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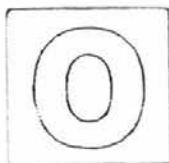
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GAN ENG SENG SCHOOL
Preliminary 1 Examination 2016



CANDIDATE
NAME

CLASS

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INDEX
NUMBER

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MATHEMATICS

Paper 2

4048/02

09 May 2016

2 hours 30 minutes

Sec 4 Express

Additional Materials: Answer Paper
Graph Paper (1 sheet)

READ THESE INSTRUCTIONS FIRST

Write your class, index number and name on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

~~Answer all~~ questions.

If working is needed for any question it must be shown with the answer.

Omission of essential working will result in loss of marks.

Calculators should be used where appropriate.

If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.

For π use either your calculator value or 3.142, unless the question requires the answer in terms of π .

The number of marks is given in brackets [] at the end of each question or part question.

The total of the marks for this paper is 100.

	For Examiner's Use
Total	100

Mathematical Formulae

Compound interest

$$\text{Total amount} = P \left(1 + \frac{r}{100} \right)^n$$

Mensuration

$$\text{Curved surface area of a cone} = \pi r l$$

$$\text{Surface area of a sphere} = 4\pi r^2$$

$$\text{Volume of a cone} = \frac{1}{3} \pi r^2 h$$

$$\text{Volume of a sphere} = \frac{4}{3} \pi r^3$$

$$\text{Area of triangle } ABC = \frac{1}{2} ab \sin C$$

$$\text{Arc length} = r\theta, \text{ where } \theta \text{ is in radians}$$

$$\text{Sector area} = \frac{1}{2} r^2 \theta, \text{ where } \theta \text{ is in radians}$$

Trigonometry

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

Statistics

$$\text{Mean} = \frac{\sum fx}{\sum f}$$

$$\text{Standard deviation} = \sqrt{\frac{\sum fx^2}{\sum f} - \left(\frac{\sum fx}{\sum f} \right)^2}$$

Answer all the questions.

1. Mrs Tay went shopping at PP Mall one Saturday afternoon.

- (a) The parking charges for PP Mall are as follows:

6 a.m. to 10 a.m.	10 a.m. to 6 p.m.
\$0.50 for 1 st hour	\$1.30 for 1 st hour
\$0.40 for next subsequent 15 minutes	\$0.40 for next subsequent 15 minutes
\$2.10 per entry from 6 p.m. to 6 a.m. the following day.	

* Parking charges are on 'per hour or part thereof basis'.

- (i) Mrs Tay parked her car in PP Mall's carpark from 9.55 a.m. to 1.22 p.m. Calculate her car park charges. [3]
- (ii) Advise her on how she can pay less for parking. [1]
- (b) Mrs Tay bought a pair of high heeled shoes. The shoes were on sale at a discount of 60%. If Mrs Tay paid \$35.96 for the pair of shoes, calculate their original retail price. [2]
- (c) Mrs Tay had her lunch at a Thai restaurant in PP Mall. She ordered the following:

Fried Olive Rice	\$9.80
Seafood Tom Yum Soup	\$12.80
Iced Red Ruby	\$4.80

* All prices are subjected to 10% service charge and an additional 7% GST.

Calculate the total amount Mrs Tay paid for lunch. [2]

- (d) As Mrs Tay was planning to go to Australia for the June holidays, she went to the Money Changer to buy some Australian dollars. The exchange rate between Singapore dollars (SGD) and Australian dollars (AUD) at the Money Changer were as follows:

Currency Name	Units	Buying Rate	Selling Rate
Australian Dollars (AUD)	1	1.0050	1.0220

If Mrs Tay exchanged SGD3000 for Australian dollars, how much Australian dollars did she get? [2]

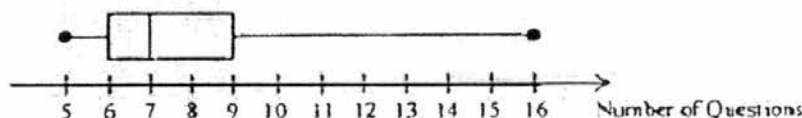
2. (a) Simplify $\left(5 - \frac{2}{y}\right)\left(\frac{2}{y} + 5\right)$. [1]
- (b) Solve $6x^2 + 7x = 20$. Hence solve $6(z-1)^2 + 7z = 27$. [3]
- (c) If $a^2 + b^2 = w$ and $(a-b)^2 = 9$, express $(a+b)^2$ in terms of w . [3]
- (d) Without drawing the graph, explain if $(-1, 3)$ is a point on the graph of $y = 6x^3 - 7x + 1$. [1]
- (e) Solve $5^{11} - 5^{10} = \frac{20}{5^{2v}}$. [3]

3. (a) $2n$ and $\frac{13n+7}{n+1}$ are two consecutive positive even integers.
- Form an equation in n and show that it reduces to $2n^2 - 9n - 5 = 0$. [2]
 - Solve the equation to find the two consecutive positive even integers. [2]
 - Explain clearly why you rejected one of the solutions for n . [1]
- (b) (i) List all the factors of 19. [1]
- (ii) Given that g and h are positive integers such that $g^2 - h^2 = 19$, find the value of $\frac{2g-2h}{g+h}$. [3]
- (iii) Explain how your answer to (b)(ii) would be different if $g^2 - h^2 = 15$ instead of 19. [1]

4. (a) Mr Phua wanted to find out how much extra mathematics practice his students do in addition to the homework questions he assigns them. The table below shows the number of extra mathematics questions done by his students per day.

Number of Extra Questions	0	1	2	3	4	5	30
Number of Students	10	8	5	$2p-2$	$p+5$	2	1

- Explain whether the mean or the median is a better measure of the average number of extra practice questions done by Mr Phua's students. [1]
 - ~~Given that the mode is 0, form inequalities and solve them to find all the possible integer values of p .~~ [3]
- (b) Mr Ng also wanted to find out how much extra mathematics practice his students do. He tasked his class' mathematics representative to collect the data. The mathematics representative presented the data collected to Mr Ng as follows:



- State the median, interquartile range and range for this data set. [3]
- Mr Ng later discovered that his mathematics representative had included his daily homework assignment of 5 questions when collecting the data. State how this error would have affected the median, interquartile range and range of the data set. [3]

5. Answer the whole of this question on a single sheet of graph paper.

A manufacturing firm makes a profit of y thousand dollars when it produces x thousand items. It is given that $y = \frac{(100-x)(x-10)}{5}$ for $0 \leq x \leq 100$.

The table below shows some values of x and the corresponding values of y .

x	0	10	20	30	40	50	60	70	80	90	100
y	-200	0	160	a	360	b	400	360	280	160	0

- (a) Find the values of a and b . [1]
- (b) Using a scale of 2 cm to represent 10 units on the x -axis and a scale of 1 cm to represent 50 units on the y -axis, draw the graph of $y = \frac{(100-x)(x-10)}{5}$ for $0 \leq x \leq 100$. [3]
- (c) Use your graph to find the
- (i) number of items the manufacturing firm must produce to maximise profit, [1]
 - (ii) the minimum number of items the manufacturing firm must produce to break even (that is, to make neither a profit nor a loss), [1]
 - (iii) the fixed operating cost of the manufacturing firm, [1]
 - (iv) the range for the number of items the manufacturing firm must produce in order to earn at least \$300 000. [1]
- (d) On the same axes, draw the line $y = 8x$.
What can you say about the profit on any point of this line? [2]
- (e) For what values of x will the graph have negative gradients?
Should the manufacturing firm continue to *increase* production when the gradient is negative? Explain your answer. [2]

- 6 The points P , Q and R are vertices of a right-angled triangle such that $\angle PQR$ is a right angle. The point P is $(-4, 4)$, point Q is $(0, 8)$ and R is $(r, 0)$ where $r > 0$.
- (a) Find the equation of PQ . [2]
- (b) Given that the gradient of QR is -1 , find the coordinates of R . [1]
- (c) It is given that $\vec{QS} = \begin{pmatrix} 4 \\ 4 \end{pmatrix}$.
- (i) Find $|\vec{QS}|$. [1]
- (ii) Explain why $\triangle RPS$ is an isosceles triangle. [2]
- (iii) Given that T is $(-7, 1)$, express $\vec{QT}$ as a column vector. [1]
- (iv) What can you deduce about T , Q and S ? Explain your answer clearly. [2]
- (v) State the ratio of Area of $\triangle TQR$: Area of $\triangle SQR$. [1]

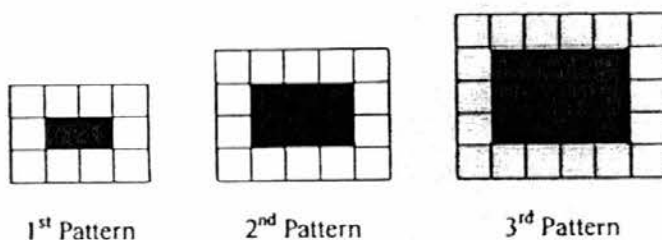
- 7(a) Rainfall is plentiful in Singapore. Based on long-term records from 1869 to 2015, it rains an average of 178 days (out of 365 days) in a year.

(Reference: <http://www.weather.gov.sg/climate-climate-of-singapore/>)

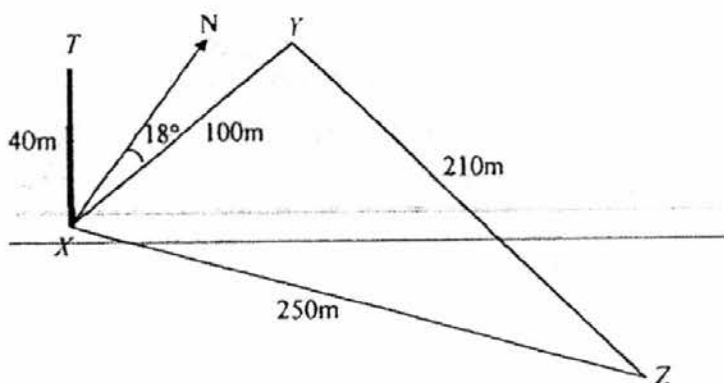
On a rainy day in Singapore, the probability that Mr Chee brings an umbrella to work is 0.75. On a non-rainy day in Singapore, the probability that he brings an umbrella to work is $\frac{1}{6}$.

- (i) Draw a tree diagram to illustrate the information. [2]
- (ii) Find the probability that on any particular day in Singapore,
- (a) it is a rainy day and Mr Chee brings an umbrella to work, [1]
- (b) Mr Chee does not bring an umbrella to work. [2]

- 7(b) The diagram below shows a sequence of tiling patterns formed by using white and black tiles.



- (i) T_n is the number of **white** tiles in the n^{th} pattern where $T_1 = 10$. Find
- (a) T_5 , [1]
- (b) T_n in terms of n . [1]
- (ii) Explain clearly why it is not possible to have 244 **black** tiles in any pattern. [3]
8. In the diagram below, XYZ represents a triangular piece of land and XT represents a vertical tower. Y is 100 m from X on a bearing of 018° and Z is 250 m from X . The length of YZ is 210 m and the height of the tower is 40 m.



- (a) Calculate the bearing of Z from X . [3]
- (b) The piece of land is to be sold at a price of \$120 000 per hectare. Given that 1 hectare is equivalent to 10 000 square metres, find the selling price of the piece of land, corrected to the nearest cent. [3]
- (c) Mdm Phoa walks along ZY , starting from Z . Calculate the largest angle of elevation of the top of the tower when viewed by Mdm Phoa as she walks along ZY produced. [3]

9. Ms Thng organised a mathematics competition for all Secondary Four and Five students. In the competition, competitors were assessed on the three content strands of mathematics: Number & Algebra, Geometry & Measurement, Statistics & Probability.

The total scores of the competitors were computed according to the weighting shown in the following table:

Content strand	Number & Algebra	Geometry & Measurement	Statistics & Probability
Weighting	3	4	3

- (a) Express the weighting information as a 3×1 matrix W . [1]

Mr Teo sent three of his students to participate in the competition. The table below shows the scores obtained by Mr Teo's students for each of the 3 content strands.

Student	Number & Algebra	Geometry & Measurement	Statistics & Probability
A	56	57	31
B	51	b	54
C	42	44	68

- (b) (i) Express the scores information in a 3×3 matrix S . [1]

- (ii) Given that $T = SW$, find the matrix T , in terms of b . [2]

- (c) To achieve a certificate of distinction, a candidate must obtain a total score greater than or equal to x hundred points. If Student C obtained a certificate of distinction while Student A did not, write down the value of x . [1]

- (d) Student B did not achieve a certificate of distinction. Find the maximum value of b . [2]

Mdm Chai also sent two of her students to participate in the competition. The table below shows the scores obtained by Mdm Chai's students for each of the 3 content strands.

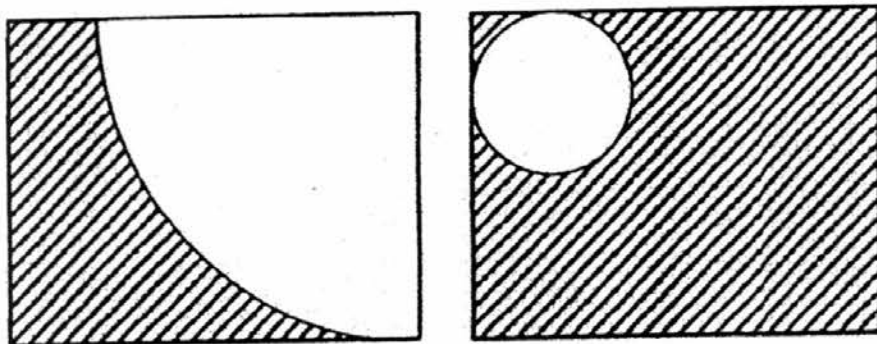
Student	Number & Algebra	Geometry & Measurement	Statistics & Probability
D	d	52	46
E	47	49	e

- (e) Express d in terms of e if both students D and E scored the same number of points. [2]

10. Miss Ngoh used 2 vanguard sheets to make a closed cone for display at the mathematics booth set up during the school's open house.

The dimension of each vanguard sheet is 25 inches by 20 inches.

After some calculations, Miss Ngoh cut a sector from one vanguard sheet and a circle from the other vanguard sheet. The shaded region shows the amount of vanguard sheet left over.



Useful Information
1 inch = 2.54 centimetres

- (a) Find the arc length of the sector, in centimetres. Leave your answer in terms of π . [1]
- (b) Calculate the total surface area and the volume of the cone made by Miss Ngoh. Give your answers correct to the nearest square centimetre and cubic centimetre respectively. [5]
- (c) Mrs Tan, the Head of Department, had originally instructed Miss Ngoh not to waste more than 40% of the vanguard sheets. Did Miss Ngoh manage to adhere to Mrs Tan's instructions? Justify your answer with calculations. [3]

End of Paper

Answer Key

Question	Answer
1	(a)(i) \$4.50
	(a)(ii) Mrs Tay can go to PP Mall after 6pm to enjoy a per entry charge of \$2.10.
	(b) \$89.90
	(c) \$32.25
	(d) AUD 2935.42
2	(a) $25 - \frac{4}{y^2}$
	(b) $x = 1\frac{1}{3}$ or $-2\frac{1}{2}$, $z = 2\frac{1}{3}$ or $-1\frac{1}{2}$
	(c) $2w - 9$
	(d) $(-1, 3)$ is not a point on the graph of $y = 6x^3 - 7x + 1$.
	(e) $y = -4.5$
3	(a)(i) $\frac{13n+7}{n+1} = 2n+2$
	(a)(ii) 10 and 12
	(a)(iii) If $n = -\frac{1}{2}$, $2n = -1$. However, since $2n$ is a positive even integer, $n \neq -\frac{1}{2}$. Hence, we reject the solution $n = -\frac{1}{2}$.
	(b)(i) 1 and 19
	(b)(ii) $\frac{2}{19}$
	(b)(iii) Since 15 has 4 factors, there will be more than one answer for $\frac{2g-2h}{g+h}$.
4	(a)(i) The median is a better measure as the mean would be affected by the outlier 30.
	(a)(ii) 1, 2, 3 and 4
	(b)(i) Median = 7 ; Interquartile range = $9 - 6 = 3$; Range = $16 - 5 = 11$
	(b)(ii) Incorrect median would be 5 more than the correct median. No difference in interquartile range. No difference in range.
5	(a) $a = 280$, $b = 400$
	(c)(i) 55 000
	(c)(ii) 10 000
	(c)(iii) \$200 000
	(c)(iv) $32\ 000 \leq \text{number of items} \leq 78\ 000$
	(d) The profit is 8 times the number of items produced.
	(e) $x > 55$ No, because when the graph has negative gradient, less profit is earned by increasing production.

6.	(a)	$y = x + 8$
	(b)	$(8, 0)$
	(c)(i)	5.66 units
	(c)(ii)	$ \vec{PQ} = \vec{QS} $ and since $\angle PQR = 90^\circ$, the height of $\triangle RPS$ bisects the base.
	(c)(iii)	$\begin{pmatrix} -7 \\ -7 \end{pmatrix}$
	(c)(iv)	Since $\vec{TQ}$ is a scalar multiple of $\vec{QS}$ and Q is a common point, T, Q and S are collinear.
	(c)(v)	7 : 4
7.	(a)(i)	
	(a)(ii)(a)	$\frac{267}{730}$
	(a)(ii)(b)	$\frac{601}{1095}$
	(b)(i)(a)	26
	(b)(i)(b)	$4n + 6$
	(b)(ii)	Since n is not an integer, it is not possible to have 244 black tiles in any pattern.
8.	(a)	073.4°
	(b)	$\$123454.10$
	(c)	22.2°
9.	(a)	$\begin{pmatrix} 3 \\ 4 \\ 3 \end{pmatrix}$
	(b)(i)	$\begin{pmatrix} 56 & 57 & 31 \\ 51 & b & 54 \\ 42 & 44 & 68 \end{pmatrix}$
	(b)(ii)	$\begin{pmatrix} 489 \\ 315 + 4b \\ 506 \end{pmatrix}$
	(c)	5

	(d)	46
	(e)	$d = e - 3$
10.	(a)	25.4π cm
	(b)	2534 cm^2 , 8308 cm^3
	(c)	Miss Ngoh did not manage to adhere to Mrs Tan's instructions