6.

Candidate's Name	Class	Register Number

BALESTIER HILL SECONDARY SCHOOL
PRELIMINARY EXAMINATION 2016
SECONDARY 4 EXPRESS

MATHEMATICS
Paper 1 **4048/01**

Thursday **18 August 2016** **2 hours**

Additional Materials:
Nil

READ THESE INSTRUCTIONS FIRST

Write your name, class and register number on all the work you hand in.
Write in dark blue or black pen.
You may use a pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** questions.
If working is needed for any question it must be shown in the space below that question.
Omission of essential working will result in loss of marks.
You are expected to use a scientific calculator to evaluate explicit numerical expressions.
If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures.
Give answers in degrees to one decimal place.
For π, use either your calculator value or 3.142, unless the question requires the answer in terms of π.

The number of marks is given in brackets [ ] at the end of each question or part question.
The total of the marks for this paper is 80.

For Examiner's use:
/80

For Examiner's Use

Mathematical Formulae

For Examiner's Use

Compound interest

$$\text{Total amount} = P\left(1 + \frac{r}{100}\right)^n$$

Mensuration

Curved surface area of a cone = $\pi r l$

Surface area of a sphere = $4\pi r^2$

$$\text{Volume of a cone} = \frac{1}{3}\pi r^2 h$$

$$\text{Volume of a sphere} = \frac{4}{3}\pi r^3$$

$$\text{Area of triangle } ABC = \frac{1}{2}ab\sin C$$

Arc length = $r\theta$, where θ is in radians

$$\text{Sector area} = \frac{1}{2}r^2\theta, \text{ where } \theta \text{ is in radians}$$

Trigonometry

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

Statistics

$$\text{Mean} = \frac{\Sigma f x}{\Sigma f}$$

$$\text{Standard deviation} = \sqrt{\frac{\Sigma f x^2}{\Sigma f} - \left(\frac{\Sigma f x}{\Sigma f}\right)^2}$$

For Examiner's Use

Answer all the questions.

For Examiner's Use

1 **(a)** Evaluate $\frac{-2a^2+0.5b}{3\sqrt{a+c}}$, where $a=-2.13$, $b=\frac{42}{5}$, and $c=3.8\pi$. Write down the first 6 digits on your calculator display.

Answer *(a)* _____ [2]

(b) Write your answer to part (a) correct to 3 significant figures.

Answer *(b)* _____ [1]

2 **(a)** Given that $90=2\times3^2\times5$, find the smallest integer n such that $90n$ is a perfect square.

Answer *(a)* _____ [1]

(b) The HCF and LCM of 2 numbers are 90 and 3780 respectively. Find the 2 numbers, given that they are not 90 nor 3780.

Answer *(b)* _____ and _____ [3]

For Examiner's Use

3 Factorise the following expressions completely.

(a) $4x^2 - 21x - 18$

Answer (a) _____ [2]

(b) $6a - 5b + 10ab - 3$

Answer (b) _____ [2]

(c) $x^2 + 2x - y^2 + 2y$

Answer (c) _____ [3]

For Examiner's Use

For Examiner's Use

4 3 interior angles of a polygon are 139°, 147°, and 164°. The remaining interior angles are $x°$. Given that the sum of the interior angles is 2880°, find the value of x.

Answer _____ ° [3]

For Examiner's Use

5 Jason invests $30 000 into bonds that give a return of 4.8% per annum compound interest. Find the amount of interest accrued at the end of 4 years.

Answer $ _____ [3]

6 Dany buys a $25 000 sailboat via hire purchase. She pays a 35% downpayment and the remaining amount by monthly installments for 5 years. If the interest rate is 6% per annum, calculate her monthly installment.

Answer $ _____ [3]

For Examiner's Use

7 (a) The temperature at a particular town is x°C at 0800. It rises at a constant rate and reaches y°C at 1500. Find an expression in terms of x and y for the temperature at 1230.

Answer (a) _____ °C [2]

(b) Beatrix bought a apples at b dollars per dozen. She sold all the apples at c cents each. Find an expression in terms of a, b, and c for her profit in dollars.

Answer (b) $_____ [2]

8 A beaker contains 7.2 cm^3 of chemical A and 12 cm^3 of chemical B. Find the ratio of chemical A : chemical B in its simplest form.

Answer _____ [2]

For Examiner's Use

9 The table below shows the charges for Uber and Comfort Taxi services in Singapore. Which service is cheaper for a 21 km journey at 8 pm on a weekday, where the total travelling time is 25 minutes, inclusive of 4 minutes of waiting time? Show your working clearly.

For Examiner's Use

	Uber	Comfort
Base Cost	$3.00	$3.20
Distance Cost	$0.50 per km	$0.22 every 400 m (up to 10 km) $0.22 every 350 m (after 10 km)
Time Cost	$0.25 per minute	$0.22 every 45 s waiting time
Peak Hour Surcharge (between 6 pm and midnight)	Nil	25% of metered fare

Answer _____ [4]

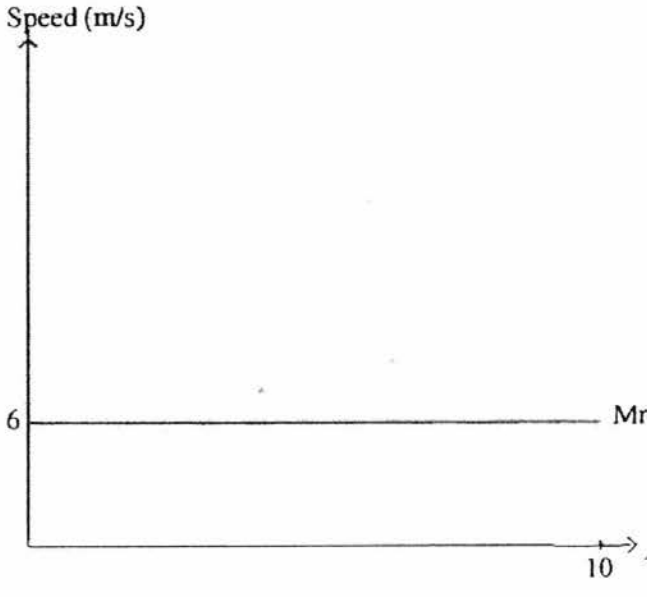
For Examiner's Use

10 A *T-rex* spots Mr Teo 60 m away and attacks, accelerating at 4 m/s^2 to its top speed of 25 m/s. At the same time, Mr Teo flees in the opposite direction, quickly reaching his top speed of 6 m/s.

For Examiner's Use

(a) Sketch the velocity-time graph of the *T-rex* for $0 \leq t \leq 10$ in the answer space below.

Answer (a)



[2]

(b) Find the distance Mr Teo can travel before he is caught and eaten.

Answer (b) _____ m [4]

For Examiner's Use

11 Make c the subject of the formula $a = \sqrt{\frac{a+c^2}{b}}$

For Examiner's Use

Answer _____ [3]

12 **(a)** Solve the inequality $\frac{x}{3}+2<\frac{3x+5}{2}\leq 7-x$.

Answer *(a)* _____ [2]

(b) Represent your answer in **(a)** on the number line in the space below.

Answer *(b)*

[1]

For Examiner's Use

13 Given that $x^2 - 8x + 3$ can be written in the form $(x-a)^2 + b$, find the values of a and b.

Answer $a =$ _____ , $b =$ _____ [2]

For Examiner's Use

14 It is given that y is inversely proportional to the cube root of x. Find the percentage decrease in y when x is increased by 500%.

Answer _____ % [3]

15 The mean, median, and modal height of 4 men are 177 cm, 176 cm, and 172 cm respectively. Write down the heights of each man in ascending order.

Answer _____ , _____ , _____ , _____ [2]

For Examiner's Use

16 The line $2y + 5x - 10 = 0$ intersects the x-axis and the y-axis at points A and B respectively.

For Examiner's Use

(a) Find the coordinates of A and B.

Answer (a) $A($ _____ , _____ $)$ [1]

$B($ _____ , _____ $)$ [1]

(b) Write down the gradient of the line AB.

Answer (b) _____ [1]

(c) Find the length of the line AB.

Answer (c) _____ units [1]

(d) A point C lies on the x-axis such that the area of triangle $ABC = 10$ units^2. Write down the x-coordinates of 2 possible positions of point C.

Answer (d) $x =$ _____ or _____ [2]

For Examiner's Use

17 Solve the following pair of simultaneous equations.

$$3x + 5y = -9$$
$$2x - 9y + 8 = 39$$

For Examiner's Use

Answer $x =$ _____

$y =$ _____ [3]

For Examiner's Use

18 Consider the number pattern below.

3, 7, 11, 15, 19, ...

(a) Write down an expression for the n^{th} term.

Answer (a) _____ [1]

(b) Hence, evaluate the 75^{th} term.

Answer (b) _____ [1]

(c) Explain clearly why 240 is not a term in the pattern.

Answer (c) _____

_____

_____ [1]

(d) Consider another number pattern below.

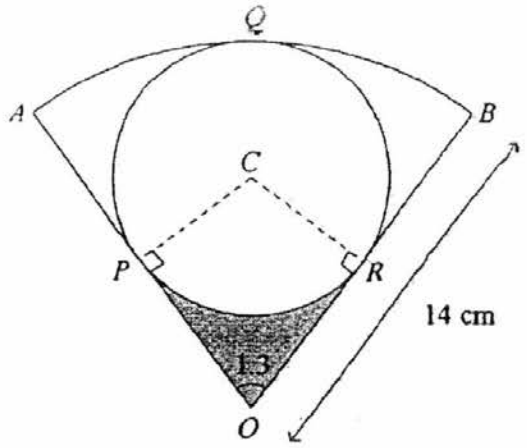
6, 21, 44, 75, 114, ...

By comparing the 2 number patterns, find an expression for the n^{th} term of the 2^{nd} pattern.

Answer (d) _____ [2]

For Examiner's Use

19 In the diagram, $OAQB$ is the sector of a circle with radius 14 cm and angle $AOB = 1.3$ rad. C is the centre of a circle PQR inscribed in the sector.



(a) Show that the radius of the circle PQR is 5.28 cm.

Answer (a)

[2]

(b) Find the perimeter of the shaded region.

Answer (b) _____ cm [3]

(c) Find the area of the shaded region.

Answer (c) _____ cm^2 [3]

For Examiner's Use

20 (a) Construct a triangle ABC such that $AB = 8$ cm, $AC = 11$ cm, and angle $ABC = 75°$. The line AB has been drawn for you.

Answer (a) [2]

For Examiner's Use

(b) By constructing 2 perpendicular bisectors, draw a circle that circumscribes points A, B, and C. Label the centre of the circle O.

Answer (b) *On diagram* [3]

(c) Measure and write down the radius of the circle.

Answer (c) _____ cm [1]

End of Paper

For Examiner's Use

For Examiner's Use

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4E MA PRELIM EXAM 2016 – PAPER 1

1(a)

$$\frac{-2a^2+0.5b}{3\sqrt{a+c}} = \frac{-2(-2.13)^2+0.5\left(\frac{42}{5}\right)}{3\sqrt{-2.13+3.8\pi}} \quad \text{-- M1}$$

$$= -0.51874 \quad \text{-- A1}$$

(or B2)

1(b) -0.519 -- B1

2(a) 10 -- B1

2(b) $3780 = 2^2 \times 3^3 \times 5 \times 7$ -- M1

The numbers are: 180 and 1890 -- A1, A1 (OE)
270 and 1260
540 and 630

3(a) $4x^2 - 21x - 18 = (4x+3)(x-6)$ -- B1, B1

3(b)

$$6a - 5b + 10ab - 3 = 3(2a-1) + 5b(2a-1) \quad \text{-- B1, B1 (OE)}$$

$$= (2a-1)(3+5b)$$

3(c)

$$x^2 + 2x - y^2 + 2y = (x+y)(x-y) + 2(x+y) \quad \text{-- M1, M1}$$

$$= (x+y)(x-y+2) \quad \text{-- A1}$$

4

$$(n-2)(180) = 2880$$

$$n = 18 \quad \text{-- M1}$$

$$139^\circ + 147^\circ + 164^\circ + 15x = 2880^\circ \quad \text{-- M1}$$

$$x = 162^\circ \quad \text{-- A1}$$

5

$$\text{Amount} = \$30000\left(1+\frac{4.8}{100}\right)^4 \quad \text{-- M1}$$

$$= \$36188.15$$

$$\text{Interest} = \$36188.15 - \$30000 \quad \text{-- M1}$$

$$= \$6188.15 \quad \text{-- A1}$$

6

$$\text{Balance} = 0.65 \times \$25000$$

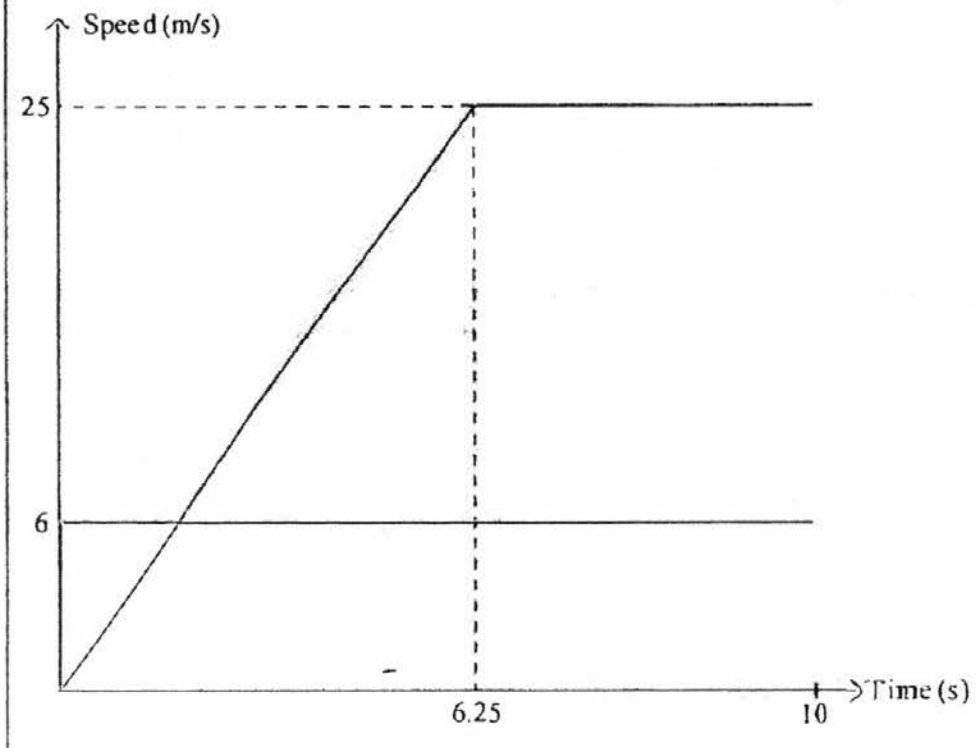
$$= \$16250 \quad \text{-- M1}$$

$$\text{Interest} = \frac{(16250)(6)(5)}{100}$$

$$= \$4875 \quad \text{-- M1}$$

$$\text{Installment} = \frac{16250+4875}{(5)(12)}$$

$$= \$352.08 \quad \text{-- A1}$$

7(a)	$\text{rate} = \frac{y-x}{7}$ – M1 $\text{temperature} = x + \left(\frac{y-x}{7}\right)(4.5)$ $= \frac{5x+9y}{14}$ – A1
7(b)	$\text{cost} = \frac{ab}{12}$ $\text{sales} = \frac{ac}{100}$ – M1 $\text{profit} = \frac{ac}{100} - \frac{ab}{12}$ – A1
8	7.2 : 12 72 : 120 -- M1 3 : 5 – A1 (or B2)
9	$\text{cost (uber)} = 3 + (0.5)(21) + (0.25)(25)$ -- M1 $= \$19.75$ $\text{cost (comfort)} = 1.25 \times \left[3.2 + (0.22)\left(\frac{10000}{400}\right) + (0.22)\left(\frac{11000}{350}\right) + (0.22)\left(\frac{(1)(60)}{45}\right)\right]$ -- M1 $= \$20.98$ -- M1 Uber -- A1
10(a)	

10(b)

dist by t-rex after 6.25s $= (0.5)(6.25)(25) = 78.125$ -- M1

dist from Mr Teo after 6.25s $= (60+(6)(6.25)) - 78.125 = 19.375$ -- M1

$$\text{time to capture} = 6.25 + \frac{19.375}{25-6} = 7\frac{41}{152} \quad \text{-- M1}$$

$$\text{total dist by Mr Teo} = 6 \times 7\frac{41}{152} = 43\frac{47}{76} \quad \text{-- A1}$$

11

$$a = \sqrt{\frac{a+c^2}{b}}$$

$$a^2 = \frac{a+c^2}{b} \quad \text{-- M1}$$

$$a^2b - a = c^2 \quad \text{-- M1}$$

$$c = \pm\sqrt{a^2b-a} \quad \text{-- A1}$$

12(a)

$$\frac{x}{3}+2 < \frac{3x+5}{2} \le 7-x$$

$$\frac{x}{3}+2 < \frac{3x+5}{2} \text{ AND } \frac{3x+5}{2} \le 7-x$$

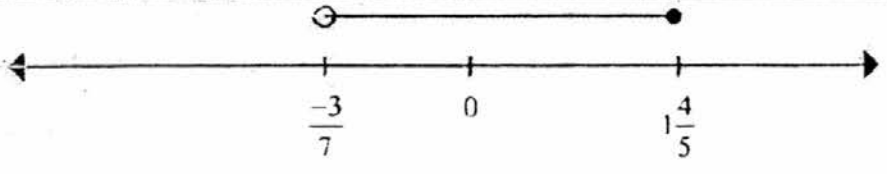
$$2x+12 < 9x+15 \text{ AND } 3x+5 \le 14-2x$$

$$7x > -3 \text{ AND } 5x \le 9$$

$$x > \frac{-3}{7} \text{ AND } x \le 1\frac{4}{5} \quad \text{-- B1, B1}$$

$$\frac{-3}{7} < x \le 1\frac{4}{5}$$

12(b)

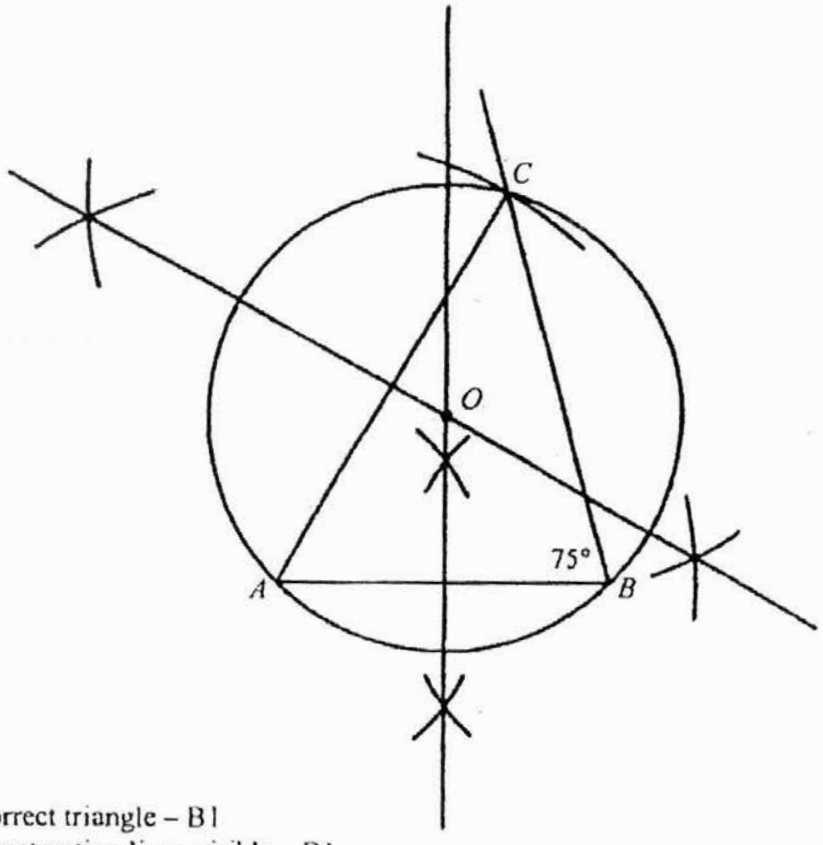


13

$a = 4$ -- B1

$b = -13$ -- B1

14	$y=\frac{k}{\sqrt[3]{x}}$ when $x=a, y=b$ $b=\frac{k}{\sqrt[3]{a}}$ $k=b\sqrt[3]{a}$ – M1 $y=\frac{b\sqrt[3]{a}}{\sqrt[3]{x}}$ when $x=6a$ $y=\frac{b\sqrt[3]{a}}{\sqrt[3]{6a}}$ $=\frac{b}{\sqrt[3]{6}}$ --M1 $\%\text{ decrease}=\frac{b-\frac{b}{\sqrt[3]{6}}}{b}\times 100$ $=45.0\%$ –A1
15	172, 172, 180, 184 -- B2
16(a)	$A(2,0)$ -- B1 $B(0,5)$ -- B1
16(b)	–2.5 -- B1
16(c)	5.39 -- B1
16(d)	6 or –2 -- B1, B1
17	$3x+5y=-9$ $2x-9y+8=39$ $2\left(\frac{-9-5y}{3}\right)-9y+8=39$ -- M1 $-18-10y-27y+24=117$ $-37y=111$ $y=-3$ – A1 $x=\frac{-9-5(-3)}{3}$ $=2$ -- A1
18(a)	$4n-1$ -- B1
18(b)	$4(75)-1=299$ -- B1
18(c)	Because $240+1$ is not a multiple of 4. ----- B1

18(d)	$(4n-1)(n+1)$ -- M1 $= 4n^2 + 3n - 1$ -- A1 (or 921)
19(a)	$\sin 0.65 = \frac{r}{14-r}$ -- M1 $r = 5.28$ -- A1
19(b)	$OP = OR = \frac{5.28}{\tan 0.65}$ -- M1 $P\hat{C}R = \pi - 1.3$ -- M1 perimeter $= (5.28)(\pi - 1.3) + 2\left(\frac{5.28}{\tan 0.65}\right)$ $= 23.6$ -- A1
19(c)	$OPCR = (2)(0.5)\left(\frac{5.28}{\tan 0.65}\right)(5.28) = 36.67224$ -- M1 $CPR = (0.5)(5.28)^2(\pi - 1.3) = 25.670328$ -- M1 shaded area $= 36.67224 - 25.670328 = 11.0$ -- A1
20(a)	 Correct triangle – B1 Construction lines visible – B1
(b)	2 perpendicular bisectors on diagram with construction lines – B1, B1 Correct circle drawn and centre of circle labelled – B1
(c)	5.6 cm – B1

Candidate's Name	Class	Register Number

BALESTIER HILL SECONDARY SCHOOL
PRELIMINARY EXAMINATION 2016
SECONDARY FOUR EXPRESS

MATHEMATICS
Paper 2

4048/02

Monday **22 August 2016** **2 hours 30 minutes**

Additional materials:
Writing paper (6 sheets)
Graph paper (1 sheet)

READ THESE INSTRUCTIONS FIRST

Write your name, class and register number in the spaces provided at the top of this page, writing papers and graph paper.
Write in dark blue or black pen.
You may use a pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** questions.
If working is needed for any question it must be shown with the answer.
Omission of essential working will result in loss of marks.
You are expected to use a scientific calculator to evaluate explicit numerical expressions.
If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures.
Give answers in degrees to one decimal place.
For π, use either your calculator value or 3.142, unless the question requires the answer in terms of π.

The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 100.

For Examiner's Use
/ 100

Mathematical Formulae

Compound interest

$$\text{Total amount} = P(1+\frac{r}{100})^n$$

Mensuration

Curved surface area of a cone = $\pi r l$

Surface area of a sphere = $4\pi r^2$

$$\text{Volume of a cone} = \frac{1}{3}\pi r^2 h$$

$$\text{Volume of a sphere} = \frac{4}{3}\pi r^3$$

$$\text{Area of triangle } ABC = \frac{1}{2}ab\sin C$$

Arc length = $r\theta$, where θ is in radians

$$\text{Sector area} = \frac{1}{2}r^2\theta, \text{ where } \theta \text{ is in radians}$$

Trigonometry

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

Statistics

$$\text{Mean} = \frac{\Sigma fx}{\Sigma f}$$

$$\text{Standard deviation} = \sqrt{\frac{\Sigma fx^2}{\Sigma f} - \left(\frac{\Sigma fx}{\Sigma f}\right)^2}$$

ALL WORKINGS MUST BE SHOWN CLEARLY ON THE WRITING PAPER.

Answer all the questions.

1 **(a)** Simplify $\left(\frac{-3a^{1}}{2b^{4}}\right)^{2} \div \frac{6a^{-2}}{4b^{-3}}$, leaving your answer in positive index notation. [3]

(b) Express $\frac{x+1}{2x^2+x-6}+\frac{7}{3-2x}$ as a single fraction in its simplest form. [3]

(c) Solve the equation $\frac{x}{x-3}-\frac{1-4x}{2x+1}=0$, leaving your answers correct to 4 significant figures. [4]

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2 Rectangular tiles measuring x cm by $(x-3)$ cm are used to line the inside edge of a rectangular section of a wall measuring 360 cm by 300 cm. All the tiles are placed vertically, with the shorter side parallel to the length of the wall section.

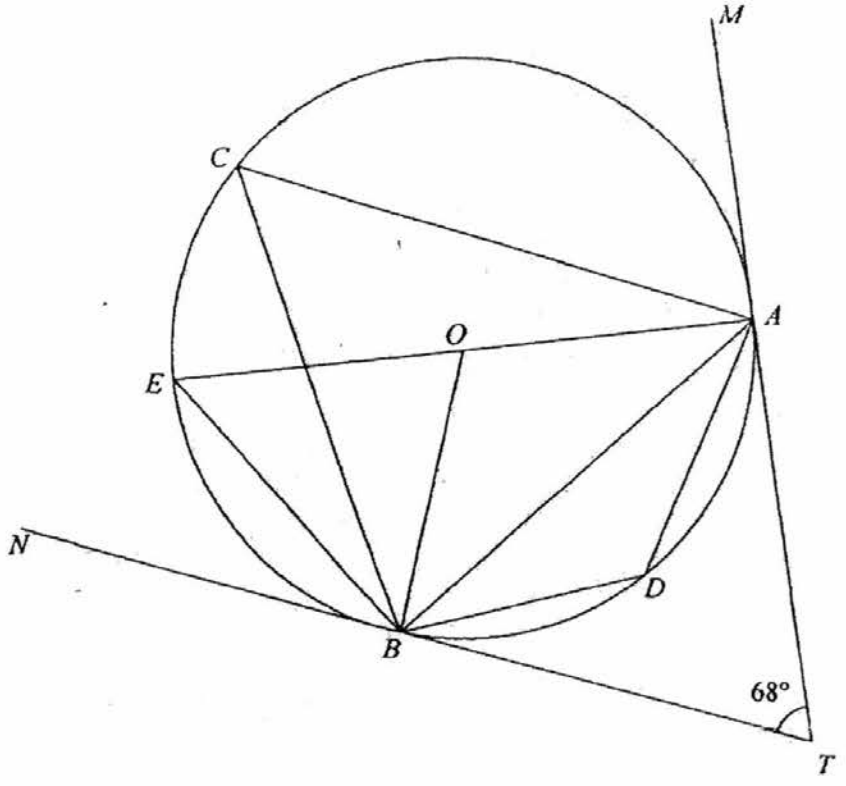
(a) Write down an expression in terms of x for the number of tiles that will fit across the length of the wall section. [1]

(b) Write down an expression in terms of x for the number of tiles that will fit across the width of the wall section. [1]

(c) Given that a total of 96 tiles are needed, form an equation and show that it reduces to $5x^2-81x+90=0$. [3]

(d) Solve the equation $5x^2-81x+90=0$. Explain why you would reject one of the answers and hence find the length of the shorter side of the tile. [4]

3 In the figure, TA and TB are tangents to a circle with centre O.



(a) Stating your reasons clearly, find

(i) angle BAT, [1]

(ii) angle AOB, [2]

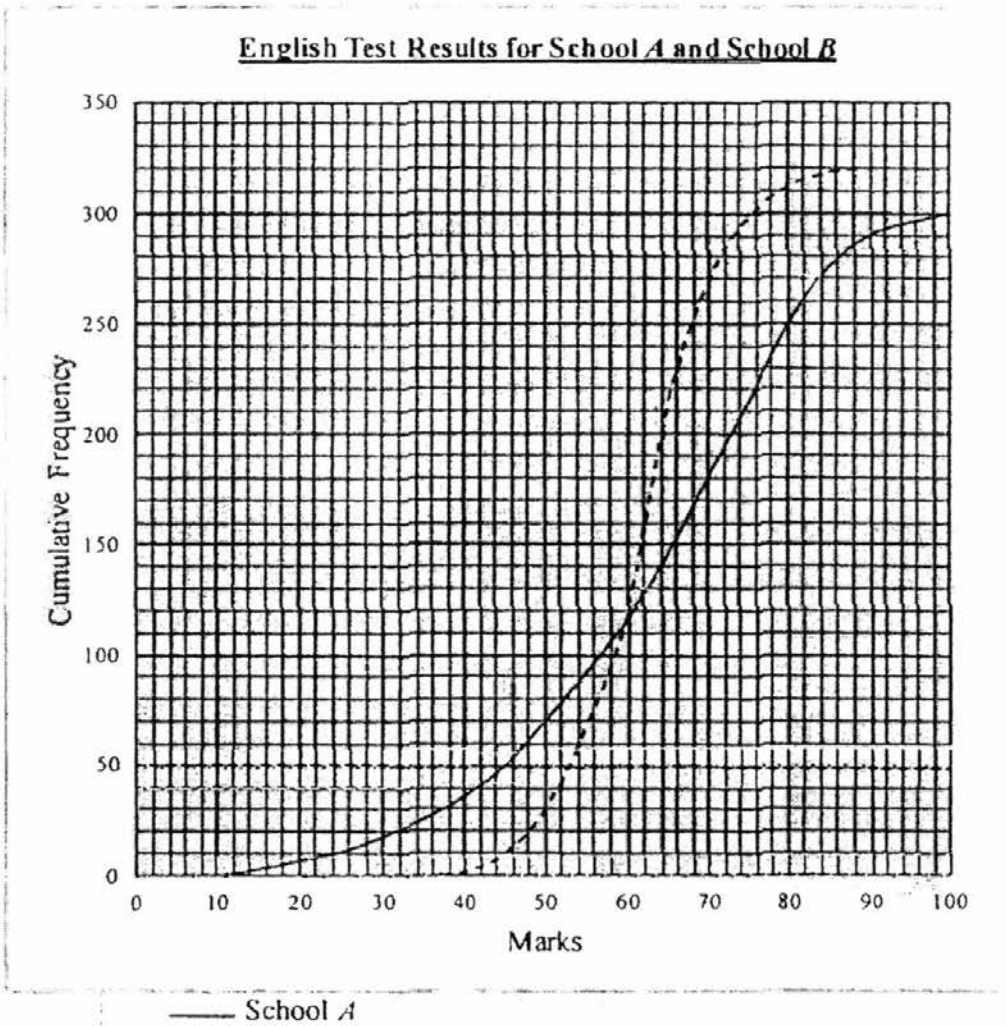
(iii) angle AEB, [1]

(iv) angle ADB, [1]

(v) angle EBN. [2]

(b) Find the radius of the circle given that the length of BT is 12 cm. [2]

4 300 students from School *A* and 320 students from School *B* took an English test. Their test results are recorded in the cumulative frequency graph below.



(a) Find the median mark for School *A* and School *B*. [2]

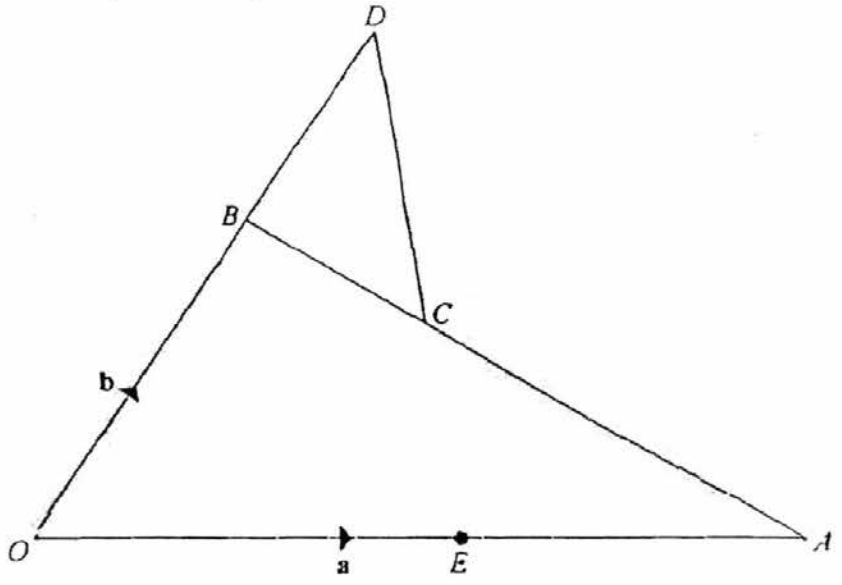
(b) Find the interquartile range for School *A* and School *B*. [2]

(c) Given that the passing mark is 50, find the percentage of students who passed the test in School *A*. [2]

(d) Find the value of x if 62.5% of the students from School *B* scored less than x marks. [2]

(e) Compare the results of the students from the 2 schools in 2 ways. [2]

5 In the diagram, OAB is a triangle and C is a point on AB such that $AC:CB=2:1$. The side OB is produced to point D such that $OB:BD=3:2$.



(a) Given that $\overrightarrow{OA}=\mathbf{a}$ and $\overrightarrow{OB}=\mathbf{b}$, express as simply as possible in terms of a and/or **b**,

(i) $\overrightarrow{AB}$, [1]

(ii) $\overrightarrow{AC}$, [1]

(iii) $\overrightarrow{OC}$, [1]

(iv) $\overrightarrow{OD}$. [1]

(b) Show that $\overrightarrow{CD}=\mathbf{b}-\frac{1}{3}\mathbf{a}$. [2]

(c) Given that E is a point on OA such that $\overrightarrow{OE}=\frac{5}{9}\mathbf{a}$, express $\overrightarrow{ED}$ in terms of a and b. [1]

(d) Write down 2 facts about the points C, D, and E. [2]

(e) Find

(i) $\dfrac{\text{area of triangle } AEC}{\text{area of triangle } OEC}$, [1]

(ii) $\dfrac{\text{area of triangle } AEC}{\text{area of triangle } OAB}$. [2]

6 **Answer the whole of this question on a sheet of graph paper.**

The variables x and y are connected by the equation $y = \frac{1}{2}\left(\frac{9}{x} + \frac{1}{x^2} - 3\right)$.

The table below shows some values of x and their corresponding y values.

x	1	1.5	2	2.5	3	4	5	6
y	3.5	1.72	0.875	0.38	0.055	a	−0.58	−0.74

(a) Find the value of a, correct to 2 decimal places. [1]

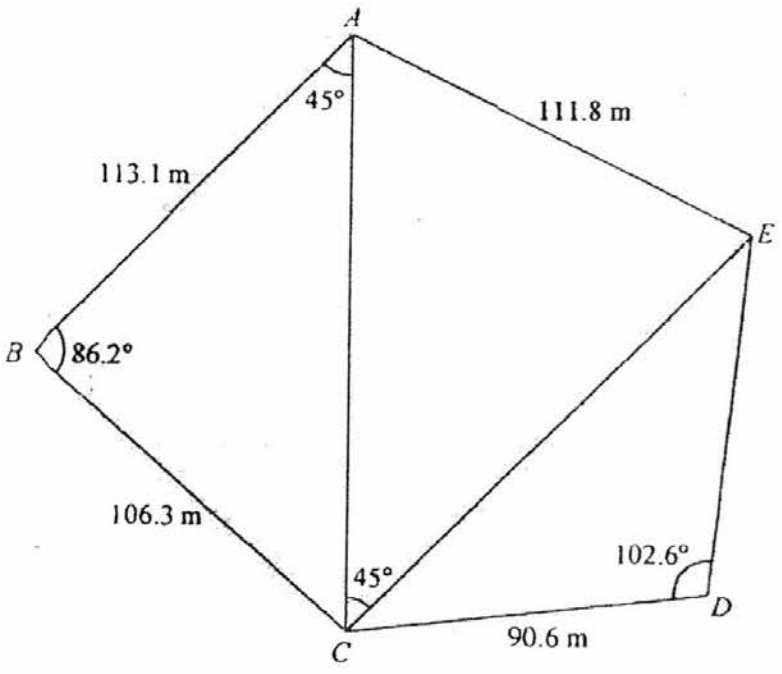
(b) Using a scale of 2 cm to represent 1 unit on the horizontal axis and 4 cm to represent 1 unit on the vertical axis, draw the graph of $y = \frac{1}{2}\left(\frac{9}{x} + \frac{1}{x^2} - 3\right)$ for $1 \le x \le 6$. [3]

(c) Using your graph, write down the solution of the equation $\frac{1}{2}\left(\frac{9}{x} + \frac{1}{x^2} - 3\right) = 0$ for $1 \le x \le 6$. [1]

(d) By drawing a tangent, find the gradient of the graph at $x = 2$. [2]

(e) By drawing an appropriate graph, find the range of values of x for which $\frac{9}{x} + \frac{1}{x^2} > 2x + 7$. [3]

7 The diagram below shows a field $ABCDE$ on level ground, where C is due south of A.



(a) Given that the bearing of E from A is 116.6°, and the bearing of D from E is 186.3°,

(i) show that angle $CED = 38.7°$. [1]

(ii) Find the length of CE. [1]

(iii) Find the total area of the field $ABCDE$. [4]

(b) John stands at D and flies a drone in a straight line to B. Given that the drone starts its flight from the ground at point D and ascends at 3 m/s at a 25° angle, find

(i) the length of BD, [3]

(ii) the height of the drone when it is directly above B, [2]

(iii) the flight time of the drone when it is directly above B, [1]

(iv) the speed of the drone. [2]

8 Choosing where to go for their tertiary education is an important choice for many students. In making their choices, students usually take into consideration several factors such as the location of the school, the quality of academic learning, and the facilities that the school has.

To help her choose which school to go to, Casey uses a scoring system to assign a value to each of the factors. After researching each school, she came up with the following table, scoring each factor from 1 to 5, with 5 being the best and 1 being the worst.

	Location	Academics	Facilities
Republic Poly	1	4	5
Singapore Poly	3	2	4
Temasek Poly	5	3	3

(a) Write down a 3×3 matrix **A** to represent the information given in the table. [1]

(b) Given that $B = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$, evaluate **AB** and state what each of the elements represents. [2]

(c) Casey does not think that all 3 factors are equally important. She assigns a weightage of 0.5 for the location, 0.3 for the academics, and 0.2 for the facilities. Write down a 3×1 matrix **C** to represent this information. [1]

(d) Hence, evaluate **AC** and state what each of the elements represents. [2]

(e) State the school that Casey will select based on her criteria. [1]

9 In a game of chance, the **expected value** is the probability-weighted average of all possible values. For example, in a game where the player randomly draws 1 of either a red or blue ball, if he wins \$2 for drawing the red ball and loses \$1 for drawing the blue ball, then the expected value of playing the game once is $\left(\frac{1}{2}\right)(2)+\left(\frac{1}{2}\right)(-1)=\frac{1}{2}$, meaning that, in the long run, after playing the game multiple times, the player can expect to win, on average, \$0.50.

Consider a game where you are given a bag with 2 black balls and 3 white balls. If you draw a black ball, you win \$1. If you draw a white ball, you lose \$1. The balls are drawn without replacement, and you may stop at any time you wish, after which the game is reset.

(a) Amelia plays the game and she decides to follow the strategy of quitting immediately if she does not draw the black ball in the first try.

(i) Write down the probability that she draws a black ball in the first try. [1]

(ii) Calculate the expected value for Amelia's strategy. [1]

(b) Benson plays the game and he decides to follow the strategy of continually drawing until he successfully draws 1 black ball.

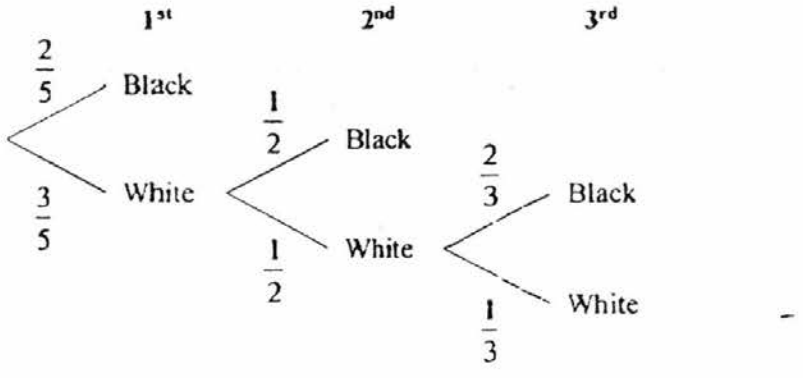
(i) Calculate the probability that Benson draws the black ball on the second try. [1]

(ii) Calculate the probability that Benson draws the black ball on the third try. [1]

(iii) Calculate the probability that Benson draws the black ball on the fourth try. [1]

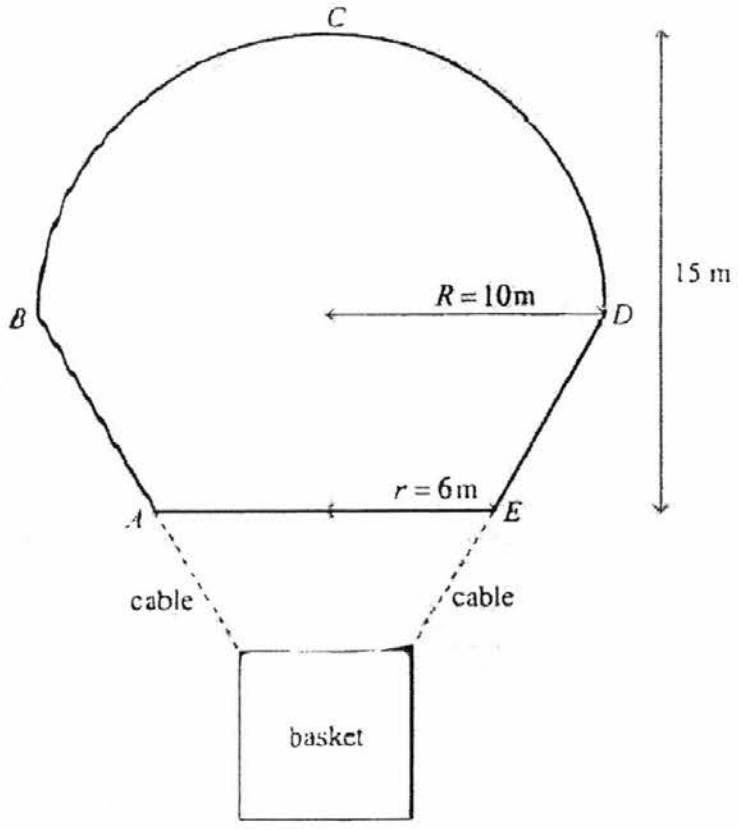
(iv) Calculate the expected value for Benson's strategy. [1]

(c) The optimal strategy for the game is to stop after drawing a black ball in the first or second try, or to stop after drawing 2 black balls, or to draw all the balls, whichever comes first. By completing the partial probability tree below, or otherwise, determine the expected value of this strategy. [2]



10 A hot air balloon consists of an envelop made of nylon with a basket suspended beneath. Heated air inside the envelope makes the balloon buoyant since the hot air has a lower density than the atmosphere outside the balloon.

The shape of the envelope can be modeled as a hemisphere on top of an open frustum. The diagram below shows the cross-section of the envelope *ABCDE*. The radius, *R*, of the hemisphere and cone is 10 m, the radius, *r*, of the narrow end of the frustum is 6 m, and the height of the envelope is 15 m.



(a) Calculate the volume of air inside the envelope. [4]

(b) By finding the surface area of the envelope, calculate the mass of the envelope in kg, given that the density of the nylon is 102 g/m^2. [3]

(c) The amount of lift generated by the hot air balloon depends on the difference in density between the heated air and the atmosphere, multiplied by the volume of the heated air. Given that the density of air in the atmosphere is 1.204 kg/m^3, and the density of air heated to 100°C is 0.949 kg/m^3, calculate the amount of lift generated by the hot air balloon in kilograms at 100°C. [2]

(d) Given that the rest of the equipment (cables, basket, fuel tanks, etc.) has a combined mass of 400 kg, find the maximum number of people that can ride the hot air balloon, if each person weighs 75 kg on average. [2]

End of Paper

4E MA PRELIM EXAM 2016 – PAPER 2

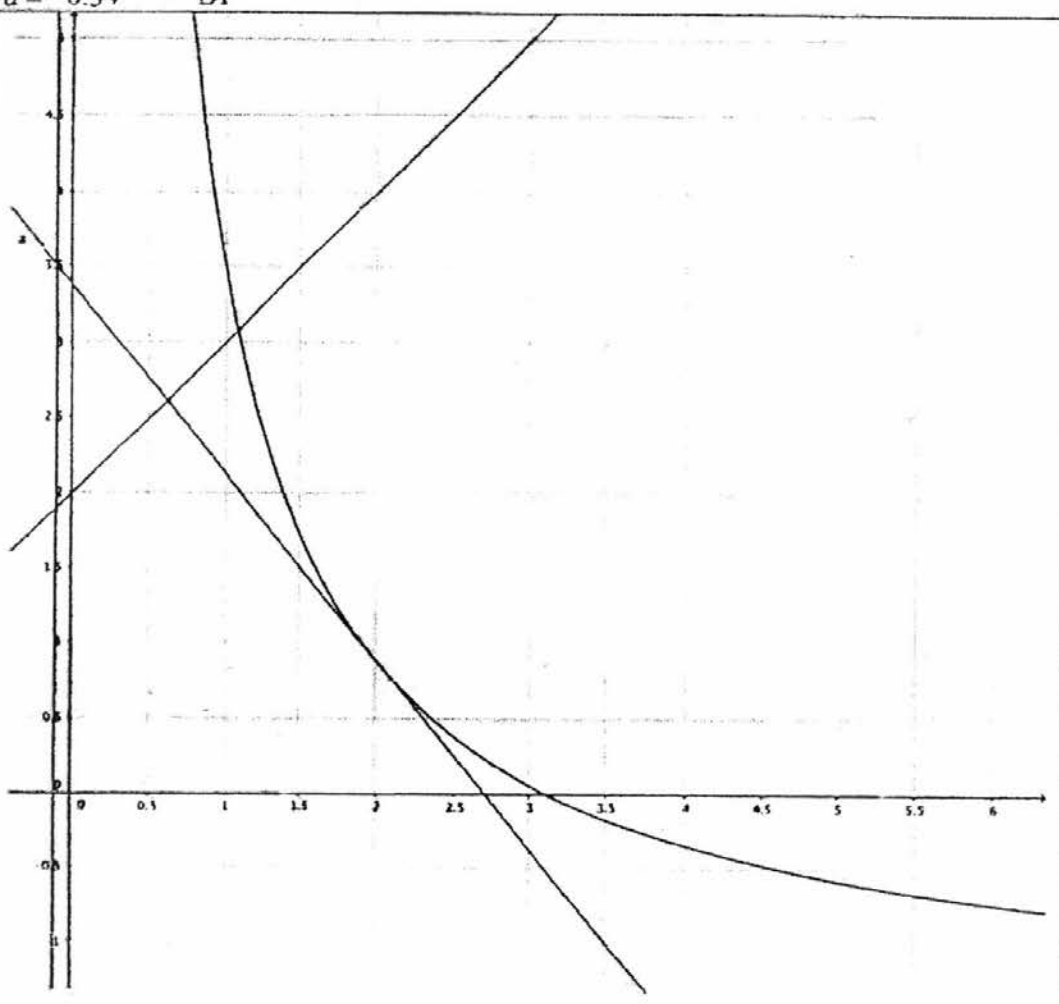
1(a)	$\left(\frac{-3a^3}{2b^4}\right)^2 \div \frac{6a^{-2}}{4b^{-3}} = \frac{9a^6}{4b^8} \div \frac{6a^{-2}}{4b^{-3}}$ -- M1 $= \frac{9a^6}{4b^8} \times \frac{4b^{-3}}{6a^{-2}}$ -- M1 $= \frac{36a^6b^{-3}}{24a^{-2}b^8}$ $= \frac{3a^8}{2b^{11}}$ -- A1	
1(b)	$\frac{x+1}{2x^2+x-6} + \frac{7}{3-2x} = \frac{x+1}{(x+2)(2x-3)} - \frac{7}{2x-3}$ -- M1 $= \frac{x+1-7(x+2)}{(x+2)(2x-3)}$ -- M1 $= \frac{-6x-13}{(x+2)(2x-3)}$ --A1	
1(c)	$\frac{x}{x-3} \quad \frac{1-4x}{2x+1} = 0$ $x(2x+1)-(1-4x)(x-3)=0$ -- M1 $2x^2+x-(x-3-4x^2+12x)=0$ $6x^2-12x+3=0$ -- M1 $x = \frac{-(-12) \pm \sqrt{(-12)^2-4(6)(3)}}{2(6)}$ $= 0.2929$ or -1.707 -- A1, A1	

2(a)	$\frac{360}{x-3}$ -- B1	
2(b)	$\frac{300}{x}$ -- B1	
2(c)	$2\left(\frac{360}{x-3}\right)+2\left(\frac{300}{x}\right)-4=96$ -- M1 $\frac{360}{x-3}+\frac{300}{x}=50$ $360x+300(x-3)=50(x)(x-3)$ -- M1 $360x+300x-900=50x^2-150x$ $50x^2-810x+900=0$ $5x^2-81x+90=0$ -- A1	
2(d)	$5x^2-81x+90=0$ $(x-15)(5x-6)=0$ -- M1 $x=15$ or $1\frac{1}{5}$ -- A1 Reject $1\frac{1}{5}$ since $(x-3)$ cannot be negative. -- B1 Length of shorter side = 12 cm -- B1	

3(ai)	$B\hat{A}T = \frac{1}{2}(180° - 68°) = 56°$ (isoceles triangle) -- B1	
3(aii)	$O\hat{A}T = O\hat{B}T = 90°$ (tangent ⊥ radius) -- M1 $A\hat{O}B = 360° - 90° - 90° - 68° = 112°$ (∠ sum of triangle) -- A1	
3(aiii)	$A\hat{E}B = \frac{1}{2}(112°) = 56°$ (∠ at centre = 2∠ at circumference) -- B1	
3(aiv)	$A\hat{D}B = 180° - 56° = 124°$ (cyclic quadrilateral) -- B1	
3(av)	$A\hat{B}E = 90°$ (∠ in semicircle) -- M1 $E\hat{B}N = 180° - 90° - 56° = 34°$ (∠s on a straight line) --- A1	
3(b)	$\tan 34° = \frac{\text{radius}}{12}$ -- M1 radius $= 12\tan 34° = 8.09$ cm -- A1	

4(a)	School A: 66 marks -- B1 School B: 62 marks -- B1	
4(b)	School A: $77-51=26$ marks -- B1 School B: $67-56=11$ marks -- B1	
4(c)	# of students $=300-70=230$ -- M1 percentage $=\frac{230}{300}\times 100\%=76\frac{2}{3}\%$ -- A1	
4(d)	# of students $=62.5\%\times 320=200$ -- M1 $x=64$ marks -- A1	
4(e)	Students from School A performed better on average because their median score is higher. -- B1 The results of students from School B is less spread because their interquartile range is lower. -- B1	

5(ai)	$\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA}$ $= \underset{\sim}{b} - \underset{\sim}{a}$ -- B1	
5(aii)	$\overrightarrow{AC} = \frac{2}{3}\overrightarrow{AB}$ $= \frac{2}{3}\underset{\sim}{b} - \frac{2}{3}\underset{\sim}{a}$ -- B1	
5(aiii)	$\overrightarrow{OC} = \overrightarrow{OA} + \overrightarrow{AC}$ $= \underset{\sim}{a} + \frac{2}{3}\underset{\sim}{b} - \frac{2}{3}\underset{\sim}{a}$ $= \frac{1}{3}\underset{\sim}{a} + \frac{2}{3}\underset{\sim}{b}$ -- B1	
5(aiv)	$\overrightarrow{OD} = \frac{5}{3}\overrightarrow{OB}$ $= \frac{5}{3}\underset{\sim}{b}$ -- B1	
5(b)	$\overrightarrow{CD} = \overrightarrow{OD} - \overrightarrow{OC}$ $= \frac{5}{3}\underset{\sim}{b} - \frac{1}{3}\underset{\sim}{a} - \frac{2}{3}\underset{\sim}{b}$ $= \underset{\sim}{b} - \frac{1}{3}\underset{\sim}{a}$	
5(c)	$\overrightarrow{ED} = \overrightarrow{EO} + \overrightarrow{OD}$ $= -\frac{5}{9}\underset{\sim}{a} + \frac{5}{3}\underset{\sim}{b}$ -- B1	
5(d)	They are collinear. – B1 $ED = \frac{5}{3}CD$ -- B1 (OE)	
5(ei)	$\frac{AEC}{OEC} = \frac{AE}{OE} = \frac{4}{5}$ -- B1	
5(eii)	$\frac{AEC}{OAB} = \frac{AEC}{AOC} \times \frac{AOC}{OAB}$ -- M1 $= \frac{4}{9} \times \frac{2}{3}$ $= \frac{8}{27}$ -- A1	

6(a)	$a = -0.34$ -- B1	
6(b)	 Scale – B1 Plot – B1 Curve – B1	
6(c)	$x = 3.1$ -- B1	
6(d)	Correct gradient line on graph – B1 $m = -1.25$ -- B1	
6(e)	Draw graph of -- A1 -- A1	

7(ai)	$C\hat{E}D = 360° - 186.3° - 63.4° - 71.6°$ $= 38.7°$ -- B1	
7(aii)	$\frac{CE}{\sin 102.6} = \frac{90.6}{\sin 38.7}$ $CE = 141.4139$ $= 141$ -- B1	
7(aiii)	$\text{area} = \left(\frac{1}{2}\right)(90.6)(141.4139)(\sin 38.7°) + \left(\frac{1}{2}\right)(141.4139)(111.8)(\sin 71.6°) + \left(\frac{1}{2}\right)(113.1)(106.3)(\sin 86.2°)$ $= 4005.335 + 7500.899 + 5998.049$ – M1, M1, M1 $= 17504.283$ $= 17500$ – A1	
7(bi)	$B\hat{C}D = 45° + 38.7° + 48.8° = 132.5°$ -- M1 $BD = \sqrt{90.6^2 + 106.3^2 - 2(90.6)(106.3)(\cos 132.5°)}$ – M1 $= 180.335$ $= 180$ -- A1	
7(bii)	$\tan 25° = \frac{\text{height}}{180.335}$ -- M1 $\text{height} = 180.335 \tan 25°$ $= 84.09159$ $= 84.1 \text{ m}$ -- A1	
7(biii)	$\text{flight time} = \frac{84.09159}{3}$ $= 28.0305$ $= 28.0 \text{ s}$ -- B1	
7(biv)	$\text{distance travelled} = \frac{180.335}{\cos 25°}$ $= 198.977$ -- M1 $\text{speed} = \frac{198.977}{28.0305}$ $= 7.09858$ $= 7.10 \text{ m/s}$ -- A1	

8(a)	$\mathbf{A} = \begin{pmatrix} 1 & 4 & 5 \\ 3 & 2 & 4 \\ 5 & 3 & 3 \end{pmatrix}$ -- B1	
8(b)	$\mathbf{AB} = \begin{pmatrix} 10 \\ 9 \\ 11 \end{pmatrix}$ -- B1 The elements represent the total score of each school respectively. – B1	
8(c)	$\mathbf{C} = \begin{pmatrix} 0.5 \\ 0.3 \\ 0.2 \end{pmatrix}$ -- B1	
8(d)	$\mathbf{AC} = \begin{pmatrix} 2.7 \\ 2.9 \\ 4 \end{pmatrix}$ -- B1 The elements represent the weighted scores of each school respectively. – B1	
8(e)	Temasek Poly. – B1	

9(ai)	$p=\frac{2}{5}$ -- B1	
9(aii)	$\text{E.V.}=\left(\frac{2}{5}\right)(1)-\left(\frac{3}{5}\right)(-1)$ $=-\frac{1}{5}$ -- B1	
9(bi)	$p=\frac{3}{5}\times\frac{2}{4}$ $=\frac{3}{10}$ -- B1	
9(bii)	$p=\frac{3}{5}\times\frac{2}{4}\times\frac{2}{3}$ $=\frac{1}{5}$ -- B1	
9(biii)	$p=\frac{3}{5}\times\frac{2}{4}\times\frac{1}{3}\times 1$ $=\frac{1}{10}$ -- B1	
9(biv)	$\text{E.V.}=\left(\frac{2}{5}\right)(1)+\left(\frac{3}{10}\right)(0)+\left(\frac{1}{5}\right)(-1)+\left(\frac{1}{10}\right)(-2)$ $=\frac{2}{5}-\frac{1}{5}-\frac{1}{5}$ $=0$ -- B1	
9(c)	$\text{E.V.}=\left(\frac{2}{5}\right)(1)+\left(\frac{3}{5}\right)\left(\frac{1}{2}\right)(0)+\left(\frac{3}{5}\right)\left(\frac{1}{2}\right)\left(\frac{2}{3}\right)\left(\frac{1}{2}\right)(0)$ $+\left(\frac{3}{5}\right)\left(\frac{1}{2}\right)\left(\frac{2}{3}\right)\left(\frac{1}{2}\right)(1)(-1)+\left(\frac{3}{5}\right)\left(\frac{1}{2}\right)\left(\frac{1}{3}\right)(1)(1)(-1)$ -- M1 $=\frac{2}{5}+0+0-\frac{1}{10}-\frac{1}{10}$ $=\frac{1}{5}$ -- A1 (or M1 for tree)	

10(a)	Let the height of the cone be $x+5$. $\frac{x}{x+5}=\frac{6}{10}$ – M1 $x=7.5$ – A1 volume $=\left(\frac{1}{2}\right)\left(\frac{4}{3}\right)(\pi)(10)^3+\left(\frac{1}{3}\right)(\pi)(10)^2(7.5+5)-\left(\frac{1}{3}\right)(\pi)(6)^2(7.5)$ – M1 $=\frac{2000}{3}\pi+\frac{1250}{3}\pi-90\pi$ $=3120.6487$ $=3120\ \text{m}^3$ – A1	
10(b)	T.S.A $=\left(\frac{1}{2}\right)(4)(\pi)(10)^2+(\pi)(10)\left(\sqrt{10^2+12.5^2}\right)-(\pi)(6)\left(\sqrt{6^2+7.5^2}\right)$ $=950.1746$ $=950\ \text{m}^2$ – M1 mass $=102\times 950.1746$ – M1 $=96917.8092$ g $=96.9178$ $=96.9$ kg – A1	
10(c)	lift $=(1.204-0.949)(3120.6487)$ – M1 $=795.7654$ $=796$ kg – A1	
10(d)	# of people $=\frac{795.7654-96.9178-400}{75}$ – M1 $=3.9846$ ≈ 3 – A1	