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	Class	Register Number
Name		

4048/01

17/S4PR1/EM/1

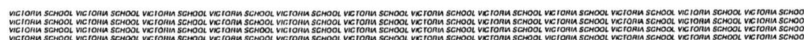
MATHEMATICS

PAPER 1

Tuesday

2 May 2017

2 hours



VICTORIA SCHOOL

**PRELIMINARY EXAMINATION ONE
SECONDARY FOUR**

Candidates answer on the Question Paper.

READ THESE INSTRUCTIONS FIRST

Write your name, class and register number on all the work you hand in.

Write in dark blue or black pen.

You may use a pencil for any diagrams or graphs.

Do not use paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.

If working is needed for any question it must be shown with the answer.

Omission of essential working will result in loss of marks.

You are expected to use a scientific calculator to evaluate explicit numerical expressions.

If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.

For π , use either your calculator value or 3.142, unless the question requires the answer in terms of π .

The number of marks is given in brackets [] at the end of each question or part question.

The total number of marks for this paper is 80.

Mathematical Formulae

Compound interest

$$\text{Total amount} = P \left(1 + \frac{r}{100} \right)^n$$

Mensuration

Curved surface area of a cone = πrl

Surface area of a sphere = $4\pi r^2$

$$\text{Volume of a cone} = \frac{1}{3}\pi r^2 h$$

$$\text{Volume of a sphere} = \frac{4}{3}\pi r^3$$

$$\text{Area of triangle } ABC = \frac{1}{2} ab \sin C$$

Arc length = $r\theta$, where θ is in radians
Sector area = $\frac{1}{2}r^2\theta$, where θ is in radians

Trigonometry

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

Statistics

$$\text{Mean} = \frac{\sum fx}{\sum f}$$

$$\text{Standard deviation} = \sqrt{\frac{\sum fx^2}{\sum f} - \left(\frac{\sum fx}{\sum f}\right)^2}$$

3

- 1 Calculate $\sqrt[3]{\frac{45^2 - 63.2}{0.896 - 1}}$, giving your answer correct to 5 significant figures.

Answer [1]

- 2 4.854×10^{-4} metres can be written as k micrometres. Find k .

Answer $k =$ [1]

- 3 When written as the product of their prime factors,
 p is $2 \times 3^2 \times 5^3$,
 q is $2^3 \times 3 \times 7^3$,
 r is $2^4 \times 3^2 \times 7^2$.

Find the

- (a) value of the square root of r ,

Answer (a) [1]

- (b) greatest number that will divide p , q and r exactly,

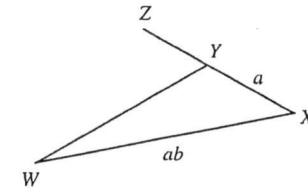
Answer (b) [1]

- (c) LCM of p , q and r , giving your answer as the product of its prime factors.

Answer (c) [1]

4

- 4 In the triangle WXY , $XY = a$ cm, $WX = ab$ cm and $WY = 2XY$. XY is produced to Z . Find $\cos \angle WYZ$ in terms of a and/or b .



Answer $\cos \angle WYZ =$ [2]

- 5 The length of a rectangular block is 30 cm and its breadth is 20 cm, both correct to the nearest centimetre.

- (a) Find the least possible base area of the block in m^2 .

Answer (a) m^2 [1]

- (b) The volume of the block is 0.03 m^3 , correct to the nearest 0.01 cubic metres. Find the greatest possible height of the block in metres.

Answer (b) m [2]

5

- 6 (a) Express $-x^2 - 2x + 1$ in the form $k - (x + h)^2$.

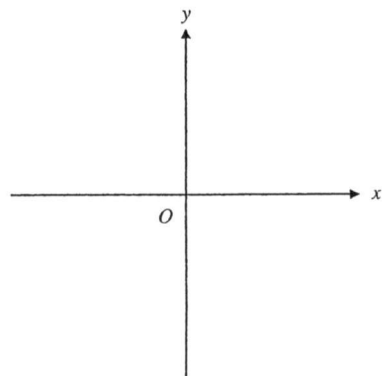
Answer (a) [2]

- (b) Hence, solve $x^2 + 2x = 1$, giving your answers to 2 decimal places.

Answer (b) $x =$ or [2]

- 7 Sketch the graph of $y = \frac{1}{x} - 1$.

Answer



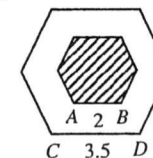
[2]

6

- 8 Evaluate $\frac{2^{1004} - 15(2^{1000}) + 1}{2^{1001} + 2}$.

Answer [2]

- 9 In a recent concert held at the Singapore Indoor Stadium, the hexagonal central stage is divided into 2 sections. The shaded area is a raised stage, similar to the central stage, while the remaining area is a water stage.



- (a) Given that $AB = 2$ m and $CD = 3.5$ m, find the value of $\frac{\text{area of raised stage}}{\text{area of water stage}}$.

Answer (a) [2]

- (b) The area of the raised stage is 4 m^2 . Find the volume of water required to fill the water stage given that the depth is 5 cm.

Answer (b) m^3 [2]

- 10 (a) It is given that,
 $\xi = \{\text{real numbers}\},$
 $N = \{\text{natural numbers}\},$
 $W = \{\text{whole numbers}\}.$

State if each of the following statements is True or False.

(i) $W \subset N,$

Answer (a)(i) [1]

(ii) $-2 \in W,$

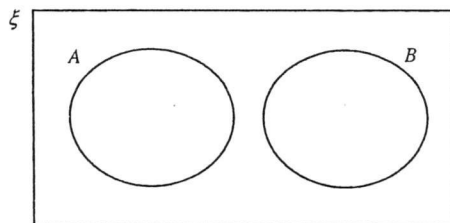
Answer (a)(ii) [1]

(iii) $n(N \cap W) = n(N).$

Answer (a)(iii) [1]

- (b) On the Venn Diagram shown in the answer space, shade the set $(A \cap B) \cup B'.$

Answer (b)



[1]

11 Simplify $\left(\frac{b^4}{2a}\right)^{-3} \div \frac{a^{\frac{1}{2}}(3b)^0}{\sqrt[3]{64b^{12}}}.$

Answer [3]

- 12 It is given that $32^x \div 4^x = 1.$

- (a) Show that $5y = 2x.$

Answer (a)

.....

.....

..... [1]

- (b) Hence or otherwise, solve the simultaneous equations,

$$32^x \div 4^x = 1,$$

$$4x - (y + x) = 13.$$

Answer $x = \dots\dots\dots$, $y = \dots\dots\dots$ [3]

13 Simplify $\frac{6m^2 - mn - 2n^2}{10m + 5n + 2mn + n^2}.$

Answer [4]

- 14 (a) Solve the inequality $\frac{4x-7}{3} < \frac{1}{2}(x+3)+1 \leq \frac{3}{4}x+8$.

Answer (a) [3]

- (b) Represent your answer to part (a) on the number line below.

Answer (b)



[1]

- 15 A map has a scale of 1 : n .

A conservation enclosure has an area of 33.8 km². It is represented by an area of 80 cm² on the map.

- (a) Find the value of n .

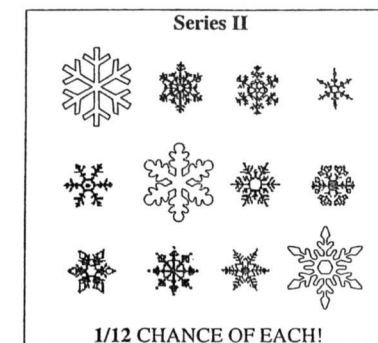
Answer (a) $n =$ [2]

- (b) The perimeter of the conservation enclosure on the map is 12 cm. Find the actual perimeter of the enclosure in kilometres.

Answer (b) km [2]

- 16 Blind boxes are sealed boxes containing collectibles from a specific series. Consumers do not know which specific design from the series they will receive from the purchased blind box. The probability of getting each design in each series is printed on the box.

Series I			
COLLECT THEM ALL!			
A	E	I	M
x	1/36	x	1/24
B	F	J	N
x	1/24	x	1/24
C	G	K	O
1/6	1/72	x	1/36
D	H	L	P
x	1/72	x	1/24



- (a) Using the information from Series I,

- (i) calculate the value of x ,

Answer (a)(i) $x =$ [1]

- (ii) from designs A to P, state which is the most common design.

Answer (a)(ii) [1]

- (b) In Series II, consumers will receive either a small black snowflake or a large white snowflake. A carton of Series II contains 12 boxes, each with a different design. Two boxes from this carton were purchased. Find the probability of receiving

- (i) 2 small black snowflakes,

Answer (b)(i) [1]

- (ii) at least 1 large white snowflake.

Answer (b)(ii) [1]

- 17 Singapore will see her first water price revision in 17 years on 1 July 2017. The price will be increased in 2 steps, on 1 July 2017 and on 1 July 2018.

Water Price Revision 2017

		Current		From 1 July 2017		From 1 July 2018	
		Water Price (\$/m ³)		Water Price (\$/m ³)		Water Price (\$/m ³)	
		0 - 40m ³	> 40m ³	0 - 40m ³	> 40m ³	0 - 40m ³	> 40m ³
Potable Water	Tariff	\$1.17	\$1.40	\$1.19	\$1.46	\$1.21	\$1.52
	Water Conservation Tax (1% of water tariff)	\$0.35 (30% of \$1.17)	\$0.63 (45% of \$1.40)	\$0.42 (35% of \$1.19)	\$0.73 (50% of \$1.46)	\$0.61 (50% of \$1.21)	\$0.99 (65% of \$1.52)
# Used Water	Waterborne Fee	\$0.28	\$0.28	\$0.78	\$1.02	\$0.92	\$1.18
	Sanitary Appliance Fee	\$2.80 per fitting*		Combined into Waterborne Fee		Combined into Waterborne Fee	
Total Price		\$1.10	\$2.61	\$2.39	\$3.21	\$2.74	\$3.69

Refers to the water consumed by the household. Waterborne Fee and Sanitary Appliance Fee go towards meeting the cost of treating used water and maintaining the used water network.

The table above is taken from Singapore's National Water Agency, Public Utilities Board's website. It was also stated that key revision to the water price includes "a 30% increase in water price, phased over 2 years, starting from 1 July 2017".

- (a) Determine how the 30% increase in water price is calculated. Show your working clearly.

Answer (a) [2]

- (b) Does this 30% increase in water price apply to all households regardless of water usage? Justify your answer.

Answer (b) [2]

- 18 A multiple choice test has a total of 50 questions. All questions must be attempted. There are 2 sections to the paper, Section A and Section B. The matrix Q shows the number of questions attempted by Victoria.

$$Q = \begin{pmatrix} \text{Correct} & \text{Incorrect} \\ 18 & 2 \\ x & 5 \end{pmatrix} \begin{matrix} \text{Section A} \\ \text{Section B} \end{matrix}$$

- (a) State the value of x .

Answer (a) $x =$ [1]

- (b) 2 marks are awarded for each correct answer and 1 mark is deducted for each incorrect answer. Represent the mark allocation in a column matrix M.

Answer (b) M = [1]

- (c) Evaluate the matrix QM.

Answer (c) QM = [1]

- (d) State what the elements in QM represent.

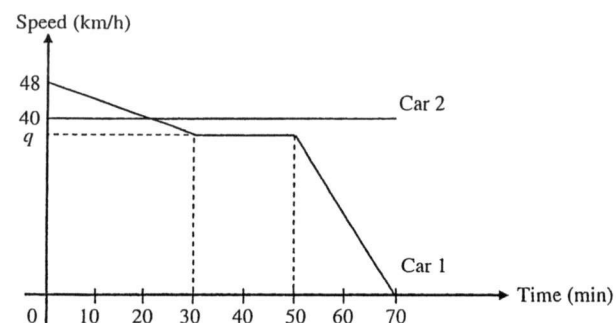
Answer (d) [1]

- (e) By matrix multiplication, find the total marks scored by Victoria in the test.

Answer (e) marks [1]

13

- 19 The diagram shows the speed-time graphs of 2 cars travelling from Town A to Town B.



- (a) Car 1 travelled at a constant deceleration of 24 km/h^2 for the first 30 minutes of the journey. Calculate the value of q .

Answer (a) $q = \dots\dots\dots$ [1]

- (b) Convert $q \text{ km/h}$ into m/s .

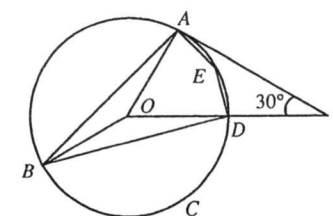
Answer (b) $\dots\dots\dots \text{ m/s}$ [1]

- (c) Find the distance from Town A when the 2 cars meet.

Answer (c) $\dots\dots\dots \text{ km}$ [3]

20

14



O is the centre of the circle passing through A, B, C, D and E .
 TA is a tangent to the circle.
 Angle $ATO = 30^\circ$.

- (a) Find, giving reasons for each answer,

- (i) angle ABD ,

Answer (a)(i) Angle $ABD = \dots\dots\dots$ [3]

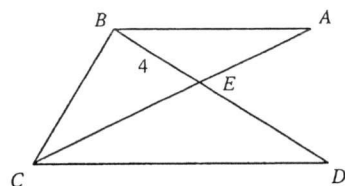
- (ii) angle AED .

Answer (a)(ii) Angle $AED = \dots\dots\dots$ [1]

- (b) Given that the radius of the circle is 2 cm and angle $BOD = \text{angle } AED$, show that the area of segment BCD is $\left(\frac{5}{3}\pi - 1\right)$ square centimetres.

Answer (b) $\dots\dots\dots$
 $\dots\dots\dots$
 $\dots\dots\dots$
 $\dots\dots\dots$ [2]

- 21 Lines AC and BD intersect at E . It is given that $BE = 4$ cm, $BD = 10$ cm and $3AE = 2EC$.



- (a) Prove a pair of similar triangles.

Answer (a)

 [3]

- (b) Prove that AB is parallel to CD .

Answer (b)
 [1]

- (c) Find the value of

(i) $\frac{\text{area of triangle } AEB}{\text{area of triangle } CED},$

Answer (c)(i) [1]

(ii) $\frac{\text{area of triangle } AEB}{\text{area of triangle } BCD}.$

Answer (c)(ii) [2]

- 22 The two towers within the One Raffles Place Complex are amongst the most iconic buildings in the Central Business District in Singapore.

Despite being constructed in the 1980s, Tower 1 (on the left) remains as one of the tallest buildings on the island. Given that the tower has a triangular cross-sectional area such that $AB = BC = 30$ m and $AC = 40$ m.



Using ruler and compass,

- (a) construct the cross-sectional of the tower using a scale of 1 cm to 5 m, [1]
 (b) construct the perpendicular bisector of BC and the angle bisector of angle ABC . [2]

Answer (a), (b) and (c)

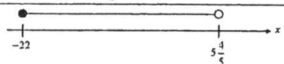
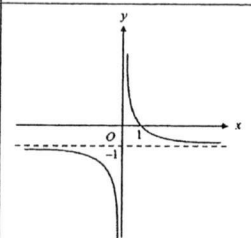
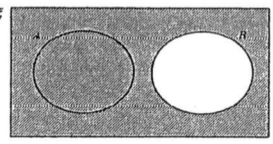
An ideal spot to view the sunset from the rooftop of the tower is equidistant from the edges AB and BC and nearer to point B .

- (c) Mark and label a possible spot, S , that is ideal to view the sunset. [1]

End of Paper

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Answer Key

1		-26.620	14	(a)	$-22 \leq x < 5\frac{4}{5}$
2		485.4		(b)	
3	(a)	84	(b)	6	
	(c)	$2^4 \times 3^2 \times 5^3 \times 7^3$			
4		$\frac{b^2 - 5}{4}$	15	(a)	65 000
				(b)	7.8 km
			16	(ai)	$\frac{1}{12}$
				(aii)	C
5	(a)	$\frac{2301}{40000} \text{ m}^2$ or 0.057525 m^2		(bi)	$\frac{6}{11}$
				(bii)	$\frac{5}{11}$
	(b)	$\frac{1400}{2301} \text{ m}$	17	(a)	$\frac{2.74 - 2.10}{2.10} \times 100\% \approx 30\%$
6	(a)	$-(x+1)^2 + 2$		(b)	For usage $> 40 \text{ m}^3$, % increase $= 41\frac{7}{13}\%$ No, usage $> 40 \text{ m}^3$, % inc $> 30\%$
	(b)	$x = -2.41$ or $x = 0.41$			
7			18	(a)	25
				(b)	$\begin{pmatrix} 2 \\ -1 \end{pmatrix}$
				(c)	$\begin{pmatrix} 34 \\ 45 \end{pmatrix}$
				(d)	Marks obtained for Section A and Section B respectively.
				(e)	$(1 \ 1) \begin{pmatrix} 34 \\ 45 \end{pmatrix} = (79)$, 79 marks
8		$\frac{1}{2}$	19	(a)	36
				(b)	10 m/s
9	(a)	$\frac{16}{33}$		(c)	30 km
	(b)	$\frac{33}{80} \text{ m}^3$ or 0.4125 m^3	20	(ai)	30°
				(aii)	150°
10	(a)	False, False, True		(b)	$\frac{150^\circ}{360^\circ} \times \pi (2^2) - \frac{1}{2} (2)(2) \sin 150^\circ$
	(b)		21	(a)	$\angle AEB = \angle CED$ (vert. opp. $\angle$ s) $\frac{AE}{CE} = \frac{2}{3}$ (given $3AE = 2EC$) $\frac{BE}{DE} = \frac{4}{10-4} = \frac{2}{3}$ $\therefore \frac{AE}{CE} = \frac{BE}{DE}$ ΔAEB & ΔCED are similar. $\angle BAE = \angle DCE$ (similar Δ) Angle properties of alternate angles, AB is parallel to CD .
11		$\frac{32a^2}{b^8}$		(ci)	$\frac{4}{9}$
				(cii)	$\frac{4}{15}$
12	(a)	$2^{5y} \div 2^{2x} = 2^0$			
	(b)	$x = 5, y = 2$			
13		$\frac{(3m-2n)}{(5+n)}$			

- 1 Calculate $\sqrt[3]{\frac{45^2 - 63.2}{0.896 - 1}}$, giving your answer correct to 5 significant figures.

$$\sqrt[3]{\frac{45^2 - 63.2}{0.896 - 1}} = -26.6199 \text{ (6sf)}$$

$$= -26.620 \text{ (5sf)} \quad \mathbf{B1}$$

Most students do get the correct answer.

Answer [1]

- 2 4.854×10^{-4} metres can be written as k micrometres. Find k .

$$4.854 \times 10^{-4} \text{ metres}$$

$$= 4.854 \times 10^3 \times 10^{-6} \text{ metres}$$

$$= 485.4 \text{ micrometres}$$

Some students took micrometers as 10^{-9} m . Some students leave the answer in standard form.

$$k = 485.4 \quad \mathbf{B1}$$

Answer $k =$ [1]

- 3 When written as the product of their prime factors,
 p is $2 \times 3^2 \times 5^3$,
 q is $2^3 \times 3 \times 7^3$,
 r is $2^4 \times 3^2 \times 7^2$.

Find the

This question was well attempted.

- (a) value of the square root of r ,

$$\text{square root of } r = 2^2 \times 3 \times 7$$

$$= 84 \quad \mathbf{A1}$$

Answer (a) [1]

- (b) greatest number that will divide p , q and r exactly,

$$\text{HCF} = 2 \times 3$$

$$= 6 \quad \mathbf{B1}$$

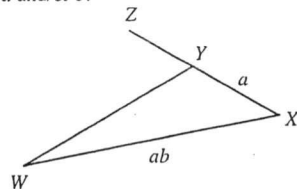
Answer (b) [1]

- (c) LCM of p , q and r , giving your answer as the product of its prime factors.

$$\text{LMC} = 2^4 \times 3^2 \times 5^3 \times 7^3 \quad \mathbf{B1}$$

Answer (c) [1]

- 4 In the triangle WXY , $XY = a$ cm, $WX = ab$ cm and $WY = 2XY$. XY is produced to Z . Find $\cos \angle WYZ$ in terms of a and/or b .



$$(ab)^2 = (2a)^2 + a^2 - 2(2a)(a)\cos \angle WYX \quad \text{B1}$$

$$4a^2 \cos \angle WYX = 5a^2 - a^2 b^2$$

$$\cos \angle WYX = \frac{a^2(5-b^2)}{4a^2}$$

$$= \frac{5-b^2}{4}$$

$$\cos \angle WYZ = \frac{b^2-5}{4} \quad \text{A1}$$

Many students apply the cos rule with respect $\cos \angle WYZ$ without considering the correct angle.

Answer $\cos \angle WYZ = \dots\dots\dots$ [2]

- 5 The length of a rectangular block is 30 cm and its breadth is 20 cm, both correct to the nearest centimetre.

- (a) Find the least possible base area of the block in m^2 .

$$\text{Least possible area} = 29.5 \text{ cm} \times 19.5 \text{ cm}$$

$$= 0.295 \text{ m} \times 0.195 \text{ m}$$

$$= \frac{2301}{40000} \text{ m}^2 \text{ or } 0.057525 \text{ m}^2 \quad \text{A1}$$

Most students did well for this qn.

Answer (a) $\dots\dots\dots \text{m}^2$ [1]

- (b) The volume of the block is 0.03 m^3 , correct to the nearest 0.01 cubic metres. Find the greatest possible height of the block in metres.

$$\text{Greatest possible height} = \frac{0.035}{0.057525} \quad \text{M1}$$

$$= \frac{1400}{2301} \text{ m} \quad \text{A1 (0.608 acceptable)}$$

Most students use max volume as 0.03 m^3

Answer (b) $\dots\dots\dots \text{m}$ [2]

- 6 (a) Express $-x^2 - 2x + 1$ in the form $k - (x+h)^2$.

$$-x^2 - 2x + 1 = -(x^2 + 2x) + 1 \quad \text{M1}$$

$$= -[x^2 + 2x + 1^2 - 1^2] + 1$$

$$= -[(x+1)^2 - 1] + 1$$

$$= -(x+1)^2 + 2 \quad \text{A1}$$

Quite a number of students attempt the qn by equating to zero.

Answer (a) $\dots\dots\dots$ [2]

- (b) Hence, solve $x^2 + 2x = 1$, giving your answers to 2 decimal places.

$$x^2 + 2x = 1$$

$$-x^2 - 2x + 1 = 0 \quad \left. \begin{array}{l} \\ \\ \end{array} \right\} \text{M1}$$

$$-(x+1)^2 + 2 = 0$$

$$(x+1)^2 = 2$$

$$x+1 = \pm\sqrt{2}$$

$$x = \pm\sqrt{2} - 1$$

$$x = -2.414 \text{ or } x = 0.414 \text{ (3dp)}$$

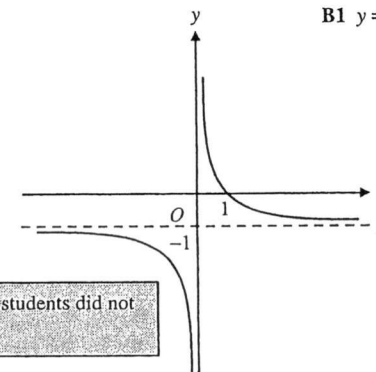
$$x = -2.41 \text{ or } x = 0.41 \text{ (2dp)} \quad \text{A1 (both)}$$

Answer (b) $x = \dots\dots\dots$ or $\dots\dots\dots$ [2]

- 7 Sketch the graph of $y = \frac{1}{x} - 1$.

Answer

B1 shape & asymptote
B1 $y = -1, x = 1$



Question was badly done. Many students did not draw the asymptote line.

[2]

8 Evaluate $\frac{2^{1004} - 15(2^{1000}) + 1}{2^{1001} + 2}$.

$$\begin{aligned}\frac{2^{1004} - 15(2^{1000}) + 1}{2^{1001} + 2} &= \frac{2^4(2^{1000}) - 15(2^{1000}) + 1}{2(2^{1000}) + 2} \\ &= \frac{2^{1000} + 1}{2(2^{1000} + 1)} \\ &= \frac{1}{2}\end{aligned}$$

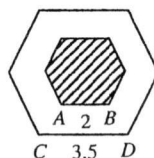
A1

B1 (same index 2^{1000})

Question was badly done. Some students write NA as they compute using calculator.

Answer [2]

- 9 In a recent concert held at the Singapore Indoor Stadium, the hexagonal central stage is divided into 2 sections. The shaded area is a raised stage, similar to the central stage, while the remaining area is a water stage.



- (a) Given that $AB = 2$ m and $CD = 3.5$ m, find the value of $\frac{\text{area of raised stage}}{\text{area of water stage}}$.

Since the stages are similar,

$$\frac{\text{area of raised stage}}{\text{area of whole stage}} = \left(\frac{2}{3.5}\right)^2 = \frac{16}{49}$$

B1 $\left(\frac{2}{3.5}\right)^2$

$$\frac{\text{area of raised stage}}{\text{area of water stage}} = \frac{16}{49 - 16}$$

$$= \frac{16}{33}$$

A1

Question was quite well attempted.

Answer (a) [2]

- (b) The area of the raised stage is 4 m^2 . Find the volume of water required to fill the water stage given that the depth is 5 cm.

$$\text{Area of water stage} = \frac{4}{16} \times 33$$

$$= 8.25 \text{ m}^2$$

$$\text{Volume of water} = 8.25 \times 0.05$$

M1

$$= \frac{33}{80} \text{ m}^3 \text{ or } 0.4125 \text{ m}^3$$

A1

Answer (b) m^3 [2]

- 10 (a) It is given that,
 $\xi = \{\text{real numbers}\}$,
 $N = \{\text{natural numbers}\}$,
 $W = \{\text{whole numbers}\}$.

Questions were badly done.

State if each of the following statements is True or False.

- (i) $W \subset N$,

Answer (a)(i) False B1 [1]

- (ii) $-2 \in W$,

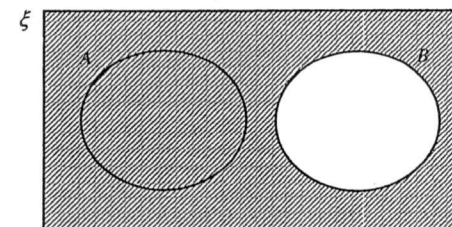
Answer (a)(ii) False B1 [1]

- (iii) $n(N \cap W) = n(N)$.

Answer (a)(iii) True B1 [1]

- (b) On the Venn Diagram shown in the answer space, shade the set $(A \cap B) \cup B'$.

Answer (b)



B1

[1]

11 Simplify $\left(\frac{b^4}{2a}\right)^{-3} \div \frac{a^{\frac{1}{2}}(3b)^0}{\sqrt[3]{64b^{12}}}$.

$$\left(\frac{b^4}{2a}\right)^{-3} \div \frac{a^{\frac{1}{2}}(3b)^0}{\sqrt[3]{64b^{12}}} = \left(\frac{2a}{b^4}\right)^3 \times \frac{4b^4}{a^{\frac{1}{2}}}$$

$$= \frac{8a^3}{b^{12}} \times \frac{4b^4}{a^{\frac{1}{2}}}$$

$$= \frac{32a^{\frac{5}{2}}}{b^8}$$

A3 $(32, a^{\frac{5}{2}}, \frac{1}{b^8} \text{ o.e.})$

Most students have no difficulty for this question except that some made careless mistakes.

Answer [3]

- 12 It is given that $32^y \div 4^x = 1$.

- (a) Show that $5y = 2x$.

Answer (a)

Questions was well attempted.

$$\left. \begin{array}{l} 32^y \div 4^x = 1 \\ 2^{5y} \div 2^{2x} = 2^0 \\ 5y - 2x = 0 \\ 5y = 2x \end{array} \right\} \text{A1}$$

[1]

- (b) Hence or otherwise, solve the simultaneous equations,

$$\begin{aligned} 32^y \div 4^x &= 1, \\ 4x - (y + x) &= 13. \end{aligned}$$

$$5y = 2x \quad \therefore y = \frac{2}{5}x$$

$$\text{Sub } y = \frac{2}{5}x, \quad 4x - \left(\frac{2}{5}x + x\right) = 13 \quad \text{M1}$$

$$\frac{13}{5}x = 13$$

$$x = 5 \quad \text{A1}$$

$$\begin{aligned} x = 5, \quad y &= \frac{2}{5}(5) \\ &= 2 \quad \text{A1} \end{aligned}$$

Answer $x = \dots\dots\dots$, $y = \dots\dots\dots$ [3]

- 13 Simplify $\frac{6m^2 - mn - 2n^2}{10m + 5n + 2mn + n^2}$.

Some students wrote the denominator as $5(2m+n)n(2m+n)$

$$\frac{6m^2 - mn - 2n^2}{10m + 5n + 2mn + n^2} = \frac{(3m-2n)(2m+n)}{5(2m+n) + n(2m+n)} \quad \text{B1}$$

$$= \frac{(3m-2n)(2m+n)}{(5+n)(2m+n)} \quad \text{M1}$$

$$= \frac{(3m-2n)}{(5+n)} \quad \text{A1}$$

Answer $\dots\dots\dots$ [4]

- 14 (a) Solve the inequality $\frac{4x-7}{3} < \frac{1}{2}(x+3)+1 \leq \frac{3}{4}x+8$.

$$\frac{4x-7}{3} < \frac{1}{2}(x+3)+1 \quad \text{and} \quad \frac{1}{2}(x+3)+1 \leq \frac{3}{4}x+8 \quad \text{B1 (split, "and")}$$

$$2(4x-7) < 3(x+3)+6 \quad 2(x+3)+4 \leq 3x+32$$

$$8x-14 < 3x+9+6 \quad 2x+6+4 \leq 3x+32$$

$$5x < 29$$

$$-22 \leq x$$

$$x < 5\frac{4}{5}$$

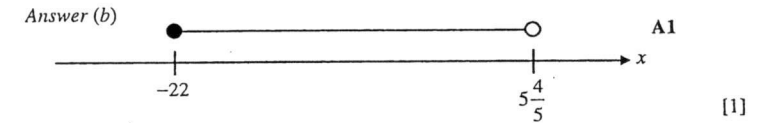
$$x \geq -22 \quad \text{A1 (both)}$$

$$\therefore -22 \leq x < 5\frac{4}{5} \quad \text{A1}$$

Some students did not write "and".

Answer (a) $\dots\dots\dots$ [3]

- (b) Represent your answer to part (a) on the number line below.



- 15 A map has a scale of 1 : n.

A conservation enclosure has an area of 33.8 km². It is represented by an area of 80 cm² on the map.

- (a) Find the value of n.

$$\begin{aligned} \text{Area Scale} &= 80 \text{ cm}^2 : 33.8 \text{ km}^2 \\ &= 400 \text{ cm}^2 : 169 \text{ km}^2 \end{aligned}$$

$$\text{Linear Scale} = \sqrt{400} \text{ cm} : \sqrt{169} \text{ km} \quad \text{M1}$$

$$= 20 \text{ cm} : 13 \text{ km}$$

$$= 1 \text{ cm} : 0.65 \text{ km}$$

$$= 1 : 65\,000$$

$$n = 65\,000 \quad \text{A1}$$

Poor presentation from majority of the students.
Prone to wrong conversions when trying to
convert 33.8 km² to cm².

Answer (a) $n = \dots\dots\dots$ [2]

- (b) The perimeter of the conservation enclosure on the map is 12 cm. Find the actual perimeter of the enclosure in kilometres.

$$\text{Perimeter} = 12 \times 0.65 \quad \text{M1}$$

$$= 7.8 \text{ km} \quad \text{A1}$$

Answer (b) $\dots\dots\dots$ km [2]

- 16 Blind boxes are sealed boxes containing collectibles from a specific series. Consumers do not know which specific design from the series they will receive from the purchased blind box. The probability of getting each design in each series is printed on the box.

Series I				Series II			
COLLECT THEM ALL!							
A	E	I	M				
x	1/36	x	1/24				
B	F	J	N				
x	1/24	x	1/24				
C	G	K	O				
1/6	1/72	x	1/36				
D	H	L	P				
x	1/72	x	1/24	1/12 CHANCE OF EACH!			

- (a) Using the information from Series I,

- (i) calculate the value of x ,

(a) Well attempted.

$$7x + \frac{1}{6} + 2\left(\frac{1}{36}\right) + 4\left(\frac{1}{24}\right) + 2\left(\frac{1}{72}\right) = 1 \quad \therefore x = \frac{1}{12} \quad \text{A1}$$

Answer (a)(i) $x = \dots\dots\dots$ [1]

- (ii) from designs A to P, state which is the most common design.

Answer (a)(ii) $\dots\dots\dots$ C B1 [1]

- (b) In Series II, consumers will receive either a small black snowflake or a large white snowflake. A carton of Series II contains 12 boxes, each with a different design. Two boxes from this carton were purchased. Find the probability of receiving

- (i) 2 small black snowflakes,

$$\text{Probability} = \frac{9}{12} \times \frac{8}{11} = \frac{6}{11} \quad \text{A1}$$

(bi) Wrong assumption, quite a number of students attempted the question assuming "with replacement".

Answer (b)(i) $\dots\dots\dots$ [1]

- (ii) at least 1 large white snowflake.

$$\text{Probability} = 1 - \frac{6}{11} = \frac{5}{11} \quad \text{A1}$$

(bii) Not many were able to see the shorter method of solving and instead went to calculate the probability of WW + WB + BW.

Answer (b)(ii) $\dots\dots\dots$ [1]

- 17 Singapore will see her first water price revision in 17 years on 1 July 2017. The price will be increased in 2 steps, on 1 July 2017 and on 1 July 2018.

Water Price Revision 2017

		Current		From 1 July 2017		From 1 July 2018	
		Water Price (\$/m ³)		Water Price (\$/m ³)		Water Price (\$/m ³)	
		0 - 40m ³	> 40m ³	0 - 40m ³	> 40m ³	0 - 40m ³	> 40m ³
Potable Water	Tariff	\$1.17	\$1.40	\$1.19	\$1.46	\$1.21	\$1.52
	Water Conservation Tax (% of water tariff)	\$0.35 (30% of \$1.17)	\$0.63 (45% of \$1.40)	\$0.42 (35% of \$1.19)	\$0.73 (50% of \$1.46)	\$0.61 (50% of \$1.21)	\$0.99 (65% of \$1.52)
# Used Water	Waterborne Fee	\$0.28	\$0.28	\$0.78	\$1.02	\$0.92	\$1.18
	Sanitary Appliance Fee	\$2.80 per fitting*		Combined into Waterborne Fee		Combined into Waterborne Fee	
Total Price		\$2.10	\$2.61	\$2.39	\$3.21	\$2.74	\$3.69

* Refers to the water consumed by the household. Waterborne Fee and Sanitary Appliance Fee go towards meeting the cost of treating used water and maintaining the used water network.

The table above is taken from Singapore's National Water Agency, Public Utilities Board's website. It was also stated that key revision to the water price includes "a 30% increase in water price, phased over 2 years, starting from 1 July 2017".

- (a) Determine how the 30% increase in water price is calculated. Show your working clearly.

Answer (a)

$$\begin{aligned} \% \text{ increase} &= \frac{2.74 - 2.10}{2.10} \times 100\% \\ &= 30\frac{10}{21}\% \\ &= 30\% \end{aligned} \quad \left. \vphantom{\begin{aligned} \% \text{ increase} &= \frac{2.74 - 2.10}{2.10} \times 100\%} \right\} \text{A1}$$

Many did not even attempt the question!
1. Do not know how to interpret the table
2. Wrong concept: addition of percentage
3. Left answer as 30.5% and not 30% as required by the question
4. Did not show $\times 100\%$

[2]

- (b) Does this 30% increase in water price apply to all households regardless of water usage? Justify your answer.

$$\begin{aligned} \text{For usage } > 40 \text{ m}^3, \quad \% \text{ increase} &= \frac{3.69 - 2.61}{2.61} \times 100\% \\ &= 41\frac{7}{13}\% \end{aligned} \quad \left. \vphantom{\begin{aligned} \% \text{ increase} &= \frac{3.69 - 2.61}{2.61} \times 100\%} \right\} \text{B1}$$

Many did not see the hint in the question "regardless of water usage". In justifying, students still missed out on answering "Yes" or "No" to the Qn.

Answer (b) No, the 30% increase does not apply to all regardless of water usage.

For households whose usage exceed 40 m³, the percentage increase

in water price per additional cubic metre is more than 30%. A1 [2]

- 18 A multiple choice test has a total of 50 questions. All questions must be attempted. There are 2 sections to the paper, Section A and Section B. The matrix Q shows the number of questions attempted by Victoria.

$$Q = \begin{pmatrix} \text{Correct} & \text{Incorrect} \\ 18 & 2 \\ x & 5 \end{pmatrix} \begin{matrix} \text{Section A} \\ \text{Section B} \end{matrix}$$

- (a) State the value of x .

$$\begin{aligned} x &= 50 - 18 - 2 - 5 \\ &= 25 \end{aligned} \quad \text{B1}$$

Answer (a) $x = \dots\dots\dots$ [1]

- (b) 2 marks are awarded for each correct answer and 1 mark is deducted for each incorrect answer. Represent the mark allocation in a column matrix M.

$$M = \begin{pmatrix} 2 \\ -1 \end{pmatrix} \quad \text{B1}$$

Surprising badly done. Students do not know what a column matrix is! [1]

Answer (b) $M = \dots\dots\dots$ [1]

- (c) Evaluate the matrix QM.

$$\begin{aligned} QM &= \begin{pmatrix} 18 & 2 \\ 25 & 5 \end{pmatrix} \begin{pmatrix} 2 \\ -1 \end{pmatrix} \\ &= \begin{pmatrix} 34 \\ 45 \end{pmatrix} \end{aligned} \quad \text{A1}$$

1. Correct QM written but wrong multiplication (need first check order of the matrices used)
2. $QM \neq MQ$

Answer (c) $QM = \dots\dots\dots$ [1]

- (d) State what the elements in QM represent.

Lack the word "respectively". Follow instructions given by subject teachers! [1]

Answer (d) The elements represent the marks obtained by Victoria in the test for

Section A and Section B respectively. B1 [1]

- (e) By matrix multiplication, find the total marks scored by Victoria in the test.

$$\begin{pmatrix} 1 & 1 \end{pmatrix} \begin{pmatrix} 34 \\ 45 \end{pmatrix} = (79)$$

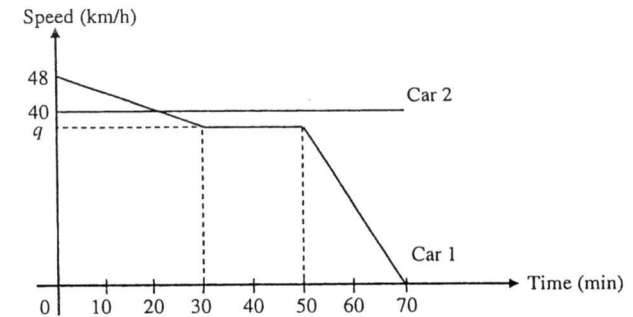
A1 (must be 1x1 matrix)

Total = 79 marks

1. Question requested for matrix multiplication!
2. Quite a number simply, $\begin{pmatrix} 1 & 1 \end{pmatrix} \begin{pmatrix} 34 \\ 45 \end{pmatrix} = 79$. The result MUST be a matrix, before giving a concluding statement.

Answer (e) $\dots\dots\dots$ marks [1]

- 19 The diagram shows the speed-time graphs of 2 cars travelling from Town A to Town B.



- (a) Car 1 travelled at a constant deceleration of 24 km/h^2 for the first 30 minutes of the journey. Calculate the value of q .

$$\begin{aligned} \frac{48 - q}{\frac{1}{2}} &= 24 \\ 48 - q &= 12 \end{aligned}$$

$$\begin{aligned} q &= 36 \end{aligned} \quad \text{A1}$$

Answer (a) $q = \dots\dots\dots$ [1]

- (b) Convert $q \text{ km/h}$ into m/s .

$$\begin{aligned} 36 \text{ km/h} &= \frac{36000 \text{ m}}{60 \times 60 \text{ s}} \\ &= 10 \text{ m/s} \end{aligned} \quad \text{B1}$$

Answer (b) $\dots\dots\dots \text{ m/s}$ [1]

- (c) Find the distance from Town A when the 2 cars meet.

Let t h be the time they meet,

$$40t = \frac{1}{2} \left(\frac{1}{2} \right) (48 + 36) + 36 \left(t - \frac{1}{2} \right)$$

B1 (left of equation)
M1 (right of equation)

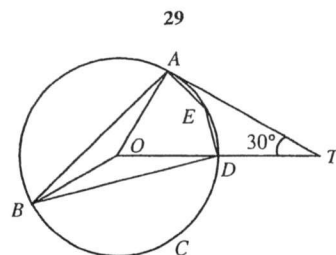
$$40t = 21 + 36t - 18$$

$$t = \frac{3}{4}$$

$$\begin{aligned} \text{Distance} &= 40 \left(\frac{3}{4} \right) \\ &= 30 \text{ km} \end{aligned} \quad \text{A1}$$

Badly done!
1. Many assumed the cars meet at $t = 20$, that is the time when they are at same speed.
2. Some even dropped a straight line on the graph to obtain $t = 20$.

Answer (c) $\dots\dots\dots \text{ km}$ [3]



O is the centre of the circle passing through A, B, C, D and E .
 TA is a tangent to the circle.
 Angle $ATO = 30^\circ$.

(a) Find, giving reasons for each answer,

(i) angle ABD ,

$$\angle TAO = 90^\circ \text{ (tan } \perp \text{ rad)}$$

B1

$$\angle AOT = 180^\circ - 90^\circ - 30^\circ \text{ (}\angle \text{ sum in } \Delta \text{)}$$

M1

$$= 60^\circ$$

$$\angle ABD = \frac{1}{2} \times 60^\circ \text{ (}\angle \text{ at centre} = 2 \times \angle \text{ at circumference)}$$

$$= 30^\circ$$

A1

Answer (a)(i) Angle $ABD = \dots\dots\dots$ [3]

(ii) angle AED .

$$\angle AED = 180^\circ - 30^\circ \text{ (opp } \angle \text{ s in cyclic quad.)}$$

$$= 150^\circ$$

A1

Answer (a)(ii) Angle $AED = \dots\dots\dots$ [1]

(b) Given that the radius of the circle is 2 cm and angle $BOD = \text{angle } AED$, show that the area of segment BCD is $\left(\frac{5}{3}\pi - 1\right)$ square centimetres.

Answer (b)

All steps must be shown – all relevant substitution into the correct formulae, etc...

$$\text{Area of segment} = \frac{150^\circ}{360^\circ} \times \pi(2^2) - \frac{1}{2}(2)(2)\sin 150^\circ$$

M1 (sector area)

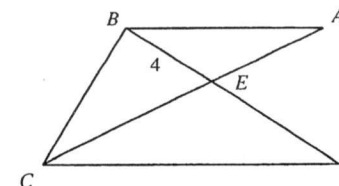
M1 (triangle area)

$$= \left(\frac{5}{3}\pi - 1\right) \text{ cm}^2$$

If angle $\neq 150^\circ$, 1M

[2]

21 Lines AC and BD intersect at E . It is given that $BE = 4$ cm, $BD = 10$ cm and $3AE = 2EC$.



(a) Prove a pair of similar triangles.

Answer (a) In $\triangle AEB$ & $\triangle CED$, $\angle AEB = \angle CED$ (vert. opp. $\angle$ s) B1

Most lost the first A1 – did not state “given” for AE/CE ratio or did not conclude the equal ratios of the 2 corr. sides.

$$\frac{AE}{CE} = \frac{2}{3} \text{ (given } 3AE = 2EC \text{)}$$

$$\frac{BE}{DE} = \frac{4}{10 - 4} = \frac{2}{3}$$

$$\therefore \frac{AE}{CE} = \frac{BE}{DE}$$

A1

A LOT of students are not naming the triangles correctly!

Hence $\triangle AEB$ & $\triangle CED$ are similar. A1 [3]

(b) Prove that AB is parallel to CD .

Answer (b) Since $\triangle AEB$ & $\triangle CED$ are similar, $\angle BAE = \angle DCE$.

Using angle properties of alternate angles, AB is parallel to CD . A1 [1]

(c) Find the value of

Students needs to learn to phrase their reasoning properly! Do NOT use short forms for such “explaining” questions.

(i) $\frac{\text{area of triangle } AEB}{\text{area of triangle } CED}$,

$$\frac{\text{area of triangle } AEB}{\text{area of triangle } CED} = \left(\frac{2}{3}\right)^2$$

$$= \frac{4}{9}$$

A1

Answer (c)(i) [1]

(ii) $\frac{\text{area of triangle } AEB}{\text{area of triangle } BCD}$.

$$\frac{\text{area of triangle } CED}{\text{area of triangle } BCD} = \frac{3}{2+3}$$

$$= \frac{3}{5} = \frac{9}{15}$$

$$\frac{\text{area of triangle } AEB}{\text{area of triangle } BCD} = \frac{4}{15}$$

A1

Badly done, many assumed the triangles are similar!

M1 (triangle with same height)

$$\frac{\text{Area } AEB}{\text{Area } BCD} = \frac{\text{Area } AEB}{\text{Area } CED} \times \frac{\text{Area } CED}{\text{Area } BCD}$$

$$= \frac{4}{9} \times \frac{3}{5} = \frac{4}{15}$$

Answer (c)(ii) [2]

- 22 The two towers within the One Raffles Place Complex are amongst the most iconic buildings in the Central Business District in Singapore.

Despite being constructed in the 1980s, Tower 1 (on the left) remains as one of the tallest buildings on the island. Given that the tower has a triangular cross-sectional area such that $AB = BC = 30$ m and $AC = 40$ m.

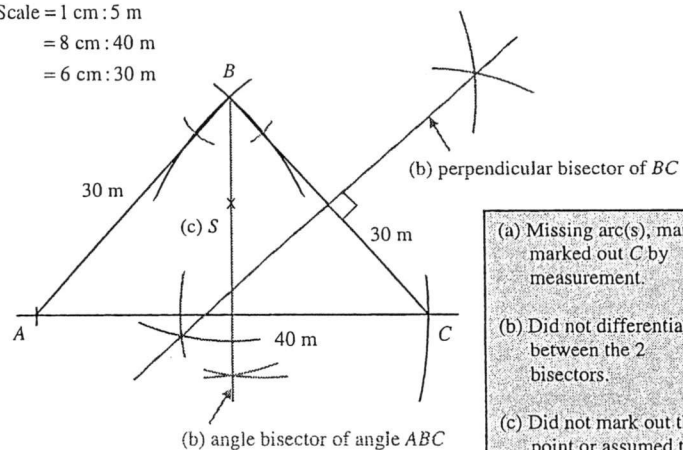


Using ruler and compass,

- (a) construct the cross-sectional of the tower using a scale of 1 cm to 5 m, [1]
 (b) construct the perpendicular bisector of BC and the angle bisector of angle ABC . [2]

Answer (a), (b) and (c)

Scale = 1 cm : 5 m
 = 8 cm : 40 m
 = 6 cm : 30 m



(a) Missing arc(s), many marked out C by measurement.

(b) Did not differentiate between the 2 bisectors.

(c) Did not mark out the point or assumed the intersection is the required point.

- (a) B1 (3 arcs + A, B)
 (b) B1 ($\perp$ bisector, 4
 B1 ($\angle$ bisector, 4
 (c) B1 (on $\angle$ bisecto

intersection of both bisectors)

An ideal spot to view the sunset from the rooftop of the tower is equidistant from the edges AB and BC and nearer to point B .

- (c) Mark and label a possible spot, S , that is ideal to view the sunset. [1]

End of Paper

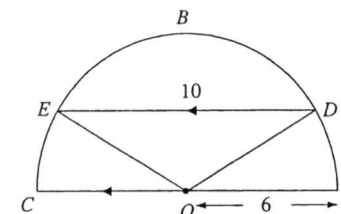
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Answer all the questions.

- 1 (a) Expand $\left(w + \frac{1}{w}\right)^2$. [1]
- (i) Given that $w + \frac{1}{w} = 6$, show that $w^2 + \frac{1}{w^2} = 34$. [1]
- (ii) Hence, find the value of $w - \frac{1}{w}$, if $0 < w < 1$. [3]
- (b) Given that $p = \sqrt{\frac{2m^2 + 3p}{m^2 - p}}$, express m in terms of p . [3]
- (c) Given $-4 \leq p \leq 10$ and $11 \leq q \leq 20$, where p and q are integers, calculate the
- (i) greatest possible value of $q - p$, [1]
- (ii) least possible value of $\frac{p}{q}$, if $q \neq 0$, [1]
- (iii) least value of $2p^2 + q^2$. [1]
-
- 2 (a) In January 2017, the price of a car was \$96 000. Over the next two months, the price increased by 25% and then decreased by 15%. Mr Ali, a potential car owner, said that the overall increase in the price of the car was 10%. Do you agree with Mr Ali? Justify your answer. [2]
- (b) Alex bought a home entertainment system during the Great Singapore Sale on hire purchase. He paid a deposit of 20% of the selling price. The balance of the payment was \$8000.
- (i) Calculate the selling price of the system. [1]
- (ii) Alex paid the balance of the payment in monthly instalments of \$380, charged at $x\%$ per annum simple interest for 2 years. Find the value of x . [3]
- (iii) Alex could have taken a loan to pay the outstanding balance. If the finance company charged him a compound interest of 6.5% per annum with a repayment period of 2 years, would you recommend that he takes up the loan instead of paying in instalments? State your reason clearly. [3]

- 3 (a) A polygon has n sides. Three of its exterior angles are 60° , 25° and 55° . The remaining angles in the polygon have interior angles of 160° . Find value of n . [2]
- (b) John claimed that he can draw a regular polygon with its exterior angle as 70° . Is he right? Explain. [2]

(c)

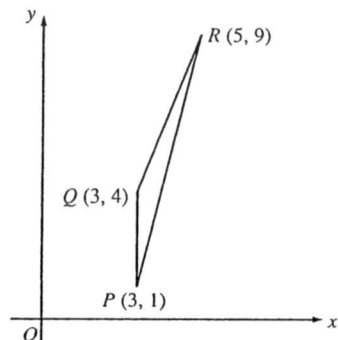


In the figure above, the chord DE , of length 10 cm, is parallel to the diameter AC of the semicircle with centre O and a radius 6 cm.

- (i) Find $\angle DOE$ in radians. [2]
- (ii) Calculate the area of the segment DBE . [3]
-
- 4 John and Tim begin to walk with a constant speed of x m/s and y m/s respectively towards each other along a pathway of length 20 metres.
- (a) Given that they walk past each other 4 seconds later, show that $y = 5 - x$. [2]
- (b) Given also that John takes 3 seconds more than Tim to walk through the pathway, form an equation in x and show that it reduces to
- $$3x^2 - 55x + 100 = 0. \quad [3]$$
- (c) Solve the equation $3x^2 - 55x + 100 = 0$, giving your answers correct to 2 decimal places. [2]
- (d) Calculate the time, to the nearest second, taken by Tim to walk through the pathway. [2]

5

- 5 The points $P(3, 1)$, $Q(3, 4)$ and $R(5, 9)$ are shown in the diagram.



Find the

- coordinates of S such that $PQRS$ is a parallelogram, [1]
- exact length of PR , [1]
- area of $\triangle PQR$ and hence the perpendicular distance from Q to PR , [4]
- equation of the line that passes through P and is parallel to the line $3y = 4x + 5$. [2]

- 6 (a) The results of two class tests are shown in the tables below. The total marks for both tests are 80.

Test 1

Marks (x)	$40 \leq x < 50$	$50 \leq x < 60$	$60 \leq x < 70$	$70 \leq x < 80$
No. of students	10	12	13	5

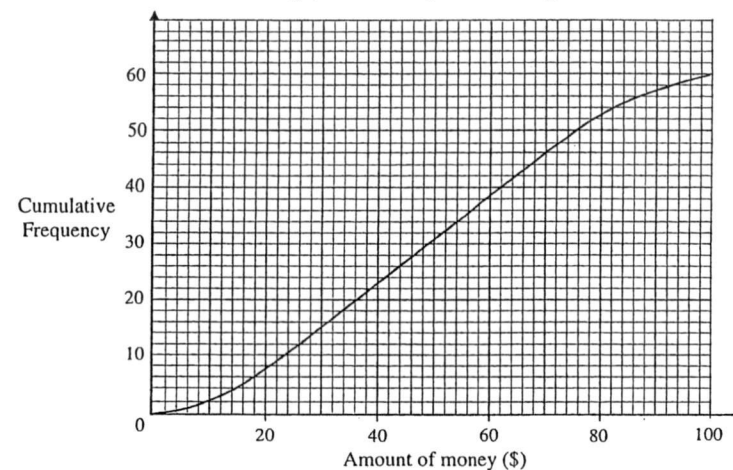
Test 2

Mean = 45
Standard deviation = 15.5

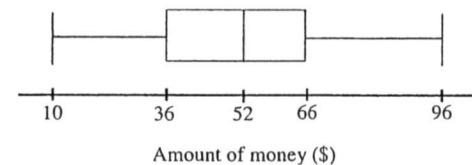
- Find the mean mark and the standard deviation of **Test 1**. [3]
- Compare and comment on the marks of the two tests in two different ways. [2]

6

- (b) The cumulative frequency curve below shows the amount of money collected by 60 students in the school's Rugby Team during a fund raising event.



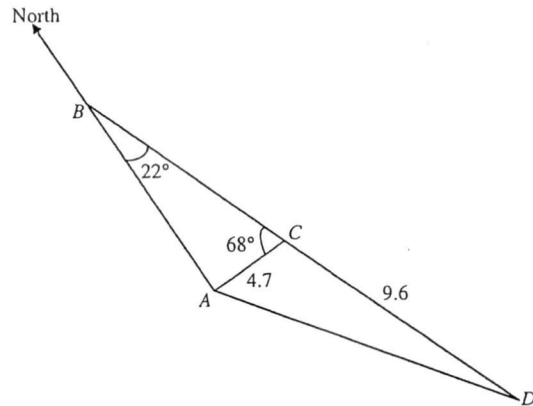
- Use the graph to find the
 - median amount of money raised, [1]
 - interquartile range of the distribution, [2]
 - tenth percentile of the distribution. [2]
- A student will be given a T-Shirt if he managed to collect at least \$ y . Find the value of y if 20% of the students were given the T-Shirt. [2]
- The box-and-whisker diagram below illustrates the amount raised by 60 students from the Outdoor Club.



Brian commented that the students in the Rugby Team raised more money as compared to the students from the Outdoor Club. Do you agree? Give a reason for your answer. [1]

7

- 7 A, B, C and D are points on level ground with A due South of B . It is given that $\angle ABC = 22^\circ$, $\angle BCA = 68^\circ$, $AC = 4.7$ km and $CD = 9.6$ km. BCD is a straight line.



Calculate

- (a) AD , [2]
 (b) $\angle CAD$, [2]
 (c) the bearing of C from D , [1]
 (d) the shortest distance from C to AD . [2]

A helicopter, H is hovering at a height of 450 m, at a point which is vertically above C .

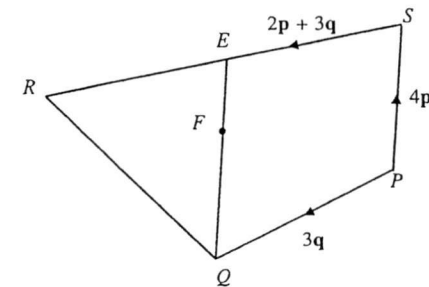
- (e) Find the smallest angle of elevation of the helicopter when observed along AD . [2]

8

- 8 (a) R is the point $(7, 1)$ and S is the point $(2, 5)$.

- (i) Find the column vector $\overrightarrow{RS}$. [1]
 (ii) Calculate the exact value of $|\overrightarrow{RS}|$. [2]
 (iii) If $\overrightarrow{AB} = \frac{1}{4}\overrightarrow{RS}$ and B is the point $(3, 6)$, find the coordinates of A . [2]

(b)



In the diagram, E is a point on SR such that $SR = 2SE$ and F is a point on QE such that $3QF = 2QE$. $\overrightarrow{PS} = 4\mathbf{p}$, $\overrightarrow{PQ} = 3\mathbf{q}$ and $\overrightarrow{SE} = 2\mathbf{p} + 3\mathbf{q}$.

- (i) Express each of the following, as simply as possible, in terms of $\mathbf{p}$ and/or $\mathbf{q}$.
 (a) $\overrightarrow{PE}$ [1]
 (b) $\overrightarrow{QE}$ [1]
 (c) $\overrightarrow{PF}$ [1]
 (d) $\overrightarrow{PR}$ [1]
 (ii) What can you deduce about P, F and R ? [1]
 (iii) Given that the area of $\triangle QRE = 24 \text{ cm}^2$, find the area of $\triangle RFE$. [2]

9 Answer the whole of this question on a sheet of graph paper.

In experiment A, water flows from a tap at a constant rate into a conical flask.

The table below shows the height of water, h cm, at various timings of the water flow.

Time (t s)	0.0	1.0	2.0	3.0	4.0	5.0	6.0
Height (h cm)	0.00	0.50	1.25	2.38	3.90	6.59	10.39

- (a) Using a scale of 2 cm to represent 1 second, draw a horizontal t -axis for $0 \leq t \leq 6$. Using a scale of 2 cm to represent 1 cm depth, draw a vertical h -axis for $0 \leq h \leq 11$. On your axes, plot a smooth graph represented by the data in the given table. [3]
- (b) Use your graph to find an estimate for the height of water in the conical flask when $t = 3.4$. [1]
- (c) By drawing a tangent at the points $t = 2$ and $t = 5$,
- find the gradient of the curve at $t = 2$ and $t = 5$, [2]
 - state briefly what these gradients represent, [1]
 - explain the significant difference between these gradients. [1]
- (d) In experiment B, another container in the shape of a cylinder is being used. Water flows from the same tap. The height of water can be described by the equation $h = 1.25t + c$, where c is a constant.
- If the cylinder is empty at the start of the experiment, determine the value of c . [1]
 - By drawing a suitable straight line on the same axes, determine the time taken for the height of water in both containers to be the same. [2]

- 10 Scuba divers use diving cylinders to help them stay underwater for a long period of time. In this question, the internal compartment of a diving cylinder, as shown in Figure 1, can be modelled as a cylinder with a hemisphere on top, as shown in Figure 2.



Figure 1

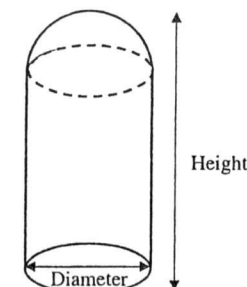


Figure 2

A diver intends to stay underwater at a depth of 20 m for 1.5 hours. He wants to determine whether a particular diving cylinder, with an internal compartment of diameter 20 cm and a height of 75 cm, is sufficient to support the dive.

- (a) The volume of the diving cylinder is measured by the volume of its internal compartment. Work out the volume, in litres, of the diving cylinder. [3]
- (b) The following equation is used to calculate the volume of oxygen in the cylinder.

$$\text{Volume of oxygen} = \frac{(\text{Volume of cylinder}) \times (\text{Pressure in cylinder})}{(\text{Atmospheric pressure})}$$

The pressure (measured in bars) in the cylinder is 210 bars and the atmospheric pressure (measured in bars) is 1.01 bars.

Find the volume, in litres, of oxygen that the diving cylinder can hold. [2]

- (c) To calculate the volume of oxygen a diver requires, the following equation is used.

$$\text{Volume of oxygen required} = (\text{Breathing rate}) \times (\text{Duration}) \times (\text{Ambient pressure})$$

Assuming that the diver's breathing rate is 18 litres per minute and that for every 5 m underwater, the ambient pressure increases by 0.5 bars from the atmospheric pressure, determine whether the diving cylinder is sufficient to support the dive.

Justify your decision with calculations. [4]

End of Paper

Answer Key

Answer Key

1	(ai)	$w^2 + 2 + \frac{1}{w^2}$				6	(bib)	Interquartile range = \$39			
	(aiii)	-5.66					(bic)	10 th percentile of the distribution = \$18			
	(b)	$m = \pm \sqrt{\frac{p(3+p^3)}{p^3-2}}$					(bii)	$y = 72$			
	(ci)	24	(cii)	$\frac{-4}{11}$	(ciii)		121	(biii)	No. The median amount raised by the Rugby Team is lower.		
2	(a)	Disagree with Mr. Ali.					(b)	47.0°			
	(bi)	\$10 000					(c)	338°			
	(bii)	$x = 7$					(d)	3.44 km			
	(biii)	Yes. As the interest/total loan is lower.					(e)	2.7°			
3	(a)	$n = 14$				8	(ai)	$\begin{pmatrix} -5 \\ 4 \end{pmatrix}$			
	(b)	No. John is wrong.					(aii)	$\sqrt{41}$ units			
	(ci)	1.97 rad					(aiii)	$\left(4\frac{1}{4}, 5\right)$			
	(cii)	18.9 cm ²									
4	(a)	Show $y = 5 - x$.					(bia)	$3(2p + q)$			
	(b)	Show $3x^2 - 55x + 100 = 0$					(bib)	6p			
	(c)	$x = 16.29$ or 2.05					(bic)	$4p + 3q$			
	(d)	7 seconds					(bid)	$2(4p + 3q)$			
						(bii)	P, F and R lie on a straight line.				
						(biii)	8 cm ²				
5	(a)	$S(5, 6)$				9	(b)	$h = 2.9$	(ci)	1.02, 3.06	
	(b)	$\sqrt{68}$ units					(cii)	The gradients represent the rate of increase of water level in the conical flask at each instant.			
	(c)	0.728 units					(ciii)	Water level is rising faster at $t = 5$ compared to $t = 2$			
	(d)	$y = \frac{4}{3}x - 3$					(di)	$c = 0$	(dii)	4.8 or 4.9	
	6	(ai)	mean = 58.25 marks SD = 9.85 marks				10	(a)	22.5 litres		
	(aii)	The mean mark of Test 1 is higher than of Test 2. This means that students scored better in Test 1 as compared to in Test 2.					(b)	4680 litres			
	(aii)	The standard deviation of Test 2 marks is higher than in Test 1. This means that the spread of marks in Test 2 is bigger/wider than that of Test 1.					(c)	No. It is insufficient.			
	(bia)	Median amount = \$49									

1 (a) $\left(w + \frac{1}{w}\right)^2 = w^2 + 2 + \frac{1}{w^2}$ ----- [B1]

Students generally did well for this question.

(i) $\left(w + \frac{1}{w}\right)^2 = w^2 + 2 + \frac{1}{w^2}$
 $\Rightarrow (6)^2 = w^2 + 2 + \frac{1}{w^2}$
 $\therefore w^2 + \frac{1}{w^2} = 36 - 2$
 $= 34$ (shown) ----- [A1]

Students generally did well for this question.

(ii) $\left(w - \frac{1}{w}\right)^2 = w^2 - 2 + \frac{1}{w^2}$ ----- [B1]
 $= w^2 + \frac{1}{w^2} - 2$ ----- [M1-for sub]
 $= 34 - 2$
 $\left(w - \frac{1}{w}\right)^2 = 32$
 $\therefore w - \frac{1}{w} = \pm\sqrt{32}$. Since $0 < w < 1$, $\Rightarrow w - \frac{1}{w} = -5.66$ (3 SF) ----- [A1]

Question was badly done. Some of the errors include:
 1) Did not use part (ai) but solve to find w instead- "Hence question"
 2) Did not reject 5.66 (or reject -5.66 instead)
 3) Careless. Note that $w^2 + \frac{1}{w^2} \neq \left(w - \frac{1}{w}\right)\left(w + \frac{1}{w}\right)$

(b) $p = \sqrt[3]{\frac{2m^2 + 3p}{m^2 - p}}$
 $p^3 = \frac{2m^2 + 3p}{m^2 - p}$ ----- [B1- taking cube on both sides]
 $p^3 m^2 - p^4 = 2m^2 + 3p$
 $m^2(p^3 - 2) = p(2m^2 + 3p)$ ----- [M1- Factorising]
 $m^2 = \frac{p(2m^2 + 3p)}{p^3 - 2}$
 $\therefore m = \pm \sqrt{\frac{p(2m^2 + 3p)}{p^3 - 2}}$ ----- [A1]

Most students were able to do this question (but lost the last 1 mark). Some of the errors include:
 1) Did not factorise $p(2m^2 + 3p)$
 2) Did not give $\pm\sqrt{\quad}$ as the final form

(c) (i) greatest possible value of $q - p = 20 - (-4) = 24$ ----- [B1]
 (ii) least possible value of $\frac{p}{q} = \frac{-4}{11}$ ----- [B1]
 (iii) least value of $2p^2 + q^2 = 2(0)^2 + 11^2 = 121$ -----

Questions were well attempted. Some students were penalized because of poor presentation of answers e.g. did not show substitutions and no statements.

- 2 (a) In January 2017, the price of a car was \$96 000. Over the next two months, the price increased by 25% and then decreased by 15%. Mr Ali, a potential car owner, said that the overall increase in the price of the car was 10%. Do you agree with Mr Ali? Justify your answer.

Disagree with Mr Ali. ----- [B1]

$$\text{The actual overall increase} = (1.25 \times 0.85 - 1) \times 100\% = 6.25\% \text{ ----- [B1]}$$

Question was well attempted with different degrees of quality. There were few outstanding reasoning. Some students were penalized because of the lack of Mathematical justifications in their answers. Need to show proper Math workings.

- (b) Alex bought a home entertainment system during the Great Singapore Sale on hire purchase. He paid a deposit of 20% of the selling price. The balance of the payment was \$8000.

- (i) Calculate the selling price of the system.

$$\text{Selling Price} = \frac{8000}{80} \times 100 = \$10\,000 \text{ ----- [B1]}$$

Question was well attempted. Some students were careless and misinterpret the question i.e. taking 20% as \$8000 resulting in an erroneous answer of \$40 000.

- (ii) Alex paid the balance of the payment in monthly instalments of \$380, charged at $x\%$ per annum simple interest for 2 years. Find the value of x . [3]

$$\text{Total repayment amount} = 380 \times 24 = \$9120$$

$$\text{Total interest amount} = 9120 - 8000 = 1120 \text{ ----- [B1]}$$

$$\therefore 1120 = \frac{8000 \times x \times 2}{100}$$

$$x = 7 \text{ ----- [A1]}$$

Question was badly done. Some students did not know the difference between Simple (ii) and Compound Interest (iii). Students did not realise the formula

$$SI = \frac{PRT}{100} \text{ (where } T \text{ is time in YEARS)}$$

- (iii) Alex could have taken a loan to pay the outstanding balance. If the finance company charged him a compound interest of 6.5% per annum with a repayment period of 2 years, would you recommend that he takes up the loan instead of paying in instalments? State your reason clearly. [3]

$$\text{Total loan amount} = 8000 \left(1 + \frac{6.5}{100} \right)^2 = \$9073.80 \text{ ----- [A1]}$$

Yes, he should take up the loan from the finance company. ----- [A1]

The interest amount is lower. / The total loan amount is lower. ----- [A1]

Question was quite well attempted. However, there were students who were penalized because;

1) Poor or wrong explanation/reasons – e.g. “The loan is cheaper.”

2) Did not understand the formula to be “Total loan amount = $8000 \left(1 + \frac{6.5}{100} \right)^2$ ”.

Instead they took $r = \frac{6.5}{2 \times 100}$ or $n = 24$.

3) Took Principal amount (P) as \$10 000 instead of \$8000.

- 3 (a) A polygon has n sides. Three of its exterior angles are 60° , 25° and 55° . The remaining angles in the polygon have interior angles of 160° . Find value of n . [2]

$$\text{Sum of exterior angles} = 360^\circ$$

$$60^\circ + 25^\circ + 55^\circ + (n-3) \times 20^\circ = 360^\circ \text{ ----- [M1]}$$

$$140 + 20n - 60 = 360$$

$$20n = 280$$

$$n = 14 \text{ ----- [A1]}$$

Question was badly done (Poor or no attempt). Some students did not know the formula for either interior or exterior angles of polygons.

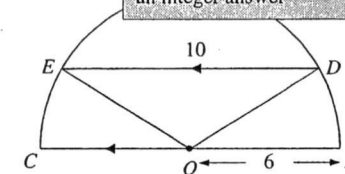
- (b) John claimed that he can draw a regular polygon with its exterior angle as 70° . Is he right? Explain. [2]

No. John is wrong. ----- [B1]

360° is not completely divisible by 70° . ----- [B1]

Most students knew how answer this question (but with different of accuracy). As a result, marks were deducted. For e.g. “ 360° cannot be divided by 70° . This is untrue. To make it complete, students should include the phrase “that will result in an integer answer”

- (c)



In the figure above, the chord DE , of length 10 cm, is parallel to the diameter AOC of the semicircle with centre O and radius 6 cm.

- (i) Find $\angle DOE$ in radians. [2]

$$\cos \angle DOE = \frac{6^2 + 6^2 - 10^2}{2(6)(6)} \text{ ----- [M1]}$$

$$\angle DOE \approx 1.9702$$

$$= 1.97 \text{ rad (3 s.f.) ----- [A1]}$$

Question was well attempted. Some common errors include;
1) Carelessness
2) Answer in degree and not in radians

- (ii) Calculate the area of the segment DBE . [3]

$$\text{Area of segment } DBE = \frac{1}{2}(6^2)(1.9702) - \frac{1}{2}(6^2)\sin 1.9702 \text{ ----- [M2]}$$

$$\approx 18.880$$

$$= 18.9 \text{ cm}^2 \text{ (3 SF) ----- [A1]}$$

Good attempt. Need to be mindful (strongly advised to use 5 SF) and not to use 3SF values in working i.e. use 1.9702 instead of just 1.97.

- 4 John and Tim begin to walk with a constant speed of x m/s and y m/s respectively towards each other along a pathway of length 20 metres.

- (a) Given that they walk past each other 4 seconds later, show that $y = 5 - x$. [2]

$$\begin{aligned} 4x + 4y &= 20 \quad \text{----- [B1]} \\ x + y &= 5 \\ y &= 5 - x \quad \text{----- [A1]} \end{aligned}$$

Question was well attempted. Most students had managed to show proper and sufficient working.

- (b) Given also that John takes 3 seconds more than Tim to walk through the pathway, form an equation in x and show that it reduces to

$$\begin{aligned} \frac{20}{x} - \frac{20}{5-x} &= 3 \quad \text{----- [M1 o.e]} \\ 100 - 20x - 20x &= 3x(5-x) \quad \text{----- [M1- single fraction]} \\ 100 - 40x &= 15x - 3x^2 \\ \therefore 3x^2 - 55x + 100 &= 0 \quad \text{----- [A1]} \end{aligned}$$

Question was well attempted. Most students had managed to show proper and sufficient working.

- (c) Solve the equation $3x^2 - 55x + 100 = 0$, giving your answers correct to 2 decimal places. [2]

$$\begin{aligned} 3x^2 - 55x + 100 &= 0 \\ \therefore x &= \frac{-(-55) \pm \sqrt{(-55)^2 - 4(3)(100)}}{2(3)} \quad \text{----- [B1]} \\ &= \frac{55 \pm \sqrt{1825}}{6} \\ \therefore x &= 16.29 \quad \text{or} \quad 2.046 \\ \Rightarrow x &= 16.29 \quad \text{or} \quad 2.05 \quad (2 \text{ DP}) \quad \text{----- [A1]} \end{aligned}$$

Question was well attempted. Students will be penalized if they did not show the substitution of $-(-55)$ into the QE formula. Some other errors include

- 1) Rejecting of 16.29 as a valid answer
- 2) Leaving answers in 3 SF and not in 2 DP as requested

- (d) Calculate the time, to the nearest second, taken by Tim to walk through the pathway. [2]

$$\begin{aligned} \therefore x &= 16.29 \quad (\text{rejected}) \\ \text{So } x &= 2.046 \Rightarrow y = 2.954 \\ \therefore \text{Time taken for Tim} &= \frac{20}{2.954} \quad \text{----- [M1]} \\ &= 6.7705 \\ &= 7 \text{ seconds (nearest seconds)} \quad \text{----- [A1]} \end{aligned}$$

Question was well attempted. Only a handful of students did not know what to do or had used the wrong formula i.e. using $\frac{20}{x}$ instead of $\frac{20}{5-x}$. Some did not show working using more than 3 SF values and some did not round answers to nearest seconds (an integer answer is required).

- 5 The points $P(3, 1)$, $Q(3, 4)$ and $R(5, 9)$ are shown in the diagram.

Find the

- (a) coordinates of S such that $PQRS$ is a parallelogram, [1]

$$S(5, 6). \quad \text{----- [B1]}$$

- (b) exact length of PR , [1]

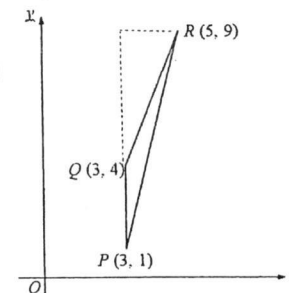
$$\begin{aligned} PR &= \sqrt{(5-3)^2 + (9-1)^2} \\ &= \sqrt{68} \text{ units (3 SF)} \quad \text{----- [B1]} \end{aligned}$$

- (c) area of $\triangle PRQ$ and hence the perpendicular distance from Q to PR , [4]

$$\begin{aligned} \text{Area } \triangle PRQ &= \frac{1}{2}(2)(3) \quad \text{----- [M1]} \\ &= 3 \text{ units}^2 \quad \text{----- [A1]} \end{aligned}$$

Let the perpendicular distance from Q to PR be h .

$$\begin{aligned} \text{Area } \triangle PRQ &= \frac{1}{2}(PR)(h) \\ 3 &= \frac{1}{2}(\sqrt{68})(h) \quad \text{----- [M1]} \\ h &= \frac{3 \times 2}{\sqrt{68}} \\ &= 0.728 \text{ units (3 SF)} \quad \text{----- [A1]} \end{aligned}$$



Question 5a, 5b were well attempted. Students will be penalized if they give answer for Q5b to 3sf instead of exact value. There is no need to reduce $\sqrt{68}$ to $2\sqrt{17}$. Some other mistakes for Q5b include

- 1) Not giving units for length.
- 2) Using the wrong formula for

$$\begin{aligned} \text{Area } \triangle PRQ &= \frac{1}{2}(2)(3) \\ &= 3 \text{ units}^2 \end{aligned}$$

Question 5c was generally well attempted. Students should avoid using "shoe lace method" to find area of triangle. Some other mistakes for Q5c include

- 1) Not writing the $\frac{1}{2}$ for the shoe lace method of finding area of triangle.
- 2) Not dividing by $\frac{1}{2}$ when finding perpendicular height.
- 3) Using 3 sf for length PR instead of $\sqrt{68}$

$$\begin{aligned} \text{Area } \triangle PRQ &= \frac{1}{2}(4-1)(5-1) \\ &= 3 \text{ units}^2 \\ \text{Perpendicular distance} &= \frac{3 \times 2}{\sqrt{68}} \\ &\approx 0.364 \text{ units} \end{aligned}$$

- (d) equation of the line that passes through P and is parallel to the line $3y = 4x + 5$.

[2]

$$3y - 4x = 5$$

$$\Rightarrow y = \frac{4}{3}x + \frac{5}{3}$$

$\therefore$ equation of the line:

$$y - 1 = \frac{4}{3}(x - 3) \text{ ----- [M1]}$$

$$y - 1 = \frac{4}{3}x - 4$$

$$y = \frac{4}{3}x - 3 \text{ ----- [A1]}$$

Question 5c was generally well attempted. Some other mistake for Q5d include

- 1) Using gradient for the line as 4

instead of $\frac{4}{3}$

- 2) The equation should not be written as $y = 1\frac{1}{3}x - 3$. Improper fraction should be used when written with x

≈ 0.727 units

$$3y = 4x + 5$$

Gradient of the line = $\frac{4}{3}$

$$y - 1 = \frac{4}{3}(x - 3)$$

$$y - 1 = \frac{4}{3}x - 4$$

$$y = \frac{4}{3}x - 3$$

Equation of line that passes through P

- 6 (a) The results of two class tests are shown in the tables below. The total marks for both tests are 80.

Test 1	Test 2	Test 3
Midvalue	45	55
Marks (x)	40 to 49	50 to 59
No. of students	10	15

Test 1

To find mid value for the following range of data.

50	50 ≤ x < 60	60 ≤ x < 70	70 ≤ x < 80
	12	13	5

$$\text{Mid value} = \frac{a+b}{2}$$

- (i) Find the mean mark and the standard deviation of Test 1.

[3]

$$\begin{aligned} \text{Mean} &= \frac{\sum fx}{\sum f} \\ &= \frac{45 \times 10 + 55 \times 12 + 65 \times 13 + 75 \times 5}{10 + 12 + 13 + 5} \\ &= \frac{2330}{40} \\ &= 58.25 \text{ marks ----- [A1]} \end{aligned}$$

Question 6a(i) was generally well attempted. Some other mistakes include

- 1) Using the wrong mid values for the marks.
- 2) Not writing units for mean and standard deviation.
- 3) Not showing substitution of values for $\sum fx^2$, $\sum fx$ for standard deviation formula.
- 4) Rounding off the mean to 3sf instead of giving exact value.

$$\begin{aligned} \text{Standard Deviation} &= \sqrt{\frac{\sum fx^2}{\sum f} - \left(\frac{\sum fx}{\sum f}\right)^2} \\ &= \sqrt{\frac{139600}{40} - \left(\frac{2330}{40}\right)^2} \text{ ----- [M1]} \\ &= \sqrt{96.9375} \\ &= 9.85 \text{ marks (3 SF) ----- [A1]} \end{aligned}$$

Must show substitution of the values!

- (ii) Compare and comment on the marks of the two tests in two different ways. [2]

The mean mark of Test 1 is higher than of Test 2. This means that students scored better in Test 1 as compared to in Test 2. [A1]

The standard deviation of Test 2 marks is higher than in Test 1. This means that the spread of marks in Test 2 is bigger/wider than that of Test 1. [A1]

Question 6a(ii) was generally well attempted. Some other mistakes include

- 1) Simply commenting the marks of test 1 is higher than test 2
- 2) Using range of marks, more varied marks.

- (b) The cumulative frequency curve below shows the amount of money collected by 60 students in the school's Rugby Team during a fund raising event.

(i) Use the graph to find the

(a) median amount of money raised, [1]

Median amount = \$49 ----- [B1]

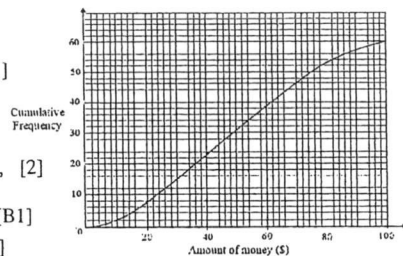
(b) interquartile range of the distribution, [2]

Interquartile range = $69 - 30 = 39$ ----- [B1]
= \$39 ----- [A1]

(c) tenth percentile of the distribution. [2]

No. of students in 10th percentile = $\frac{10}{100} \times 60 = 6$ ----- [B1]

10th percentile of the distribution = \$18 ----- [A1]



Must show no. of students at 10th percentile.

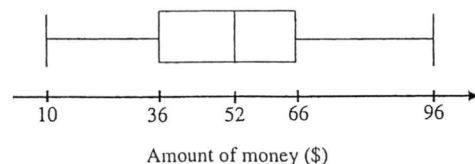
- (ii) A student will be given a T-Shirt if he managed to collect at least \$y. Find the value of y if 20% of the students were given the T-Shirt. [2]

No. of students = $\frac{20}{100} \times 60 = 12$ ----- [B1]

Value of y = 72 ----- [A1]

Must show no. of students at 20%.

- (iii) The box-and-whisker diagram below illustrates the amount raised by 60 students from the Outdoor Club.



Brian commented that the students in the Rugby Team raised more money as compared to the students from the Outdoor Club. Do you agree? Give a reason for your answer. [1]

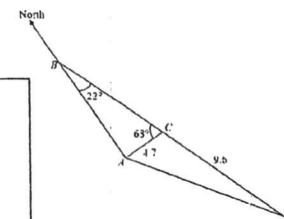
No. The median amount raised by the Rugby Team is lower as compared to the median amount raised by the Outdoor Club students. ----- [B1]

Some students carelessly use mean instead of median.

- 7 A, B, C and D are points on level ground, with A due South of B. It is given that $\angle ABC = 22^\circ$, $\angle BCA = 68^\circ$, $AC = 4.7$ km and $CD = 9.6$ km. BCD is a straight line. Calculate

(a) AD, [2]

$$\begin{aligned} AD^2 &= DC^2 + AC^2 - 2(AC)(DC) \cos \angle DCA \\ &= (9.6)^2 + (4.7)^2 - 2(9.6)(4.7) \cos (180^\circ - 68^\circ) \text{ ----- [M1]} \\ AD &\approx 12.168 \\ &= 12.2 \text{ km (3 SF)} \text{ ----- [A1]} \end{aligned}$$



(b) $\angle CAD$, [2]

$$\begin{aligned} \frac{9.6}{\sin \angle CAD} &= \frac{AD}{\sin 112^\circ} \text{ ----- [M1]} \\ \sin \angle CAD &= 0.73151 \\ \angle CAD &= 47.012^\circ \\ &= 47.0^\circ \text{ (1 DP)} \text{ ----- [A1]} \end{aligned}$$

Generally, well attempted for Q7. Common mistakes include

1. Wrong use of cosine rule.
2. Not using the correct sf in intermediate steps resulting in losing accuracy for final answers.
3. Not providing reasons for 7c.

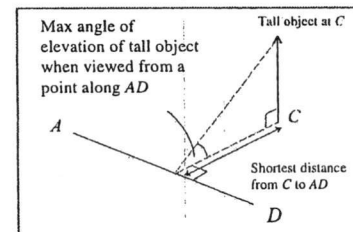
(c) the bearing of C from D, [1]

$$\begin{aligned} \text{Bearing of C from D} &= 360^\circ - 22^\circ \text{ (}\angle\text{s at a point)} \\ &= 338^\circ \text{ ----- [A1]} \end{aligned}$$

(d) the shortest distance from C to AD. [2]

Need to have 4 to 5 sf

$$\begin{aligned} \text{Let the shortest distance from C to AD be } h \text{ km.} \\ \frac{1}{2} \times AD \times h &= \frac{1}{2} \times DC \times AC \times \sin \angle DCA \\ \frac{1}{2} \times 12.168 \times h &= \frac{1}{2} \times 4.7 \times 9.6 \times \sin 112^\circ \text{ ----- [M1]} \\ h &\approx 3.4381 \\ &= 3.44 \text{ km (3 SF)} \text{ ----- [A1]} \end{aligned}$$



A helicopter, H, is hovering at a height of 450 m, at a point which is vertically above C.

- (e) Find the smallest angle of elevation of the helicopter when observed along AD. [2]

The smallest angle of elevation occurs when it is observed from point D .
Let the angle of elevation be θ .

$$\tan \theta = \frac{450}{9600} \quad \text{----- [M1]}$$

$$\theta = 2.6838^\circ$$

$$= 2.7^\circ \text{ (1 DP)} \quad \text{----- [A1]}$$

The most common mistake students make was to assume the smallest angle of elevation occurs at shortest distance from C to AD .

Final answer to be given to 1 d.p instead of 2 d.p.

- 8 (a) R is the point $(7, 1)$ and S is the point $(2, 5)$.

- (i) Find the column vector $\overrightarrow{RS}$. [1]

$$\begin{aligned}\overrightarrow{RS} &= \overrightarrow{OS} - \overrightarrow{OR} \\ &= \begin{pmatrix} 2 \\ 5 \end{pmatrix} - \begin{pmatrix} 7 \\ 1 \end{pmatrix} \\ &= \begin{pmatrix} -5 \\ 4 \end{pmatrix} \quad \text{----- [A1]}\end{aligned}$$

- (ii) Calculate the exact value of $|\overrightarrow{RS}|$. [2]

$$\begin{aligned}|\overrightarrow{RS}| &= \sqrt{(-5)^2 + 4^2} \quad \text{----- [M1]} \\ &= \sqrt{41} \text{ units} \quad \text{----- [A1]}\end{aligned}$$

- (iii) If $\overrightarrow{AB} = \frac{1}{4}\overrightarrow{RS}$ and B is the point $(3, 6)$, find the coordinates of A . [2]

$$\begin{aligned}\overrightarrow{AB} &= \frac{1}{4}\overrightarrow{RS} \\ \overrightarrow{OB} - \overrightarrow{OA} &= \frac{1}{4}\begin{pmatrix} -5 \\ 4 \end{pmatrix} \quad \text{----- [M1]} \\ \overrightarrow{OA} &= \overrightarrow{OB} - \overrightarrow{AB} \\ &= \begin{pmatrix} 3 \\ 6 \end{pmatrix} - \begin{pmatrix} -\frac{5}{4} \\ 1 \end{pmatrix} \\ &= \begin{pmatrix} 4\frac{1}{4} \\ 5 \end{pmatrix}\end{aligned}$$

$$\therefore \text{The coordinates of } A \text{ are } \left(4\frac{1}{4}, 5\right). \quad \text{----- [A1]}$$

Generally, well attempted for Q8a. Common mistakes include

1. Not giving exact value for $|\overrightarrow{RS}|$.
2. Unclear method when finding $\overrightarrow{OA}$. Some simply add or subtract directly the elements of the vector.
3. Leaving answer in column vector instead of providing the coordinates of A .

- (b) In the diagram, E is a point on SR such that $SR = 2SE$ and F is a point on QE such that $3QF = 2QE$. $\overrightarrow{PS} = 4\mathbf{p}$, $\overrightarrow{PQ} = 3\mathbf{q}$ and $\overrightarrow{SE} = 2\mathbf{p} + 3\mathbf{q}$.

- (i) Express each of the following, as simply as possible, in terms of $\mathbf{p}$ and/or $\mathbf{q}$.

(a) $\overrightarrow{PE}$, [1]

$$\begin{aligned}\overrightarrow{PE} &= 4\mathbf{p} + 2\mathbf{p} + 3\mathbf{q} \\ &= 6\mathbf{p} + 3\mathbf{q} \\ &= 3(2\mathbf{p} + \mathbf{q}) \text{ ----- [A1]}\end{aligned}$$

(b) $\overrightarrow{QE}$, [1]

$$\begin{aligned}\overrightarrow{QE} &= -3\mathbf{q} + 6\mathbf{p} + 3\mathbf{q} \\ &= 6\mathbf{p} \text{ ----- [A1]}\end{aligned}$$

(c) $\overrightarrow{PF}$, [1]

$$\begin{aligned}\overrightarrow{PF} &= 3\mathbf{q} + \frac{2}{3}(6\mathbf{p}) \\ &= 4\mathbf{p} + 3\mathbf{q} \text{ ----- [A1]}\end{aligned}$$

(d) $\overrightarrow{PR}$, [1]

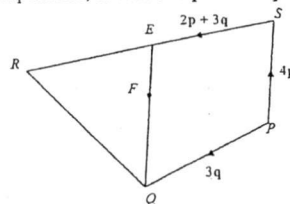
$$\begin{aligned}\overrightarrow{PR} &= 4\mathbf{p} + 2(2\mathbf{p} + 3\mathbf{q}) \\ &= 8\mathbf{p} + 6\mathbf{q} \\ &= 2(4\mathbf{p} + 3\mathbf{q}) \text{ ----- [A1]}\end{aligned}$$

- (ii) What can you deduce about P , F and R ? [1]

$$\begin{aligned}\overrightarrow{PR} &= 2(4\mathbf{p} + 3\mathbf{q}) \\ &= 2\overrightarrow{PF}\end{aligned}$$

Since $\overrightarrow{PR}$ is some scalar multiple of $\overrightarrow{PF}$, $\overrightarrow{PR}$ is parallel to $\overrightarrow{PF}$. P is a common point.
 $\therefore P$, F and R lie on a straight line. ----- [A1]

- (iii) Given that the area of $\triangle QRE = 24 \text{ cm}^2$, find the area of $\triangle RFE$. [2]



Generally, well attempted for Q8b. Common mistakes include

1. Not factorizing the common scalar multiple for the final answer.
2. Not writing the vector with the tilde symbol.
3. Not providing supporting evidence for making deduction about P , F and R .

$$4\mathbf{p} + 3\mathbf{q}$$

tilde, ~

Wrong use

$$\begin{aligned}\overrightarrow{PR} &= \overrightarrow{PS} + \overrightarrow{SE} = \overrightarrow{PE} \\ \overrightarrow{PR} &= \overrightarrow{PS} + \overrightarrow{SE} \\ \overrightarrow{PR} &= 4\mathbf{p} + 2\mathbf{p} + 3\mathbf{q} \\ \overrightarrow{PR} &= 6\mathbf{p} + 3\mathbf{q}\end{aligned}$$

Generally, well attempted for Q8biii. Common mistake include

1. Assuming the 2 triangles are similar.

$\triangle RFE$ and $\triangle QRE$ share a common height,

$$\frac{\text{Area of } \triangle RFE}{\text{Area of } \triangle QRE} = \frac{FE}{QE}$$

$$\frac{\text{Area of } \triangle RFE}{24} = \frac{1}{3} \text{ ----- [M1]}$$

$$\text{Area of } \triangle RFE = \frac{1}{3} \times 24 = 8 \text{ cm}^2 \text{ ----- [A1]}$$

- 9 Answer the whole of this question on a sheet of graph paper.

In experiment A, water flows from a tap at a constant rate into a conical flask.

The table below shows the height of water, h cm, at various timings of the water flow.

Time (t s)	0.0	1.0	2.0	3.0	4.0	5.0	6.0
Height (h cm)	0.00	0.50	1.25	2.38	3.90	6.59	10.39

- (a) Using a scale of 2 cm to represent 1 second, draw a horizontal t -axis for $0 \leq t \leq 6$. Using a scale of 2 cm to represent 1 cm depth, draw a vertical h -axis for $0 \leq h \leq 11$. On your axes, plot a smooth graph represented by the data in the given table. [3]

- (b) Use your graph to find an estimate for the height of water in the conical flask when $t = 3.4$. [1]

$$h = 2.9 \text{ cm} \text{ ----- [B1]}$$

Common mistake - not having units

- (c) By drawing a tangent at the points $t = 2$ and $t = 5$,

- (i) find the respective gradient of the curve at $t = 2$ and $t = 5$, [2]

$$\begin{aligned}\text{At } t = 2: \text{ Gradient} &= \frac{3.35 - 0}{4 - 0.7} = 1.02 \text{ ----- [M1 - to find the gradient]} \\ \text{At } t = 5: \text{ Gradient} &= \frac{9.5 - 4}{6 - 4.2} = 3.06 \text{ ----- [M1 - to find the gradient]}\end{aligned}$$

- (ii) state briefly what these gradients represent, [1]

The gradients represent the rate of increase of water level in the conical flask at each instant. ----- [B1]

- (iii) explain the significant difference between these gradients. [1]

Water level is rising faster at $t = 5$ compared to $t = 2$. ----- [B1]

- (d) In experiment B, another container in the shape of a cylinder is being used. Water flows from the same tap. The height of water can be described by the equation $h = 1.25t + c$, where c is a constant.

Common mistake
- not showing the working for gradients.
- not drawing the tangent line touching the point on the curve.

Wrong to say rate of water flow

Only 2 points are needed. To draw the line! (0, 0) and (6, 7.5)