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3.

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Class: _____



南橋中學

NAN CHIAU HIGH SCHOOL

**PRELIMINARY EXAMINATION (2) 2016
SECONDARY FOUR EXPRESS**

For Marker's Use

MATHEMATICS

4048/01

PAPER 1

4 May 2016, Wednesday

2 hours

Candidates answer on Question paper.

READ THESE INSTRUCTIONS FIRST

Write your name, class and index number at the top of the cover page.

Write in dark blue or black pen.

You may use a pencil for any diagrams or graphs.

Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer all questions.

If working is needed for any question it must be shown with the answer.

Omission of essential working will result in loss of marks.

Calculators should be used where appropriate.

If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.

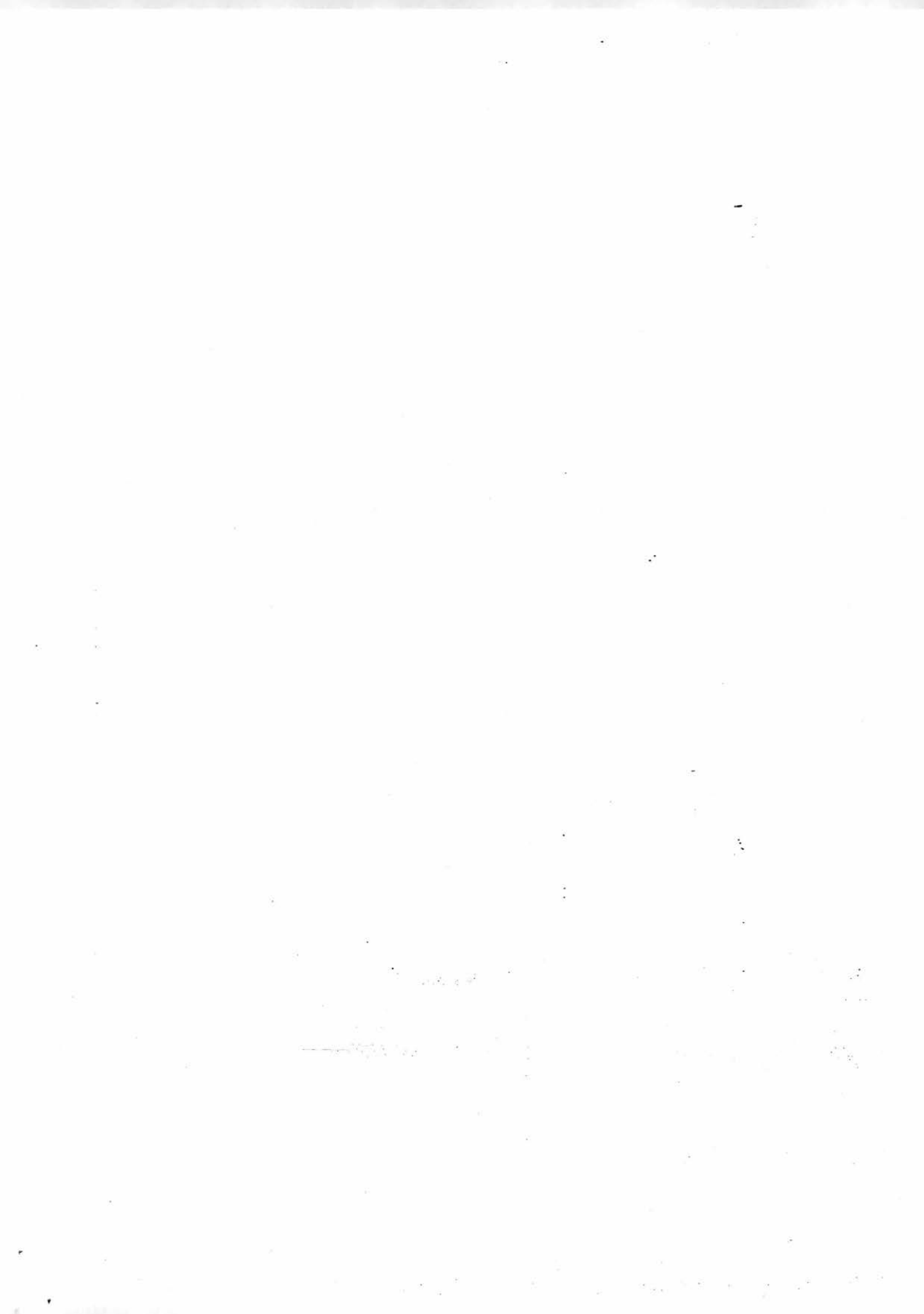
For π , use either your calculator value or 3.142, unless the question requires the answer in terms of π .

The number of marks is given in brackets [] at the end of each question or part question.

The total marks for this paper is 80.

Setter: Mdm Tang PL

This document consists of 18 printed pages including the cover page.



Mathematical Formulae

Compound interest

$$\text{Total amount} = P \left(1 + \frac{r}{100} \right)^n$$

Mensuration

$$\text{Curved surface area of a cone} = \pi r l$$

$$\text{Surface area of a sphere} = 4\pi r^2$$

$$\text{Volume of a cone} = \frac{1}{3} \pi r^2 h$$

$$\text{Volume of a sphere} = \frac{4}{3} \pi r^3$$

$$\text{Area of triangle } ABC = \frac{1}{2} ab \sin C$$

$$\text{Arc length} = r\theta, \text{ where } \theta \text{ is in radians}$$

$$\text{Sector area} = \frac{1}{2} r^2 \theta, \text{ where } \theta \text{ is in radians}$$

Trigonometry

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

Statistics

$$\text{Mean} = \frac{\sum fx}{\sum f}$$

$$\text{Standard deviation} = \sqrt{\frac{\sum fx^2}{\sum f} - \left(\frac{\sum fx}{\sum f} \right)^2}$$

Answer all the questions.

1 (a) Expand the expression $(-3a - \frac{7}{4}b)^2$.

(b) Factorise the expression $25x^2 + 4 - 20x - 4y^2$ completely.

Answer (a) _____ [1]

(b) _____ [2]

2 Solve $\frac{3}{27^{x-2}} = 0.1$.

Answer _____ [3]

3 Given that $2x - 0.4(4 - 3x) \geq 10.5$, find

- (a) the smallest possible value of x .
- (b) the smallest possible value of x if x is a factor of 28.

Answer (a) $x =$ _____ [2]

(b) $x =$ _____ [1]

4 (a) Given that $8^a = 3 \times 10^{-3}$, $16^b = 10^2$ and $4^c = 50$, find the value of $2^{3a+4b-2c}$, leaving your answer in standard form.

- (b) A factory produces 5.3 giga forks per year. Each fork has a mass of 3×10^{-2} kg and it was found that 0.08% of the fork produced has a defect. Calculate the total mass of defective forks produced in a year, leaving your answer in standard form.

Answer (a) _____ [2]

(b) _____ kg [2]

5 A quadratic curve passes through the points $A(-3, 0)$, $B(5, 0)$ and $C(0, 7.5)$.

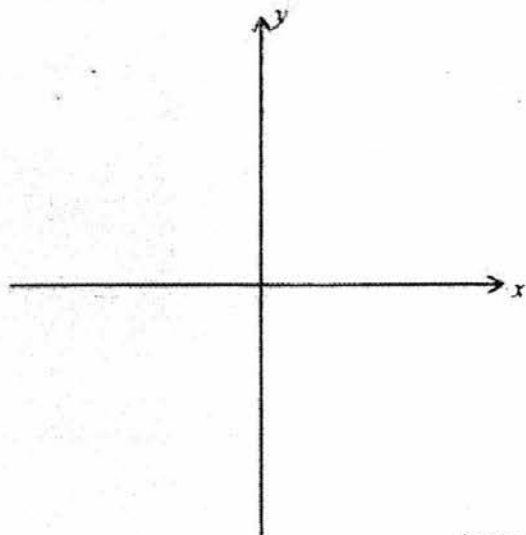
(a) Find the equation of the quadratic curve.

(b) State the line of symmetry of the curve.

(c) Hence, sketch the quadratic curve showing clearly the x -intercept, y -intercept and the coordinate of the turning point.

[2]

(c)



Answer (a) _____ [3]

(b) _____ [1]

- 6 The frequency table shows the number of countries of a group of students had visited.

Number of countries	0	1	2	3	4	5	6
Number of students	2	8	6	7	5	x	4

- (a) Given that the mean number of countries is more than 3, find the smallest possible value of x .
- (b) Given that the median number of countries visited is 4, find the largest possible value of x .
- (c) Given that the mode is 1, state the largest possible value of x .

Answer (a) _____ [2]
(b) _____ [1]
(c) _____ [1]

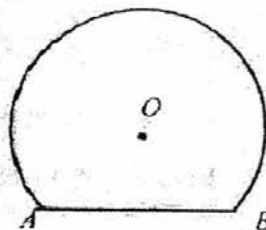
7 A straight line crosses the x -axis at $P(6, 0)$ and the y -axis at $Q(0, -3)$.

- (a) Find the exact length of OR , such that R lies on line PQ and is nearest to the origin, O .
- (b) Given that the area of $\triangle ZPQ$ is 9 square units and PQ is the base of the $\triangle ZPQ$. Find the two possible straight line equations where the point Z should lie on.

Answer (a) _____ [3]

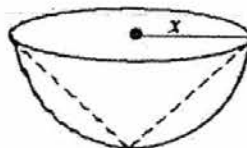
(b) _____ [2]

- 8 The figure shows a major segment of a circle, with centre O and radius r . Given that $\angle AOB = 120^\circ$, find the exact area of the figure in terms of r .



Answer _____ units² [3]

- 9 A right circular cone is removed from a hemispherical solid as shown. Both the cone and the hemisphere have the same circular base of radius x cm. Find the ratio of the volume of hemisphere to the volume of the leftover solid.

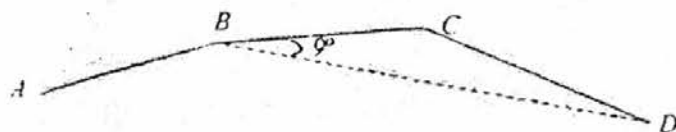


Answer _____ [2]

- 10 In an exhibition, a prototype of a product is displayed. The prototype is constructed using a scale factor of 0.4 of the actual product.
- (a) Find the ratio of the total surface area of prototype to the total surface area of actual product.
 - (b) Both the prototype and actual product are built with wood. The mass of the prototype is 1.24 kg, find the mass of the actual product.

Answer (a) _____ [2]
(b) _____ kg [2]

- 11 The diagram shows 3 sides of a part of a regular polygon, AB , BC and CD .



Given that $\angle CBD = 9^\circ$, calculate the number of sides of the polygon.

Answer _____ [2]

- 12 $E = \{x: x \text{ is a whole number less than } 12\}$
 $P = \{x: x \text{ is a prime number}\}$
 $S = \{x: x \text{ is the solution of } 3x - 5 \leq 18\}$

- (a) Draw a Venn diagram to illustrate the information.
 (b) State the value of $n(P \cup S)$.
 (c) List the elements in $S \cap P^c$.

Answer (a)

[2]

Answer (b) _____ [1]

(c) _____ [1]

- 13 The length of each side of a cube is increased by 40%.
 Find the percentage increase in the volume of the cube.

Answer _____ % [2]

14 The table shows the heights of 33 boys.

Height h (cm)	Frequency
$140 \leq h < 150$	5
$150 \leq h < 155$	6
$155 \leq h < 160$	9
$160 \leq h < 170$	10
$170 \leq h < 180$	3

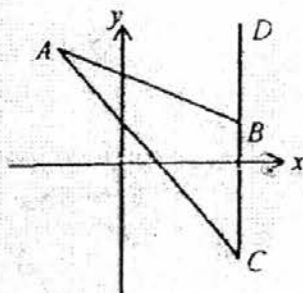
- (a) Calculate an estimate for
- (i) the mean height of the boys,
 - (ii) the standard deviation of their heights.
- (b) Find the probability that if one boy is chosen at random, his height is more than or equal to 160 cm.

Answer (a) (i) _____ cm [1]

(ii) _____ cm [1]

(b) _____ [1]

- 15 The points $A(-3, 10)$, $B(5, 2)$ and $C(5, -8)$ are shown in the diagram. D is a point on the line CB produced.



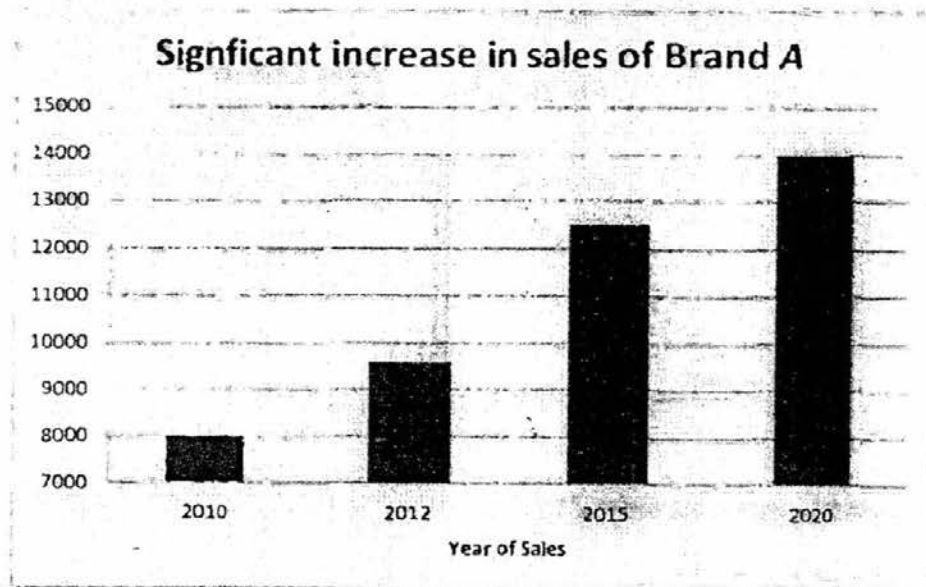
- Find the length of line AC .
- Find the equation of line AC .
- Find the exact value of $\cos \angle ABC$.

Answer (a) _____ units [2]

(b) _____ [2]

(c) _____ [2]

- 16 The graph shows the sales of Brand A over a period of time.



List down 3 things that are misleading in the above statistical representation.

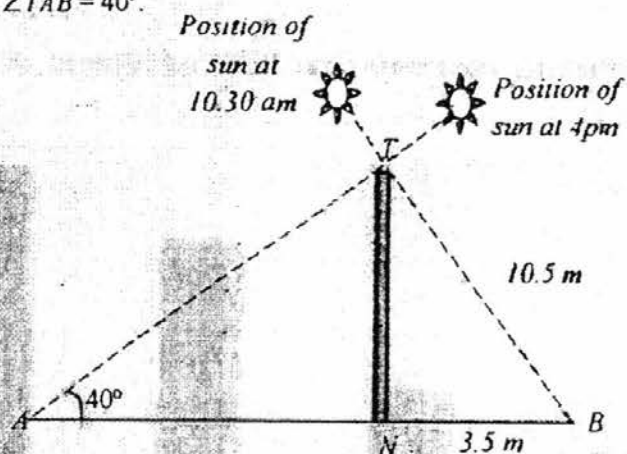
1. _____

2. _____

3. _____

[3]

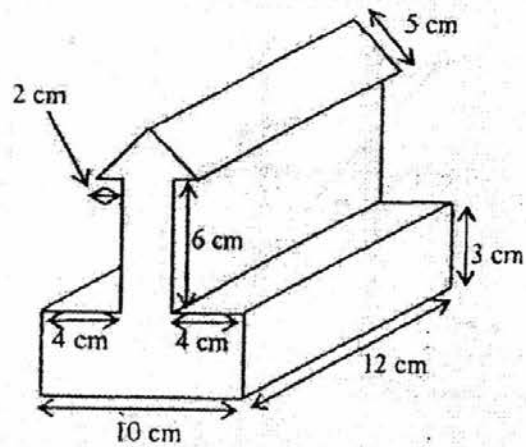
- 17 The diagram shows a tower TN and its shadow NB of length 3.5 m at 10.30 am . At 4 pm , the shadow of the tower is represented by AN . Given that the distance of TB is 10.5 m and $\angle TAB = 40^\circ$.



- (a) Find the height of the tower, TN .
 (b) Find the distance AB .

- (a) _____ m [2]
 (b) _____ m [2]

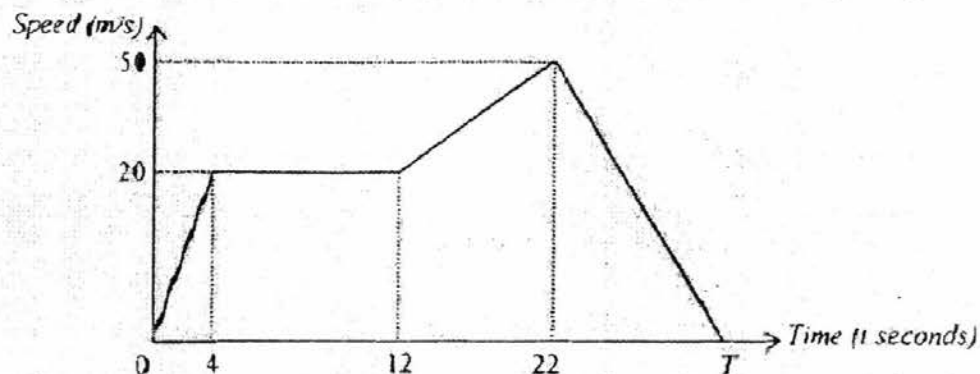
- 18 The diagram below shows a prism where the symmetrical cross-section is made up of two rectangles and an isosceles triangle.



- (a) Find the volume of the prism.
 (b) Find the total surface area of the prism.

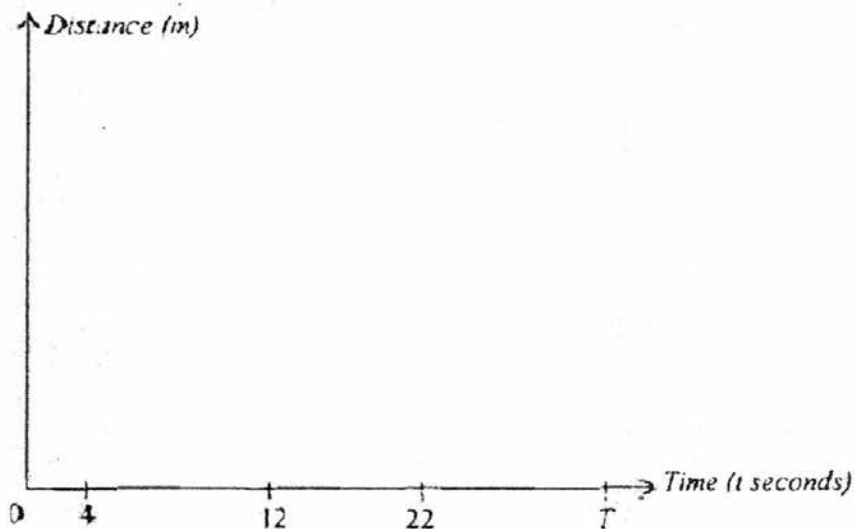
- (a) _____ cm^3 [3]
 (b) _____ cm^2 [2]

- 19 The diagram shows the speed-time graph for the first T seconds of a car's journey.



- (a) Find the speed of the car when $t = 15$ s.
 (b) The car decelerate at 3 m/s^2 , find the value of T .
 (c) On the answer space provided, sketch the distance-time graph of the car.

Answer (c)



[3]

Answer (a) _____ m/s [2]

(b) _____ [2]

20 A manufacturer plans to produce a new medicine for quick relieve of headache.

Each tablet has the shape of a cylinder with base radius 4 mm and height, h mm.

Give that the volume of each tablet is 65 mm^3 , find

- (a) the value of h ,
- (b) the total surface area of the tablet.

The manufacturer wishes to change the tablet into a spherical shape. Without changing the volume of each tablet,

- (c) find the surface area of the new tablet,
- (d) hence, explain which shape the manufacturer should adopt for the new medicine.

Answer (a) _____ mm [2]

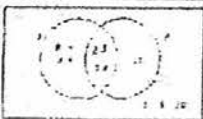
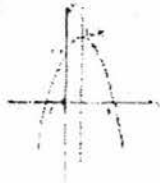
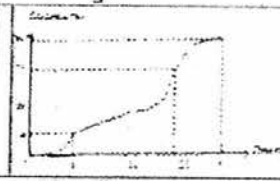
(b) _____ mm^2 [2]

(c) _____ mm^2 [2]

(d) _____
_____ [1]

~ End of Paper ~

Answer Sheet

1a	$9a^2 + \frac{21}{2}ab + \frac{49}{16}b^2$	12a	
b	$(5x - 2 + 2y)(5x - 2 - 2y)$	b	$n(P \cup S) = 9$
2	$x = 3$	c	$S \cap P = \{0, 1, 4, 6\}$
3a	3.78125 or $3\frac{25}{32}$	13	174.4% or $174\frac{2}{5}\%$
b	$x = 4$	14a	159 cm
4a	6×10^{-3}	b	8.48 cm
b	$1.272 \times 10^5 \text{ kg}$	c	$\frac{13}{33}$ or 0.39
5a	$y = -\frac{1}{2}(x+3)(x-5)$	15a	19.7 units
b	$x = 1$	b	$y = -\frac{9}{4}x + \frac{13}{4}$
c		c	$-\frac{1}{\sqrt{2}}$
6a	6	16	Any logical reasoning
b	23	17a	9.90 m
c	7	b	15.3 m
7a	$\frac{6}{\sqrt{3}}$ units	18a	648 cm ³
b	$y = \frac{1}{2}x$ & $y = \frac{1}{2}x - 6$	b	708 cm ²
8	$r^2 \left(\frac{2}{3}\pi + \frac{\sqrt{3}}{4} \right)$	19a	29 m/s
9	2:1	b	$t = 38\frac{2}{3} \text{ s}$ or 38.7 s
10a	4:25	c	
b	19.375 kg or $19\frac{3}{8} \text{ kg}$	20a	$h = 1.29 \text{ mm}$
11	20	b	133 mm ²
		c	78.2 mm ²
		d	Any logical reasoning

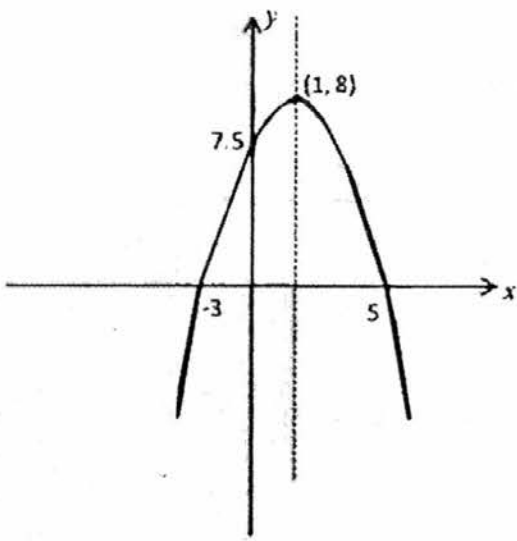
Secondary Four
Elementary Mathematics Preliminary Two
Paper 1
Marking Scheme

Qns	Marking Scheme	Remarks
1(a)	$\left(-3a - \frac{7}{4}b\right)^2 = (-3a)^2 + 2(-3a)\left(-\frac{7}{4}b\right) + \left(-\frac{7}{4}b\right)^2$ $= 9a^2 + \frac{21}{2}ab + \frac{49}{16}b^2 \quad \text{----- [B1]}$	
(b)	$25x^2 + 4 - 20x - 4y^2 = 25x^2 - 20x + 4 - 4y^2$ $= (5x - 2)^2 - (2y)^2 \quad \text{----- [M1]}$ $= (5x - 2 + 2y)(5x - 2 - 2y) \quad \text{----- [A1]}$	

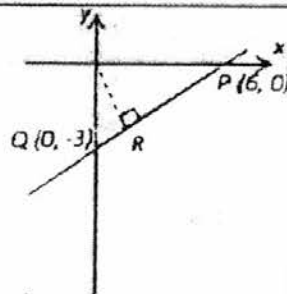
Qns	Marking Scheme	Remarks
2	$\frac{3}{27x-2} = 0.1$ $\frac{3}{3^{3x-4}} = \frac{1}{3^2} \quad \text{----- [M1]}$ $3^{1-3x+6} = 3^{-2} \quad \text{----- [M1]}$ $7 - 3x = -2$ $x = 3 \quad \text{----- [A1]}$	

Qns	Marking Scheme	Remarks
3(a)	$2x - 0.4(4 - 3x) \geq 10.5$ $2x - 1.6 + 1.2x \geq 10.5$ $3.2x \geq 12.1$ ----- [M1] $x \geq 3.78125$ or $3\frac{25}{32}$ The smallest value is 3.78125 or $3\frac{25}{32}$ ----- [A1]	
(b)	Factor, $x = 4$ ----- [B1]	

Qns	Marking Scheme	Remarks
4 (a)	$2^{3a+4b-2c} = \frac{2^{3a} \times 2^{4b}}{2^{2c}}$ $= \frac{3 \times 10^{-3} \times 10^2}{50}$ ----- [M1] $= \frac{0.3}{50}$ $= 6 \times 10^{-3}$ ----- [A1]	
(b)	total mass of defective forks $= 5.3 \times 10^9 \times 3 \times 10^{-2} \times 0.08 \times 10^{-2}$ ----- [M1] $= 127200$ $= 1.272 \times 10^5 \text{ kg}$ ----- [A1]	

Qns	Marking Scheme	Remarks
5(a)	From Point A (-3, 0), Point B (5, 0) $y = a(x + 3)(x - 5) \text{ ----- [M1]}$ Considering Point C (0, 7.5) $7.5 = a(3)(-5)$ $a = -\frac{1}{2} \text{ ----- [A1]}$ $y = -\frac{1}{2}(x + 3)(x - 5) \text{ ----- [A1]}$	
(b)	line of symmetry, $x = \frac{-3+5}{2}$ $x = 1 \text{ ----- [B1]}$ * Must write the answer as an equation at the answer space	
(c)	When $x = 1$, $y = -\frac{1}{2}(1 + 3)(1 - 5)$ $= 8$  [G1] correct shape [G1] correctly indication of x and y intercept & coordinate of turning points	

Qns	Marking Scheme	Remarks
6(a)	$\text{Mean} > 3$ $\frac{(0)(2) + (1)(8) + (2)(6) + (3)(7) + (4)(5) + 5x + (6)(4)}{2 + 8 + 6 + 7 + 5 + x + 4} > 3 \quad \text{----- [M1]}$ $\frac{85 + 5x}{32 + x} > 3$ $85 + 5x > 96 + 3x$ $2x > 11$ $x > 5.5$ $\therefore x = 6 \quad \text{----- [A1]}$	
(b)	$2 + 8 + 6 + 7 + 4 = x + 4$ $x = 23 \quad \text{----- [B1]}$	
(c)	$x = 7 \quad \text{----- [B1]}$	

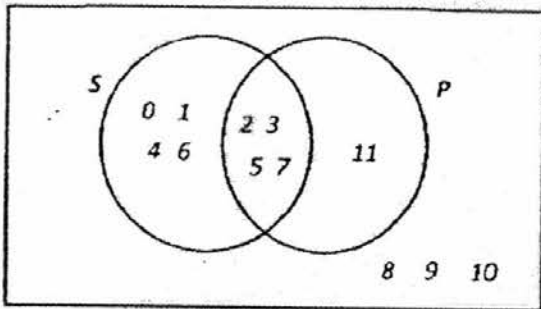
Qns	Marking Scheme	Remarks
7(a)	Let the shortest distance be OR. $PQ = \sqrt{6^2 + 3^2}$ $= \sqrt{45} \text{ units} \quad \text{----- [M1]}$ $\frac{1}{2}(6)(3) = \frac{1}{2}(\sqrt{45})(OR) \quad \text{----- [M1]}$ $OR = \frac{18}{\sqrt{45}}$ $= \frac{6}{\sqrt{5}} \text{ units} \quad \text{----- [A1]}$	
(b)	Gradient if $PQ = \frac{1}{2}$ The possible equations are $y = \frac{1}{2}x \quad \text{and} \quad \text{----- [B1]}$ $y = \frac{1}{2}x - 6 \quad \text{----- [B1]}$	

Qns	Marking Scheme	Remarks
Q8	Area of segment = Area of sector + Area of triangle $= \frac{360-120}{360} \times \pi r^2 + \frac{1}{2} r^2 \sin 120 \text{ ----- [M1]}$ $= \frac{2}{3} \pi r^2 + \frac{1}{2} r^2 \left(\frac{\sqrt{3}}{2} \right) \text{ ----- [M1]}$ $= \frac{2}{3} \pi r^2 + \frac{\sqrt{3}}{4} r^2$ $= r^2 \left(\frac{2}{3} \pi + \frac{\sqrt{3}}{4} \right) \text{ ----- [A1]}$	

Qns	Marking Scheme	Remarks
Q9	Volume of hemisphere = $\frac{2}{3} \pi x^3$ Volume of cone = $\frac{1}{3} \pi x^2 (x) = \frac{1}{3} \pi x^3$ ratio of the volume of hemisphere to the volume of leftover solid $= \frac{2}{3} \pi x^3 : \frac{1}{3} \pi x^3$ $= 2 : 1 \text{ ----- [A1]}$	

Qns	Marking Scheme	Remarks
10 (a)	$\frac{\text{total surface area of prototype}}{\text{total surface area of actual product}} = \left(\frac{0.4}{1} \right)^2 \text{ ----- [M1]}$ $= \left(\frac{2}{5} \right)^2$ $= \frac{4}{25}$ Ratio is 4: 25 ----- [A1]	
(b)	$\frac{\text{mass of prototype}}{\text{mass of actual product}} = \left(\frac{2}{5} \right)^3$ $\frac{1.24}{\text{mass of actual product}} = \frac{8}{125} \text{ ----- [M1]}$ mass of actual product = $\frac{1.24 \times 125}{8}$ $= 19.375 \text{ kg or } 19\frac{3}{8} \text{ kg ----- [A1]}$	

Qns	Marking Scheme	Remarks
Q11	$\angle CBD = 180^\circ - 9^\circ - 9^\circ$ ($\angle$ s sum of Δ) Interior angle of polygon = 162° Exterior angle of polygon = $180^\circ - 162^\circ$ ($\angle$ on a str line) = 18° ----- [M1] Or Exterior angle of polygon = $180^\circ - 9^\circ - 9^\circ$ (2 int $\angle$ = 1 ext $\angle$) = 18° ----- [M1] the number of sides of the polygon = $\frac{360}{18}$ = 20 ----- [A1]	

Qns	Marking Scheme	Remarks
12		
(a)	$\epsilon = \{x: x \text{ is a whole number less than } 12\}$ $P = \{x: x \text{ is a prime number}\}$ = $\{2, 3, 5, 7, 11\}$ $S = \{x: x \text{ is the solution of } 3x - 5 \leq 18\}$ = $\{0, 1, 2, 3, 4, 5, 6, 7\}$  1 mark deducted for every 1 errors ----- [B2] * there shd not be any 'comma'	
(b)	$n(P \cup S) = 9$ ----- [B1]	
(c)	$S \cap P' = \{0, 1, 4, 6\}$ ----- [B1]	

Qns	Marking Scheme	Remarks
13	$V = l^3$ $V = (1.4l)^3$ $= 2.744l^3$ ----- [M1] percentage increase in $y = \frac{2.744-1}{1} \times 100\%$ $= 174.4\%$ or $174\frac{2}{5}\%$ ----- [A1]	

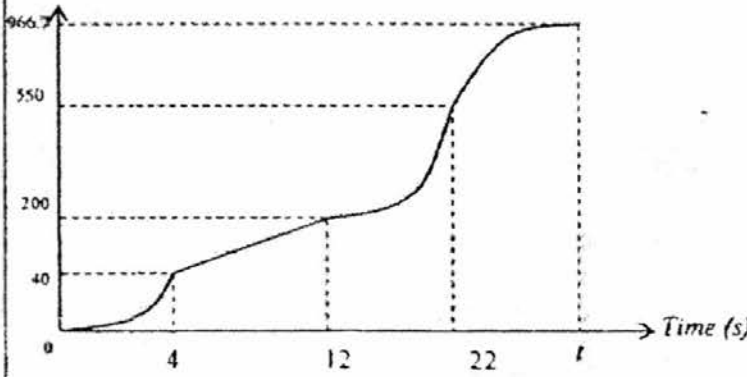
Qns	Marking Scheme	Remarks
14	mean height of the students = 159 cm ----- [B1]	
(a)		
(i)		
(a)	standard deviation of their heights = 8.48 cm ----- [B1]	
(ii)		
(b)	$P(\geq 160) = \frac{13}{33}$ or 0.39 ----- [B1]	

Qns	Marking Scheme	Remarks
15	$AC = \sqrt{(-3-5)^2 + (10+8)^2}$ ----- [M1]	
(a)	$= \sqrt{64 + 324}$ $= \sqrt{388}$ $= 19.7$ units ----- [A1]	
	Gradient = $\frac{10-(-8)}{-3-5}$ ----- [M1] $= -\frac{9}{4}$ Equation of line AC $y + 8 = -\frac{9}{4}(x - 5)$ $y = -\frac{9}{4}x + \frac{13}{4}$ ----- [A1]	
(b)	$AB^2 = 8^2 + 8^2$ $AB = \sqrt{128}$ ----- [M1] $\cos \angle ABC = -\frac{8}{\sqrt{128}}$ $= -\frac{1}{\sqrt{2}}$ ----- [A1]	

Qns	Marking Scheme	Remarks
16	<i>Any of the possible logical reasons ----- [B3]</i> 1. The choice of word of 'Significant' for the title might mislead the audience. In fact, the biggest difference is only 6000 units. 2. The vertical scale did not start from 0, hence the sale in year 2020 will look like it is 7 times the sales in 2010. 3. The years indicated on the horizontal axis are not consistent. As data from several years are missing; hence, actual data might not be increasing throughout the years. 4. The sale in year 2020 is a predicted value instead of actual value; hence the sales might not be increased in 2020. 5. The vertical scale is not labelled; hence the vertical axis may represent the number of items or the prices of the item or the total sales of the year and etc. Not clear in the intention.	

Qns	Marking Scheme	Remarks
17 (a)	$TN^2 = 10.5^2 - 3.5^2$ ----- [M1] $TN = \sqrt{98}$ $= 9.90\text{ m}$ ----- [A1]	
(b)	$\tan 40^\circ = \frac{\sqrt{98}}{AN}$ ----- [M1] $AN = 11.80\text{ m}$ $AB = 11.80 + 3.5$ $= 15.3\text{ m}$ ----- [A1]	

Qns	Marking Scheme	Remarks
18		
(a)	Cross-sectional Area $= (10)(3) + (2)(6) + \frac{1}{2}(6)(4) \text{ ----- [M1, M1(area of } \Delta)]$ $= 54 \text{ cm}^2$ Volume $= 54 \times 12$ $= 648 \text{ cm}^3 \text{ ----- [A1]}$	
(b)	Perimeter of cross-sectional area $= 5+5+2+2+6+6+4+4+3+3+10$ $= 50 \text{ cm ----- [M1]}$ total surface area $= 2(54) + 50(12)$ $= 708 \text{ cm}^2 \text{ ----- [A1]}$	

Qns	Marking Scheme	Remarks
19 (a)	Let the speed of the car at $t = 15$ s be v $\frac{50-20}{22-12} = \frac{v-20}{15-12} \text{ ----- [M1]}$ $\frac{30}{10} = \frac{v-20}{3}$ $v - 20 = 9$ $v = 29 \text{ m/s} \text{ ----- [A1]}$	
(b)	Acceleration = -3 m/s^2 $\frac{0-50}{t-22} = -3 \text{ ----- [M1]}$ $-50 = -3t + 66 \quad \text{or duration} = 16\frac{2}{3}$ $3t = 116$ $t = 38\frac{2}{3} \text{ s or } 38.7 \text{ s} \text{ ----- [A1]}$	
(c)	Distance from 0s to 4s = $\frac{1}{2} \times 20 \times 4$ = 40 m Distance from 0s to 12s = $40 + 20 \times (12 - 4)$ = 200 m Distance from 0s to 22s = $200 + \frac{1}{2} \times 20 \times (12 + 22)$ = 550 m Distance from 0s to 38.7 s = $540 + \frac{1}{2} \times (38.7 - 22) \times 50$ = $966\frac{2}{3} \text{ s or } 966.7 \text{ m}$ Distance (m)  [G3] – 1 mark for every pair of correctly drawn shape of line / curve segment 1 mark for correct vertical axis with the correct values	

Qns	Marking Scheme	Remarks
20	$V = \pi r^2 h$	
(a)	$65 = \pi(4)^2 h$ ----- [M1] $h = 1.2931$ $h = 1.29 \text{ mm}$ ----- [A1]	
(b)	total surface area = $2\pi r^2 + 2\pi r h$ $= 2\pi(4)^2 + 2\pi(4)(1.2931)$ ----- [M1] $= 133 \text{ mm}^2$ ----- [A1]	
(c)	$V = \frac{4}{3}\pi r^3$ $65 = \frac{4}{3}\pi r^3$ $r = 2.4943 \text{ mm}$ ----- [M1] surface area of the spherical medicine = $4\pi r^2$ $= 4\pi(2.4943)^2$ $= 78.2 \text{ mm}^2$ ----- [A1]	
(d)	<i>Any of the possible logical reasons</i> ----- [B1] 1. The manufacturer should make the medicine into cylinder shape. For a medicine use for quick relieve of pain/ headache, the medicine should have a greater surface area for medicine to be dissolve easier and absorption. 2. The manufacturer should make the medicine into spherical shape. For a smaller surface area, the cost spend on packaging will be lesser.	

Name: _____

Register Number: _____

Class: _____



南僑中學

NAN CHIAU HIGH SCHOOL

PRELIMINARY EXAMINATION (2) 2016
SECONDARY FOUR EXPRESS

For Marker's Use

MATHEMATICS

4048/02

PAPER 2

5 May 2016, Thursday

2 hours 30 minutes

Additional materials: Writing Papers (6 sheets)
Graph Paper (1 sheet)

READ THESE INSTRUCTIONS FIRST

Write your name, class and index number at the top of the cover page.

Write in dark blue or black pen.

You may use a pencil for any diagrams or graphs.

Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer all questions.

If working is needed for any question it must be shown with the answer.

Omission of essential working will result in loss of marks.

Calculators should be used where appropriate.

If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.

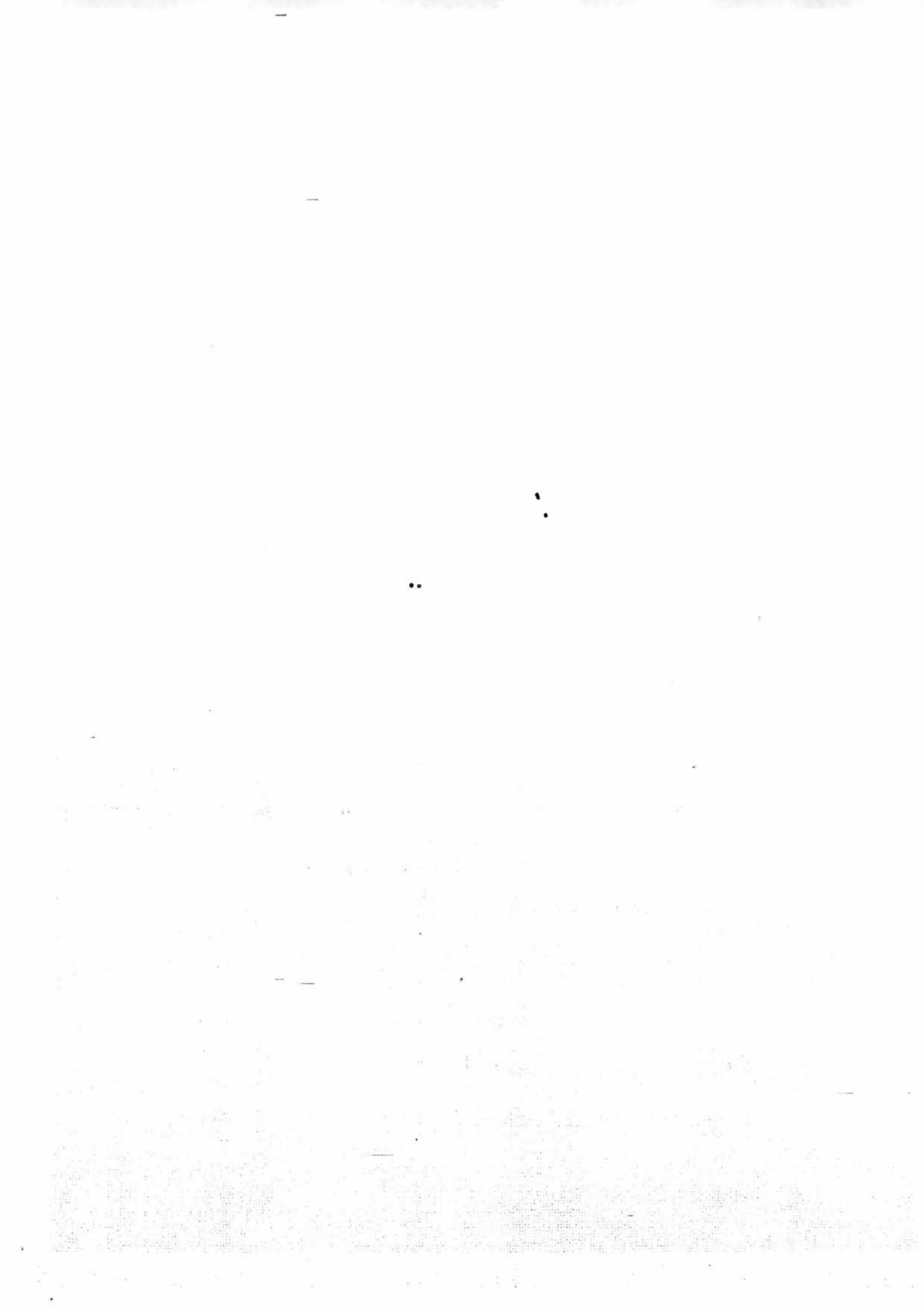
For π , use either your calculator value or 3.142, unless the question requires the answer in terms of π .

At the end of the examination, fasten all your work securely together.

The number of marks is given in brackets [] at the end of each question or part question.

The total marks for this paper is **100**.

Setter: Ms Ting Shi Yun



Mathematical Formulae

Compound interest

$$\text{Total amount} = P \left(1 + \frac{r}{100} \right)^n$$

Mensuration

$$\text{Curved surface area of a cone} = \pi r l$$

$$\text{Surface area of a sphere} = 4\pi r^2$$

$$\text{Volume of a cone} = \frac{1}{3} \pi r^2 h$$

$$\text{Volume of a sphere} = \frac{4}{3} \pi r^3$$

$$\text{Area of triangle } ABC = \frac{1}{2} ab \sin C$$

$$\text{Arc length} = r\theta, \text{ where } \theta \text{ is in radians}$$

$$\text{Sector area} = \frac{1}{2} r^2 \theta, \text{ where } \theta \text{ is in radians}$$

Trigonometry

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

Statistics

$$\text{Mean} = \frac{\sum fx}{\sum f}$$

$$\text{Standard deviation} = \sqrt{\frac{\sum fx^2}{\sum f} - \left(\frac{\sum fx}{\sum f} \right)^2}$$

Answer all questions.

1 (a) Given that $x^2 - y^2 = 2x - 1$, express x in terms of y [3]

(b) Simplify $\frac{3x+1}{2x^2-x-6} + \frac{1}{2-x}$ [3]

(c) Given that $31 \times 417 = 12927$, without using a calculator, evaluate $31 \times 0.417 + \frac{12927}{4170}$ [2]

(d) Soh is riding an electric scooter at a constant speed of x km/h while Wang is riding an electrical bicycle at a constant speed of 15 km/h slower. Both travelled the same distance of 10 km and their difference in time, reaching the same destination, is 15 minutes.

(i) Write down an equation in x to represent the above information and show that it can be reduced to $x^2 - 15x - 600 = 0$. [2]

(ii) Find the time taken by Wang. Correct your answer to the nearest second. [3]

2 The items that Coffee Shop A and Coffee Shop B sold in an hour are represented in a 2×3 matrix N .

$$N = \begin{matrix} & \begin{matrix} \text{coffee} & \text{tea} & \text{coke} \end{matrix} \\ \begin{pmatrix} 135 & 104 & 75 \\ 128 & 115 & 50 \end{pmatrix} & \begin{matrix} A \\ B \end{matrix} \end{matrix}$$

(i) The cost price of a cup of coffee is 35 cents, that of a cup of tea is 30 cents and a can of coke is 45 cents. Represent this information as Matrix C . [1]

(ii) Evaluate NC and explain what the elements of the matrix represent. [3]

(iii) Each Coffee Shop sells \$1 for a cup of coffee, 90 cents for a cup of tea and \$1.40 for a can of coke. Represent this information as Matrix P . [1]

(iv) Evaluate $N(P - C)$ and explain what the elements of the result represent. [3]

3 Answer the whole of this question on a sheet of graph paper.

The variables x and y are connected by the equation

$$y = \frac{x^3}{2} + 2x^2 - 2.$$

Some corresponding values of x and y are given in the table below.

x	-4	-3.5	-3	-2.5	-2	-1.5	-1	-0.5	0	0.5	1	1.5
y	p	1.06	2.5	2.69	2	0.81	-0.5	-1.56	-2	-1.44	0.5	4.19

- (a) Find the value of p . [1]

- (b) Using a scale of 2 cm to represent 1 unit on each axis, draw a horizontal x -axis for $-4 \leq x \leq 2$ and a vertical y -axis for $-3 \leq y \leq 5$.

On your axes, plot the points given in the table and join them with a smooth curve. [3]

- (c) By drawing a tangent, find the gradient of the curve at the point $(-3, 2.5)$. [2]

- (d) Use your graph to find

- (i) the solution of $x^3 + 4x^2 - 4 = 0$, [2]

- (ii) the range of values of x for which $\frac{x^3}{2} + 2x^2 + \frac{x}{2} - 1 < 0$. [3]

4 The tuition fee for a three-year Accountancy course at a University in Singapore is \$48000. As a Singaporean, Fang Min enjoys a government subsidy of 55% off her tuition fee. Fang Min's parents agree to pay 20% of the subsidised tuition fees.

- (i) Find the remaining amount that Fang Min has to pay for her three-year Accountancy course. [2]

Fang Min decides to take a study loan to pay the remaining tuition fee. There are two loan schemes available for her to choose.

Scheme A: Simple interest of 5% per annum, repayment period of 6 years.

Scheme B: Compound interest of 4.5% per annum, repayment period of 5 years.

- (ii) Find the total amount payable for Scheme A. [2]

- (iii) Find the total amount payable for Scheme B. [2]

- (iv) Fang Min is able to afford a maximum instalment of \$325 per month. Support with workings, which scheme she should choose. [3]

- 5 (a) The sequence, S , shows the sum of n consecutive positive even numbers. The first 4 terms are given below.

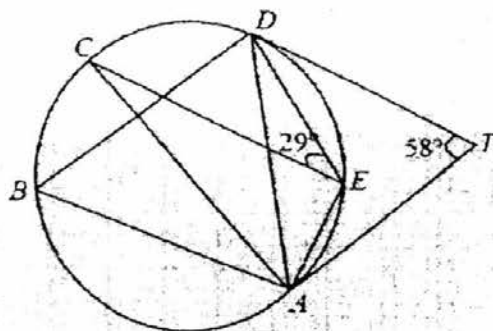
$$\begin{aligned} S_1 &= 2 & &= 2 \\ S_2 &= 2 + 4 & &= 6 \\ S_3 &= 2 + 4 + 6 & &= 12 \\ S_4 &= 2 + 4 + 6 + 8 & &= 20 \end{aligned}$$

- (i) Find the expression, S_n , in terms of n , for the n^{th} term of the sequence [2]
 (ii) Hence, find S_{50} . [1]

- (b) The sequence, T , shows the sum of n consecutive positive odd numbers. The first 4 terms are given below:

$$\begin{aligned} T_1 &= 1 & &= 1^2 = 1 \\ T_2 &= 1 + 3 & &= 2^2 = 4 \\ T_3 &= 1 + 3 + 5 & &= 3^2 = 9 \\ T_4 &= 1 + 3 + 5 + 7 & &= 4^2 = 16 \\ & & &\vdots \\ T_n &= 1 + 3 + 5 + \dots + a & &= b^2 = c \end{aligned}$$

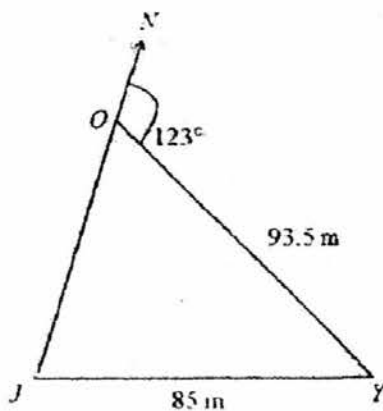
- (i) Find the expression, T_n , in terms of n , for the n^{th} term of the sequence [1]
 (ii) Find an expression of a , in terms of n . [1]
 (iii) Hence, find the value of $1 + 3 + 5 + \dots + 99$. [2]
- (c) Using your answers in part (a) and (b), find the value of [2]
 $1 - 2 + 3 - 4 + \dots + 99 - 100$.
-



The diagram shows a circle $ABCDE$. AT and DT are tangents to the circle at point A and D respectively. Angle $ATD = 58^\circ$ and angle $CED = 29^\circ$.

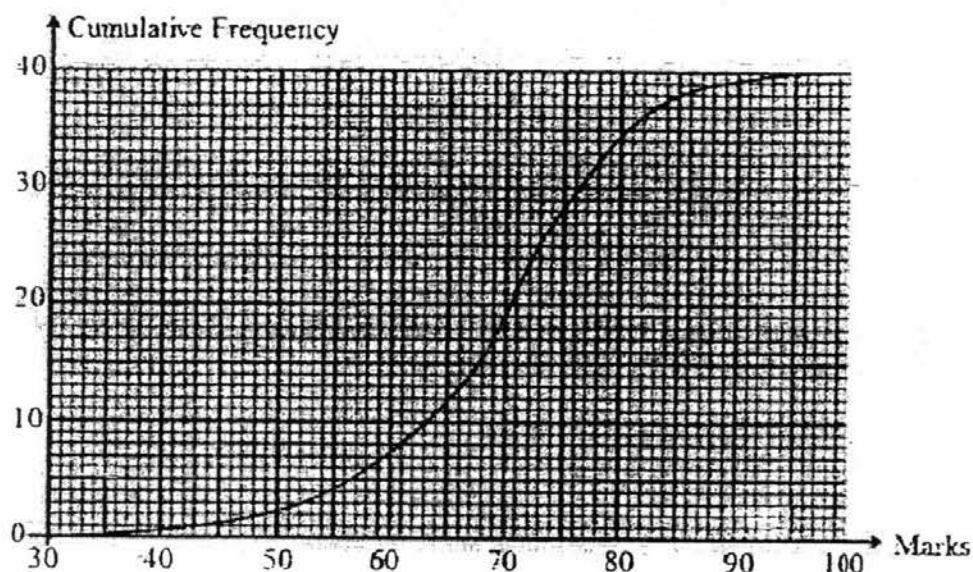
- (a) Find, giving reasons for each answer,
 - (i) angle CAD , [1]
 - (ii) angle ABD . [3]
- (b) Justify, with reasons, if AC is the diameter of the circle. [2]
- (c) Another circle with centre T passes through points A and D . Given that the perimeter of the minor sector DAT is 18.07 cm. Find the value of AT . [3]

- 7 Ofelia runs 93.5 m from her house O to Y , where Y is at a bearing of 123° from O . Janice left her house at J and runs 85 m to meet Ofelia at Y .

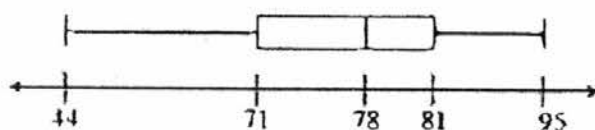


- (a) Find the bearing of Y from J . [3]
- (b) Find the distance OJ . [3]
- (c) Calculate the area of $\triangle JOY$. [2]
- (d) Janice saw an aeroplane flying along OY just before she was about to leave her house. Calculate the greatest angle of elevation of Janice, given that the aeroplane is 10 km above ground. [3]

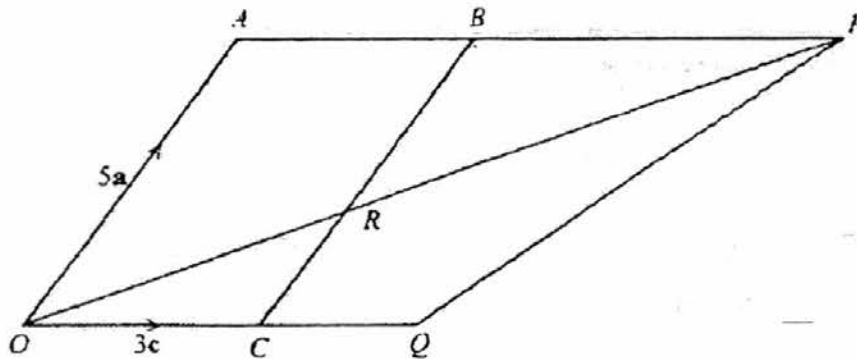
- 8 The cumulative frequency curve below shows the distribution of the Mathematics marks in Preliminary Examination One for a class.



- (i) Use the curve to estimate the
- median mark. [1]
 - interquartile range of the marks. [2]
 - value of x , given only 30% of the students scored at least x marks. [2]
- (ii) Two of the students from the class are selected at random.
Find the probability that both of them scored at least 65 marks. [2]
- (iii) The Mathematics marks in Preliminary Examination Two for this class had the same median as the Mathematics marks in Preliminary Examination One marks but are more consistent.
Describe how the cumulative frequency curve for Preliminary Examination Two may differ from the given curve above. [2]
- (iv) The Mathematics marks in Preliminary Examination Three for this class are recorded and the box and whiskers plot below shows the distribution of the marks



Make two comparisons between the performance of the class in Mathematics Preliminary Examination One and Three. [2]



In the diagram, $OABC$ is a parallelogram. $\overrightarrow{OA} = 5\mathbf{a}$ and $\overrightarrow{OC} = 3\mathbf{c}$.
 P is produced from AB and Q is produced from OC , such that $OC : OQ$ is $3 : 5$.
 BC intersects OP at point R such that $CR : BR$ is $2 : 3$.

- (a) Show that $\triangle OCR$ is similar to $\triangle PBR$. [2]
- (b) Express, in terms of $\mathbf{a}$ and $\mathbf{c}$,
- $\overrightarrow{CR}$ [1]
 - $\overrightarrow{AR}$ [1]
 - $\overrightarrow{PC}$ [2]
- (c) Haryl commented that A , R and Q are collinear. Do you agree with him? Support your answer with reasons. [3]
- (d) Find
- $\frac{\text{area of } \triangle CRQ}{\text{area of } \triangle BRP}$, [1]
 - $\frac{\text{area of } \triangle OCR}{\text{area of } \triangle APQ}$. [2]

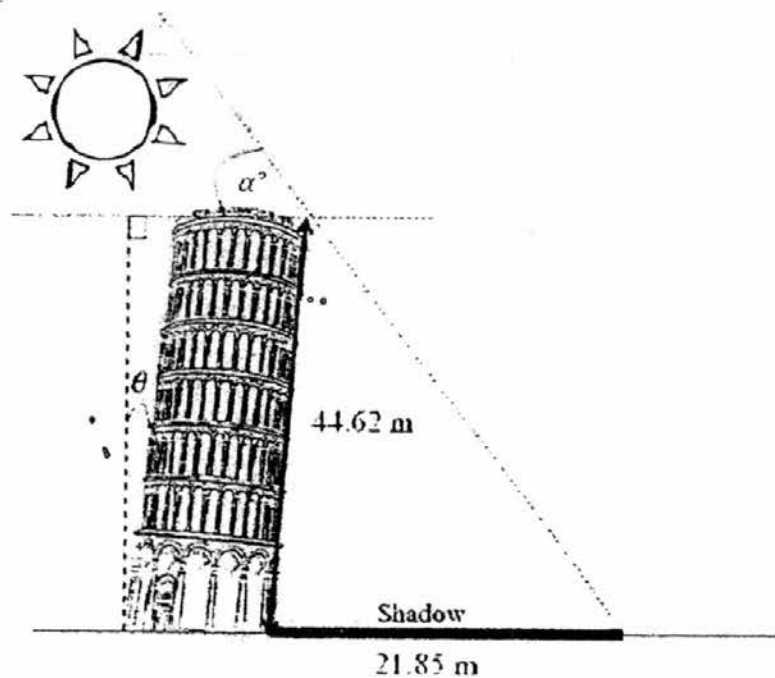
- 10 (a) Bag X contains 7 black marbles and 5 white marbles.

Bag Y contains 3 black marbles and 1 white marble.

A marble is drawn from Bag X and placed into Bag Y. A marble is then drawn from Bag Y. By drawing a tree diagram, find the probability of getting a black marble from Bag Y.

[3]

- (b) The Leaning Tower of Pisa in Italy was constructed since 1173 and was only completed in 1372. The tower began to lean after the construction had progressed to the second floor in 1178. This was due to weak foundation of the ground that caused the design of the tower to be flawed from the beginning. The leaning angle is defined as θ and the top of the building is parallel to the ground.



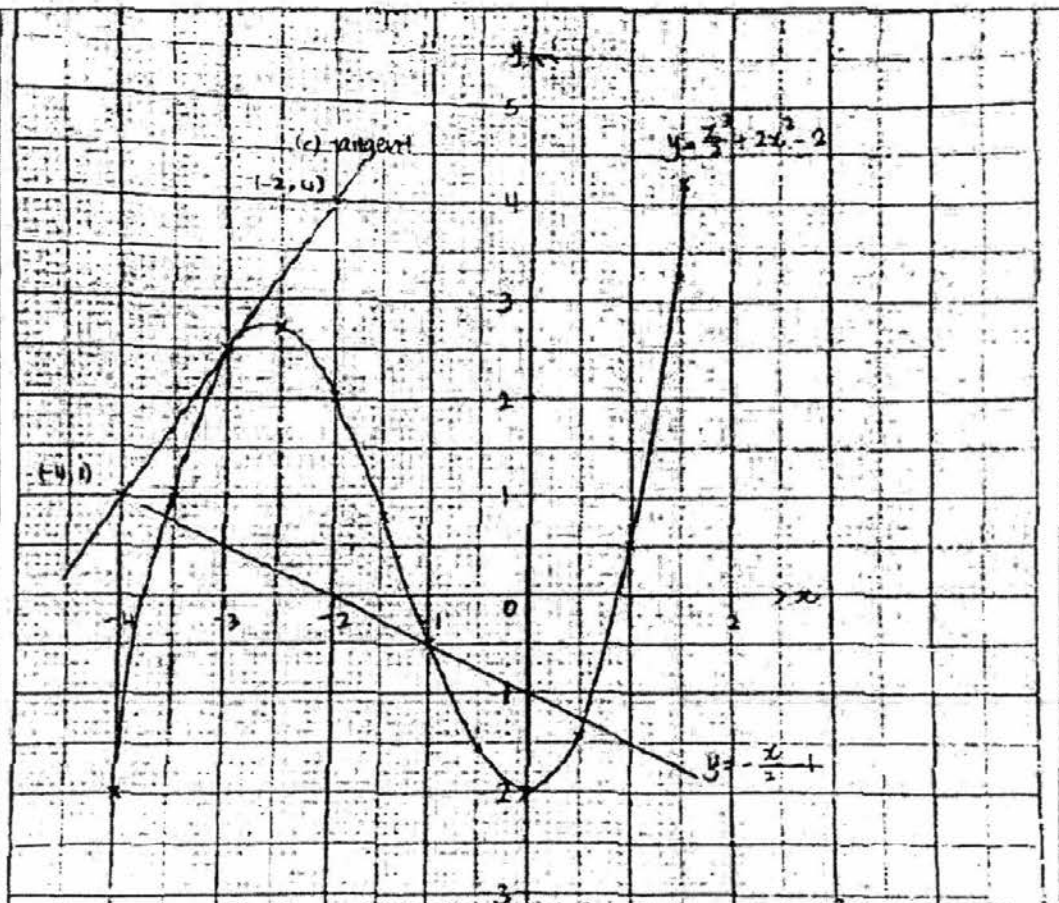
The diagram above shows the tower in 1319. There was 7 storeys built and the height of the shorter side of the tower is 44.62 m, as shown. Given that the shadow of the tower is 21.85 m when α is 68° , find the leaning angle.

[4]

— End of paper —

1 (a) $x^2 - y^2 = 2x - 1$ $-x^2 - 2x + 1 = y^2$ $(x-1)^2 = y^2$ $x = 1 \pm y$	2 i) $C = \begin{pmatrix} 35 \\ 30 \\ 45 \end{pmatrix}$ or $\begin{pmatrix} 0.35 \\ 0.3 \\ 0.45 \end{pmatrix}$
(b) $\frac{3x+1}{(2x+3)(x-2)} + \frac{1}{2-x}$ $= \frac{3x+1}{(2x+3)(x-2)} - \frac{1}{x-2}$ $= \frac{3x+1-2x-3}{(2x+3)(x-2)}$ $= \frac{x-2}{(2x+3)(x-2)}$ $= \frac{1}{2x+3}$	ii) $NC = \begin{pmatrix} 135 & 104 & 75 \\ 128 & 115 & 50 \end{pmatrix} \begin{pmatrix} 35 \\ 30 \\ 45 \end{pmatrix}$ $= \begin{pmatrix} 11220 \\ 10180 \end{pmatrix}$ or $\begin{pmatrix} 112.20 \\ 101.80 \end{pmatrix}$ The elements represent the total cost price in dollars/cents for the coffee, tea and coke sold in an hour in Coffee Shop A and B respectively.
(c) $31 \times 0.417 + \frac{12927}{4170}$ $= \frac{31 \times 417 + 12927}{1000} \times \frac{1}{10}$ $= 12.927 + 3.1$ $= 16.027$	iii) $P = \begin{pmatrix} 100 \\ 90 \\ 140 \end{pmatrix}$ or $\begin{pmatrix} 1 \\ 0.9 \\ 1.4 \end{pmatrix}$
(d) (i) $\frac{10}{x-15} - \frac{10}{x} = \frac{15}{60}$ $\frac{150}{x^2 - 15x} = \frac{1}{4}$ $x^2 - 15x - 600 = 0$ (ii) $x = \frac{15 \pm \sqrt{15^2 - 4(-600)}}{2}$ $= 33.117$ or -18.117 (rej) Time taken for Wang $= \frac{10}{33.117 - 15}$ $= 0.55196 \text{ h}$ $= 33.118 \text{ min}$ $= 33 \text{ min } 7 \text{ s or } 1987 \text{ s}$	iv) $N(P - C) = \begin{pmatrix} 135 & 104 & 75 \\ 128 & 115 & 50 \end{pmatrix} \begin{pmatrix} 100 - 35 \\ 90 - 30 \\ 140 - 45 \end{pmatrix}$ $= \begin{pmatrix} 22140 \\ 19970 \end{pmatrix}$ or $\begin{pmatrix} 221.40 \\ 199.70 \end{pmatrix}$ The elements represent the total profit in dollars/cents for the coffee, tea and coke sold in an hour in Coffee Shop A and B respectively.

3



$$(a) p = -2$$

$$(c) \text{ grad} = \frac{4-1}{-2-(-4)} = 1.5$$

$$(di) x^3 + 4x^2 - 9 = 0$$

$$\frac{x^3}{2} + 2x^2 - 2 = 0 \quad y=0$$

$$\therefore x = -3.75, -1.2, 0.95$$

$$(dii) \frac{x^3}{2} + 2x^2 - 2 < -\frac{x}{2} - 1$$

$$\text{Plot } y = -\frac{x}{2} - 1$$

$$\therefore -4 < x < -3.55$$

$$\text{and } -1 < x < 0.55$$

$$a) p = -2$$

BI

b) scale, points, label curve, smooth curve, turning points $(-2.65, 2.75)$ and $(0, -2)$

c) gradient $= -1.5 (\pm 0.1)$

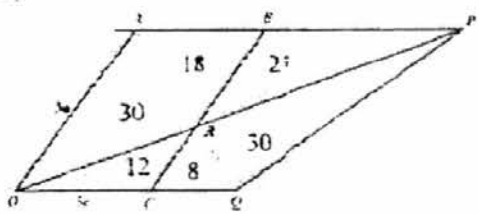
d) i) $y = 0, x = -3.7, -1.2, 0.9 (\pm 0.1)$

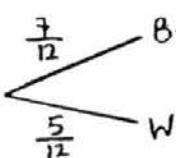
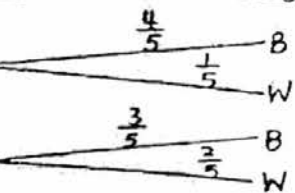
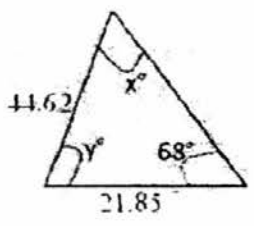
$$d) ii) \frac{x^3}{2} + 2x^2 - 2 < -\frac{x}{2} - 1$$

$$\text{Plot } y = -\frac{1}{2}x - 1$$

$$-4 < x < -3.55 \text{ or } -1 < x < 0.55$$

4	i) $48000 \times \frac{45}{100} \times \frac{80}{100} = \\17280 ii) $17280 + \frac{17280 \times 5 \times 6}{100} = \\22464 iii) $17280 \left(1 + \frac{4.5}{100}\right)^5 = \\21534.02 iv) For Scheme A, $22464 \div 72 = \\$312 < \\325 For Scheme B, $21534.02 \div 60 = \\$358.90 > \\325 Fang Min should choose Scheme A. Or For Scheme A, $325 \times 72 = 23400 > 22464$ For Scheme B, $325 \times 60 = 19500 < 21534.02$ Fang Min should choose Scheme A.	5	(a)(i) Angle CAD = 29° ($\angle$ in same segment) (a)(ii) Let O be the centre of the circle. Angle TDO = Angle TAO = 90° (tan $\perp$ rad) Angle AOD = $360 - 90 - 90 - 58$ = 122° ($\angle$ sum in quad.) Angle ABD = $122 \div 2 = 61^\circ$ ($\angle$ at centre = $2 \angle$ at circumference) Or Angle ABD = Angle ADT or DAT = $(180 - 58) \div 2$ = 61° (Alternate Segment Theorem) (tangent from ext. pt. / Equal tangents) (b) Angle AEC = $180 - 61 - 29 = 90^\circ$ ($\angle$ in opp segment) By converse of angle in semicircle, AC is a diameter. (c) $\frac{58}{360} \times 2\pi r + 2r = 18.07$ $2r \left(\frac{58\pi}{360} + 1\right) = 18.07$ $r = 6.00 \text{ cm}$
5	a) $S_n = n^2 + n$ or $n(n+1)$ ai) $S_{50} = 50^2 + 50 = 2550$ bi) $T_n = n^2$ bii) $a = 2n - 1$ biii) $2n - 1 = 99, n = 50, T_{50} = 2500$ c) $1 - 2 + 3 - 4 + \dots + 99 - 100$ = $1 + 3 + 5 + \dots + 99 - (2 + 4 + 6 + \dots + 100)$ = $T_{50} - S_{50}$ = -50 or $n^2 - (n^2 + n) = -n = -50$		

7 a) Angle $JOY = 180 - 123 = 57$ $\frac{\sin \angle OJY}{93.5} = \frac{\sin(57^\circ)}{85}$ $\angle OJY = \sin^{-1}\left(\frac{93.5 \sin(57^\circ)}{85}\right) = 67.3^\circ$ Bearing = 067.3°	9 a) $\angle OCR = \angle BPR$ (alt angle, $OC \parallel BP$) $\angle ORC = \angle BRP$ (vert. opp. angle) By AA similarity, $\triangle OCR$ is similar to $\triangle BPR$.
b) $OJ^2 = 93.5^2 + 85^2 - 2(93.5)(85)\cos(55.7^\circ)$ $OJ^2 = 7010.003$ $OJ = 83.7$ m OR $\frac{\sin(55.7^\circ)}{OJ} = \frac{\sin(57^\circ)}{85}$ $OJ = \frac{85 \sin(55.7^\circ)}{\sin(57^\circ)}$ $OJ = 83.7$ m	b) i) $\overline{CR} = 2a$ ii) $\overline{AR} = 3c - 3a$ iii) $\overline{PC} = -7.5c - 5a + 3c = -5a - 4.5c$ c) $\overline{RQ} = -2a + 2c = 2(c - a)$ $RQ = \frac{2}{3}AR$ and R is a common point, A, Q and R are collinear. Therefore, I agree with Haryl.
c) Area $\triangle JOY =$ $\frac{1}{2}(93.5)(85)\sin 55.7^\circ = 3282.7 = 3280 \text{ m}^2$	d) i) Area of $\triangle CRQ : \triangle OCR : \triangle BPR$ $2 : 3$ $4 : 9$ $8 : 12 : 27$ $\frac{\text{Area of } \triangle CRQ}{\text{Area of } \triangle BPR} = \frac{8}{27}$ OR $\frac{\frac{1}{2}(CR)(CQ)\sin \angle C}{\frac{1}{2}(BR)(BP)\sin \angle B} = \frac{CR}{BR} \times \frac{CQ}{BP}$ $= \frac{2}{3} \times \frac{2}{4.5} = \frac{8}{27}$
d) Shortest distance from J to OY $= 85 \sin 55.7^\circ = 70.218$ Greatest angle of elevation $= \tan^{-1} \frac{10000}{70.218} = 89.6^\circ$	
8 ia) Median = 70.5 marks ib) IQR = $76 - 63 = 13$ marks or 13.5 marks ic) $\frac{70}{100} \times 40 = 28$ students, $\therefore 75$ marks	
ii) $\frac{28}{40} \times \frac{27}{39} = \frac{63}{130}$	ii)  $\frac{\text{Area of } \triangle OCR}{\text{Area of } OAPQ} = \frac{12}{125}$
iii) The 2 curves <u>intersect at the same median</u> but Prelim 2 graph is <u>steeper</u> than Prelim 1 at the middle 50% of the distribution.	
iv) The class performed better for Prelim 3 as the median mark is higher. The class performance is more consistent for Prelim 3 as the IQR is smaller.	

10(a)	<u>Bag X</u>  $\frac{7}{12}$ B $\frac{5}{12}$ W <u>Bag Y</u>  $\frac{4}{5}$ B $\frac{1}{5}$ W $\frac{3}{5}$ B $\frac{2}{5}$ W $\frac{7}{12} \times \frac{4}{5} + \frac{5}{12} \times \frac{3}{5} = \frac{43}{60}$
10(b)	 $\frac{\sin(x^\circ)}{21.85} = \frac{\sin(68^\circ)}{44.62}$ $x = 27.0^\circ$ $y = 180 - 27.0 - 68 = 85.0^\circ$ $\theta = 90 - 85.0 = 5.0^\circ$ (1dp)