SERANGOON GARDEN SECONDARY SCHOOL

Vision: Critical Thinkers, Thoughtful Leaders
Mission: Love to Learn, Learn to Lead

PRELIMINARY EXAMINATION 2017

CANDIDATE NAME

CLASS | REGISTER NUMBER

MATHEMATICS 4048/02
Paper 2 22 August 2017
Secondary 4 Express/ 5 Normal Academic 2 hours 30 minutes
0800 – 1030

Additional Materials: Writing Paper
Graph Paper (1 sheet)

READ THESE INSTRUCTIONS FIRST

Write your name, class and class register number on all the work you hand in.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer all questions.
If working is needed for any question, it must be shown with the answer.
Omission of essential working will result in loss of marks.
The use of an approved scientific calculator is expected, where appropriate.
If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.
For π, use either your calculator value or 3.142, unless the question requires the answer in terms of π.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total of the marks for this paper is 100.

Areas for Improvement		
Error	Penalty	Qn. No.(s)
Accuracy of non-exact answers	– 1	
Missing/ wrong units (for Paper 2 only)	– 1	
Presentation/ Not using ink	– 1	

Name/Signature of Parent/Guardian _____ Date _____

FOR MARKER'S USE

100

This question paper consists of 13 printed pages and 1 blank page.

Setter: Mr Ng HJ Vetter: Mr Ko TH

MATHEMATICAL FORMULAE

Compound Interest

$$\text{Total amount} = P\left(1+\frac{r}{100}\right)^n$$

Mensuration

$$\text{Curved surface area of cone} = \pi rl$$

$$\text{Surface area of a sphere} = 4\pi r^2$$

$$\text{Volume of a cone} = \frac{1}{3}\pi r^2 h$$

$$\text{Volume of sphere} = \frac{4}{3}\pi r^3$$

$$\text{Area of triangle ABC} = \frac{1}{2}ab\sin C$$

$$\text{Arc length} = r\theta, \text{ where } \theta \text{ is in radians}$$

$$\text{Sector area} = \frac{1}{2}r^2\theta, \text{ where } \theta \text{ is in radians}$$

Trigonometry

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

Statistics

$$\text{Mean} = \frac{\sum fx}{\sum f}$$

$$\text{Standard Deviation} = \sqrt{\frac{\sum fx^2}{\sum f} - \left(\frac{\sum fx}{\sum f}\right)^2}$$

Answer all the questions.

1 (a) n is a positive integer. Show that $n^2 + n$ is always even. [2]

(b) Solve the equation $p^2 - 7p + 12 = 0$. [2]

Hence solve the equation $q^4 - 7q^2 + 12 = 0$. [2]

(c) A 2.5 km^2 lake has an area of 40 cm^2 on a map.

(i) If the scale of the map is such that 1 cm represents n km, find the value of n. [2]

(ii) The distance between the hospital and the village town on the map is 30 cm. Find the actual distance, in kilometres, between the hospital and the village town. [1]

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2 Mr Kia is going on a business trip to a province in the same country. There are two options for him to go to the province: by domestic flight or by car.

- If he decides to drive, he would cover a distance of 400 km at a speed of x km/h.
- If he decides to take a domestic flight, he would cover a distance of 300 km at a speed of $(x + 250)$ km/h.

(i) Find an expression, in terms of x, for the time taken to travel from home to the province if Mr Kia decides to drive. [1]

(ii) Find an expression, in terms of x, for the time taken to travel from home to the province if Mr Kia decides to take a domestic flight. [1]

(iii) If the flight time is 210 minutes less than the driving time, form an equation in x and show that it reduces to $7x^2 + 1550x - 200000 = 0$. [3]

(iv) Solve the equation $7x^2 + 1550x - 200000 = 0$, giving your answers correct to 1 decimal place. [3]

(v) If Mr Kia needs to meet his client punctually at 1400, find the latest time that he needs to leave home if he decides to drive. **Assume that time has been factored in for the usual traffic conditions.** [2]

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3 (a) A set of 10 cards is made as shown.

S	T	A	T	I	S	T	I	C	S

The cards are shuffled and placed face down on a desk. A card is drawn at random from the set of cards. It is then replaced and shuffled again before another card is being drawn again.

Calculate the probability that

(i) both cards show the letter T, [2]

(ii) exactly one of the cards shows the letter T. [2]

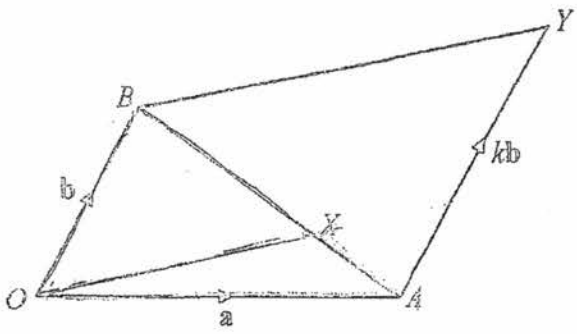
(b) The table shows the ages of 1100 people who entered a 10-km run

Age (x years)		$20 \leq x < 30$	$30 \leq x < 40$	$40 \leq x < 50$
Frequency	Men	375	186	99
	Women	250	122	68

(i) One person is chosen at random. Find, as a fraction in its lowest term, the probability that the person is a man aged less than 40 years old. [1]

(ii) Two persons are chosen at random. Find the probability that both of them are women aged 30 or more. [2]

4 In the diagram, $\overrightarrow{OA} = \mathbf{a}$, $\overrightarrow{OB} = \mathbf{b}$ and $\overrightarrow{AY} = k\mathbf{b}$. X lies on the line AB such that $\overrightarrow{AX} = \frac{1}{3}\overrightarrow{AB}$.



(i) Express $\overrightarrow{AX}$ and $\overrightarrow{OX}$ in terms of $\mathbf{a}$ and $\mathbf{b}$. [2]

(ii) Express $\overrightarrow{BY}$ in terms of k, $\mathbf{a}$ and $\mathbf{b}$. [1]

(iii) Given that OX is parallel to BY, find the value of k. [2]

(iv) The line OX when produced, meets AY at Z. Express $\overrightarrow{AZ}$ in terms of $\mathbf{b}$. [1]

(v) Find the value of

(a) $\dfrac{\text{area of } \Delta OAX}{\text{area of } \Delta OBX}$, [1]

(b) $\dfrac{\text{area of } \Delta AXZ}{\text{area of quadrilateral } XBYZ}$. [2]

5 The following shows the work done by a student in calculating the sum of the first n natural numbers.

n	Series	Sum	Formula
1	1	1	$\frac{1}{2}(1)(1+1)$
2	$1+2$	3	$\frac{1}{2}(2)(2+1)$
3	$1+2+3$	6	$\frac{1}{2}(3)(3+1)$
4	$1+2+3+4$	10	$\frac{1}{2}(4)(4+1)$
⋮	⋮	⋮	⋮
6	$1+2+3+4+5+6$	a	b
⋮	⋮	⋮	⋮
n	$1+2+3+\cdots+n$	c	

(i) Study the pattern and write down the values of a and b. [2]

(ii) Find in terms of n, the value of c. [1]

After doing some additional calculations, the student realised that

$$1^3+2^3+3^3=36=6^2,$$
$$1^3+2^3+3^3+4^3=100=10^2.$$

(iii) Determine the sum of the series

(a) $1^3+2^3+3^3+4^3+5^3+6^3$, [1]

(b) $1^3+2^3+3^3+\cdots+n^3$ in terms of n. [1]

(iv) Hence, using (iii)(b), determine the exact value of the sum of the series

$$3^3+6^3+9^3+12^3+\cdots+300^3.$$ [2]

6 (a) State the order and name of each matrix. [2]

	Matrix	Order	Name of matrix
(i)	$\begin{pmatrix} 2 \\ 5 \\ 12 \end{pmatrix}$		
(ii)	$\begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$		

(b) The Tan family owns two cars. Every week (Monday to Friday) on average, Mr Tan spends \$150, \$70 and \$10 on petrol, carpark charges and road pricing (ERP) respectively. Every week (Monday to Friday) on average, Mrs Tan spends \$80, \$45 and \$30 on petrol, carpark charges and road pricing (ERP) respectively.

The information can be represented by the matrix

$$\mathbb{P} = \begin{matrix} \text{Mr Tan} & \text{Mrs Tan} \\ \begin{pmatrix} 150 & 80 \\ 70 & 45 \\ 10 & 30 \end{pmatrix} & \begin{matrix} \text{Petrol} \\ \text{Carpark charges} \\ \text{Road pricing (ERP)} \end{matrix} \end{matrix}.$$

During weekends, the Tan family drives the weekend car and spends on average \$20, \$10 and \$2 on petrol, carpark charges and ERP respectively.

In a year, on average, both Mr Tan and Mrs Tan work for 48 weeks.

(i) Represent the average weekend car expenses of the Tan family by a matrix $\mathbb{R}$. [1]

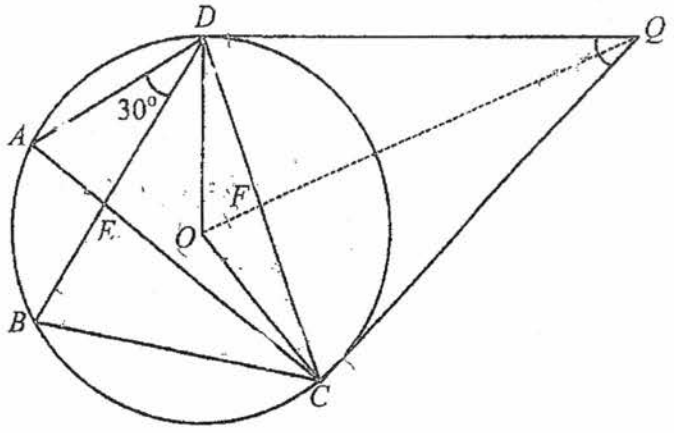
(ii) Evaluate $\mathbb{Q} = \mathbb{P}\begin{pmatrix} 1 \\ 1 \end{pmatrix}$ and $\mathbb{S} = 48\mathbb{Q} + 52\mathbb{R}$. [3]

(iii) State what the elements of $\mathbb{S}$ represent. [1]

(iv) The matrix $\mathbb{T}$ is given by $\mathbb{T} = \begin{pmatrix} 1 & 1 & 1 \end{pmatrix}\mathbb{S}$. Evaluate matrix $\mathbb{T}$ and describe in a sentence what the element(s) of the matrix $\mathbb{T}$ represent. [2]

(v) A recent credit card promotion entitles Mr and Mrs Tan 12.5% savings on petrol every time they pump petrol.

Calculate the new expenses for petrol, carpark charges and ERP for the Tan family in a year. [2]



In the diagram above, AEC and BED are chords of the circle with centre O. $\angle ADE = 30^\circ$ and $\angle CQD = 50^\circ$. CQ and DQ are tangents to the circle and F is the midpoint of chord CD.

(i) Explain why ΔADE is similar to ΔBCE. [2]

(ii) Name a pair of congruent triangles. [1]

(iii) Find, stating your reasons clearly,

(a) $\angle DAC$, [2]

(b) $\angle BEC$. [1]

(iv) Is it possible to draw a circle that passes through C, O, D and Q? Explain your answer clearly. [1]

8 **Answer the whole of this question on a single sheet of graph paper.**

The table below gives the values of x- and y-coordinates of some points on the graph of $y = \frac{ax}{x+b}$.

x	-0.5	0	1	2	3	4	5
y	-2	0	2	3	3.6	4	4.3

(a) By formulating two equations, find the values of a and b. [3]

(b) Using a scale of 2 cm to represent 1 unit on both the x-axis and y-axis, plot the points given in the table and join them with a smooth curve for $-0.5 \le x \le 5$. [3]

(c) By drawing a suitable tangent, find the gradient of the curve at the point $x = 1.5$. [2]

Using the values of a and b found in (a),

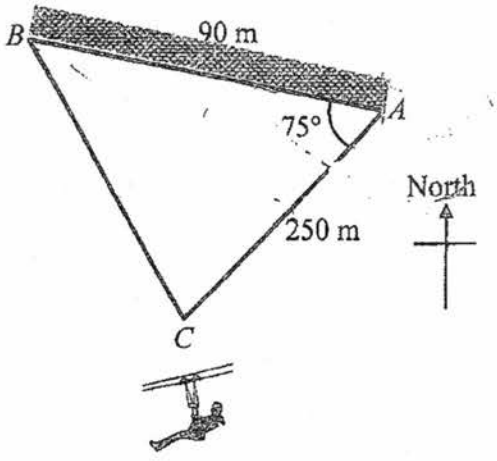
(d) find the solution(s) of the equation

$$\frac{ax}{x+b} = -\frac{1}{3}x + 1,$$

by drawing a suitable straight line on the same axes, [2]

(e) find the range of values of x such that $\frac{ax}{x+b} < 2.5$. [2]

9 Points A and B are points at the bottom a cliff 50 metres tall in height. Point C on a flat ground is 250 metres away from A with AB making an angle of $75°$ with the line AC. The bearing of C from A is $217°$ and A and B are 90 m apart.



Calculate the

(a) bearing of B from A, [1]

(b) area of the land formed by the points A, B and C, [2]

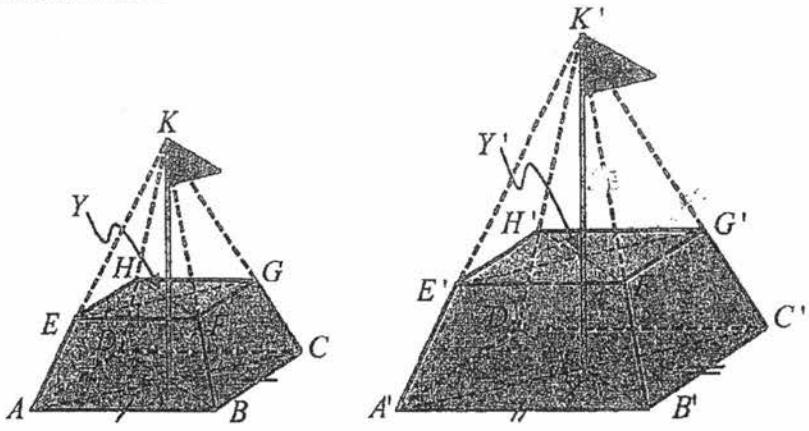
(c) shortest distance from C to the bottom of the cliff. [2]

An outdoor adventure company wants to build a flying fox using a metal cable with the starting point X on the cliff and the landing point at C.

(d) Find the distance away from B vertically below X such that the slope is the greatest. [2]

(e) Find the angle that the metal cable makes with the ground at point C. [2]

10 A company manufactures geometrically similar flagpole bases of two different sizes as shown below.



The bases are made of cement and are in the shape of truncated right pyramids. If each pyramid could be completed, its vertex would be the top of the flagpole at K and K' respectively. The height of the flagpole for the bigger-sized base is 2.5 metres and the ratio of the side length of the bottom surfaces $ABCD$ and $A'B'C'D'$ is $3:5$.

(a) The area of the bottom surface $A'B'C'D'$ is $2500\,\text{cm}^2$. What is the area of the bottom surface $ABCD$? [2]

(b) Given that $E'F' = F'G' = 40\,\text{cm}$, find the length $K'Y'$ and the volume of the base (**as represented by the shaded part**) for the bigger-sized flagpole base. [3]

(c) Hence, find the volume of the base for the smaller-sized flagpole base. [2]

(d) If it costs \$15 to buy a smaller-sized flagpole base and \$25 to buy a bigger-sized flagpole base, which flagpole base is more value for money? Explain with clear working. [2]

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11 The concert band of a school intends to rent a concert venue for their annual performance as their school hall is undergoing a renovation.

Information that the chairperson Peter and his committee need is on the opposite page.

As shown in **Figure 1**, seats in the concert hall are arranged along arcs of concentric circles of equal spacing. There are three rows of seats in front and one row of limited seats behind the stage.

(i) Show that angle $COD = 1.55$ radians and find the area taken up by the stage. [3]

(ii) Each normal concert chair takes up 80 cm of the arc. Show that row 1 can fit a maximum of 47 normal concert chairs. [2]

Peter and his committee decide that they will have a total of 3 rehearsals (including the rehearsal on the actual performance day) and a total of 30 VIP guests. They need to decide whether they should take up Package A or Package B of the concert hall rental offered by the venue management.

(iii) Assuming that Peter and his committee decide to charge \$20, \$15, \$12 and \$25 for Row 1, 2, 3 and 4 respectively, help Peter to decide which package he should take up. Justify the decision with clear calculations and assumption(s) so that Peter can present the proposal to his teacher-in-charge. [5]

Details of the stage

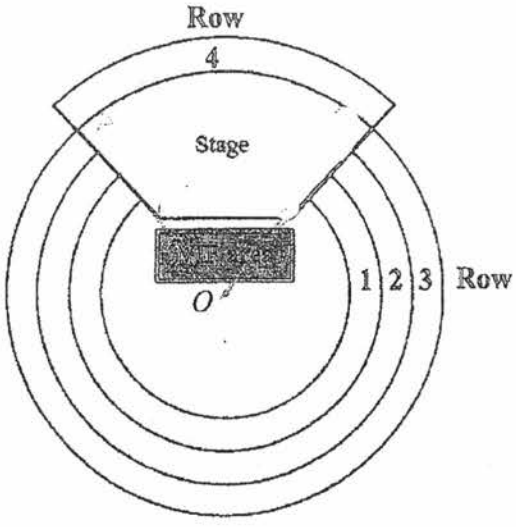


Figure 1

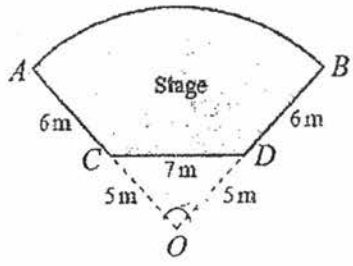


Figure 2

Cost of rental of items

Package	Details (All prices in this column are nett prices)	Cost of renting one normal concert chair (excluding 7% GST)	Cost of renting one VIP concert chair (excluding 7% GST)
A	▫ Basic rental cost: $2800 ▫ Freebies: ◦ Free 150 normal concert chairs ◦ Free 25 VIP chairs ◦ 1st rehearsal (unlimited time usage on day of event): $100 ◦ 2nd rehearsal: 20% off normal rehearsal price ◦ 3rd rehearsal and beyond: 10% off normal rehearsal price	$8	$18
B	▫ Basic rental cost: $1500 ▫ Freebies: ◦ Free 100 normal concert chairs ◦ Free 10 VIP chairs ▫ All rehearsals cost $120 each with unlimited time usage ▫ Terms and Condition: Row 4 cannot be opened for selling of tickets.	$12	$20

END OF PAPER

Sec 4E/5NA Prelims P2 Suggested Mark Scheme

Qn	Solution	
Algebra		
1(a)	$n^2+n=n(n+1)$ If n is odd, then $(n+1)$ is even. If n is even, then $(n+1)$ is odd. Product of an odd number and an even number is even. Thus $n(n+1)$ is even. **Alternative:** If n is odd, then n^2 is odd. Then sum of two odd numbers n and n^2 is even. If n is even, then n^2 is even. Then sum of two even numbers n and n^2 is even.	
(b)	$p^2-7p+12=0$ $(p-3)(p-4)=0$ $p=3$ or $p=4$ $q^4-7q^2+12=0$ Let $p=q^2$. $q^2=3\Rightarrow q=\pm\sqrt{3}$ or $q^2=4\Rightarrow q=\pm 2$	
(c) (i)	$40\,\text{cm}^2: 2.5\,\text{km}^2$ $1\,\text{cm}^2: 0.0625\,\text{km}^2$ $1\,\text{cm}: 0.25\,\text{km}$ $n=0.25$	
(ii)	Actual distance between the hospital and the village town $=30\times 0.25\,\text{km}$ $=7.5\,\text{km}$	
		Total for Q1: 9

Word problem and quadratic equations		
2(i)	Time taken to travel from home to the province if Mr Kia decides to drive = $\frac{400}{x}$ h.	
(ii)	Time taken to travel from home to the province if Mr Kia decides to take a domestic flight = $\frac{300}{x+250}$ h.	

(iii)	$\frac{400}{x}-\frac{300}{x+250}=\frac{7}{2}$ $400(x+250)-300(x)=\frac{7}{2}(x)(x+250)$ $400x+100000-300x=\frac{7}{2}x^2+875x$ $\frac{7}{2}x^2+775x-100000=0$ $7x^2+1550x-200000=0$ (shown)	
(iv)	$7x^2+1550x-200000=0$ $x=\frac{-1550\pm\sqrt{(1550)^2-4(7)(-200000)}}{14}$ $=\frac{-1550\pm\sqrt{8002500}}{14}$ $=94.919$ or -312.776 $=94.9$ or -312.8 (1d.p.)	
(v)	x must be positive, thus $x=94.919$ If Mr Kia drives, time taken $=\frac{400}{94.919}\text{h}=4.2141\text{h}$ 0947 hrs $\xrightarrow{13\text{ minutes}}$ 1000 hrs $\xrightarrow{4\text{ hr}}$ 1400 hrs He must leave home latest by 0947.	
		Total for Q2: 10

Probability

S	T	A	T	I	S	T	I	C	S

3S, 3T, A, 2I, C

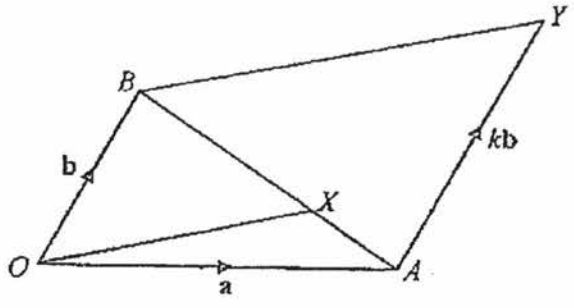
3(a) (i)	P(both cards show the letter T) $=\frac{3}{10}\times\frac{3}{10}$ $=\frac{9}{100}$	
(ii)	P(exactly one of the cards shows the letter T) $=\frac{3}{10}\times\frac{7}{10}+\frac{7}{10}\times\frac{3}{10}$ $=\frac{42}{100}=\frac{21}{50}$	
(b) (i)		

Age (x years)		$20\le x<30$	$30\le x<40$	$40\le x<50$
Frequency	Men	375	186	99
	Women	250	122	68

Probability that the person is a man aged less than 40 years old

	$=\frac{375+186}{1100}$ $=\frac{561}{1100}$ $=\frac{51}{100}$	
(ii)	Probability that both of them are women aged 30 or more $=\frac{122+68}{1100}\times\frac{122+68-1}{1099}$ $=\frac{190}{1100}\times\frac{189}{1099}$ $=\frac{513}{17270}=0.0297$	
		Total for Q3: 7

Vectors



4(i)	$\overrightarrow{AX}=\frac{1}{3}\overrightarrow{AB}=\frac{1}{3}(\mathbf{b}-\mathbf{a})$ $\overrightarrow{OX}-\overrightarrow{OA}=\frac{1}{3}(\mathbf{b}-\mathbf{a})$ $\overrightarrow{OX}=\frac{1}{3}(\mathbf{b}-\mathbf{a})+\mathbf{a}=\frac{1}{3}(2\mathbf{a}+\mathbf{b})$	
(ii)	$\overrightarrow{BY}=-\mathbf{b}+\mathbf{a}+k\mathbf{b}=\mathbf{a}+(k-1)\mathbf{b}$	
(iii)	OX is parallel to $BY \Rightarrow m\overrightarrow{OX}=\overrightarrow{BY}$ $\frac{1}{3}m(2\mathbf{a}+\mathbf{b})=\mathbf{a}+(k-1)\mathbf{b}$ $\begin{cases}\frac{2}{3}m=1\Rightarrow m=\frac{3}{2}\\ \frac{1}{3}m=k-1\Rightarrow k=1+\frac{1}{3}(\frac{3}{2})=\frac{3}{2}\end{cases}$	
(iv)	$\overrightarrow{AZ}=\frac{1}{2}\mathbf{b}$ since $\overrightarrow{OZ}=\overrightarrow{BY}$ and $\overrightarrow{OB}=\overrightarrow{ZY}$.	
(v)(a)	$\frac{\text{area of }\Delta OAX}{\text{area of }\Delta OBX}=\frac{AX}{BX}=\frac{(\frac{1}{3})}{(\frac{2}{3})}=\frac{1}{2}$	
(b)	$\frac{\text{area of }\Delta AXZ}{\text{area of }\Delta ABY}=\left(\frac{1}{3}\right)^2=\frac{1}{9}$ $\frac{\text{area of }\Delta AXZ}{\text{area of quadrilateral }XBYZ}=\frac{1}{8}$	
		Total for Q4: 9

Number patterns

n	Series	Sum	Formula
1	1	1	$\frac{1}{2}(1)(1+1)$
2	$1+2$	3	$\frac{1}{2}(2)(2+1)$
3	$1+2+3$	6	$\frac{1}{2}(3)(3+1)$
4	$1+2+3+4$	10	$\frac{1}{2}(4)(4+1)$
⋮	⋮	⋮	⋮
6	$1+2+3+4+5+6$	a	b
⋮	⋮	⋮	⋮
n	$1+2+3+\cdots+n$	c	

5(i) $b=\frac{1}{2}(6)(6+1)=21$

$a=21$

(ii) $c=\frac{1}{2}(n)(n+1)$

(iii)(a) $1^3+2^3+3^3+4^3+5^3+6^3=21^2=441$

(b) $1^3+2^3+3^3+\cdots+n^3=\left[\frac{1}{2}n(n+1)\right]^2$

(iv)

$$3^3+6^3+9^3+12^3+\cdots+300^3$$
$$=(3\times1)^3+(3\times2)^3+(3\times3)^3+(3\times4)^3+\cdots+(3\times100)^3$$
$$=3^3\left[1^3+2^3+3^3+4^3+\cdots+100^3\right]$$
$$=27\left[\frac{1}{2}(100)(101)\right]^2$$
$$=688567500$$

Total for Q5: 7

Matrices

6(a)

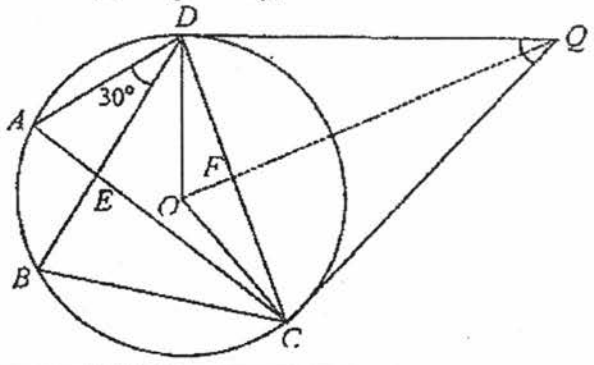
	Matrix	Order	Name of matrix
(i)	$\begin{pmatrix}2\\5\\12\end{pmatrix}$	3×1 **OR** 3 by 1	Column matrix
(ii)	$\begin{pmatrix}0&0\\0&0\end{pmatrix}$	2×2 **OR** 2 by 2	Square matrix **OR** Null matrix **OR** Zero matrix

(b)(i) $\mathbf{R}=\begin{pmatrix}20\\10\\2\end{pmatrix}$

(ii) Given $\mathbf{P}=\begin{pmatrix}150&80\\70&45\\10&30\end{pmatrix}$

	$\mathbf{Q} = \mathbf{P}\begin{pmatrix}1\\1\end{pmatrix} = \begin{pmatrix}150 & 80\\70 & 45\\10 & 30\end{pmatrix}\begin{pmatrix}1\\1\end{pmatrix} = \begin{pmatrix}230\\115\\40\end{pmatrix}$ $\mathbf{S} = 48\mathbf{Q} + 52\mathbf{R} = 48\begin{pmatrix}230\\115\\40\end{pmatrix} + 52\begin{pmatrix}20\\10\\2\end{pmatrix} = \begin{pmatrix}12080\\6040\\2040\end{pmatrix}$	
(iii)	The elements 12080, 6040 and 2040 represent the Tan family's yearly car expenses on petrol, carpark charges and ERP respectively.	
(iv)	$\mathbf{T} = \begin{pmatrix}1 & 1 & 1\end{pmatrix}\mathbf{S}$ $= \begin{pmatrix}1 & 1 & 1\end{pmatrix}\begin{pmatrix}12080\\6040\\2040\end{pmatrix} = (12080+6040+2040)$ $= (20160)_{1\times1}$ It represents the Tan family's total car expenses in a year.	
(v)	<u>Method 1:</u> New yearly expenses for petrol = $0.875\times12080 = \$10570$ carpark charges = \$6040 ERP = \$2040 <u>Method 2:</u> Given $\mathbf{P}_{new} = \begin{pmatrix}131.25 & 70\\70 & 45\\10 & 30\end{pmatrix}$ $\mathbf{Q}_{new} = \mathbf{P}_{new}\begin{pmatrix}1\\1\end{pmatrix} = \begin{pmatrix}131.25 & 70\\70 & 45\\10 & 30\end{pmatrix}\begin{pmatrix}1\\1\end{pmatrix} = \begin{pmatrix}201.25\\115\\40\end{pmatrix}$ $\mathbf{S}_{new} = 48\mathbf{Q}_{new} + 52\mathbf{R}_{new} = 48\begin{pmatrix}201.25\\115\\40\end{pmatrix} + 52\begin{pmatrix}17.5\\10\\2\end{pmatrix} = \begin{pmatrix}10570\\6040\\2040\end{pmatrix}$	
		Total for Q6: 11

Circle Properties (& similarity, congruency)



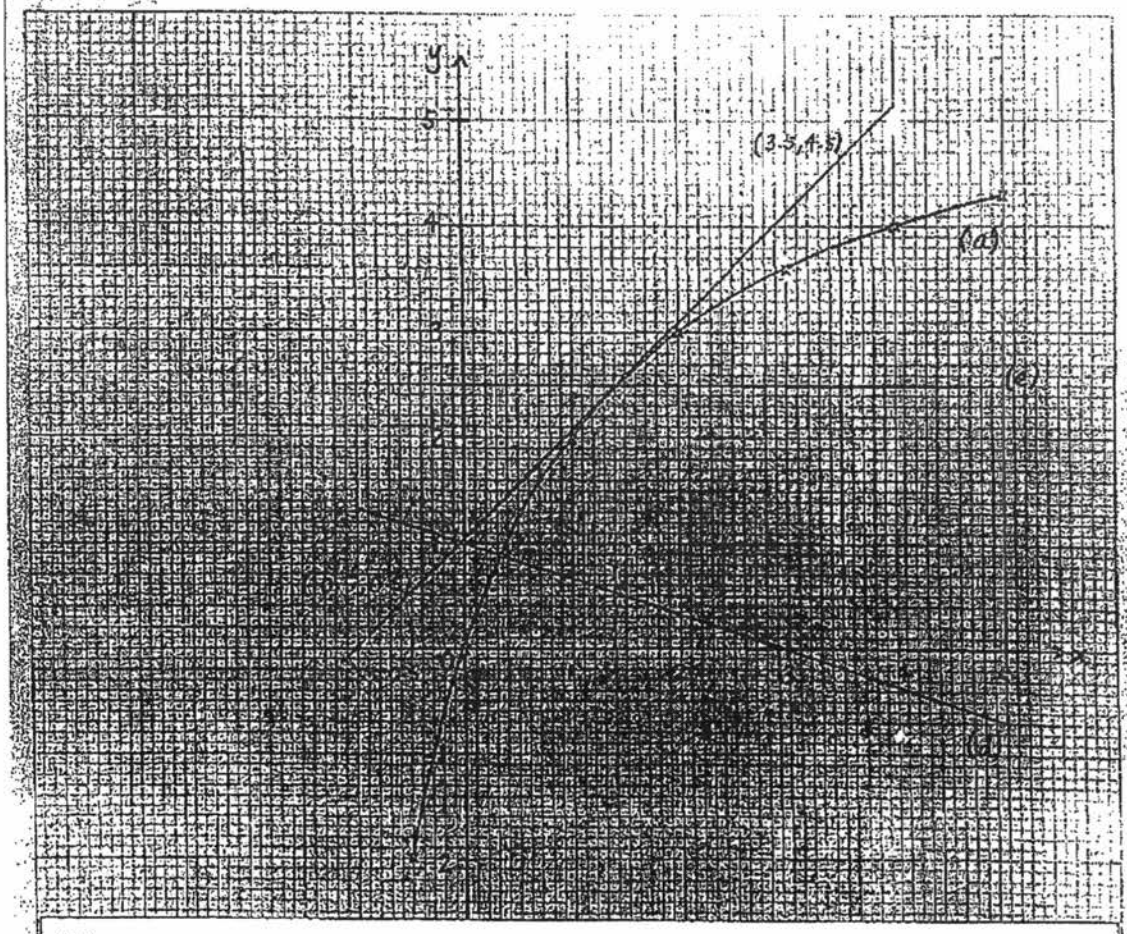
7(i)	$\angle ADE = \angle BCE$ (angle in the same segment) $\angle DAE = \angle CBE$ (angle in the same segment) By the **AA similarity test**, ΔADE is similar to ΔBCE.	
(ii)	Any of the following answers: ▫ ΔDOF is congruent to ΔCOF **OR** ▫ ΔDOQ is congruent to ΔCOQ **OR** ▫ ΔDFQ is congruent to ΔCFQ	
(iii) (a)	$\angle ODQ = \angle OCQ = 90°$ **(tan ⊥ rad)** $\angle DOQ = 360° - 90° - 90° - 50° = 130°$ $\angle DAC = \frac{1}{2}\angle DOC$ $= 65°$ **(angle at centre = twice angle at circumference)**	
(b)	$\angle BEC = \angle AED$ (vertically opposite angles) $= 180° - 30° - 65°$ ($\angle$ sum of triangle) $= 85°$	
(iv)	It is possible to draw a circle that passes through C, O, D and Q since $\angle ODQ = \angle OCQ = 90°$ **(tan ⊥ rad) and angle in a semicircle**. In this case, OQ is the diameter of the circle.	
		Total for Q7: 7

Graph

8(a)	$y = \frac{ax}{x+b}$ (1,2): $2 = \frac{a}{1+b} \Rightarrow a - 2b = 2$ (2,3): $3 = \frac{2a}{2+b} \Rightarrow 2a - 3b = 6$ Solving the two equations simultaneously, $a = 6, b = 2$	
(b)	Refer to graph on page 7.	
(c)		
(d)		
(e)		
		Total for Q8: 12

Name: _____

Subject: _____



(c)

Gradient at $x = 1.5$

$$= \frac{4.5 - 0.5}{3.5 - (-0.5)}$$

$$= \frac{4}{4}$$

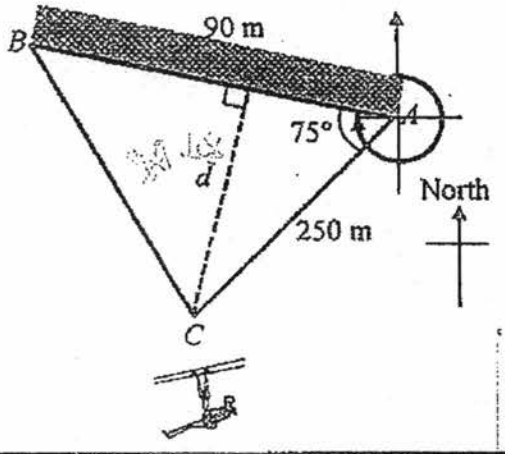
$$= 1$$

(d) $x = 0.35$

(e) $-0.5 \leq x < 1.4$

Trigonometry

9(a) Bearing of B from $A = 217° + 75° = 292°$



(b) Area of the land formed by the points A, B and C

$= \frac{1}{2}(90)(250)\sin 75°$

$= 10866.67$

$= 10900\ \text{m}^2$ (3 s.f.)

(c) Shortest distance from C to the bottom of the cliff

$= d$

$= 250\sin 75°$

$= 241.48$

$= 241\ \text{m}$ (3 s.f.)

(d) Slope is greatest when angle of elevation is the greatest from C.

Distance away from B

$= 90 - 250\cos 75°$

$= 25.295$

$= 25.3\ \text{m}$ (3 s.f.)

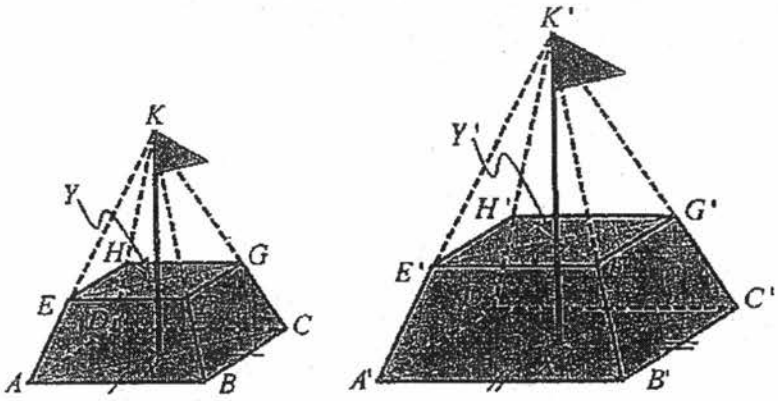
(e) Required angle

$= \tan^{-1}\left(\frac{50}{241.48}\right)$

$= 11.7°$ (1 d.p.)

Total for Q9: 9

Mensuration and similarity involving areas and volumes



10 (a)

$$\frac{\text{area of } ABCD}{\text{area of } A'B'C'D'} = \left(\frac{3}{5}\right)^2$$

$$\frac{\text{area of } ABCD}{2500} = \left(\frac{3}{5}\right)^2$$

$$\Rightarrow \text{area of } ABCD = \frac{9}{25} \times 2500 = 900 \text{ cm}^2$$

(b)

$$\frac{K'Y'}{K'X'} = \frac{40}{50} \Rightarrow K'Y' = \frac{4}{5} \times 2.5 = 2 \text{ m}$$

$$\text{Volume of the base} = \frac{1}{3} \times 2500 \times 250 - \frac{1}{3} \times 1600 \times 200$$

$$= 101666\frac{2}{3} \text{ cm}^3$$

(c)

$$\frac{\text{volume of smaller base}}{\text{volume of bigger base}} = \left(\frac{3}{5}\right)^3$$

$$\frac{\text{volume of smaller base}}{101666\frac{2}{3}} = \left(\frac{3}{5}\right)^3$$

$$\Rightarrow \text{volume of smaller base} = 21960 \text{ cm}^3$$

(d)

1 cm^3 of the smaller base costs $\$\frac{15}{21960} \approx \$0.000683$

1 cm^3 of the bigger base costs $\$\frac{25}{101666\frac{2}{3}} \approx \$0.000246$

Since 1 cm^3 of the bigger base costs cheaper, the bigger-sized flagpole base is more value for money.

Total for Q10: 9

PRWC – Arc length and area of sector, segment

Details of the stage

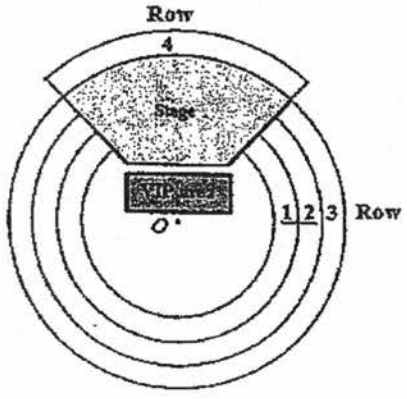


Figure 1

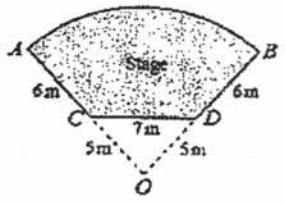


Figure 2

11 (i)

$$\angle COD = \cos^{-1}\left(\frac{5^2+5^2-7^2}{2(5)(5)}\right)$$

$$= \cos^{-1}\left(\frac{1}{50}\right)$$

$$= 1.5508 \text{ rad}$$

$$= 1.55 \text{ rad (3 s.f.)} \quad \text{(shown)}$$

$$\text{Area of stage} = \frac{1}{2}(11)^2(1.5508) - \frac{1}{2}(5)(5)\sin 1.5508$$

$$= 81.326 \text{ m}^2$$

$$= 81.3 \text{ m}^2$$

(ii) Length of first row

$$= (5+3)(2\pi - 1.5508)$$

$$= 37.859 \text{ m}$$

Number of normal concert seats that can be accommodated in the first row $= \frac{37.859 \text{ m}}{0.80 \text{ m}} = 47.324 \approx 47$ (shown)

(iii)

Row	Length of row	No. of normal concert chairs	Cost of earnings
1	37.859 m	47	47×20=940
2	44.958 m	56	56×15=840
3	52.056 m	65	65×12=780
4	12.5(1.5 =19.38		24×25=600
Total earnings if package A taken up			$3160
Total earnings if package B taken up			$2560

Total number of normal concert chairs needed
= 47 + 56 + 65 + 24 = 192

Package	Cost of renting VIP chairs	Cost of renting normal concert chairs	Cost of rehearsals
A	5×18×1.07 = $96.30	(192−150)×8 ×1.07 = $359.52	100+80+90 = $270
Total cost for using package A = $2800 + 96.30 + 359.52 + 270 = $3525.82			
B	20×20×1.07 = $428	(192−100)×12 ×1.07 = $1181.28	120×3 = $360
Total cost for using package B = $1500 + 428 + 1181.28 + 360 = $3469.28			

Profit after taking up package A
= $3160 − 3525.82
= −$365.82

Profit after taking up package B
= $2560 − 3469.28
= −$909.28

Although package B seems cheaper than package A, taking into consideration the earnings, package A has a smaller loss than package B. Thus Peter and his committee should take up package A.

Assumptions:
- Other factors are not taken into consideration. The decision is made purely based on the profit made.

Total for Q11: 10