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Name:	Class	Class Register Number/ Centre No./Index No.
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中正中學

CHUNG CHENG HIGH SCHOOL (MAIN)

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Parent's Signature

PRELIMINARY EXAMINATION 2016
SECONDARY 4

Mathematics

4048/01

Paper 1

Thursday 14 July 2016

2 hours

READ THESE INSTRUCTIONS FIRST

Do not open this booklet until you are told to do so.

Write your name, class and index number clearly in the spaces provided at the top of this page.
Write in dark blue or black pen.

You may use a pencil for any diagrams, graphs or rough working.

Do not use highlighters or correction fluid.

Answer all questions.

If working is needed for any question it must be shown with the answer.

Omission of essential working will result in loss of marks.

The use of an approved scientific calculator is expected, where appropriate.

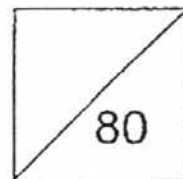
If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.

For π , use either your calculator value or 3.142, unless the question requires the answer in terms of π .

At the end of the examination, fasten all your work securely together.

The number of marks is given in brackets [] at the end of each question or part question.

The total of the marks for this paper is 80.



This document consists of 16 printed pages

[Turn over

Mathematical Formulae**Compound interest**

$$\text{Total amount} = P \left(1 + \frac{r}{100} \right)^n$$

Mensuration

$$\text{Curved surface area of a cone} = \pi r l$$

$$\text{Surface area of a sphere} = 4 \pi r^2$$

$$\text{Volume of a cone} = \frac{1}{3} \pi r^2 h$$

$$\text{Volume of a sphere} = \frac{4}{3} \pi r^3$$

$$\text{Area of triangle } ABC = \frac{1}{2} a b \sin C$$

$$\text{Arc length} = r \theta, \text{ where } \theta \text{ is in radians}$$

$$\text{Sector area} = \frac{1}{2} r^2 \theta, \text{ where } \theta \text{ is in radians}$$

Trigonometry

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2 b c \cos A$$

Statistics

$$\text{Mean} = \frac{\sum f x}{\sum f}$$

$$\text{Standard deviation} = \sqrt{\frac{\sum f x^2}{\sum f} - \left(\frac{\sum f x}{\sum f} \right)^2}$$

Answer all the questions.

- 1 Calculate the value of $\frac{\sqrt{0.99} \times 59.98}{24.9 - 1.01^2}$, giving your answer correct to 3 significant figures.

Answer [1]

- 2 If a man sells an art piece at \$250, he would make a loss of 20%. Evaluate the selling price of the art piece if he wants to make a profit of 15%.

Answer [2]

- 3 Given that $\left(\frac{1}{3}\right)^{-2x} = 27^{\frac{1}{2}} + 9^x$, find the value of x .

Answer [2]

- 4 Given that $\frac{3a-b}{2b} = \frac{1}{4}$, find the value of $\frac{a}{b}$.

Answer [2]

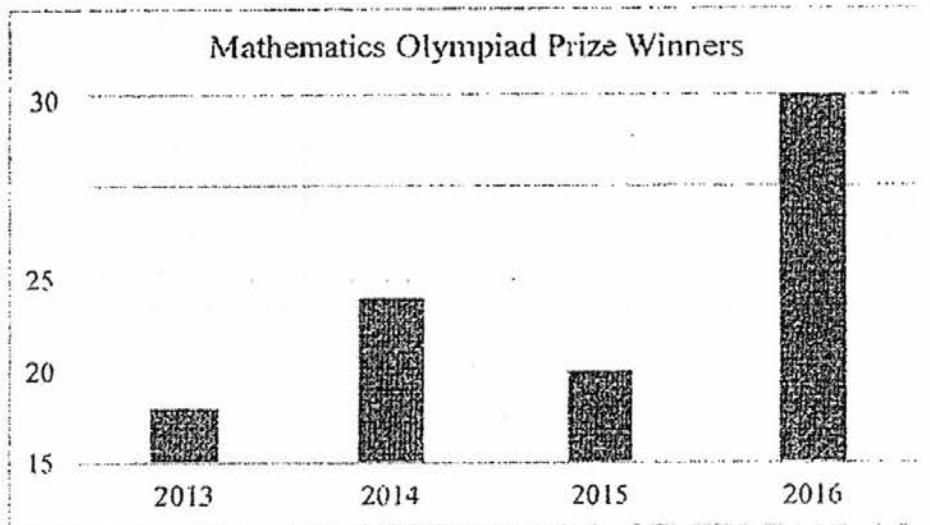
- 5 A set of four even numbers has a mean of 13, median of 14 and mode of 18. Find the four numbers.

Answer [2]

- 6 Factorise $15ax - 21bx - 10ay + 14by$.

Answer [2]

- 7 The graph shows the number of prize winners from Chung Cheng High School (Main) at the Mathematics Olympiad over a number of years.



Explain one way in which the graph is misleading.

Answer

.....

.....

.....

[1]

- 8 Solve the equation $x(x-2)=3$.

Answer $x = \dots\dots\dots$ and $\dots\dots\dots$ [2]

- 9 The scale of a map is 1 : 60 000. A park is represented by an area of 4 cm^2 on the map. Calculate the actual area of the park in square kilometres.

Answer $\dots\dots\dots \text{ km}^2$ [2]

- 10 The table shows the battery lifespan of 40 laptops.

Time (x in hours)	No. of laptops
$0 \leq x < 1$	2
$1 \leq x < 2$	3
$2 \leq x < 3$	12
$3 \leq x < 4$	16
$4 \leq x < 5$	6
$5 \leq x < 6$	1

- (a) Find the percentage of laptops that have battery lifespan of at least 4 hours.

Answer (a) $\dots\dots\dots$ [1]

- (b) Find the mean and standard deviation of the battery lifespan of the laptops.

Answer (b) Mean = $\dots\dots\dots$ hours [1]

Standard Deviation = $\dots\dots\dots$ hours [2]

- 11 Benson has 480 strawberry-flavoured lollipops and 560 cola-flavoured lollipops. He puts them into packets, each containing the same number of each type of lollipops and with no remainder. If he would like to pack the lollipops in as many packets as possible, find the total number of lollipops in each packet.

Answer [2]

- 12 Consider the sequence $1^3 - 4, 2^3 - 6, 3^3 - 8, 4^3 - 10, \dots$

(a) Write down the 5th term of the sequence.

Answer (a) [1]

(b) Write down and simplify, in terms of n , an expression for the n^{th} term of the sequence.

Answer (b) [1]

(c) Evaluate the 20th term of the sequence.

Answer (c) [1]

- 13 It is given that $\overrightarrow{AB} = \begin{pmatrix} 4 \\ 5 \end{pmatrix}$, $\overrightarrow{OC} = \begin{pmatrix} 2 \\ p \end{pmatrix}$ and the position vector of A is $\begin{pmatrix} -2 \\ 1 \end{pmatrix}$.

(a) Find $|\overrightarrow{AB}|$.

Answer (a) [2]

(b) Find the coordinates of B .

Answer (b) [2]

(c) Given that AB and OC are the opposite parallel sides of a trapezium $OCBA$, find the value of p .

Answer (c) $p =$ [1]

- 14 (a) Solve the inequalities $x - 2 \leq \frac{2x}{5} < \frac{x}{2} + \frac{1}{5}$.

Answer (a) [3]

- (b) Write down all the integers that satisfy $x - 2 \leq \frac{2x}{5} < \frac{x}{2} + \frac{1}{5}$.

Answer (b) [1]

- 15 (a) The universal set, ε , contains three sets, A , B and C . The three sets satisfy the following conditions:

$$A \subset B \text{ and } B \cap C = \emptyset$$

Draw a Venn diagram to illustrate these sets.

[2]

Answer (a)

- (b) It is given that

$$\varepsilon = \{x : x \text{ is a whole number and } 1 \leq x < 10\}$$

$$P = \{x : x \text{ is a prime number}\}$$

$$Q = \{x : x \text{ is a factor of } 20\}$$

- (i) List the elements of $P \cap Q'$.

Answer (b)(i) [1]

- (ii) Find $n(P \cup Q)$.

Answer (b)(ii) [1]

- 16 Two interior angles of a n -sided polygon are 160° and 40° while the remaining interior angles are each 140° . Find the value of n .

Answer [2]

- 17 A quadratic curve $y = (h-x)(x+k)$ meets the x -axis at $(-0.5, 0)$ and $(6, 0)$.

(a) Find the equation of the curve.

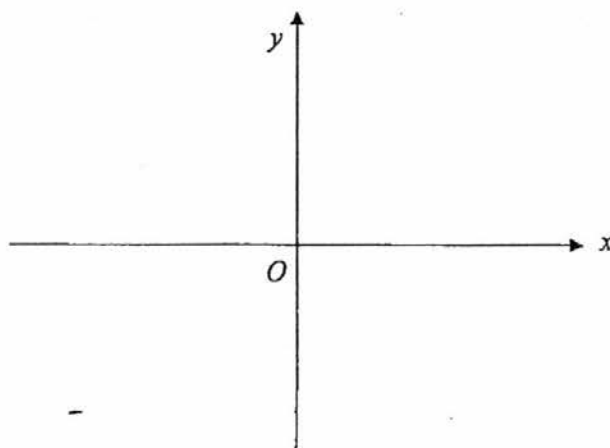
Answer (a) [1]

(b) Write down the equation of the line of symmetry of the graph.

Answer (b) [1]

(c) Hence, sketch the graph of $y = (h-x)(x+k)$.

Answer



[2]

- 18 Alex has \$ x and Ben has \$ y . If Ben gives Alex \$5, Alex will have twice as much money as Ben.

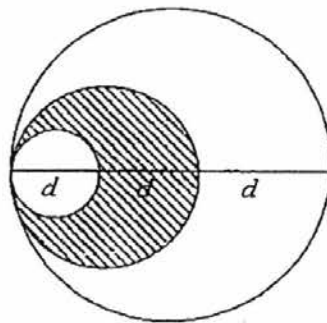
(a) Form an equation in x and y .

Answer (a) [1]

- (b) If they have a total of \$48, form another equation in x and y . Hence, find the values of x and y .

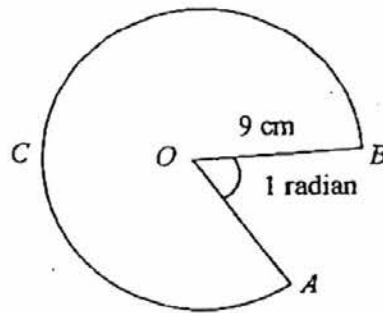
Answer (b) $x = \dots\dots\dots$, $y = \dots\dots\dots$ [3]

- 19 The diagram shows three circles with diameters d cm, $2d$ cm and $3d$ cm respectively. Find, in terms of π and d , the difference in area between the unshaded region and the shaded region.



Answer cm^2 [3]

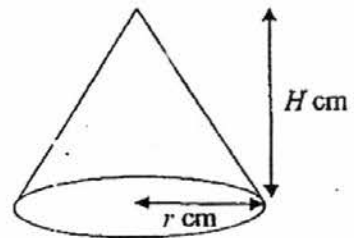
- 20 $OACB$ is a sector of a circle with centre O and radius 9 cm, angle $AOB = 1$ radian.



- (a) Find the length of arc ACB .

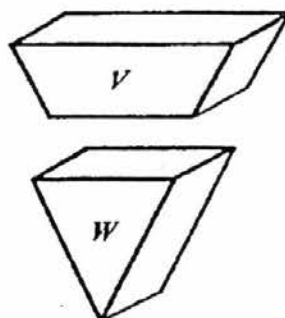
Answer cm [2]

- (b) The edges OA and OB are joined together to form the cone below.
Find r , the radius, and H , the height, of the cone.



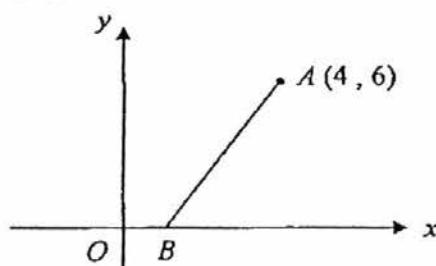
Answer $r = \dots\dots\dots$, $H = \dots\dots\dots$ [4]

- 21 The diagram shows the top part V of a prism with a triangular cross-section being cut horizontally across and removed to leave the lower part W .
The vertical heights of V and W are in the ratio of 2 : 3 respectively.
If the volume of W is 54 m^3 , find the volume of the solid V .



Answer m^3 [2]

- 22 In the figure, O is the origin and A is the point $(4, 6)$. B is a point on the x -axis such that the gradient of AB is 2.



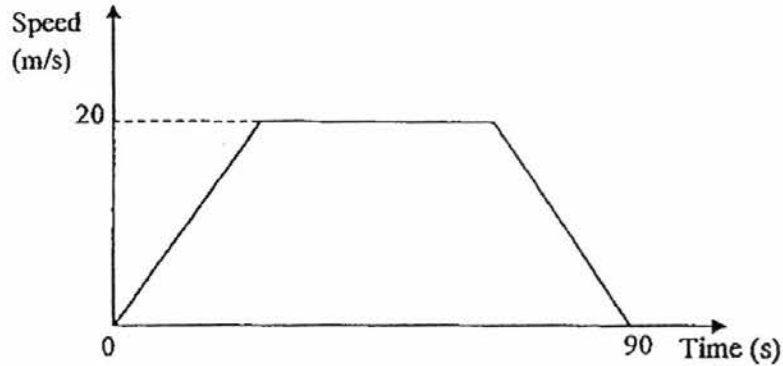
- (a) Find the coordinates of B .

Answer (a) [2]

- (b) C is another point on the x -axis such that $AB = AC$. Find the coordinates of C .

Answer (b) [1]

- 23 The speed time graph shows the journey of a train over a period of 90 s from Paya Lebar Station to Eunos Station. The train reaches a maximum speed of 20 m/s.



- (a) Express 20 m/s in km/h.

Answer (a) km/h [1]

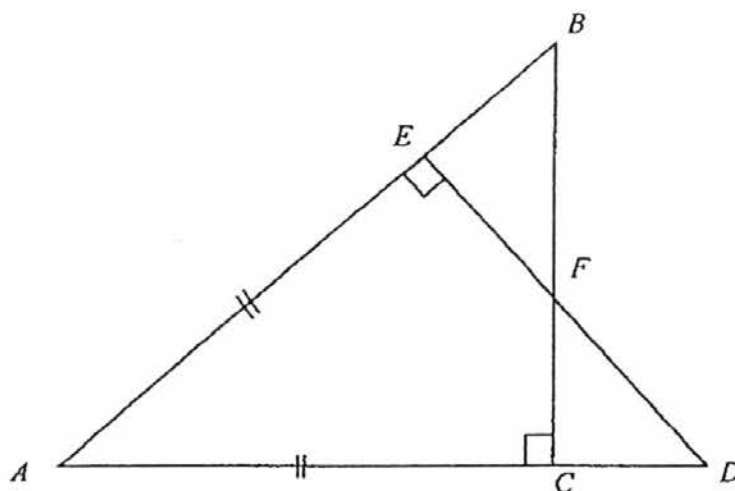
- (b) Given that the acceleration in the first part of the journey was 0.8 m/s^2 , calculate the time taken, in seconds, for the train to reach its maximum speed.

Answer (b) seconds [2]

- (c) The total distance travelled during the 90 s was 1300 m. Calculate the duration that the train was travelling at its maximum speed.

Answer (c) seconds [2]

- 24 In the diagram, $AE = AC$, and triangles ACB and AED are right angled triangles. Show that triangles ACB and AED are congruent.



Answer

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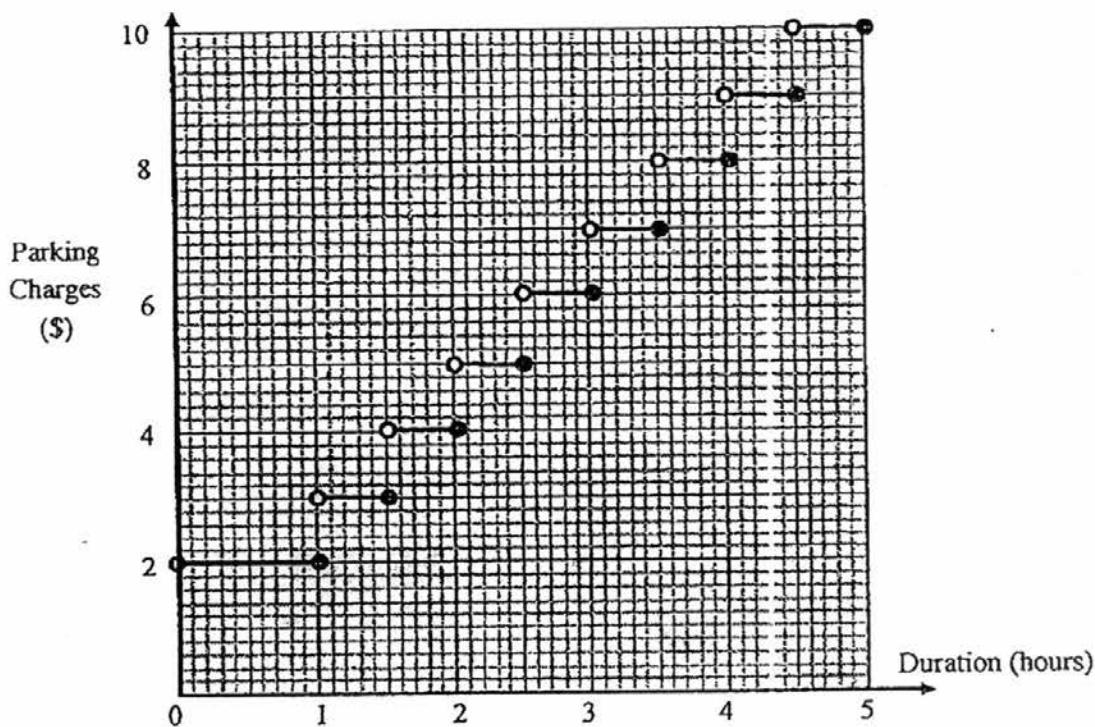
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[2]

- 25 The step-function graph shows the parking charges for the first 5 hours at Carpark A.



- (a) Write down the parking charges for a car that is parked at Carpark A for
 (i) 4 hour 15 minutes (ii) $2\frac{1}{2}$ hours.

Answer (a) (i) \$ (ii) \$ [2]

- (b) Another nearby carpark, Carpark B, offers the parking charges.

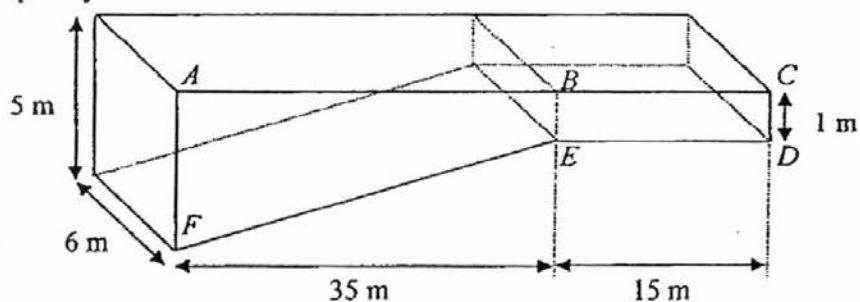
\$1 upon entry,
 3 cents per minute thereafter
 (Maximum charges of \$8.20)

Kate wishes to park her car for $2\frac{1}{2}$ hours at one of these carparks.

- (i) On the same axes, draw the graph of parking charges offered at Carpark B. [2]
 (ii) State the carpark Kate should choose to park her car.

Answer (b)(ii) [1]

- 26 The diagram shows a swimming pool, with its cross section consisting of a trapezium, $ABEF$, and a rectangle, $BCDE$. The pool is initially empty. Water is pumped into the pool at a constant rate, and it takes 6 hours to fill the empty pool completely with water.



- (a) Calculate the volume of the pool.

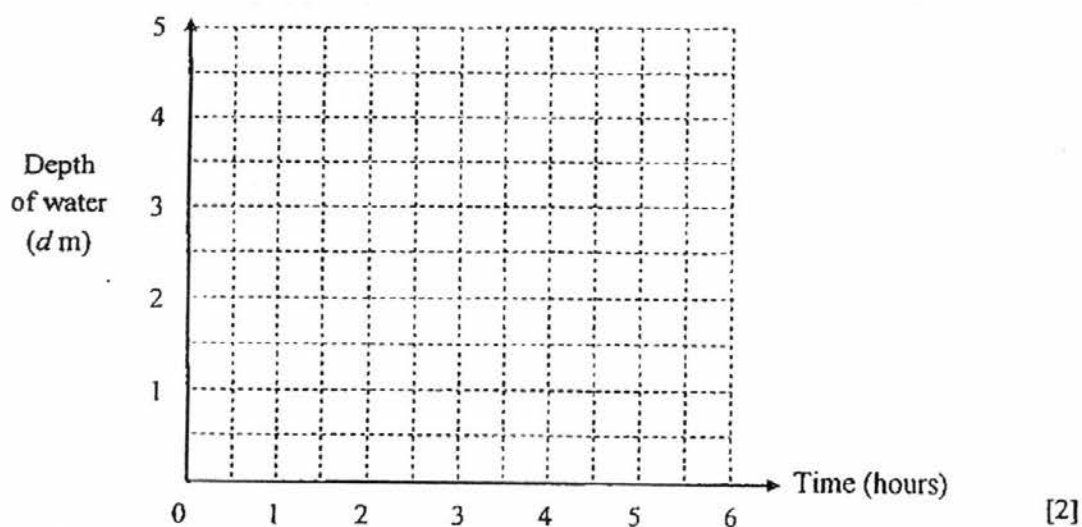
Answer (a) m^3 [2]

- (b) Find the time taken to fill the pool to a depth of 4 m at the deep end.

Answer (b) hours [2]

- (c) On the axes in the answer space, sketch a graph to represent how the depth of water at the deep end of the pool changes with time.

Answer



Mathematical Formulae**Compound interest**

$$\text{Total amount} = P \left(1 + \frac{r}{100} \right)^n$$

Mensuration

$$\text{Curved surface area of a cone} = \pi r l$$

$$\text{Surface area of a sphere} = 4 \pi r^2$$

$$\text{Volume of a cone} = \frac{1}{3} \pi r^2 h$$

$$\text{Volume of a sphere} = \frac{4}{3} \pi r^3$$

$$\text{Area of triangle } ABC = \frac{1}{2} a b \sin C$$

$$\text{Arc length} = r \theta, \text{ where } \theta \text{ is in radians}$$

$$\text{Sector area} = \frac{1}{2} r^2 \theta, \text{ where } \theta \text{ is in radians}$$

Trigonometry

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

Statistics

$$\text{Mean} = \frac{\sum f x}{\sum f}$$

$$\text{Standard deviation} = \sqrt{\frac{\sum f x^2}{\sum f} - \left(\frac{\sum f x}{\sum f} \right)^2}$$

- 1 Calculate the value of $\frac{\sqrt{0.99 \times 59.98}}{24.9 - 1.01^2}$, giving your answer correct to 3 significant figures.

$$\begin{aligned}\frac{\sqrt{0.99 \times 59.98}}{24.9 - 1.01^2} &= 2.4991 \\ &= 2.50 \text{ (3sf)}\end{aligned}$$

Answer 2.50 [1]

- 2 If a man sells an art piece at \$250, he would make a loss of 20%. Evaluate the selling price of the art piece if he wants to make a profit of 15%.

$$\begin{aligned}\text{Selling Price} &= \frac{115}{80} \times \$250 \\ &= \$359.38 \text{ (nearest cent)}\end{aligned}$$

OR

$$\begin{aligned}\text{Original Selling Price} &= \frac{100}{80} \times \$250 = \$312.50 \\ \text{Selling Price at profit} &= \frac{115}{100} \times \$312.50 = \$359.38 \text{ (nearest cent)}\end{aligned}$$

Answer [2]

- 3 Given that $\left(\frac{1}{3}\right)^{-2x} = 27^{\frac{1}{2}} \div 9^x$, find the value of x .

$$\begin{aligned}\left(\frac{1}{3}\right)^{-2x} &= 27^{\frac{1}{2}} \div 9^x \\ 3^{2x} &= 3^{\frac{3}{2}} \div 3^{2x} \\ 3^{4x} &= 3^{\frac{3}{2}} \\ 4x &= \frac{3}{2} \\ x &= \frac{3}{8}\end{aligned}$$

Answer [2]

- 4 Given that $\frac{3a-b}{2b} = \frac{1}{4}$, find the value of $\frac{a}{b}$.

$$\begin{aligned}\frac{3a-b}{2b} &= \frac{1}{4} \\ 12a - 4b &= 2b \\ 12a &= 6b \\ \frac{a}{b} &= \frac{1}{2}\end{aligned}$$

Answer [2]

- 5 A set of four even numbers has a mean of 13, median of 14 and mode of 18. Find the four numbers.

Let x be the last number.

$$\frac{x+10+18+18}{4} = 13$$

$$x+46 = 52$$

$$x = 6$$

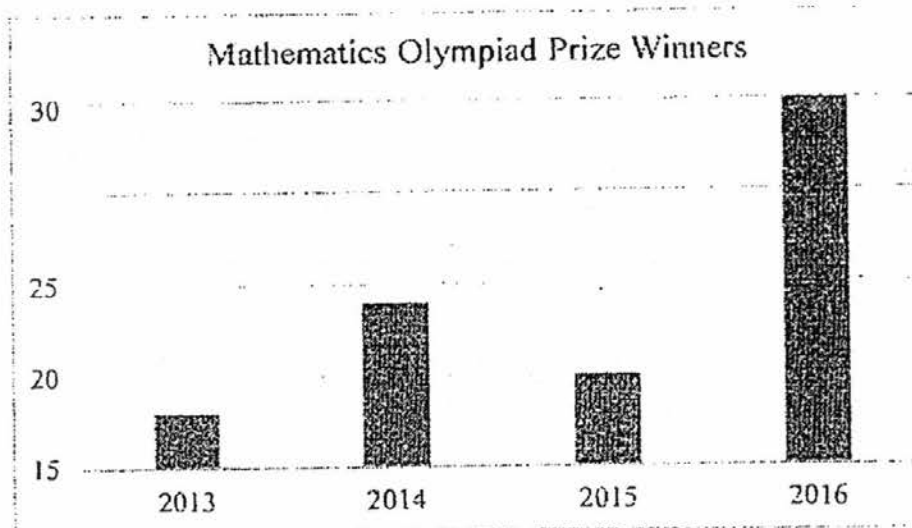
Answer 6, 10, 18, 18 [2]

- 6 Factorise $15ax - 21bx - 10ay + 14by$.

$$\begin{aligned} & 15ax - 21bx - 10ay + 14by \\ &= 3x(5a - 7b) - 2y(5a - 7b) \\ &= (3x - 2y)(5a - 7b) \end{aligned}$$

Answer [2]

- 7 The graph shows the number of prize winners from Chung Cheng High School (Main) at the Mathematics Olympiad over a number of years.



Explain one way in which the graph is misleading.

Answer The y-axis (does not start at 0) starts at 15 OR The scale of the

y-axis is not uniform so the length of each bar is not representative of [1]

the number of prize winners.

- 8 Solve the equation
- $x(x-2)=3$
- .

$$x(x-2)=3$$

$$x^2 - 2x - 3 = 0$$

$$(x-3)(x+1) = 0$$

$$x = 3 \text{ or } x = -1$$

Answer $x = \dots\dots\dots$ and $\dots\dots\dots$ [2]

- 9 The scale of a map is 1 : 60 000. A park is represented by an area of
- 4cm^2
- on the map. Calculate the actual area of the park in square kilometres.

Map Scale

1 cm : 60 000 cm

1 cm : 0.6 km

Area Scale - 1 cm^2 : 0.36 km^2

4 cm^2 : 1.44 km^2

Answer (a) $\dots\dots\dots \text{km}^2$ [2]

- 10 The table shows the battery lifespan of 40 laptops.

Time (x in hours)	No. of laptops
$0 \leq x < 1$	2
$1 \leq x < 2$	3
$2 \leq x < 3$	12
$3 \leq x < 4$	16
$4 \leq x < 5$	6
$5 \leq x < 6$	1

- (a) Find the percentage of laptops that have battery lifespan of at least 4 hours.

$$\text{Percentage} = \frac{7}{40} \times 100\% = 17.5\%$$

Answer (a) 17.5 % [1]

- (b) Find the mean and standard deviation of the battery lifespan of the laptops.

$$\text{Mean} = \frac{2(0.5) + 3(1.5) + 12(2.5) + 16(3.5) + 6(4.5) + 1(5.5)}{40}$$

$$= \frac{124}{40}$$

$$= 3.1 \text{ hours}$$

$$\begin{aligned} \text{Standard Deviation} &= \sqrt{\frac{430}{40} - (3.1)^2} \\ &= 1.07 \text{ hours (3sf)} \end{aligned}$$

Answer (b) Mean = $\dots\dots\dots$ hours [1]

Standard Deviation = $\dots\dots\dots$ hours [2]

- 11 Benson has 480 strawberry-flavoured lollipops and 560 cola- flavoured lollipops. He puts them into packets, each containing the same number of each type of lollipops and with no remainder. If he would like to pack the lollipops in as many packets as possible, find the total number of lollipops in each packet.

2	480	560
2	240	280
2	120	140
2	60	70
5	30	35
	6	7

Total number of lollipops in each packet = $6+7$

$$= 13$$

Answer [2]

- 12 Consider the sequence $1^3 - 4, 2^3 - 6, 3^3 - 8, 4^3 - 10, \dots$

- (a) Write down the 5th term of the sequence.

Answer (a) $5^3 - 12$ [1]

- (b) Write down and simplify, in terms of n , an expression for the n^{th} term of the sequence.

$$n^{\text{th}} \text{ term} = n^3 - 2(n+1)$$

$$= n^3 - 2n - 2$$

Answer (b) [1]

- (c) Evaluate the 20th term of the sequence.

$$20^{\text{th}} \text{ term} = (20)^3 - 2(20) - 2$$

$$= 7958$$

Answer (c) [1]

- 13 It is given that $\overrightarrow{AB} = \begin{pmatrix} 4 \\ 5 \end{pmatrix}$, $\overrightarrow{OC} = \begin{pmatrix} 2 \\ p \end{pmatrix}$ and the position vector of A is $\begin{pmatrix} -2 \\ 1 \end{pmatrix}$.

(a) Find $|\overrightarrow{AB}|$.

$$\begin{aligned} |\overrightarrow{AB}| &= \sqrt{4^2 + 5^2} \\ &= 6.40 \text{ units (3sf)} \end{aligned}$$

Answer (a) [2]

(b) Find the coordinates of B .

$$\begin{aligned} \overrightarrow{AB} &= \overrightarrow{OB} - \overrightarrow{OA} \\ \begin{pmatrix} 4 \\ 5 \end{pmatrix} &= \overrightarrow{OB} - \begin{pmatrix} -2 \\ 1 \end{pmatrix} \\ \overrightarrow{OB} &= \begin{pmatrix} 4 \\ 5 \end{pmatrix} + \begin{pmatrix} -2 \\ 1 \end{pmatrix} \\ &= \begin{pmatrix} 2 \\ 6 \end{pmatrix} \end{aligned}$$

Coordinates of $B = (2, 6)$

Answer (b) [2]

(c) Given that AB and OC are the opposite parallel sides of a trapezium $OCBA$, find the value of p .

$$\begin{aligned} \overrightarrow{OC} &= k\overrightarrow{AB} \\ \begin{pmatrix} 2 \\ p \end{pmatrix} &= k\begin{pmatrix} 4 \\ 5 \end{pmatrix} \\ \therefore k &= \frac{1}{2}, p = 2.5 \end{aligned}$$

Answer (c) $p = 2.5$ [1]

- 14 (a) Solve the inequalities $x - 2 \leq \frac{2x}{5} < \frac{x}{2} + \frac{1}{5}$.

$$x - 2 \leq \frac{2x}{5} < \frac{x}{2} + \frac{1}{5}$$

$$x - 2 \leq \frac{2x}{5} \quad \text{and} \quad \frac{2x}{5} < \frac{x}{2} + \frac{1}{5}$$

$$5x - 10 \leq 2x$$

$$4x < 5x + 2$$

$$3x \leq 10$$

$$-x < 2$$

$$x \leq 3\frac{1}{3}$$

$$x > -2$$

$$\therefore -2 < x \leq 3\frac{1}{3}$$

Answer (a) [3]

- (b) Write down all the integers that satisfy $x - 2 \leq \frac{2x}{5} < \frac{x}{2} + \frac{1}{5}$.

Answer (b) -1, 0, 1, 2, 3 [1]

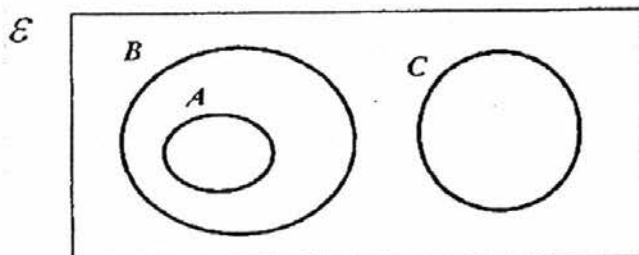
- 15 (a) The universal set, $\mathcal{E}$, contains three sets, A , B and C . The three sets satisfy the following conditions:

$$A \subset B \text{ and } B \cap C = \emptyset$$

Draw a Venn diagram to illustrate these sets.

[2]

Answer (a)



- (b) It is given that
 $\mathcal{E} = \{x : x \text{ is a whole number and } 1 \leq x < 10\}$
 $P = \{x : x \text{ is a prime number}\}$
 $Q = \{x : x \text{ is a factor of } 20\}$

- (i) List the elements of $P \cap Q'$.

Answer (b)(i) $P \cap Q' = \{3, 7\}$ [1]

- (ii) Find $n(P \cup Q)$.

Answer (b)(ii) 6 [1]

- 16 Two interior angles of a n -sided polygon are 160° and 40° while the remaining interior angles are each 140° . Find the value of n .

$$140^\circ + 20^\circ + (n-2)(40^\circ) = 360^\circ$$

$$(n-2)(40) = 200$$

$$n-2 = 5$$

$$n = 7$$

OR

$$200^\circ + (n-2)(140^\circ) = (n-2)180^\circ$$

$$(n-2)(40) = 200$$

$$n-2 = 5$$

$$n = 7$$

Answer [2]

- 17 A quadratic curve, with equation $y = (h-x)(x+k)$, meets the x -axis at $(-0.5, 0)$ and $(6, 0)$.

- (a) Find the equation of the curve.

$$y = -(x+0.5)(x-6)$$

$$= (6-x)(x+0.5)$$

$$= -x^2 + 5.5x + 3$$

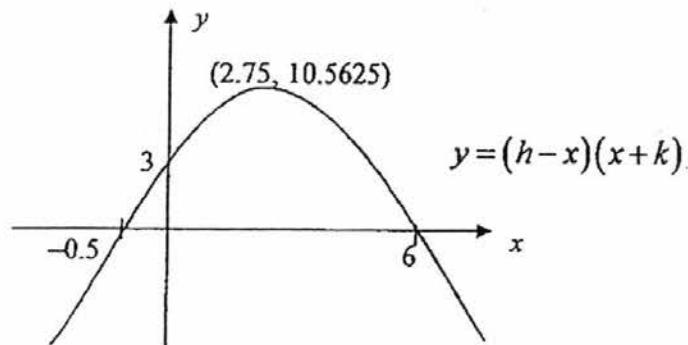
Answer (a) [1]

- (b) Write down the equation of the line of symmetry of the graph.

Answer (b) $x = 2.75$ [1]

- (c) Hence, sketch the graph of $y = (h-x)(x+k)$.

Answer



[2]

- 18 Alex has \$ x and Ben has \$ y . If Ben gives Alex \$5, Alex will have twice as much money as Ben.

(a) Form an equation in x and y .

$$2(y - 5) = x + 5$$

$$2y - 10 = x + 5$$

$$2y - x = 15$$

Answer (a) [1]

- (b) If they have a total of \$48, form another equation in x and y . Hence, find the values of x and y .

$$x + y = 48 \quad \text{--- (1)}$$

$$2y - x = 15 \quad \text{--- (2)}$$

$$(1) + (2):$$

$$3y = 63$$

$$y = 21$$

$$\text{When } y = 21,$$

$$x = 48 - 21$$

$$= 27$$

$$\therefore x = 27, y = 21$$

Answer (b) $x = \dots\dots\dots$, $y = \dots\dots\dots$ [3]

- 19 The diagram shows three circles with diameters d cm, $2d$ cm and $3d$ cm respectively. Find, in terms of π and d , the difference in area between the unshaded region and the shaded region.

$$\text{Area of shaded region} = \pi d^2 - \pi \left(\frac{d}{2}\right)^2$$

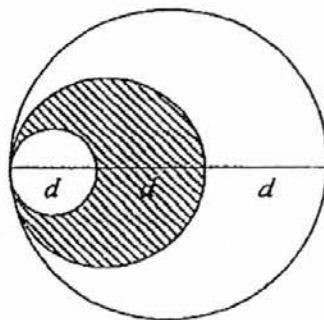
$$= \frac{3}{4} \pi d^2$$

$$\text{Area of unshaded region} = \pi \left(\frac{3d}{2}\right)^2 - \frac{3}{4} \pi d^2$$

$$= \frac{3}{2} \pi d^2$$

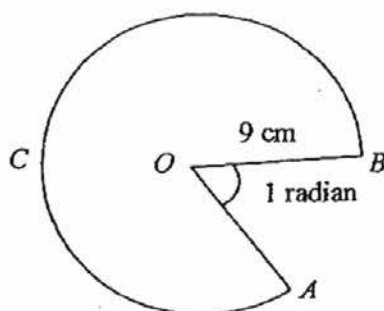
$$\text{Difference} = \frac{3}{2} \pi d^2 - \frac{3}{4} \pi d^2$$

$$= \frac{3}{4} \pi d^2$$



Answer cm^2 [3]

- 20 $OACB$ is a sector of a circle with centre O and radius 9 cm. Angle $AOB = 1$ radian.



- (a) Find the length of arc ACB .

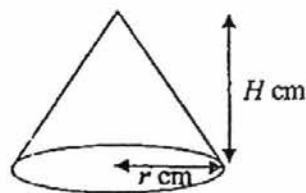
$$\begin{aligned}\text{Length of arc } ACB &= 9(2\pi - 1) \\ &= 18\pi - 9 \\ &= 47.5486 \\ &= 47.5 \text{ cm (3sf)}\end{aligned}$$

Answer cm [2]

- (b) The edges OA and OB are joined together to form the cone below. Find r , the radius, and H , the height of the cone.

Length of arc ACB = circumference of base

$$\begin{aligned}9(2\pi - 1) &= 2\pi r \\ r &= \frac{9(2\pi - 1)}{2\pi} \\ &= 7.56760 \\ &= 7.57 \text{ (3sf)}\end{aligned}$$



By pythagoras' theorem,

$$\begin{aligned}H^2 &= 9^2 - r^2 \\ &= 81 - (7.56760)^2 \\ &= 23.7314 \\ H &= 4.87149 \\ &= 4.87 \text{ (3sf)}\end{aligned}$$

Answer $r = \dots\dots\dots$, $H = \dots\dots\dots$ [3]

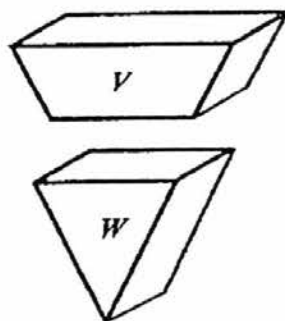
- 21 The diagram shows the top part V of a prism with a triangular cross-section being cut horizontally across and removed to leave the lower part W .
The vertical heights of V and W are in the ratio of 2 : 3 respectively.
If the volume of W is 54 m^3 , find the volume of the solid V .

$$\frac{\text{Volume of } V+W}{\text{Volume of } W} = \left(\frac{5}{3}\right)^2$$

$$\frac{\text{Volume of } V + 54}{54} = \frac{25}{9}$$

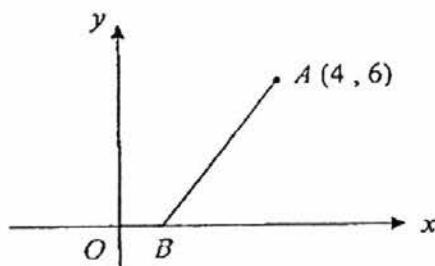
$$\text{Volume of } V + 54 = 150$$

$$\text{Volume of } V = 96 \text{ m}^3$$



Answer m^3 [2]

- 22 In the figure, O is the origin and A is the point $(4, 6)$. B is a point on the x -axis such that the gradient of AB is 2.



- (a) Find the coordinates of B .

$$B(x, 0)$$

$$\text{gradient of } AB = \frac{6-0}{4-x}$$

$$\frac{6}{4-x} = 2$$

$$6 = 8 - 2x$$

$$2x = 2$$

$$x = 1$$

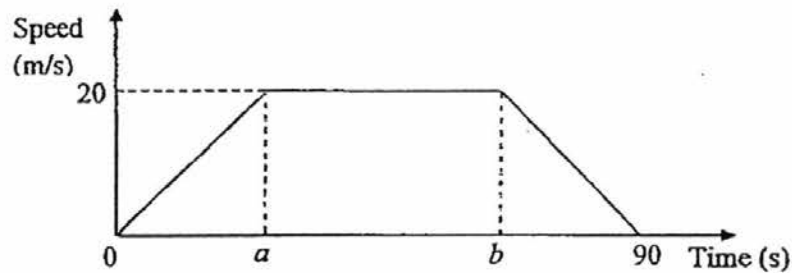
$$\therefore B(1, 0)$$

Answer (a) [2]

- (b) C is another point on the x -axis such that $AB = AC$. Find the coordinates of C .

Answer (b) $C(7, 0)$ [1]

- 23 The speed time graph shows the journey of a train over a period of 90 s from Paya Lebar Station to Eunos Station. The train reaches a maximum speed of 20 m/s.



- (a) Express 20 m/s in km/h.

$$\begin{aligned} 20 \text{ m/s} &= \frac{20 \times 1000}{3600} \\ &= 72 \text{ km/h} \end{aligned}$$

Answer (a) [1]

- (b) Given that the acceleration in the first part of the journey was 0.8 m/s^2 , calculate the time taken, in seconds, for the train to reach its maximum speed.

$$\begin{aligned} 0.8 &= \frac{20 - 0}{a - 0} \\ 0.8a &= 20 \\ a &= 25 \end{aligned}$$

Answer (b) seconds [2]

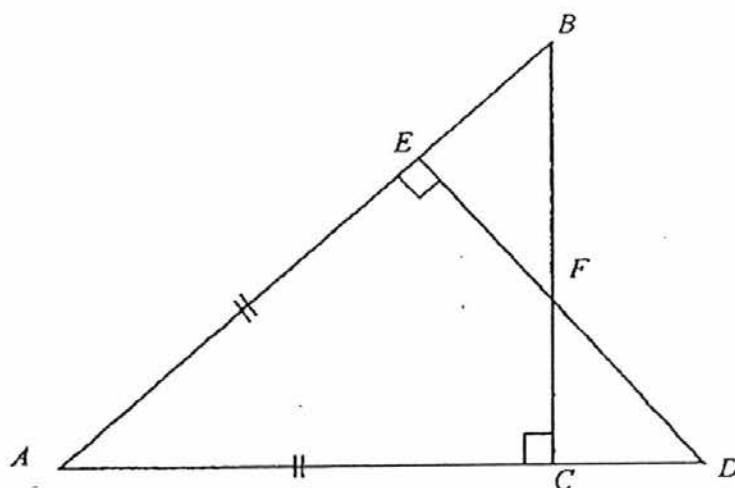
- (c) The total distance travelled during the 90 s was 1300 m. Calculate the duration that the train was travelling at its maximum speed.

$$\begin{aligned} \frac{1}{2}(20)(25) + \frac{1}{2}(65 + b - 25)(20) &= 1300 \\ 250 + 10(40 + b) &= 1300 \\ 10(40 + b) &= 1050 \\ 40 + b &= 105 \\ b &= 65 \end{aligned}$$

Length of time at maximum speed = 40 s

Answer (c) seconds [2]

- 24 In the diagram, $AE = AC$, and triangles ACB and AED are right angled triangles. Show that triangles ACB and AED are congruent.

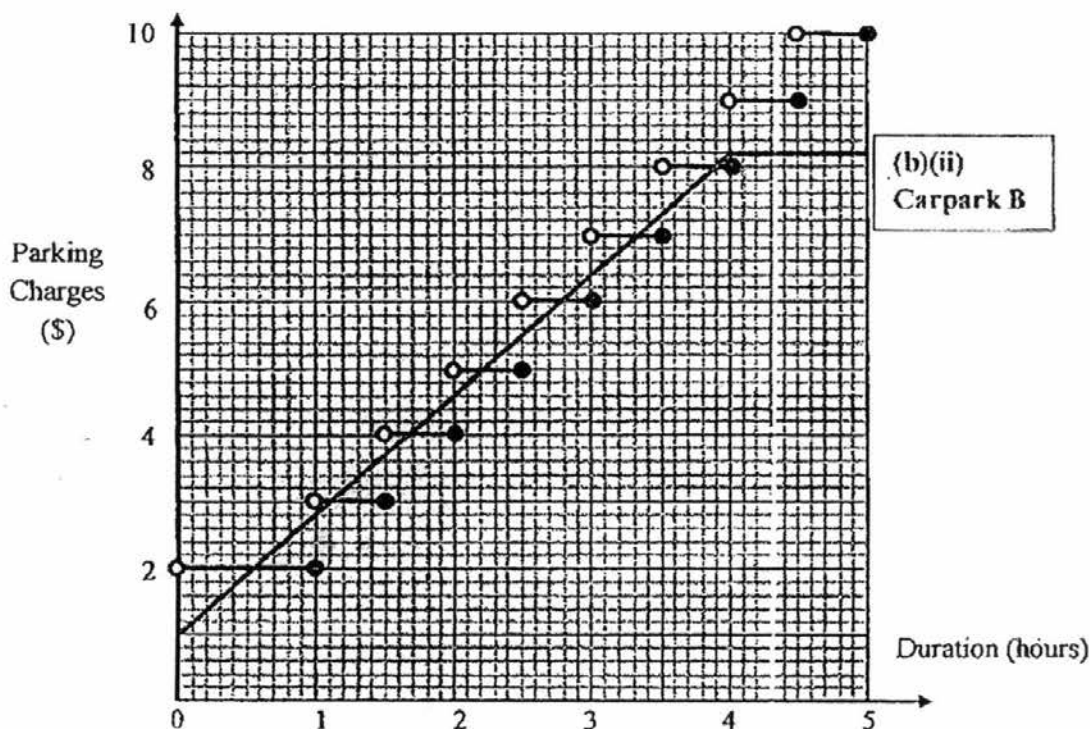


Answer

$$\begin{aligned} AC &= AE \text{ (given)} \\ \angle ACB &= \angle AED \text{ (given)} \\ \angle BAC &= \angle DAE \text{ (common angle)} \\ \therefore \triangle ACB &\equiv \triangle AED \text{ (ASA)} \end{aligned}$$

[2]

- 25 The step-function graph shows the parking charges for the first 5 hours at Carpark A.



- (a) Write down the parking charges for a car that is parked at Carpark A for
 (i) 4 hour 15 minutes (ii) $2\frac{1}{2}$ hours.

Answer (a) (i) \$ 9 (ii) \$ 5 [2]

- (b) Another nearby carpark, Carpark B, offers the parking charges.

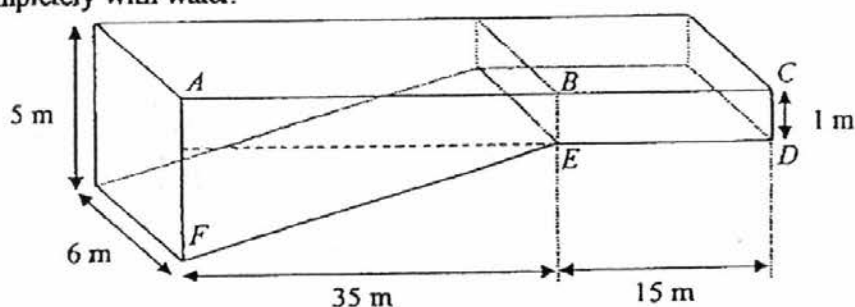
\$1 upon entry,
3 cents per minute thereafter
(Maximum Charges of \$8.20)

Kate wishes to park her car for $2\frac{1}{2}$ hours at one of these carparks.

- (i) On the same axes, draw the graph of parking charges at Carpark B. [2]
 (ii) State the carpark Kate should choose to park her car.

Answer (b)(ii) Carpark A

- 26 The diagram shows a swimming pool, with its cross section consisting of a trapezium, $ABEF$, and a rectangle, $BCDE$. The pool is initially empty. Water is pumped into the pool at a constant rate, and it takes 6 hours to fill the empty pool completely with water.



- (a) Calculate the volume of the pool.

$$\begin{aligned}\text{Volume of pool} &= \frac{1}{2}(1+5)(35)(6) + (1)(15)(6) \\ &= 720 \text{ m}^3\end{aligned}$$

Answer (a) [2]

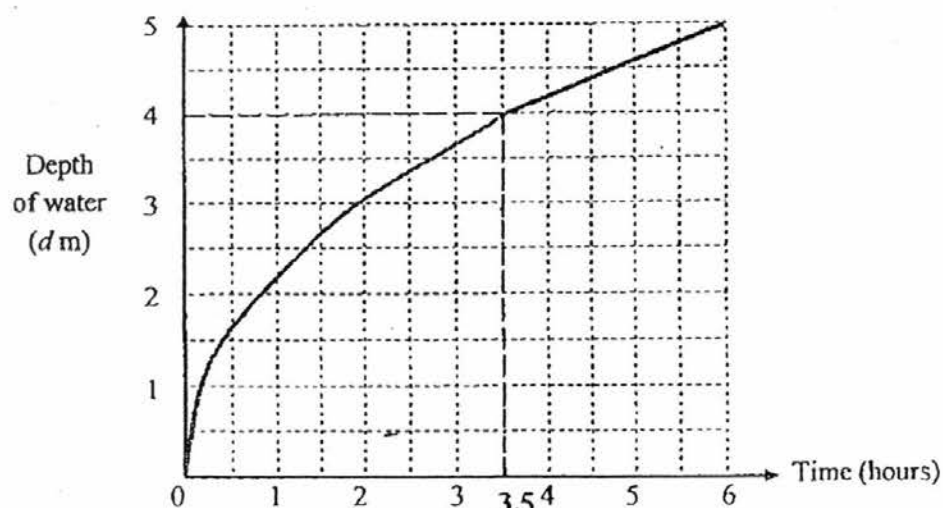
- (b) Find the time taken to fill the pool to a depth of 4 m at the deep end.

$$\begin{aligned}\text{Time taken} &= \frac{\frac{1}{2}(4)(35)(6)}{720} \times 6 \\ &= 3.5 \text{ hours}\end{aligned}$$

Answer (b) 3.5 hours [2]

- (c) On the axes in the answer space, sketch a graph to represent how the depth of water at the deep end of the pool changes with time.

Answer



[2]

Mathematical Formulae**Compound interest**

$$\text{Total amount} = P \left(1 + \frac{r}{100} \right)^n$$

Mensuration

$$\text{Curved surface area of a cone} = \pi r l$$

$$\text{Surface area of a sphere} = 4 \pi r^2$$

$$\text{Volume of a cone} = \frac{1}{3} \pi r^2 h$$

$$\text{Volume of a sphere} = \frac{4}{3} \pi r^3$$

$$\text{Area of triangle } ABC = \frac{1}{2} a b \sin C$$

$$\text{Arc length} = r \theta, \text{ where } \theta \text{ is in radians}$$

$$\text{Sector area} = \frac{1}{2} r^2 \theta, \text{ where } \theta \text{ is in radians}$$

Trigonometry

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2 b c \cos A$$

Statistics

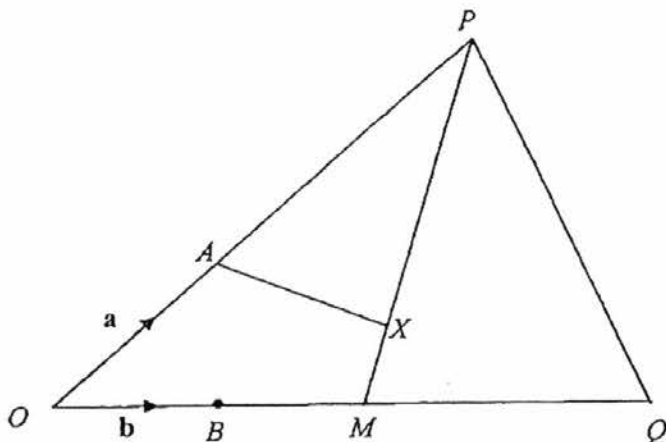
$$\text{Mean} = \frac{\sum f x}{\sum f}$$

$$\text{Standard deviation} = \sqrt{\frac{\sum f x^2}{\sum f} - \left(\frac{\sum f x}{\sum f} \right)^2}$$

Answer all the questions.

- 1 (a) Simplify $\frac{4y}{3x^3} \div \frac{12y^4}{x}$. [1]
- (b) Simplify $\frac{4x-8}{7x^2-12x-4}$. [2]
- (c) Factorise completely $20m^2 - 45n^2$. [2]
- (d) Solve the equation $\frac{4x+5}{3x-2} = 4$. [2]
- (e) The volume, $V \text{ m}^3$, of a certain object is given by the formula $V = \frac{b^2}{2}(a+3h)$, where h is the height of the object in metres and a and b are constants. [1]
- (i) Make b the subject of the formula. [2]
- (ii) Hence find b , when the volume and height of the object are 15 m^3 and 2.5 m respectively, and $a = 0.25$. [2]
- 2 (a) The annual tuition fees for a three-year Business degree at a local university in Singapore is \$12 000. [1]
- (i) Wayne receives a bursary of \$5 000 per year for 3 years.
Find the remaining amount that Wayne has to pay.
- Wayne decides to take a study loan from the bank to pay the remaining tuition fees. There are two loan packages available for him to choose.
- | | |
|-----------|--|
| Package A | Compound interest of 4.7% per annum, repayment period of 5 years |
| Package B | Simple interest of 4.9% per annum, repayment period of 6 years. |
- (ii) Find the total repayment amount for Package A. [2]
- (iii) Find the total repayment amount for Package B. [2]
- (iv) Wayne is able to afford a maximum repayment of \$400 per month.
Explain, with working, which package Wayne should choose. [3]
- (b) In this part, use the fact that $1 \text{ light year} = 9.46 \times 10^{15} \text{ metres}$.
The distance of the star Sirius from the Sun is 8.6 light years.
A space probe travels at $70\,000 \text{ km/h}$.
Calculate the time taken for the probe to travel from the Sun to Sirius.
Give your answer in years, correct to three significant figures. [3]

3

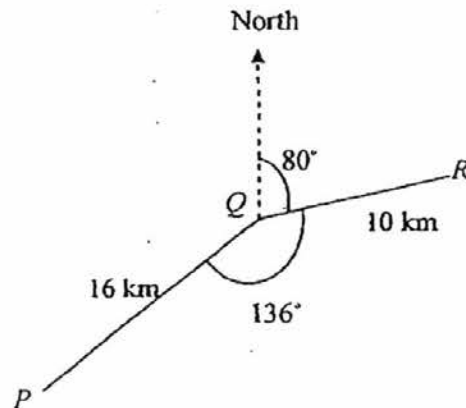


In the diagram, $\overrightarrow{OA} = \frac{1}{3}\overrightarrow{OP}$, and $\overrightarrow{OB} = \frac{1}{4}\overrightarrow{OQ}$. M is the midpoint of OQ , and $MX = \frac{1}{5}MP$.

- (a) Given that $\overrightarrow{OA} = \mathbf{a}$ and $\overrightarrow{OB} = \mathbf{b}$, express in terms of $\mathbf{a}$ and/or $\mathbf{b}$,
- (i) $\overrightarrow{AQ}$, [1]
 - (ii) $\overrightarrow{MP}$, [1]
 - (iii) $\overrightarrow{MX}$, [1]
 - (iv) $\overrightarrow{AX}$. [2]
- (b) Prove that AX , when produced, will pass through Q . [2]
- (c) Given that the area of triangle OPQ is 40 cm^2 , calculate the area of
- (i) triangle PMQ , [1]
 - (ii) triangle PQX , [1]
 - (iii) triangle OAB . [2]

- 4 A Mercedo car uses x litres of petrol to travel 240 km from Singapore to Malacca.
A Yotamo car uses 5 litres less than a Mercedo car to travel the same route.

- (a) Write down an expression, in terms of x , for
- (i) the distance travelled per litre by the Mercedo car, [1]
 - (ii) the distance travelled per litre by the Yotamo car. [1]
- (b) The mean value of the distance travelled per litre by both cars is 10 km/litre.
Form an equation in x to represent this information, and show that it reduces to $x^2 - 29x + 60 = 0$. [3]
- (c) Solve the equation $x^2 - 29x + 60 = 0$, giving your answers correct to 2 decimal places. [3]
- (d) Hence, find the distance travelled per litre by the Mercedo car. [2]



A ship sails 16 km from P to Q . It then sails 10 km from Q to R on a bearing of 080° .

- (a) Given that angle $PQR = 136^\circ$, calculate
- (i) the bearing of Q from P , [2]
 - (ii) the distance PR . [2]
- (b) The ship finally sails from R to a position T , which is due North of Q .
Given that angle $QTR = 35^\circ$, calculate
- (i) the distance RT , [2]
 - (ii) the shortest distance between the point Q and the ship as it sails from R to T . [2]
- (c) A helicopter is hovering vertically above point R , 2 km from sea level.
Calculate the angle of depression of P from the helicopter. [1]

- 6 A group of students took a multiple choice test containing 40 questions. The mean number of questions correctly answered, wrongly answered and unattempted are 28, 4 and 4 respectively.

Ashley attempted all questions with $(40 - x)$ questions answered correctly and x questions answered wrongly. Compared to Ashley, Benny answered 2 more questions correctly, answered 5 fewer questions wrongly and left 3 more questions unattempted.

The information can be represented by the matrix $Q = \begin{matrix} & \begin{matrix} \text{Mean} & A & B \end{matrix} \\ \begin{pmatrix} 28 & 40-x & 2 \\ 4 & x & -5 \\ 4 & 0 & 3 \end{pmatrix} & \begin{matrix} C \\ W \\ U \end{matrix} \end{matrix}$.

- (a) For every correct answer, 2 marks were awarded and for every wrong answer 1 mark was deducted.
No mark was awarded or deducted for unattempted questions.
Write down a 1×3 matrix R to represent this information. [1]
- (b) Find $S = RQ$, leaving your answer in terms of x . [2]
- (c) Benny claims that his score is better than Ashley's. Is his claim correct?
Justify your answer. [1]
- (d) Ashley's score is 4 marks higher than the mean mark. Find the value of x . [2]

7 Answer the whole of this question on a sheet of graph paper.

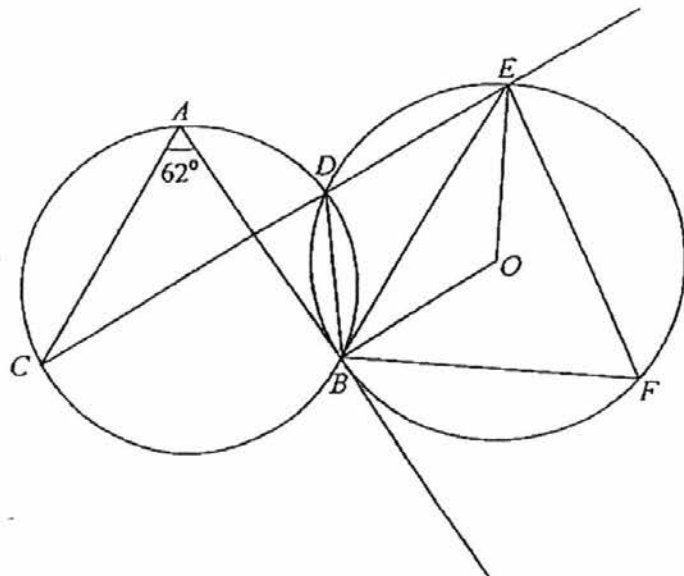
The variables x and y are connected by the equation $y = 0.25x^2(x - 6) + 3$.

The table below shows the corresponding values of x and y .

x	-1	0	1	2	2.5	3	4	5	5.5	6
y	1.25	3	1.75	-1	-2.47	-3.75	-5	h	-0.78	3

- (a) Calculate the value of h . [1]
- (b) Using a scale of 2 cm to represent 1 unit, draw a horizontal axis for $-1 \leq x \leq 6$.
Using a scale of 2 cm to represent 1 unit, draw a vertical axis for $-5 \leq y \leq 3$.
On your axes, plot the points given in the table and join them with a smooth curve. [3]
- (c) Use your graph to find the solutions of the equation $0.25x^2(x - 6) = -1$. [2]
- (d) By drawing a tangent, find the gradient of the curve at the point $x = 1$. [2]
- (e) By drawing a suitable straight line on your graph, find the solutions of the equation $0.25x^2(x - 6) + x + 1 = 0$. [3]

- 8 In the diagram, AB is a tangent to the circle with centre O , angle $CAB = 62^\circ$ and CDE is a straight line.



- (a) Stating your reasons clearly, find
- (i) angle CDB , [1]
 - (ii) angle BDE , [1]
 - (iii) angle EFB , [1]
 - (iv) angle BOE , [2]
 - (v) angle EBO . [2]
- (b) Find angle ABE . Hence, write a statement about the lines CA and BE . Give a reason for your answer. [3]

- 9 (a) The stem-and-leaf diagram shows the mass of 20 Secondary Four Judo students.

3	8	9					
4	1	2	4	5	5	8	9
5	0	1	1	2	3	5	<i>m</i>
6	0	1	2	6			

Key 4 | 1 means 41 kg

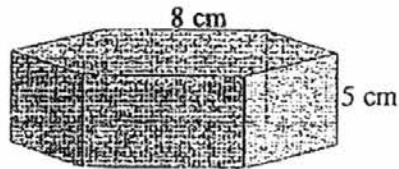
- (i) The interquartile range of the above distribution is 11.5 kg.
Show that the value of *m* is 7. [3]
- (ii) It was discovered that the masses of the students were recorded wrongly.
The correct masses were all 2 kg more than those recorded.
Explain how the interquartile range and the standard deviation of the mass
have been affected by this error. [2]
- (b) The 20 Secondary Four Judo students are divided into 2 groups.

Group A	Students whose mass is not more than 51 kg
Group B	Students whose mass is more than 51 kg

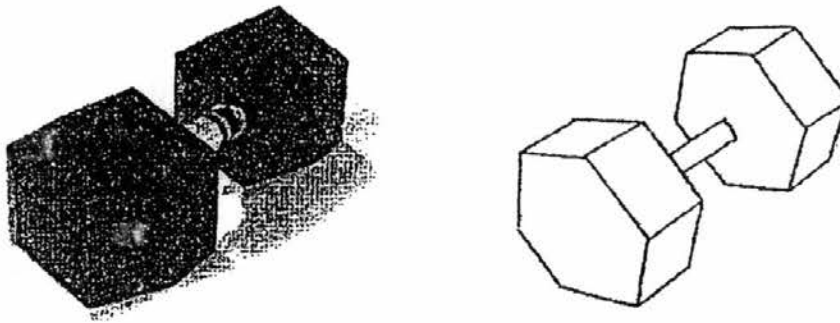
Mr Ng, the Judo instructor, selects two students at random, one after the other.

- (i) Draw a tree diagram to show the probabilities of the possible outcomes. [2]
- (ii) Find, as a fraction in its simplest form, the probability that
- (a) the first student is from Group B and the second student from Group A, [2]
- (b) at most one of the students is from Group B. [2]

- 10 The diagram shows a prism with a cross-section in the shape of a regular hexagon with sides 8 cm.
The prism has thickness 5 cm.



- (a) Show that the volume of the prism is 831.4 cm^3 , correct to 4 significant figures. [3]
- (b) A dumbbell can be modelled using a cylinder and two prisms from (a).
The dumbbell shown in the diagram below is made entirely of cast iron.
The density of cast iron is 7000 kg/m^3 .
1 kg is 2.204 pounds.



- (i) Given that the cylinder has radius 1.5 cm and the total length of the dumbbell is 25 cm, calculate the total volume of the dumbbell. [2]
- (ii) Shawn wants to do weight-lifting and his physical trainer advises him not to carry more than 25 pounds.

Determine if Shawn is able to use this dumbbell. Justify your decision with calculations. [5]

End of Paper

Mathematical Formulae

Compound interest

$$\text{Total amount} = P \left(1 + \frac{r}{100} \right)^n$$

Mensuration

$$\text{Curved surface area of a cone} = \pi r l$$

$$\text{Surface area of a sphere} = 4 \pi r^2$$

$$\text{Volume of a cone} = \frac{1}{3} \pi r^2 h$$

$$\text{Volume of a sphere} = \frac{4}{3} \pi r^3$$

$$\text{Area of triangle } ABC = \frac{1}{2} a b \sin C$$

$$\text{Arc length} = r \theta, \text{ where } \theta \text{ is in radians}$$

$$\text{Sector area} = \frac{1}{2} r^2 \theta, \text{ where } \theta \text{ is in radians}$$

Trigonometry

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

Statistics

$$\text{Mean} = \frac{\sum fx}{\sum f}$$

$$\text{Standard deviation} = \sqrt{\frac{\sum fx^2}{\sum f} - \left(\frac{\sum fx}{\sum f} \right)^2}$$

Answer all the questions.

- 1 (a) Simplify $\frac{4y}{3x^3} \div \frac{12y^4}{x}$. [1]
- (b) Simplify $\frac{4x-8}{7x^2-12x-4}$. [2]
- (c) Factorise completely $20m^2 - 45n^2$. [2]
- (d) Solve the equation $\frac{4x+5}{3x-2} = 4$. [2]
- (e) The volume, $V \text{ m}^3$, of a certain object is given by the formula $V = \frac{b^2}{2}(a+3h)$, where h is the height of the object in metres and a and b are constants.
- (i) Make b the subject of the formula. [2]
- (ii) Hence find b , when the volume and height of the object are 15 m^3 and 2.5 m respectively, and $a = 0.25$. [2]

$$\begin{aligned} \text{(a)} \quad \frac{4y}{3x^3} \div \frac{12y^4}{x} &= \frac{4y}{3x^3} \times \frac{x}{12y^4} \\ &= \frac{1}{9x^2y^3} \quad \boxed{\text{B1}} \end{aligned}$$

$$\begin{aligned} \text{(b)} \quad \frac{4x-8}{7x^2-12x-4} &= \frac{4(x-2)}{(7x+2)(x-2)} \quad \boxed{\text{M1: Factorise numerator/denominator}} \\ &= \frac{4}{7x+2} \quad \boxed{\text{A1}} \end{aligned}$$

$$\begin{aligned} \text{(c)} \quad 20m^2 - 45n^2 &= 5(4m^2 - 9n^2) \quad \boxed{\text{M1}} \\ &= 5(2m+3n)(2m-3n) \quad \boxed{\text{A1}} \end{aligned}$$

$$\begin{aligned} \text{(d)} \quad \frac{4x+5}{3x-2} &= 4 \\ 12x-8 &= 4x+5 \quad \boxed{\text{M1}} \\ 8x &= 13 \\ x &= 1\frac{5}{8} \quad \boxed{\text{A1}} \end{aligned}$$

$$\text{(e)(i)} \quad V = \frac{b^2}{2}(a+3h)$$

$$b^2(a+3h) = 2V$$

$$b^2 = \frac{2V}{a+3h} \quad \boxed{\text{M1}}$$

$$b = \pm \sqrt{\frac{2V}{a+3h}} \quad \boxed{\text{A1}}$$

$$\begin{aligned} \text{e(ii)} \quad b &= \pm \sqrt{\frac{2(15)}{0.25+3(2.5)}} \quad \boxed{\text{M1}} \\ &= 1.97 \text{ or } -1.97 \text{ (3 s.f.)} \quad \boxed{\text{A1}} \end{aligned}$$

- {11}

Package A	Compound interest of 4.7% per annum, repayment period of 5 years
Package B	Simple interest of 4.9% per annum, repayment period of 6 years.

- (b) In this part, use the fact that $1 \text{ light year} = 9.46 \times 10^{15} \text{ metres}$.
The distance of the star Sirius from the Sun is 8.6 light years.
A space probe travels at 70 000 km/h.
Calculate the time taken for the probe to travel from the Sun to Sirius.
Give your answer in years, correct to three significant figures. [3]

Therefore Wayne should take Package B as he can afford to pay the repayment amount within the loan period.

Alternatively

$$\text{For Package A, amount Wayne has to pay per month} = \frac{\$26421.21}{5 \times 12} \quad \boxed{\text{M1}}$$

$$= \$440.35 \text{ (nearest cent)}$$

$$\text{For Package B, amount Wayne has to pay per month} = \frac{\$27174}{6 \times 12} \quad \boxed{\text{M1}}$$

$$= \$377.42 \text{ (nearest cent)}$$

Since Wayne can only afford a maximum of \$400 per month, he should choose Package B. $\boxed{\text{A1}}$

Alternatively

$$\text{For Package A, no. of months required} = \frac{26421.21}{400} \quad \boxed{\text{M1}}$$

$$= 66.05 > 60$$

$$\text{For Package B, no. of months required} = \frac{27174}{400} \quad \boxed{\text{M1}}$$

$$= 67.935 < 72$$

Wayne should choose Package B as he will be able to pay in full ahead of schedule (72 months) as compared to Package A where he will have to prolong the loan period by 6.05 months. $\boxed{\text{A1}}$

(b) Distance of Sirius from Sun = $8.6 \times 9.46 \times 10^{15} \text{ m}$ $\boxed{\text{M1: convert to m}}$

$$= 8.1356 \times 10^{16} \text{ m}$$

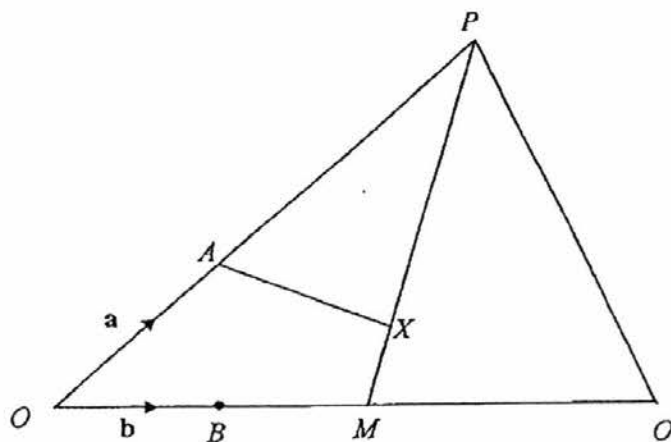
$$= 8.1356 \times 10^{13} \text{ km}$$

$$\text{Time taken by probe} = \frac{8.1356 \times 10^{13}}{70\,000} \text{ h}$$

$$= \frac{8.1356 \times 10^{13}}{70\,000 \times 24 \times 365} \text{ years} \quad \boxed{\text{M1: convert to years}}$$

$$= 132\,674.4 \dots \text{ years}$$

$$= 133\,000 \text{ years (3 sig fig)} \quad \boxed{\text{A1}}$$



In the diagram, $\overrightarrow{OA} = \frac{1}{3}\overrightarrow{OP}$, and $\overrightarrow{OB} = \frac{1}{4}\overrightarrow{OQ}$. M is the midpoint of OQ , and $MX = \frac{1}{5}MP$.

- (a) Given that $\overrightarrow{OA} = \mathbf{a}$ and $\overrightarrow{OB} = \mathbf{b}$, express in terms of $\mathbf{a}$ and/or $\mathbf{b}$,
- (i) $\overrightarrow{AQ}$, [1]
 - (ii) $\overrightarrow{MP}$, [1]
 - (iii) $\overrightarrow{MX}$, [1]
 - (iv) $\overrightarrow{AX}$. [2]
- (b) Prove that AX , when produced, will pass through Q . [2]
- (c) Given that the area of triangle OPQ is 40 cm^2 , calculate the area of
- (i) triangle PMQ , [1]
 - (ii) triangle PQX , [1]
 - (iii) triangle OAB . [2]

- (a) (i) $\overrightarrow{AQ} = 4\mathbf{b} - \mathbf{a}$ [B1]
- (ii) $\overrightarrow{MP} = \overrightarrow{OP} - \overrightarrow{OM}$
 $= 3\mathbf{a} - 2\mathbf{b}$ [B1]
- (iii) $\overrightarrow{MX} = \frac{1}{5}\overrightarrow{MP}$
 $= \frac{1}{5}(3\mathbf{a} - 2\mathbf{b})$ [B1]
- (iv) $\overrightarrow{AX} = \overrightarrow{AP} - \overrightarrow{XP}$
 $= 2\mathbf{a} - \frac{4}{5}(3\mathbf{a} - 2\mathbf{b})$ [M1]
 $= -\frac{2}{5}\mathbf{a} + \frac{8}{5}\mathbf{b}$
 $= \frac{2}{5}(4\mathbf{b} - \mathbf{a})$ [A1]

(b) $\overline{AQ} = 4b - a$

$$\overline{AX} = \frac{2}{5}(4b - a) \quad \boxed{\text{M1}}$$

$$= \frac{2}{5} \overline{AQ}$$

Hence $\overline{AQ} \parallel \overline{AX}$ and since A is a common point, A , X and Q are collinear.

A1

Thus AX , when produced, will pass through Q .

(c) (i) Area of $\triangle PMQ = \frac{1}{2} \times 40 \text{ cm}^2$
 $= 20 \text{ cm}^2$

B1

(ii) Area of $\triangle PQX = \frac{4}{5}$ Area of $\triangle PMQ$
 $= \frac{4}{5} \times 20 \text{ cm}^2$
 $= 16 \text{ cm}^2$

B1

(iii) Area of $\triangle OAB = \frac{1}{3}$ Area of $\triangle OPB$
 $= \frac{1}{3} \left(\frac{1}{4} \text{ Area of } \triangle OPQ \right)$
 $= \frac{1}{12} \times 40 \text{ cm}^2$
 $= 3\frac{1}{3} \text{ cm}^2$

M1

A1

- 4 A Mercedo car uses x litres of petrol to travel 240 km from Singapore to Malacca.
A Yotamo car uses 5 litres less than a Mercedo car to travel the same route.

(a) Write down an expression, in terms of x , for

(i) the distance travelled per litre by the Mercedo car, [1]

(ii) the distance travelled per litre by the Yotamo car. [1]

(b) The mean value of the distance travelled per litre by both cars is 10 km/litre.
Form an equation in x to represent this information, and show that it reduces to $x^2 - 29x + 60 = 0$. [3]

(c) Solve the equation $x^2 - 29x + 60 = 0$, giving your answers correct to 2 decimal places. [3]

(d) Hence, find the distance travelled per litre by the Mercedo car. [2]

(a) (i) Distance = $\frac{240}{x}$ km/litre

B1

(ii) Distance = $\frac{240}{x-5}$ km/litre

B1

(b) $\frac{1}{2} \left(\frac{240}{x} + \frac{240}{x-5} \right) = 10$

M1: Forming equation

$$\left(\frac{240}{x} + \frac{240}{x-5} \right) = 20$$

$$\frac{240(x-5) + 240x}{x(x-5)} = 20$$

M1: Common denominator

$$240x - 1200 + 240x = 20x^2 - 100x$$

$$20x^2 - 580x + 1200 = 0$$

M1 for simplification

$$x^2 - 29x + 60 = 0$$

AG

Hence shown.

(c) $x^2 - 29x + 60 = 0$

$$x = \frac{-(-29) \pm \sqrt{(-29)^2 - 4(1)(60)}}{2(1)}$$

M1: Substitution of values

$$= \frac{29 \pm \sqrt{601}}{2}$$

$$= 26.757\dots \text{ or } 2.2423\dots$$

$$= 26.76 \text{ or } 2.24 \text{ (2 dec pl)}$$

A1

A1

(d) Reject $x = 2.24$ as $x - 5$ will be negative.

B1

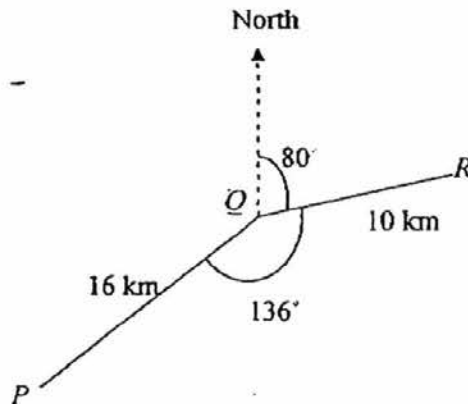
$$\text{Distance travelled per litre by the Mercedo car} = \frac{240}{26.757\dots} \text{ km}$$

$$= 8.96939\dots \text{ km}$$

$$= 8.97 \text{ km (3 sig fig)}$$

A1

5



A ship sails 16 km from P to Q . It then sails 10 km from Q to R on a bearing of 080° .

- (a) Given that angle $PQR = 136^\circ$, calculate
- (i) the bearing of Q from P , [2]
 - (ii) the distance PR . [2]
- (b) The ship finally sails from R to a position T , which is due North of Q . Given that angle $QTR = 35^\circ$, calculate
- (i) the distance RT , [2]
 - (ii) the shortest distance between the point Q and the ship as it sails from R to T . [2]
- (c) A helicopter is hovering vertically above point R , 2 km from sea level. Calculate the angle of depression of P from the helicopter. [1]

(a) (i) $\angle NQP = 360^\circ - 80^\circ - 136^\circ$ ($\angle$ s at a point) M1
 $= 144^\circ$

$\angle N_1PQ = 180^\circ - 144^\circ$ (int. $\angle$ s, $N_1P \parallel NQ$)
 $= 36^\circ$
 $\therefore$ Bearing of Q from P is 036° . A1

(ii) Distance $PR = \sqrt{16^2 + 10^2 - 2(16)(10)\cos 136^\circ}$ M1
 $= 24.211\dots \text{km}$
 $= 24.2 \text{ km (3 sig fig)}$ A1

(b) (i) Using sine rule,
 $\frac{RT}{\sin 80^\circ} = \frac{10}{\sin 35^\circ}$ M1
 $RT = \frac{10 \sin 80^\circ}{\sin 35^\circ}$
 $= 17.1695\dots \text{km}$
 $= 17.2 \text{ km}$ A1

- (ii) Let the shortest distance be
- QX
- .

$$\angle QRT = 180^\circ - 80^\circ - 35^\circ$$

$$= 65^\circ$$

$$\sin 65^\circ = \frac{QX}{10}$$

M1

$$QX = 10 \sin 65^\circ$$

$$= 9.06 \text{ km (3 sig fig)}$$

A1

- (c) Let the angle of depression of
- P
- from the helicopter be
- θ
- .

$$\tan \theta = \frac{2}{24.211\dots}$$

$$\theta = 4.7^\circ \text{ (1 dec pl)}$$

B1

- 6 A group of students took a multiple choice test containing 40 questions.

The mean number of questions correctly answered, wrongly answered and attempted are 28, 4 and 4 respectively.

Ashley attempted all questions with $(40 - x)$ questions answered correctly and x questions answered wrongly. Compared to Ashley, Benny answered 2 more questions correctly, answered 5 fewer questions wrongly and left 3 more questions attempted.

Mean A B

$$\text{The information can be represented by the matrix } Q = \begin{pmatrix} 28 & 40-x & 2 \\ 4 & x & -5 \\ 4 & 0 & 3 \end{pmatrix} \begin{matrix} C \\ W \\ U \end{matrix}$$

- (a) For every correct answer, 2 marks were awarded and for every wrong answer 1 mark was deducted.

No mark was awarded or deducted for attempted questions.

Write down a 1×3 matrix R to represent this information.

[1]

- (b) Find
- $S = RQ$
- , leaving your answer in terms of
- x
- .

[2]

- (c) Benny claims that his score is better than Ashley's. Is his claim correct?

Justify your answer.

[1]

- (d) Ashley's score is 4 marks higher than the mean mark. Find the value of
- x
- .

[2]

$$(a) R = \begin{pmatrix} 2 & -1 & 0 \end{pmatrix}$$

B1

$$(b) S = RQ$$

$$= \begin{pmatrix} 2 & -1 & 0 \end{pmatrix} \begin{pmatrix} 28 & 40-x & 2 \\ 4 & x & -5 \\ 4 & 0 & 3 \end{pmatrix}$$

M1

$$= \begin{pmatrix} 52 & 80-3x & 9 \end{pmatrix}$$

A1

- (c) Yes. Benny's score is 9 marks higher than Ashley's.

B1

(d) $80 - 3x = 52 + 4$ M1
 $3x = 24$
 $x = 8$ A1

7 Answer the whole of this question on a sheet of graph paper.

The variables x and y are connected by the equation $y = 0.25x^2(x - 6) + 3$.

The table below shows the corresponding values of x and y .

x	-1	0	1	2	2.5	3	4	5	5.5	6
y	1.25	3	1.75	-1	-2.47	-3.75	-5	h	-0.78	3

- (a) Calculate the value of h . [1]
- (b) Using a scale of 2 cm to represent 1 unit, draw a horizontal axis for $-1 \leq x \leq 6$.
 Using a scale of 2 cm to represent 1 unit, draw a vertical axis for $-5 \leq y \leq 3$.
 On your axes, plot the points given in the table and join them with a smooth curve. [3]
- (c) Use your graph to find the solutions of the equation $0.25x^2(x - 6) = -1$. [2]
- (d) By drawing a tangent, find the gradient of the curve at the point $x = 1$. [2]
- (e) By drawing a suitable straight line on your graph, find the solutions of the equation $0.25x^2(x - 6) + x + 1 = 0$. [3]

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(Curve crosses by 2 small sq)

(a) $y = 0.25x^2(x-6) + 3$
 when $x=5$, $y = 0.25(5)^2(5-6) + 3$
 $= -3.25$ or (B1)

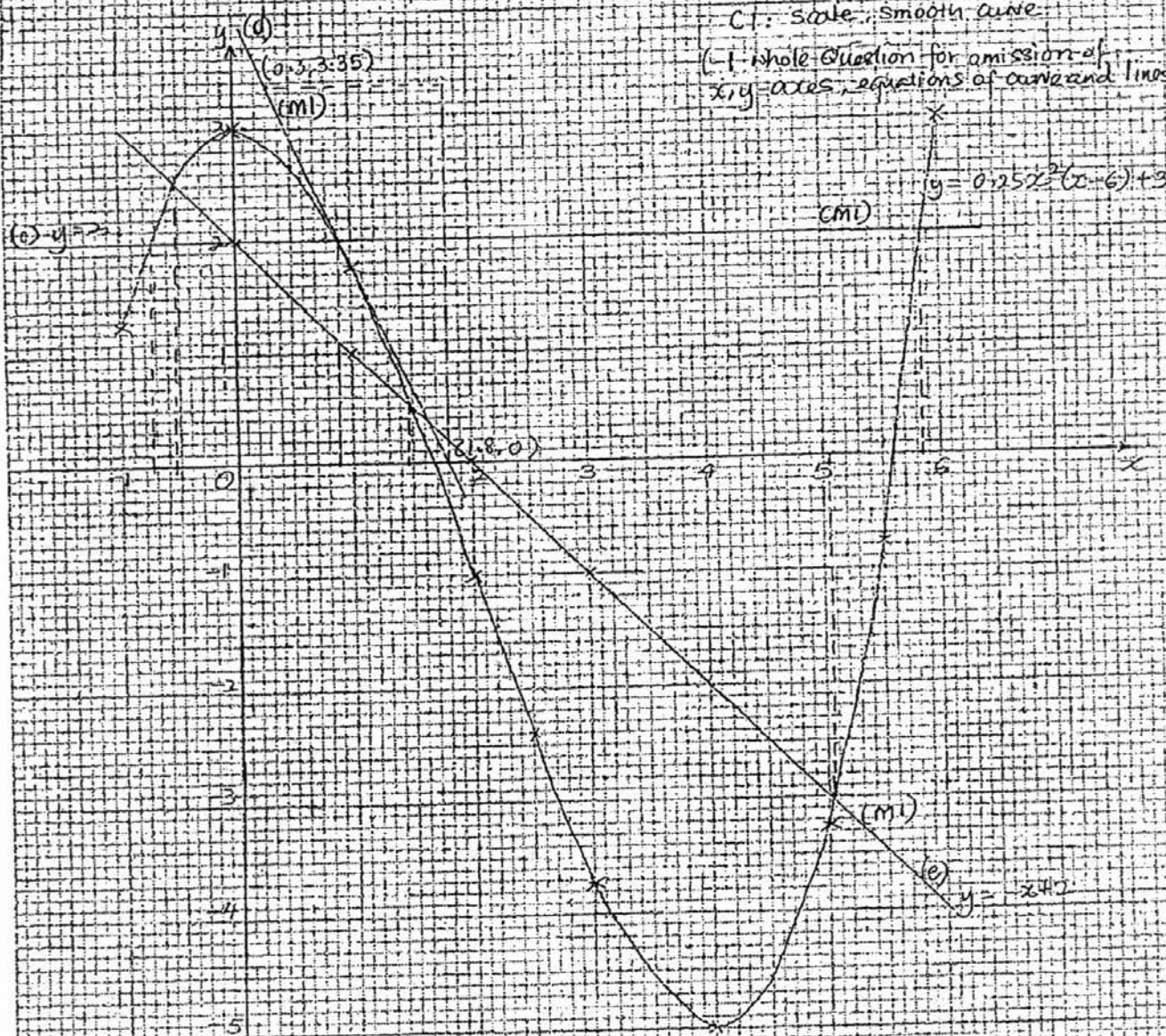
(b) Graph

P2: All points correctly plotted

P1: At least 6 points correctly plotted

C1: scale, smooth curve

(1 whole question for omission of x, y axes, equations of curve and lines)



(c) $0.25x^2(x-6) = -1$
 $0.25x^2(x-6) + 3 = -1 + 3$

Draw $y = 2$

from the graph, $x = 0.75, 0.9$ or 5.85 (A1 for 3 answers)

(d) Gradient $= \frac{3.35 - 0}{0.3 - 0} = 11.17$
 $= 11.17$ (3 sf) (A1)

(e) $0.25x^2(x-6) + x + 1 = 0$

$0.25x^2(x-6) + 3 + x + 1 = 3 = 0$

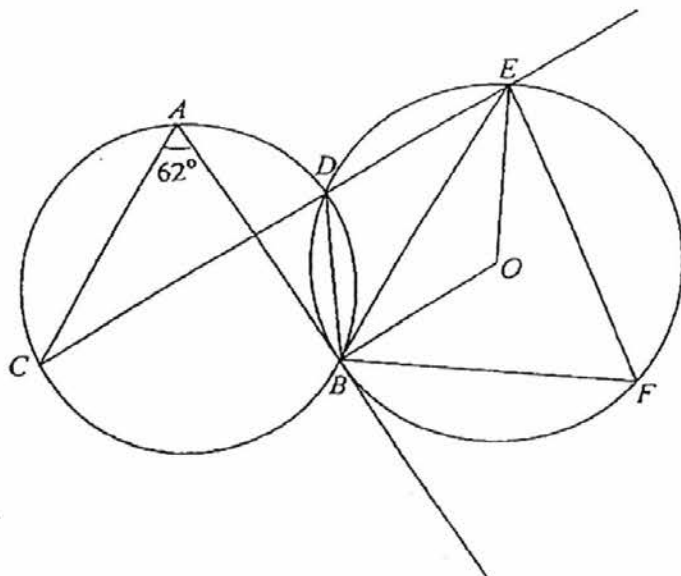
Draw $y = -x + 2$

from the graph, $x = 0.55, 1.45$ or 5.05

(A1)

for all 3 answers

- 8 In the diagram, AB is a tangent to the circle with centre O , angle $CAB = 62^\circ$ and CDE is a straight line.



- (a) Stating your reasons clearly, find
- (i) angle CDB , [1]
 - (ii) angle BDE , [1]
 - (iii) angle EFB , [1]
 - (iv) angle BOE , [2]
 - (v) angle EBO . [2]
- (b) Find angle ABE . Hence, write a statement about the lines CA and BE . Give a reason for your answer. [3]

(a) (i) $\angle CDB = \angle CAB$ ($\angle$ s in the same segment)
 $= 62^\circ$ B1

(ii) $\angle BDE = 180^\circ - 62^\circ$ (adj. $\angle$ s on a str line)
 $= 118^\circ$ B1

(iii) $\angle EFB = 180^\circ - \angle BDE$ ($\angle$ s in opp segments)
 $= 180^\circ - 118^\circ$
 $= 62^\circ$ B1

(iv) Reflex $\angle BOE = 2 \times 118^\circ$ ($\angle$ at centre $= 2 \times \angle$ at circumference)
 $= 236^\circ$ M1

Therefore $\angle BOE = 360^\circ - 236^\circ$
 $= 124^\circ$ A1

$$(v) \quad \angle EBO = \frac{180^\circ - 124^\circ}{2} \text{ (base } \angle\text{s of isos. } \Delta) \quad \boxed{\text{M1}}$$

$$= 28^\circ \quad \boxed{\text{A1}}$$

$$(b) \quad \angle OBF = 90^\circ \text{ (tan } \perp \text{ rad)} \quad \boxed{\text{M1}}$$

$$\angle ABE + 28^\circ + 90^\circ = 180^\circ \text{ (adj } \angle\text{s on a str. line)}$$

$$\angle ABE = 62^\circ \quad \boxed{\text{A1}}$$

Alternatively,

$$\angle OBE = 28^\circ \text{ (base angles of isosceles } \Delta)$$

$$\angle ABE = 90^\circ - 28^\circ$$

$$= 62^\circ \quad \boxed{\text{A1}}$$

Since $\angle CAB = \angle ABE = 62^\circ$, lines CA and BE are parallel, due to alternate angles. $\boxed{\text{B1}}$

- 9 (a) The stem-and-leaf diagram shows the mass of 20 Secondary Four Judo students.

3	8	9					
4	1	2	4	5	5	8	9
5	0	1	1	2	3	5	<i>m</i>
6	0	1	2	6			

Key 4 | 1 means 41 kg

- (i) The interquartile range of the above distribution is 11.5 kg.
Show that the value of m is 7. [3]
- (ii) It was discovered that the masses of the students were recorded wrongly. The correct masses were all 2 kg more than those recorded.
Explain how the interquartile range and the standard deviation of the mass have been affected by this error. [2]

- (b) The 20 Secondary Four Judo students are divided into 2 groups.

Group A	Students whose mass is not more than 51 kg
Group B	Students whose mass is more than 51 kg

Mr Ng, the Judo instructor, selects two students at random, one after the other.

- (i) Draw a tree diagram to show the probabilities of the possible outcomes. [2]
- (ii) Find, as a fraction in its simplest form, the probability that
- (a) the first student is from Group B and the second student from Group A, [2]
- (b) at most one of the students is from Group B. [2]

(a) (i) Lower Quartile = $\frac{44+45}{2}$ kg

= 44.5 kg

Upper Quartile = 44.5 + 11.5 kg

= 56 kg

$\frac{55+x}{2} = 56$

$x = 112 - 55$

= 57

Therefore $m = 7$

M1

M1

M1

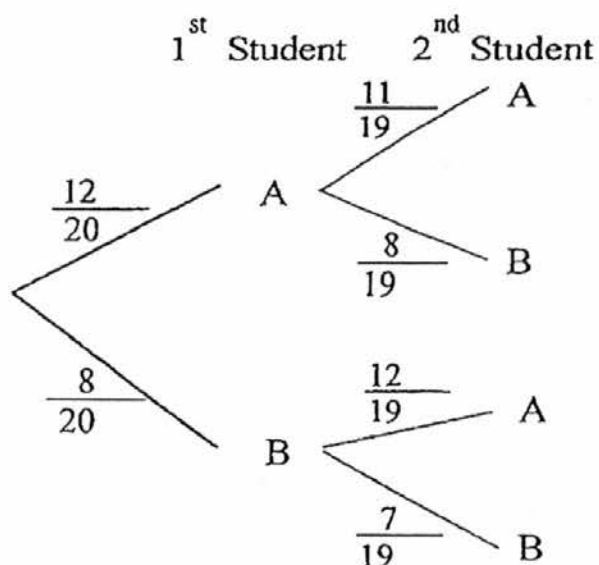
AG

(ii) Both will not be affected by the error.

B1

B1

(b) (i)



B2

(ii)(a) $P(\text{1st student from Group B and 2nd from Group A}) = \frac{8}{20} \times \frac{12}{19}$

M1

$= \frac{24}{95}$

A1

(ii)(b) $P(\text{at most one student from Group B}) = 1 - \frac{8}{20} \times \frac{7}{19}$

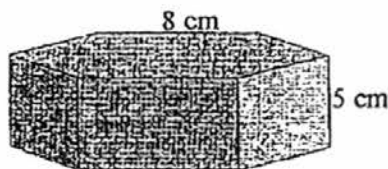
M1

$= 1 - \frac{14}{95}$

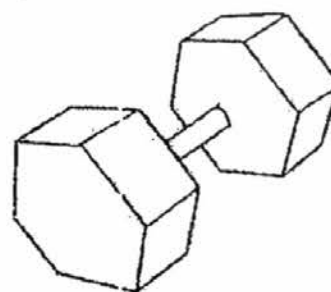
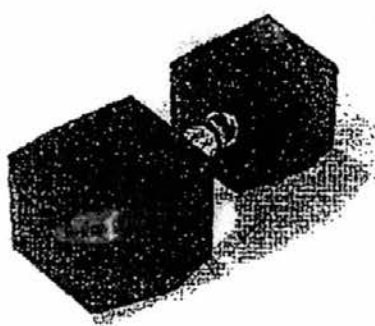
$= \frac{81}{95}$

A1

- 10 The diagram shows a prism with a cross-section in the shape of a regular hexagon with sides 8 cm.
The prism has thickness 5 cm.



- (a) Show that the volume of the prism is 831.4 cm^3 , correct to 4 significant figures. [3]
- (b) A dumbbell can be modelled using a cylinder and two prisms from (a).
The dumbbell shown in the diagram below is made entirely of cast iron.
The density of cast iron is 7000 kg/m^3 .
1 kg is 2.204 pounds.



- (i) Given that the cylinder has radius 1.5 cm and the total length of the dumbbell is 25 cm, calculate the total volume of the dumbbell. [2]
- (ii) Shawn wants to do weight-lifting and his physical trainer advises him not to carry more than 25 pounds.

Determine if Shawn is able to use this dumbbell. Justify your decision with calculations. [5]

- (a) The hexagon is made up of 6 congruent triangles.

$$\begin{aligned}\text{Area of hexagon} &= 6 \times \left(\frac{1}{2} \times 8 \times 8 \times \sin 60^\circ \right) \text{ cm}^2 && \boxed{\text{M1}} \\ &= 192 \sin 60^\circ \text{ cm}^2 \\ &= 166.276... \text{ cm}^2 \\ &= 166 \text{ cm}^2 \text{ (3 sig fig)}\end{aligned}$$

$$\begin{aligned}\text{Volume of prism} &= 192 \sin 60^\circ \times 5 \text{ cm}^3 && \boxed{\text{M1}} \\ &= 831.384... \text{ cm}^3 && \boxed{\text{A1}} \\ &= 831.4 \text{ cm}^3\end{aligned}$$

- (b) (i) Total volume of dumbbell = $831.384... \times 2 + \pi(1.5)^2(15) \text{ cm}^3$ M1: vol of cylinder
 $= 1768.79... \text{ cm}^3$
 $= 1770 \text{ cm}^3 \text{ (3 sig fig)}$ A1

- (ii) Mass of dumbbell = $\frac{1768.79...}{100^3} \times 7000 \text{ kg}$ M1: conversion of cm^3 to m^3
M1: $\times 7000$ to find mass
 $= 12.3816 \text{ kg}$

$$\frac{25}{2.204} = 11.343... \text{ kg} \quad \boxed{\text{M1: convert pounds to kg}}$$

Since $12.3816 > 11.343$, Shawn cannot use this dumbbell. A1: Conclusion

M1

OR

$$12.3816 ... \times 2.204 = 27.289 \text{ pounds} \quad \boxed{\text{M1: convert kg to pounds}}$$

Since $27.289 > 25$, Shawn cannot use this dumbbell. A1: Conclusion

M1

End of Paper