JUNYUAN SECONDARY SCHOOL
PRELIMINARY EXAMINATION 2017
SECONDARY FOUR EXPRESS / FIVE NORMAL (ACADEMIC)

CANDIDATE NAME

CLASS | | | | INDEX NUMBER | | |

MATHEMATICS **4048/01**

Paper 1 7 August 2017

2 hours

Candidates answer on the Question Paper.

READ THESE INSTRUCTIONS FIRST

Write your name, class and index number on all the work you hand in.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use paper clips, highlighters, glue or correction fluid.

Answer **all** questions.
If working is needed for any question it must be shown with the answer.
Omission of essential working will result in loss of marks.
The use of an approved scientific calculator is expected, where appropriate.
If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.
For π, use either your calculator value or 3.142, unless the question requires the answer in terms of π.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total of the marks for this paper is 80.

For Examiner's Use

This document consists of **19** printed pages (including the Cover Sheet).

[Turn over

Mathematical Formulae

Compound interest

$$\text{Total amount} = P\left(1+\frac{r}{100}\right)^n$$

Mensuration

$$\text{Curved surface area of a cone} = \pi rl$$

$$\text{Surface area of a sphere} = 4\pi r^2$$

$$\text{Volume of a cone} = \frac{1}{3}\pi r^2 h$$

$$\text{Volume of a sphere} = \frac{4}{3}\pi r^3$$

$$\text{Area of triangle } ABC = \frac{1}{2}ab\sin C$$

$$\text{Arc length} = r\theta, \text{ where } \theta \text{ is in radians}$$

$$\text{Sector area} = \frac{1}{2}r^2\theta, \text{ where } \theta \text{ is in radians}$$

Trigonometry

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

Statistics

$$\text{Mean} = \frac{\sum fx}{\sum f}$$

$$\text{Standard deviation} = \sqrt{\frac{\sum fx^2}{\sum f} - \left(\frac{\sum fx}{\sum f}\right)^2}$$

1 Solve $0.5\left(4-\frac{x}{3}\right)=1$.

Answer $x=$ ..... [2]

2 Write as a single fraction $c-d+\frac{1}{c}+\frac{1}{d}-\frac{c^2-d^2}{c+d}$.

Answer ..... [2]

3 Brad invested \$4 000 into an account which pays r% per annum interest compounded monthly. His account tripled in value after 320 months.

Find r.

Answer $r=$ ..... [2]

4 An interior angle of a regular polygon is 120° bigger than its exterior angle.

Find the number of sides of the polygon.

Answer ..... [2]

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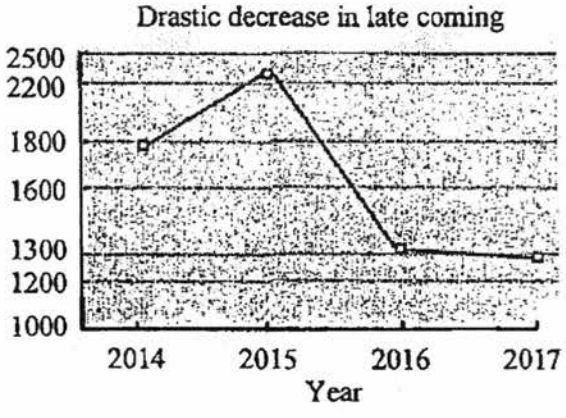
5 The total count of student late coming occurrences in a school is represented by a line graph as shown.



State and explain how the graph can be modified to give a more accurate representation of the late coming occurrences in the school.

Answer .....

.....

..... [2]

6 $\xi = \{\text{even integers } x : 2 < x \leq 14\}$
$A = \{\text{perfect squares}\}$
$B = \{\text{factors of } 12\}$

(a) Draw a Venn diagram to illustrate this information.

Answer

[2]

(b) List the element(s) contained in the set $A \cap B'$.

Answer ..... [1]

---

7 Singapore's tourism hit a record high in 2016, where tourism numbers grew by 7.7% and tourism spending rose by 13.9%.

Some information about the number of tourists and tourism spending are given in the table.

Year	2016	2015
Number of Tourists	1.64×10^7	
Tourism Spending		S$21.4 billion

Estimate, to the nearest dollar, the average tourism spending in 2016.

Answer $ ..... [3]

8 Factorise completely

(a) $(d+e)^2 - 2(d+e) - 8$,

Answer ..... [1]

(b) $1 + x - 2a - 2ax$.

Answer ..... [2]

---

9 Two solid metal cones, which are geometrically similar, have surface areas A_1 and A_2 such that $9A_1 = 16A_2$.

If the volume of the larger cone is 32 cm^3, find the volume of the smaller cone.

Answer ..... cm^3 [3]

10 **(a)** Express 2 700 as a product of its prime factors.

Answer ..... [1]

(b) Using your answer to part **(a)**, explain why $2\,700b$ is a perfect cube when $b = 10$.

Answer .....

..... [1]

(c) Find the smallest value of p so that $2\,700 \times \sqrt{p}$ is divisible by 14.

Answer $p =$ ..... [1]

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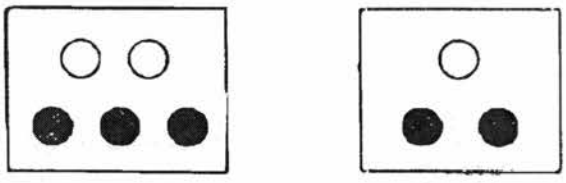
11 The length of a rectangle is 3 metres more than its width.
Its perimeter is equal in value to its area.

Find the dimensions of this rectangle.

Answer ... . m by ..... m [3]

12



Box A Box B

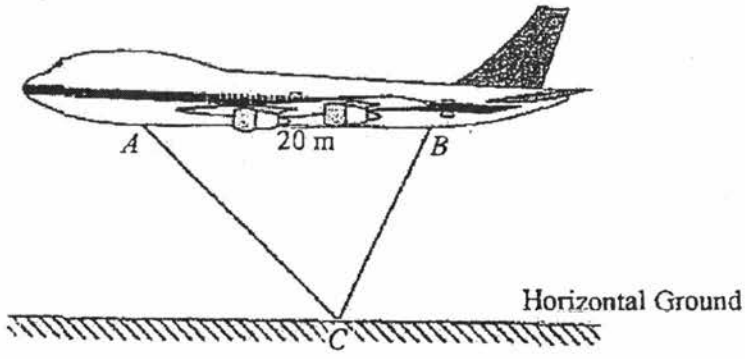
In Box A, there are 3 black balls and 2 white balls.
In Box B, there are 2 black balls and 1 white ball.

Ravi takes at random a ball from Box A and places it in Box B.
He then takes at random a ball from Box B.

Work out the probability that the ball he takes from Box B will be black.

Answer ..... [3]

13



An aeroplane is flying parallel to the ground.
Lights have been fitted at A and B as shown.
When the aeroplane is flying at a certain height, the beams from these lights meet exactly on the ground at C.

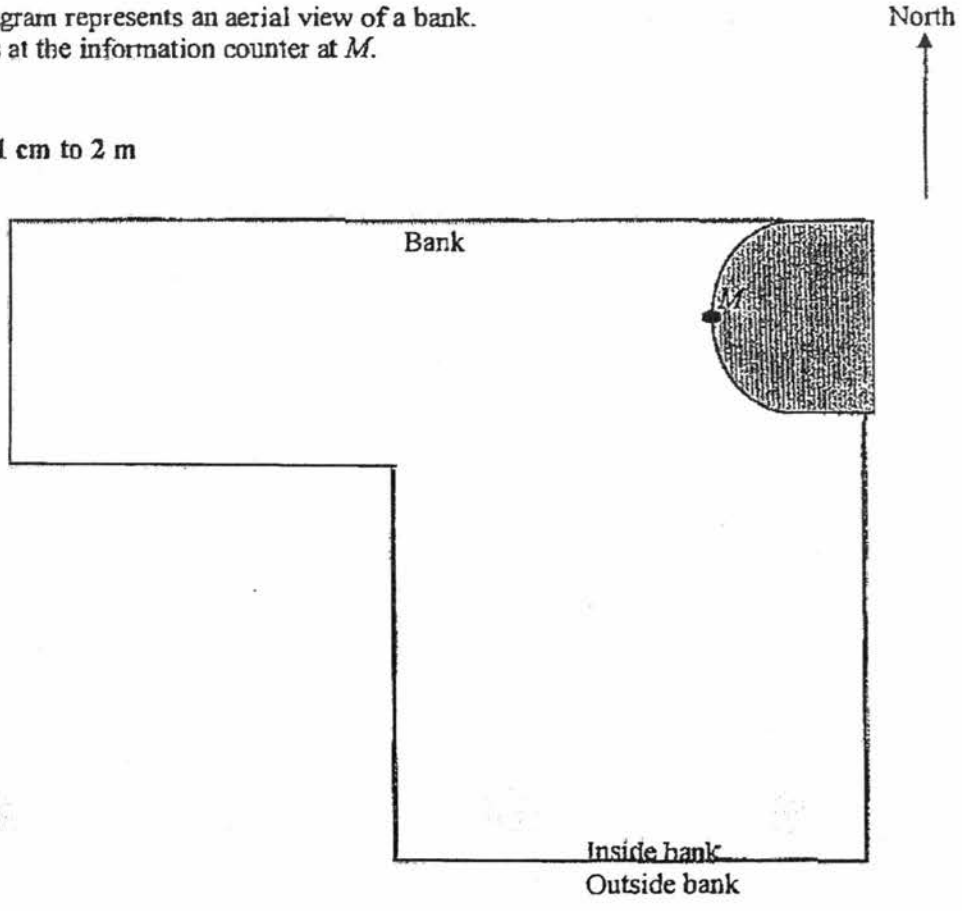
The angle of depression of the beam of light from A to C is 50°.
The angle of depression of the beam of light from B to C is 70°.
The distance AB is 20 metres.

Find the height of the aeroplane from the ground when the lights meet at C.

Answer ..... m [3]

14 The diagram represents an aerial view of a bank.
Mark is at the information counter at M.

Scale: 1 cm to 2 m



(a) Mark tethered his dog to a lamp post outside the bank, by means of a leash, at a bearing of 220° and 15 m from M.

On the diagram, mark out the location outside the bank where the dog is tethered to and label this point X.

[1]

(b) Jasmine is keeping a lookout for the dog inside the bank.
She is standing at a point that is equidistant from points M and X.

By showing your working clearly, work out one possible position Jasmine is standing at and label this point J. [1]

(c) The dog is unable to enter the bank.
The leash is 2 m long.

Draw the boundary of the region in which the dog can roam. [1]

15 (a) $(-5, 2)$ is the maximum point of a quadratic curve.

Write the equation of the graph in the form $y = p - (x+q)^2$.

Answer ..... [1]

(b) A straight line on the xy–axes passes through $(-5, 2)$ and cuts the x-axis at $x = 1$.

Find the equation of the straight line.

Answer ..... [2]

16 Ron exchanged 2 000 Singapore Dollars (SGD) for US Dollars (USD) in New York.
The exchange rate was x USD to 1 SGD.

After his trip, he had 8% of his total USD left.
He exchanged the remaining USD for SGD at the rate of 1 USD = 1.46 SGD and received 170 SGD.

Find x.

Answer $x =$ ..... [3]

17 (a) Write down all the integers satisfying the inequalities $-11 < 1 - 3x \leq 2$.

Answer ..... [3]

(b) Given $-6 \leq a \leq -1$ and $2 \leq b \leq 6$, find the range of possible values of $\frac{b}{a}$.

Answer ..... [1]

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18 (a) Simplify $x\left(2x^{\frac{1}{4}}\right)^4$.

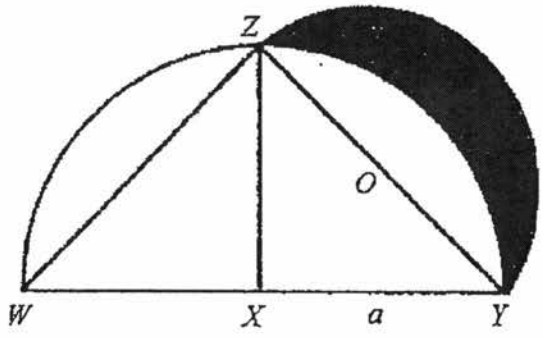
Answer ..... [2]

(b) Evaluate $\frac{3^{n+2}}{5(3^{n-1})}$.

Answer ..... [2]

19 WY is the diameter of a semi-circle with centre X and radius a cm.
Z is on the circumference and angle ZXY is a right angle.
A smaller semi-circle, centred at O, is drawn with ZY as diameter.

Find the area of the shaded region, in terms of a, in its simplest form.



Answer ..... cm^2 [4]

20 The area A of a television screen varies proportionally to the square of its diagonal d. A television set with a diagonal of 30 cm has an area of 440 cm^2.

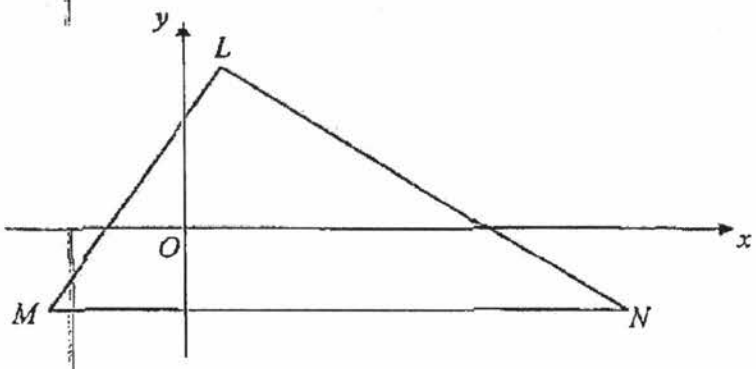
(a) Find the area of a television screen with a diagonal of 75 cm.

Answer ..... cm^2 [3]

(b) State the percentage change in A when d is decreased by 15%.

Answer ..... [1]

21 The diagram below, not drawn to scale, shows triangle LMN.



The equations of the lines LM and LN are $2y = 3x + 5$ and $x + 4y = 24$ respectively.

(a) Find the coordinates of L.

Answer (....., ...) [3]

(b) The coordinates of M are $(-3, -2)$ and MN is parallel to the x-axis.

Write the equation of line MN.

Answer ..... [1]

22 Justin is locked out of his house.
He intends to borrow a ladder.

The only open window is on the second floor, 8 m above the ground.
There is a bush along the edge of the house, 1 m away from the house and 2 m in height.

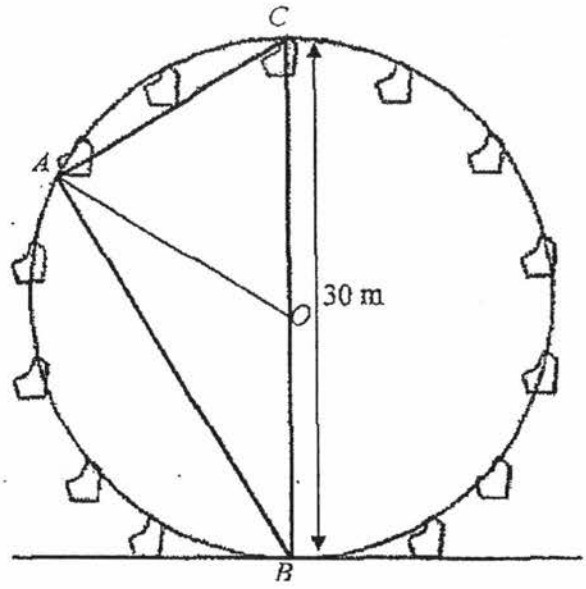
The bush is too thick for Justin to pass through on foot or climb through along the ladder.

What is the **minimum** length of the ladder Justin needs in order for him to reach the window?

Answer ..... m [4]

23 The diagram shows a circle which represents a ferris wheel with centre O.
The diameter is 30 m.

NOT TO SCALE



(a) A seat starts at B and travels one-third of the circumference to A.

Explain why angle AOB is equal to $\frac{2\pi}{3}$ radian.

Answer ..... [1]

(b) Find the exact value, in radian, of angle ABO.

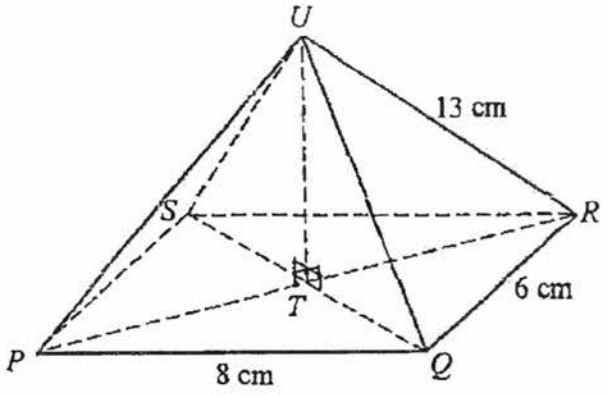
Answer ..... radian [2]

(c) It takes 2.5 minutes for a seat to travel from position B to A.

Find the average speed, in metres per second, of the wheel.

Answer ..... m/s [3]

24 **NOT TO SCALE**



The diagram shows a pyramid on a horizontal rectangular base $PQRS$.
The diagonals of $PQRS$ meet at T.
U is vertically above T.
$PQ = 8$ cm, $QR = 6$ cm and $UR = 13$ cm.

(a) Calculate angle URP.

Answer ..... ° [3]

(b) Find the volume of the pyramid.

Answer ..... cm^3 [2]

(c) Show that triangle PTQ is congruent to triangle RTS.

Answer .....

.....

.....

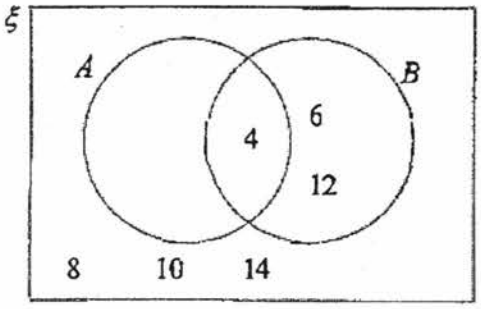
.....

..... [2]

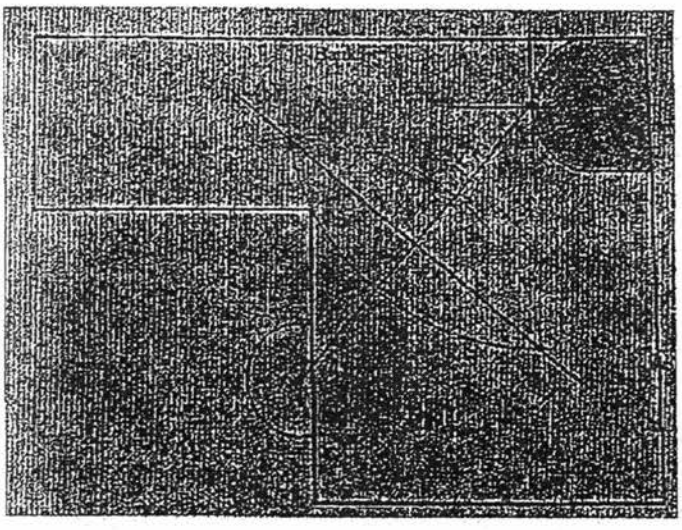
End of Paper

JYSS 4E5N Prelim 2017 Paper 1

No.	Answer	Workings	Marks	*Remarks
1	$x=6$	$0.5(4-\frac{x}{3})=1$ $2-\frac{x}{6}=1$ $\frac{x}{6}=1$ $x=6$	M1 A1	Alternative: $4-\frac{x}{3}=2$ $\frac{x}{3}=2$ $x=6$
2	$\frac{c+d}{cd}$	$c-d+\frac{1}{c}+\frac{1}{d}-\frac{c^2-d^2}{c+d}$ $=c-d+\frac{1}{c}+\frac{1}{d}-\frac{(c+d)(c-d)}{c+d}$ $=\frac{1}{c}+\frac{1}{d}$ $=\frac{c+d}{cd}$	M1 A1	M1 – For any correct method that eliminates $c-d-\frac{c^2-d^2}{c+d}$ to 0.
3	$r=4.13$	$12000=4000(1+\frac{r/12}{100})^{320}$ $3=(1+\frac{r}{1200})^{320}$ $r=4.1268$ $=4.13$	M1 A1	Award M1 for correct substitution of values
4	12	Let x be the size of an exterior angle. $2x+120°=180°$ $x=30°$ $\frac{360°}{30°}=12$	M1 A1	
5	The title is biased and does not allow readers to make their own judgement. It should only state "Late coming occurrences in the past 4 years". or The vertical axis has to start from zero so that it does not exaggerate the differences between the number of counts of late-coming. or The scale of the vertical axis have to be consistent and the intervals between the values on the vertical axis have to be equal. This prevents distortion of the graph.		B2	B1 - State the modification B1 – Explain how the modification will make the graph a better representation.

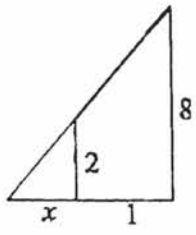
6	(a)	$\xi = \{4, 6, 8, 10, 12, 14\}$ $A = \{4\}$ $B = \{4, 6, 12\}$  		B2	Deduct 1m for each mistake
	(b)	$\{\ \}$ or ϕ		B1	
7		\$1486	Tourism spending in 2016 $= \frac{113.9}{100} \times 21.4 \times 10^9$ $= S\$2.43746 \times 10^{10}$ Average visitor spending $= \frac{2.43746 \times 10^{10}}{1.64 \times 10^7}$ = \$1486.256 = \$1486 (nearest dollar)	M1 M1 A1	No mark is awarded to finding number of visitors in 2015 as this information is not needed.
8	(a)	$(d+e+2)(d+e-4)$	$(d+e)^2 - 2(d+e) - 8$ $= (d+e+2)(d+e-4)$	B1	
	(b)	$(1-2a)(1+x)$	$1+x-2a-2ax$ $= 1+x-2a(1+x)$ $= (1-2a)(1+x)$	M1 A1	
9		$13.5 cm^3$	$\frac{A_1}{A_2} = \frac{16}{9}$ $\frac{L_1}{L_2} = \sqrt{\frac{16}{9}} = \frac{4}{3}$ Using $\frac{V_1}{V_2} = (\frac{L_1}{L_2})^3$, $\frac{32}{V_1} = (\frac{4}{3})^3$ $\Rightarrow V_1 = 13.5\ cm^3$ (3 s.f.)	M1 M1 A1	

No.		Answer	Workings	Marks	*Remarks
10	(a)	$2700 = 2^2 \times 3^3 \times 5^2$		B1	
	(b)	When $b = 10$, $2700 \times 10 = 2^2 \times 3^3 \times 5^2 \times (2 \times 5) = 2^3 \times 3^3 \times 5^3$ Since all the powers of 2700×10 are multiples of 3, $\sqrt[3]{2700 \times 10} = \sqrt[3]{2^3 \times 3^3 \times 5^3} = 2 \times 3 \times 5$, it is a perfect cube.		B1	Accept any correct explanation that $2^3 \times 3^3 \times 5^3$ is a perfect cube.
	(c)	$p = 49$	$\frac{2700 \times \sqrt{p}}{14}$ $= \frac{2^2 \times 3^3 \times 5^2 \times \sqrt{p}}{2 \times 7}$ $\Rightarrow \sqrt{p} = 7$ $\Rightarrow p = 49$	B1	
11		3 m by 6 m	Let width = x m length = $(x+3)$ m Perimeter = Area $2(x + x + 3) = x(x + 3)$ $4x + 6 = x^2 + 3x$ $x^2 - x - 6 = 0$ $(x+2)(x-3) = 0$ $x = 3$ or $x = -2$ (rej) Width = 3 m Length = 6 m	M1 M1 A1	Correct expression for both area and perimeter Correct factorization Correct values for both width and length
12		$\frac{13}{20}$	Black from Box A then black from Box B: $\frac{3}{5} \times \frac{3}{4} = \frac{9}{20}$ White from Box A then black from Box B: $\frac{2}{5} \times \frac{2}{4} = \frac{1}{5}$ Total probability $= \frac{9}{20} + \frac{1}{5}$ $= \frac{13}{20}$	M1 M1 A1	 Accept 0.65

No.		Answer	Workings	Marks	*Remarks
13		16.6 m	$\angle ACB = 60°$ By Sine Rule, $\frac{20}{\sin 60°} = \frac{BC}{\sin 50°}$ $BC = 17.6910\,\text{m}$ $\sin 70° = \frac{h}{17.6910}$ $h = 16.6\,\text{m}$	M1 M1 A1	
14	(a)			B1	
	(b)			B1 No mark for no working	Accept position of J anywhere along the correct perpendicular bisector drawn and inside the bank.
	(c)			B1	Accept boundary as the arc of 1 cm outside the bank. No mark given if arc extends inside the bank.
15	(a)	$y = 2-(x+5)^2$		B1	
	(b)	$y = \frac{1}{3}x - \frac{1}{3}$	Gradient of line passing through (–5, 2) and (1, 0) $= \frac{2-0}{-5-1} = -\frac{1}{3}$ $y = -\frac{1}{3}x + c$ $0 = -\frac{1}{3}(1) + c$ $c = \frac{1}{3}$ $\Rightarrow \quad y = -\frac{1}{3}x + \frac{1}{3}$	M1 A1	Accept $\frac{1}{3} = 0.333$

No.		Answer	Workings	Marks	*Remarks
16		$x = 0.73$	SGD 1.46 = 1 USD SGD 170 $= \frac{170}{1.46} = 116.438$ USD Total amount of USD $= \frac{116.438}{8} \times 100$ = 1455.475 USD 2000 SGD = 1455.475 USD 1 SGD = 0.7277 USD $x = 0.73$ (2 d.p.)	M1 M1 A1	
17	(a)	$x = 0, 1, 2, 3$	$-11 < 1 - 3x \leq 2$ $-1 \leq 3x < 12$ $-\frac{1}{3} \leq x < 4$ $\Rightarrow x = 0, 1, 2, 3$	M1 M1 A1	Award M2 for any correct method to get $-\frac{1}{3} \leq x < 4$.
	(b)	$-6 \leq \frac{b}{a} \leq -\frac{1}{3}$		B1	Accept $\frac{1}{3} = 0.333$
18	(a)	16	$x(2x^{\frac{1}{4}})^4$ $= x(16x^{-1})$ $= 16$	M1 A1	
	(b)	5.4	$\frac{3^{n+2}}{5(3^{n-1})}$ $= \frac{3^{n+2-n+1}}{5}$ $= \frac{3^3}{5}$ $= 5.4$	M1 A1	Also accept final answer as $\frac{27}{5}$ or $5\frac{2}{5}$.

No.		Answer	Workings	Marks	*Remarks
19		$\frac{1}{2}a^2$	Area of quadrant $= \frac{1}{4}\pi a^2$ Area of triangle ZXY $= \frac{1}{2}a^2$ Area of segment ZY $= \frac{1}{4}\pi a^2 - \frac{1}{2}a^2$	M1	
			Diameter ZY $= \sqrt{a^2 + a^2}$ $= \sqrt{2}a$	M1	
			Area of semi-circle $= \frac{1}{2}\pi\left(\frac{\sqrt{2}a}{2}\right)^2$ $= \frac{1}{4}\pi a^2$	M1	
			Area of shaded region $= \frac{1}{4}\pi a^2 - \left(\frac{1}{4}\pi a^2 - \frac{1}{2}a^2\right)$ $= \frac{1}{2}a^2$	A1	Accept $0.5a^2$
20	(a)	2750 cm^2	$A = kd^2$ $440 = k(30)^2$ $k = \frac{22}{45}$	M1	
			$A = \frac{22}{45}(75)^2$	M1	
			$= 2750$ cm^2	A1	
	(b)	When d decreases by 15%, A becomes $(0.85)^2 = 0.7225$. A decreases by 27.75%.		B1	May use calculator to verify
21	(a)	(2, 5.5)	$2y = 3x + 5$ --- (1) $x + 4y = 24$ --- (2) $x = 24 - 4y$ --- (3) Sub (3) into (1), $2y = 3(24 - 4y) + 5$	M1	1m for correct substitution or elimination method 1m for correct x and y
			$14y = 77$ $y = 5.5$	M1	1m for correct coordinates
			$x = 2$	A1	(accept $5\frac{1}{2}$ or $\frac{11}{2}$)

	(b)	$y=-2$		B1	
22		8.11 m	Let x be the distance from the bush to the ladder. Using similar triangles, $\frac{2}{x}=\frac{8}{x+1}$ $6x=2$ $x=\frac{1}{3}$  Length of ladder $=\sqrt{(1\frac{1}{3})^2+8^2}$ $=8.11$ m	M1 M1 M1 A1	
23	(a)	$\frac{2\pi}{3}$ radian is one third of one revolution (2π).		B1	
	(b)	$\frac{\pi}{6}$ radian	angle ABO $=\frac{180°-120°}{2}$ $=30°$ $=\frac{\pi}{6}$	M1 A1	Accept radian method $\frac{\pi-\frac{2\pi}{3}}{2}=\frac{\pi}{6}$
	(c)	0.209 m/s	Total time $=2.5\times60=150$ s Total distance $=r\theta$ $=\frac{2\pi}{3}(15)$ $=10\pi$ Average speed $=\frac{10\pi}{150}$ $=0.209$ m/s	M1 M1 A1	
24	(a)	67.4°	$TR=\sqrt{3^2+4^2}=5$ $\cos URT=\frac{5}{13}$ Angle $URT=67.4°$	M1 M1 A1	
	(b)	192 cm^3	$UT=\sqrt{13^2-5^2}=12$ Volume $=\frac{1}{3}(\quad)(\quad)12$	M1 A1	

	(c)	SR = PQ (sides of rectangle) ST = TQ (T is the midpoint of diagonal) TR = TP (T is the midpoint of diagonal) By SSS Test, triangle *PTQ* is congruent to triangle *RTS*.	M1 A1	Accept correct SAS and ASA tests too.

1 (a) Simplify $\frac{5(x-y)^4}{(x+y)^2} \div \frac{(x-y)^3}{6x+6y}$. [2]

(b) Express $\frac{5}{x-1} - \frac{2}{x^2-1}$ as a single fraction in its simplest form. [2]

(c) It is given that $z = \frac{x^2 - y^2}{y}$.

(i) Make x the subject of the formula. [2]

(ii) If $x = 2$ and $z = 3$, find the value(s) of y. [3]

(d) Given that $\frac{x+3y}{5x-4y} = \frac{2}{3}$, find the ratio $x : y$. [3]

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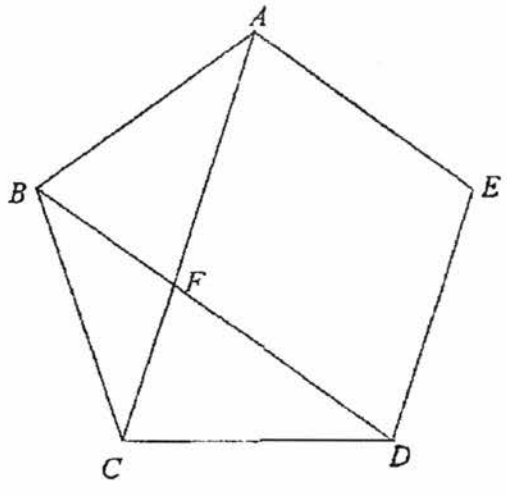
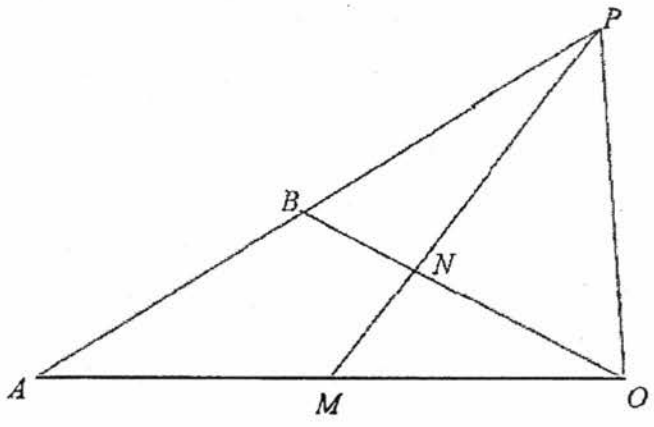
2 (a) The diagram below shows a regular pentagon $ABCDE$.
AC and BD intersect at F.



(i) Find the value of angle CDF, [2]

(ii) Explain why angle $DFA = 108°$. [2]

(b) In the triangle OAB, M is the midpoint of OA.
N is a point on OB such that $ON : NB = 2 : 1$.
MN is produced to P so that $MN : NP = 1 : 2$.



It is given that $\overrightarrow{OA} = \mathbf{a}$ and $\overrightarrow{OB} = \mathbf{b}$.

(i) Express, in terms of **a** and/or **b**,

(a) $\overrightarrow{NB}$, [1]

(b) $\overrightarrow{MN}$, [1]

(c) $\overrightarrow{NP}$. [1]

(ii) Express $\overrightarrow{AB}$ and $\overrightarrow{BP}$ in terms of **a** and **b**. [2]

(iii) Write down two facts about points A, B and P. [2]

(iv) Find $\dfrac{\text{area of triangle } PMB}{\text{area of triangle } PMA}$. [1]

3 (a) The n^{th} term of a sequence is given by $T_n = \frac{n(n+3)}{2}$.

(i) Which term of the sequence has value 275? [2]

(ii) Explain why each term of the sequence is a whole number. [1]

(b) The diagram shows a sequence of shapes T_1, T_2, T_3, ...
Each shape consists of a number of shaded and unshaded triangles.

T_1 T_2 T_3 T_4

The letter r represents the number of rows of triangles in each shape

The letters S, U and N represent the number of shaded triangles, unshaded triangles and total number of triangles respectively.

The data is recorded in the table below.

Shape		T_1	T_2	T_3	T_4	T_5
Number of rows	r	1	2	3	4	5
Number of shaded triangles	S	0	1	3	6	a
Number of unshaded triangles	U	1	3	6	10	b
Total number of triangles	N	1	4	9	16	c

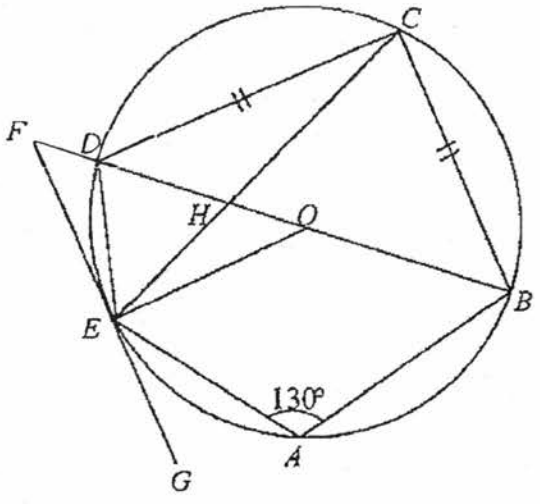
(i) Write down the value of a, of b and of c. [3]

(ii) Write a formula for the total number of triangles in the r^{th} shape, N_r. [1]

(iii) Write a formula for the number of unshaded triangles in the r^{th} shape, U_r. [1]

(iv) Find the number of shaded triangles in shape T_{50}. [2]

4 In the diagram, which is not drawn to scale, O is the centre of the circle.
Points A, B, C, D and E lie on the circumference.



BD is a diameter.
The tangent at E meets BD produced at F.
EC meets BD at H.
$BC = CD$ and angle $EAB = 130°$.

(a) Stating your reasons clearly, find

(i) reflex angle EOB, [1]

(ii) angle ECB, [1]

(iii) angle CBD, [1]

(iv) angle DOE, [1]

(v) angle OFE. [1]

(b) Is the line ED parallel to line BC?
Justify your answer with clear working. [2]

5 The distance between Town P and Town Q is 150 km.
An express bus travels from Town P to Town Q at the average speed of x km/h.
If the average speed of the bus is increased by 15 km/h, the time taken would be 21 minutes less.

(a) Express, in terms of x,

(i) the time taken by the bus at its original speed, [1]

(ii) the time taken by the bus when the speed is increased by 15 km/h. [1]

(b) Form an equation in x and show that it can be reduced to $7x^2 + 105x - 45\,000 = 0$. [3]

(c) Solve the equation in part (b) and hence find the original time taken in hours and minutes, correct to the nearest minute. [4]

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6 At the end of a semester, the final grade of the students is recorded based on their marks obtained from tests, projects, homework and quizzes.
The marks obtained by three students, Aaron, Beatrice and Carly, are given in the following table.

	Tests	Projects	Homework	Quizzes
Aaron	82	95	89	60
Beatrice	72	85	65	57
Carly	88	91	70	64

(a) (i) Write down a 3×4 matrix **M** that represents the information in the table. [1]

(ii) The weightage for each component are as follows:

Tests	50%
Projects	20%
Homework	10%
Quizzes	20%

Represent the weightage as a decimal number in a 4×1 matrix **X**. [1]

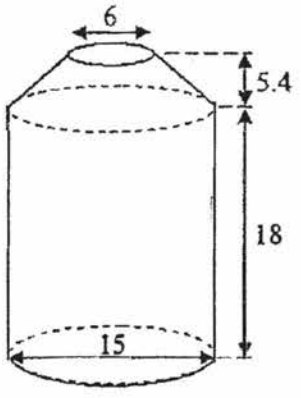
(iii) Evaluate the matric $\mathbf{F} = \mathbf{MX}$. [2]

(iv) State what the elements of **F** represent. [1]

(b) Overall, the cohort did better in projects than in quizzes.

Suggest how the weightage for each component could change so as to improve the final grade of the students. [1]

7 The diagram, not drawn to scale, shows an open container which is made up of a cylinder and a frustum.
A frustum is a cone with part of its top removed.
The cylinder has height 18 cm and diameter of 15 cm.
The conical section has base diameter 15 cm, top diameter 6 cm and height 5.4 cm.



(a) Show that the height of the cone before its top was removed is 9 cm. [2]

(b) The container is filled to its brim with water.

Calculate

(i) the volume of the water in the container, [2]

(ii) the total surface area of the container in contact with water. [3]

(c) All the water in the container is poured into a rectangular tank with a base area of 120 cm^2.

Find the minimum height of the tank so that the water **does not overflow**.
Give your answer as a whole number. [2]

8 The variables x and y are connected by the equation $y = 2x + \frac{50}{x} - 30$, where $x \neq 0$.

Some corresponding values of x and y are given in the following table, corrected to 2 decimal places.

x	1	1.5	3	5	7	9	11	13	15	16
y	22.00	p	−7.33	−10.00	−8.86	−6.44	q	−0.15	3.33	5.13

(a) Find the value of p and of q. [2]

(b) Using a scale of 1 cm to 1 unit on the horizontal x-axis and 2 cm to 5 units on the vertical y-axis, draw the graph of $y = 2x + \frac{50}{x} - 30$ for $1 \leq x \leq 16$. [3]

(c) By drawing a tangent, find the gradient of the curve at the point $x = 10$. [2]

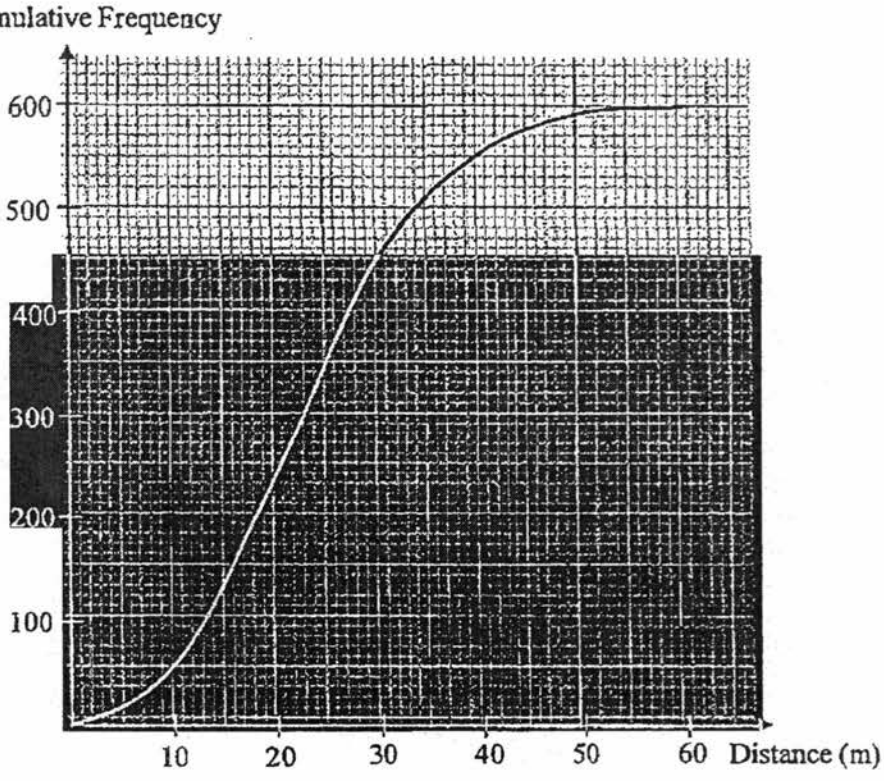
(d) (i) On the same axes, draw the line $y = 3 - 2x$. [1]

(ii) From the graph, state the x-coordinate of the points where this line intersects the curve. [2]

(iii) These values of x are the solutions of the equation $Ax^2 - 33x + B = 0$.

Find the value of A and of B. [2]

9 (a) A group of 600 young children was tested to find the distance that each of them was able to swim in an indoor swimming pool.
The results of the test are shown on the cumulative frequency curve below.



(i) Using the given curve, find for this distribution,

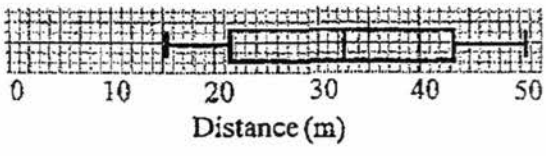
(a) the median, [1]

(b) the interquartile range. [2]

(ii) The distance to pass the test was 35 metres.

Estimate the percentage of children who passed the test. [2]

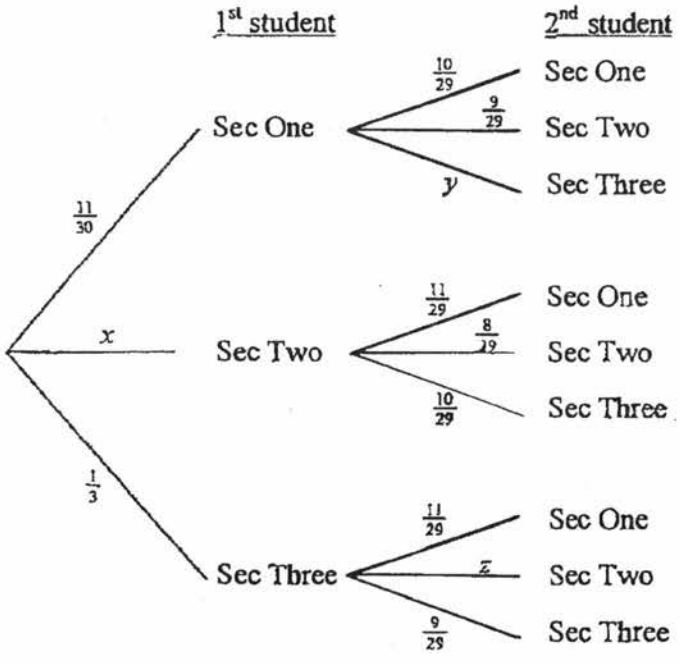
(iii) The same group of children was tested to swim in the outdoor swimming pool.
The box-and-whisker plot shows the distribution of the test.



Make two comparisons between the performances of the children in the two tests. [2]

(b) A room consists of 30 students.
The students are selected from three different levels.
There are 11 students from Sec One, 9 students from Sec Two and 10 students from Sec Three.

Two students are selected from the room to compete with other students from another room.
The tree diagram below shows the possible outcomes and some of their probabilities.



(i) Calculate the value of x, of y and of z as shown on the tree diagram. [3]

(ii) Expressing your answers in fractions in its lowest term, calculate the probability that

(a) both students are from the same level, [1]

(b) both students are of different levels, [1]

(c) one student will be from Sec One and the other from Sec Three. [1]

10 Sarah wants to sell chocolate cupcakes at the next neighbourhood Food Fair.
She intends to bake 180 to 200 cupcakes.
She bakes in batches of 16 cupcakes.
Information that Sarah needs is provided below.

(a) How many times must she bake in order to have a total of 180 to 200 cupcakes? [2]

Sarah needs to decide how much to charge customers for a box of 6 chocolate cupcakes.
She must make sure that she charges enough money to cover all of her costs.

(b) Using your answer from (a), find the number of boxes she will need for the packaging. [1]

(c) Suggest a sensible amount for her to charge for a box of 6 cupcakes.
Justify the decision you make and show your calculations clearly. [7]

Ingredients Recipe makes 16 cupcakes
114 g butter
2 eggs
160 g caster sugar
100 g plain flour
60 g cocoa powder
125 ml evaporated milk
Chocolate cream frosting

Baking supplies		
Items	**Description**	**Unit cost**
Butter	Pack of 500 g	$4.95
Eggs	Pack of 30 eggs	$3.85
Caster sugar	Pack of 800 g	$2.65
Plain flour	Pack of 1 kg	$1.70
Cocoa powder	Pack of 250 g	$4.10
Evaporated Milk	Can of 350 ml	$1.60
Chocolate Cream Frosting for 50 cupcakes	1 tub	$18
Cupcake liners	Pack of 100 pieces	$4.00
Cupcake boxes	Pack of 5 boxes	$3.00

Booth Rental Fee: $100

END OF PAPER

4E5N Prelim 2017 Paper 2
Answer Key

1a	$\frac{30(x-y)}{x+y}$
1b	$\frac{5x+3}{(x-1)(x+1)}$
1c(i)	$x=\pm\sqrt{y^2+yz}$ or $x=\pm\sqrt{y(y+z)}$
1c(ii)	$y=-4$ or $y=1$
1d	17 : 7
2a(i)	36°
2b(i)	(a) $\frac{1}{3}\mathbf{b}$ (b) $\frac{2}{3}\mathbf{b}-\frac{1}{2}\mathbf{a}$ (c) $\frac{4}{3}\mathbf{b}-\mathbf{a}$
2b(ii)	$\overrightarrow{AB}=\mathbf{b}-\mathbf{a}$ $\overrightarrow{BP}=\mathbf{b}-\mathbf{a}$
2b(iv)	$\frac{1}{2}$
3a(i)	22nd term / T_{22}
3b(i)	$a=10$ $b=15$ $c=25$
3b(ii)	$N_r=r^2$
3b(iii)	$U_r=\frac{r(r+1)}{2}$
3b(iv)	1 225
4a(i)	260°
4a(ii)	50°
4a(iii)	45°
4a(v)	10°
4a(iv)	80°
4b	Not parallel
5a(i)	$\frac{150}{x}$ h
5a(ii)	$\frac{150}{x+15}$ h

5c	2 hours 3 mins
6a(i)	$\mathbf{M}=\begin{pmatrix}82 & 95 & 89 & 60\\ 72 & 85 & 65 & 57\\ 88 & 91 & 70 & 64\end{pmatrix}$
6a(ii)	$\mathbf{X}=\begin{pmatrix}0.5\\ 0.2\\ 0.1\\ 0.2\end{pmatrix}$
6a(iii)	$\begin{pmatrix}80.9\\ 70.9\\ 82\end{pmatrix}$
7b(i)	3 680 cm^3 (3 sf)
7b(ii)	1 260 cm^2 (3 sf)
7c	31 cm
8a	$p=6.33$ $q=-3.45$
8c	1.5 (±1)
8d(ii)	$x=2.2(\pm0.2)$ $x=6.25(\pm0.2)$
8d(iii)	$A=4$ $B=50$
9a(i)	**(a)** 23 m **(b)** 14 m
9a(ii)	14.2 %
9b(i)	$x=\frac{3}{10}$, $y=\frac{10}{29}$, $z=\frac{9}{29}$
9b(ii)	**(a)** $\frac{136}{435}$ **(b)** $\frac{299}{435}$ **(c)** $\frac{22}{87}$
10a	12 times
10b	32 boxes
10c	minimum \$7.90 per box so as to cover cost.

Marking Scheme
4E5N Prelim 2017 Paper 2

Qn	Answer Key		Marks
1a	$\frac{30(x-y)}{x+y}$	$\frac{5(x-y)^4}{(x+y)^2} \div \frac{(x-y)^3}{6x+6y}$	
		$= \frac{5(x-y)^4}{(x+y)^2} \times \frac{6(x+y)}{(x-y)^3}$	M1
		$\frac{30(x-y)}{x+y}$	A1
1b	$\frac{5x+3}{(x-1)(x+1)}$	$\frac{5}{x-1} - \frac{2}{x^2-1}$	
		$= \frac{5(x+1)}{(x-1)(x+1)} - \frac{2}{(x-1)(x+1)}$	M1
		$= \frac{5x+5-2}{(x-1)(x+1)}$	
		$= \frac{5x+3}{(x-1)(x+1)}$	A1
1c(i)	$x = \pm\sqrt{y^2+yz}$ or $x = \pm\sqrt{y(y+z)}$	$z = \frac{x^2-y^2}{y}$	
		$yz = x^2 - y^2$	M1
		$x^2 = y^2 + yz$	
		$x = \pm\sqrt{y^2+yz}$	A1
1c(ii)	$y=-4$ or $y=1$	$3 = \frac{(2)^2-y^2}{y}$	M1
		$3y = 4 - y^2$	
		$y^2 + 3y - 4 = 0$	
		$(y+4)(y-1) = 0$ *factorisation*	M1
		$y=-4$ or $y=1$	A1
1d	17:7	$\frac{x+3y}{5x-4y} = \frac{2}{3}$	
		$3(x+3y) = 2(5x-4y)$	M1
		$3x+9y = 10x-8y$	
		$17y = 7x$	M1
		$\frac{x}{y} = \frac{17}{7}$	
		$x:y = 17:7$	A1

2a(i)	$36°$	$\angle BCD$ = Each interior angle $= \frac{(5-2)\times 180}{5} = 108°$ $\angle CDF$ $= \frac{180° - 108°}{2}$ $= 36°$ (base $\angle$ of isos. Δ)	M1 A1
2a(ii)		$\angle ACB = 36°$ (symmetry / congruent triangles) $\angle BFC = 180° - 36° - 36° = 108°$ (isos. Δ) $\angle DFA = 108°$ (vert. opp $\angle$s)	B1 B1
2b(i)	(a) $\frac{1}{3}\mathbf{b}$ (b) $\frac{2}{3}\mathbf{b} - \frac{1}{2}\mathbf{a}$ (c) $\frac{4}{3}\mathbf{b} - \mathbf{a}$	(a) $\overrightarrow{NB} = \frac{1}{3}\overrightarrow{OB} = \frac{1}{3}\mathbf{b}$ (b) $\overrightarrow{MN} = \overrightarrow{ON} - \overrightarrow{OM}$ $= \frac{2}{3}\mathbf{b} - \frac{1}{2}\mathbf{a}$ *(preferred answer)* $= \frac{1}{6}(4\mathbf{b} - 3\mathbf{a})$ (c) $\overrightarrow{NP} = 2\overrightarrow{MN}$ $= \frac{4}{3}\mathbf{b} - \mathbf{a}$ *(preferred answer)* $= \frac{1}{3}(4\mathbf{b} - 3\mathbf{a})$	B1 B1 B1
2b(ii)	$\overrightarrow{AB} = \mathbf{b} - \mathbf{a}$ $\overrightarrow{BP} = \mathbf{b} - \mathbf{a}$	$\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA} = \mathbf{b} - \mathbf{a}$ $\overrightarrow{BP} = \overrightarrow{NP} - \overrightarrow{NB}$ $= \frac{4}{3}\mathbf{b} - \mathbf{a} - \frac{1}{3}\mathbf{b}$ $= \mathbf{b} - \mathbf{a}$	B1 B1
2b(iii)		1. B is the midpoint of line ABP. 2. A, B and P are collinear/ABP lies on a straight line	B2
2b(iv)	$\frac{1}{2}$	$\frac{\text{area of triangle } PMB}{\text{area of triangle } PMA}$ $= \frac{\frac{1}{2}(PM)(PB)\sin P}{\frac{1}{2}(PM)(PA)\sin P} = \frac{PB}{PA} = \frac{1}{2}$ *Alternative Mtd* *Use common height*	B1
3a(i)	22^{nd} term / T_{22}	$\frac{n(n+3)}{2} = 275$ $n^2 + 3n - 550 = 0$ $(n-22)(n+25) = 0$ $n = 22$ or $n = -25$ (reject $\because n > 0$) *Alternative Mtd* *Quadratic Formula*	 M1 A1
3a(ii)		For any integer value of n, either n is even or $(n+3)$ is even. Hence, $n(n+3)$ is always divisible by 2.	B1 *Accept equivalent reasoning*

3b(i)	$a=10$ $b=15$ $c=25$		B1 B1 B1
3b(ii)	$N_r=r^2$		B1
3b(iii)	$U_r=\frac{r(r+1)}{2}$		B1
3b(iv)	1 225	$N_{50}=50^2=2500$ triangles in total $U_{50}=\frac{50(50+1)}{2}=1275$ triangles unshaded Number of shaded triangles $=2500-1275=1225$	M1 A1
4a(i)	260°	reflex $\angle EOB$ $=130^\circ\times 2$ ($\angle$ at centre $=2\angle$ at circumference) $=260^\circ$	B1
4a(ii)	50°	$\angle ECB=180^\circ-130^\circ$ (angles in opposite segment) $=50^\circ$	B1
4a(iii)	45°	$\angle CBD=(180^\circ-90^\circ)\div 2$ (isosceles triangle, $BC=CD$) $=45^\circ$	B1
4a(iv)	80°	$\angle DOE=260^\circ-180^\circ$ $=80^\circ$	B1
4a(v)	10°	$\angle OFE=180^\circ-90^\circ-80^\circ$ (tangent $\perp$ radius) $=10^\circ$	B1√
4b	Not parallel	$\angle BDE=50^\circ$ (angles in same segment) $\angle CBD=45^\circ$ (from aiii) $\angle CBD\neq\angle BDE$ $\therefore$ Line ED is not parallel to line BC [accept any other mathematically logical method]	M1 A1
5a(i)	$\frac{150}{x}$ h		B1
5a(ii)	$\frac{150}{x+15}$ h		B1
5b		$\frac{150}{x}-\frac{150}{x+15}=\frac{21}{60}$ $\frac{150(x+15)-150x}{x(x+15)}=\frac{7}{20}$ $\frac{2250}{x(x+15)}=\frac{7}{20}$ $45000=7x(x+15)$ $7x^2+105x-45000=0$	M1 M1 A1

5c	2 hours 3 mins	$7x^2 + 105x - 45000 = 0$ $x = \dfrac{-105 \pm \sqrt{105^2 - 4(7)(-45000)}}{2(7)}$ $x = 73.028$ or $x = -88.028$ Original time taken $= \dfrac{150}{73.028} = 2.054$ hours $= 2$ hours 3 minutes	M1 A1 M1 A1
6a(i)	$\mathbf{M} = \begin{pmatrix} 82 & 95 & 89 & 60 \\ 72 & 85 & 65 & 57 \\ 88 & 91 & 70 & 64 \end{pmatrix}$		B1
6a(ii)	$\mathbf{X} = \begin{pmatrix} 0.5 \\ 0.2 \\ 0.1 \\ 0.2 \end{pmatrix}$		B1
6a(iii)	$\begin{pmatrix} 80.9 \\ 70.9 \\ 82 \end{pmatrix}$	$\mathbf{F} = \mathbf{MX}$ $= \begin{pmatrix} 82 & 95 & 89 & 60 \\ 72 & 85 & 65 & 57 \\ 88 & 91 & 70 & 64 \end{pmatrix} \begin{pmatrix} 0.5 \\ 0.2 \\ 0.1 \\ 0.2 \end{pmatrix}$ $= \begin{pmatrix} 80.9 \\ 70.9 \\ 82 \end{pmatrix}$	B2 All correct B1 1 mistake 0 More than 1 mistake
6a(iv)		The elements in F represent the <u>respective final combined score/grade</u> of Aaron, Beatrice and Carly at the end of the semester.	B1
6b		The teacher could increase the weightage of projects and decrease the weightage of quizzes. *(Any other suitable suggestions)*	B1
7a		Let the height of the original cone be x cm. Using similarity, $\dfrac{x-5.4}{x} = \dfrac{6}{15}$ $15x - 81 = 6x$ $x = 9$ (shown)	M1 A1
7b(i)	3 680 cm^3 (3 sf)	Height of cone that has been removed $9 - 5.4 = 3.6$ cm Volume of water in cylinder $= \pi(7.5)^2(18)$ 3181 cm^3 $= 1012.5\pi$ cm^3	M1 for

		Volume in cone (with top removed) $= \frac{1}{3}\pi(7.5)^2(9) - \frac{1}{3}\pi(3)^2(3.6)$ 496.2 cm^3 $= 157.95\pi$ cm^3 Volume of water in container $= 1012.5\pi + 157.95\pi$ $= 3\,677.1$ $= 3\,680$ cm^3	either of the two A1
7b(ii)	1 260 cm^2 (3 sf)	Slant height, $L = \sqrt{9^2 + 7.5^2} = 11.72$ cm (4 sf) Slant height $l = \sqrt{3.6^2 + 3^2} = 4.686$ cm (4 sf) Surface area of cylinder in contact with water $= 2\pi(7.5)(18) + \pi(7.5)^2$ $= 326.25\pi$ cm^2 1025 cm^2 Surface area of cone in contact with water $= \pi(7.5)(11.72) - \pi(3)(4.686)$ $= 231.98$ cm^2 Required surface area $= 326.25\pi + 231.98$ $= 1256.92$ $= 1260$ cm^2	M1 (either one) M1 for either of the two A1
7c	31 cm	Height $= \frac{3677.1}{120} = 30.64$ cm Minimum height = 31 cm (whole number)	M1 A1
8a	$p = 6.33$ $q = -3.45$		B1 B1
8b		See attached: Correct Scale Plotted points Smooth Curve	S1 P1 C1
8c	1.5 (±1)	Draw a tangent at $x = 10$. Gradient = 1.5 (±0.1)	1 1
8d(i)		Draw the line $y = 3 - 2x$.	1
8d(ii)	$x = 2.2(\pm 0.2)$ $x = 6.25(\pm 0.2)$		B1 B1

8d(iii)	$A=4$ $B=50$	$2x+\frac{50}{x}-30=3-2x$ $2x^2+50-30x=3x-2x^2$ $4x^2-33x+50=0$ $\therefore A=4$ and $B=50$	M1 A1
9a(i)	(a) 23 m (b) 14 m	Median = 23 m Interquartile Range $= Q_3 - Q_1$ $= 30-16$ $= 14$ m	B1 M1 A1
9a(ii)	14.2 %	Number of students who passed the test $= 600 - 515^*$ $= 85$ Percentage of students $= \frac{85}{600}\times 100\%$ $= 14.2\ \%$ (3 s.f)	M1 (*accept 80 or 90) *Possible answers: 13.3% or 15% A1
9a(iii)		Children on average swam **further** at the outdoor swimming pool as can be seen from the **bigger median** of 32 m. The **maximum distance** that was covered in the outdoor swimming pool is 50 m, which is lower than the maximum distance covered in the indoor swimming pool which is 60 m. The **interquartile range** for the test in the outdoor swimming pool is 22 m, which is **more than** the test in the indoor swimming pool, 14 m indicating that the distance covered at the indoor swimming pool is **more consistent**.	B2 Any two
9b(i)		$x=\frac{9}{30}=\frac{3}{10}$, $y=\frac{10}{29}$, $z=\frac{9}{29}$	B1 B1 B1
9b(ii)	(a) $\frac{136}{435}$ (b) $\frac{299}{435}$ (c) $\frac{22}{87}$	(a) $\left(\frac{11}{30}\right)\left(\frac{10}{29}\right)+\left(\frac{3}{10}\right)\left(\frac{8}{29}\right)+\left(\frac{1}{3}\right)\left(\frac{9}{29}\right)=\frac{136}{435}$ (b) $1-\frac{136}{435}=\frac{299}{435}$ (c) $\left(\frac{11}{30}\right)\left(\frac{10}{29}\right)+\left(\frac{1}{3}\right)\left(\frac{11}{29}\right)=\frac{22}{87}$	B1 B1 B1
10a	12 times	Number of batches $= \frac{200}{16}=12.5\approx 12$ If she bakes 12 times, she will get $12\times 16=192$ cupcakes	M1 A1 Equivalent working acceptable

10b	32 boxes	With 192 cupcakes, she needs $\frac{192}{6} = 32$ boxes	B1
10c	B2 Quantity of ingredients for 192 cupcakes (shaded box) • 1 mistake (–1m) • > 1 mistake (–2m) B2√ Cost of ingredients for 192 cupcakes (expenses) • 1 mistake (–1m) • > 1 mistake (–2m) B1 Add Booth Rental Fee B1√ Find cost price of 6 cupcakes ($7.85) B1 Sensible amount with justification. Profit, transport costs etc are additional consideration. So long as it makes sense and covers the basic cost price. The minimum amount should be $7.90 per box.	How much of each ingredient is needed: (see table below) Cost breakdown (see table below) Total expenses ≈ $250. Cost price for 6 cupcakes = $\frac{\$251.35}{192} \times 6 = \$7.85$ Consider: • transport costs, • electricity costs, • profit to be made	7

To follow through from their (a) answer

Item	1 batch	For 12 batches	Sold as:	Need to get:
Butter	114 g	1368 g	500 g	3
Eggs	2	24	30	1
Caster sugar	160 g	1920 g	800 g	3
Plain flour	100 g	1200 g	1 kg	2
Cocoa powder	60 g	720 g	250 g	3
Evaporated Milk	125 ml	1500 ml	350 ml	5
Chocolate Cream Frosting	16 cupcakes	192 cupcakes	50 cupcakes	4
Cupcake liners	16 cupcakes	192 liners	100 liners	2
Cupcake boxes		32 boxes	5 boxes	7

Item	Unit Price	Qty	Expenses
Butter	$4.95	3	$14.85
Eggs	$3.85	1	$3.85
Caster sugar	$2.65	3	$7.95
Plain flour	$1.70	2	$3.40
Cocoa powder	$4.10	3	$12.30
Evaporated Milk	$1.60	5	$8.00
Chocolate Cream Frosting	$18.00	4	$72.00
Cupcake liners	$4.00	2	$8.00
Cupcake boxes	$3.00	7	$21.00
		Sub Total	$151.35
		Booth rental	$100.00

Question 8