

Answer:

1(a) $-2x + 2$ (b) $\frac{2}{5}x^{\frac{15}{2}}$

2(a) $7.1 \times 10^8 \text{ m}^2$ (b) $m = 0, n = -2$

3(a) $\frac{2}{3}, -1$ (b) $\frac{5}{9}, 0$

4 9

5(a) $2^3 \times 3 \times 5 \times 7^2$ (b) 40 (c) 22

6(a) $4 : 13$ (b) 67.4

7(a) $(2x - 7y)^2$ (b) $x = \frac{7y}{2}$

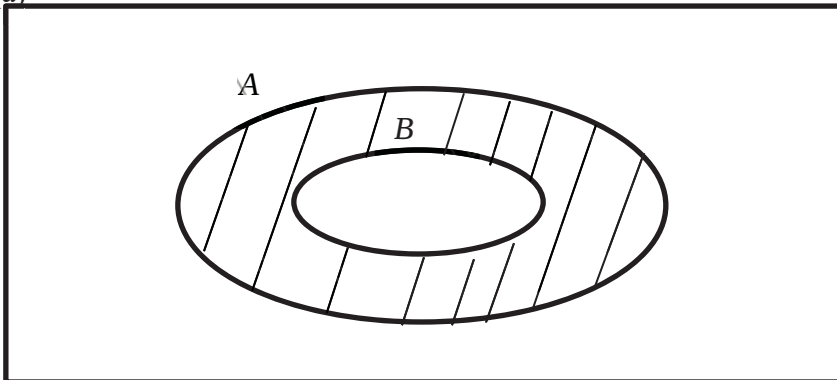
8(a) 12 (b) 18 (c) 7

9(a) $1 : 250\,000$ (b) 1.5 cm

10(a) 480 cm^3 (b) 45 cm^3

11(a) any negative odd integer (b) any exponential equation

12(a)



(b) $\{1, 27\}$

13(a) $\frac{1}{5}$ (b) 15

14(a) $\binom{8}{10}$ (b) $\binom{\frac{47}{6}}{\frac{1}{2}}$ (c) $8\frac{1}{3} \text{ km}$

15 Mean = 153 g, SD = 4.29 g

16 SD of Fati = 0.816, SD of Bala = 1.73, Mean of Fati = Mean of Bala = 2

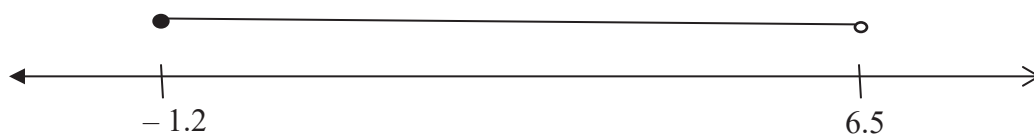
Select Fati as he is more consistent in his score.

17(a) 10 units (b) $\begin{pmatrix} 12 \\ 10 \end{pmatrix}$

18 4 : 1

19 10, -4

20(a)



(b) -1

21 \$8494.10

22(a) $\frac{9}{17}, \frac{8}{17}, \frac{9}{17}, \frac{8}{17}, \frac{9}{16}, \frac{7}{16}$ (b) $\frac{7}{34}$

23 Scale was not state in the question.

Height of similey face or area of similey face could be used to represent the readings from each year which may lead to misinterpretation of data.

24(a) $\angle EAD = \angle BAC$ (common angle)

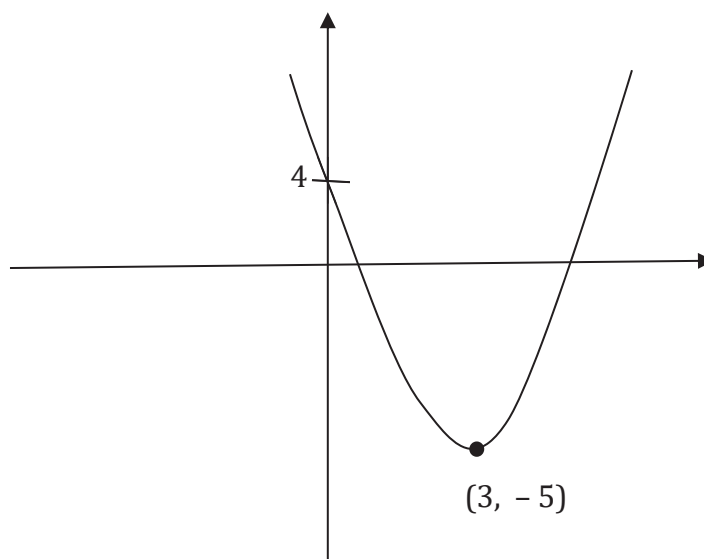
$\angle ADE = \angle ACB$ (given)

Triangle ADE is similar to triangle ACB (AA similarity test)

(b) 10

(c) 1 : 3

25(a) $(x - 3)^2 - 5$ (b)





ST. MARGARET'S SECONDARY SCHOOL

Mid-Year Examinations 2019

CANDIDATE NAME

CLASS

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REGISTER NUMBER

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MATHEMATICS

4048/02

Paper 2

6 May 2019

Secondary 4 Express

2 hours 30 minutes

Additional Materials: Writing papers
Graph paper (1 sheet)

READ THESE INSTRUCTIONS FIRST

Write your name, registration number and class on all the work you hand in.

Write in **dark blue** or **black pen**.

You may use a soft pencil for any diagrams or graphs.

Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.

If working is needed for any question it must be shown with the answer.

Omission of **essential working** will result in loss of marks.

The use of an **approved scientific** calculator is expected, where appropriate.

If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.

For π , use either your calculator value or 3.142, unless the question requires the answer in terms of π .

At the end of the examination, fasten all your work securely together.

The number of marks is given in brackets [] at the end of each question or part question.

The total of the marks for this paper is 100.

This document consists of 22 printed pages

Mathematical Formulae

Compound Interest

$$\text{Total amount} = P \left(1 + \frac{r}{100} \right)^n$$

Mensuration

$$\text{Curved surface area of a cone} = \pi r l$$

$$\text{Surface area of a sphere} = 4\pi r^2$$

$$\text{Volume of a cone} = \frac{1}{3}\pi r^2 h$$

$$\text{Volume of a sphere} = \frac{4}{3}\pi r^3$$

$$\text{Area of triangle } ABC = \frac{1}{2}ab \sin C$$

$$\text{Arc length} = r\theta, \text{ where } \theta \text{ is in radians}$$

$$\text{Sector area} = \frac{1}{2}r^2\theta, \text{ where } \theta \text{ is in radians}$$

Trigonometry

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

Statistics

$$\text{Mean} = \frac{\sum fx}{\sum f}$$

$$\text{Standard deviation} = \sqrt{\frac{\sum fx^2}{\sum f} - \left(\frac{\sum fx}{\sum f} \right)^2}$$

1 The graph $y = (x - 1)(x + 3)$ cuts the x -axis at A and B and the y -axis at C .

(a) Find the coordinates of C .

[1]

$$y = (x - 1)(x + 3)$$

At C , $x = 0$,

$$y = (0 - 1)(0 + 3)$$

$$= -3$$

$\therefore$ Coordinates of C are $(0, -3)$.

(b) Find the coordinates of M , the minimum point of the curve.

[2]

When $y = 0$, $x = -3, 1$

$$x\text{-coordinate of } M = \frac{-3+1}{2}$$

$$= -1$$

$$y\text{-coordinate of } M = (-1 - 1)(-1 + 3)$$

$$= -4$$

$\therefore$ Coordinates of M are $(-1, -4)$

Hence,

(c) (i) find the equation of the line MC , and

[2]

$$M(-1, -4), \quad C(0, -3)$$

$$\text{Gradient } MC = \frac{-4+3}{-1-0}$$

$$= 1$$

$$\text{Equation of } MC : y = x - 3$$

(ii) the length of the line joining M to C .

[2]

$$M(-1, -4), \quad C(0, -3)$$

$$\text{Length of } MC = \sqrt{(0 + 1)^2 + (-3 + 4)^2}$$

$$= \sqrt{2}$$

$$= 1.41 \text{ units}$$

2 (a) Factorise completely

(i) $2p^2 + 6p - pq - 3q$, [2]

$$\begin{aligned} & 2p^2 + 6p - pq - 3q \\ &= 2p(p + 3) - q(p + 3) \\ &= (p + 3)(2p - q) \end{aligned}$$

(ii) $24 - 54x^2$. [2]

$$\begin{aligned} & 24 - 54x^2 \\ &= 6(4 - 9x^2) \\ &= 6(2 + 3x)(2 - 3x) \end{aligned}$$

(b) Express $\frac{y-1}{y^2-1} - \frac{2}{1-y}$ as a single fraction in its simplest form. [3]

$$\begin{aligned} & \frac{y-1}{y^2-1} - \frac{2}{1-y} \\ &= \frac{y-1}{(y+1)(y-1)} + \frac{2}{y-1} \\ &= \frac{y-1+2(y+1)}{(y+1)(y-1)} \\ &= \frac{3y+1}{(y+1)(y-1)} \end{aligned}$$

- (c) Given that $b = a + \frac{c}{x}$ and $d = \frac{c}{b}$, express x in terms of a , c and d . [3]

$$b = a + \frac{c}{x} \text{ ----- (1)}$$

$$d = \frac{c}{b} \text{ ----- (2)}$$

From (2) $b = \frac{c}{d}$

Substitute into (1)

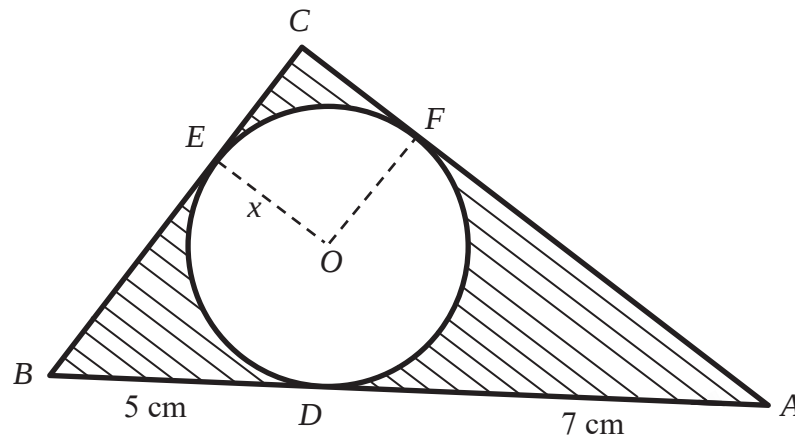
$$\frac{c}{d} = a + \frac{c}{x}$$

$$\frac{c}{x} = \frac{c}{d} - a$$

$$\frac{c}{x} = \frac{c-ad}{d}$$

$$x = \frac{cd}{c-ad}$$

- 3 In the diagram, the circle of radius x cm with centre O is touching the sides of the right angled triangle ABC at D, E, F , where angle ACB is 90° .



- (a) Find the length of CE in terms of x . State the circle **properties** used in your working.

[2]

$\angle ACB = 90^\circ$ (given)
 $\Rightarrow \angle ECO = 45^\circ$ (tangents from external point)
 $\angle CEO = 90^\circ$ (tangent is perpendicular to radius)
 $\Rightarrow \angle EOC = 45^\circ$ (angle sum of triangle)

$\angle ECO = \angle EOC \Rightarrow$ triangle CEO is isosceles

Hence $CE = OE$
 $= x$

- (b) Given also that $AD = 7$ cm and $BD = 5$ cm, form an equation in x , and show that it simplifies to $x^2 + 12x - 35 = 0$.

[3]

$CF = CE$ (tangents from external point)
 $= x$

By Pythagoras' Theorem

$$\begin{aligned}
 (5 + x)^2 + (7 + x)^2 &= (5 + 7)^2 \\
 25 + 10x + x^2 + 49 + 14x + x^2 &= 144 \\
 2x^2 + 24x - 70 &= 0 \\
 x^2 + 12x - 35 &= 0 \text{ (shown)}
 \end{aligned}$$

- (c) Solve the equation $x^2 + 12x - 35 = 0$, giving both answers correct to 2 decimal places. [3]

$$\begin{aligned}
 x^2 + 12x - 35 &= 0 \\
 x &= \frac{-12 \pm \sqrt{12^2 - 4(1)(-35)}}{2(1)} \\
 &= \frac{-12 \pm \sqrt{284}}{2} \\
 &= 2.426 \quad \text{or} \quad -14.426 \\
 &= 2.43 \quad \text{or} \quad -14.43 \quad (\text{to 2 d.p.})
 \end{aligned}$$

- (d) Hence find the shaded area. [2]

$$\begin{aligned}
 \text{Since } x > 0, \therefore x &= 2.426 \\
 \text{Shaded area} &= \frac{(5+2.426)(7+2.426)}{2} - \pi(2.426)^2 \\
 &= 16.5 \text{ cm}^2 \text{ (to 3 s.f.)}
 \end{aligned}$$

- 4 4 toothpicks are arranged to form a square. The diagram shows the first three of a sequence of figures that are formed. All the squares are of the same size.



Figure 1

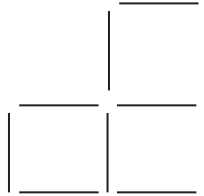


Figure 2

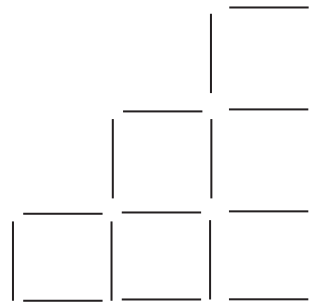


Figure 3

Figure Number (n)	Number of vertical toothpicks (V)	Total number of toothpicks (S)
1	2	4
2	5	10
3	9	18
$\vdots$	$\vdots$	$\vdots$
7	v	s

The number of vertical sides (V) and the total number of sides (S) are recorded in the table above.

- (a) Find the value of v and of s .

[2]

$$\begin{array}{ccccccc}
 2 & \frown & 5 & \frown & 9 & \frown & 14 & \frown & 20 & \frown & 27 & \frown & 35 \\
 3 & & 4 & & 5 & & 6 & & 7 & & 8
 \end{array}$$

$$\begin{array}{ccccccc}
 4 & \frown & 10 & \frown & 18 & \frown & 28 & \frown & 40 & \frown & 54 & \frown & 70 \\
 6 & & 8 & & 10 & & 12 & & 14 & & 16
 \end{array}$$

$$\therefore v = 35, \quad s = 70$$

- (b) Write down a formula that shows the relationship between S and n .

[1]

$$n = 1, s = (1)(1 + 3) = 4; \quad n = 2, s = (2)(2 + 3) = 10$$

$$\therefore s = n(n + 3)$$

- (c) If the total number of sides is 700, find the value of n . [2]

$$n(n + 3) = 700$$

$$n^2 + 3n - 700 = 0$$

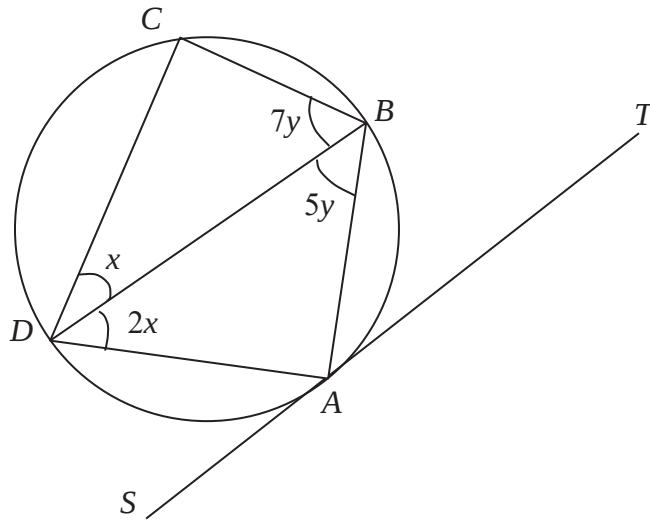
$$(n + 28)(n - 25) = 0$$

$$n = -28 \text{ is rejected.} \quad \therefore n = 25$$

- (d) Give a reason why the value of S cannot be a prime number. [1]

Values of S start from 4 and are even numbers, hence it cannot be a prime number.

- 5 In the diagram, DB is a diameter of the circle and ST is a tangent to the circle at A .
 $\angle BDA = 2x$, $\angle BDC = x$, $\angle CBD = 7y$ and $\angle DBA = 5y$.



- (a) Explain why angle $BCD = 90^\circ$. [1]

Since BD is a diameter, by “right angle in a semi-circle” property, $\angle BCD = 90^\circ$.

- (b) Find the value of x and of y . [4]

$$x + 2x + 7y + 5y = 180^\circ \text{ (angles in opposite segments)}$$

$$\Rightarrow x + 4y = 60^\circ \text{ ----- (1)}$$

$$x + 7y + 90^\circ = 180^\circ \text{ (angle sum of triangle)}$$

$$\Rightarrow x + 7y = 90^\circ \text{ ----- (2)}$$

$$(2) - (1) \quad 3y = 30^\circ$$

$$y = 10^\circ$$

$$\text{Substitute into (1)} \quad x = 60^\circ - 4(10^\circ)$$

$$= 20^\circ$$

(c) Given that M is the mid-point of BD ,

(i) determine if $\angle MAT$ is a right angle. Explain your answer. [1]

M is the mid-point of BD

$\Rightarrow M$ is the centre of the circle

$\Rightarrow MA$ is a radius of the circle

Since ST is a tangent, by “tangent is perpendicular to radius” property

$\angle MAT$ is a right angle.

(ii) Find $\angle DMA$. [2]

$\triangle DMA$ is isosceles, hence

$$\angle MAD = \angle MDA$$

$$= 2(20^\circ)$$

$$= 40^\circ$$

$$\angle DMA = 180^\circ - 2 \times 40^\circ \text{ (angle sum of triangle)}$$

$$= 100^\circ$$

- 6 (a) Find the values of the unknown in each of the following.

$$\begin{pmatrix} 1 & 0 & 4 \\ 5 & 2 & -3 \end{pmatrix} \begin{pmatrix} 1 \\ 2 \\ a \end{pmatrix} = \begin{pmatrix} 8 \\ b \end{pmatrix}. \quad [3]$$

$$\begin{pmatrix} 1 + 0 + 4a \\ 5 + 4 - 3a \end{pmatrix} = \begin{pmatrix} 8 \\ b \end{pmatrix}$$

$$1 + 4a = 8$$

$$a = 1.75$$

$$b = 5 + 4 - 3(1.75) \\ = 3.75$$

- (b) If $\mathbf{P} = \begin{pmatrix} 1 & 2 \\ 4 & 3 \end{pmatrix}$ and $\mathbf{Q} = \begin{pmatrix} -2 & 1 \\ 5 & 9 \end{pmatrix}$, find the matrix $\mathbf{P}^2 + 3\mathbf{Q}$. [3]

$$\mathbf{P}^2 + 3\mathbf{Q}$$

$$= \begin{pmatrix} 1 & 2 \\ 4 & 3 \end{pmatrix} \begin{pmatrix} 1 & 2 \\ 4 & 3 \end{pmatrix} + 3 \begin{pmatrix} -2 & 1 \\ 5 & 9 \end{pmatrix}$$

$$= \begin{pmatrix} 9 & 8 \\ 16 & 17 \end{pmatrix} + \begin{pmatrix} -6 & 3 \\ 15 & 27 \end{pmatrix}$$

$$= \begin{pmatrix} 3 & 11 \\ 31 & 44 \end{pmatrix}$$

- (c) The following table shows the working hours of 3 clerks in a week.

	Regular Working Hours	Overtime
Fatimah	40	4
Vani	45	3
June	38	7

The hourly wages of these clerks are \$20 for regular working hours and \$30 for overtime.

- (i) Represent the working hours of the clerks by a matrix **A** and the hourly wage rates by a matrix **B** such that **AB** exists. [2]

$$\mathbf{A} = \begin{pmatrix} 40 & 4 \\ 45 & 3 \\ 38 & 7 \end{pmatrix}, \quad \mathbf{B} = \begin{pmatrix} 20 \\ 30 \end{pmatrix}$$

- (ii) Find **AB**. [2]

$$\begin{pmatrix} 40 & 4 \\ 45 & 3 \\ 38 & 7 \end{pmatrix} \begin{pmatrix} 20 \\ 30 \end{pmatrix} = \begin{pmatrix} 800 + 120 \\ 900 + 90 \\ 760 + 210 \end{pmatrix} \\ = \begin{pmatrix} 920 \\ 990 \\ 970 \end{pmatrix}$$

- (iii) Explain what your answer in (ii) represents. [1]

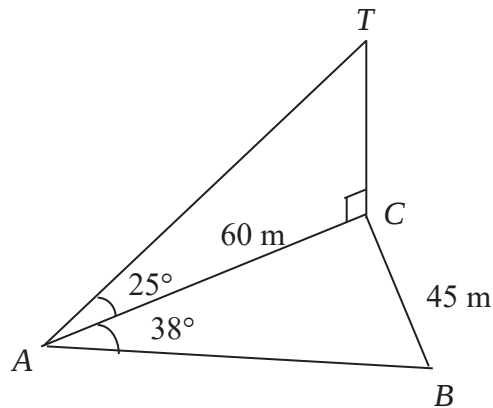
The elements in **AB** represent the wages earned by Fatimah, Vani and June respectively.

- (iv) The hourly wage for regular working hours increased by 20% and overtime increased by 15% respectively, but the working hours remain the same. Calculate the increase in the company's expense in wages paid to the three clerks using matrix multiplication. [3]

$$\begin{pmatrix} 1 & 1 & 1 \end{pmatrix} \begin{pmatrix} 40 & 4 \\ 45 & 3 \\ 38 & 7 \end{pmatrix} \begin{pmatrix} 4 \\ 4.5 \end{pmatrix} \\ = \begin{pmatrix} 123 & 14 \end{pmatrix} \begin{pmatrix} 4 \\ 4.5 \end{pmatrix} \\ = (555)$$

The increase in wages paid is \$555.

7 (a)



In the above diagram, C represents the foot of a vertical tower CT . The points A , B and C are on horizontal ground, where angle $CAB = 38^\circ$, $AC = 60$ m and $BC = 45$ m.

Given that the angle of elevation of T from A is 25° , calculate

- (i) the height of the tower, [2]

$$\tan 25^\circ = \frac{CT}{60}$$

$$CT = 60 \tan 25^\circ$$

$$= 27.978$$

$$= 28.0 \text{ (to 3 s.f.)}$$

Height of tower is 28.0 m

- (ii) angle ABC , [2]

$$\frac{\sin \angle ABC}{60} = \frac{\sin 38^\circ}{45}$$

$$\angle ABC = \sin^{-1} \frac{60 \sin 38^\circ}{45}$$

$$= 55.173^\circ$$

$$= 55.2^\circ \text{ (to 1 d.p.)}$$

- (iii) the angle of depression of B from T . [2]

Let θ be the angle of depression

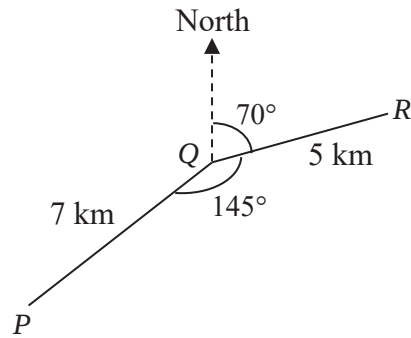
$$\tan \theta = \frac{27.978}{45}$$

$$\theta = \tan^{-1} \frac{27.978}{45}$$

$$= 31.870^\circ$$

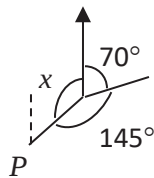
$$= 31.9^\circ \text{ (to 1 d.p.)}$$

(b)



A ship sails 7 km from P to Q . It then sails 5 km from Q to R on a bearing of 070° . Given that angle $PQR = 145^\circ$, calculate

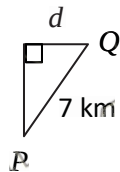
- (i) the bearing of Q from P , [2]



$$\begin{aligned} x &= 360^\circ - 70^\circ - 145^\circ \\ &= 145^\circ \end{aligned}$$

$$\begin{aligned} \text{Bearing of } Q \text{ from } P \\ &= 180^\circ - 145^\circ \\ &= 035^\circ \end{aligned}$$

- (ii) how far Q is east of P , [2]



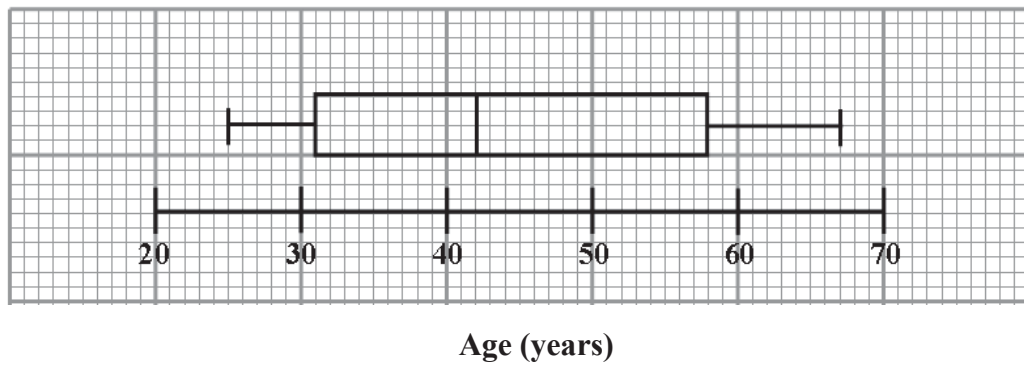
$$\begin{aligned} \sin 35^\circ &= \frac{d}{7} \\ d &= 7 \sin 35^\circ \\ &= 4.01503 \\ &= 4.02 \text{ km} \end{aligned}$$

Ans: Q is 4.20 km east of P .

- (iii) the distance PR . [2]

$$\begin{aligned} PR^2 &= 7^2 + 5^2 - 2(7)(5) \cos 145^\circ \\ PR &= \sqrt{74 - 70 \cos 145^\circ} \\ &= 11.5 \text{ km (to 3 s.f.)} \end{aligned}$$

- 8 (a) The box-and-whisker plot below shows the distribution of the ages (in years) of 50 members from the Rainbow Country Club.



- (i) State the median and find the interquartile range. [3]

Median = 42 years

Interquartile range = $58 - 31$
 $= 27$ years

- (ii) The table below shows the ages of 50 members from Sunshine Country Club.

Age (years)	30	35	40	45	50	55	60
Number of Members	8	7	15	8	6	4	2

Find the median and the interquartile range. [3]

Median = 40 years

Interquartile range = $45 - 35$
 $= 10$ years

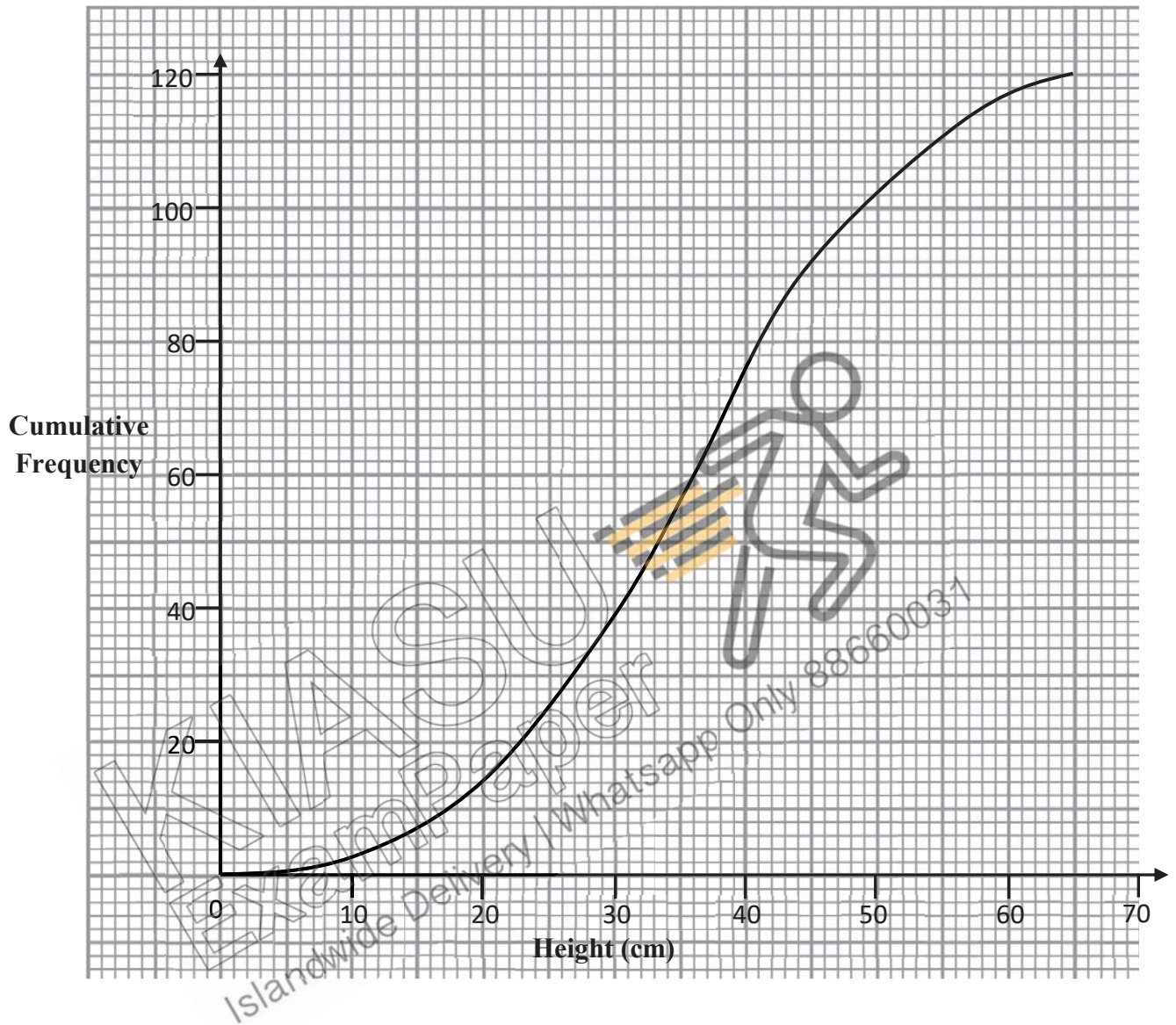
- (iii) Sunshine Club claims that their members are generally younger than those from Rainbow Club. Do you agree? Give a reason for your answer. [1]

I agree that members of Sunshine Club are generally younger than those from Rainbow Club because the median age of Sunshine Club (40 years) is lower than the median age of Rainbow Club (42 years).

- (iv) Which country club has a wider spread of ages among its members? Give a reason for your answer. [1]

Rainbow Club has a wider spread of ages among its members because its interquartile range (27 years) is bigger/larger than that of Sunshine Club (10 years).

- 8 (b) A school has two gardens, Famous Garden and Eco-Garden.
 The heights of 120 plants from the Eco-Garden were measured.
 The cumulative frequency diagram below shows the distribution of these heights.



Use the graph to estimate the

- (i) 40th percentile,

[1]

$$120 \times 0.4 = 48$$

From the graph, 40th percentile = 33 cm.

- (ii) percentage of the plants whose height is at least 45 cm. [2]

$$\begin{aligned}\text{No. of plants with length at least 45 cm} &= 120 - 92 \\ &= 28\end{aligned}$$

$$\begin{aligned}\text{Percentage of plants whose height is at least 45 cm} \\ &= \frac{28}{120} \times 100\% \\ &= 23\frac{1}{3}\%\end{aligned}$$

The heights of 120 plants from the Famous Garden were also measured.
It was found that the height measured has the same median but a smaller interquartile range compared to that of the Eco-Garden.

- (iii) Describe how the cumulative frequency curve of the Famous Garden may differ from that of the Eco-Garden given above. [2]

The cumulative frequency curve of the Famous Garden will be steeper and it will intersect the graph of the Eco-Garden at the median height.

- 9 There are 120 commercial buildings in the Sunflower City. A commercial building generates solid waste at a rate of 2.75 m^3 per day.

The solid waste of commercial buildings to that of the entire city is in the ratio 4 : 10.

The Seashell landfill has a capacity of $1.2 \times 10^7 \text{ m}^3$. Currently, 50% of the landfill is filled.

- (a) Calculate the volume of solid waste the city generates per week. (Assume 5 working days in a week) [2]

$$\begin{aligned} \text{Volume of waste generated by a commercial building per week} \\ &= 120 \times 5 \times 2.75 \\ &= 1650 \text{ m}^3 \end{aligned}$$

$$\begin{aligned} \text{Volume of solid waste generated by the city per week} \\ &= \frac{10}{4} \times 1650 \\ &= 4125 \text{ m}^3 \end{aligned}$$

- (b) Burning the solid waste in incinerators reduces its volume by 90%. However, only 60% of the solid waste can be burned. Determine how long more, in years, will the Seashell landfill be completely filled. [6]

$$\begin{aligned} \text{60\% of solid waste that can be burned} \\ &= 0.6 \times 4125 \\ &= 2475 \text{ m}^3 \end{aligned}$$

$$\begin{aligned} \text{Remain of solid waste after incineration} \\ &= 0.1 \times 2475 \\ &= 247.5 \text{ m}^3 \end{aligned}$$

$$\begin{aligned} \text{40\% of waste that cannot be burned} \\ &= 0.4 \times 4125 \\ &= 1650 \text{ m}^3 \end{aligned}$$

$$\begin{aligned} \text{50\% of landfill that is not used} \\ &= 0.5 \times 1.2 \times 10^7 \\ &= 6 \times 10^6 \text{ m}^3 \end{aligned}$$

$$\begin{aligned} \text{No. of years left where the landfill will be completely filled} \\ &= 6 \times 10^6 \div (1650 + 247.5) \div 52 \\ &= 3162.055 \div 52 \\ &= 60.8 \text{ years} \end{aligned}$$

The landfill will be completely filled in another 60.8 years.

- (c) Norman commented that incineration is a good way of getting rid of rubbish. Do you agree? Give a reason for your answer. [2]

Agree.

Reason: It is a more efficient use of space.

OR

The process of incineration can produce electricity that can be used.

Disagree.

Reason: Incineration facilities are costly.

OR

The process of incineration emits / produces hazardous pollutants that are harmful.

10 Answer the whole of this question on a sheet of graph paper.

The following is a table of values for the graph of $y = x + \frac{6}{x}$.

x	1	1.5	2	2.5	3	4	5	6	8
y	7	5.5	5	4.9	5	p	6.2	7	8.8

- (a) Calculate the value of p . [1]
- (b) Using a scale of 2 cm to 1 unit on each axis, draw the graph of $y = x + \frac{6}{x}$ for $1 \leq x \leq 8$. [3]
- (c) Use your graph to find the values of x for which $y = 6.5$. [2]
- (d) By drawing a suitable tangent to your curve, find the coordinates of the point at which the gradient of the tangent is equal to -2 . [2]
- (e) By drawing another suitable line on the same axes, use your graph to find the solutions of the equation $\frac{3x}{2} + \frac{6}{x} = 7$. [3]
-

