2]
- (iii) Hence explain why the difference between any two consecutive terms in the sequence is always an odd number. [1]
-

- 3 The fares for taxis from 3 companies are shown in the table below.

| Company                                   | Epic   | Toyo   | Crystal |
|-------------------------------------------|--------|--------|---------|
| Flag-Down (inclusive of first km or less) | \$3.60 | \$3.80 | \$5.00  |
| Every 1 km thereafter or less up to 10 km | \$0.40 | \$0.40 | \$0.60  |
| Every 1 km thereafter or less after 10 km | \$0.80 | \$0.80 | \$1.20  |

The information can be represented by matrix  $A = \begin{pmatrix} 3.6 & 3.8 & 5.0 \\ 0.4 & 0.4 & 0.6 \\ 0.8 & 0.8 & 1.2 \end{pmatrix}$ .

Bobby travels a distance of 13 km by taxi. The breakdown of the distance he travelled can be represented by matrix  $B = \begin{pmatrix} 1 & 9 & 3 \end{pmatrix}$ .

- (a) Evaluate matrix  $C$  such that  $C = BA$ . [1]

- (b) Explain what the elements in  $C$  represent. [1]

Clara travels a distance of 20 km by taxi every day. During peak hour, the peak hour surcharge for Epic, Toyo and Crystal are 30%, 20% and 10% of the total fare respectively.

- (c) (i) Write down a matrix  $D$  similar to  $B$  to represent the breakdown of the distance she travelled. [1]

- (ii) Evaluate  $DA$ . [1]

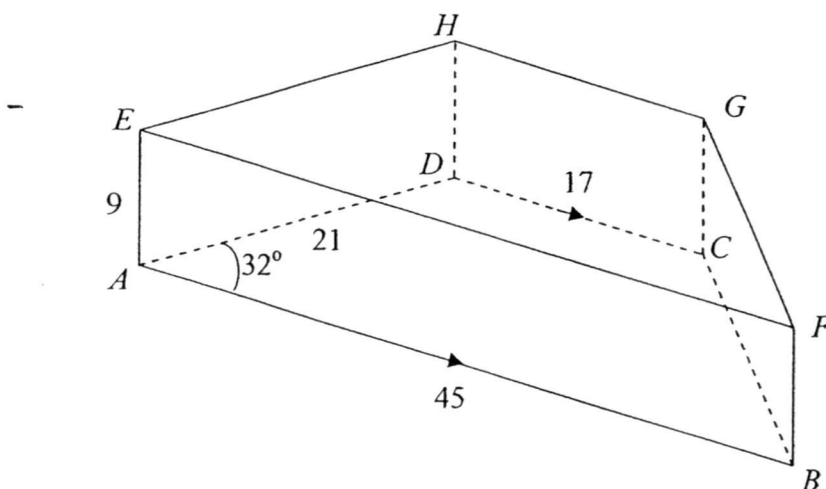
- (d) (i) Write down a matrix  $E$  such that the product  $DAE$  will give the amount after surcharge that Clara will have to pay for a taxi from each of the companies. [1]

- (ii) Evaluate  $DAE$ . [1]

- (e) Explain which company would be the most economical choice during peak hour and non-peak hour respectively. [1]
-

- 4 (a) Solve the simultaneous equations
- $$3x + 5y = 5,$$
- $$\frac{1}{2}x = \frac{1}{3}y + 2. \quad [3]$$
- (b) Andy, Bernard and Charlie were running on a 400 m circular track. Andy started running at point  $O$  in an anti-clockwise direction at a speed of  $x$  m/s. At the same time, Bernard and Charlie also started running at point  $O$ , but moved in a clockwise direction at speeds of  $(x + 3)$  m/s and  $(x - 4)$  m/s respectively.
- (i) Show that the time passed before Andy and Bernard meet each other on the track is  $\frac{400}{2x + 3}$  seconds. [1]
- (ii) Find, in terms of  $x$ , the time passed before Andy and Charlie meet each other on the track. [1]
- (iii) Given that Andy meets Charlie  $23\frac{1}{3}$  seconds after passing Bernard, form an equation in terms of  $x$  and show that it simplifies to  $2x^2 - x - 66 = 0$ . [3]
- (iv) Solve the equation  $2x^2 - x - 66 = 0$ . [3]
- (v) Find the time taken for Andy to run one round around the track. [1]

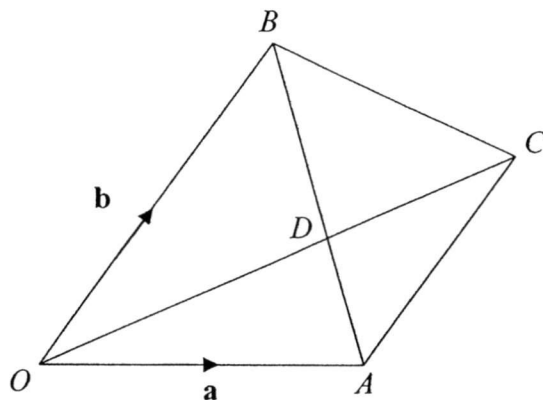
- 5 In the diagram,  $ABCDEFGH$  shows a lecture theatre in the shape of a trapezoidal prism. The floor of the theatre,  $ABCD$ , is on flat ground and  $ABFE$ ,  $BCGF$ ,  $DCGH$  and  $ADHE$  are vertical walls.  $AB = 45$  m,  $CD = 17$  m,  $AD = 21$  m,  $AE = 9$  m,  $\angle BAD = 32^\circ$  and  $AB$  is parallel to  $DC$ .



- (a) Find the length of  $BD$ . [3]
- (b) Find the angle of depression of  $D$  from  $F$ . [2]

- (c) Show that  $\angle ADB = 125.74^\circ$ , correct to two decimal places. [2]
- (d) Calculate the area of  $\triangle BCD$ . [2]
- (e)  $J$  is a point on  $AB$  such that  $DJ$  and  $AB$  are perpendicular. Find  $\angle DFJ$ . [3]

- 6 (a)  $A$  is the point  $(4, 6)$ ,  $B$  is the point  $(6, k)$  and  $C$  is the point  $(-3, 5)$ .
- (i) Find  $|\overrightarrow{AC}|$ . [2]
- (ii) Express  $\overrightarrow{AC}$  as a column vector. [1]
- (iii) Find the value of  $k$  if  $OA$  and  $OB$  are parallel. [2]
- (b) In the diagram,  $OC$  and  $AB$  intersect at  $D$ ,  $\overrightarrow{OA} = \mathbf{a}$ ,  $\overrightarrow{OB} = \mathbf{b}$ ,  $\overrightarrow{DC} = \frac{2}{5}\overrightarrow{OC}$  and  $3AD = 2DB$ .



- (i) Express, as simply as possible, in terms of  $\mathbf{a}$  and  $\mathbf{b}$ ,
- (a)  $\overrightarrow{AB}$ , [1]
- (b)  $\overrightarrow{OD}$ , [1]
- (c)  $\overrightarrow{AC}$ . [2]
- (ii) Write down two facts about  $OB$  and  $AC$ . [1]
- (iii) Find  $\frac{\text{area of } \triangle OAD}{\text{area of } \triangle CBD}$ . [2]

## Section B (39 marks)

Begin this section on a fresh sheet of paper.

7 Answer the whole of this question on a sheet of graph paper.

The variables  $x$  and  $y$  are connected by the equation

$$y = 5x - 3 + \frac{1}{2x}.$$

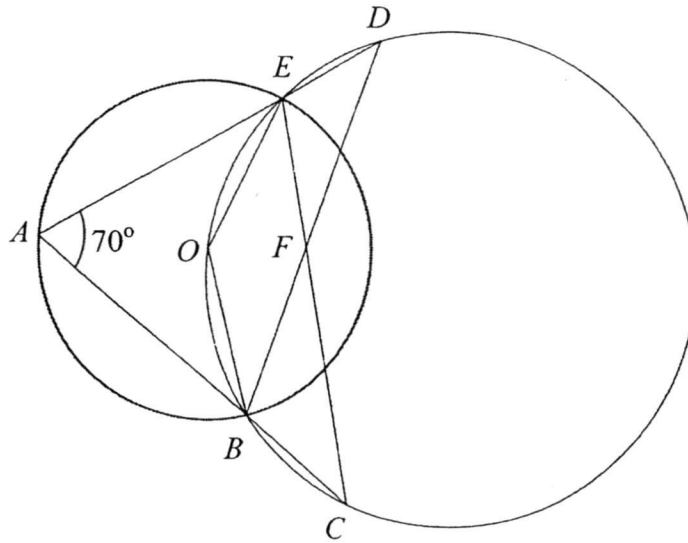
The table below shows some values of  $x$  and the corresponding values of  $y$  correct to 2 decimal places.

|     |      |      |      |      |      |     |      |      |
|-----|------|------|------|------|------|-----|------|------|
| $x$ | 0.05 | 0.1  | 0.2  | 0.3  | 0.4  | 0.5 | 0.6  | 0.7  |
| $y$ | 7.25 | 2.50 | 0.50 | 0.17 | 0.25 | $a$ | 0.83 | 1.21 |

- (a) Calculate the value of  $a$ . [1]
- (b) Using a scale of 2 cm to represent 0.1 unit, draw a horizontal  $x$ -axis for  $0 < x \leq 0.7$ . Using a scale of 2 cm to represent 1 unit, draw a vertical  $y$ -axis for  $0 \leq y \leq 8$ .  
  
On your axes, plot the points given in the table and join them with a smooth curve. [3]
- (c) By drawing a tangent, find the gradient of the curve at  $(0.2, 0.5)$ . [2]
- (d) By drawing a suitable line, find the solutions of  $10x^2 - 8x + 1 = 0$  in the range  $0 < x \leq 0.7$ . [2]
- (e) The line  $y = kx$  touches the curve  $y = 5x - 3 + \frac{1}{2x}$  at exactly one point in the range  $0 < x \leq 0.7$ .

By drawing a suitable straight line on the same axes, use your graph to find the value of  $k$ . [2]

- 8 The diagram shows two circles with the bigger circle passing through the centre,  $O$ , of the smaller circle. The circles intersect at  $B$  and  $E$ .  $A$  lies on the smaller circle while  $C$  and  $D$  lie on the bigger circle.  $AED$  and  $ABC$  are straight lines.  $CE$  and  $BD$  intersect at  $F$  and  $\angle BAE = 70^\circ$ .



- (a) Find, stating the reasons clearly,
- (i)  $\angle BOE$ , [1]
  - (ii)  $\angle BDE$ , [1]
  - (iii)  $\angle ABD$ , [1]
  - (iv)  $\angle CED$ , [2]
  - (v)  $\angle DFC$ . [1]
- (b) A point  $G$  lies on the same side of  $BE$  as  $A$ . Given that  $\angle BGE = 75^\circ$ , explain whether point  $G$  lies inside, outside or on the circumference of the smaller circle. [1]
-

- 9 (a) The table below shows the marks obtained in a Mathematics test by 200 students.

| Mark ( $x$ ) | $0 < x \leq 20$ | $20 < x \leq 40$ | $40 < x \leq 60$ | $60 < x \leq 80$ | $80 < x \leq 100$ |
|--------------|-----------------|------------------|------------------|------------------|-------------------|
| Frequency    | 4               | 43               | 55               | 64               | 34                |

- (i) State the median class. [1]
- (ii) Calculate
- (a) the mean mark, [1]
- (b) the standard deviation. [2]
- (b) Asha and Brennan are sitting for a test. The probability that Asha passes the test is 0.9 and the probability that Brennan passes the test is 0.8.
- (i) Find the probability that both Asha and Brennan pass the test. [1]
- A similar test is set every week and Asha and Brennan can take the test as many times as they like. They decide to stop taking the test only when both of them succeed in passing the test in the same week.
- (ii) Find the probability that they
- (a) do not succeed in the first week but succeed in the second, [1]
- (b) succeed in either the first or second week, [1]
- (c) do not succeed in the first three weeks, [1]
- (d) succeed in one of the first three weeks. [1]
- (iii) Find the probability, in terms of  $n$ , that they
- (a) do not succeed in the first  $n$  weeks, [1]
- (b) succeed in one of the first  $n$  weeks. [1]

- 10 The corn in Diagram I is modelled as a frustum with a hemisphere on top as shown in Diagram II. The diameter of the top of the frustum is 4 cm and the diameter of the bottom of the frustum is 6 cm. The height of the frustum is 20 cm.

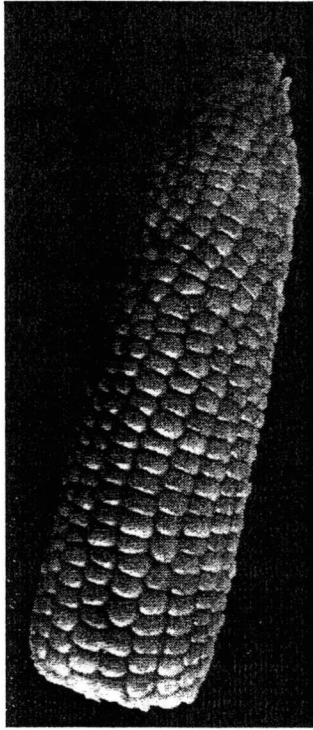


Diagram I

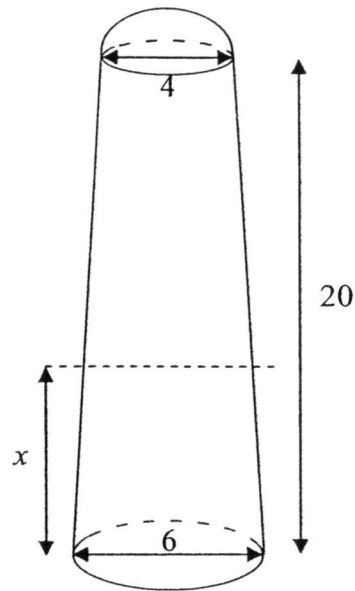


Diagram II

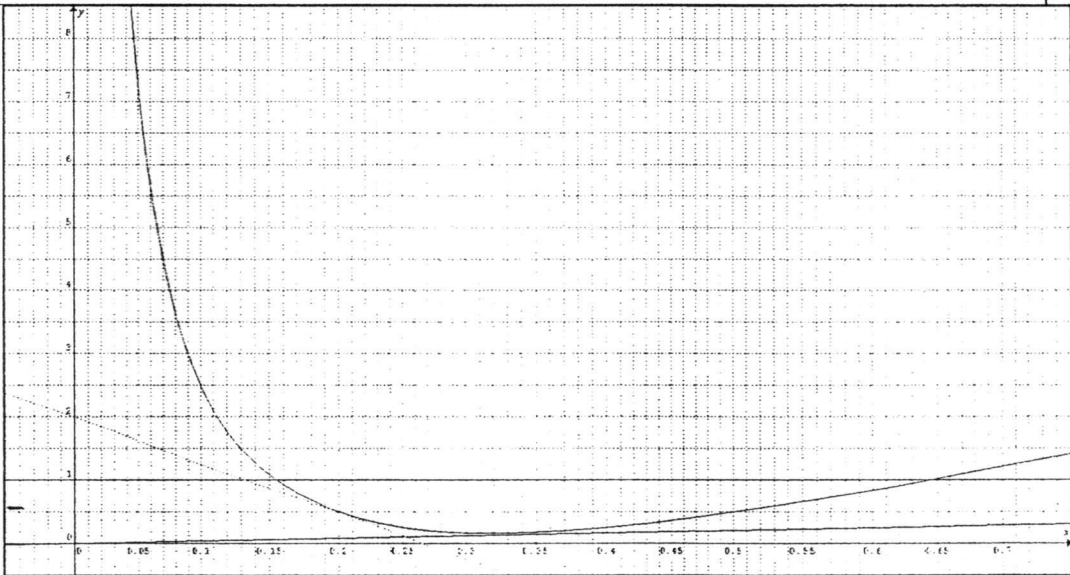
- (a) Calculate the volume of the corn. [4]
- (b) The curved surface area of the corn is covered by grains but the circular base is not. Find the total curved surface area of the corn. [3]
- (c) The number of grains on the corn is directly proportional to the curved surface area of the corn. The corn is to be cut horizontally into two lengths such that the number of grains on both lengths is the same. Calculate the height from the base,  $x$  cm, at which the corn should be cut. [4]

---

End of paper

**Mathematics Paper 2 (100 marks)**

| Qn. # | Solution                                                                                                                                                  |
|-------|-----------------------------------------------------------------------------------------------------------------------------------------------------------|
| 1a    | $(ab - 2)(a^2 + 2b^2)$                                                                                                                                    |
| 1b    | $\frac{-3x - 13}{(3x - 4)(x + 5)}$                                                                                                                        |
| 1c    | $x = \pm \sqrt{\frac{3yz}{4y^2 + 12y + 10}}$                                                                                                              |
| 1di   | $-y^2$                                                                                                                                                    |
| 1dii  | $-9$                                                                                                                                                      |
| 2ai   | $3780 = 2^2 \times 3^3 \times 5 \times 7$                                                                                                                 |
| 2aai  | $p \approx 2450$                                                                                                                                          |
| 2aiii | $q = 5$                                                                                                                                                   |
| 2aiv  | $k = 120$                                                                                                                                                 |
| 2bi   | $n^2 + 2$                                                                                                                                                 |
| 2bii  | $2n + 1$                                                                                                                                                  |
| 2biii | Since $n$ is an integer, $2n$ is always even and $2n + 1$ will always be an odd number                                                                    |
| 3a    | $C = \begin{pmatrix} 9.6 & 9.8 & 14 \end{pmatrix}$                                                                                                        |
| 3b    | Elements in $C$ represent the taxi fare Bobby would have paid if he took a taxi from Epic, Toyo or Crystal respectively.                                  |
| 3ci   | $D = \begin{pmatrix} 1 & 9 & 10 \end{pmatrix}$                                                                                                            |
| 3cii  | $DA = \begin{pmatrix} 15.2 & 15.4 & 22.4 \end{pmatrix}$                                                                                                   |
| 3di   | $E = \begin{pmatrix} 1.3 & 0 & 0 \\ 0 & 1.2 & 0 \\ 0 & 0 & 1.1 \end{pmatrix}$                                                                             |
| 3dii  | $DAE = \begin{pmatrix} 19.76 & 18.48 & 24.64 \end{pmatrix}$                                                                                               |
| 3e    | Epic will be the most economical during non-peak hour and Toyo most economical during peak hours as their prices are cheapest during the respective hours |
| 4a    | $y = -1, x = 3\frac{1}{3}$                                                                                                                                |
| 4bi   | $t = \frac{400}{2x + 3}$                                                                                                                                  |
| 4bii  | $t = \frac{400}{2x - 4}$                                                                                                                                  |
| 4biv  | $x = -5.5$ or $x = 6$                                                                                                                                     |
| 4bv   | Time taken = 66.7 s                                                                                                                                       |
| 5a    | $BD = 29.4$ m                                                                                                                                             |
| 5b    | Angle of depression = $17.0^\circ$                                                                                                                        |
| 5c    | $\angle ADB = 125.74$ (2 d.p.)                                                                                                                            |
| 5d    | Area $BCD = 94.6$ m <sup>2</sup>                                                                                                                          |

| Qn. # | Solution                                                                                                                                                                     |  |
|-------|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|--|
| 5e    | $\angle DFJ = 21.2^\circ$                                                                                                                                                    |  |
| 6ai   | $ \overrightarrow{AC}  = 7.07 \text{ units}$                                                                                                                                 |  |
| 6aia  | $\overrightarrow{AC} = \begin{pmatrix} -7 \\ -1 \end{pmatrix}$                                                                                                               |  |
| 6aiii | $k = 9$                                                                                                                                                                      |  |
| 6bia  | $\overrightarrow{AB} = \mathbf{b} - \mathbf{a}$                                                                                                                              |  |
| 6bib  | $\overrightarrow{OD} = \frac{1}{5}(3\mathbf{a} + 2\mathbf{b})$                                                                                                               |  |
| 6bic  | $\overrightarrow{AC} = \frac{2}{3}\mathbf{b}$                                                                                                                                |  |
| 6bii  | $OB \parallel AC$<br>$OB = \frac{3}{2}AC$                                                                                                                                    |  |
| 6biii | $\frac{\text{area of } \triangle OAD}{\text{area of } \triangle CBD} = 1$                                                                                                    |  |
| 7a    | $a = 0.5$                                                                                                                                                                    |  |
| 7b    |  <p>B1 – Correct axes and scale<br/>B1 – Correct points plotted<br/>B1 – Smooth curve</p> |  |
| 7c    | Gradient = $-7.5 \pm 1.5$                                                                                                                                                    |  |
| 7d    | $x = 0.155 \pm 0.01$ or $x = 0.64 \pm 0.01$                                                                                                                                  |  |
| 7e    | $k = 0.409 \pm 0.15$                                                                                                                                                         |  |
| 8ai   | $\angle BOE = 140^\circ$                                                                                                                                                     |  |
| 8aia  | $\angle BDE = 40^\circ$                                                                                                                                                      |  |
| 8aiii | $\angle ABD = 70^\circ$                                                                                                                                                      |  |
| 8aiv  | $\angle CED = 110^\circ$                                                                                                                                                     |  |

| Qn. #  | Solution                                                                                                                                                                                                                                              |  |
|--------|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|--|
| 8av    | $\angle DFC = 150^\circ$                                                                                                                                                                                                                              |  |
| 8b     | Any point $X$ on the same side of $BE$ as $A$ , lying on the circumference of the circle will form an angle $\angle BXE = 70^\circ$ (angles in same segment). Since $\angle BGE = 75^\circ$ which is larger than $70^\circ$ , $G$ lies in the circle. |  |
| 9ai    | $40 < x \leq 60$                                                                                                                                                                                                                                      |  |
| 9aiia  | Mean mark = 58.1 marks                                                                                                                                                                                                                                |  |
| 9aiib  | Standard deviation = 21.3 marks                                                                                                                                                                                                                       |  |
| 9bi    | 0.72                                                                                                                                                                                                                                                  |  |
| 9biia  | 0.2016                                                                                                                                                                                                                                                |  |
| 9biib  | 0.9216                                                                                                                                                                                                                                                |  |
| 9biic  | 0.021952                                                                                                                                                                                                                                              |  |
| 9biid  | 0.978048                                                                                                                                                                                                                                              |  |
| 9biiaa | 0.28"                                                                                                                                                                                                                                                 |  |
| 9biibb | $1 - 0.28"$                                                                                                                                                                                                                                           |  |
| 10a    | Total volume = $415 \text{ cm}^3$                                                                                                                                                                                                                     |  |
| 10b    | Total curved surface area = $340 \text{ cm}^2$                                                                                                                                                                                                        |  |
| 10c    | $x = 9.80$                                                                                                                                                                                                                                            |  |

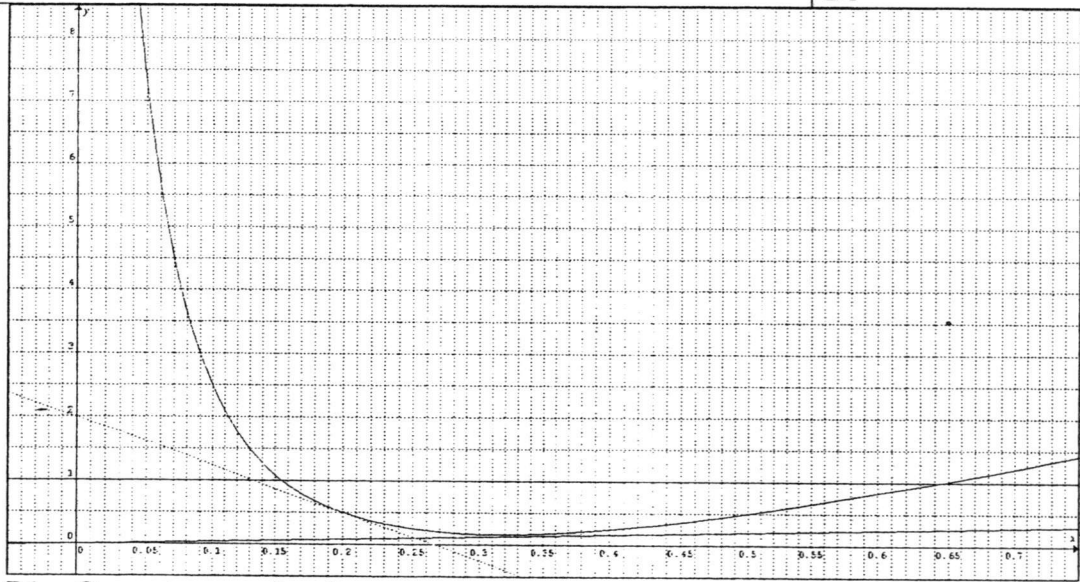
## Mathematics Paper 2 (100 marks)

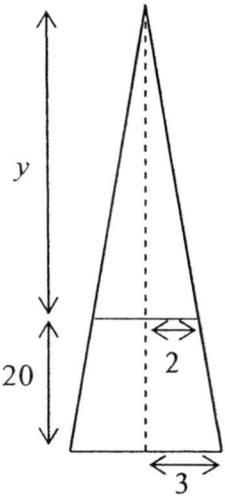
| Qn. # | Solution                                                                                                                                                                                                           | Mark Allocation                |
|-------|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|--------------------------------|
| 1a    | $a^3b + 2ab^3 - 2a^2 - 4b^2$ $= ab(a^2 + 2b^2) - 2(a^2 + 2b^2)$ $= (ab - 2)(a^2 + 2b^2)$                                                                                                                           | M1<br>A1                       |
| 1b    | $\frac{2}{3x^2 + 11x - 20} - \frac{3}{3x - 4}$ $= \frac{2}{(3x - 4)(x + 5)} - \frac{3}{3x - 4}$ $= \frac{2 - 3(x + 5)}{(3x - 4)(x + 5)}$ $= \frac{-3x - 13}{(3x - 4)(x + 5)}$                                      | M1<br><br>M1<br><br>A1         |
| 1c    | $2xy + 3x = \sqrt{3yz - x^2}$ $4x^2y^2 + 12x^2y + 9x^2 = 3yz - x^2$ $4x^2y^2 + 12x^2y + 10x^2 = 3yz$ $x^2(4y^2 + 12y + 10) = 3yz$ $x^2 = \frac{3yz}{4y^2 + 12y + 10}$ $x = \pm \sqrt{\frac{3yz}{4y^2 + 12y + 10}}$ | M1<br><br><br>M1<br><br><br>A1 |
| 1di   | $(2x + y)(2x - y) - 4x^2$ $= 4x^2 - y^2 - 4x^2$ $= -y^2$                                                                                                                                                           | B1                             |
| 1dii  | $23 \times 17 - 400$ $= [2(10) + 3][2(10) - 3] - 4(10)^2$ $= -(3)^2$ $= -9$                                                                                                                                        | A1                             |
| 2ai   | $3780 = 2^2 \times 3^3 \times 5 \times 7$                                                                                                                                                                          | B1                             |
| 2aii  | $p = 2 \times 5^2 \times 7^2 = 2450$                                                                                                                                                                               | B1                             |
| 2aiii | $q = 5$                                                                                                                                                                                                            | B1                             |
| 2aiv  | $k = 2^3 \times 3 \times 5 = 120$                                                                                                                                                                                  | B1                             |
| 2bi   | $n^2 + 2$                                                                                                                                                                                                          | B1                             |
| 2bii  | $(n + 1)^2 + 2 - (n^2 + 2)$ $= 2n + 1$                                                                                                                                                                             | M1<br>A1                       |
| 2biii | Since n is an integer, 2n is always even and 2n+1 will always                                                                                                                                                      |                                |

| Qn. # | Solution                                                                                                                                                                                                                                                       | Mark Allocation |
|-------|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-----------------|
|       | be an odd number                                                                                                                                                                                                                                               | B1              |
| 3a    | $C = \begin{pmatrix} 1 & 9 & 3 \end{pmatrix} \begin{pmatrix} 3.6 & 3.8 & 5.0 \\ 0.4 & 0.4 & 0.6 \\ 0.8 & 0.8 & 1.2 \end{pmatrix} = \begin{pmatrix} 9.6 & 9.8 & 14 \end{pmatrix}$                                                                               | B1              |
| 3b    | Elements in C represent the taxi fare Bobby would have paid if he took a taxi from Epic, Toyo or Crystal respectively.                                                                                                                                         | B1              |
| 3ci   | $D = \begin{pmatrix} 1 & 9 & 10 \end{pmatrix}$                                                                                                                                                                                                                 | B1              |
| 3cii  | $DA = \begin{pmatrix} 1 & 9 & 10 \end{pmatrix} \begin{pmatrix} 3.6 & 3.8 & 5.0 \\ 0.4 & 0.4 & 0.6 \\ 0.8 & 0.8 & 1.2 \end{pmatrix} = \begin{pmatrix} 15.2 & 15.4 & 22.4 \end{pmatrix}$                                                                         | B1              |
| 3di   | $E = \begin{pmatrix} 1.3 & 0 & 0 \\ 0 & 1.2 & 0 \\ 0 & 0 & 1.1 \end{pmatrix}$                                                                                                                                                                                  | B1              |
| 3dii  | $DAE = \begin{pmatrix} 15.2 & 15.4 & 22.4 \end{pmatrix} \begin{pmatrix} 1.3 & 0 & 0 \\ 0 & 1.2 & 0 \\ 0 & 0 & 1.1 \end{pmatrix}$ $= \begin{pmatrix} 19.76 & 18.48 & 24.64 \end{pmatrix}$                                                                       |                 |
| 3e    | Epic will be the most economical during non-peak hour and Toyo most economical during peak hours as their prices are cheapest during the respective hours                                                                                                      | B1              |
| 4a    | $3x + 5y = 5 \quad \text{--- (1)}$<br>$\frac{1}{2}x = \frac{1}{3}y + 2 \quad \text{--- (2)}$<br>$(2) \times 6: 3x = 2y + 12 \quad \text{--- (3)}$<br>Sub (3) to (1):<br>$2y + 12 + 5y = 5$<br>$7y = -7$<br>$y = -1$<br>$3x = 2(-1) + 12$<br>$x = 3\frac{1}{3}$ | M1<br>A1<br>A1  |
| 4bi   | $xt + (x + 3)t = 400$<br>$t = \frac{400}{2x + 3}$                                                                                                                                                                                                              | A1              |
| 4bii  | $xt + (x - 4)t = 400$<br>$t = \frac{400}{2x - 4}$                                                                                                                                                                                                              |                 |

| Qn. # | Solution                                                                                                                                                                                               | Mark Allocation |
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| 4biii | $\frac{400}{2x-4} - \frac{400}{2x+3} = 23\frac{1}{3}$ $1200 + 1600 = \frac{70}{3}(4x^2 - 2x - 12)$ $280x^2 - 140x - 9240 = 0$ $2x^2 - x - 66 = 0$                                                      | M1<br>M1<br>A1  |
| 4biv  | $2x^2 - x - 66 = 0$ $(2x+11)(x-6) = 0$ $x = -5.5 \text{ or } x = 6$                                                                                                                                    | M1<br>A2        |
| 4bv   | $\text{Time taken} = \frac{400}{6} = 66.7 \text{ s}$                                                                                                                                                   | B1              |
| 5a    | $BD^2 = 45^2 + 21^2 - 2(45)(21)\cos 32$ $BD = 29.38008$ $BD = 29.4 \text{ m}$                                                                                                                          | M2<br>A1        |
| 5b    | $\tan(\text{angle of depression}) = \frac{9}{29.38008}$ $\text{Angle of depression} = 17.03140 = 17.0^\circ$                                                                                           | M1<br>A1        |
| 5c    | $\frac{\sin \angle ADB}{45} = \frac{\sin 32}{29.38008}$ $\sin \angle ADB = \frac{45 \sin 32}{29.38008}$ $\angle ADB = 54.2575 \text{ (N.A.) or } 180 - 54.2575$ $\angle ADB = 125.74 \text{ (2 d.p.)}$ | M1<br>A1        |
| 5d    | $\text{Area } BCD$ $= \frac{1}{2} \times 17 \times 29.38008 \times \sin(180 - 32 - 125.74)$ $= 94.6052$ $= 94.6 \text{ m}^2$                                                                           | M1<br>A1        |
| 5e    | $DJ = 21 \sin 32 = 11.12830$ $JB = 45 - 21 \cos 32$ $JF = \sqrt{(45 - 21 \cos 32)^2 + 9^2}$ $= 28.64175$ $\tan \angle DFJ = \frac{11.12839}{28.64175}$ $\angle DFJ = 21.2^\circ$                       | M1<br>M1<br>A1  |
| 6ai   | $ \overrightarrow{AC}  = \sqrt{(4+3)^2 + (6-5)^2}$ $= 7.07 \text{ units}$                                                                                                                              | M1<br>A1        |
| 6aii  | $\overrightarrow{AC} = \begin{pmatrix} -7 \\ -1 \end{pmatrix}$                                                                                                                                         | B1              |

| Qn. # | Solution                                                                                                                                                | Mark Allocation                          |
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| 6aiii | $\begin{pmatrix} 4 \\ 6 \end{pmatrix} = m \begin{pmatrix} 6 \\ k \end{pmatrix}$ $4 = 6m$ $m = \frac{2}{3}$ $6 = \frac{2}{3}k$ $k = 9$                   | M1<br><br><br><br><br><br><br><br><br>A1 |
| 6bia  | $\overrightarrow{AB} = \mathbf{b} - \mathbf{a}$                                                                                                         | B1                                       |
| 6bib  | $\overrightarrow{OD} = \mathbf{a} + \frac{2}{5}(\mathbf{b} - \mathbf{a})$ $= \frac{1}{5}(3\mathbf{a} + 2\mathbf{b})$                                    | B1                                       |
| 6bic  | $\overrightarrow{AC} = -\mathbf{a} + \frac{5}{3} \times \frac{1}{5}(3\mathbf{a} + 2\mathbf{b})$ $= \frac{2}{3}\mathbf{b}$                               | M1<br><br>A1                             |
| 6bii  | $OB \parallel AC$ $OB = \frac{3}{2}AC$                                                                                                                  | B1                                       |
| 6biii | $\triangle OAD : \triangle ADC : \triangle CBD$ $3 : 2$ $: 2 : 3$ $3 : 2 : 3$ $\frac{\text{area of } \triangle OAD}{\text{area of } \triangle CBD} = 1$ | M1<br><br><br><br>A1                     |

| Qn. # | Solution                                                                                                                                                                                                                                              | Mark Allocation         |
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| 7a    | $a = 0.5$                                                                                                                                                                                                                                             | B1                      |
| 7b    |  <p>           B1 – Correct axes and scale<br/>           B1 – Correct points plotted<br/>           B1 – Smooth curve         </p>                                 |                         |
| 7c    | Draw tangent<br>Gradient = $-7.5 \pm 1.5$                                                                                                                                                                                                             | M1<br>A1                |
| 7d    | $10x^2 - 8x + 1 = 0$<br>$5x - 4 + \frac{1}{2x} = 0$<br>$5x - 3 + \frac{1}{2x} = 1$<br>Draw $y = 1$<br>$x = 0.155 \pm 0.01$ or $x = 0.64 \pm 0.01$                                                                                                     | M1<br>A1                |
| 7e    | At $x = 0.33$ , tangent passes through origin<br>Gradient = $0.409 \pm 0.2$<br>$k = 0.409 \pm 0.15$                                                                                                                                                   | M1 – Draw tangent<br>A1 |
| 8ai   | $\angle BOE = 2 \times 70 = 140^\circ$<br>(angle at centre is twice angle at circumference)                                                                                                                                                           | B1                      |
| 8aii  | $\angle BDE = 180 - 140 = 40^\circ$ (opp. angles of cyclic quad)                                                                                                                                                                                      | B1                      |
| 8aiii | $\angle ABD = 180 - 70 - 40 = 70^\circ$ (angle sum of triangle)                                                                                                                                                                                       | B1                      |
| 8aiv  | $\angle CBD = 180 - 70 = 110^\circ$ (angles on a straight line)<br>$\angle CED = 110^\circ$ (angles in same segment)                                                                                                                                  | M1<br>A1                |
| 8av   | $\angle DFC = 40 + 110 = 150^\circ$ (ext. angle of a triangle)                                                                                                                                                                                        | B1                      |
| 8b    | Any point $X$ on the same side of $BE$ as $A$ , lying on the circumference of the circle will form an angle $\angle BXE = 70^\circ$ (angles in same segment). Since $\angle BGE = 75^\circ$ which is larger than $70^\circ$ , $G$ lies in the circle. | B1                      |

| Qn. # | Solution                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                       | Mark Allocation                                                           |
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| 9ai   | $40 < x \leq 60$                                                                                                                                                                                                                                                                                                                                                                                                                                                                                               | B1                                                                        |
| 9aia  | Mean mark = $\frac{11620}{200} = 58.1$ marks                                                                                                                                                                                                                                                                                                                                                                                                                                                                   | B1                                                                        |
| 9aiib | Standard deviation = $\sqrt{\frac{765600}{200} - \left(\frac{11620}{200}\right)^2}$<br>= 21.3 marks                                                                                                                                                                                                                                                                                                                                                                                                            | M1<br>A1                                                                  |
| 9bi   | $0.9 \times 0.8 = 0.72$                                                                                                                                                                                                                                                                                                                                                                                                                                                                                        | B1                                                                        |
| 9biia | $0.28 \times 0.72 = 0.2016$                                                                                                                                                                                                                                                                                                                                                                                                                                                                                    | B1                                                                        |
| 9biib | $0.72 + 0.28 \times 0.72 = 0.9216$                                                                                                                                                                                                                                                                                                                                                                                                                                                                             | B1                                                                        |
| 9biic | $0.28^3 = 0.021952$                                                                                                                                                                                                                                                                                                                                                                                                                                                                                            | B1                                                                        |
| 9biid | $1 - 0.021952 = 0.978048$                                                                                                                                                                                                                                                                                                                                                                                                                                                                                      | B1                                                                        |
| 9biia | $0.28^n$                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                       | B1                                                                        |
| 9biib | $1 - 0.28^n$                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                   | B1                                                                        |
| 10a   | <p>Let the height of imaginary cone above frustum be <math>y</math></p> $\frac{y}{2} = \frac{y+20}{3}$ $y = 40$ <p>Volume of frustum</p> $= \frac{1}{3} \times \pi \times (3)^2 (60) - \frac{1}{3} \times \pi \times (2)^2 (40)$ $= 397.93507$ <p>Volume of hemisphere</p> $= \frac{2}{3} \times \pi \times (2)^3$ $= 16.75516$ <p>Total volume = <math>397.93507 + 16.75516</math></p> $= 414.69023$ $= 415 \text{ cm}^3$  | M1<br>M1<br>M1<br>A1                                                      |
| 10b   | <p>Curved surface area of frustum</p> $\pi \times (3)(\sqrt{3^2 + 60^2}) - \pi \times (2)(\sqrt{2^2 + 40^2})$ $= 314.55172$ <p>Total curved surface area</p> $= 2\pi(2)^2 + 314.55172$ $= 339.68446$ $= 340 \text{ cm}^2$                                                                                                                                                                                                                                                                                      | M1 – Pythagoras<br>M1 – large curved surface – small curved surface<br>A1 |
| 10c   | <p>Curved area of bottom part</p> $= \pi \times (3)(\sqrt{3^2 + 60^2}) - \left(\frac{60-x}{60}\right)^2 \left[\pi \times (3)(\sqrt{3^2 + 60^2})\right]$ $= 18.87310x - 0.15728x^2$ <p>Curved area of bottom part = <math>339.68446 \div 2</math></p>                                                                                                                                                                                                                                                           | M1 – Area of top or bottom in terms of a variable<br>M1 – Equate to half  |

| Qn. # | Solution                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                        | Mark Allocation                                                                                           |
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|       | $18.87310x - 0.15728x^2 = 169.84223$<br>$0.15728x^2 - 18.87310x + 169.84223 = 0$<br>Using quadratic formula,<br>$x = 110.19739$ or $9.79943$ cm<br>$x = 9.80$                                                                                                                                                                                                                                                                                                                                                                                                   | of earlier area found<br>M1 – quadratic formula<br>A1                                                     |
| 10c   | Curved area of bottom part<br>$= \pi \times (3)(\sqrt{3^2 + 60^2}) - \left(\frac{60-x}{60}\right)^2 \left[\pi \times (3)(\sqrt{3^2 + 60^2})\right]$<br>$= 18.87310x - 0.15728x^2$<br>Curved area of top part<br>$= 2\pi \times (2)^2 + (314.55172 - (18.87310x - 0.15728x^2))$<br>$= 0.15728x^2 - 18.87310x + 339.68446$<br>Curved area of top = Curved area of bottom<br>$0.15728x^2 - 18.87310x + 339.68446 = 18.87310x - 0.15728x^2$<br>$0.31456x^2 - 37.7462x + 339.68446 = 0$<br>Using quadratic formula,<br>$x = 110.19739$ or $9.79943$ cm<br>$x = 9.80$ | M1<br><br><br><br><br><br><br><br><br><br><br>M1<br><br><br><br><br><br><br><br><br><br><br>M1<br>°<br>A1 |